

INTERFACIAL CRACK BEHAVIOR IN THE STATIONARY TEMPERATURE FIELD CONDITIONS

by

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The brittle coatings, made of different materials, when subjected to elevated temperatures and in the heat exchange conditions, are susceptible to delamination. Those coatings, as well as thin films, can be used for various thermo insulating deposits, e.g. in turbines of thermal power plants. Due to environmental temperature change, in layers made of materials having different thermal expansion coefficients, appear thermal stresses. In this paper driving forces causing delamination of one layer from the other are analyzed i.e. the interfacial fracture in the two-layered, bi-material sample. This analysis was limited to considering the sample behavior when exposed to the stationary temperature field. The energy release rate G , which is the driving force for this interfacial fracture, is changing with temperature and that variation is increasing with increase of the temperature difference between the environment and the sample. Analysis of this relation, between the G variation and temperature difference, can be used to predict the maximal temperature difference, which the two-layered sample can be subjected to, without appearance of delamination between layers.

Key words: *interfacial crack, thermal stresses, two-layered sample, stationary temperature field.*

Introduction

Thin films, coatings or multi-layer samples, made of different materials, could be used for various purposes. The most common examples of application are the ceramic coatings on the metal substrate, metal layers on the polymer substrate, where the temperature at which these layers are applied is significantly higher than the working temperature; the thermo-insulating coatings like Al_2O_3 on Ni-Cr-Al and Fe-Cr-Al alloys, hard transparent coatings on optic polymers, metal fibers on the polymer substrate in electronic modules or the photo-electric actuators.

The brittle coatings operating at elevated temperatures are susceptible to delamination and spalling. The most widely investigated were the thermal barrier coatings used in turbines for power generation. Analyses of mechanisms capable of providing sufficient energy release rate to drive delamination are given in [1, 2, 10]. Thermal barrier coating (TBC) systems are susceptible to delamination failures in the presence of a large thermal gradient, [11, 12]. The three possible causes of internal delamination are analyzed in [3]: "(a) One mechanism relates

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to exfoliation of an internal separation in the TBC due to a through thickness heat flux. (b) Another is concerned with edge-related delamination within a thermal gradient. (c) The third is a consequence of sintering-induced stresses. The results of these analyses, when used in combination with available properties for the TBC, strongly suggest that the second mechanism (b) predominates in all reasonable scenarios." Delamination of coatings, initiated by small cracks paralleling the free surface, under conditions of high thermal flux associated with a through-thickness temperature gradient, is investigated in [4].

In layers made of different materials, during the environmental temperature change, thermal stresses appear, which are the result of difference in the thermal expansion coefficients. Those stresses are causing the appearance of an interfacial crack. When such a crack is formed, the energy release rate G , being the driving force for the crack propagation, depends on intensities of stresses in both layers. If one assumes that the layers were made of the elastic isotropic materials, the stresses would depend on the elastic and thermal characteristics of the two layers' materials, as well as on the temperature variations.

Problem formulation

The two-layered, bimaterial sample, shown in Fig. 1, has a crack of length $2a$ subjected to plane strain conditions. The crack is located at a distance H_1 from the upper surface and at a distance H_2 from the bottom surface of an infinitely large two-layer plate. The plate thickness obviously is $H=H_1+H_2$.

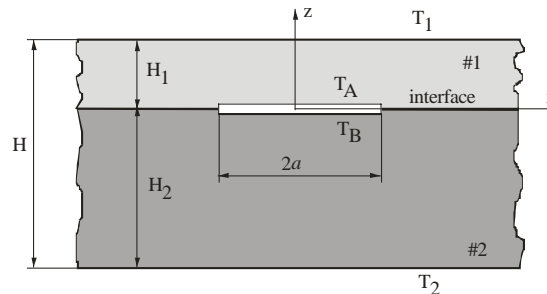


Figure 1. The two-layer sample under thermal loading containing a crack.

The upper surface of the sample is exposed to a uniform temperature T_1 and the lower surface to temperature T_2 . This means that the crack is opened. Heat flow across the crack surface, at any point, satisfies the equation:

$$q_z = h_c(T_B - T_A), \quad (1)$$

where $T_A = T(x, 0^+)$, $T_B = T(x, 0^-)$ and h_c is the conductivity across the interface. The conductivity depends on the heat conduction mechanisms across the crack and will depend on the crack opening. Here the assumption is made that h_c is constant along the crack; thus, h_c should be the average value. With k_z as thermal conductivity of the body in the z direction, the temperature gradient of each crack surfaces must satisfy the equation

$$-k_z \partial T / \partial z = q_z. \quad (2)$$

The two-layer sample has homogeneous characteristics and it is orthotropic with respect to the (x, y, z) axes. The stress-strain equations for the given problem are:

$$\begin{aligned}\varepsilon_{xx} &= \frac{1}{E_x} \sigma_{xx} - \frac{\nu_{xy}}{E_y} \sigma_{yy} - \frac{\nu_{xz}}{E_z} \sigma_{zz} + \alpha_x \Delta T \\ \varepsilon_{yy} &= -\frac{\nu_{xy}}{E_y} \sigma_{xx} + \frac{1}{E_y} \sigma_{yy} - \frac{\nu_{yz}}{E_z} \sigma_{zz} + \alpha_y \Delta T \\ \varepsilon_{zz} &= -\frac{\nu_{xz}}{E_z} \sigma_{xx} - \frac{\nu_{yz}}{E_z} \sigma_{yy} + \frac{1}{E_z} \sigma_{zz} + \alpha_z \Delta T; \quad \varepsilon_{xz} = \frac{1}{2G_{xz}} \sigma_{xz},\end{aligned}\quad (3)$$

where, without loss of generality, the temperature on the lower surface is taken as a reference, such that $\Delta T = T - T_2$.

The problem shown in Fig. 1 is discussed under the plane strain constraint, i.e. $\varepsilon_{yy} = 0$. The stress σ_{yy} does not induce any change in the crack tip fields, thus eqs. (3) are reduced to:

$$\varepsilon_{xx} = \left(\frac{1}{E_x} - \frac{\nu_{xy}^2}{E_y} \right) \sigma_{xx} + (\alpha_x + \nu_{xy} \alpha_y) \Delta T \equiv \frac{1}{\bar{E}} \sigma_{xx} - \bar{\alpha} \Delta T, \quad (4)$$

where $\bar{E} = \frac{E_x E_y}{(E_y - \nu_{xy}^2 E_x)}$ and $\bar{\alpha} = \alpha_x + \nu_{xy} \alpha_y$.

The idea of this analysis is to determine the boundaries for which the crack is long enough that the temperature and stress distribution ahead of the crack tip depend only on z . By determining this dependence, the energy release rate and stress intensity factor can be calculated by application of the linear elastic fracture mechanics (LEFM) concept to interfacial fracture. In areas of the two-layer sample far ahead of the crack tip, the temperature distribution within each layer is linear in z . In the sample without a crack, to the right of the tip, temperature is:

$$T = \frac{(T_1 - T_2)z}{H} + \frac{(H_1 T_2 + H_2 T_1)}{H}. \quad (5)$$

In the area with a central crack, away from the crack tip:

$$T = \frac{(T_1 - T_A)z}{H_1} + T_A, \quad \text{for } z > 0; \quad T = \frac{(T_B - T_2)z}{H_2} + T_B, \quad \text{for } z < 0 \quad (6)$$

where T_A and T_B are the temperatures at the upper and lower surface of crack. There are:

$$T_A = \frac{T_1(1 + \eta) + B_c(T_1 + \eta T_2)}{(1 + B_c)(1 + \eta)}; \quad T_B = \frac{T_2(1 + \eta) + B_c(T_1 + \eta T_2)}{(1 + B_c)(1 + \eta)}, \quad (7)$$

where $\eta = H_1 / H_2$ and $B_c = H h_c / k_z$ is the dimensionless Biot number. The temperature jump across the crack is given by:

$$T_A - T_B = \frac{T_1 - T_2}{1 + B_c}, \quad (8)$$

such that the heat flow impeded by the crack is $q_z/(1+B_c)$. The Biot number, B_c controls the heat flow through crack surface. When $B_c=0$, the crack is perfectly insulated so that $T_A = T_1$ and $T_B = T_2$. When B_c tends to infinity, the crack does not interrupt the heat flow, thus $T_A = T_B = (T_1 + \eta T_2)/(1 + \eta)$, and eq. (6) is reduced to eq. (5).

Based on [5-9] and [13], the energy release rate for the problem shown in fig. 1 can be written as:

$$G = \frac{\eta(1+\eta^3)}{2(1+B_c)^2(1+\eta)^5} \cdot \bar{E}H \cdot [\bar{\alpha} \cdot (T_1 - T_2)]^2, \quad (9)$$

while the stress intensity factors are:

$$K_I = \bar{E} \cdot \sqrt{H} \bar{\alpha} \cdot (T_1 - T_2) \cdot F \sqrt[4]{\lambda} \cdot \left[\frac{\cos \omega}{\sqrt{U}} - \frac{\eta^2(1+\eta)}{2(1+\eta^3)} \cdot \frac{\sin(\omega + \gamma)}{\sqrt{V}} \right] \\ K_{II} = \bar{E} \cdot \sqrt{H} \bar{\alpha} \cdot (T_1 - T_2) \cdot F \cdot \left[\frac{\sin \omega}{\sqrt{U}} + \frac{\eta^2(1+\eta)}{2(1+\eta^3)} \cdot \frac{\cos(\omega + \gamma)}{\sqrt{V}} \right], \quad (10)$$

where: $F = \frac{\sqrt{\lambda}}{\sqrt[4]{2(1+\rho)}} \cdot \frac{\sqrt{\eta} \cdot (1+\eta^3)}{(1+B_c) \cdot \sqrt{(1+\eta)^9}}$, $\lambda = \frac{E_z}{E_x}$, $\rho = \frac{1}{G_{xz} \cdot \sqrt{\lambda}} - \nu_{xz}$, $\omega \cong 52.1 - 3\eta$ [°],

$V = \frac{1}{12 \cdot (1+\eta^3)}$, $U = \frac{1}{1 + (4\eta + 6\eta^2 + 3\eta^3)}$ and $\gamma = \arcsin \cdot (6\eta^2 \cdot (1+\eta) \cdot \sqrt{UV})$.

The mode mixity, that measures the relative size of the Mode II with respect to Mode I, is:

$$\psi = \arctg \frac{K_{II}}{K_I} = \arctg \left[\frac{4\sqrt{\lambda} \cdot 2(1+\eta^3) \sqrt{V} \sin \omega + \eta^2(1+\eta) \sqrt{U} \cos(\omega + \gamma)}{2(1+\eta^3) \sqrt{V} \cos \omega - \eta^2(1+\eta) \sqrt{U} \sin(\omega + \gamma)} \right]. \quad (11)$$

In order to eliminate the Biot's number, B_c , as an unknown from eq. (9), the relationship is used between the energy release rate, G , and the size of the crack opening, δ . This relationship for a crack of length $2a$, lying along one of the main axis in an infinite orthotropic body is:

$$\delta = \frac{4n \cdot \cos \psi}{\sqrt[4]{\lambda^3} \cdot \sqrt{\pi}} \cdot \sqrt{\frac{Ga}{E}}, \quad (12)$$

where $n = ((1 + \rho) / 2)^{1/2}$. The $\cos\psi$ factor reflects the fact that only the component K_I , the stress intensity factor for Mode I, at the crack tip, influences on the opening. If the sample is isotropic i.e. $\lambda = \rho = 1$, based on eq. (9), the Biot's number is:

$$B_c = \frac{k_g H}{k_z \delta} = \frac{k_g H \cdot \sqrt[4]{\lambda^3}}{4k_z \cdot n \cos\psi} \cdot \sqrt{\frac{\pi E}{Ga}}, \quad (13)$$

where k_g is the conductivity of the gas. Eliminating B_c in eq. (8) results in relation between the energy release rate, G and the temperature loading, $(T_1 - T_2)$, as:

$$\left(1 + \frac{k_g H \cdot \sqrt[4]{\lambda^3}}{4k_z \cdot n \cos\psi} \cdot \sqrt{\frac{\pi E}{Ga}} \right)^2 G = \frac{\eta(1 + \eta^3)}{2(1 + \eta)^5} \cdot \bar{E}H \cdot [\bar{\alpha} \cdot (T_1 - T_2)]^2. \quad (14)$$

Results and discussion

The energy release rate curves as functions of the crack distance from the upper sample surface, based on eq. (9), for different values of B_c are shown in Fig. 2. Diagrams were obtained with the help of the programming package *Mathematica*[®], and for the case of an isotropic sample. Figure 3 shows results for the mixed mode that does not depend on the Biot's number, for $T_1 > T_2$.

Figure 2 shows that the highest value of the energy release rate is for a crack located at approximately one fifth of the sample thickness, i.e. for $H_1 / H = 0.211$, what means that it is the most likely that the crack, which causes the sample delamination lies at this distance.

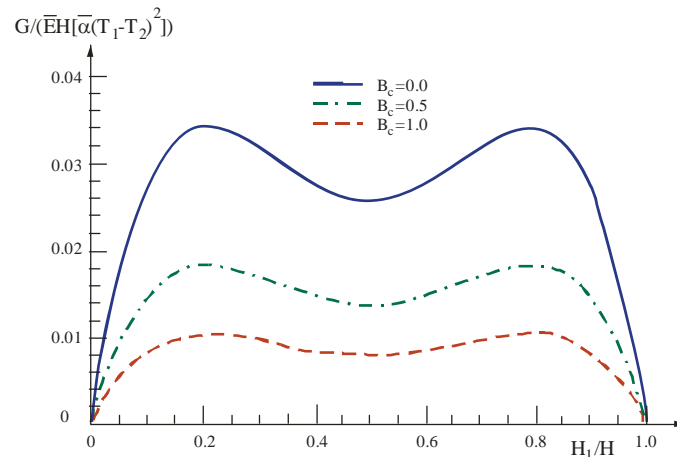


Figure 2. Dependence of the energy release rate on the crack distance from the upper sample surface for different values of B_c .

As shown in Fig. 3, the crack tip is opened with a positive stress intensity factor for Mode I, as long as the crack is above or in the middle of the sample. Crack tip is closed in terms of pure Mode II, when the crack is below the mid-thickness of the sample, i.e. when $H_1 / H > 0.5$.

In Figs. 4 and 5 dependences of stress intensity factors for Mode I and Mode II of crack propagation on Biot's number are shown, respectively. From Fig. 4 one can notice that the value of dimensionless stress intensity factor for Mode I decreases with increase of Biot's number. The same holds for the case of Mode II crack propagation, fig. 5. However, if one compares figs. 4 and 5, it is obvious that variation of Biot's number, B_c , has stronger effect on stress intensity factor for Mode II. For the case of the homogeneous material ($\lambda=1$, $\rho=1$ – red - full line) only Mode II exists. On the other hand, for the case of mixed mode of crack propagation, the value of stress intensity factor for Mode I is much lower than the value for Mode II.

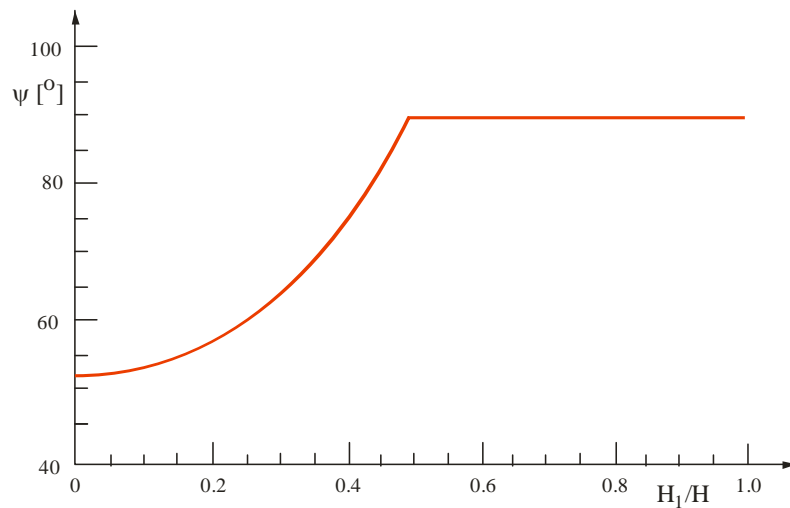


Figure 3. The dependence of the mode mixity on the crack distance from the upper surface of the sample.

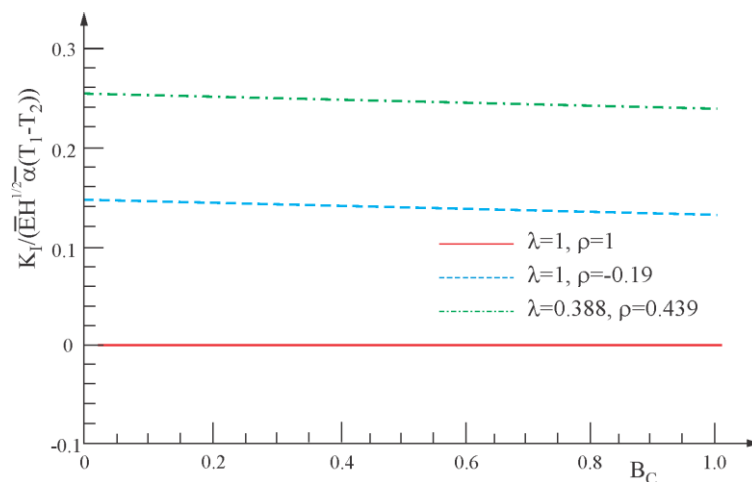


Figure 4. Variation of dimensionless stress intensity factor for Mode I in terms of Biot's number B_c for various material combinations

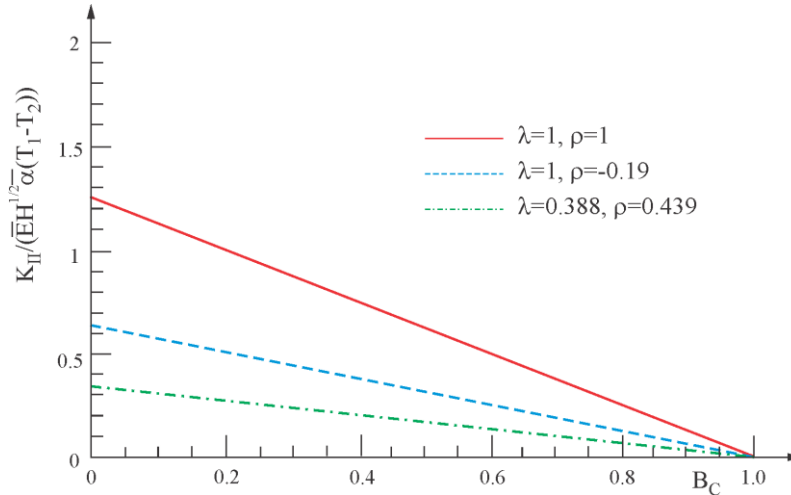


Figure 5. Variation of dimensionless stress intensity factor for Mode II in terms of Biot's number B_c for various material combinations

Figure 6 shows the dependence of the energy release rate, G , on the temperature loading ($T_1 - T_2$) for $H_1/H = 0.211$, and different values of B_c . As shown in Fig. 6, the Biot's number, B_c plays important role in heat flow through the crack. At temperatures below 1500 K, the dominant mechanism of heat transfer through the crack is due to gaseous transport. Approximate formula for B_c , when the size of the crack opening δ , is larger than 0.1 μm , is

$$B_c = k_z H / k_g \delta, \quad (15)$$

where k_g is the gas conductivity.

From Fig. 6 follows that for large values of the Biot's number, the energy release rate G has a relatively lower value, which means that the crack opening is small. On the other hand, when B_c is small, the energy release rate will be large, as well as the crack opening, i.e. crack is completely and totally isolated, and as such is suitable for delamination of the sample if the temperature gradient is sufficiently large.

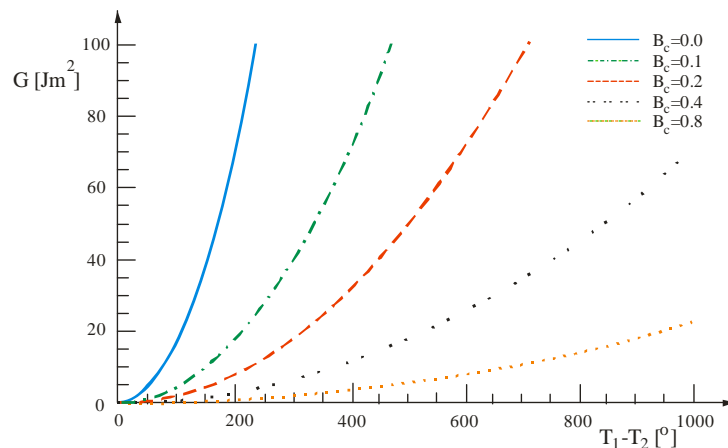


Figure 6. Dependence of the energy release rate on the temperature loading for different values of B_c .

The relationship between the energy release rate and the temperature loading is shown in fig. 7, for three different values of k_g for an isotropic sample. The crack length $2a$ is equal to the sample thickness H . A surprising feature of these curves is that when once the threshold of $(T_1 - T_2)$ is reached, the energy release rate below it becomes zero, while for a very small increase in $(T_1 - T_2)$ the energy release rate becomes large very quickly.

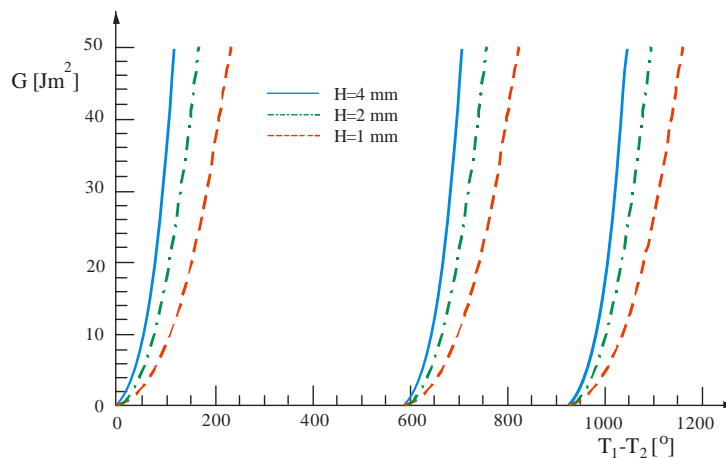


Figure 7. Dependence of the energy release rate on the temperature loading for different values of the sample thickness.

In Fig. 7 it is shown that with increasing temperature on the crack the threshold of the temperature difference also increases, while k_g increases with temperature. If the threshold exceeds the temperature difference, to which the sample is exposed, delamination of the sample should not be expected. Heat flow through the crack can be significant and the assumption of a perfectly isolated crack would be wrong. Another loading, causing a significant crack opening, would further reduce the heat transfer through the crack.

Conclusions

In this paper the theoretical basis for determining the driving forces of interfacial crack propagation in a two-layer bi-material specimen is presented, under conditions when the temperatures of the outer layers' surfaces are different. The analysis is limited to the fact that the two-layer bi-material sample is exposed to a stationary temperature field.

The driving force of the interfacial fracture, in this case, is the energy release rate G , which is determined as a function of the temperature loading. It was noticed that the energy release rate tends to increase with increasing temperature difference. This relation can be used to predict the maximum temperature differences the two-layer sample can sustain without delamination.

The highest value of the energy release rate is for a crack located at approximately one fifth of the sample thickness, what means that it is the most likely that the crack, which causes the sample delamination lies at this distance.

Variation of the Biot's number imposes significantly higher influence on Mode II stress intensity factor and in the case of the mixed mode crack propagation the value of dimensionless stress intensity factor for Mode I is much smaller than the value of this factor for Mode II. The

Biot's number, B_c plays an important role in heat flow through the crack. At temperatures below 1500K, the dominant mechanism of heat transfer through the crack is due to gaseous transport.

For large values of the Biot's number, the energy release rate G has a relatively lower value, which means that the crack opening is small. However, when B_c is small, the energy release rate is large, as well as the crack opening, namely crack is completely and totally isolated, and suitable for delamination of the sample if the temperature gradient is sufficiently large.

The relationship between the energy release rate and the temperature loading that when once the threshold of $(T_1 - T_2)$ is reached, the energy release rate below it becomes zero, while for a very small increase in $(T_1 - T_2)$ the energy release rate becomes large very quickly.

The threshold of the temperature difference also increases with temperature. If the threshold exceeds the imposed temperature difference, delamination of the sample should not be expected. The heat flow through the crack can be significant and the assumption of a perfectly isolated crack would be wrong.

For the future analysis remains investigation of the case when the two-layer bimaterial sample is subjected to an unsteady temperature field.

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Note

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Nomenclature

a	– crack length, [m]
B_c	– Biot number, [–]
E_i	– Young modulus, [Nm ⁻²]
\bar{E}_i	– reduced elasticity modulus, [Nm ⁻²]
\bar{G}	– energy release rate, [Jm ⁻²]
H_i	– thickness of layer, [m]
h_c	– conductivity across the interface,
k_z	– thermal conductivity, [W/(mK)]
k_g	– thermal gas conductivity, [W/(mK)]
K_i	– stress intensity factor, [MPam ^{1/2}]
q_z	– heat flow across the surface crack
T_i	– temperature, [K]

Greek letters

α_i	– coefficient of thermal expansion, [K ⁻¹]
$\bar{\alpha}_i$	– reduced coefficient of thermal expansion, [K ⁻¹]
ν_i	– Poisson's ratio, [–]
ε_{ii}	– strain component, [–]
η	– relative layer thickness, [–]
σ_{ii}	– normal stress, [MPa]
ψ	– phase load angle, [°]

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