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Some New Results on Iterative Learning Control of Noninteger Order

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Iterative learning control (ILC) is one of the recent topics in control theories and it is a powerful control concept that iteratively improves the behavior of processes repetitive in their nature. ILC is suitable for controlling a wider class of mechatronic systems - it is especially suitable for the motion control of robotic systems that attract and hold an important position in technical systems involving control applications, military industry, etc. This paper addresses the problem of iterative learning control (ILC) for fractional nonlinear time delay systems. Particularly, we study fractional order time delay systems in the state space form with unknown bounded constant time delay as well as time-varying delay. Sufficient conditions for the convergence of a proposed PD^{α} type of a learning control algorithm for a class of fractional state space time delay systems are presented in the time domain. Also, a feedback-feedforward PD^{α} type robust iterative learning control (ILC) of the given fractional order uncertain time delay system is considered. We consider fractional order time delay systems in the state space form with uncertain bounded constant time delay in particular. Sufficient conditions for the convergence in the time domain of the proposed PD^{α} ILC are given by the corresponding theorem together with its proof.Finally, a simulation example shows the feasibility and effectiveness of the proposed approach

Key words: **theory of control, iterative control, learning control, fractional order, nonlinear system, time delay, robotic system.**

Introduction

ITERATIVE learning control (ILC) is one of the recent topics in control theories and it is a powerful intelligent control concept that iteratively improves the behavior of processes that are repetitive in their nature,[1-3]. Since the early 80s, ILC [4, 5] has been one of very effective control strategies in dealing with repeated tracking control with the aim of improving tracking performance for systems that work in a repetitive mode. In 1978, the concept of ILC was originally proposed by Uchiyama when he presented the initial explicit formulation of ILC in Japanese [4]. In 1984, Arimoto et al. first introduced this method in English [5] where they proposed ILC for accurate tracking of robot trajectories. One motivation for the development of ILC is the industrial robot which repeats the same task from trial to trial. To overcome this problem, Arimoto, one of the inventors of ILC, [4-6] suggested that both the information from the previous tasks or “trials” and the current task should be used to improve the control action during the current trial. In other words, the controller should learn iteratively the correct control actions in order to minimize the difference between the output of the system and the given reference signal. He called this method “*betterment process*” [5]. Regarding the past of ILC, it is clear that the pioneering work of Arimoto and his colleagues stimulated a new approach to controlling certain types of repetitive systems. The concept of iterative learning is quite natural but had not been expressed in the algorithmic form of ILC until the early 1980s. In many practical control systems, the tasks are executed within a finite time interval while the same tasks are repeatedly operated. Examples for such systems are, more generally, the class of repetitive tracking systems such as process plants, robotic systems, etc. It is well known that conventional control algorithms do not take advantage of the repetitiveness. As opposed to tradicional controllers, ILC is a simple and effective control and can progressively reduce tracking errors and improve system performance from iteration to iteration, [5, 7]. The ILC approach is more or less an imitation of the learning process of every intelligent being. Intelligent beings tend to learn by performing a trial (i.e. selecting a control input) and observing what was the end result of this control input selection. After that, they try to change their behavior in order to get an improved performance during the next trial. By emulating human learning, ILC uses the knowledge obtained from the previous trial to adjust the control input for the current trial so that a better performance can be achieved. In that way, ILC incorporates past control information, such as tracking errors and their corresponding control input signals, into constructing the present control action. Also, there has been a great deal of study to overcome limitations of conventional controllers against uncertainty due to inaccurate modeling and/or parameter variations.The first ILC approaches used only the error from the previous run and thus could only handle repetitive disturbances,[1, 5, 7-9].The addition of current cycle feedback has been proposed to handle non-repetitive disturbances [10]. Therefore, ILC is a recursive control method that relies on less calculation and requires less *a priori* knowledge about the controlled system than many other kinds of control. Owing to its simplicity and effectiveness, ILC has been found to be a good alternative in

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many areas and applications, e.g. see recent surveys [3, 11] for detailed results.

Besides, in terms of how to use the tracking error signal of the previous iteration to form the control signal of the current iteration, ILC updating schemes can be classified as P-type, D-type, PD-type, and PID type. The ILC system operates in two dimensions; one is in the time domain and the other is in the iteration domain. The conventional ILC is an *open-loop* strategy, which refines the current iteration control input by only employing information from the previous iterations and, hence, cannot improve the tracking performance along the time axis within a single cycle. Moreover, a typical ILC in the time domain is a simple open-loop control (*off-line* ILC) and it cannot suppress unanticipated, non-repeating disturbances.

In real applications, to overcome such drawbacks, an ILC scheme is usually performed together with a proper feedback controller for compensation, where we often design a learning operator for the closed-loop (*on*-line ILC) systems that have achieved a good performance. Algorithms that only use information of the past trial are called first order algorithms, and can be distinguished from higher order algorithms that use multiple past trials or current trial algorithms, which incorporate a feedback loop.

Therefore, ILC is a technique of controling systems operating in a repetitive mode with the additional requirement that a specified output trajectory $y_d(t)$ in an interval $[0,T]$ be followed to a high precision and through improving the performance from trial to trial in the sense that the tracking error is sequentially reduced. The basic strategy is to use an iteration of the form $u_{i+1}(t) = f(u_i(t), e_i(t)),\ e_i(t) = y_d(t) - y_i(t)$, where $f(.,.)$ defines the learning algorithm and remains to be specified, $y_i(t)$ is the output at the i-th operation resulting from an input $u_i(t)$, and $y_d(t)$ represents the desired output. The new control input $u_{i+1}(t)$ should make the system closer to the desired result in the next execution cycle. Namely, the intuitive notion of "improving performance progressively" can be refined to a convergence condition on the error, i.e., (in some norm topology) $\lim_{i\to\infty}\|e_i(.)\| = 0$, $e_i(t) = y_d(t) - y_i(t)$. The original ILC scheme in English is proposed by [5] for a better control of systems performing repetitive tasks as D-type, i.e., $u_{i+1}(t) = u_i(t) + \Pi(de_i/dt)$.

Recently, increasing attentions have been paid to fractional calculus (FC) and its application in various science and engineering fields, [12-14]. Fractional calculus is a mathematical topic with more than 300- year old history, but its application to physics, mathematics, and engineering has been reported only in the recent years [13, 15-17]. The fractional integro-differential operators are a generalization of integration and derivation to non-integer order (fractional) operators [12, 14, 18, 19]. All fractional operators consider the entire history of the process being considered, thus being able to model the nonlocal and distributed effects often encountered in natural and technical phenomena.The theory of FC is a well-adapted tool for modeling many physical phenomena, allowing the description to take into accounts some peculiarities that classical integer-order models simply neglect, [14]. For example, wide and fruitful applications can be found in rheology, viscoelasticity, acoustics, optics, chemical and statistical physics, robotics, control theory, electrical and mechanical engineering, bioengineering, etc. As important applications of FC, fractional-order control systems [20, 21] and fractional-order modeling [22, 23] have attracted more and more interests in the last several years to enhance the robustness and performance of the proposed systems.

Particularly, the application of ILC to the fractional-order system has become a new topic, where authors [24] were the first to propose the fractional order *D*-type iterative learning control algorithm and the convergence was proved in the frequency domain. Then, the time domain analyses of fractional-order ILC are obtained and presented in the papers, [25-30], as well as for a class of fractional-order nonlinear time-delay systems [31, 32] and in a survey/overview [33-35].

Motivated by the mentioned investigations of ILC algorithms for ILC fractional order control in the tracking problems of these systems, a new robust iterative learning feedforward control as well as feedback ILC control for a particular class of fractional order uncertain time delay systems are suggested in this paper.This paper extends the results obtained in papers [27, 34] to consider more general systems i.e. fractional order uncertain time delay systems (including constant but unknown delay as well as time-varying delay) described in the form of state space and output equations. Sufficient convergent conditions of the proposed ILC will be derived in time-domain and formulated by the theorems. Finally, the simulation results are presented to illustrate the performance of the proposed robust PD^{α} ILC scheme.

Preliminaries and basics of fractional calculus

The λ -norm, maximum norm, induced norm

For a later use in proving the convergence of the proposed learning control, the following norms are introduced [35] for the n-dimensional Euclidean space R^n: the sup-norm $\|x\|_\infty = \sup_{1\le k\le n}|x_k|$, $x=[x_1, x_2, \ldots x_n]^T$, $|x_k|$ - absolute value; the maximum norm $\|x\|_s = \max_{0\le t\le T}|x(t)|$, $x(t)=[x_1(t), x_2(t),\ldots,x_n(t)]^T$; the matrix norm as $\|A\|_\infty = \max_{1\le k\le m}\left(\sum_{j=1}^{n}|g_k|\right)$, $A=[a_{kj}]_{mxn}$ and the λ -norm for a real function:

$$\begin{gathered} h(t), \quad (t\in[0,T]),\ h:[0,T]\to\Re^n \\ \|h(t)\|_\lambda = \sup_{t\in[0,T]} e^{-\lambda t}\|h(t)\|, \quad \lambda>0 \end{gathered} \tag{1}$$

A useful property associated with the λ-norm is the following inequality.

Property 1: λ norm has the next property

$$\begin{gathered} \sup_{t\in[0,T]} e^{-\lambda t}\int_0^t \|f(.)\|e^{a(t-\tau)}d\tau = \\ \sup_{t\in[0,T]}\int_0^t e^{-\lambda t}\|f(.)\|e^{a(t-\tau)(a-\lambda)}d\tau \le \frac{1-e^{(a-\lambda)T}}{\lambda-a}\|f(.)\|_\lambda \end{gathered} \tag{2}$$

The induced norm of the matrix A is defined as:

$$\|A\| = \sup\left\{\frac{\|Ax\|}{x} : x \in X \text{ with } \|x\| \neq 0\right\} \text{ with,} \tag{3}$$

where $\|(.)\|$ denotes an arbitrary vector norm. In case $\|(.)\|_\infty$ it follows that

$$\|Ax\|_\infty \leq \|A\|_\infty \|x\|_\infty , \tag{4}$$

where $\|A\|_\infty$ denotes the maximum value of the matrix A. For the previous norms, note that

$$\|h(t)\|_\lambda \leq \|h(t)\|_\infty \leq e^{\lambda T}\|h(t)\|_\lambda . \tag{5}$$

The λ - norm is thus equivalent to the ∞ - norm. For simplicity, in applying the norm $\|(.)\|_\infty$ the index ∞ will be omitted. Before giving the main results, we first give the following Lemma 1, [21].

Lemma 1. Suppose a real positive series $\{a_n\}_1^\infty$ satisfies

$$\begin{aligned} &a_n \leq \rho_1 a_{n-1} + \rho_2 a_{n-2} + \ldots + \rho_N a_{n-N} + \varepsilon \\ &(n = N+1,\ N+2,\ \ldots,\ ) \end{aligned} \tag{6}$$

where $\rho_1 \geq 0\ (i = 1,2,\ldots,N)$ $\varepsilon = 0$ and $\rho = \sum_{i=1}^{N} \rho_i < 1$. Then the following holds:

$$\lim_{n\to\infty} a_n \leq \varepsilon / (1-\rho) . \tag{7}$$

Basics of fractional calculus

Fractional calculus (*FC*) is a generalization of classical calculus concerned with the operations of integration and the differentiation of non-integer (fractional) order. The concept of fractional operators has been introduced almost simultaneously with the development of the classical ones. This question consequently attracted the interest of many well- known mathematicians, including Euler, Liouville, Laplace, Riemann, Grünwald, Letnikov and many others. Since the 19th century, the theory of fractional calculus developed rapidly, mostly as a foundation for a number of applied disciplines, including fractional geometry, fractional differential equations (*FDE*) and fractional dynamics. The applications of *FC* are very wide nowadays, [11-13], [22-25]. It is safe to say that almost no discipline of modern engineering and science in general remains untouched by the tools and techniques of fractional calculus. For example, wide and fruitful applications can be found in rheology, viscoelasticity, acoustics, optics, chemical and statistical physics, robotics, control theory, electrical and mechanical engineering, bioengineering, etc. The main reason for the success of *FC* applications is that these new fractional-order models are often more accurate than integer-order ones, i.e. there are more degrees of freedom in the fractional order model than in the corresponding classical one. All fractional operators consider the entire history of the process being considered, thus being able to model the non-local and distributed effects often encountered in natural and technical phenomena.

There exist today many different forms of fractional integral operators, ranging from divided-difference types to infinite-sum types, Riemann-Liouville fractional derivative, Grunwald–Letnikov fractional derivative, Caputo's, Weyl's and Erdely-Kober left and right fractional derivatives, etc. [12]. The three most frequently used definitions for the general fractional differintegral are: the Grunwald-Letnikov (*GL*) definition, the Riemann-Liouville (*RL*) and the Caputo definitions, [11, 12]. Also, fractional order dynamic systems and controllers have been increasing in interest in many areas of science and engineering in the last few years. In most cases, our objective of using fractional calculus is to apply the fractional order controller to enhance the system control performance, [13, 25].

Here, we review some basic properties of fractional integrals and derivatives, which we will need later in the obtaining *ILC* algorithms schemes. Sets of natural, real, integer real and complex numbers are denoted, respectively, by N, R, Z, C . Also $L^p((a,b)) = L^p([a,b])$, $p \geq 1$, is the space of the measurable functions for which $\left(\int_a^b |f(x)|^p\, dx\right)^{1/p} < \infty$.

Definition 1. The left Riemann-Liouville fractional integral of the order $\alpha \in C$ is given by

$$\begin{aligned} &{}_aI_t^\alpha f(t) = \frac{1}{\Gamma(\alpha)}\int_a^t (t-\tau)^{\alpha-1} f(\tau)d\tau, \\ &t \in [a,b],\ \operatorname{Re}\alpha > 0. \end{aligned} \tag{8}$$

In the special case of a positive real $\alpha\ (\alpha \in R_+)$ and $f \in L^1(a,b)$, the integral ${}_aI_t^\alpha f$ exists for almost all $t \in [a,b]$ as well as ${}_aI_t^\alpha f \in L^1(a,b)$, [26,37].

Defintion 2. The left RL fractional derivatives ${}_aD_t^\alpha f$, of the order $\alpha \in C,\ \operatorname{Re}\alpha \geq 0,\ n-1 \leq \operatorname{Re}\alpha < n,\ n \in N$, is defined as

$$\begin{aligned} {}^{RL}_{a}D_t^\alpha f(t) &= \frac{d^n}{dt^n}\left({}_aI_t^{n-\alpha} f(t)\right) = \\ &= \frac{1}{\Gamma(n-\alpha)}\frac{d^n}{dt^n}\int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}}d\tau,\ t \in (a,b), \end{aligned} \tag{9}$$

Also, for the special case $0 \leq \alpha < 1$ where $t > a$ we have

$${}^{RL}_{a}D_t^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_a^t \frac{f(\tau)}{(t-\tau)^\alpha}d\tau, \tag{10}$$

Definition 3. The left Caputo fractional derivative of a function of the order α, denoted by ${}^C_aD_t^\alpha f$, is given, [36]

$${}^C_aD_t^\alpha f(t) = {}_aI_t^{n-\alpha}\left(\frac{d^n}{dt^n} f(t)\right) \tag{11}$$

where ${}_aI_t^{n-\alpha}$ is the left RL fractional integral (8) or in the explicit form as follows:

$${}^C_aD_t^\alpha f(t) = \begin{cases} \dfrac{1}{\Gamma(n-\alpha)}\displaystyle\int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}}d\tau, & n-1 \leq \alpha < n, \\ \dfrac{d^n}{dt^n} f(t), & \alpha = n, \end{cases} \tag{12}$$

The Riemann-Liouville fractional derivatives and the Caputo fractional derivatives are connected with each other by the following relations:

$$ {}_{a}^{RL}D_t^{\alpha} f(t) = {}_{a}^{C}D_t^{\alpha} f(t) + \sum_{k=0}^{n-1} \frac{(-1)^k f^{(k)}(a)}{\Gamma(k-\alpha+1)} (t-a)^{k-\alpha}, \quad (13) $$

$$ {}_{t}^{RL}D_b^{\alpha} f(t) = {}_{t}^{C}D_b^{\alpha} f(t) + \sum_{k=0}^{n-1} \frac{(-1)^k f^{(k)}(b)}{\Gamma(k-\alpha+1)} (b-t)^{k-\alpha}, \quad (14) $$

The Caputo and Riemann-Liouville formulations coincide when the initial conditions are zero, [13].

Lemma 2. If the function $f(t,x)$ is continuous, then the initial value problem

$$ \begin{cases} {}_{t_0}^{C}D_t^{\alpha} x(t) = f(t, x(t)), & 0<\alpha<1 \\ x(t_0) = x(0) \end{cases} \quad (15) $$

is the equivalent to the following nonlinear Volterra integral equation:

$$ x(t) = x(0) + \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t-s)^{\alpha-1} f(s, x(s)) ds \quad (16) $$

and its solutions are continuous, [38].

PD^{α} - type iterative learning control for the fractional order uncertain time delay system

System description

The non-integer (fractional) order uncertain system with time delay described in the form of state space and output equations is considered here. In his Phd. Matignon [39], gives the model of pressure wave transmission through an air-filled tube with viscothermic perturbation and discusses the stability of the transfer function *as an example of a fractional delay system*. In their paper, authors [40] considered finite-dimensional fractional time delay systems. This description is convenient for simple models of systems with only one Caputo fractional-order derivation

$$ \begin{aligned} &{}_{0}^{C}D_t^{\alpha} \mathbf{x}_i(t) = A_0 \mathbf{x}_i(t) + A_1 \mathbf{x}_i(t-\tau) + B\mathbf{u}_i(t) + f(x_i(t)), \\ &\mathbf{y}_i(t) = C\mathbf{x}_i(t), \qquad 0<\alpha<1 \end{aligned} \quad (17) $$

and with the associated function of the initial state

$$ x(t) = \psi_x(t), \qquad -\tau_M \le t \le 0. \quad (18) $$

where t is the time in the operation interval $J=[0,T], J\subset R$, as well as $0<\alpha<1$ fractional order derivative, A_0, A_1, B and C are the matrices with appropriate dimensions and τ denotes a pure time delay. Moreover, τ is an unknown time delay but a bounded parameter which satisfies

$$ 0<\tau\le\tau_M\le \mathrm{T}, \quad \forall t\in J,\ J=[0,T], J\subset R \quad (19) $$

Also, the initial conditions of fractional differential equations which were compared to the given fractional derivatives were considered by authors [41], and it is assumed that there is no difficulty with questions of existence, uniqueness, and continuity of solutions with respect to the initial data. The following assumptions on the system (17, 18) are imposed.

A1. The desired trajectories $y_d(t), x_d(t)$ are continuously differentiable on $[0,T]$.

A2. The system (17) is causal and when $t<0$ it is assumed $\psi_{\mathbf{i}}(t) = \psi_d(t), \quad \forall t\in[-\tau_M, 0]$, where $x_{\mathbf{i}}(t) = \psi_i(t), \quad \forall t\in[-\tau_M, 0]$ is the initial function of the system (17).

A3. The input-output coupling matrix CB is of full column rank.

A4. The vector function f presents the nonlinear parameter perturbation of the system in respect to $\boldsymbol{x}(t)$ and it is uniformly globally Lipschitz in terms of x in $[0,T]$, i.e.

$$ \| f_0(x_1(t)) - f_0(x_2(t)) \| \le k_f \| x_1(t) - x_2(t) \|, \quad t\in[0,t] \quad (20) $$

where $k_f>0$ is some finite constant.

PD^{α} - type iterative learning control

In ILC, the convergence aspects have always been a key issue, i.e. guaranteeing that the systems output trajectory is converging to the desired one within a prescribed desired accuracy as the number of ILC iterations increases. Here, it is suggested the learning control scheme PD^{α} - type ILC updates the law for the given system (17) such as:

$$ u_{i+1}(t) = u_i(t) + \Gamma e_i(t) + \Pi\, {}_{0}^{C}D_t^{\alpha} e_i(t), \quad (21) $$

where Γ, Π are the gain matrices appropriate dimensions. A sufficient condition for the convergence of a proposed ILC is given by Theorem 1 and proved as follows.

Theorem 1: Suppose that the updated law (21) is applied to the system (17) and that the initial function (18) at each iteration satisfies $\psi_{\mathbf{i}}(t) = \psi_d(t), \quad \forall t\in[-\tau, 0]$. If the matrix Π exists such that

$$ \| [I - \Pi CB] \| \le \rho < 1, \quad (22) $$

then, when $i\to\infty$ the bounds of the tracking errors $\|x_d(t)-x_i(t)\|$, $\|y_d(t)-y_i(t)\|$, $\|u_d(t)-u_i(t)\|$, converge asymptotically to zero.

Proof. Let

$$ \begin{aligned} &\delta h_i = h_d(t) - h_i(t), \quad h_i^{(\alpha)} = h_d^{(\alpha)}(t) - h_i^{(\alpha)}(t), \\ &h = x, x_d, u, u_d, f \end{aligned} \quad (23) $$

The tracking error can be obtained as follows:

$$ \begin{aligned} e_i^{(\alpha)}(t) &= \frac{d^{(\alpha)}}{dt^{(\alpha)}} (y_d(t) - y_i(t)) = C\delta x_i^{(\alpha)}(t) \\ &= CA_0 \delta x_i(t) + CA_1 \delta x_i(t-\tau) + CBu_i(t) + C\delta f_i(t) \end{aligned}, \quad (24) $$

Taking the proposed control law gives:

$$ \begin{aligned} \delta u_{i+1} &= u_d - u_{i+1} = u_d - u_i - \Pi e_i^{(\alpha)} - \Gamma e_i = \\ &= \delta u_i - \Pi e_i^{(\alpha)} - \Gamma e_i \end{aligned} \quad (25) $$

or, taking (24) it yields:

$$\begin{aligned}\delta u_{i+1}(t) = \left[I - \Pi CB\right]\delta u_i(t) - \left[\Gamma C + \Pi CA_0\right]\delta x_i(t) \\ - \left[\Pi CA_1\right]\delta x_i(t-\tau) - \Pi C\delta f_i(t)\end{aligned}, \quad (26)$$

Estimating the norms of (26) with $\|(.)\|$ and using the condition of Theorem 1 implies:

$$\begin{aligned}&\|\delta u_{i+1}\| \le \rho\|\delta u_i\| + \beta_0\|\delta x_i\| + \\ &+\beta_1\|\delta x_i(t-\tau)\| + \beta_2\|\delta f_i\|, \\ &\beta_0 = \|\Gamma C + \Pi CA_0\|, \\ &\beta_1 = \|\Pi CA_1\|, \\ &\beta_2 + \|\Pi C\|\end{aligned} \quad (27)$$

Also, one can write the solutions of (17) in the form of the equivalent Volterra integral equations,[13] using assumption A2, as:

$$\begin{aligned}x_i(t) &= \psi_i(t) + \\ &+ \frac{1}{\Gamma(a)}\int_0^t (t-s)^{\alpha-1}\begin{pmatrix}A_0x_i(s) + A_1x_i(s-\tau) \\ +Bu_i(s) + f_i(x_i(s))\end{pmatrix}ds \\ x_d(t) &= \psi_d(t) + \\ &+ \frac{1}{\Gamma(a)}\int_0^t (t-s)^{\alpha-1}\begin{pmatrix}A_0x_d(s) + A_1x_d(s-\tau) \\ +Bu_d(s) + f_d(x_d(s))\end{pmatrix}ds\end{aligned} \quad (28)$$

or

$$\delta x_i(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}\begin{pmatrix}A_0\delta x_i(s) + A_1\delta x_i(s-\tau) + \\ +B\delta u_i(s) + \delta f_i(x_i(s))\end{pmatrix}ds \quad (29)$$

Applying the norms and using the Lipschitz condition with respect to x (20), if it is the uniqueness solution [13], it yields:

$$\begin{aligned}\|\delta x_i(t)\| &\le \frac{1}{\Gamma(\alpha)}\|A_0\|\int_0^t \left|(t-s)^{\alpha-1}\right|\|\delta x_i(s)\|ds + \\ &+\frac{1}{\Gamma(\alpha)}\|A_1\|\int_0^t \left|(t-s)^{\alpha-1}\right|\|\delta x_i(s-\tau)\|ds + \\ &+\frac{1}{\Gamma(\alpha)}\|B_0\|\int_0^t \left|(t-s)^{\alpha-1}\right|\|\delta u_i(s)\|ds + \\ &+\frac{k_f}{\Gamma(\alpha)}\int_0^t \left|(t-s)^{\alpha-1}\right|\|\delta x_i(s)\|ds \\ &\le \frac{1}{\Gamma(\alpha)}(a_0 + k_f)\int_0^t \left|(t-s)^{\alpha-1}\right|\|\delta x_i(s)\|ds + \\ &+\frac{a_1}{\Gamma(\alpha)}\int_0^t \left|(t-s)^{\alpha-1}\right|\|\delta x_i(s-\tau)\|ds + \\ &+\frac{b_0}{\Gamma(\alpha)}\int_0^t \left|(t-s)^{\alpha-1}\right|\|\delta u_i(s)\|ds\end{aligned} \quad (30)$$

Moreover, applying the λ-norm to both sides of the previous equation (30), it follows

$$\|\delta x_i(t)\|_\lambda \le \sup_{0\le t\le T}\left\{\int_0^t e^{-\lambda t}\frac{|(ts)|^{\alpha-1}}{\Gamma(\alpha)}\begin{bmatrix}(a_0+k_f)\|\delta x_i(s)\| + \\ a_1\|\delta x_i(s-\tau_M)\| + b_0\|\delta u_i(s)\|\end{bmatrix}ds\right\}$$

$$\begin{aligned}&\le \sup_{0\le t\le T}\int_0^t e^{-\lambda(t-s)}\frac{|(t-s)|^{\alpha-1}}{\Gamma(\alpha)}\cdot \\ &\cdot\left(\sup_{0\le t\le T} e^{-\lambda s}\begin{bmatrix}(a_0+k_f)\|\delta x_i(s)\| + \\ a_1\|\delta x_i(s-\tau_M)\| + b_0\|\delta u_i(s)\|\end{bmatrix}\right)ds \\ &\le \begin{pmatrix}(a_0+k_f)\|\delta x_i(s)\|_\lambda + \\ a_1e^{-\lambda\tau}\|\delta x_i(s-\tau_M)\|_\lambda + b_0\|\delta u_i(s)\|_\lambda\end{pmatrix}\cdot \\ &\left(\sup_{0\le t\le T}\int_0^t e^{-\lambda(t-s)}ds \sup_{0\le t\le T}\int_0^t \frac{|(t-s)|^{\alpha-1}}{\Gamma(\alpha)}ds)\right)\end{aligned} \quad (31)$$

Due to the fact that $\|x(t-\tau_M)\|_\lambda \le \|x(t)\|_\lambda$,one can find that (31)

$$\begin{aligned}\|\delta x_i(t)\|_\lambda &\le \begin{pmatrix}(a_0 + a_1e^{-\lambda\tau_M} + k_f)\|\delta x_i(s)\|_\lambda + \\ +b_0\|\delta u_i(s)\|_\lambda\end{pmatrix} \\ &\cdot\left(\sup_{0\le t\le T}\int_0^t e^{-\lambda(t-s)}ds \sup_{0\le t\le T}\int_0^t \frac{|(t-s)|^{\alpha-1}}{\Gamma(\alpha)}ds\right) \\ &= \begin{pmatrix}(a_0 + a_1e^{-\lambda\tau_M} + k_f)\|\delta x_i(s)\|_\lambda + \\ +b_0\|\delta u_i(s)\|_\lambda\end{pmatrix}\frac{\left(1-e^{-\lambda T}\right)}{\lambda}\frac{T^\alpha}{\Gamma(\alpha+1)}\end{aligned} \quad (32)$$

Defining $O\left(\lambda^{-1}\right)$, as

$$O\left(\lambda^{-1}\right) = \frac{\left(1-e^{-\lambda T}\right)}{\lambda}\frac{T^\alpha}{\Gamma(\alpha+1)} \quad (33)$$

where (32) simplifies to

$$\|\delta x_i(t)\|_\lambda \le \begin{pmatrix}(a_0 + a_1e^{-\lambda\tau_M} + k_f)\|\delta x_i(s)\|_\lambda + \\ +b_0\|\delta u_i(s)\|_\lambda\end{pmatrix}O\left(\lambda^{-1}\right) \quad (34)$$

or, one may conclude

$$\begin{aligned}\|\delta x_i(t)\|_\lambda &\le \frac{b_0O\left(\lambda^{-1}\right)}{\left(1-\left(a_0 + a_1e^{-\lambda\tau} + k_f\right)O\left(\lambda^{-1}\right)\right)}\|\delta u_i(t)\|_\lambda \le \\ &\le O_\eta\left(\lambda^{-1}\right)\|\delta u_i(t)\|_\lambda\end{aligned} \quad (35)$$

where, if a sufficiently large λ is used, one can obtain that:

$$\lambda\Gamma(\alpha+1) - \left(a_0 + a_1e^{-\lambda\tau_M} + k_f\right)\left(1-e^{-\lambda T}\right)T^\alpha > 0 \quad (36)$$

Taking the λ-norm of (27) with the substitution of (35) simply yields:

$$\begin{aligned}\|\delta u_{i+1}\|_\lambda &\le \rho\|\delta u_i\|\lambda + \beta_0\|\delta x_i\|_\lambda + \\ &+\beta_1e^{-\lambda\tau}\|\delta x_i(t-\tau_M)\|_\lambda + \beta_2k_f\|\delta x_i\|_\lambda,\end{aligned} \quad (37)$$

$$\|\delta u_{i+1}\|_\lambda \le \rho\|\delta u_i\|_\lambda + \beta\|\delta x_i\|_\lambda \quad (38)$$

where is

$$\beta = \left[\beta_0 + \beta_1e^{-\lambda\tau_M} + \beta_2k_f\right] \quad (39)$$

or

$$\|\delta u_{i+1}\|_\lambda \le \left(\rho + \beta O_\eta\left(\lambda^{-1}\right)\right) = \rho'\|\delta u_i\|_\lambda \quad (40)$$

So that there exists a sufficient large λ satisfying

$$\rho' = \left[\rho + \beta_0 O_\eta\left(\lambda^{-1}\right)\right] < 1, \tag{41}$$

Therefore, according to Lemma1 Moore,[1] it can be concluded that:

$$\lim_{i\to\infty} \left\|\delta u_i\right\|_\lambda \to 0 \tag{42}$$

This completes the proof of Theorem 1.Moreover, due to the uniqueness and existence theorem for fractional order time delay systems,V.Lakshmikantham [42], one can conclude that

$$\lim_{i\to\infty} x_i(t) = x_d(t)\,, \ \lim_{i\to\infty} y_i(t) = y_d(t) \tag{43}$$

The case of the fractional order time delay system with time-varying delay is also discussed here:

$$\begin{aligned} &\boldsymbol{x}_i^{(\alpha)}(t) = A_0\boldsymbol{x}_i(t) + A_1\boldsymbol{x}_i(t-\tau(t)) + B\boldsymbol{u}_i(t) + f\left(x_i(t)\right), \\ &\boldsymbol{y}_i(t) = C\boldsymbol{x}_i(t), \qquad 0<\alpha<1 \end{aligned} \tag{44}$$

with the associated function of the initial state

$$x(t) = \psi_x(t), \qquad -\tau_M \le t \le 0. \tag{45}$$

In the case of the time varying time delay $\tau(t)$ we introduce the next assumption A5.

$$\text{A5: } 0 \le \tau(t) \le \tau_M\,, \quad \tau_M < T, \quad \forall t \in J,\ J = [0,T], \tag{46}$$

Theorem 2. For the fractional order time-varying delay system (44) and the given initial function (45), with the PD^α-type ILC scheme (21), and the assumptions A1-A5 where the convergence condition is given by (22), then we have

$$\lim_{i\to\infty} y_i(t) = y_d(t) \tag{47}$$

Proof: The proof immediately follows from the proof ofTheorem 1 and (46).

***Remark* 1.** By comparing it with [34], the author extends the main results of [34].

***Remark* 2**. If the time delay $\tau = const$ is known, one may obtain similar results as formulated in Theorems 1,2.

***Remark* 3**. In the case of the non-perturbed system $f\left(x_i(t)\right) = 0,$ and a known time delay τ, one may obtain the results which are obtained and presented in papers [27,34].

***Remark* 4**. In the case of the non-perturbed system $f\left(x_i(t)\right) = 0,$ and without the time delay τ, one may obtain the results which are obtained and presented in papers [26,30].

Feedback-feedforward PD^α type iterative learning control

Here, the feedback-feedforward fractional order PD^α learning algorithm which comprises two types of control laws: a feed-forward PD^α control law and a PD^α feedback law for given system (17) is assumed. In the feed-forward control loop it is proposed that a PD^α - type ILC updating law for the given system is:

$$u_{fi+1}(t) = u_i(t) + \Gamma_2\left({}_C D_{0,t}^\alpha e_i(t) + \Pi_2 e_i(t)\right). \tag{48}$$

where $e_i(t) = y_d(t) - y_i(t)$ is the trajectory tracking error in the $i-$th iteration and $y_d(t) = Cx_d(t)$ denotes a desired output trajectory. In the feedback loop, the PD^α controller provides stability of the system and keeps its state errors within uniform bounds. Besides, the introduced feedback control is as follows:

$$u_{fbi+1}(t) = \Gamma_1({}_C D_{0,t}^\alpha e_{i+1}(t) + \Pi_1\, e_{i+1}(t))\,, \tag{49}$$

where $\Gamma_i,\ \Pi_i,\ i=1,2$ are the gain matrices of appropriate dimensions. Moreover, it was shown in [29] that the tracking speed was the fastest when the system order and the order of ILC PD^α are of the same order i.e. α . In that way, the open-closed-loop fractional order PD^α learning algorithm takes the form

$$\begin{aligned} u_{i+1}(t) &= u_{fi+1}(t) + u_{fbi+1}(t) = u_i(t) + \\ &+\Gamma_1\left({}_C D_{0,t}^\alpha e_{i+1}(t) + \Pi_1 e_{i+1}(t)\right) + \\ &+\Gamma_2\left({}_C D_{0,t}^\alpha e_i(t) + \Pi_2 e_i(t)\right) \end{aligned} \tag{50}$$

A sufficient condition for the convergence of the proposed feedback ILC is given by Theorem 3 and proved as follows.

Theorem 3. Suppose that the update law (50) is applied to the system (17) and that the initial function (18) at each iteration satisfies $\psi_{\mathbf{i}}(t) = \psi_d(t),\ \forall t \in [-\tau, 0)$. If there exist the matrices $\Gamma_i, i=1,2$ such that

$$\left\|\left[I + \Gamma_1 CB\right]^{-1}\left[I - \Gamma_2 CB\right]\right\| \le \rho < 1, \tag{51}$$

then, if $i \to \infty$, the bounds of the tracking errors $\left\|x_d(t) - x_i(t)\right\|, \left\|y_d(t) - y_i(t)\right\|, \quad \left\|u_d(t) - u_i(t)\right\|$ converge asymptotically to zero.

Proof. The fractional derivative order α of $e_i(t)$ is obtained as follows:

$$\begin{aligned} e_i^{(\alpha)}(t) &= \frac{d^{(\alpha)}}{dt^{(\alpha)}}\left(y_d(t) - y_i(t)\right) = C\delta x_i^{(\alpha)}(t) \\ &= CA_0\delta x_i(t) + CA_1\delta x_i(t-\tau) + \\ &+CB\delta u_i(t) + C\delta f_i(t) \end{aligned} \tag{52}$$

Taking the proposed control law gives:

$$\begin{aligned} \delta u_{i+1} &= u_d - u_{i+1} = \\ &= \delta u_i - \Gamma_1\left(e_{i+1}^{(\alpha)}(t) + \Pi_1\, e_{i+1}(t)\right) -, \\ &-\Gamma_2\left(e_i^{(\alpha)} + \Pi_2 e_i(t)\right) \end{aligned} \tag{53}$$

or, taking (52) it yields:

$$\begin{aligned} \delta u_{i+1}(t) &= \left[I + \Gamma_1 CB\right]^{-1}\left[I - \Gamma_2 CB\right]\delta u_i(t) - \\ &-\left[I + \Gamma_1 CB\right]^{-1}\begin{pmatrix} \left[\Gamma_1\left(CA_0 + \Pi_1 C\right)\right]\delta x_{i+1}(t) + \\ \left[\Gamma_2\left(CA_0 + \Pi_2 C\right)\right]\delta x_i(t) + \\ \left[\Gamma_1 CA_1\right]\delta x_{i+1}(t-\tau) + \\ \left[\Gamma_2 CA_1\right]\delta x_i(t-\tau) + \\ \left[\Gamma_1 C\right]\delta f_{i+1}(t) + \left[\Gamma_2 C\right]\delta f_i(t) \end{pmatrix} \end{aligned} \tag{54}$$

Estimating the norms of (54) with $\left\|(.)\right\|$ and using the condition of Theorem 1 implies:

$$\begin{aligned}\|\delta u_{i+1}\| \le \rho\|\delta u_i\| + \beta_0\|\delta x_{i+1}\| + \beta_1\|\delta x_i\| \\ +\beta_2\|\delta x_{i+1}(t-\tau)\| + \beta_3\|\delta x_i(t-\tau)\| \\ +\beta_4\|\delta f_{i+1}\| + \beta_5\|\delta f_i\|,\end{aligned} \tag{55}$$

where are

$$\begin{aligned}\beta_0 &= \left\|[I+\Gamma_1 CB]^{-1}[\Gamma_1(SA_0+\Pi_1 C)]\right\|,\\ \beta_1 &= \left\|[I+\Gamma_1 CB]^{-1}[\Gamma_2(SA_0+\Pi_2 C)]\right\|,\\ \beta_2 &= \left\|[I+\Gamma_1 CB]^{-1}[\Gamma_1 SA_1]\right\|,\\ \beta_3 &= \left\|[I+\Gamma_1 CB]^{-1}[\Gamma 2SA_1]\right\|,\\ \beta_4 &= \left\|[I+\Gamma_1 CB]^{-1}[\Gamma_1 SA_1]\right\|,\\ \beta_5 &= \left\|[I+\Gamma_1 CB]^{-1}[\Gamma_2 C]\right\|\end{aligned} \tag{56}$$

Further, applying the same procedure as in the proof of Theorem 3, one can also conclude that:

$$\lim_{i\to\infty}\|\delta u_i\|_\lambda \to 0, \tag{57}$$

and taking into account the uniqueness and existence theorem for fractional order time delay system, V.Lakshmikantham [42], it follows

$$\lim_{i\to\infty} x_i(t) = x_d(t),\ \lim_{i\to\infty} y_i(t) = y_d(t). \tag{58}$$

Simulation example

In this section, an example is presented to show the effectiveness of the proposed PD^α iterative learning feedback-feedforward controller. Consider the following fractional order uncertain time delay system in the state space form described by

$$\begin{aligned}\mathbf{x}_i^{(0.5)}(t) = 0.1x_i(t) + 0.1x_i(t-\tau) + u_i(t) + \\ +0.15\sin(x_i^2(t)+0.05)\end{aligned}$$

$$\mathbf{y}_i(t) = \mathbf{x}_i(t), \tag{59}$$

where $t\in[0,1]$, $\alpha = 0.5$, $\tau \le \tau_M = 0.1$ and $\psi_d(t) = 0.1$, $-0.1 \le t < 0$. The desired output trajectory is given by $y_d(t) = 2t(1-t)$ and the parameter perturbation $f_i = 0.15\sin\left(x_i^2 + 0.05\right)$. We apply the PD^α-type ILC updating law

$$\begin{aligned}u_{i+1}(t) = u_i(t) + 0.7\left(e_{i+1}^{0.5}(t) + 0.7e_{i+1}(t)\right) + \\ +0.5\left(e_i^{0.5}(t) + 0.5e_i(t)\right)\end{aligned} \tag{60}$$

with the initial control $u_0(t) = 0$.

The simulation results in Figures 1 and 2 show the effectiveness of the developed ILC control scheme for the system (59). The ILC rule (50) is used and Fig.1 shows the tracking performance of the ILC system outputs on the interval $t\in[0,1]$. Also, we can find, (see Fig.2), that the proposed requirement of the tracking performance is achieved at the ninth iteration.

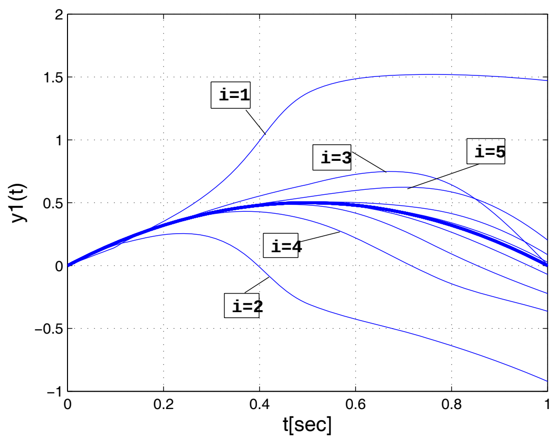

Figure 1. The tracking performance of the system output (y/(t)): solid line, $y_d(t)$: bold line)

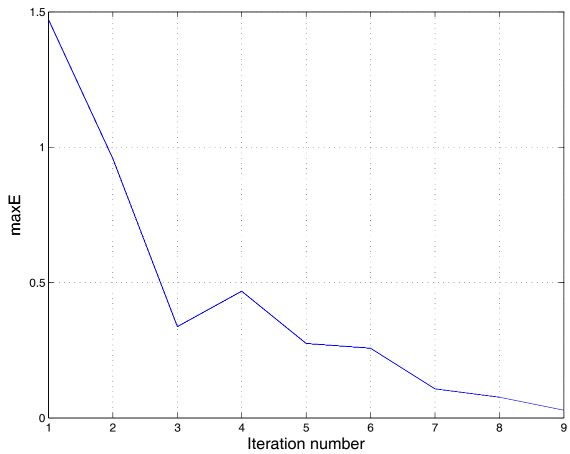

Figure 2. The sup-norm of the tracking error $e(t)$ in each iteration

Conclusion

Iterative learning control, which can be categorized as an intelligent control methodology, is an approach for improving the transient performance of systems that operate repetitively over a fixed time interval such as motion control of robotic systems, etc. In this paper, a PD^α type of robust ILC is proposed for a given class of fractional order uncertain unknown and bounded time delay systems. Particularly, we considered two cases of time delay: unknown bounded constant time delay as well as time-varying delay. Sufficient conditions for the convergence in the time domain of a proposed ILC were given by the corresponding theorems and proved. In the second part of this paper, the applications of the feedback-feedforward PD^α type of ILC are proposed for the given class of fractional order uncertain time delay systems. Finally, the sufficient conditions for the convergence in the time domain of the proposed ILC were given by the corresponding theorem together with its proof. The theoretical results have also been verified through a numerical simulation.

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Neki novi rezultati iterativnog upravljanja učenjem necelog reda

Iterativno upravljanje putem učenja (ILC) predstavlja jedno od važnih oblasti u teoriji upravljanja i ono je moćan koncept upravljanja koji na iterativan način poboljšava ponašanje procesa koji su po prirodi ponovljivi. ILC je pogodno za upravljanje šire klase mehatroničkih sistema i posebno su pogodni za upravljanje na primer kretanja robotskih sistema koji imaju važnu ulogu u tehničkim sistemima koji uključuju sisteme upravljanja, primenu u vojnoj industriji itd. Ovaj se rad bavi problemom ILC upravljanja za nelinearne sisteme necelog reda sa vremenskim kašnjenjem. Posebno, ovde se proučavaju sistemi necelog reda sa nepoznatim ograničenim vremenskim kašnjenjem u prostoru stanja kao i slučaj vremenski promenljivog kašnjenja. Pri tome, dovoljni uslovi za konvergenciju u vremenskom domenu predloženog PD^{α} ILC upravljanja za datu klasu necelog reda sistema sa kašnjenjem su prezentovani i dati u vremenskom domenu.Takođe, robusno PD^{α} ILC upravljanje u direktnoj-povratnoj sprezi za dati sistem sa kašnjenjem je razmatrano.Posebno, razmatra se sistem necelog reda sa nepoznatim ali ograničenim konstantnim vremenskim kašnjenjem. Dovoljni uslovi za konvergenciju u vremenskom domenu predloženog PD^{α} ILC upravljanja su dati odgovarajućom teoremom sa pratećim dokazom. Konačno, simulacioni primer pokazuje izvodljivost i efikasnost predloženog pristupa.

Ključne reči: **teorija upravljanja, iterativno upravljanje, upravljanje učenjem, necelobrojni red, nelinearni sistem, vremensko kašnjenje, robotski sistem.**

Некоторые новые результаты итеративного управления обучением дробного порядка

Итеративное управление процессом обучения (ИУПО) является одним из важных направлений в теории управления, и это мощная концепция управления, которая при помощи итеративного метода улучшает поведение процессов, которые по своей природе повторяются. ИУПО идеально подходит для управления широких класс мехатронных систем и особенно подходит для управления, например, движения роботизированных систем, которые играют важную роль в технических системах, которые включают в себя системы управления, применение в военной промышленности и так далее. Эта статья имеет дело с ИУПО управлением нелинейными системами дробного порядка с запаздыванием. В частности, здесь изучаются системы дробного порядка с неизвестной ограниченного периода задержки времени в пространстве состояний, а также и при изменяющихся во времени задержек. При этом, достаточные условия для сходимости во временной области предлагаемой PD^{α} ИУПО управления для данного класса дробного порядка систем с запаздыванием времени заказа представлены и даны во временной области. Также рассматривано надёжное PD^{α} ИУПО управление в прямой и обратной связи для даной системы с задержкой времени. Отдельно рассматривается система дробного порядка с неизвестным, но ограниченным постоянным запаздыванием. Достаточные условия для сходимости во временной области предлагаемого PD^{α} ИУПО управления получили и соответствующую теорему с подтверждающими доказательствами. Наконец, пример моделирования показывает целесообразность и эффективность предложенного подхода.

Ключевые слова: **Теория управления, итеративное управление, управления обучением, дробной порядок, нелинейная система, время задержки, роботизированная система.**

Quelques nouveaux résultats du contrôle itératif par l'étude de l'ordre fractionnel

Le contrôle itératif par l'étude (CIE) est un domaine important dans la théorie de contrôle. C'est un concept puissant qui à la façon itérative améliore le comportement des processus itératifs par leur nature. Le contrôle itératif par l'étude est convenable pour le contrôle d'une large classe des systèmes mécatroniques notamment pour le contrôle des mouvement des systèmes robotiques qui jouent un rôle important dans les systèmes techniques comprenant les systèmes de contrôle, l'application dans l'industrie militaire, etc. Ce travail considère le problème du contrôle CIE chez les systèmes de l'ordre fractionnel à délai temporel. On étudie ici en particulier les systèmes de l'ordre fractionnel à délai temporel limités et inconnus dans l'espace d'état ainsi que le cas du délai temporel variable. Les conditions suffisantes pour la convergence dans le domaine temporel PD^{α} du contrôle CIE pour la classe donnée de l'ordre fractionnel du système à délai ont été présentées et donnée dans le domaine temporel. Le contrôle CIE PD^{α} robuste aux réactions directes pour le système à délai a été traité aussi. On a considéré surtout le système de l'ordre fractionnel à délai temporel inconnu et limité constamment. Les conditions suffisantes pour la convergence dans le domaine temporel du CIE PD^{α} contrôle proposé ici ont été présentés par la théorème accompagnée de preuve. Finalement l'exemple simulé démontre l'efficacité et la réalisabilité de l'approche proposée.

Mots clé: **théorie de contrôle , contrôle itératif, contrôle par étude, ordre fractionnel, système non linéaire, délai temporel, système robotique.**