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# Some coincidence and common fixed point theorems for ordered Prešić-Reich type contractions

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## Abstract

The purpose of this paper is to prove some coincidence and common fixed point theorems for ordered Prešić-Reich type contractions in ordered metric spaces. Results of this paper generalize and extend several known results from metric spaces into product spaces when the underlying space is an ordered metric space. An example illustrates the case when new results can be applied while old ones cannot.

**Keywords:** Prešić type mapping; coincidence point; common fixed point; ordered space

## 1 Introduction and preliminaries

The well-known Banach contraction mapping principle states that if  $(X, d)$  is a complete metric space and  $f : X \rightarrow X$  is a self-mapping such that

$$d(fx, fy) \leq \lambda d(x, y) \quad (1)$$

for all  $x, y \in X$ , where  $0 \leq \lambda < 1$ , then there exists a unique  $x \in X$  such that  $fx = x$ . This point  $x$  is called the fixed point of the mapping  $f$ . On the other hand, for mappings  $f : X \rightarrow X$ , Kannan [1] introduced the contractive condition

$$d(fx, fy) \leq \lambda [d(x, fx) + d(y, fy)] \quad (2)$$

for all  $x, y \in X$ , where  $\lambda \in [0, \frac{1}{2})$  is a constant and proved a fixed point theorem using (2) instead of (1). Conditions (1) and (2) are independent, as it was shown by two examples in [2].

Reich [3], for mappings  $f : X \rightarrow X$ , generalized Banach and Kannan fixed point theorems using the contractive condition

$$d(fx, fy) \leq \alpha d(x, y) + \beta d(x, fx) + \gamma d(y, fy) \quad (3)$$

for all  $x, y \in X$ , where  $\alpha, \beta, \gamma$  are nonnegative constants with  $\alpha + \beta + \gamma < 1$ . An example in [3] shows that condition (3) is a proper generalization of (1) and (2).

In 1965, Prešić [4, 5] extended the Banach contraction mapping principle to mappings defined on product spaces and proved the following theorem.

**Theorem 1.1** Let  $(X, d)$  be a complete metric space,  $k$  be a positive integer and  $f : X^k \rightarrow X$  be a mapping satisfying the following contractive type condition:

$$d(f(x_1, x_2, \dots, x_k), f(x_2, x_3, \dots, x_{k+1})) \leq \sum_{i=1}^k q_i d(x_i, x_{i+1}) \quad (4)$$

for every  $x_1, x_2, \dots, x_{k+1} \in X$ , where  $q_1, q_2, \dots, q_k$  are nonnegative constants such that  $q_1 + q_2 + \dots + q_k < 1$ . Then there exists a unique point  $x \in X$  such that  $f(x, x, \dots, x) = x$ . Moreover, if  $x_1, x_2, \dots, x_k$  are arbitrary points in  $X$  and for  $n \in \mathbb{N}$ ,  $x_{n+k} = f(x_n, x_{n+1}, \dots, x_{n+k-1})$ , then the sequence  $\{x_n\}$  is convergent and  $\lim x_n = f(\lim x_n, \lim x_n, \dots, \lim x_n)$ .

Note that condition (4) in the case  $k = 1$  reduces to the well-known Banach contraction mapping principle. So, Theorem 1.1 is a generalization of the Banach fixed point theorem. Some generalizations and applications of the Prešić theorem can be seen in [4–19].

The existence of a fixed point in partially ordered sets was investigated by Ran and Reurings [20] and then by Nieto and Lopez [21, 22]. Fixed point results in ordered metric spaces were obtained by several authors (see, e.g., [6, 18, 23–32]). The following version of the fixed point theorem was proved, among others, in these papers.

**Theorem 1.2** (see [22] and references therein) Let  $(X, \preceq)$  be a partially ordered set, and let  $d$  be a metric on  $X$  such that  $(X, d)$  is a complete metric space. Let  $f : X \rightarrow X$  be a nondecreasing map with respect to  $\preceq$ . Suppose that the following conditions hold:

- (i) there exists  $k \in (0, 1)$  such that  $d(fx, fy) \leq kd(x, y)$  for all  $x, y \in X$  with  $y \preceq x$ ;
- (ii) there exists  $x_0 \in X$  such that  $x_0 \preceq fx_0$ ;
- (iii)  $f$  is continuous.

Then  $f$  has a fixed point  $x^* \in X$ .

Păcurar [10] introduced the Prešić-Kannan type contraction and proved some common fixed point theorems for such contractions. Very recently, in [18] (see also [33]) authors introduced the ordered Prešić type contraction and generalized the result of Prešić and proved some fixed point theorems for such mappings. In this paper, we introduce the ordered Prešić-Reich type contraction and prove some common fixed point theorems for such type of mappings in ordered metric spaces. Our results generalize and extend the results of Prešić [4, 5], Păcurar [10], Malhotra *et al.* [18], Luong and Thuan [33], Nieto and López [21] and several known results of metric spaces. An example, which illustrates the case when new results can be applied while old ones cannot, is included.

The following definitions will be needed in the sequel.

**Definition 1.3** Let  $X$  be a nonempty set,  $k$  be a positive integer and  $f : X^k \rightarrow X$  be a mapping. If  $f(x, x, \dots, x) = x$ , then  $x \in X$  is called a fixed point of  $f$ .

**Definition 1.4** (see [13]) Let  $X$  be a nonempty set,  $k$  be a positive integer,  $f : X^k \rightarrow X$  and  $g : X \rightarrow X$  be mappings.

- (a) An element  $x \in X$  is said to be a coincidence point of  $f$  and  $g$  if  $gx = f(x, \dots, x)$ .
- (b) If  $w = gx = f(x, \dots, x)$ , then  $w$  is called a point of coincidence of  $f$  and  $g$ .
- (c) If  $x = gx = f(x, \dots, x)$ , then  $x$  is called a common fixed point of  $f$  and  $g$ .

- (d) Mappings  $f$  and  $g$  are said to be commuting if  $g(f(x, \dots, x)) = f(gx, \dots, gx)$  for all  $x \in X$ .
- (e) Mappings  $f$  and  $g$  are said to be weakly compatible if they commute at their coincidence points.

Remark that the above definition in the case  $k = 1$  reduces to the usual definitions of commuting and weakly compatible mappings in the sense of [34] (for details, see the Introduction from [34]).

**Definition 1.5** (see [18]) Let a nonempty set  $X$  be equipped with a partial order ' $\preceq$ ' such that  $(X, d)$  is a metric space, then  $(X, \preceq, d)$  is called an ordered metric space. A sequence  $\{x_n\}$  in  $X$  is said to be nondecreasing with respect to ' $\preceq$ ' if  $x_1 \preceq x_2 \preceq \dots \preceq x_n \preceq \dots$ . Let  $k$  be a positive integer and  $f : X^k \rightarrow X$  be a mapping, then  $f$  is said to be nondecreasing with respect to ' $\preceq$ ' if for any finite nondecreasing sequence  $\{x_n\}_{n=1}^{k+1}$  we have  $f(x_1, x_2, \dots, x_k) \preceq f(x_2, x_3, \dots, x_{k+1})$ . Let  $g : X \rightarrow X$  be a mapping.  $f$  is said to be  $g$ -nondecreasing with respect to ' $\preceq$ ' if for any finite nondecreasing sequence  $\{gx_n\}_{n=1}^{k+1}$  we have  $f(x_1, x_2, \dots, x_k) \preceq f(x_2, x_3, \dots, x_{k+1})$ .

**Remark 1.6** For  $k = 1$ , the above definitions reduce to usual definitions of fixed point and nondecreasing mapping in a metric space.

**Definition 1.7** Let  $X$  be a nonempty set equipped with partial order ' $\preceq$ ' and let  $g : X \rightarrow X$  be a mapping. A nonempty subset  $\mathcal{A}$  of  $X$  is said to be well ordered if every two elements of  $\mathcal{A}$  are comparable. Elements  $a, b \in \mathcal{A}$  are called  $g$ -comparable if  $ga$  and  $gb$  are comparable.  $\mathcal{A}$  is called  $g$ -well ordered if for all  $a, b \in \mathcal{A}$ ,  $a$  and  $b$  are  $g$ -comparable, i.e.,  $ga$  and  $gb$  are comparable.

**Example 1.8** Let  $X = \{0, 1, 2, 3\}$ , ' $\preceq$ ' be a partial order relation on  $X$  defined by  $\preceq = \{(0, 0), (1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$ . Let  $\mathcal{A} = \{0, 1, 3\}$  and  $g : X \rightarrow X$  be defined by  $g0 = 1, g1 = 2, g2 = 3, g3 = 3$ . Then it is clear that  $\mathcal{A}$  is not well ordered but it is  $g$ -well ordered.

Let  $(X, \preceq, d)$  be an ordered metric space. Let  $k$  be a positive integer and  $f : X^k \rightarrow X$  be a mapping.  $f$  is said to be an ordered Prešić type contraction if

$$d(f(x_1, x_2, \dots, x_k), f(x_2, x_3, \dots, x_{k+1})) \leq \sum_{i=1}^k \alpha_i d(x_i, x_{i+1}) \tag{5}$$

for all  $x_1, x_2, \dots, x_{k+1} \in X$  with  $x_1 \preceq x_2 \preceq \dots \preceq x_{k+1}$ , where  $\alpha_i$  are nonnegative constants such that  $\sum_{i=1}^k \alpha_i < 1$ . If (5) is satisfied for all  $x_1, x_2, \dots, x_{k+1} \in X$ , then  $f$  is called a Prešić type contraction.

$f$  is said to be an ordered Prešić-Kannan type contraction (see [10] for details) if  $f$  satisfies following condition:

$$d(f(x_1, x_2, \dots, x_k), f(x_2, x_3, \dots, x_{k+1})) \leq \beta \sum_{i=1}^{k+1} d(x_i, f(x_i, x_i, \dots, x_i)) \tag{6}$$

for all  $x_1, x_2, \dots, x_{k+1} \in X$  with  $x_1 \preceq x_2 \preceq \dots \preceq x_{k+1}$ , where  $0 \leq \beta k(k+1) < 1$ . If (6) is satisfied for all  $x_1, x_2, \dots, x_{k+1} \in X$ , then  $f$  is called a Prešić-Kannan type contraction.

$f$  is said to be an ordered Prešić-Reich type contraction (see also [16]) if  $f$  satisfies the following condition:

$$d(f(x_1, x_2, \dots, x_k), f(x_2, x_3, \dots, x_{k+1})) \leq \sum_{i=1}^k \alpha_i d(x_i, x_{i+1}) + \sum_{i=1}^{k+1} \beta_i d(x_i, f(x_i, x_i, \dots, x_i)) \quad (7)$$

for all  $x_1, x_2, \dots, x_{k+1} \in X$  with  $x_1 \preceq x_2 \preceq \dots \preceq x_{k+1}$ , where  $\alpha_i, \beta_i$  are nonnegative constants such that  $\sum_{i=1}^k \alpha_i + k \sum_{i=1}^{k+1} \beta_i < 1$ . If (7) is satisfied for all  $x_1, x_2, \dots, x_{k+1} \in X$ , then  $f$  is called a Prešić-Reich type contraction.

Note that the Prešić-Reich type contraction is a generalization of Prešić type and Prešić-Kannan type contractions. Indeed, for  $\beta_i = 0, 1 \leq i \leq k+1$ , a Prešić-Reich type contraction reduces into a Prešić type contraction and for  $\alpha_i = 0, 1 \leq i \leq k$ , and  $\beta_i = \beta, 1 \leq i \leq k+1$ , a Prešić-Reich type contraction reduces into a Prešić-Kannan type contraction. Also, for  $k = 1$ , a Prešić-Reich type contraction reduces into a Reich contraction, so it generalizes the Banach and Kannan contractions.

Now we can state our main results.

## 2 Main results

**Theorem 2.1** *Let  $(X, \preceq, d)$  be an ordered complete metric space. Let  $k$  be a positive integer,  $f : X^k \rightarrow X$  and  $g : X \rightarrow X$  be two mappings such that  $f(X^k) \subset g(X)$ ,  $g(X)$  is a closed subset of  $X$  and*

$$d(f(x_1, x_2, \dots, x_k), f(x_2, x_3, \dots, x_{k+1})) \leq \sum_{i=1}^k \alpha_i d(gx_i, gx_{i+1}) + \sum_{i=1}^{k+1} \beta_i d(gx_i, f(x_i, x_i, \dots, x_i)) \quad (8)$$

for all  $x_1, x_2, \dots, x_{k+1} \in X$  with  $gx_1 \preceq gx_2 \preceq \dots \preceq gx_{k+1}$ , where  $\alpha_i, \beta_i$  are nonnegative constants such that

$$\sum_{i=1}^k \alpha_i + k \sum_{i=1}^{k+1} \beta_i < 1. \quad (9)$$

Suppose that the following conditions hold:

- (I) there exists  $x_0 \in X$  such that  $gx_0 \preceq f(x_0, x_0, \dots, x_0)$ ;
- (II)  $f$  is  $g$ -nondecreasing;
- (III) if a nondecreasing sequence  $\{gx_n\}$  converges to  $gu \in X$ , then  $gx_n \preceq gu$  for all  $n \in \mathbb{N}$  and  $gu \preceq ggu$ .

Then  $f$  and  $g$  have a point of coincidence. If, in addition,  $f$  and  $g$  are weakly compatible, then  $f$  and  $g$  have a common fixed point  $v \in X$ . Moreover, the set of common fixed points of  $f$  and  $g$  is  $g$ -well ordered if and only if  $f$  and  $g$  have a unique common fixed point.

*Proof* Starting with given  $x_0 \in X$ , we define a sequence  $\{y_n\}$  as follows: let  $y_0 = gx_0$ ,  $y_1 = f(x_0, x_0, \dots, x_0)$ . As  $f(X^k) \subset g(X)$ , there exists  $x_1 \in X$  such that  $f(x_0, x_0, \dots, x_0) = gx_1$ . Therefore  $y_1 = gx_1 = f(x_0, x_0, \dots, x_0)$  as  $gx_0 \preceq f(x_0, x_0, \dots, x_0)$ , we have  $gx_0 \preceq gx_1$ , that is,  $y_0 \preceq y_1$ . Again, as  $f$  is  $g$ -nondecreasing and  $gx_0 \preceq gx_1$ , we have  $f(x_0, x_0, \dots, x_0) \preceq f(x_0, \dots, x_0, x_1) \preceq f(x_0, \dots, x_0, x_1, x_1) \preceq \dots \preceq f(x_1, x_1, \dots, x_1)$ . Choose  $x_2 \in X$  such that  $y_2 = gx_2 = f(x_1, x_1, \dots, x_1)$  (which is possible since  $f(X^k) \subset g(X)$ ). So,  $gx_0 \preceq gx_1 \preceq gx_2$ , that is,  $y_0 \preceq y_1 \preceq y_2$ . Continuing this process, we obtain

$$gx_0 \preceq gx_1 \preceq \dots \preceq gx_n \preceq gx_{n+1} \preceq \dots,$$

that is,

$$y_0 \preceq y_1 \preceq y_2 \preceq \dots \preceq y_n \preceq y_{n+1} \preceq \dots$$

and  $y_n = gx_n = f(x_{n-1}, x_{n-1}, \dots, x_{n-1})$  for  $n = 1, 2, \dots$ . Thus,  $\{y_n\} = \{gx_n\}$  is nondecreasing with respect to ' $\preceq$ ', that is,  $\{x_n\}$  is  $g$ -nondecreasing with respect to ' $\preceq$ '. We shall show that  $\{y_n\} = \{gx_n\}$  is a Cauchy sequence in  $g(X)$ . If  $y_n = y_{n+1}$  for any  $n$ , then

$$\begin{aligned} d(y_{n+1}, y_{n+2}) &= d(f(x_n, \dots, x_n), f(x_{n+1}, \dots, x_{n+1})) \\ &\leq d(f(x_n, \dots, x_n), f(x_n, \dots, x_n, x_{n+1})) \\ &\quad + d(f(x_n, \dots, x_n, x_{n+1}), f(x_n, \dots, x_n, x_{n+1}, x_{n+1})) + \dots \\ &\quad + d(f(x_n, x_{n+1}, \dots, x_{n+1}), f(x_{n+1}, \dots, x_{n+1})). \end{aligned}$$

As  $gx_n \preceq gx_{n+1}$ , using (8), the above inequality implies that

$$\begin{aligned} d(y_{n+1}, y_{n+2}) &\leq \alpha_k d(gx_n, gx_{n+1}) + \beta_1 d(gx_n, f(x_n, \dots, x_n)) + \dots \\ &\quad + \beta_k d(gx_n, f(x_n, \dots, x_n)) \\ &\quad + \beta_{k+1} d(gx_{n+1}, f(x_{n+1}, \dots, x_{n+1})) \\ &\quad + \alpha_{k-1} d(gx_n, gx_{n+1}) + \beta_1 d(gx_n, f(x_n, \dots, x_n)) + \dots \\ &\quad + \beta_{k-1} d(gx_n, f(x_n, \dots, x_n)) \\ &\quad + \beta_k d(gx_{n+1}, f(x_{n+1}, \dots, x_{n+1})) \\ &\quad + \beta_{k+1} d(gx_{n+1}, f(x_{n+1}, \dots, x_{n+1})) + \dots \\ &\quad + \alpha_1 d(gx_n, gx_{n+1}) + \beta_1 d(gx_n, f(x_n, \dots, x_n)) \\ &\quad + \beta_2 d(gx_{n+1}, f(x_{n+1}, \dots, x_{n+1})) + \dots \\ &\quad + \beta_{k+1} d(gx_{n+1}, f(x_{n+1}, \dots, x_{n+1})), \end{aligned}$$

that is,

$$\begin{aligned} d(y_{n+1}, y_{n+2}) &\leq \left[ \sum_{i=1}^k \alpha_i \right] d(y_n, y_{n+1}) + \beta_1 d(y_n, y_{n+1}) + \dots + \beta_k d(y_n, y_{n+1}) \\ &\quad + \beta_{k+1} d(y_{n+1}, y_{n+2}) + \beta_1 d(y_n, y_{n+1}) + \dots + \beta_{k-1} d(y_n, y_{n+1}) \end{aligned}$$

$$\begin{aligned} &+ \beta_k d(y_{n+1}, y_{n+2}) + \beta_{k+1} d(y_{n+1}, y_{n+2}) + \cdots + \beta_1 d(y_n, y_{n+1}) \\ &+ \beta_2 d(y_{n+1}, y_{n+2}) + \cdots + \beta_{k+1} d(y_{n+1}, y_{n+2}) \\ &\leq [k\beta_{k+1} + (k-1)\beta_k + \cdots + \beta_2] d(y_{n+1}, y_{n+2}) \end{aligned}$$

since  $y_n = y_{n+1}$ . In view of (9), we have  $k\beta_{k+1} + (k-1)\beta_k + \cdots + \beta_2 = \sum_{i=2}^{k+1} (i-1)\beta_i \leq k \sum_{i=2}^{k+1} \beta_i < 1$ , therefore it follows from the above inequality that  $d(y_{n+1}, y_{n+2}) = 0$ , that is,  $y_{n+1} = y_{n+2}$ . Similarly, it can be shown that

$$y_n = y_{n+1} = y_{n+2} = y_{n+3} = \cdots$$

Therefore  $\{y_n\}$  is a Cauchy sequence. If  $y_n \neq y_{n+1}$  for all  $n$ , then for any  $n \geq 0$ , we have

$$\begin{aligned} d(y_n, y_{n+1}) &= d(gx_n, gx_{n+1}) \\ &= d(f(x_{n-1}, \dots, x_{n-1}), f(x_n, \dots, x_n)) \\ &\leq d(f(x_{n-1}, \dots, x_{n-1}), f(x_{n-1}, \dots, x_{n-1}, x_n)) \\ &\quad + d(f(x_{n-1}, \dots, x_{n-1}, x_n), f(x_{n-1}, \dots, x_{n-1}, x_n, x_n)) + \cdots \\ &\quad + d(f(x_{n-1}, x_n, \dots, x_n), f(x_n, \dots, x_n)). \end{aligned}$$

As  $\{x_n\}$  is  $g$ -nondecreasing, using (8), the above inequality implies that

$$\begin{aligned} d(y_n, y_{n+1}) &\leq \alpha_k d(gx_{n-1}, gx_n) + \beta_1 d(gx_{n-1}, f(x_{n-1}, \dots, x_{n-1})) + \cdots \\ &\quad + \beta_k d(gx_{n-1}, f(x_{n-1}, \dots, x_{n-1})) \\ &\quad + \beta_{k+1} d(gx_n, f(x_n, \dots, x_n)) \\ &\quad + \alpha_{k-1} d(gx_{n-1}, gx_n) + \beta_1 d(gx_{n-1}, f(x_{n-1}, \dots, x_{n-1})) + \cdots \\ &\quad + \beta_{k-1} d(gx_{n-1}, f(x_{n-1}, \dots, x_{n-1})) \\ &\quad + \beta_k d(gx_n, f(x_n, \dots, x_n)) \\ &\quad + \beta_{k+1} d(gx_n, f(x_n, \dots, x_n)) + \cdots \\ &\quad + \alpha_1 d(gx_{n-1}, gx_n) + \beta_1 d(gx_{n-1}, f(x_{n-1}, \dots, x_{n-1})) \\ &\quad + \beta_2 d(gx_n, f(x_n, \dots, x_n)) + \cdots \\ &\quad + \beta_{k+1} d(gx_n, f(x_n, \dots, x_n)), \end{aligned}$$

that is,

$$\begin{aligned} d(y_n, y_{n+1}) &\leq \left[ \sum_{i=1}^k \alpha_i \right] d(y_{n-1}, y_n) \\ &\quad + \beta_1 d(y_{n-1}, y_n) + \cdots + \beta_k d(y_{n-1}, y_n) + \beta_{k+1} d(y_n, y_{n+1}) \\ &\quad + \beta_1 d(y_{n-1}, y_n) + \cdots + \beta_{k-1} d(y_{n-1}, y_n) + \beta_k d(y_n, y_{n+1}) \\ &\quad + \beta_{k+1} d(y_n, y_{n+1}) + \cdots \\ &\quad + \beta_1 d(y_{n-1}, y_n) + \beta_2 d(y_n, y_{n+1}) + \cdots + \beta_{k+1} d(y_n, y_{n+1}). \end{aligned}$$

Setting  $d_n = d(y_n, y_{n+1})$ , we obtain

$$\begin{aligned} d_n &\leq \left[ \sum_{i=1}^k \alpha_i \right] d_{n-1} + [k\beta_1 + (k-1)\beta_2 + \dots + 2\beta_{k-1} + \beta_k] d_{n-1} \\ &\quad + [k\beta_{k+1} + (k-1)\beta_k + \dots + 2\beta_3 + \beta_2] d_n, \\ d_n &\leq \left[ \sum_{i=1}^k \alpha_i + \sum_{i=1}^k (k+1-i)\beta_i \right] d_{n-1} + \left[ \sum_{i=2}^{k+1} (i-1)\beta_i \right] d_n, \\ d_n &\leq \frac{\sum_{i=1}^k \alpha_i + \sum_{i=1}^{k+1} (k+1-i)\beta_i}{1 - \sum_{i=1}^{k+1} (i-1)\beta_i} d_{n-1}, \\ d_n &\leq \lambda d_{n-1} \quad (\text{say}). \end{aligned}$$

Let  $A = \sum_{i=1}^k \alpha_i$ ,  $B = k \sum_{i=1}^{k+1} \beta_i$ ,  $C = \sum_{i=1}^{k+1} (i-1)\beta_i$ , then in view of (9) we have  $A + B < 1$ . Therefore

$$\lambda = \frac{\sum_{i=1}^k \alpha_i + \sum_{i=1}^{k+1} (k+1-i)\beta_i}{1 - \sum_{i=1}^{k+1} (i-1)\beta_i} = \frac{A + B - C}{1 - C} < 1.$$

By repeating this process, we obtain

$$d_n \leq \lambda^n d_0. \tag{10}$$

Let  $m, n \in \mathbb{N}$  and  $m > n$ , then it follows from (10) that

$$\begin{aligned} d(y_n, y_m) &\leq d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + d(y_{n+2}, y_{n+3}) + \dots + d(y_{m-1}, y_m) \\ &\leq d_n + d_{n+1} + d_{n+2} + \dots \\ &\leq \lambda^n d_0 + \lambda^{n+1} d_0 + \lambda^{n+2} d_0 + \dots \\ &= [1 + \lambda + \lambda^2 + \dots] \lambda^n d_0, \\ d(y_n, y_m) &\leq \frac{\lambda^n}{1 - \lambda} d_0. \end{aligned}$$

As  $\lambda < 1$ , we have  $\frac{\lambda^n}{1-\lambda} d_0 \rightarrow 0$  as  $n \rightarrow \infty$ . Therefore, it follows from the above inequality that  $\lim_{n,m \rightarrow \infty} d(y_n, y_m) = 0$ . Therefore  $\{y_n\} = \{gx_n\}$  is a Cauchy sequence. As  $g(X)$  is closed, there exist  $v \in g(X)$ ,  $u \in X$  such that

$$\lim_{n \rightarrow \infty} y_n = \lim_{n \rightarrow \infty} gx_n = gu = v.$$

We shall show that  $v$  is a point of coincidence of  $f$  and  $g$ . For any  $n \in \mathbb{N}$ , we obtain

$$\begin{aligned} d(gu, f(u, \dots, u)) &\leq d(gu, y_{n+1}) + d(y_{n+1}, f(u, \dots, u)) \\ &\leq d(gu, y_{n+1}) + d(f(x_n, \dots, x_n), f(u, \dots, u)) \\ &\leq d(gu, y_{n+1}) + d(f(x_n, \dots, x_n), f(x_n, \dots, x_n, u)) \\ &\quad + d(f(x_n, \dots, x_n, u), f(x_n, \dots, x_n, u, u)) + \dots \\ &\quad + d(f(x_n, u, \dots, u), f(u, \dots, u)). \end{aligned}$$

By (III) we have  $gx_n \leq gu$  for all  $n \in \mathbb{N}$ , also, as  $y_n = gx_n = f(x_n, \dots, x_n)$ ,  $d_n = d(gx_n, gx_{n+1})$  and  $gu = v$ . Therefore, using (8) in the above inequality, we obtain

$$\begin{aligned} d(v, f(u, \dots, u)) &\leq \alpha_k d(y_n, v) + \beta_1 d_n + \beta_2 d_n + \dots + \beta_k d_n \\ &\quad + \beta_{k+1} d(v, f(u, \dots, u)) \\ &\quad + \alpha_{k-1} d(y_n, v) + \beta_1 d_n + \beta_2 d_n + \dots \\ &\quad + \beta_{k-1} d_n + \beta_k d(v, f(u, \dots, u)) + \beta_{k+1} d(v, f(u, \dots, u)) \\ &\quad + \dots \\ &\quad + \alpha_1 d(y_n, v) + \beta_1 d_n + \beta_2 d(v, f(u, \dots, u)) + \dots \\ &\quad + \beta_{k+1} d(v, f(u, \dots, u)) + d(v, y_{n+1}) \\ &= \left[ \sum_{i=1}^k \alpha_i \right] d(y_n, v) + \left[ \sum_{i=1}^k (k+1-i)\beta_i \right] d_n \\ &\quad + \left[ \sum_{i=2}^{k+1} (i-1)\beta_i \right] d(v, f(u, \dots, u)) + d(v, y_{n+1}), \end{aligned}$$

that is,

$$\begin{aligned} (1-C)d(v, f(u, \dots, u)) &\leq Ad(y_n, v) + (B-C)d(y_n, y_{n+1}) + d(v, y_{n+1}), \\ (1-C)d(v, f(u, \dots, u)) &\leq (A+B-C)d(y_n, v) + (1+B-C)d(y_{n+1}, v). \end{aligned}$$

As  $\lim_{n \rightarrow \infty} d(y_n, v) = 0$  and  $1-C = 1 - \sum_{i=1}^{k+1} (i-1)\beta_i > 0$ , therefore it follows from the above inequality that

$$d(v, f(u, \dots, u)) = 0, \quad \text{that is, } f(u, \dots, u) = v = gu. \tag{11}$$

Thus,  $u$  is a coincidence point and  $v$  is a corresponding point of coincidence of  $f$  and  $g$ . Suppose,  $f$  and  $g$  are weakly compatible, then by (11) we have

$$f(v, \dots, v) = f(gu, \dots, gu) = g(f(u, \dots, u)) = gv.$$

Again, by (III),  $gu \leq ggu = gv$ ; therefore using (8) and a similar process as several times before, we obtain

$$\begin{aligned} d(v, f(v, \dots, v)) &= d(f(u, \dots, u), f(v, \dots, v)) \\ &\leq d(f(u, \dots, u), f(u, \dots, u, v)) \\ &\quad + d(f(u, \dots, u, v), f(u, \dots, u, v, v)) + \dots \\ &\quad + d(f(u, v, \dots, v), f(v, \dots, v)) \\ &\leq Ad(gu, gv) + (B-C)d(gu, f(u, \dots, u)) \\ &\quad + Cd(gv, f(v, \dots, v)) \\ &= Ad(v, f(v, \dots, v)). \end{aligned}$$



As  $A = \sum_{i=1}^k \alpha_i < 1$ , we obtain

$$d(v, f(v, \dots, v)) = 0, \quad \text{that is, } f(v, \dots, v) = v = gv.$$

Thus  $v$  is a common fixed point of  $f$  and  $g$ . Suppose that the set of common fixed points is  $g$ -well ordered. We shall show that the common fixed point is unique. Assume on the contrary that  $v_0$  is another common fixed point of  $f$  and  $g$ , that is,  $v_0 = f(v_0, \dots, v_0) = gv_0$  and  $v \neq v_0$ . As  $v$  and  $v_0$  are  $g$ -comparable, let for example  $gv \preceq gv_0$ . From (8), it follows that

$$\begin{aligned} d(v, v_0) &= d(f(v, \dots, v), f(v_0, \dots, v_0)) \\ &\leq d(f(v, \dots, v), f(v, \dots, v, v_0)) \\ &\quad + d(f(v, \dots, v, v_0), f(v, \dots, v, v_0, v_0)) + \dots \\ &\quad + d(f(v, v_0, \dots, v_0), f(v_0, \dots, v_0)) \\ &\leq Ad(gv, gv_0) + (B - C)d(gv, f(v, \dots, v)) + Cd(gv_0, f(v_0, \dots, v_0)) \\ &= Ad(v, v_0). \end{aligned}$$

As  $A = \sum_{i=1}^k \alpha_i < 1$ , we obtain  $d(v, v_0) = 0$ , that is,  $v = v_0$ , a contradiction. Therefore the common fixed point is unique. For converse, if a common fixed point of  $f$  and  $g$  is unique, then the set of common fixed points of  $f$  and  $g$  is singleton, and thus  $g$ -well ordered.  $\square$

**Remark 2.2** Let  $(X, \preceq, d)$  be an ordered metric space, and let  $f, g : X \rightarrow X$  be two mappings. Then  $f$  is called an ordered  $g$ -weak contraction if

$$d(fx, fy) \leq \alpha_1 d(gx, gy) + \alpha_2 d(fx, gx) + \alpha_3 d(fy, gy)$$

for all  $x, y \in X$  with  $x \preceq y$ , where  $\alpha_1, \alpha_2, \alpha_3$  are nonnegative constants such that  $\alpha_1 + \alpha_2 + \alpha_3 < 1$ . If the above inequality is satisfied for all  $x, y \in X$ , then  $f$  is called a  $g$ -weak contraction (see [35]). For  $k = 1$  in Theorem 2.1, we get a fixed point result for an ordered  $g$ -weak contraction in metric spaces.

The following is a fixed point result for ordered Prešić-Reich type mappings in metric spaces and can be obtained by taking  $g = I_X$  (that is, the identity mapping of  $X$ ) in Theorem 2.1.

**Corollary 2.3** Let  $(X, \preceq, d)$  be an ordered complete metric space. Let  $k$  be a positive integer,  $f : X^k \rightarrow X$  be a mapping such that the following conditions hold:

- (I)  $f$  is an ordered Prešić-Reich type contraction;
- (II) there exists  $x_0 \in X$  such that  $x_0 \preceq f(x_0, x_0, \dots, x_0)$ ;
- (III)  $f$  is nondecreasing (with respect to ' $\preceq$ ');;
- (IV) if a nondecreasing sequence  $\{x_n\}$  converges to  $u \in X$ , then  $x_n \preceq u$  for all  $n \in \mathbb{N}$ .

Then  $f$  has a fixed point. Moreover, the set of fixed points of  $f$  is well ordered if and only if  $f$  has a unique fixed point.

The following corollary is a generalization of the result of Prešić in an ordered metric space and can be obtained by taking  $\beta_i = 0$  for  $1 \leq i \leq k + 1$  in Theorem 2.1.

**Corollary 2.4** Let  $(X, \preceq, d)$  be an ordered complete metric space. Let  $k$  be a positive integer,  $f : X^k \rightarrow X$  and  $g : X \rightarrow X$  be two mappings such that  $f(X^k) \subset g(X)$ ,  $g(X)$  is a closed subset of  $X$  and

$$d(f(x_1, x_2, \dots, x_k), f(x_2, x_3, \dots, x_{k+1})) \leq \sum_{i=1}^k \alpha_i d(gx_i, gx_{i+1}) \quad (12)$$

for all  $x_1, x_2, \dots, x_{k+1} \in X$  with  $gx_1 \preceq gx_2 \preceq \dots \preceq gx_{k+1}$ , where  $\alpha_i$  are nonnegative constants such that  $\sum_{i=1}^k \alpha_i < 1$ . Suppose that the following conditions hold:

- (I) there exists  $x_0 \in X$  such that  $gx_0 \preceq f(x_0, x_0, \dots, x_0)$ ;
- (II)  $f$  is  $g$ -nondecreasing;
- (III) if a nondecreasing sequence  $\{gx_n\}$  converges to  $gu \in X$ , then  $gx_n \preceq gu$  for all  $n \in \mathbb{N}$  and  $gu \preceq ggu$ .

Then  $f$  and  $g$  have a point of coincidence. If, in addition,  $f$  and  $g$  are weakly compatible, then  $f$  and  $g$  have a common fixed point  $v \in X$ . Moreover, the set of common fixed points of  $f$  and  $g$  is  $g$ -well ordered if and only if  $f$  and  $g$  have a unique common fixed point.

The following corollary generalizes the result of Păcurar [10] in ordered metric spaces and can be obtained by taking  $\alpha_i = 0$  for  $1 \leq i \leq k$  in Theorem 2.1.

**Corollary 2.5** Let  $(X, \preceq, d)$  be an ordered complete metric space. Let  $k$  be a positive integer,  $f : X^k \rightarrow X$  and  $g : X \rightarrow X$  be two mappings such that  $f(X^k) \subset g(X)$ ,  $g(X)$  is a closed subset of  $X$  and

$$d(f(x_1, x_2, \dots, x_k), f(x_2, x_3, \dots, x_{k+1})) \leq \sum_{i=1}^{k+1} \beta_i d(gx_i, f(x_i, x_i, \dots, x_i)) \quad (13)$$

for all  $x_1, x_2, \dots, x_{k+1} \in X$  with  $gx_1 \preceq gx_2 \preceq \dots \preceq gx_{k+1}$ , where  $\beta_i$  are nonnegative constants such that  $k \sum_{i=1}^{k+1} \beta_i < 1$ . Suppose that the following conditions hold:

- (I) there exists  $x_0 \in X$  such that  $gx_0 \preceq f(x_0, x_0, \dots, x_0)$ ;
- (II)  $f$  is  $g$ -nondecreasing;
- (III) if a nondecreasing sequence  $\{gx_n\}$  converges to  $gu \in X$ , then  $gx_n \preceq gu$  for all  $n \in \mathbb{N}$  and  $gu \preceq ggu$ .

Then  $f$  and  $g$  have a point of coincidence. If, in addition,  $f$  and  $g$  are weakly compatible, then  $f$  and  $g$  have a common fixed point  $v \in X$ . Moreover, the set of common fixed points of  $f$  and  $g$  is  $g$ -well ordered if and only if  $f$  and  $g$  have a unique common fixed point.

The following example illustrates that an ordered Prešić-Reich type contraction may not be an ordered Prešić type or ordered Prešić-Kannan type or Prešić-Reich type contraction; moreover, that the fixed point of an ordered Prešić-Reich type contraction may not be unique (when the set of fixed points of  $f$  is not well-ordered).

**Example 2.6** Let  $X = [0, 2]$  and order relation ' $\preceq$ ' be defined by

$$\preceq = \{(x, y) : x, y \in [0, 1] \text{ with } y \leq x\} \cup \{(x, y) : x, y \in (1, 2) \text{ with } y \leq x\} \cup \{(2, 2)\}$$

and let  $d$  be the usual metric on  $X$ . Then  $(X, \preceq, d)$  is an ordered complete metric space. For  $k = 2$ , define  $f : X^2 \rightarrow X$  by

$$f(x, y) = \begin{cases} \frac{1}{12} & \text{if } x = y = 1, \\ 2 & \text{if } x = y = 2, \\ \frac{x+y}{2} - 1 & \text{if } (x, y) \in [0, 1) \times \{2\} \cup \{2\} \times [0, 1), \\ \frac{x+y}{6} & \text{otherwise.} \end{cases}$$

Then:

- (a)  $f$  is not an ordered Prešić type contraction;
- (b)  $f$  is not an ordered Prešić-Kannan type contraction;
- (c)  $f$  is not a Prešić-Reich type contraction;
- (d)  $f$  is an ordered Prešić-Reich type contraction with  $\alpha_1 = \alpha_2 = \frac{1}{6}$ ,  $\beta_1 = \beta_2 = \beta_3 = \frac{1}{10}$ .

*Proof* (a) For  $k = 2$ , (5) becomes

$$d(f(x_1, x_2), f(x_2, x_3)) \leq \alpha_1 d(x_1, x_2) + \alpha_2 d(x_2, x_3) \tag{14}$$

for all  $x_1, x_2, x_3 \in X$  with  $x_1 \preceq x_2 \preceq x_3$ , where  $\alpha_1, \alpha_2$  are nonnegative constants such that  $\alpha_1 + \alpha_2 < 1$ . Note that  $1 \preceq 1 \preceq \frac{9}{10}$ , therefore for  $x_1 = x_2 = 1, x_3 = \frac{9}{10}$ , (14) becomes

$$\begin{aligned} d\left(f(1, 1), f\left(1, \frac{9}{10}\right)\right) &\leq \alpha_1 d(1, 1) + \alpha_2 d\left(1, \frac{9}{10}\right), \\ d\left(\frac{1}{12}, \frac{19}{60}\right) &\leq \alpha_2 \frac{1}{10}, \\ \frac{7}{3} &\leq \alpha_2. \end{aligned}$$

But  $\alpha_1 + \alpha_2 < 1$  and therefore the above inequality will never hold. Thus  $f$  is not an ordered Prešić type contraction.

(b) For  $k = 2$ , (6) becomes

$$\begin{aligned} d(f(x_1, x_2), f(x_2, x_3)) &\leq \beta [d(x_1, f(x_1, x_1)) + d(x_2, f(x_2, x_2)) \\ &\quad + d(x_3, f(x_3, x_3))] \end{aligned} \tag{15}$$

for all  $x_1, x_2, x_3 \in X$  with  $x_1 \preceq x_2 \preceq x_3$ , where  $\beta$  is a nonnegative constant such that  $\beta < \frac{1}{6}$ . Note that  $x \preceq 0 \preceq 0$  for all  $x \in (0, 1)$  and therefore for  $x_2 = x_3 = 0, x_1 = x \in (0, 1)$ , (15) becomes

$$\begin{aligned} d(f(x, 0), f(0, 0)) &\leq \beta_1 d(x, f(x, x)) + \beta_2 d(0, f(0, 0)) + \beta_3 d(0, f(0, 0)), \\ d\left(\frac{x}{6}, 0\right) &\leq \beta_1 d\left(x, \frac{x}{3}\right) + \beta_2 d(0, 0) + \beta_3 d(0, 0), \\ \frac{x}{6} &\leq \beta_1 \frac{2x}{3}. \end{aligned}$$

But  $\beta < \frac{1}{6}$ , and therefore the above inequality will never hold. Thus  $f$  is not an ordered Prešić-Kannan type contraction.

(c) For  $k = 2$ , (7) becomes

$$\begin{aligned} d(f(x_1, x_2), f(x_2, x_3)) &\leq \alpha_1 d(x_1, x_2) + \alpha_2 d(x_2, x_3) \\ &\quad + \beta_1 d(x_1, f(x_1, x_1)) \\ &\quad + \beta_2 d(x_2, f(x_2, x_2)) + \beta_3 d(x_3, f(x_3, x_3)), \end{aligned} \tag{16}$$

where  $\alpha_1, \alpha_2, \beta_1, \beta_2, \beta_3$  are nonnegative constants such that  $\sum_{i=1}^2 \alpha_i + 2 \sum_{i=1}^3 \beta_i < 1$ . If  $f$  is a Prešić-Reich type contraction, then inequality (16) must be satisfied for all  $x_1, x_2, x_3 \in X$ . Note that for  $x_1 = x_2 = 2, x_3 = 0$ , (16) becomes

$$\begin{aligned} d(f(2, 2), f(2, 0)) &\leq \alpha_1 d(2, 2) + \alpha_2 d(2, 0) \\ &\quad + \beta_1 d(2, f(2, 2)) + \beta_2 d(2, f(2, 2)) + \beta_3 d(0, f(0, 0)), \\ d(2, 0) &\leq \alpha_2 d(2, 0) + \beta_1 d(2, 2) + \beta_2 d(2, 2) + \beta_3 d(0, 0), \\ 2 &\leq 2\alpha_2. \end{aligned}$$

But  $\sum_{i=1}^2 \alpha_i + 2 \sum_{i=1}^3 \beta_i < 1$ , and therefore the above inequality will never hold. Thus  $f$  is not a Prešić-Reich type contraction.

(d) If  $f$  is an ordered Prešić-Reich type contraction, then inequality (16) must be satisfied for all  $x_1, x_2, x_3 \in X$  with  $x_1 \leq x_2 \leq x_3$ . Indeed, we have to check the validity of (16) only for  $x_1, x_2, x_3 \in [0, 1], x_1, x_2, x_3 \in (1, 2)$  and  $x_1 = x_2 = x_3 = 2$ . If  $x_1 = x_2 = x_3 = 1$  or  $x_1 = x_2 = x_3 = 2$ , then (16) is satisfied trivially. If  $x_1, x_2, x_3 \in [0, 1)$  or  $x_1, x_2, x_3 \in (1, 2)$  with  $x_1 \leq x_2 \leq x_3$ , that is,  $x_3 \leq x_2 \leq x_1$ , then (16) becomes

$$\begin{aligned} d\left(\frac{x_1 + x_2}{6}, \frac{x_2 + x_3}{6}\right) &\leq \alpha_1 d(x_1, x_2) + \alpha_2 d(x_2, x_3) \\ &\quad + \beta_1 d\left(x_1, \frac{x_1}{3}\right) + \beta_2 d\left(x_2, \frac{x_2}{3}\right) + \beta_3 d\left(x_3, \frac{x_3}{3}\right), \\ \frac{x_1 - x_3}{6} &\leq \alpha_1(x_1 - x_2) + \alpha_2(x_2 - x_3) + \frac{2x_1}{3}\beta_1 + \frac{2x_2}{3}\beta_2 + \frac{2x_3}{3}\beta_3, \end{aligned}$$

which is valid for  $\alpha_1 = \alpha_2 = \frac{1}{6}, \beta_1 = \beta_2 = \beta_3 = \frac{1}{10}$ . If any one of  $x_1, x_2, x_3$  is equal to 1, then with a similar process one can verify the same result. If any two of  $x_1, x_2, x_3$  are equal to 1, for example, let  $x_1 = x_2 = 1, x_3 \in [0, 1)$ , then (16) becomes

$$\begin{aligned} d\left(\frac{1}{12}, \frac{1 + x_3}{6}\right) &\leq \alpha_1 d(1, 1) + \alpha_2 d(1, x_3) \\ &\quad + \beta_1 d\left(1, \frac{1}{12}\right) + \beta_2 d\left(1, \frac{1}{12}\right) + \beta_3 d\left(x_3, \frac{x_3}{3}\right), \\ \frac{1 + 2x_3}{12} &\leq \alpha_2(1 - x_3) + \frac{11}{12}\beta_1 + \frac{11}{12}\beta_2 + \frac{2x_3}{3}\beta_3, \end{aligned}$$

which is valid for  $\alpha_1 = \alpha_2 = \frac{1}{6}, \beta_1 = \beta_2 = \beta_3 = \frac{1}{10}$ . Similarly, in all possible cases, (16) is satisfied for  $\alpha_1 = \alpha_2 = \frac{1}{6}, \beta_1 = \beta_2 = \beta_3 = \frac{1}{10}$ . Thus,  $f$  is an ordered Prešić-Reich type contraction. All the conditions of Corollary 2.3 (except the set of fixed points of  $f$  is well ordered) are satisfied and the set of fixed points of  $f$  is  $\mathcal{F} = \{0, 2\}$ . Note that the set of fixed points of  $f$ , that is  $\mathcal{F}$ , is not well ordered (as  $(0, 2), (2, 0) \notin \leq$ ) and fixed point  $f$  is not unique.  $\square$

### Competing interests

All authors of the present paper disclose no actual potential conflict of interests including any financial, personal or other relationships with people or organizations.

### Authors' contributions

All authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

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