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BUCKLING ANALYSIS OF 3D MODEL OF SLENDER PILE IN INTERACTION WITH SOIL USING FINITE ELEMENT METHOD

ANALIZA STABILNOSTI VITKOG 3D MODELA ŠIPA U INTERAKCIJI SA TLOM METODOM KONAČNIH ELEMENATA

Originalni naučni rad / Original scientific paper
UDK /UDC: 624.155.114.046.3:519.673
Rad primljen / Paper received: 16.11.2012.

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Keywords
- slender pile
- stability analysis
- finite element analysis
- solid finite elements
- contact elements link

Ključne reči
- vitki šip
- analiza stabilnosti
- metoda konačnih elemenata
- solid konačni elementi
- kontaktni link elementi

Abstract

The paper describes the modelling and stability analysis of slender piles using finite element method. The concept of a modified stability analysis of 3D model pile–soil–pile cap is formulated; it is formed from solid finite element models for the two types of soil, and a single-pile. Connection in pile–soil contact is modelled using link elements. The terms of the level of normalised critical load P_{cr}/P_E 3D model of pile–soil–pile cap are derived using regression analysis of the power function.

On the basis of performed numerical tests and regression analyses expressions are derived for the buckling length coefficient β as a function of pile length, pile stiffness and soil stiffness. Research has shown that the use of sophisticated mathematical models and numerical analysis is justified and necessary in order to gain better insight into the behaviour of slender piles in terms of stability. The paper points to the complex shape of the buckling of slender piles with a number of half waves.

Izvod

U radu je prikazano modeliranje i analiza rezultata stabilnosti vitkih šipova primenom metode konačnih elemenata. Formulisan je koncept za modifikovanu analizu stabilnosti 3D modela šip–naglavna ploča–tlo formiran od solid konačnih elemenata za dva tipa modela tla: jednoslojni i dvoslojni. Veza šip–tlo je modelirana primenom kontaktnih link elemenata. Izrazi za nivo normalizovane vrednosti kritične sile P_{cr}/P_E 3D modela šip–naglavna ploča–tlo izvedeni su primenom regresionih analiza za stepenu funkciju.

Na osnovu sprovedenih numeričkih testova i regresionih analiza izvedeni su izrazi za koeficijent dužine izvijanja β u funkciji dužine šipa, krutosti šipa i krutosti tla. Istraživanjem je pokazano da je primena sofisticiranih matematičkih modela i numeričkih analiza opravdana i neophodna u cilju kvalitetnijeg uvida u ponašanje vitkog šipa sa aspekta stabilnosti. Ukazano je na kompleksan oblik izvijanja vitkog šipa sa većim brojem polutalasa.

INTRODUCTION

Foundations are one of the most important support elements of building structures. Their role is to transfer loads which structures are subjected to onto the load-bearing layers of soil and to ensure sufficient structural stability. In case of weak capacity soil, piles are used as foundations. Depending on how they transfer loads, piles can be classified as standing that transfer loads with their bases, and floating that transfer loads by friction on pile's mantle.

When designing piles, their number, layout and length is affected by soil quality and required foundation and single pile bearing capacity. Pile length, as well as their bearing capacity, primarily depend on the depth of load bearing layers of soil, soil properties along its depth and on shaping of bases and lateral surfaces of piles. Designing piles subjected to static load requires fulfilling and proving of bearing criteria, stability and usability. Bearing capacity treatment is referred to fulfilling the criteria according to which the bearing force of the pile is larger than the exploitation force for the corresponding situation. Usability is determined according to vertical and horizontal pile strains, i.e. allowed sinking in accordance to technical regulations. Pile stability is far more unknown to the expert community in comparison to the previous two criteria, since it was not involved in a lot of research. Even though the bearing criteria is, in most cases, more accurate than the stability criteria, the latter can be dominant in case of slender piles, especially on soil susceptible to liquefaction /8/, and should be considered in such situations.

One of the most important classifications of pile foundations is based on whether single or group piles are in question. It is common to study pile foundations as single, whereas group piles are also considered on a basis of single ones, including specifics that apply to such piles.

GENERAL TREATMENT IN ANALYSIS OF PILES SUBJECTED TO AXIAL LOAD

The major problem in pile behaviour analysis is caused by the nature of the load. The pile receives axial and lateral forces, bending and torsion moments through a headset panel. Transfer of these loads depends on conditions in connections of piles and the soil itself, as well as on soil properties. Friction resistance along the mantle is formed along with lateral and torsional stresses. A more accurate analytical treatment of soil-pile interaction (SPI) would be too complex, hence various approximations of piles, soils or loads, are often introduced. Piles are most commonly treated to axial loading as the most dominant, which is transferred to the soil via the pile's base and mantle.

Load bearing mechanics of piles, i.e. elements that ensure a single pile's bearing capacity include, /7/:
- load bearing by friction along the mantle (*side resistance*) – shear resistance to friction and cohesion along the pile,
- load bearing by vertical loads on the base (*end bearing*) – vertical resistance to base pressure,
- load bearing by shear loads on the base (*base shear*) – horizontal resistance to shear and base cohesion,
- lateral earth pressure – horizontal resistance to lateral soil pressure.

Piles subjected to axial forces are calculated based on boundary states of bearing capacity and stability, i.e. buckling for any given level of pile slenderness. Pile stability is affected by following parameters: length, boundary conditions (mantle and base), cross-section properties, material and soil quality and type of load, while the buckling length of the piles is a function of pile real length, boundary conditions and soil stiffness.

Research given in paper /12/ shows that critical buckling force of the pile is derived from an analytical procedure, using Euler's principle for critical force analysis:

$$P_E = \frac{\pi^2 E_p I_p}{(0.7L_e)^2} \tag{1}$$

where E_p is pile's elasticity module; I_p is the moment of inertia for pile's cross-section; L_e is equivalent buckling length, /1/:

$$L_e = L_u + L_f\,, \quad L_f = 1.8\left(\frac{E_p I_p}{n_h}\right)^{0.2} \tag{2}$$

where L_u is the unsupported (free) pile length; L_f is the depth of pile fixing in sandy soil.

A pile subjected to axial forces becomes deformed, i.e. buckles, in such a way that its axis changes its position compared to the initial state with no loads. In this case, the real level of critical force is smaller, hence, with certain corrections, Eq.(1) can be written using coefficient α as:

$$P_E = \alpha\frac{\pi^2 E_p I_p}{(0.7L_e)^2} \tag{3}$$

Extensive experimental testing of a large number of piles examined the deformed states of piles through incremental situations (Fig. 1.) By comparing theoretical and experimental critical force values of piles, it is determined that analytical procedures result in lower values of critical force (about 10–25% lower).

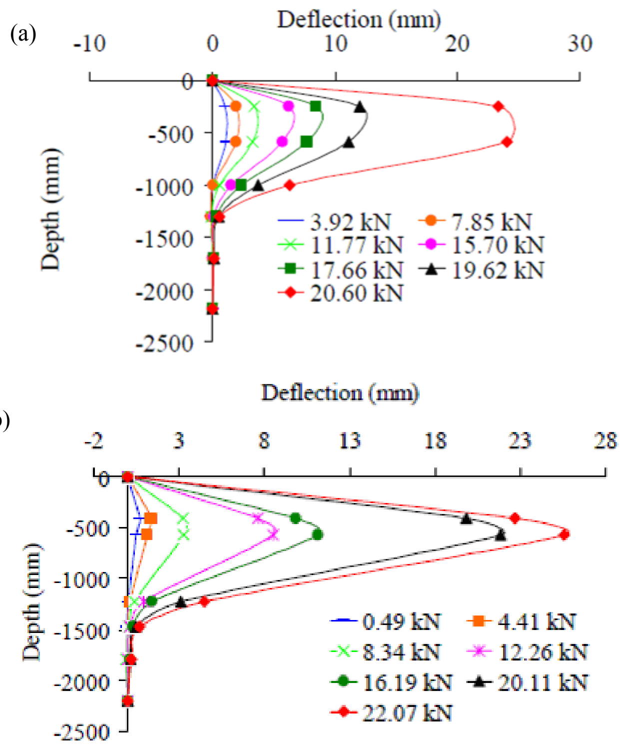


Figure 1. Lateral strain of pile: a) L_u = 1 m, e = 60 mm, RD = 30%, b) L_u = 1.1 m, e = 40 mm, RD = 70%, /12/.
Slika 1. Bočna deformacija šipa: a) L_u = 1 m, e = 60 mm, RD = 30%, b) L_u = 1.1 m, e = 40 mm, RD = 70%, /12/

Compared to the analytical treatment of slender piles in soil, numerical analysis offers significantly a more complex treatment of the SPI problem. However, performing complex numerical experiments requires parallel experimental testing, as a form of quality verification of obtained results. Commonly used and a most efficient approach in presenting mathematical models of piles for numerical treatment is the application of rod elements, i.e. line systems /10/, while soil models are compensated using elastic springs. Figure 2 shows a soil-pile system subjected to axial loading and an element subjected to according cross-section forces. Springs, whose role is to represent active and passive effects of soil are placed on lateral sides (mantle) along the pile, including a single spring placed at the pile's base. Spring stiffness depends on the way the pile is made, for example drilled or compacted, and on soil type and stiffness, as well as the contact between the soil and the surface of the pile.

The medium containing the pile can be of various characteristics, ranging from single to multi-layered environment with different geomechanical properties. In most cases, when single-layered systems, or systems with multiple layers with similar geomechanical properties are involved, standard buckling length corresponds to pile lengths through given layers, /18/. This is caused by the fact that piles buckle in form of a sine semi-waves through all layers of soil. In case there is a significant difference in the geomechanical properties of certain layers in comparison to the rest, a noticeably smaller pile length can occur as a result due to the buckling taking the form of a larger number of sine semi-waves (Fig. 3).

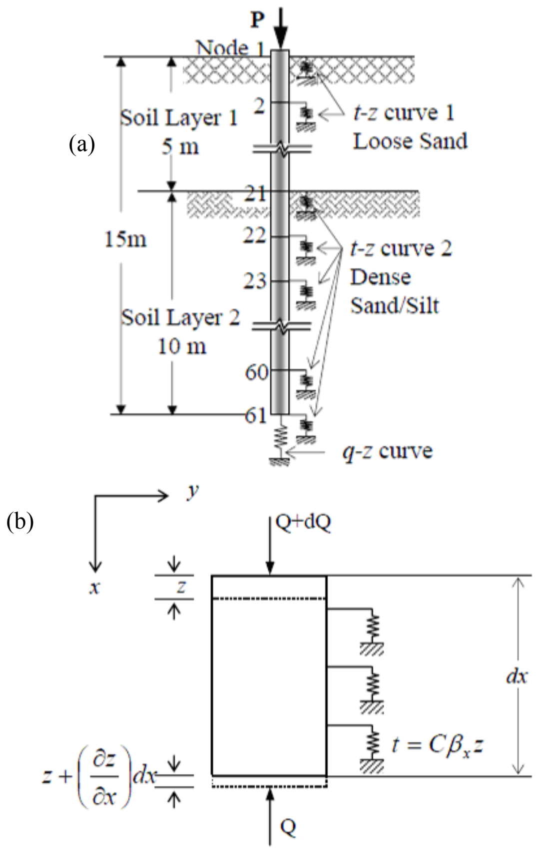


Figure 2. a) A soil-pile system subjected to an axial force; b) pile element affected by cross-section forces, /10/.
Slika 2. a) Sistem tlo-šip opterećen aksijalnom silom; b) element šipa pod uticajem sila u poprečnom preseku, /10/

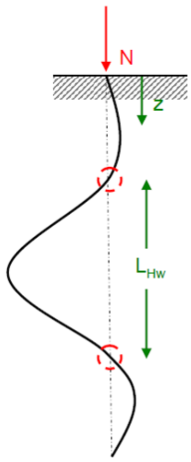


Figure 3. Pile buckling in the form of a larger number of sine semi-waves in case of a large difference in geomechanical properties of certain layers of soil, /18/.
Slika 3. Izvijanje šipa u obliku velikog broja sinusnih polu-talasa za slučaj velike razlike u geomehaničkim osobinama pojedinih slojeva tla, /18/

Research, /9/, has considered effects of various boundary conditions on pile stability by modelling lateral soil reactions using a degree function for soil distribution along the pile (Fig. 4a). The critical force level is determined according to:

$$P_{cr} = \frac{\pi^2 E_p I_p}{L^2} P' \tag{4}$$

as a function of equivalent buckling length of pile L_e:

$$P_E = \frac{\pi^2 E_p I_p}{L_e^2} \tag{5}$$

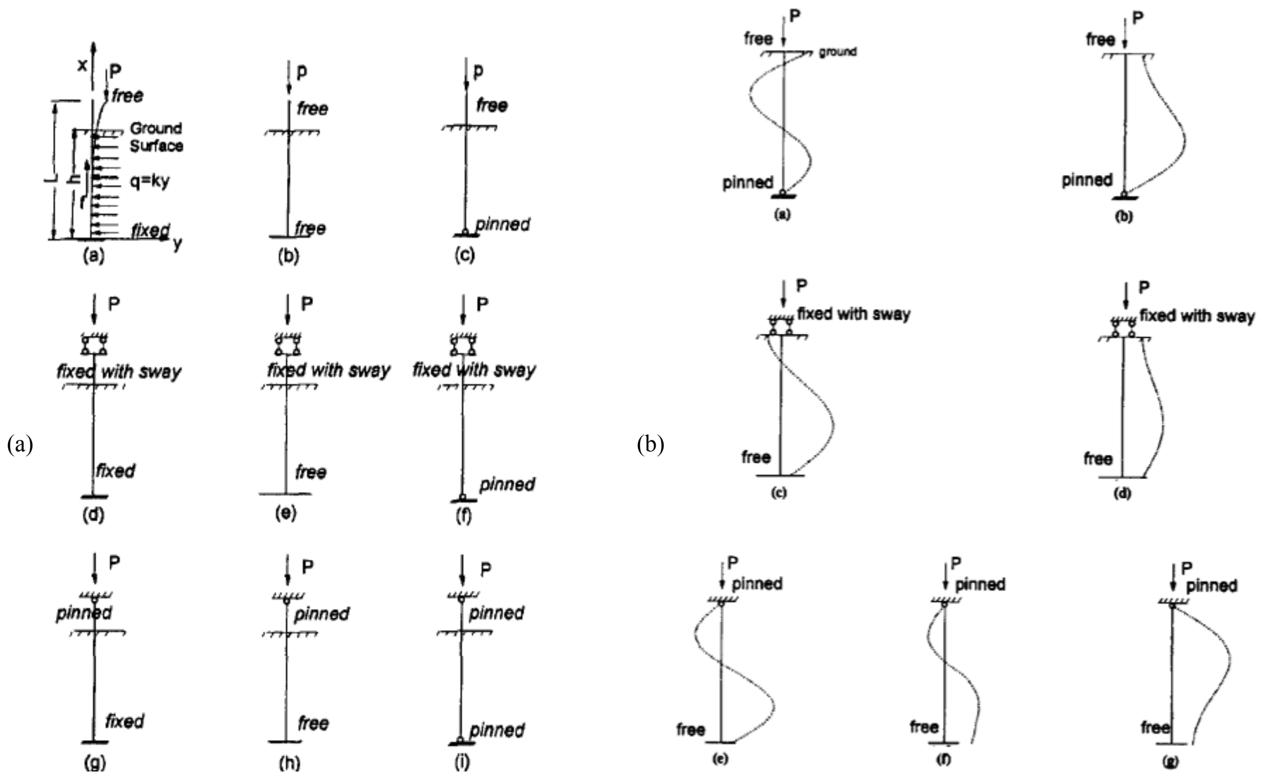


Figure 4. a) Boundary conditions for various situations of pile stability calculations, b) corresponding forms of buckling for specific boundary conditions, /9/.
Slika 4. a) Granični uslovi za različite situacije proračuna stabilnosti šipa; b) odgovarajući oblici izvijanja za specifične granične uslove, /9/

Figure 4b shows the forms of pile buckling for corresponding boundary conditions.

An analysis of bearing capacity and pile stability in soil by applying a linear model and in case of the development of geometric and material non-linearity with the addition of non-linear material behaviour of soil, indirectly presented by springs, is given in paper /3/ (Fig. 5). In this case the geomechanical soil model is shown as three-component bilinear elastic-plastic. The pile is modelled by applying linear finite elements, with plastic hinges placed at the end of elements. In order to enable plastification along the pile, a sufficiently large number of finite elements is used, which directly determined the number of plastic hinges. However, increasing the finite element number also increased the size of the stiffness matrix, leading to a bigger scale numerical analysis. On the other hand, reducing the number of finite elements also reduces the effects of non-linear behaviour.

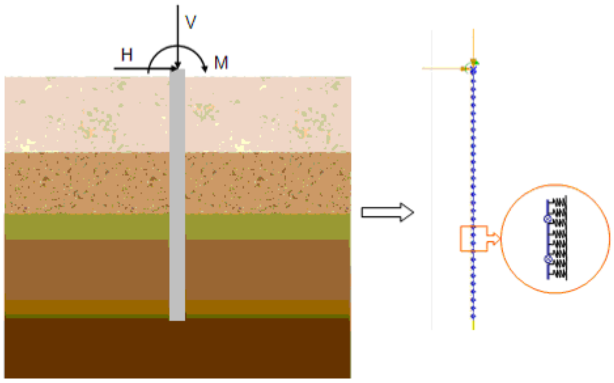


Figure 5. Pile model in realistic conditions and a numerical model made of finite elements, /3/.
Slika 5. Model šipa u realnim uslovima i numerički model sačinjen od konačnih elemenata, /3/

In order to control the incremental growth of load, non-linear pile analysis is used in accordance with the Newton-Raphson procedure. The incremental-iterative method enables an incremental growth of load and an analysis of static effects within the pile and soil reactions. The load is divided into a specific number of increments, wherein parameter **0** corresponds to the unburdened state (no load), an parameter **1** corresponds to 100% load where maximum value of increment numbers is reached. The load parameter is divided into ten parts which are used to keep track of static effects distribution in the pile. Figure 6 shows the horizontal component of pile displacement U_h along with the bending moment M for different load parameters at specific depths z.

In comparison to the linear pile model that uses the beam finite element, a better and more advanced level of modelling includes consideration of soil as a two-dimensional surface system. In this case, the soil is modelled for plane strain state, while the pile can be modelled by using linear or even surface finite elements (Fig. 7), /13/. Stability analysis, and therefore the problem of buckling in numerical models defined this way, takes place in a plane, which leads to forms of buckling that are typical for a planar problem.

The next, higher level in SPI modelling is about three-dimensional modelling, /6, 17/, which is the most complicated and demands much greater hardware resources (Fig. 8). Domain discretization of soil and piles is preformed using the solid prism or tetrahedral finite elements, whereas in case of single piles, a rotationally symmetrical treatment in geometrical modelling is applied.

Modelling of the transitional, i.e. contact (interface) zone between the pile and soil in practical calculations is either eliminated or heavily approximated. On the other hand, this zone can be introduced as a softening zone with corrected geomechanical properties of the soil. Numerical treatment of the interfacial zone can be performed by applying special nodal, linear (link) or solid finite elements. In case of nodal or linear finite elements, connections are established through nodes of solid elements of soil and pile, so that the node compatibility is the key condition in generating of the finite element mesh. When solid finite elements are used for interface zone modelling, the connection is established using solid element nodes for soil and pile (Figs. 9, 10). Applying solid finite elements in interface zone modelling gives more realistic and a better description of given transitional conditions.

In general, it can be concluded that the methodology of analysing and modelling a soil–structure interaction (SSI) includes the following disciplines:
– soil–pile–cap interaction (SPCI),
– soil–foundation–structure interaction (SFSI).

In the case of soil–pile–cap interaction considered are the effects of a separate above-ground structure model, while in the case of a soil–foundation–structure interaction, complete effects and phenomena that appear for the given spatial models are considered.

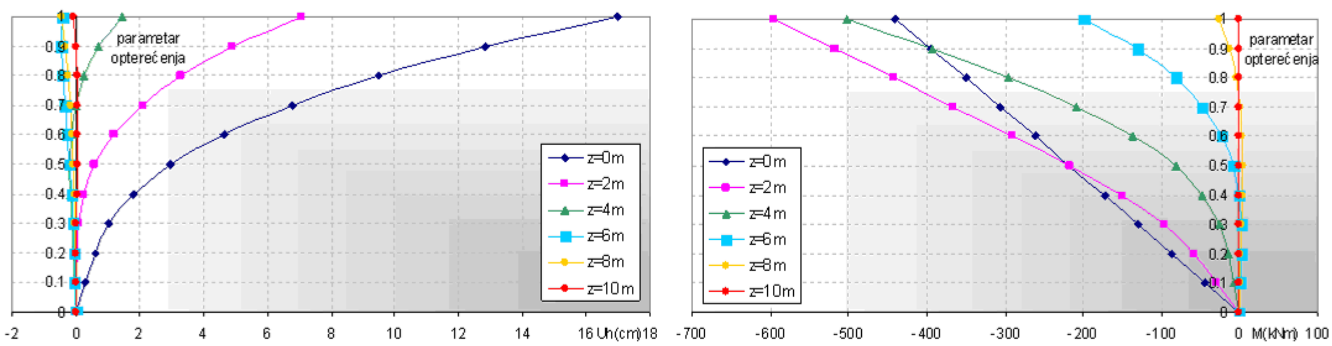


Figure 6. Horizontal displacements U_h and bending moments of pile on specific depths z, depending on load parameters, /3/.
Slika 6. Horizontalna pomeranja U_h i momenti savijanja šipa sa pojedinim dubinama z, u zavisnosti od parametara opterećenja, /3/

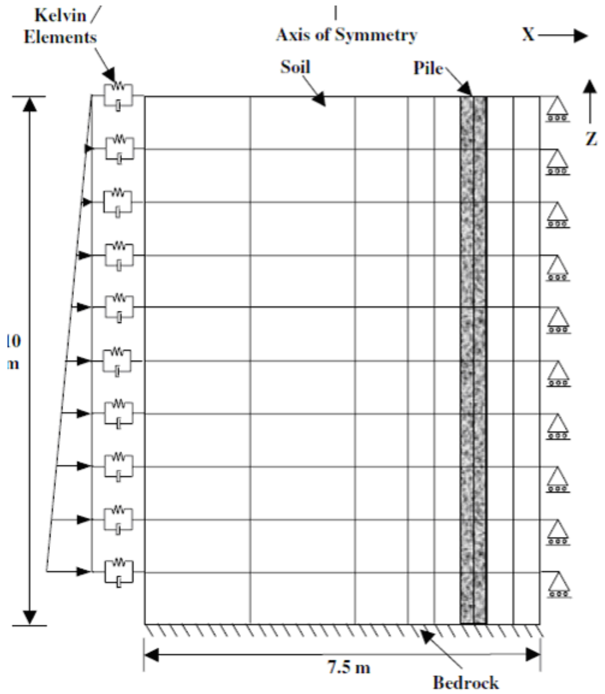


Figure 7. Two-dimensional surface model of soil and pile, /13/.
Slika 7. Dvodimenzionalni površinski model tla i šipa, /13/

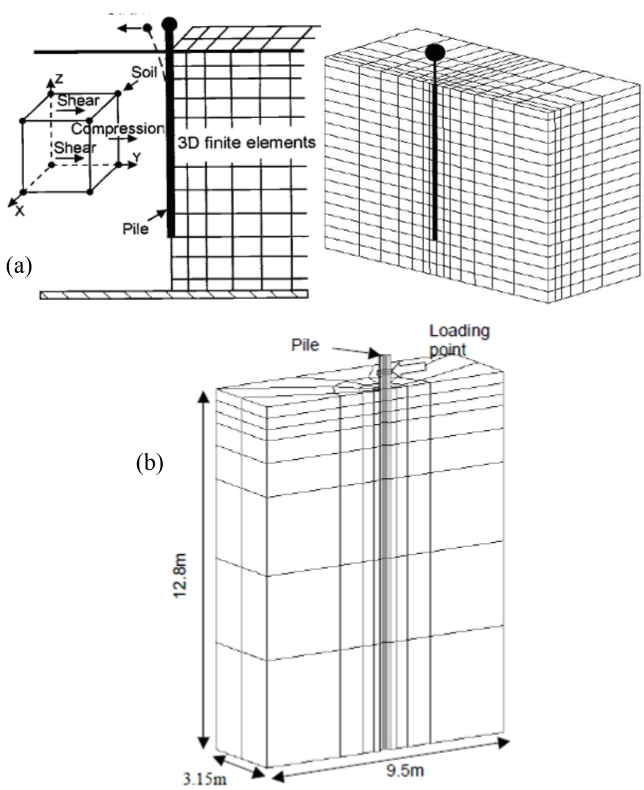


Figure 8. 3D spatial model of soil and piles: a) /6/, b) /17/.
Slika 8. 3D prostorni model tla i šipova, a) /6/, b) /17/

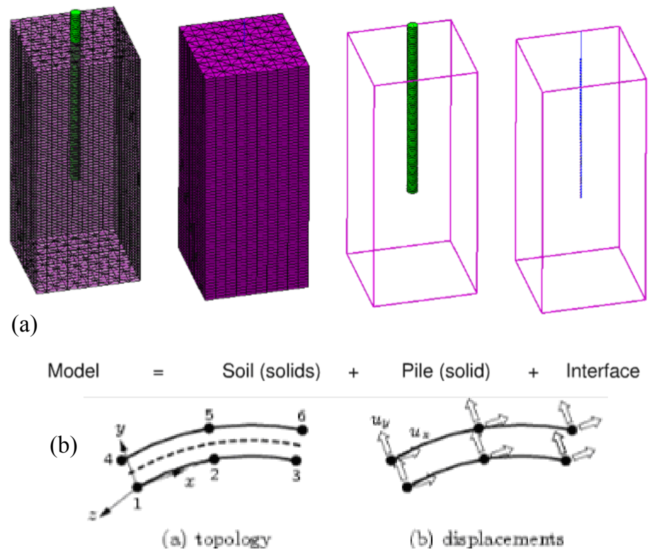


Figure 9. 3D spatial model of soil and piles with a transitional (interface): a) complete solid model, b) linear interface element for modelling a frictional interaction between soil and pile, /16/.
Slika 9. 3D prostorni model tla i šipova sa prelazom (interfejs), a) potpuni solid model, b) linearni interfejs element za modeliranje interakcije trenja između tla i šipa, /16/

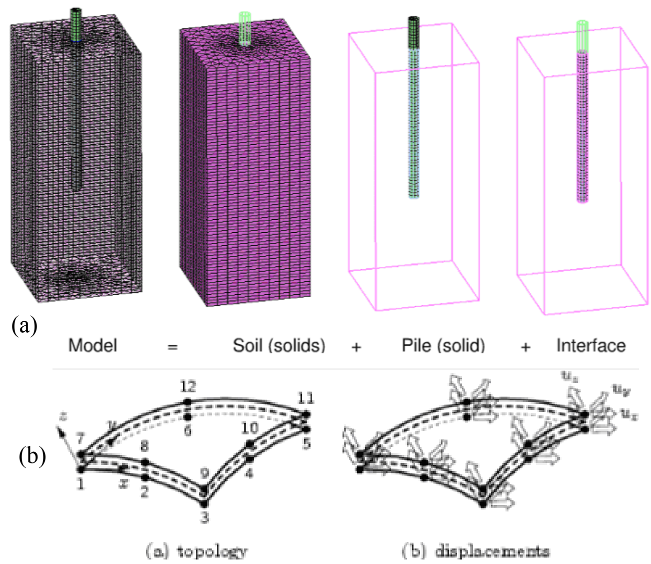


Figure 10. 3D spatial model of soil and piles with a transitional (interface): a) complete solid model, b) triangular interface element for modelling a frictional interaction between soil and pile, /16/.
Slika 10. 3D prostorni model tla i šipova sa prelazom (interfejs), a) potpuni solid model, b) trouglasti interfejs element za modeliranje interakcije trenja između tla i šipa, /16/

STABILITY ANALYSIS OF SLENDER PILES USING FINITE DIFFERENCE METHOD

The problem of modelling and analysing pile stability is previously presented, while mentioning that the application of FEM (Finite element Method) is favoured as of lately. However, prior to developing FEM, the FDM (Finite Difference Method) is used, and has proven to be very effective, giving results reliable enough for practical purposes. The solution to the stability problem presented here is based on FDM, wherein the soil is modelled in a far more realistic way, compared to the Winkler's soil model. Specifically, the soil is modelled as a homogenous, elastic, isotropic semi-space (HEIS), that is located on a solid rocky base. The pile is vertical, linear-elastic, with length of L, diameter d and whose base lies on a rocky base, whereas it is discretized with $n + 1$ elements. The connection between the soil and the pile is ideal, so the bearing capacity to sliding is not exceeded along the soil-pile contact surface.

Lateral soil displacement along the pile length can be expressed as (Fig. 11), /14/:

$$\{{}_s\rho\} = \frac{d}{E_s}[{}_sI]\{p\} \tag{6}$$

where $\{{}_s\rho\}$ is the soil displacement vector, $[{}_sI]$ is an $(n + 1) \times (n + 1)$ matrix of the soil displacement influence factor, $\{p\}$ is the soil pressure vector, and E_s is the soil elasticity modulus.

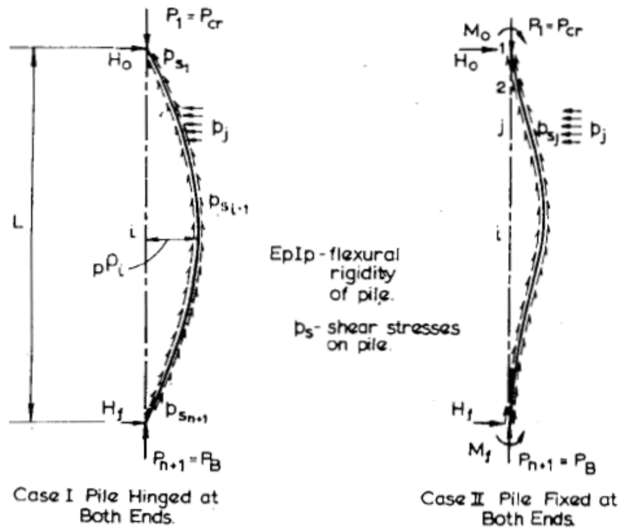


Figure 11. Pile model, boundary conditions and loads used for FDM calculations, /14/.

Slika 11. Model šipa, granični uslovi i opterećenja za FDM proračune, /14/

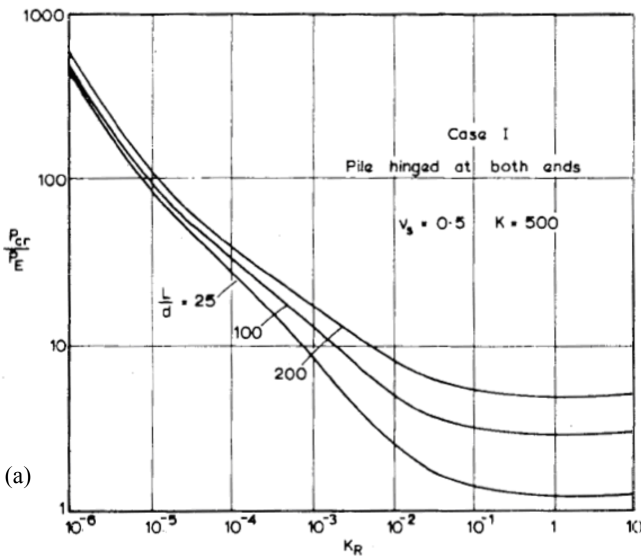
Critical load factor, i.e. critical buckling force of piles, P_{cr} is determined according to:

$$\left[-[\xi]^{-1}[A]+\left(P_{cr}/E_sL^2\right)[1]\right]\{\rho\}=0 \tag{7}$$

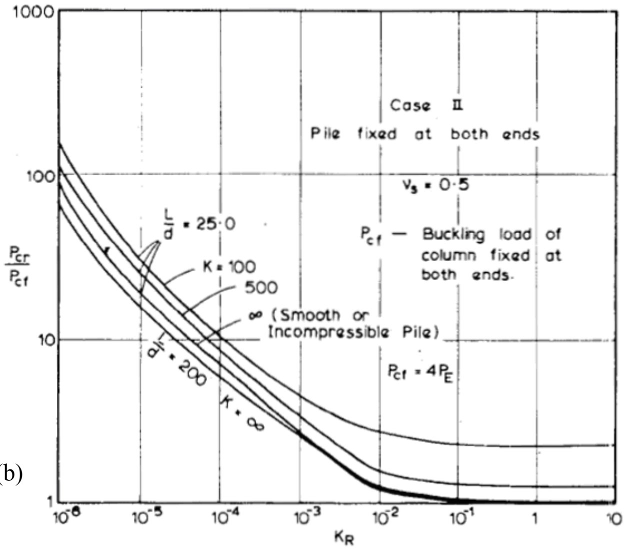
Figure 12 represents the change in normalized value of critical force P_{cr}/P_E as a function of stiffness factor K_R, in cases where hinge supports are placed on pile ends (Figure 12a), and where fixed supports are placed as well (Figure 12b).

STABILITY ANALYSIS OF SLENDER PILES USING FINITE ELEMENT METHOD

Modelling of slender pile-headset panel-soil interactions, required for this study's research, is performed using 3D finite elements, whereas complete analysis is performed using FEM in SAP 2000 software, /15/. During the modelling of complex slender pile–headset panel–soil systems, several phases are taken into consideration separately: pre-processing, processing and post processing.



(a)



(b)

Figure 12. Change of normalized value of critical force P_{cr}/P_E as a function of stiffness factor K_R: a) hinge supports placed on pile ends, b) fixed supports placed on pile ends, /14/.

Slika 12. Promena normalizovanih vrednosti kritičnog opterećenja P_{cr}/P_E u funkciji faktora krutosti K_R: a) zglobni oslonci na krajevima šipa, b) nepokretni oslonci na krajevima šipa, /14/

Basic aspects of the pre-processing phase are the approximation and discretization. In the process of approximation, the considered slender pile–headset panel–soil domain is modelled by selecting a finite element type, whereas in this case 3D finite elements are used. The discretization aspect refers to the forming of a finite element mesh, i.e. the selection of mesh quality and finite element quantity, which directly affects the accuracy of the obtained results. On the other hand, the amount of finite elements generated in the pre-processing phase directly affects the time it takes for the calculation to be completed in the processing phase, as well as the time needed to interpret the obtained results in the post processing phase. Important properties for a numerical slender pile–headset panel–soil model include:

– highly accurate geometrical models (ideally corresponding to the realistic physical model),

– application of 3D finite elements in modelling of slender pile–headset panel–soil domain, including the use of link elements for contact (interface) zone,
– approximation by using the following finite elements: 3D solid finite elements with 8 nodes and 24 degrees of freedom,
– discretization using finite elements: free-field zone, transitional zone and increased density zone,
– soil model: single-layered, homogenous, elastic, isotropic semi-space (HEIS) lying on a solid base,
– pile boundary conditions: headset–panel, contact with soil along the pile mantle with a connection through the base,
– soil model boundary conditions: base and lateral sides (mantle).

Spatial three-dimensional solid finite elements possess three dominant dimensions. These finite elements are also mathematically three-dimensional because the appropriate considerations are related to a three-axial coordinate system. Finite element mesh is generated in an enclosed area that defines the slender pile–headset panel–soil domain by applying hexahedral solid finite elements, with 8 nodes used for defining displacements located at the angles. Solid finite elements use a 2 × 2 × 2 numerical integration via Gaussian quadrature, /5/. Modelling of soil–pile contacts is performed using link elements, i.e. by applying contact elements (gap element) for which special stiffness properties for pressure are defined, whereas tensile stresses are eliminated. The contact element is used for modelling the contact between two points in a model, and is characterised by two states: active (contact is achieved, very large stiffness) and non-active (no contact, very small stiffness), /11/. Figure 13 shows a force–displacement diagram of a contact element with active pressure. Applying contact elements in modelling of a transitional soil–pile zone requires the application of a geometrically nonlinear incremental-iterative analysis, /3/. Due to nonlinear behaviour of contact elements, where change of states is followed by a significant change in stiffness, serious difficulties in maintaining nonlinear solution convergence can occur.

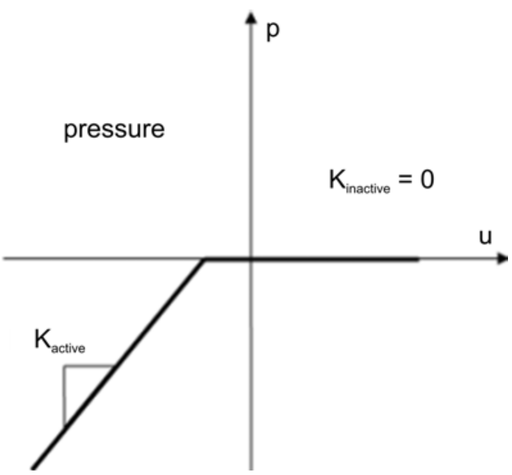


Figure 13. Force–displacement diagram of a pressure active contact element, /11/.
Slika 13. Dijagram sila–pomeranje za aktivni pritisni kontaktni element, /11/

Based on the previously defined types of finite and link elements, a numerical model is generated and analysed. However, prior to initializing the numerical analysis and processing phase, defining of all possible nonlinear analysis which include link elements is performed (Fig. 14). The pile domain is denoted as *P*, the soil domain as *S*, and the element domain as *L*. Development of geometrical nonlinearity is assumed for every analysis, whereas the material nonlinearity development is denoted as *mn*, and material linearity development as *ml*.

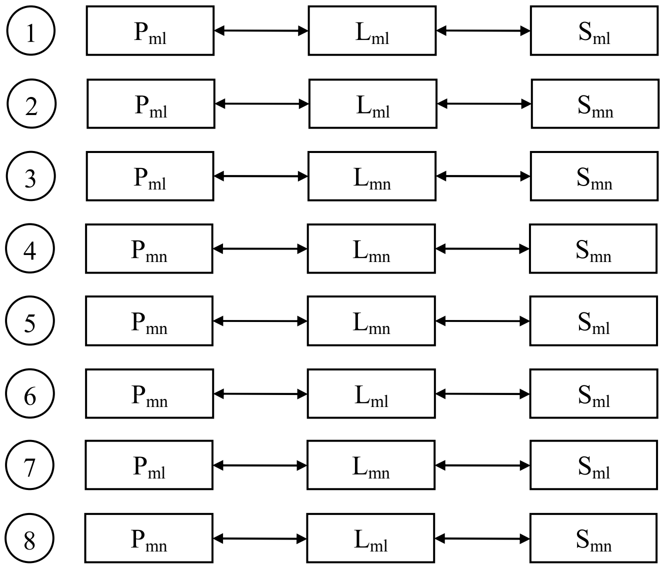


Figure 14. Types of nonlinear analysis involving link elements.
Slika 14. Tipovi nelinearne analize sa veznim elementima

In case of both geometrical and material nonlinearity (type 4: $P_{mn} + L_{mn} + S_{mn}$), the analysis is entirely nonlinear, whereas in every other case, the analysis is partially nonlinear. For the purpose of this research and stability analysis of slender pile–headset panel–soil assessment, a special procedure is applied. First, a geometrical nonlinear analysis including development of nonlinear strain in contact link elements (type 7: $P_{mn} + L_{mn} + S_{mn}$) for the effects of axial load on the pile. The final stiffness matrix that is obtained by this analysis is used as an initial stiffness matrix for the purpose of analysing the buckling of a pile–headset panel–soil. In a specific case, the load from the first analysis is not transferred into the second, instead a new load is applied. Since the modelling of soil–pile connection is performed using link elements, the Newton-Raphson incremental-iterative concept is used.

According to FEM, a nonlinear problem is formulated by a system of nonlinear equations, /2/. Consistent application of the incremental concept in nonlinear analysis can be described as linearization in increments, and the solution can be described as the sum of incremental linear solutions. Equations are solved for a series of separate incremental loads, as opposed to a total load. Within every increment, it is assumed that the equation system is linear. This results in a solution that can be obtained as a sum of series of linear (incremental) solutions. A nonlinear problem can be represented as:

$$[K_t]\{\Delta u\}+\lambda\{F\}=0 \tag{8}$$

i.e.:

$$\{P\}+\lambda\{F\}=0 \tag{9}$$

where $\{u\}$ are the unknown displacement parameters, $\{F\}$ are the generalized external loads in system nodes, $\{P\}$ is the internal generalized force vector of the model which is a function of generalized displacements $\{u\}$, λ is the incremental load parameter.

Differentiating Eq.(8) by variable λ results in:

$$\frac{d\{P\}}{d\{u\}}\frac{d\{u\}}{d\lambda}+\{F\}=[K_t]\frac{d\{u\}}{d\lambda}+\{F\}=0\,,$$

$$\frac{d\{u\}}{d\lambda}=-[K_t]^{-1}\{F\} \tag{10}$$

wherein the tangent stiffness matrix of the model is:

$$[K_t]=\frac{d\{P\}}{d\{u\}}\,. \tag{11}$$

The residual load vector can be expressed as equilibrium deviation:

$$\{\Delta R\}_i=\{\Delta F\}_i-[K_t]_{i+1}\{\Delta u\}_i\,. \tag{12}$$

Error correction is achieved by adding residual load to the external load in the next increment:

$$\{\Delta F\}_{i+1}^{R}=\{\Delta F\}_{i+1}+\{\Delta R\}_i \tag{13}$$

In a strictly incremental procedure, residual load is added to the external load in the next increment, which reduces, but does not eliminate, the error. Best results are obtained by combining the incremental and the iterative procedure.

The second phase of the calculation includes the stability analysis of pile–headset panel–soil with a stiffness matrix from the previous one, wherein the level of critical load is determined according to /4/:

$$\left|\left[K_{e,corr}\right]+\lambda\left[K_{g,corr}\right]\right|=0 \tag{14}$$

where $[K_{e,corr}]$ is the corrected elastic stiffness matrix of the system. According to the previously described procedure, and as a result of geometrical nonlinearity development of the system, along with link element nonlinearity, corrected stiffness matrices are obtained ($[K_{e,corr}]$ and $[K_{g,corr}]$). By applying an iterative procedure, the critical load factor λ can be determined (the number of lowest positive and real values of critical load factor). The maximum number of buckling forms of a given system is equal to the number of degrees of freedom. In engineering terms, the lowest value of critical force is most important.

Figure 15a shows the vertical cross-section of a pile–headset panel–soil 3D model made of solid finite elements, while Fig. 15b shows a detail of a soil–pile connection and a transitional finite elements zone. A complete pile–headset panel–soil 3D model is generated using 7250 solid finite elements and 980 contact link elements (Fig. 16). The maximal solid finite element length is 1 m, whereas the pile domain and the zone surrounding it used a more dense mesh, resulting in a maximal element length of 25 cm.

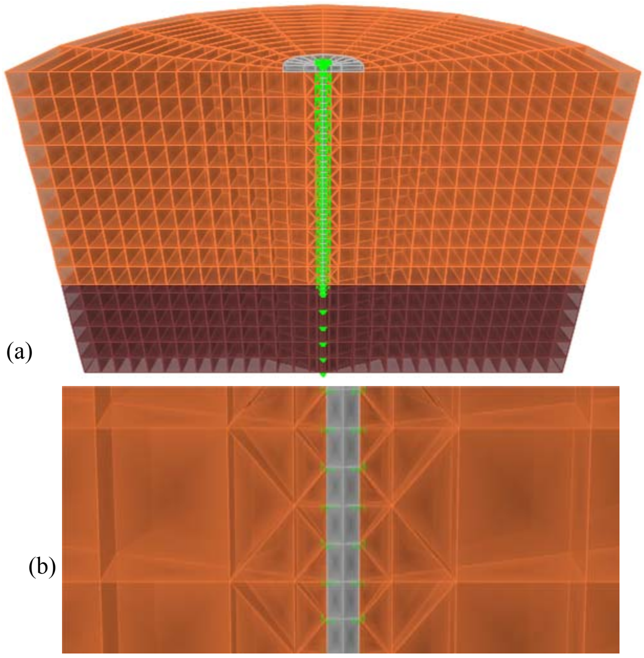


Figure 15. Pile–headset panel–soil model made of solid finite elements: a) vertical cross-section, b) a soil–pile connection detail including the finite element transition zone.
Slika 15. Model šip–ispuna–tlo od solid konačnih elemenata: a) vertikalni poprečni presek, b) detalj veze tlo–šip sa prelaznom zonom konačnih elemenata

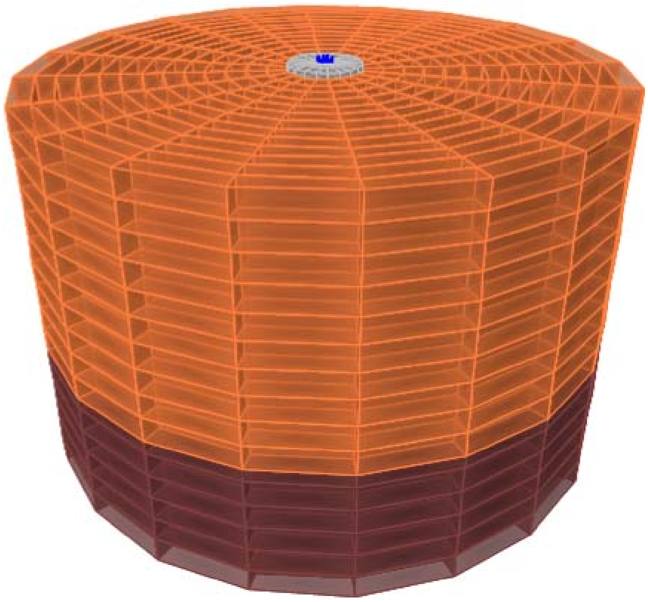


Figure 16. A complete pile–headset panel–soil 3D model made of solid finite elements.
Slika 16. Kompletni 3D model šip–ispuna–tlo od solid konačnih elemenata

NUMERICAL AND REGRESSION ANALYSIS

Numerical analysis is preformed on a generated 3D pile–headset panel–soil model made of solid finite elements for two types of soil. The first model is a two-layered system in which the bottom layer simulates a rigid base, while the second model is single-layered. In case of the two-layered model, the top layer parameters are varied. Parameter analysis is performed for five different L/d ratios (where L is pile length, and d is its diameter):

$$L/d = \{25, 50, 75, 100, 150\} . \tag{15}$$

The length parameter is kept constant throughout all analysis, hence the number of finite elements generated for all of the models remains the same, with the number of equilibrium equations for the entire system of 22226. The diameter of the pile is determined based on the ratio between the length and the value from Eq.(15). In addition, the elasticity modulus value E_s as a function of stiffness K_R, is varied, /14/:

$$K_R = \frac{E_p I_p}{E_s L^4} \tag{16}$$

where:

$$K_R = \{10^{-6}, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, 1, 10\} . \tag{17}$$

The total amount of numerical analysis performed is 80. A representative critical buckling force is determined from the first buckling form, since it gave the lowest critical load factors.

Figure 17a shows a typical first form of buckling for $L/d = 50$ and $K_R = 10^{-6}$ of a two-layered system. The shape of buckled piles is formed from a greater number of semi-waves, instead of the usual sine semi-wave. This is the result of a very low K_R in this specific case, which leads to only 2/3 of the pile being affected by bucking. Due to this, the pile transfers the load with both its base and mantle. Such principle of load transfer applies to both single and two-layered systems (Figs. 17a and b).

(a)

(b)

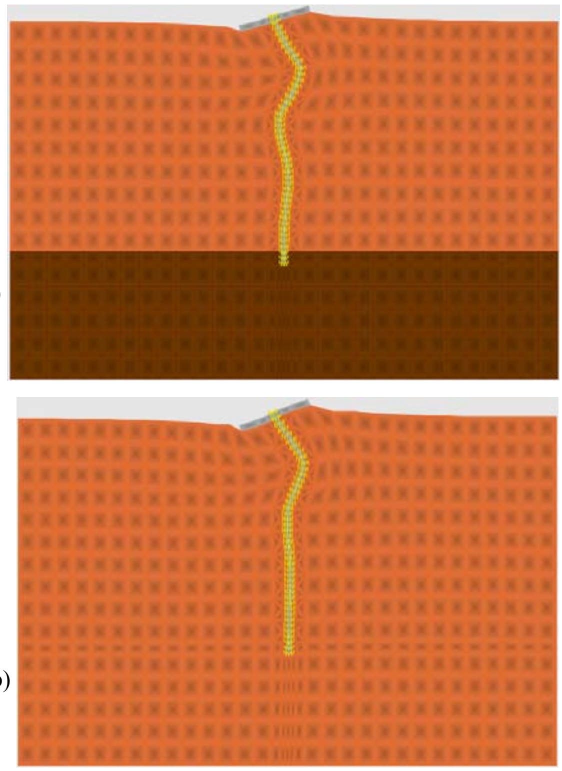


Figure 17. First form of buckling for $L/d = 50$ and $K_R = 10^{-6}$: a) two-layered system, b) single-layered system.
Slika 17. Prvi oblik izvijanja za $L/d = 50$ i $K_R = 10^{-6}$: a) dvoslojni sistem, b) jednoslojni sistem

Figure 18a shows the typical first form of buckling for $L/d = 50$ and $K_R = 10^{-4}$ of a two-layered system, while Figure 18b shows the same for a single-layered system. Compared to the previous situation, a form in the shape of a sine semi-wave has developed, hence due to lower value of K_R, the load is transferred more by the base, and less by the mantle.

(a)

(b)

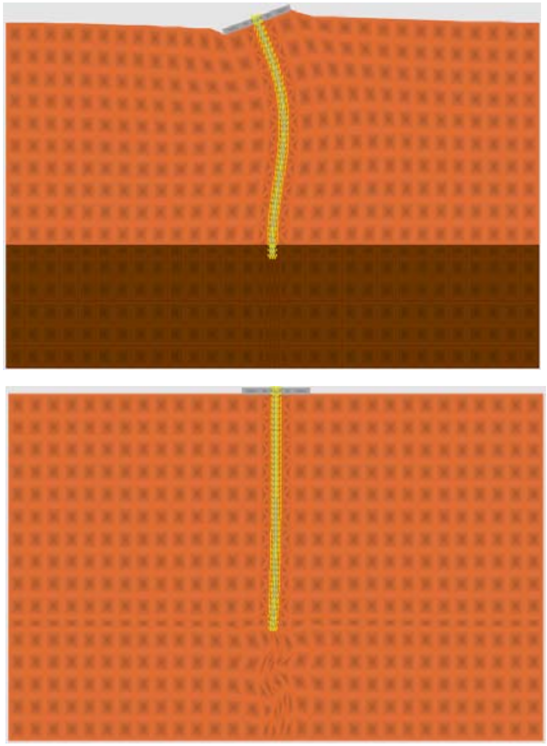


Figure 18. First form of buckling for $L/d = 50$ and $K_R = 10^{-4}$: a) two-layered system, b) single-layered system.
Slika 18. Prvi oblik izvijanja za $L/d = 50$ i $K_R = 10^{-4}$: a) dvoslojni sistem, b) jednoslojni sistem

In order to determine the expression for the level of critical buckling force in the 3D pile–headset panel–soil model, regression analyses are performed. The total amount of analyses performed is 15. During preliminary research, a large number of various functions is considered, such as:

– the exponential function:

$$P_{cr} / P_E = a e^{b K_R} \tag{18}$$

– a linear function:

$$P_{cr} / P_E = a + b K_R \tag{19}$$

– the logarithmic function:

$$P_{cr} / P_E = a + b \ln K_R \tag{20}$$

– a degree function:

$$P_{cr} / P_E = a K_R^b \tag{21}$$

where a, b are coefficients determined by regression analysis, P_{cr}/P_E is the normalized value of critical buckling force, P_{cr} is the critical buckling force, P_E is the Euler's critical buckling force:

$$P_E = \frac{\pi^2 E_p I_p}{L^2} \tag{22}$$

The optimal type of a regression function is determined by taking into account the coefficient of correlation r^2. Higher values of this coefficient points toward a better fitting of numerically determined values according to the FEM and the values of regression analysis. Table 1 shows a typical example of regression analysis application in determining of the normalized value of critical force P_{cr}/P_E as a function of stiffness K_R for $L/d = 25$ in a two-layered system.

Highest values of correlation coefficient r^2 are obtained for degree functions, that are used in further research. Expressions for normalized values of critical buckling force P_{cr}/P_E, for degree functions of two-layered and single-layered systems, derived from regression analysis are given in Table 2.

Table 1. Applied regression analysis in determining a normalized value of P_{cr}/P_E as a function of K_R for $L/d = 25$ in a two-layered system.
Tabela 1. Primena regresione analize u određivanju normalizovanih vrednosti P_{cr}/P_E u funkciji K_R za $L/d = 25$ kod dvoslojnog sistema

	exponential	linear	logarithmic	degree
a	74.77	169.23	–443.46	0.007
b	–3881.26	–182401.29	–54.24	–0.786
r^2	0.644	0.216	0.723	0.998

Table 2. Values of degree function coefficients for two- and single-layered soil used for determining the critical force.
Tabela 2. Vrednosti koeficijenata stepene funkcije za dvo- i jednoslojno tlo korišćeni za određivanje kritične sile

pile slenderness	two-layered system	single-layered system
$L/d = 25$	$P_{cr}/P_E = 0.007 \cdot K_R^{-0.786}$, $r^2 = 0.998$	$P_{cr}/P_E = 0.0004 \cdot K_R^{-0.996}$, $r^2 = 0.999$
$L/d = 50$	$P_{cr}/P_E = 0.006 \cdot K_R^{-0.759}$, $r^2 = 0.995$	$P_{cr}/P_E = 0.0003 \cdot K_R^{-0.984}$, $r^2 = 0.999$
$L/d = 75$	$P_{cr}/P_E = 0.005 \cdot K_R^{-0.742}$, $r^2 = 0.994$	$P_{cr}/P_E = 0.0003 \cdot K_R^{-0.968}$, $r^2 = 0.999$
$L/d = 100$	$P_{cr}/P_E = 0.005 \cdot K_R^{-0.724}$, $r^2 = 0.994$	$P_{cr}/P_E = 0.0002 \cdot K_R^{-0.958}$, $r^2 = 0.998$
$L/d = 150$	$P_{cr}/P_E = 0.005 \cdot K_R^{-0.707}$, $r^2 = 0.995$	$P_{cr}/P_E = 0.0002 \cdot K_R^{-0.952}$, $r^2 = 0.998$

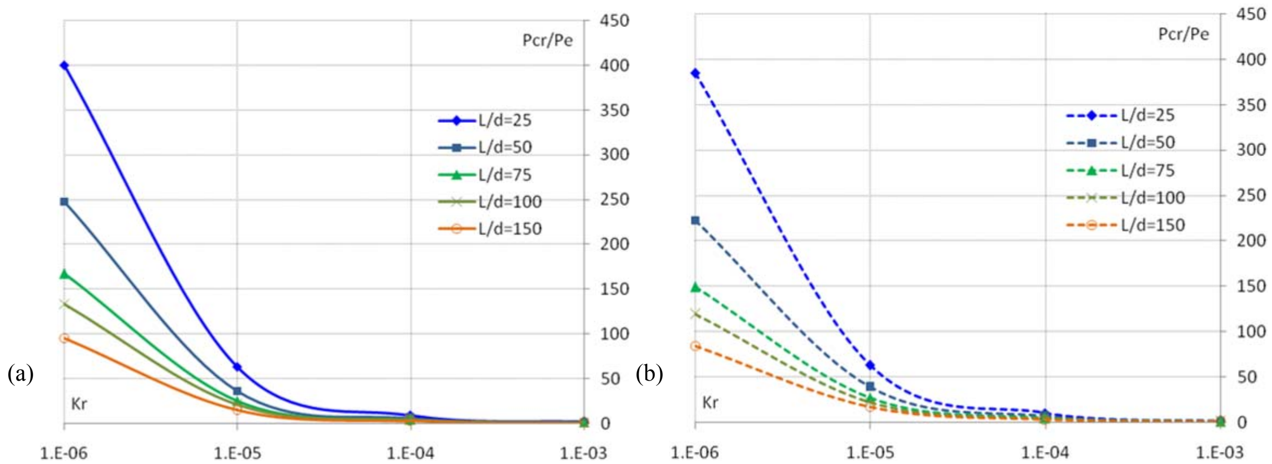


Figure 19. Normalized values P_{cr}/P_E as a function of stiffness K_R for a two-layered system: a) according to FEM, b) by regression analysis.
Slika 19. Normalizovane vrednosti P_{cr}/P_E u funkciji krutosti K_R za dvoslojni sistem: a) prema FEM, b) regresionom analizom

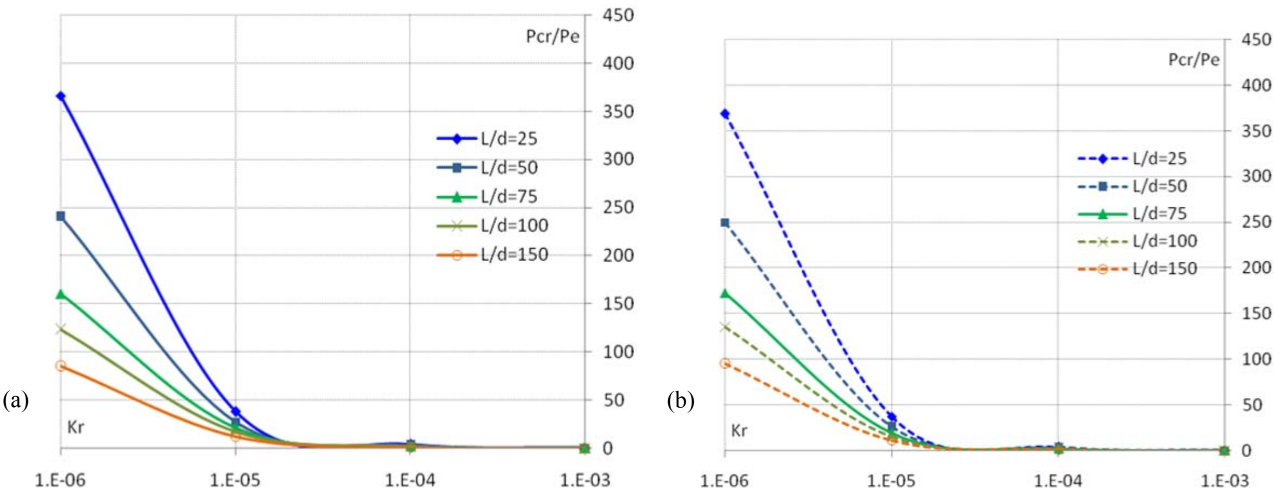


Figure 20. Normalized values of P_{cr}/P_E as a function of stiffness K_R for a single-layered system: a) FEM, b) regression analysis.
Slika 20. Normalizovane vrednosti P_{cr}/P_E u funkciji krutosti K_R za jednoslojni sistem: a) FEM, b) regresiona analiza

Figure 19a shows the changes to the normalized values of P_{cr}/P_E as a function of stiffness K_R for a two-layered system according to the finite element method, and Fig. 19b shows these values determined by using regression analysis for degree functions. Figure 20a shows the changes to the normalized values of P_{cr}/P_E as a function of stiffness K_R for a single-layered system according to the finite element method, and Fig. 19b shows these values determined by using regression analysis for degree functions. Values on the x-axis are given in the logarithmic scale. There is an obvious increase in normalized critical force P_{cr}/P_E with reduced stiffness K_R. This is caused by the fact that the elasticity modulus of soil, E_s is inversely proportional to K_R. Also evident are the obtained higher levels of normalized values of critical forces in a two-layered system in comparison to a single-layered. By applying a degree function in regression analysis it is possible to exceptionally describe the dependence of P_{cr}/P_E from K_R, since the provided correlation between the two is at a very high level.

The final phase of research has reviewed pile buckling length L_i as a function of normalized value of P_{cr}/P_E, which are determined by regression analysis. An analogy with the buckling length is established, hence the expressions derived can be used for practical purposes. The expression for Euler's critical buckling force, Eq.(22), is given as a function of a rod's total length L, wherein the buckling length coefficient is $\beta = 1$. The expression (22), therefore, can be fully represented as a function of buckling length L_i:

$$P_E = \frac{\pi^2 E_p I_p}{L_i^2} \tag{23}$$

where:

$$L_i = \beta L \, . \tag{24}$$

By equalizing expressions (23) and the one used for obtaining the level of critical force by means of regression analysis with degree functions:

$$\frac{\pi^2 E_p I_p}{\beta^2 L^2} = \frac{\pi^2 E_p I_p}{L^2} a K_R^b \, , \tag{25}$$

we can derive the expression for buckling length coefficient β as a function of K_R:

$$\beta = (a K_R^b)^{-0.5} \, . \tag{26}$$

Expressions for buckling length coefficient β derived from regression analysis, for degree functions, are given in Table 3.

Table 3. Values of buckling length coefficient β for single and two-layered systems.
Tabela 3. Vrednosti koeficijenta dužine izvijanja β za jedno- i dvoslojne sisteme

pile slenderness	two-layered system	single-layered system
$L/d = 25$	$\beta = 11.95 \cdot K_R^{0.393}$	$\beta = 50.00 \cdot K_R^{0.498}$
$L/d = 50$	$\beta = 12.91 \cdot K_R^{0.379}$	$\beta = 57.74 \cdot K_R^{0.492}$
$L/d = 75$	$\beta = 14.14 \cdot K_R^{0.371}$	$\beta = 57.74 \cdot K_R^{0.484}$
$L/d = 100$	$\beta = 14.14 \cdot K_R^{0.362}$	$\beta = 70.71 \cdot K_R^{0.479}$
$L/d = 150$	$\beta = 14.14 \cdot K_R^{0.354}$	$\beta = 70.71 \cdot K_R^{0.476}$

Figure 21a shows the changes in buckling length coefficient β as a function of K_R for a two-layered system according to FEM, and Fig. 21b shows these values as determined by regression analysis for degree functions. Figure 22a shows the changes in buckling length coefficient β as a function of K_R for a singe-layered system according to FEM, and Fig. 22b shows these values as determined by regression analysis for degree functions. Values on x-axis are given in logarithmic scale. Two-layered systems gave lower results for buckling length coefficient β in comparison to single-layered ones. This is due to a direct correlation between buckling length coefficient β and the normalized critical force P_{cr}/P_E.

Expressions derived for the normalized value of critical buckling force P_{cr}/P_E (Table 2), as well as for buckling length coefficient β (Table 3) can be directly applied in practice, in order to analyse the stability of slender piles.

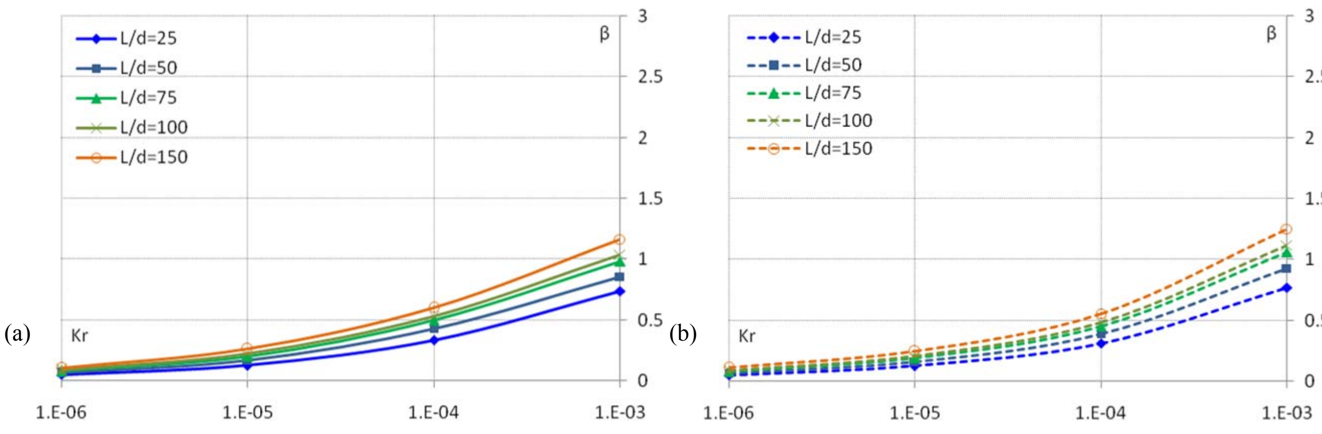


Figure 21. Changes in buckling length coefficient β as a function of K_R for a two-layered system:
a) according to FEM, b) determined by regression analysis.
Slika 21. Promene koeficijenta dužine izvijanja β u funkciji krutosti K_R za dvoslojni sistem:
a) prema FEM, b) regresionom analizom

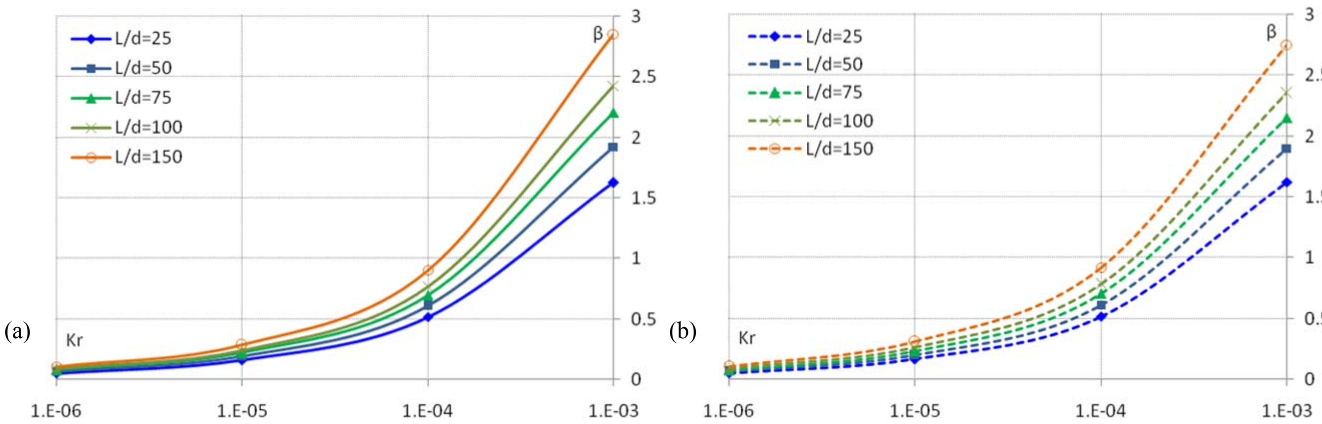


Figure 22. Changes in buckling length coefficient β as a function of K_R for a single-layered system: a) according to FEM, b) determined by regression analysis.
Slika 22. Promene koeficijenta dužine izvijanja β u funkciji krutosti K_R za jednoslojni sistem: a) prema FEM, b) regresionom analizom

CONCLUSION

Existing methods and numerical models presented in the scientific research, rarely mentioned in foreign publications, treat the issue of slender pile stability by using mathematical model with certain simplifications. This paper considers the complex issue of stability analysis of a slender pile by using the finite element method. A concept is formulated for the purpose of a modified stability analysis of a 3D pile–headset panel–soil model made of solid finite elements for two types of soil: single-layered and two-layered.

By applying extensive numerical and regression analysis, expressions for normalized values of critical force P_{cr}/P_E (Table 2), are obtained along with buckling length coefficients (Table 3) as a function of stiffness. These expressions can be directly applied for practical purposes, in order to analyse slender pile stability.

Research has shown that the application of sophisticated mathematical models and numerical analysis is justified and necessary for the purpose of high quality analysis of slender pile stability. Assuming that pile buckling can be considered as a separate rod with added soil-pile interaction improvements may lead to conservative solutions.

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