

FLUID BOUNDARIES SHAPING USING THE METHOD OF KINETIC BALANCE

by

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Fluid flow in curved channels with various cross-sections, as a common problem in theoretical and applied fluid mechanics, is a very complex and quite undiscovered phenomenon. Defining the optimum shape of the fluid flow boundaries, which would ensure minimum undesirable phenomena, like "dead water" zones, unsteady fluid flow, etc., is one of the crucial hydraulic engineering's task. Method of kinetic balance is described and used for this purpose, what is illustrated with few examples.

Key words: *kinetic balance method, fluid flow, optimal geometry*

Introduction

Determination of the optimal hydraulic boundary which results in stable fluid flow without separation and transient phenomena is often solved with the help of numerous experiments, sometimes followed by unsuccessful trials. Here is presented the method of kinetic balance, which is the basis of the computational solving procedure for this problem.

The inner fluid flow current is at hydraulic equilibrium, containing "sound flow", boundary layer, and maybe "dead water", zones. The latter ones are zones of slowly moving or fluid at rest, separated from the "sound flow" by the discontinuity surface. They occur as the result of fluid separation, which happens, either due to the boundary layer thickening or significant fluid inertia. Boundary layer suction is the solution for the first reason, used in for example aerodynamics, but the latter one cannot be avoided in such a manner. Strscheletzky [1] named it "inertial separation".

In the rotating fluid, that will occur in the problems discussed in this paper, close to the rotation axis, is formed swirl core, which can be treated as "dead water". It is surrounded by the "sound flow" region, where the flow can be assumed homogeneous, fluid ideal, and incompressible, which is neighbored by the boundary layer.

Fluid boundaries shaping problem, which will ensure stable fluid flow with the lack or minimum presence of the undesirable phenomena, was studied by, fore mentioned, Strscheletzky [1, 2], who has developed the method of kinetic balance based on

the Euler flow equation ideal, incompressible fluid. This is the theoretical approach to the problem of the optimal flow field boundary shapes.

Presented method is confirmed in many problems, of which some are introduced later on.

Theoretical background

Navier-Stokes equation for incompressible fluid states [3]:

$$\frac{D\vec{c}}{Dt} = \vec{F} - \frac{1}{\rho} \text{grad} p - \nu \Delta \vec{c} \quad (1)$$

where \vec{c} and p are local velocity and pressure, respectively, \vec{F} – volume forces, ρ – density, and ν – kinematic viscosity.

Introducing assumption for the volume forces to be conservative, *i. e.* $\vec{F} = \text{grad} U$, momentum eq. (1) for elementary fluid mass ($dm_i = \rho dV_i$), is transformed to:

$$\rho \frac{D\vec{c}}{Dt} dV_i - \rho \text{grad} U dV_i - \text{grad} p dV_i - \eta \vec{c} dV_i = 0 \quad (2)$$

The whole flow domain contains n elementary volumes. The “sound flow” and “dead water” zones are separated with the surfaces of zero, second or higher order of discontinuity. Virtual work of forces acting on the fluid in the volume V_i at the moment t , for the virtual displacement $\delta \vec{r}$ is:

$$\rho \frac{D\vec{c}}{Dt} \delta \vec{r} dV_i - \rho \text{grad} U \delta \vec{r} dV_i - \text{grad} p \delta \vec{r} dV_i - \eta \vec{c} \delta \vec{r} dV_i = 0 \quad (3)$$

Introducing assumption for ideal (non-viscous), incompressible fluid, for the whole flow domain V , we get:

$$\sum_{i=1}^n \rho \frac{D\vec{c}}{Dt} \delta \vec{r} dV_i - \sum_{i=1}^n \text{grad} U \delta \vec{r} dV_i - \sum_{i=1}^n \text{grad} p \delta \vec{r} dV_i = 0 \quad (4)$$

Since the integrals are additive and $V = \sum_{i=1}^n V_i$, it follows from the last equation:

$$\int_V \rho \frac{D\vec{c}}{Dt} \delta \vec{r} dV - \int_V \text{grad} U \delta \vec{r} dV - \int_V \text{grad} p \delta \vec{r} dV = 0 \quad (5)$$

The last eq. (5), states the Lagrange’s principle of virtual work: Flow equilibrium in the volume V at the moment t , is achieved when the sum of virtual works of the forces, acting on the fluid, equals zero.

Introducing the kinetic $dE_k = c^2\rho/2 dV_i$ and potential $dE_p = Udm + pdV$ energy, it follows from eq. (5), that:

$$\int_{t_1}^{t_2} \delta \int_V (dE_k - dE_p) dt = 0 \quad (6)$$

Equation (6) represents balance condition for the fluid in motion, which states that non-viscous and incompressible fluid is in equilibrium if the difference between potential and kinetic energy is at minimum. Knowing that the total energy of virtual moving does not change, equilibrium condition (6) is expressed as the variation of the sum of integrals of action I_i , formed for the characteristic flow domain zones V_i :

$$\delta I = \sum_{i=1}^n \delta I_i = \sum_{i=1}^n \int_{s_1}^{s_2} \delta \rho \bar{c} d\bar{s} dV_i = 0 \quad (7)$$

where \bar{c} is the local flow velocity, V_i – i -th fluid flow region, dV_i – elementary volume bounded by the inflow A_{ei} and outflow A_{oi} control surfaces (fig. 1), as well as by the given boundaries; s_1 and s_2 – representative positions of the fluid particle at the moments t_1 and t_2 , respectively, with $d\bar{s} = \bar{c} dt$.

Equation (7), is more convenient statement of the equilibrium condition for the fluid in motion, for the purpose of defining optimum fluid flow boundaries, than eq. (6). This results in fact that optimally defined geometry of fluid flow boundary differs from other solutions, by having the minimum value of the action integral I .

Usually main the inner flow is consisted of one “sound flow” and one or many closed secondary flow regions, separated from the main flow by the free boundaries, which are vorticity dissipative layers in the real fluid, or discontinuity surfaces, of different order, for the ideal fluid flow, that is model discussed here. Variational conditions can be applied to the “sound flow” region, but action integral for the “dead water”, where fluid is at rest or moves very slowly, equals zero. It follows from eq. (7) that:

$$\delta I = \int_{s_1}^{s_2} \delta \rho \bar{c} dV d\bar{s} = 0 \quad (8)$$

This equation has analytical solution only in special cases. For this reason numerical, or previously grapho-analytical, method is used. It is well known that elliptic partial differential equations describe equilibrium phenomena, the one is needed here. Stream lines and the lines of the same potential are mutually normal and they form a curvilinear grid. Considering this, the whole computational, fluid flow domain, between two control surfaces, should be divided into finitely small volumes: $\Delta V^{(q,p)}$. Equation (8) is then applied to these finite elements of the q -th stream tube, where $q \in [1, m]$. Each of $\Delta V^{(q,p)}$ is divided into p ($p \in [1, k]$) elementary volumes. The action integral is approximated as:

$$I = \sum_{q=1}^m \rho \sum_{p=1}^k \bar{c}(q,p) \Delta V^{(q,p)} \quad (9)$$

where $\bar{c}(q, p)$ is a local flow velocity corresponding to the mean streamline of the q -th stream tube divided into k parts, $s^{(q, p)}$ is the distance between the two respective positions $s^{(q, p)}$ and $s^{(q, p+1)}$, along the mean streamline of the q -th stream tube (fig. 1).

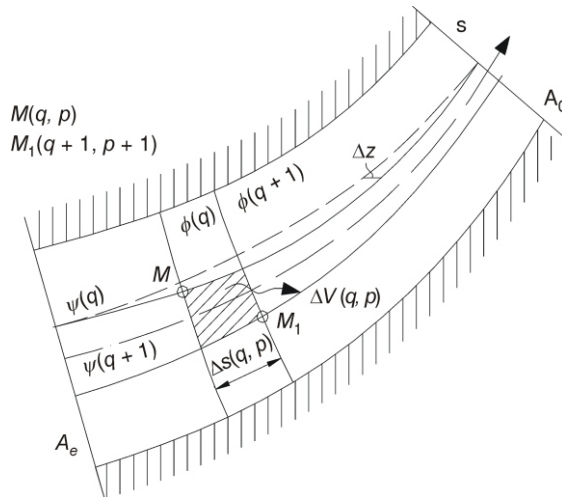


Figure 1. Stream and potential line grid for action integral calculation

In practice, defining flow field boundaries reduces to the variation of one boundary, while others save its position, as well as the control surface. The action integral is computed for each case, and the one with minimum integral is adopted as the final solution.

Application of the kinetic balance method is illustrated in the next flow field boundaries forming examples.

Application of the kinetic balance method

Forming inner curved contour of the diffuser with parallel lateral walls [1]

Diffuser model is shown in fig. 2a. Variation of the inner boundary form is defined by the ratio h/b_1 values defined with arithmetic progression in interval $[0.7, 1.1]$, with the step 0.1, as it is shown in fig. 2a. The other fluid flow boundaries are fixed. The action integral is calculated, using the expression (9), for each geometry.

Values of dimensionless action integral I/I_0 for variety of geometry is presented in fig. 2b. Function I/I_0 has minimum for the value $h/b_1 = 0.815$, so this contour was found to be the optimal. These theoretical results were confirmed by the conducted experiments, presented in [1].

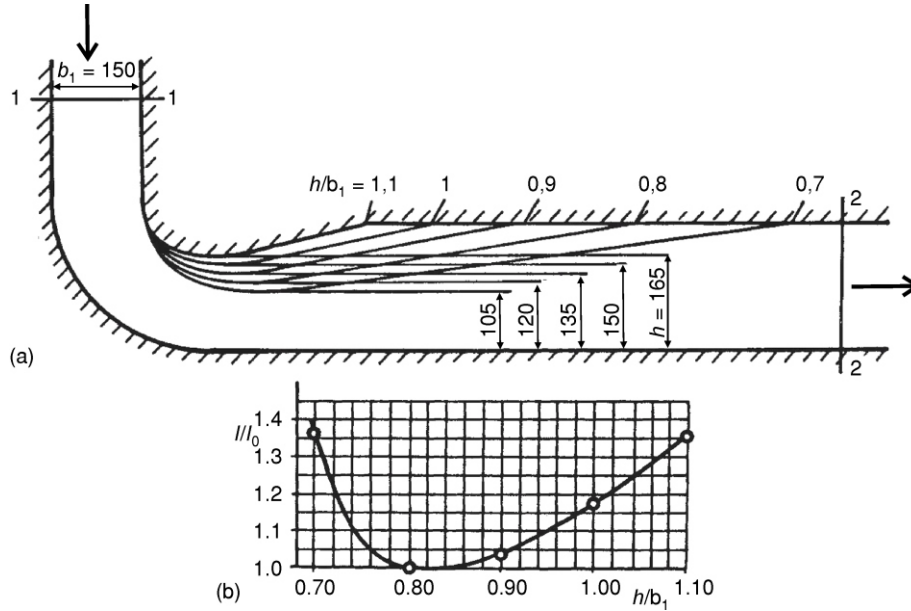


Figure 2. Diffuser with parallel lateral walls
(a) geometry, (b) action integral values

Defining the aeration duct of the bottom outlets of the Haditha dam in Iraq [4]

The geometrical form of the aeration duct is shown in fig. 3a. The only variable boundary is the inner one, which forms and positions are defined by the value $r/b_u = 1.57$, 1.37, and 1.14. Computed values of dimensionless action integral I/I_0 are shown in fig. 3b. Function I/I_0 has minimum value for the ratio $r/b_u = 1.37$, which is denoted as the case II in fig. 3b.

Determination of the intake structure lower contour of the additional hydro-turbine plant on the spillway of the HPP "Djerdap II" [4]

The kinetic balance method, presented in this paper, has been used for defining optimal shape of the lower contour of the intake structure, with geometry defined in fig. 4a. Discussed geometry variations are denoted by numbers 4-10. Not only geometry changes, but various head water levels: 63.0, 65.0, and 69.5 have been taken into consideration. The computation results are shown in fig. 4b. The minimum values of dimensionless action integral I/I_0 , for various head water levels are, as it is shown in fig. 4b, in interval (6, 8). These theoretically gained results, have been confirmed by experiments.

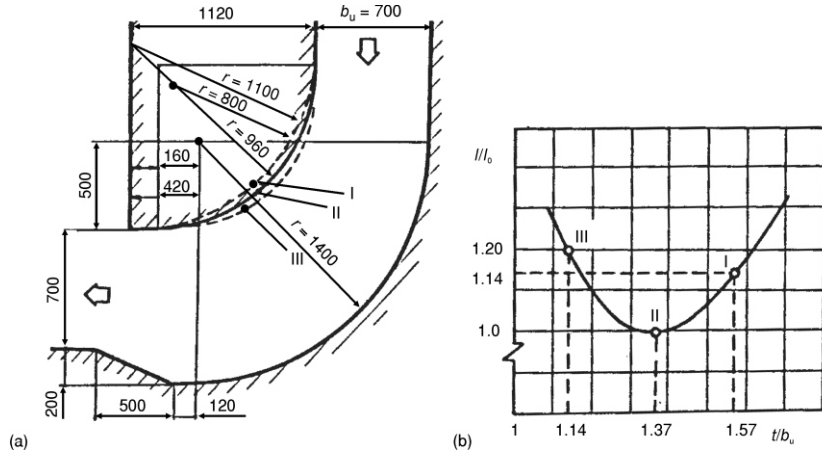


Figure 3. Aeration duct of the bottom outlets
 (a) geometry definition, (b) action integral values

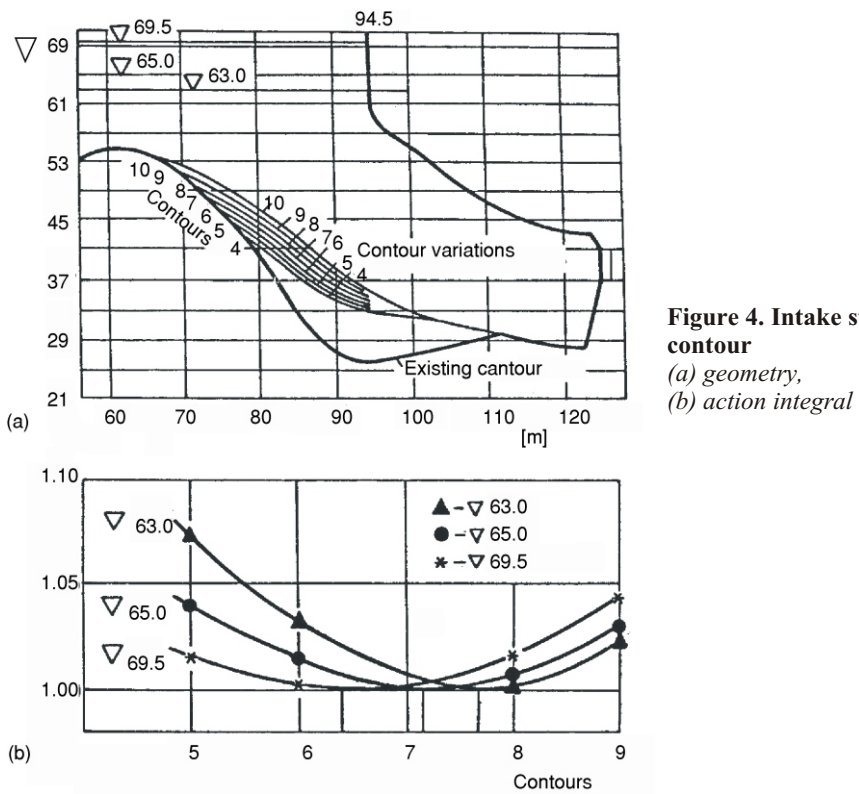


Figure 4. Intake structure contour
 (a) geometry, (b) action integral

Flow boundaries shaping under the inlet bell mouth of the axial pump in pumping station "Gradištanski rit" [4]

After installing larger capacity axial pumps in the pumping station "Gradištanski rit", for the existing suction level in the sump, some undesired phenomena occurred. In order to prevent rotational flow, establish uniform velocity field at suction pipe mouth and prevent air entraining vortices, sump geometry has been analyzed. It was necessary, besides some other manipulations, to form the fluid region below the larger bell mouth diameter. Shape of axis-symmetrical fluid flow domain between the suction bell and the guide cone is shown in the fig. 5a.

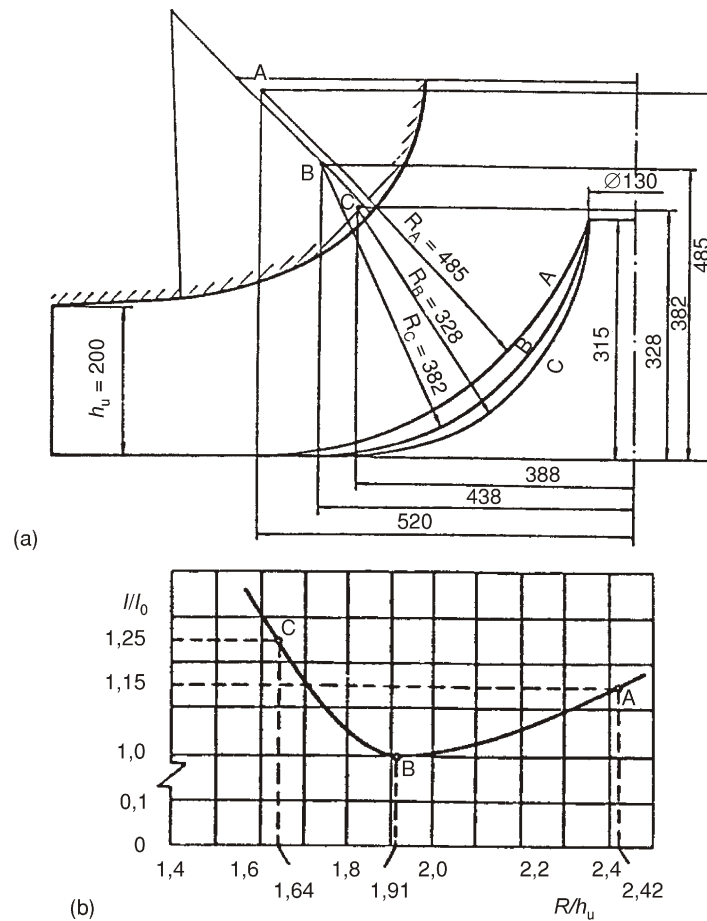


Figure 5. The flow passage
(a) geometry, (b) action integral

Like in all the previous examples one boundary is changeable, the other remain unchanged. In this case inner concave boundary of suction cone was varied, while the shape of the bell was already defined and kept constant.

Dimensionless action integrals I/I_0 have been computed for the three shapes of suction bell contour, defined with the ratio: $R/h_u = 2.42, 1.91,$ and 1.64 , which is denoted in the fig. 5 as A, B, and C, respectively. Computation results are shown in fig. 5b. It was found, and in this diagram obvious, that the function I/I_0 has its minimum for the ratio $R/h_u = 1.91$, corresponding to the curve B.

The suction cone was built in the shape of the curve B. No fore mentioned problems appeared, so once again, successful use of the kinetic balance method was confirmed in reality.

Defining the optimal shape of the Banki turbine, cross-flow turbine, semi-spiral case [5]

Shaping optimal Banki turbine semi-spiral case has been done by using the method of kinetic equilibrium. Impeller, semi-spiral case (intake chamber) and wicket gate are main parts of the Banki turbine fluid flow geometry. Semi-spiral case directs water to the impeller under defined angle, with as much as possible lower energy losses. Turbine inflow is regulated by the wicket gate blade. This is working principle of this, like other action turbines, where water kinetic energy is used.

The most convenient construction of the wicket gate, from hydraulic point of view, is the hydraulically shaped blade, built in as console, rounded at the end.

Fluid flow boundaries, *i. e.* water passage geometry, of the Banki turbine is given in fig. 6, preceded by the tab. 1, where radii for three various constructions, denoted with i , are given. Value of the clasping angle of the semi-spiral case is here 90° , though it may have various values.

Integral of action has been calculated for each possible geometry of the intake chamber.

Table 1. Geometry parameters (curvature radii) for turbine semi-spiral case definition

i	R_1 [mm]	R_2 [mm]	R_3 [mm]	R_4 [mm]	R_5 [mm]	R_6 [mm]	R_7 [mm]
I	253,7	235,3	227,0	216,1	202,5	185,7	167,6
II	253,7	235,3	218,3	202,5	187,9	174,3	161,7
III	253,7	227,3	205,9	190,5	175,5	164,1	157,7

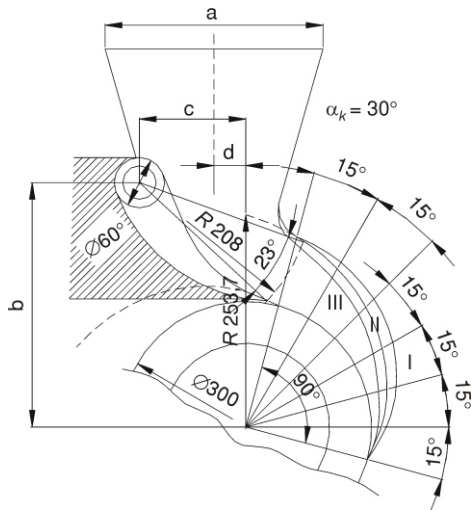


Figure 6. Various constructions of the Banki turbine intake chamber

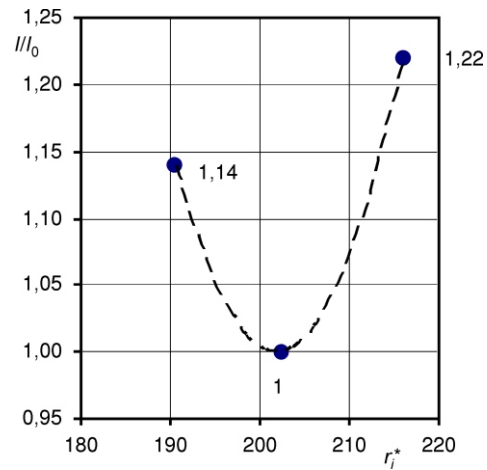


Figure 7. Action integral values for various constructions of the semi-spiral case

Relative values of action integral in the function of contour radius r^* at angle $\alpha_k = 30^\circ$ are presented in fig. 7. Dimensionless action integral I/I_0 has a minimum value equals 1, for the construction II. It comes out, according to the condition for fluid flow stability, that construction II has optimal shape of the intake chamber.

Conclusions

Presented results introduce method of kinetic balance as a reliable, helpful tool for defining optimum shape of the fluid flow boundaries. This analytic approach, with the help of numerical methods, leads to the most appropriate geometry, what is confirmed with numerous experiments conducted with fluid flow boundaries constructed according to the numerically obtained shapes. Consequently, number of various constructions which should be tested experimentally decreases in a great percentage by the application of this method. The influence of viscosity, which is neglected in this study, should be checked experimentally for the final definition of the fluid flow boundaries. The method is simple, and since equilibrium is defined with elliptic equations, potential flow solution is probably the easiest to use. This method could be used in a variety of problems, not just like examples described in this paper, but also, for example flow passage forming and determination of vortex core radius [6].

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Nomenclature

ϕ – equipotential line
 ψ – stream line

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