Temperature and Stress Fields in Thin Metallic Partially Fixed Plate Induced by Harmonic Electromagnetic Wave

Vesna Milošević-Mitić

Assistant Professor
University of Belgrade
Faculty of Mechanical Engineering

In the paper the behavior of thin elastic metallic plate influenced by the harmonic electromagnetic plane wave is considered. The plate is simply supported along three edges and fixed along the fourth one. As a result of time-varying electromagnetic field the conducting currents appear in the plate. Distribution of eddy-currents and hysterisis power losses across the plate thickness are obtained by use of complex analysis. There after, by treating this power stemming from a volume heat source, differential equations governing distribution of the temperature field are formulated and solved by using integral-transform technique. The influence of the plate thickness, wave frequency and hysterisis factor on the temperature field are considered, as well. Strain and stress fields are obtained by using finite element method (FEM) and integral- transform technique.

Keywords: *electromagnetic field, temperature, plate, induction, heat, stress, finite element.*

1. INTRODUCTION

Electro-magneto-thermoelasticity investigates the interaction between temperature, strain, stress and electromagnetic fields in a solid elastic body. On metallic deformable solids subjected to electromagnetic fields two types of forces are reacted [1]. Forces of the first type are between the stationary magnetic field and the magnetized material and the second type are volume dynamic forces on the conducting currents, which appear in electric conductors as a result of their motion or a time change of the magnetic field. The elastic field influences the magnetic field through the modified Ohm's law [2].

In the paper is assumed that the plates material is elastic, isotropic, soft ferromagnetic, possessing a good electric conductivity. Many nickel-iron alloys used for building the magnetic circuits of motors, generators, inductors, transformers are of this type.

The vibrations of the thin metallic plates from soft ferromagnetic materials are described using four coupled systems of differential equations based on the classical theory of thin plates and linear theory of thermoelasticity [3].

The first system is a system of Maxwell's equations relating to slowly moving media and the modified Ohm's low [2]:

$$\text{rot}\,\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad , \quad \text{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}, \qquad (1)$$

$$\text{div}\,\vec{D} = 0 \quad , \quad \text{div}\,\vec{B} = 0,$$

$$\vec{D} = \varepsilon_0 \left(\vec{E} + \dot{\vec{u}} \times \vec{B} \right), \ \vec{B} = \mu \left(\vec{H} - \dot{\vec{u}} \times \vec{D} \right),$$

$$\vec{J} = \sigma \left(\vec{E} + \dot{\vec{u}} \times \vec{B} \right),$$

where the following notation is applied: H – intensity of magnetic field, E – intensity of electric field, B – magnetic flux density (magnetic induction), D – electric induction, J – current density, u – deflection, μ_0 – permeability of vacuum, σ - electric conductivity, ε_0 – dielectric constant of vacuum, t –time.

The linear theory of thermoelasticity takes the assumption that the temperature linearly changes across the thickness of the plate. By using coordinate system presented on Figure 1, temperature field distribution $\theta(x_1, x_2, x_3, t)$ can be described by using the temperature (τ_0) in the middle surface of the plate and the rate of temperature (τ_1) across the plate thickness [4]:

$$\theta(x_1, x_2, x_3, t) = \tau_0(x_1, x_2, t) + x_3 \tau_1(x_1, x_2, t), \quad (2a)$$

where θ [°C, K] = T-T_0 and T_0 is the temperature of the plate in its natural state.

The second system of differential equations describes the temperature field in a thin plate. It consists of two partial differential equations [3]:

$$(\nabla_1^2 - \beta_k - \frac{1}{\kappa}\frac{\partial}{\partial t})\tau_k + \frac{{\beta_k}^k}{h}\left[x_3^k \frac{\partial \theta}{\partial x_3} \right]_{-\frac{h}{2}}^{\frac{h}{2}} - \eta^* \varepsilon_k = -\frac{{\beta_k}^k W_k}{h \lambda_0}$$

$$(k=0,1), \qquad (2b)$$

$$\beta_k = \begin{cases} 0, & k=0 \\ 12/h^2, & k=1 \end{cases}, \qquad \varepsilon_k = \begin{cases} \varepsilon', & k=0 \\ \nabla_1^2 \ddot{w}, & k=1 \end{cases},$$

$$W_k = \int_{-h/2}^{h/2} W(x_1, x_2, x_3, t)\, x_3^k \,\text{d}x_3 \ , \ W = W_E + W_H + \frac{J^2}{\sigma} \ ,$$

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Correspondence to: Vesna Milošević-Mitić,
Faculty of Mechanical Engineering, University of Belgrade,
27. Marta 80, 11120 Belgrade, SCG
E-mail: vmilosevic@mas.bg.ac.yu

where κ is the coefficient of thermal intensity, η^* is the coupling between the temperature and the deformation fields, ε' is deformation in the middle surface of the plate, h is the plate thickness, λ_0 is heat conduction coefficient, w is deflection of the plate in x_3-direction and ∇_1^2 is Laplace operator. Quantity of heat generated in unite volume and unit time (heat source intensity) $W(x_1, x_2, x_3, t)$ consists of three parts: intensity of external heat source W_E, hysterisis losses W_H and Joule's heat (eddy-current losses).

In the consideration of the plate vibrations, we shall take the assumption that the longitudinal vibrations are independent of the transverse vibrations.

Vibrations of the plate middle surface are defined with equation:

$$\Delta_2^2\left(\Delta_1^2 F + E\,h\,\alpha_t \tau_0\right) = 0\,, \qquad (3a)$$

$$\Delta_i^2 \equiv \nabla_1^2 - \frac{1}{C_i^2}\frac{\partial^2}{\partial t^2}\,, \qquad (i=1,2),$$

$$C_1 = \sqrt{E/\rho(1-\nu^2)}\,, \quad C_2 = \sqrt{E/2\rho(1+\nu)}\,,$$

where E is modulus of elasticity, α_t is coefficient of thermal expansion, ρ is the plate density, ν is Poisson ratio and F is the Airy-s Stress function. Forces N_{ij} can be expressed in tensor notation as [5]:

$$N_{ij} = -F_{,ij} + \delta_{ij}(\nabla_1^2 - \frac{1}{2C_2^2}\partial_t^2)F \qquad (i,j=1,2),$$

$$N_{kk} = -F_{,kk} + 2(\nabla_1^2 - \frac{1}{2C_2^2}\partial_t^2)F = \Delta_2^2 F\,, \qquad (3b)$$

where ∂_t is the time derivative.

Transverse vibrations can be obtained by using the following differential equation [3]:

$$D\nabla_1^4 w + D(1+\nu)\alpha_t \nabla_1^2 \tau_1 - \rho h \ddot{w} + \frac{\rho h^3}{12}\ddot{w} =$$

$$= \left(\sigma_{33}^+ - \sigma_{33}^-\right) + \left(T_{33}^+ - T_{33}^-\right) + \frac{h}{2}\frac{\partial}{\partial x_i}\left(\sigma_{i3}^+ + \sigma_{i3}^-\right) +$$

$$+ \frac{h}{2}\frac{\partial}{\partial x_i}\left(T_{i3}^+ + T_{i3}^-\right) + \int_{-\frac{h}{2}}^{\frac{h}{2}}\left(X_{i,i} + f_{i,i}\right)x_3 \mathrm{d}x_3 + \int_{-\frac{h}{2}}^{\frac{h}{2}} X_3 \mathrm{d}x_3\,,$$

$$(i=1,2), \qquad (4)$$

where: D - flexural rigidity of the plate, X - mechanical force, f - Lorenz force.

σ_{ij} and T_{ij} denote mechanical and magnetic stress tensors (σ_{ij}^+, T_{ij}^+ are stress components on the upper and σ_{ij}^-, T_{ij}^- on the lower side of the plate).

Of course, presented system of equations has to be accomplished with the appropriate set of boundary and initial conditions.

2. CONDUCTING CURRENTS, JOULE'S HEAT AND HYSTERISIS LOSSES

Electromagnetic wave with complex time- varying fields can be represented as sum of simple plane waves. In this paper is given the analytical solution for harmonic plane wave with E_{10} and H_{20} components on the upper surface of the plate. It is assumed that all field components vary in time t as $\exp(j\omega t)$, where ω is the angular frequency.

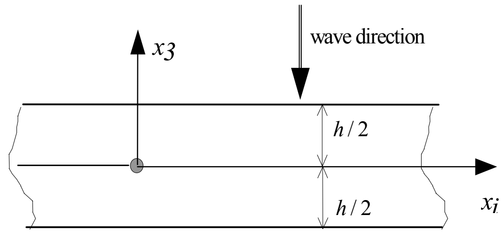

Figure 1. Coordinate system (middle surface of the plate).

In the case of the high plate conductivity, the dielectric current can be neglected in comparation with the conducting current. So, for the homogeneous, isotropic and linear magnetic medium the system of Maxwell's equations (1) can be presented in the form [6]:

$$\begin{aligned} &\mathrm{rot}\,\vec{H} = \sigma\vec{E}\,, \quad \mathrm{div}\,\vec{E} = 0\,, \\ &\mathrm{rot}\,\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}\,, \quad \mathrm{div}\,\vec{H} = 0\,. \end{aligned} \qquad (5a)$$

Using symbolic-complex representation of vectors $\vec{A} = \left\{\vec{E},\, \vec{H}\right\}$ ($\vec{A} = \underline{\vec{A}}\,e^{j\omega t}$) we obtain the equations:

$$\begin{aligned} &\mathrm{rot}\,\underline{\vec{\boldsymbol{H}}} = \sigma\underline{\vec{\boldsymbol{E}}}\,, \quad \mathrm{div}\,\underline{\vec{\boldsymbol{E}}} = 0\,, \\ &\mathrm{rot}\,\underline{\vec{\boldsymbol{E}}} = -\mu(j\omega)\underline{\vec{\boldsymbol{H}}}\,, \quad \mathrm{div}\,\underline{\vec{\boldsymbol{H}}} = 0\,. \end{aligned} \qquad (5b)$$

If the direction of the wave propagation is x_3 -axis (negative) and if the field components are independent of x_1 and x_2, then from the equations of divergence (5*b*) we conclude that the components $\underline{H}_3$ and $\underline{E}_3$ must be zero. In the case of the plane wave, only normal components of the electric and magnetic field depend of each other [7]. So, we will make the analysis only for one wave with components $\underline{E}_1$ and $\underline{H}_2$. Let they have next values in the plane $x_3=h/2$:

$$\vec{\boldsymbol{E}} = E_1\vec{\boldsymbol{i}}_1 = E_0\cos(\omega t)\vec{\boldsymbol{i}}_1\,, \qquad (6)$$

$$\vec{\boldsymbol{H}} = H_2\vec{\boldsymbol{i}}_2 = H_0\cos(\omega t)\vec{\boldsymbol{i}}_2\,, \quad H_0 = \sqrt{\frac{\varepsilon}{\mu}}E_0\,.$$

Then the Maxwell's equations (5*b*) obtain the following form:

$$-\frac{\partial\underline{H}_2}{\partial x_3} = \sigma\,\underline{E}_1\,, \quad \frac{\partial\underline{E}_1}{\partial x_3} = -\mu j\omega\,\underline{H}_2\,,$$

or

$$\frac{\partial^2\underline{H}_2}{\partial x_3^2} - \gamma^2\underline{H}_2 = 0\,, \quad \underline{E}_1 = -\frac{1}{\sigma}\frac{\partial\underline{H}_2}{\partial x_3}\,, \qquad (7)$$

where

$$\gamma^2 = j\sigma\mu\omega\,, \quad \gamma = \alpha + j\beta\,, \quad \alpha = \beta = \sqrt{\frac{\sigma\mu\omega}{2}}\,.$$

If we want to find the solution of (7) in the case when the skin-depth of a progressive wave is small compared to the plate thickness, we use the basic solution of (7) which can be represented as follows:

$$\underline{\boldsymbol{H}}_2 = \underline{\boldsymbol{C}}\,e^{\gamma x_3}\,, \qquad \underline{E}_1 = \frac{\gamma}{\sigma}\underline{H}_2\,. \qquad (8)$$

By using the boundary condition for $x_3 = h/2$ we obtain:

$$\underline{C} = H_0\, e^{-\gamma h/2}.$$

The characteristic impedance is:

$$\underline{Z}_c = \frac{\underline{E}}{\underline{H}} = \frac{\gamma}{\sigma} = \sqrt{\frac{\mu\omega}{2\sigma}}\,(1+j) = \sqrt{\frac{\omega\mu}{\sigma}}\; e^{j\pi/4} \quad (9)$$

and the result for the field components obtain the form:

$$\underline{H}_2 = H_0 e^{-\alpha h/2} e^{-j\beta h/2} e^{\alpha x_3} e^{j\beta x_3},$$

$$\underline{E}_1 = H_0 \sqrt{\frac{\omega\mu}{\sigma}}\, e^{j\frac{\pi}{4}}\, e^{-\alpha h/2} e^{-j\beta h/2} e^{\alpha x_3} e^{j\beta x_3}, \quad (10)$$

or

$$H_2 = \mathrm{Re}\left[\underline{H}_2 e^{j\omega t}\right] = H_0 e^{\alpha(x_3 - h/2)} \cos(\omega t + \beta x_3 - \beta \frac{h}{2})$$

$$E_1 = \mathrm{Re}\left[\underline{E}_1 e^{j\omega t}\right] =$$

$$= H_0 \sqrt{\frac{\omega\mu}{\sigma}} e^{\alpha(x_3 - h/2)} \cos(\omega t + \beta x_3 - \beta \frac{h}{2} + \frac{\pi}{4}).$$

Electromagnetic wave (10) is accompanied by the conducting currents of density:

$$\underline{J}_1 = \sigma \underline{E}_1 = H_0 \sqrt{\omega\mu\sigma}\, e^{\alpha(x_3 - h/2)} e^{j\beta(x_3 - h/2)} e^{j\frac{\pi}{4}}. \quad (11)$$

Fields and current amplitudes are the exponentially decreasing function, along the trajectory of wave propagation. The constant of wave penetration correspond to the decay of one Neper (0.368) and its value is [7]:

$$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\sigma\mu\pi f}}, \qquad \left(\omega = \frac{2\pi}{T} = 2\pi f\right). \quad (12)$$

Skin-depth decreases with increasing of frequency f, conductivity and permeability. The origin for this phenomenon is heat losses in metal.

Distribution of the Joule's heat (P_v) can to be determined in the following way:

$$P_v(x_3) = \frac{1}{2\sigma}\|J\|^2 = \frac{1}{2} H_0^2 \omega\mu\, e^{2\alpha(x_3 - h/2)}. \quad (13)$$

Distribution of the eddy-currents power (13) across the plate thickness is

$$P_v(x_3) = \frac{1}{2} H_0^2 \omega\mu\, e^{-h\sqrt{\frac{\sigma\mu\omega}{2}}}\, e^{x_3\sqrt{2\sigma\mu\omega}}, \quad (14)$$

and it can be treated as a volume heat source having the intensity $P_v(x_3)$.

In the case of a nonlinear magnetic material, in the presented analysis must be included the factor which involves heat losses due to hysterisis. For the most of soft ferromagnetic materials the basic curve of magnetization is nearly linear. This fact assures that the middle value for permeability μ_{sr} can be used in calculation [6].

In this paper will be presented one method for analytical obtaining the influence of the hysterisis on the temperature field in the plate.

The hysterisis losses power P_H is proportional to the square of the magnetic field amplitude and frequency f:

$$P_H \approx H^2 f, \quad P_H(x_3) = k_H\, \mu H_0^2 f\, e^{2\alpha(x_3 - h/2)},$$

which approves that the distribution of P_H is the same as distribution of the eddy-current losses. Coefficient k_H is the hysterisis factor of known material characteristics.

So, density of the power of the heat losses is approximately given by:

$$P(x_3) = \frac{1}{2} H_0^2 \omega\mu\, e^{-h\sqrt{\frac{\sigma\mu\omega}{2}}} \left(1 + \frac{k_H}{\pi}\right) e^{x_3\sqrt{2\sigma\mu\omega}}. \quad (15)$$

Expression (15) shows that the heat source intensity decreases exponentially with the plate thickness. Gradient of the exponential curve increases with increasing of the wave frequency, permeability and electric conductivity of a material.

The phenomenon of concentration of conducting currents on the plate surface, valid for conductors with very high electric conductivity and magnetic permeability subjected to high frequency wave, is known as the skin-effect.

3. TEMPERATURE FIELD

Let the rectangular plate dimensions $a \times b \times h$ be isolated on the upper and the lower surface and the temperature along the lateral sides is equal to initial temperature T_0 ($\theta = T - T_0 = 0$). The initial and the boundary conditions have the form:

$$\theta|_{t=0} = 0, \quad \theta|_{x_1=0,a} = 0, \quad \theta|_{x_2=0,b} = 0, \quad \left.\frac{\partial\theta}{\partial x_3}\right|_{x_3=\pm\frac{h}{2}} = 0. \quad (16)$$

By using (15) the power of the heat source can be expressed as function of time t as follows:

$$W(x_3, t) = P\, e^{2\alpha x_3} H(t),$$

$$P = \frac{1}{2} H_0^2 \omega\mu\, e^{-h\sqrt{\frac{\sigma\mu\omega}{2}}} \left(1 + \frac{k_H}{\pi}\right) \quad (17)$$

Subjected to the boundary conditions (16) the equation (2) can be solved by using the integral-transform technique. Applying double Fourier finite-sine transform (signed as nm) and Laplace transform (signed by *, $t \to p$) [4, 5] we arrive to the transform functions of temperature field:

$$\overset{*}{\tau}_{knm} = \frac{4\beta_k^k C_k \kappa}{\lambda_0 h \alpha_n \alpha_m} \frac{1}{p\left[p + \kappa(\beta_k + \Delta_{mn})\right]},$$

where

$$C_k = \begin{cases} \dfrac{P}{2\alpha^2}\left[\alpha h\, \mathrm{ch}(\alpha h) - \mathrm{sh}(\alpha h)\right], & k = 1 \\ \dfrac{P}{\alpha}\mathrm{sh}(\alpha h), & k = 0 \end{cases}$$

$$\Delta_{mn} = \alpha_n^2 + \alpha_m^2 = (n\pi/a)^2 + (m\pi/b)^2.$$

The inverse Laplace transform gives:

$$\tau_{knm} = \frac{4\beta_k^k C_k}{\lambda_0 h \alpha_n \alpha_m} \frac{1 - e^{-\kappa(\beta_k + \Delta_{mn})t}}{(\beta_k + \Delta_{mn})} H(t). \quad (18)$$

The solution to the temperature field now obtains its final form by using inverse double Fourier finite-sine transform:

$$\tau_k = \frac{4}{ab}\sum_{m=1,3,\ldots}^{\infty}\sum_{n=1,3,\ldots}^{\infty}\frac{4\beta_k^k C_k}{\lambda_0 h\,\alpha_n\alpha_m(\Delta_{mn}+\beta_k)}\times$$

$$\times\left[1-e^{-\kappa\left(\beta_k+\Delta_{mn}\right)t}\right]\sin\alpha_n x_1 \sin\alpha_m x_2 H(t)\,. \quad (19)$$

Now, a numerical example will be given for the steel rectangular plate having dimensions a=50 cm, b=30 cm. Material constants are: λ_o=0.5 W/cmK, $\sigma=7.710^{8}$ S/m and μ_r=1000. Electromagnetic field parameters are: H_0=2000A/m and f=0.5÷2.5 MHz. The appropriate skin-depth (12) is in range 1.16 to 2.56 μm.

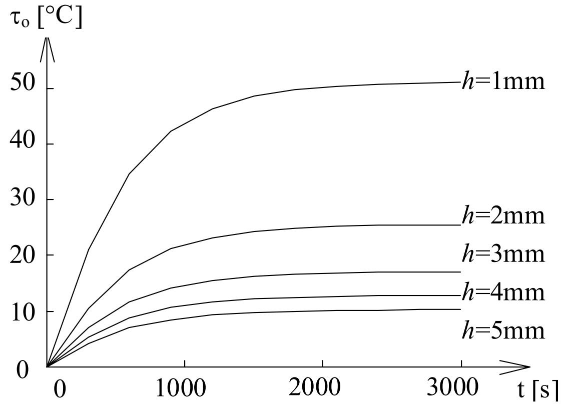

Figure 2. Temperature in the middle point as function of time and plate thickness (f=2 MHz, k_H=1).

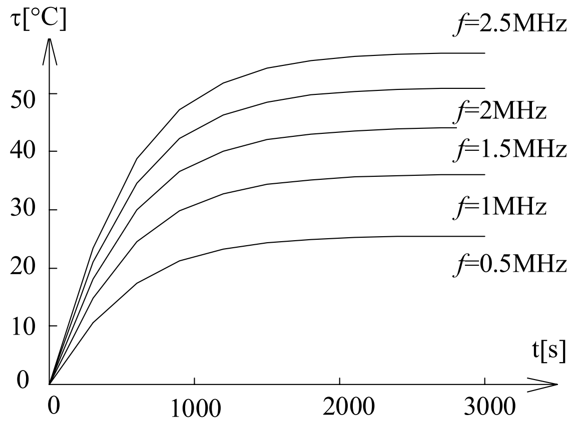

Figure 3. Temperature in the middle point as function of time and wave frequency (h=1mm, k_H=1).

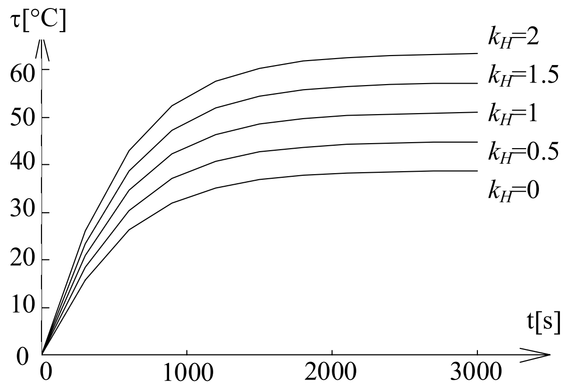

Figure 4. Temperature in the middle point as function of time and hysterisis factor (h=1mm, f=2MHz).

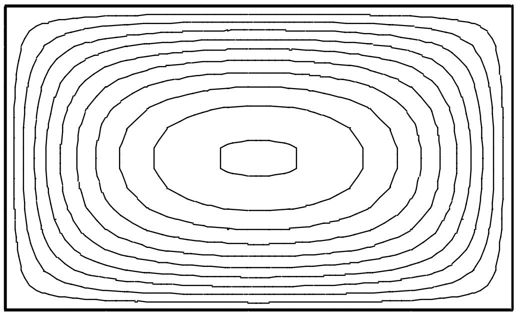

Figure 5. Isotherm lines in the middle surface of plate τ_o=0÷50.7 (step 5 °C).

In figures 2 to 4 the temperature in the middle point of the plate is presented as a function of time, plate thickness, wave frequency and hysterisis factor. Figure 5 shows distribution of the isotherm lines in the middle surface for the stationary state of plate.

4. VIBRATIONS. STRESS FIELD

4.1. Transversal vibrations

Let the plate be simply supported along the three edges ($x_1=a$, $x_2=0$, b) and fixed along the fourth edge ($x_1=0$) (Fig. 6). The boundary conditions have the form

$$w\big|_{x_1=0,a}=0\,,\qquad w\big|_{x_2=0,b}=0\,,$$

$$M_{11}\big|_{x_1=a}=\left[\frac{\partial^2 w}{\partial x_1^2}+\nu\frac{\partial^2 w}{\partial x_2^2}+(1+\nu)\alpha_t\tau_1\right]D\Bigg|_{x_1=a}=0\,,$$

$$M_{22}\big|_{x_2=0,b}=\left[\nu\frac{\partial^2 w}{\partial x_1^2}+\frac{\partial^2 w}{\partial x_2^2}+(1+\nu)\alpha_t\tau_1\right]D\Bigg|_{x_2=0,b}=0$$

$$\partial w/\partial x_1\big|_{x_1=0}=0\,. \quad (20)$$

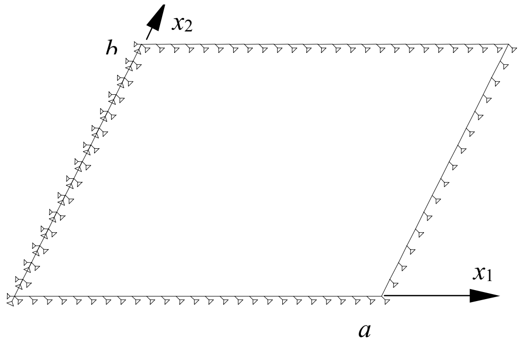

Figure 6. Boundary conditions.

Initial conditions are responsible for the natural undeformed state:

$$w\big|_{t=0}=0\ ,\ \ \partial w/\partial t\big|_{t=0}=0\,. \quad (21)$$

Problem with nonhomogeneous boundary conditions is not very suitable for obtaining the solution in analytical form. So, in this section of the paper one way for soluting the problem by using only simple integral transformations will be presented. Differential equation describing transverse vibrations (4) is adapted to form which enabling very easy simulation of the bending moments along the fixed edges.

Using equation (4) we can form the appropriate equation for stationary problem ($t\to\infty$) by simulating the moment along the edge $x_1=0$ through the stress σ_{13} in the following way

$$\nabla_1^4 w + (1+\nu)\alpha_t \nabla_1^2 \tau_1 = \frac{h}{D}\frac{\partial}{\partial x_1}\left[\sigma_{13}(x_2)\delta(x_1)\right] , \quad (22)$$

where $\delta(x_1)$ is Dirac delta-function.

By using (18), double Fourier finite-sine transform and the relation for the first derivative of δ function:

$$\int_0^a \frac{\partial\delta(x_1)}{\partial x_1}\sin(\alpha_n x_1)\,\mathrm{d}x_1 = -\alpha_n ,$$

the solution for transversal vibrations can be represented in form:

$$w(x_1,x_2,t) = \frac{4}{ab}\sum_{m=1,3,\ldots}^{\infty}\sum_{n=1}^{\infty} w_{nm}\sin\alpha_n x_1 \sin\alpha_m x_2 ,$$

$$w_{nm} = \frac{\alpha_t(1+\nu)\tau_{1nm}}{\Delta_{mn}} - \frac{h}{D}\frac{\alpha_n}{\Delta_{mn}^2}\sigma_{13m} . \quad (23)$$

Using the boundary condition for the edge $x_1=0$ we can calculate the “moment stress” $\sigma_{13}(x_2)$:

$$\sigma_{13m} = \frac{\sum_{n=1}^{\infty}\frac{\alpha_t(1+\nu)\tau_{1nm}\alpha_n}{\Delta_{mn}}}{\frac{h}{D}\sum_{n=1}^{\infty}\frac{\alpha_n^2}{\Delta_{mn}^2}} , \quad (24)$$

$$\sigma_{13}(x_2) = \frac{2}{b}\sum_{m=1,3,\ldots}^{\infty}\sigma_{13m}\sin\alpha_m x_2 .$$

Presented method is very suitable for obtaining analytical solutions for problems with moving boundary conditions.

For the numerical example presented in section 3, deformation calculated from (23) and (24) is nearly 10^{-5}cm due to the small temperature gradient across the plate thickness.

For geometrically more complex problems and nonlinear temperature distribution across the plate thickness, finite element method has to be involved in analysis. The stiffness matrix and the load matrix for the plate element can be formed by using the analogy with the finite element of the composite plate. Corresponding values for deflection obtained from the analytical solution and the numerical solution are the same, but there is a large discrepancy between the stress distributions based on the real and the reduced FEM model [8].

4.2. Deformation of the middle surface. Stress field

Vibrations and stress in the middle surface of the plate are calculated by using the program package KOMIPS [9], based on the finite element method. Input file was done on the analytical solution (19).

Deformation in the middle surface of the plate is depicled in Fig. 7. Maximal deformation is calculated to be 0.171cm.

Figure 8 shows the appropriate stress field. Maximal temperature in the middle point is θ=50.7 K and maximal stress value calculated by the finite element method (FEM) is 7.524 kN/cm^2 (on the edges x_2=0, b).

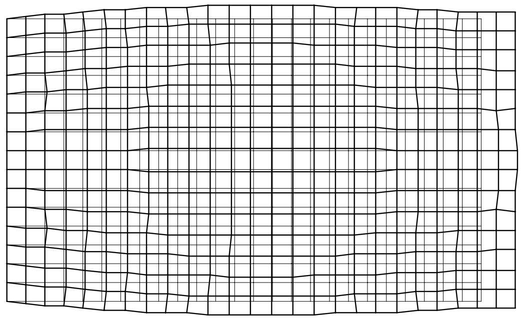

Figure 7. Deformation (middle surface) f_{max}=0.171cm.

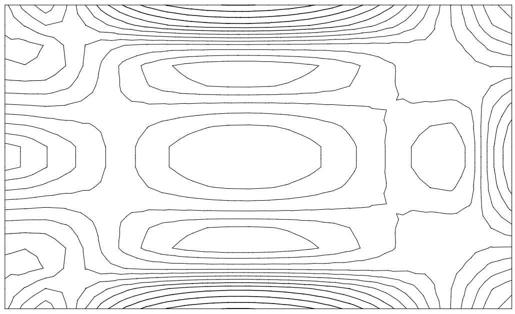

Figure 8. Stress field, σ = 0÷7.524/0.5 kN/cm^2

5. CONCLUSION

The magneto-thermoelasticity has received the considerable attention in recent years because of the possibility for application in detection of flaws in ferrous metals, optical acoustics, levitation by superconductors, magnetic fusion and many other electro-mechanical devices.

The problem of the thin metallic plate subjected to transversal line propagation of simple harmonic electromagnetic wave can be described through four systems of differential equations. In this case the most influence on the stress field has the increasing of temperature. This is the result of the time- varying electromagnetic field that is accompanied with the appearance of eddy-current and hysterisis losses. Intensity of the losses decreases exponentially with the plate thickness. As it is shown in paper, temperature in the plate increases with increasing of the wave frequency, increasing of the hysterisis factor and decreasing of the plate thickness. When the frequency and conductivity are very high, the problem can be treated as a thermal shock problem because of the small skin-effect depth.

Very suitable method for solving the considered problem in analytical form, as has been shown in the paper, is the integral-transform technique. But, for dynamic and geometrically more complex problems with non-homogeneous boundary conditions it is very difficult to find vibrations and stress in analytical form. Then, the finite element method has to been involved in calculation.

REFERENCES

[1] Pao Y.H., Yeh C.S., A Linear Theory for Soft Ferromagnetic Elastic Solids, Int. J. Enging. Sci., Vol. 11, pp. 415-436,1973.

[2] Parkus H, *Electromagnetic Interactions in Elastic Solids*, Springer-Verlag, (W. Nowacki, On Magneto Elasticity), 1979 .

[3] Milošević-Mitić V., *Magneto-thermo-elastic bending of thin plates,* Zadužbina Andrejević, Belgrade, 1999 (in Serbian).

[4] Čukić R., Milošević V., Thermoelastic Vibrations of The Thin Plate Induced by Moving Heat Source, J. of Theor. and Applied Mechanics 2, 31, Warszava, pp. 403-412, 1993.

[5] Čukić R., Naerlović N., Šumarac D., *Thermoelasticity*, Faculty of Mechanical Engineering – Belgrade. 1993 (in Serbian).

[6] Milošević-Mitić V., Temperature, Strain and Stress Fields Produced by Impulsiv Electromagnetic Radiation in the Thin Metallic Plate, Presented on XIXth ICTAM, Kyoto (1996), Printed in FACTA UNIVERSITATIS Series "Mechanics, Automatic Control and Robotics", Vol. 3, No. 12, pp. 405-416 , 2002.

[7] Surutka J., *Electromagnetic,* Građevinska knjiga, Belgrade, 1975 (in Serbian).

[8] Čukić R., Maneski T., Milošević V.., The Problem of the Nonlinear Temperature Distribution Across the Thickness of the Plate Produced by Electro-magnetic Field, Thermal Stresses 97, Rochester, New York, pp. 719-722, 1997.

[9] Maneski T., Milošević-Mitić V., Ostrić D., *Postulates for strength of constructions,* Faculty of Mechanical Engineering University of Belgrade, 2002, (in Serbian).

ТЕМПЕРАТУРСКО И НАПОНСКО ПОЉЕ ТАНКЕ МЕТАЛНЕ ДЕЛИМИЧНО УКЛЕШТЕНЕ ПЛОЧЕ ИНДУКОВАНО ХАРМОНИЈСКИМ ЕЛЕКТРОМАГНЕТСКИМ ТАЛАСОМ

Весна Милошевић-Митић

У раду се разматра понашање танке еластичне металне плоче изазвано хармонијским електро-магнетским раванским таласом. Плоча је слободно ослоњена дуж три ивице и уклештена дуж четврте. Као резултат дејства временски променљивог електро-магнетског поља појављују се кондукционе струје у плочи. Расподела снаге кондукционих и хистерезисних губитака по дебљини плоче одређена је применом комплексног рачуна. У даљем прорачуну та снага је третирана као запремински извор топлоте. Диференцијалне једначине поља температуре решене су методом интегралних трансформација. Разматран је и утицај дебљине плоче, фреквенције таласа и фактора хистерезисних губитака на температурно поље. Поља напона и деформације одређена су применом методе коначних елемената и методе интегралних трансформација.