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## P5_10 A Dizzying Demonstration of Cartographical Centrality


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**Abstract**

This paper aims to determine the angular velocity that must be transferred to an Earth-mass globe, situated at the center of the Earth and spinning in the same orientation as the Earth, to extend it's rotational period by one hour. This new 25 hour day would necessitate an angular velocity of $\omega_g = 7.679 \times 10^8$ rad $\cdot$ s$^{-1}$ for the globe. An explorer spinning the globe along it's equator would need to move their hand at a tangential velocity of $2.34 \times 10^8$ ms$^{-1}$.


### Introduction

An explorer, inspired by Jules Verne, decided to make the intrepid journey to the very centre of the Earth. In order to prepare for this latest excursion, he invested his profits obtained whilst exploring Atlantis to create the most realistic globe possible: a shrunken Earth. However, during the shrinking process the original mass of the Earth was retained, giving the globe a mass of $M_g = M_\otimes$. After reaching the central point, he became lost and so decided to consult his globe. Upon realising that it was of no use to him after all in this situation, he became enraged and began to spin the globe frantically, in the same direction as the rotation of the Earth about it's axis. Suddenly an idea dawned upon him: could he give the Earth an additional hour each day by stealing it's angular momentum? Yes! By the principle of conservation of momentum, he could give himself more time to explore each corner of the planet. Faster and faster he span the globe. Until the effect of the rotation acted to elongate the length of a day.

### Discussion

Modelling the globe as a thin spherical shell and the Earth as a solid sphere, the moments of inertia for each can be calculated, finally leading to a consideration of conservation of angular momentum and a quantification of the slowing of Earth's rotation. Firstly, calculating the moment of inertia of the Earth:

$$I_\otimes = \frac{2}{5} M_\otimes R_\otimes^2 \tag{1}$$

where $M_\otimes$ is the mass of Earth and $R_\otimes$ is the radius of Earth. These value were found to be $M_\otimes = 5.9722 \times 10^{24}$ kg and $R_\otimes = 6.3781 \times 10^6$ m [1]. Inputting these value into (1) gives a value of $I_\otimes = 9.7180 \times 10^{37}$ kgm$^2$. Next, it is possible to calculate the angular momentum of the Earth with its 24 hour rotational period as follows:

$$L_\otimes = \omega_\otimes I_\otimes. \tag{2}$$

Using a value of

$$\omega_\otimes = \frac{2\pi}{86400s} = 7.27 \times 10^{-5} \text{ rad} \cdot \text{s}^{-1} \tag{3}$$

a value of $L_\otimes = 7.0671 \times 10^{33}$ kgm$^2$s$^{-1}$ was found.

It is then possible to calculate the angular momentum of the Earth in the scenario of a 25 hour rotational period. Following the same process as was done above, a value of $L_{\otimes 25} = 2.8256 \times 10^{33}$ kgm$^2$s$^{-1}$ was determined. Finding the change in angular momentum, $L_{diff} = L_\otimes - L_{\otimes 25} = 2.8256 \times 10^{32}$ kgm$^2$s$^{-1}$ was required to slow the Earth by a single hour .

The angular velocity required of the globe in order to make this change is calculated as follows. Initially, the moment of inertia of the globe can be determined by

$$I_g = \frac{2}{3} M_\otimes R_g^2, \tag{4}$$

where a value of 12 inches was assumed for the radius of the globe. Inputting this into equation (4), a value of $I_g = 3.6795 \times 10^{23}$ kgm$^2$ was determined.

To calculate the angular velocity required of the globe, the value $I_g$ must be used along with the value $L_{diff}$:

$$\omega_g = \frac{L_{diff}}{I_g}. \tag{5}$$

After inputting numerical values, a value of $\omega_g = 7.679 \times 10^8$ rad·s$^{-1}$ was determined. Next this will be converted to a tangential velocity in order to determine the speed at which the explorer's hand must be moving along the equator of the globe to spin it. This is done via the equation

$$v_g = \omega_g R_g. \tag{6}$$

From this, it was determined that his hand must have been moving at $2.34 \times 10^8$ ms$^{-1}$. This is 78% of the speed of light.

## Conclusion

In this paper, the idea has been tested that a globe spinning at the center of the Earth could slow the Earth's rotation, therefore giving it's population another hour every day. By consideration of conservation of angular momentum it has been determined that an angular velocity of $\omega_g = 7.679 \times 10^8$ rad $\cdot$ s$^{-1}$ would be necessary. However, this would mean an equatorial tangential velocity of $2.34 \times 10^8$ ms$^{-1}$ for the explorer's hand. Due to the fact that this is 78% of the speed of light, special relativistic effects may need to be taken into consideration for a more complete treatment of the problem.

## References

[1] E. E. Mamajek et al. *IAU 2015 Resolution B3 on Recommended Nominal Conversion Constants for Selected Solar and Planetary Properties.* 2015. DOI: `10.48550/ARXIV.1510.07674`. URL: `https://arxiv.org/abs/1510.07674`.