Journal of Physics Special Topics
An undergraduate physics journal

P4_5 A Rogue Earth

R.Rowe, K. Byworth, J. Garner and D. Lawrence-Morgan

Department of Physics and Astronomy, University of Leicester, Leicester, LE1 7RH

December 12, 2022

Abstract

The Earth decides a giant mirror would be the best defence against the Death Star's laser. Due to the conservation of momentum, when the mirror reflects the laser the mirror's velocity and hence the Earth's velocity must increase to 0.9998617325c. This will result in the Earth escaping from the Sun's gravitational pull and becoming a rogue planet. As a result, this strategy would be infeasible as the destruction of everyone on planet Earth would still be guaranteed.

Introduction

After seeing Alderaan, Ref.[1], being destroyed Earth decides to develop a defence system against the Death Star's laser. It is decided that the best way to prevent annihilation by the Empire is to build a giant mirror and reflect the laser away from the Earth. As momentum must be conserved, the Earth will gain momentum from the reflection of the laser. The Earth orbits the Sun, the other planets in the solar system will be ignored to make the calculations easier. In the film, the Death Star and Alderaan are both shown in the important scene, however there is no sign of a star as a result a hit tangential to the current orbit of the Earth will be considered. The aim of the paper is to investigate how an attack from a Death Star laser would effect the orbit of the Earth.

Calculating Death Star Energy

First, the energy of the Death Star's laser must be calculated. As the Earth is the planet targeted it will be used to approximate the energy of the Death Star. This is calculated by considering the gravitational potential on a thin shell which is then integrated through the radius of the Earth. The mass of the shell is given by $m_{shell} = 4\pi r^2 \rho \delta r$. Where r is the current radius from the centre of the Earth, ρ is the density of the Earth, δr is the radius of the thin shell. The mass of the interior and Eq. (1) use the same symbols. The mass of the interior can be calculated using, $m_{interior} = \frac{4}{3}\pi r^3 \rho$. The mass of the shell and the mass of the interior can be combined with the equation of gravitational potential of a sphere then integrated over the radius of the Earth to form Eq. (1).

$$U_B = -\frac{16G\pi^2\rho^2R^5}{15} \tag{1}$$

Where U_B is the gravitational binding energy, G is gravitational constant, R is the radius of the Earth. The same symbols are used in Eq.(3). Finally subbing in $\rho = \frac{3M}{4\pi R^3}$, using the same convention as before leads to Eq. (2).

$$U_B = -\frac{3GM^2}{5R} \tag{2}$$

Where G is $6.67 \times 10^{-11}\text{m}^3\text{kg}^{-1}\text{s}^{-2}$, M is 5.97×10^{24}kg and R is 6371km taken from Ref. [2].

The gravitational binding energy of the Earth is found to be 2.24×10^{32}J. As the Death Star was meant to destroy any planet and there is no suggestion of it having a variable laser energy, we will use a range of $1.00U_B$ to $10.0U_B$.

Calculating Velocity Change of Earth

Secondly, the velocity change of Earth must be calculated to do this the laser will be considered to instantly provide sufficient photons to destroy Earth as suggested by Ref. [1]. This leads to a scenario where the reflection of the photons causes the velocity of the mirror to increase. The new velocity of the mirror and hence the Earth can be found by conserving the momentum of the system. Assuming, the Death Star hits the centre of the Earth, this means there is no change in the angular momentum of the Earth-laser system so it can be ignored. Eq. (3) considers only the impact of one photon.

$$\frac{2h}{\lambda} + mv_0 = mv \tag{3}$$

Where h is the Planck's constant, m is the mass of the Earth, v_0 is the initial velocity of the Earth and v is the new velocity of the Earth. The symbols will remain the same for the rest of this section. The relativistic mass of the Earth after must be taken into account.

$$m_{rel} = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4}$$

Where m_{rel} is the relativistic mass of the Earth, this will be used for the rest of the paper. The equation for the laser can be made by multiplying the first term with the number of photons in the laser. However as we have already worked out the energy of the laser, we can get an expression for momentum of $p = E_B c$. This can be used in conjunction with Eq. (3) and Eq. (4) to form Eq. (5).

$$2E_B c + mv_0 = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}} v \tag{5}$$

Where E_B is the binding energy of the Earth, this symbol will be used for the remainder of the paper. The mass of the mirror itself is not important as the materials to construct the mirror would come from the Earth therefore have no impact on mass involved in the calculation. Using the same value for the mass of the Earth as before, leads to a new velocity of Earth being 0.9998617325c, this has been shown with all of its significant figures to illustrate how close this speed is to the speed of light. If the energy of the Death Star is even larger than the Earth's new speed will get even closer to the speed of light.

Effect on the orbit

Lastly, the effect on the orbit of the Earth must be calculated. To do this the escape velocity from the surface of the Sun can be calculated by the use of Eq. (6).

$$v_e = \sqrt{\frac{2GM_s}{R_s}} \tag{6}$$

Where v_e is the escape velocity, G is the gravitational constant, M_s is the mass of the Sun and R_s is the radius of the Sun. Where M_s is 1.989×10^{30}kg and R_s is 696340km, the escape velocity from the Sun is found to be 2.06×10^{-3}c. This is less than the new velocity of the Earth. Therefore, the Earth would now be a rogue planet and the Moon would also no longer be within the Earth's orbit.

Conclusion

This paper calculated the binding energy of the Earth and used it to estimate the energy of the Death Star's laser, the proposed idea of a mirror reflecting the Death Star's laser would be an infeasible idea. Whilst the planet would not be destroyed by the laser it would have a high enough velocity to escape the Sun's orbit becoming a rogue planet.

References

[1] Star Wars IV: A New Hope. Directed by George Lucas, Twentieth Century Fox, 1977.

[2] P.A. Tipler and G. Mosca, Physics For Scientists and Engineers, (W.H. Freeman and Company, New York, 2008), Sixth Edition.