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P1_8 Ballet Dancer

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Abstract

In this paper, we investigated the effect of a female dancer's arms on their rotational kinetic energy and therefore how fast they are turning during a pirouette. We did this by calculating the rotational kinetic energy change of a dancer when they have their arms fully outstretched and when they have their arms 50% closer to their rotation axis. In this case, the arm length of the dancer is the dancer's radius when turning. Using the dancer's moment of inertia in both scenarios, we found that the rotational kinetic energy change was a 320% (2 s.f) increase when the dancer's radius was halved. This implies that the dancer's angular velocity will also increase.

Introduction

Ballet is an artistic dance form performed to music, that originated during the Italian Renaissance in the fifteenth century [1]. The most common ballet spin is a pirouette, in which a dancer is tuning on one foot whilst the other is off the ground. During a ballet performance, the faster a dancer turns and therefore the number of turns in a set time period, the more impressed the audience will likely be. Therefore, we have conducted an investigation into how the dancer's arms/radius affects how fast they are turning during a pirouette. In our calculations, we have assumed the dancer is the average mass of a female of 77 kg (2 s.f) [2] and has a human wingspan the same length as their height, 1.6 m (2 s.f) [3]. They are turning on a smooth horizontal floor and rotating around their own axis.

Theory

The dancer's moment of inertia can be calculated using,

$$I = MR^2, \tag{1}$$

where I is the moment of inertia, M is the dancer's mass and R is the dancer's radius.

Rotational kinetic energy is defined as,

$$KE = \frac{1}{2}I\omega^2, \tag{2}$$

where KE is the rotational kinetic energy, I is the moment of inertia and ω is angular velocity.

Rotational kinetic energy can also be expressed in terms of angular momentum,

$$KE = \frac{L^2}{2I}, \tag{3}$$

where KE is the rotational kinetic energy, L is angular momentum and I is the moment of inertia.

As there is no external torque on the dancer, angular momentum is said to be conserved. Therefore, rotational kinetic energy is said to be inversely proportional to the moment of inertia,

$$KE \propto \frac{1}{I}, \tag{4}$$

where KE is the rotational kinetic energy and I is the moment of inertia.

Method

When considering the dancer as a point mass and by substituting a radius of 0.80 m (half of the human wingspan, 2 s.f) and 0.40 m (arms brought 50% to the dancer's vertical axis, 2 s.f) into equation 1, we calculated values for the moment of inertia of 49 kgm^2 (2 s.f) and 12 kgm^2 (2 s.f), respectively. This gives us a percentage decrease in the moment of inertia of 76% (2 s.f). Therefore, we have said that the moment of inertia changes from I to 0.24 I (2 s.f) when the radius is halved. By substituting 0.24 I (2 s.f) into equation 4, the corresponding new rotational kinetic energy will be 4.2 KE (2 s.f), when the radius is 0.40 m (2 s.f). The change in rotational kinetic energy will be:

$$\triangle KE = 4.2KE - KE, \quad (5)$$

where $\triangle KE$ is the change in rotational kinetic energy and KE is the rotational kinetic energy when the radius is 0.80 m (2 s.f).

The change in rotational kinetic energy is 3.2 KE (2 s.f), which means there is a 320% (2 s.f) increase in the rotational kinetic energy of the dancer, when their radius is halved.

Discussion

It is shown in equation 2, that the rotational kinetic energy is proportional to the square of angular velocity. This implies that the 320% (2 s.f) increase in the dancer's rotational kinetic energy is going to increase the dancer's angular velocity dramatically when they bring their arms 50% closer to their own vertical axis. As a result, the dancer will be able to do more pirouettes in a set time period, which is more impressive to the audience.

In this investigation, we have assumed a smooth surface and not taken into account the friction between the dancer's foot and floor. The friction would decrease the change in rotational kinetic energy and also the dancer's angular velocity. Furthermore, the dancer could potentially wobble during the pirouette and move off their own vertical axis.

Conclusion

In this paper, we calculated two values for the dancer's moment of inertia of 49 kgm^2 (2 s.f) and 12 kgm^2 (2 s.f) for the radii of 0.80 m (2 s.f) and 0.40 m (2 s.f) respectively. This is a percentage decrease in the moment of inertia of 76% (2 s.f). Finally, we determined there is a 320% (2 s.f) increase in the rotational kinetic energy of the dancer, when the radius is halved.

References

[1] https://en.wikipedia.org/wiki/Ballet [Accessed 26 October 2022]

[2] https://www.medicalnewstoday.com/articles/321003 [Accessed 26 October 2022]

[3] https://www.healthline.com/health/womens-health/average-height-for-women[Accessed 26 October 2022]