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P4_3 It's a Mad Lego World

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Abstract

In this paper, we investigate what would happen to the orbital radius of the Moon if the Earth was made of Lego. We found that for an Earth of equal radius and therefore equal volume, and keeping orbital velocity constant, the orbital radius of the Moon would decrease from 3.84×10^8m to 3.35×10^7m.

Introduction

Lego is a plastic construction toy, famous throughout the world for it's ability to create almost anything that the user can think of. It's interlocking bricks allow anybody to assemble and disassemble their creation in order to build something completely different. In this paper, we investigate what would happen to the orbital distance of the Moon if the Earth was replaced by Lego whilst keeping the volume of the Earth constant.

Method

To begin with, we need to calculate the volume of Earth using the radius 6.37×10^6m. Using the equation for volume of a sphere

$$V = \frac{4}{3}\pi r^3 \tag{1}$$

We calculate the volume of the Earth to be $1.08 \times 10^{21}\text{m}^3$. Next we need to find the volume of a Lego brick. For this thought experiment we assume that the Earth is made of 1 certain type of brick, in this case the 2x4 brick. We can't use standard length, width and height measurements to work out the volume of a Lego brick as a significant portion is made of air and the shape isn't equal for all points on a surface. However once the Lego bricks are connected these factors can be ignored and standard length, height and width factors can be considered. We can also use the height of a Lego brick without taking into account the studs that sit on top of the brick as when 2 bricks are interlocked, the studs no longer have any effect on the volume of the shape. Using known values for the bricks measurements (1.58×10^{-2}m x 3.18×10^{-2}m x 0.96×10^{-2}m [1]) we can calculate the volume using:

$$V = LWH \tag{2}$$

This gave a value of $4.82 \times 10^{-6}\text{m}^3$. Therefore the amount of Lego bricks needed to make an Earth of equal radius and volume can be calculated by dividing the volume of Earth by the volume of a Lego brick. This gives a value of 2.24×10^{26} bricks. Using this value we can work out the mass of this Lego Earth by multiplying the number of Lego bricks needed by their mass (2.32g [1]). Therefore, the mass of the Lego Earth is 5.21×10^{23}kg. For this paper we have considered that the orbital velocity of the Moon remains constant over the change from the Earth to the Lego Earth. Because of this, we can use the equation for orbital velocity to find the new orbital radius of the Moon after the change from Earth to Lego Earth. Orbital velocity, v can be defined as:

$$v = \sqrt{\frac{GM}{r}} \tag{3}$$

Where G is the gravitational constant ($6.67 \times 10^{-11}\text{m}^3\text{kg}^{-1}\text{s}^{-2}$), M is the mass of the body the object is orbiting and r is the orbital radius. Rearranging to find orbital radius we get:

$$r = \frac{GM}{v^2} \tag{4}$$

By keeping orbital velocity in this situation constant, we can simply relate the mass and orbital radius as v and G are constants:

$$r \propto M \tag{5}$$

This relation can then be used to form a ratio relation between the orbital radius of the Earth-Moon system now and the Earth-Moon system after the change to a Lego Earth:

$$\frac{r_{Earth}}{r_{Lego}} = \frac{M_{Earth}}{M_{Lego}} \tag{6}$$

Where r_{Earth} is the current orbital radius of the Earth-Moon system, r_{Lego} is the new orbital radius of the Lego Earth-Moon system, M_{Earth} is the mass of the Earth and M_{Lego} is the mass of the Lego Earth. Using (6), the mass of the Earth (5.97×10^{24}kg), the mass calculated earlier for the Lego Earth and the current orbital radius of the Moon (3.84×10^8m[2]) we can then calculate the new orbital distance of the system given a constant orbital velocity. This was calculated to be 3.35×10^7m.

Discussion and Conclusion

In conclusion we can see that if the Earth was made of Lego, the Moon would orbit at a much shorter distance then it currently does. This agrees with what we expect as the mass of the Earth has decreased from 5.97×10^{24}kg to 5.21×10^{23}kg and thus, the Moon would have to orbit at a shorter distance in order to maintain the constant orbital velocity that we have assumed. The current orbital distance of the Moon is 3.84×10^8m [2] and according to our calculations, this would, decrease to 3.35×10^7m. A further exploration of these findings that could be made would be to investigate how this decrease in orbital distance would affect the tides on this Lego Earth or what would happen to the distance the Moon orbits at, if we consider the orbital velocity to change as the Earth changes to a Lego Earth. Moreover, the orbital radius of the Sun-Earth system could be explored.

References

[1] Christoph Bartneck (2019). LEGO Brick Dimensions and Measurements - Christoph Bartneck, Ph.D. [online] Christoph Bartneck, Ph.D. Available at: `https://www.bartneck.de/2019/04/21/lego-brick-dimensions-and-measurements/` [Accessed 29 Nov. 2022].

[2] Moon: NASA Science. (2021). Overview — Inside & Out – Moon: NASA Science. [online] Available at : `https://moon.nasa.gov/inside-and-out/overview/` [Accessed 8 October 2022]