Ordered Tomlinson-Harashima Precoding in G.fast Downstream

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***Abstract*—G.fast is an upcoming next generation DSL standard envisioned to use bandwidth up to 212 MHz. Far-end crosstalk (FEXT) at these frequencies greatly overcomes direct links. Its cancellation based on non-linear Tomlinson-Harashima Precoding (THP) proved to show significant advantage over standard linear precoding. This paper proposes a novel THP structure in which ordering of successive interference pre-cancellation can be optimized for downstream with non-cooperating receivers. The optimized scheme is compared to existing THP structure denoted as equal-rate THP which is widely adopted in wireless downlink. Structure and performance of both methods differ significantly favoring the proposed scheme. The ordering that maximizes the minimum rate (max-min fairness) for each tone of the discrete multi-tone modulation is the familiar V-BLAST ordering. However, V-BLAST does not lead to the global maximum when applied independently on each tone. The proposed novel Dynamic Ordering (DO) strategy takes into account asymmetric channel statistics to yield the highest minimum aggregated rate.**

I. Introduction

Digital Subscriber Line (DSL) is dominant broadband access technology due to its ability to fulfill demands for reliable high-data-rate connectivity in a cost-effective way by exploiting the existing infrastructure of twisted-pair copper lines. Upcoming next generation DSL standard, G.fast [1], proceeds in the trend of shortening copper lines (up to 250 m) between Central Office (CO) and Consumer Premised Equipment (CPE) aiming at fiber-like connection (up to 1 Gbps). Short lines enable the usage of wider bandwidth (initially up to 106 MHz extended later to 212 MHz) than used by the foregoing Very-high-bit-rate DSL (VDSL2) standard operating up to 30 MHz. Cancellation of crosstalk between the lines by multi-user processing (denoted as vectoring or signal coordination) has a major impact on system performance. Far-End-Crosstalk (FEXT) is the crosstalk affecting the other end of the line w.r.t. the transmitter as shown for Downstream (DS) in Fig. 1. FEXT is typically canceled by suitable transmitter precoding. If signal coordination is restricted (e.g. multiple non-cooperating providers in the same cable bundle), spectrum coordination (dynamic spectrum management) is applied [2]. Diagonal Precoding (DP) is a linear FEXT cancellation precoding adopted in VDSL2. It performs at the information theoretical limits in VDSL2 band where FEXT channel is much weaker than direct lines [3]. Non-linear FEXT cancellation based on Tomlinson-Harashima Precoding (THP) [4] provides significant gains over linear precoding in G.fast band where FEXT is often as strong as direct lines [5].

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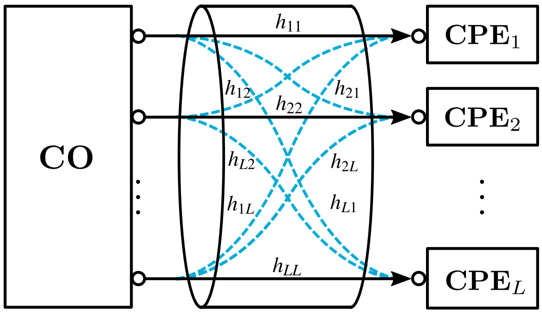

Fig. 1. Downstream FEXT channel model.

This paper modifies the THP proposed in [4] by introducing an ordering of successive interference pre-cancellation performed by CO that is optimized for non-cooperating CPEs. Ordering optimization has been already presented in [6], [7] for a different THP structure which we denote as Equal-Rate THP (ER-THP). ER-THP provides constant Signal-to-Noise Ratio (SNR) at each line. Any type of ordering can be concatenated with the proposed THP scheme. The ordering which maximizes the minimum rate on a single tone of Discrete Multi-Tone (DMT) modulation is V-BLAST (VB) ordering [8], [9]. However, VB does not provide the maximal minimum of aggregated rates when applied on all DMT tones. We propose a novel Dynamic Ordering (DO) strategy that takes into account the asymmetry of G.fast channel statistics. The proposed scheme together with DO provides the maximal minimum rate of $\sim$ 955 Mbps over tested 100 m long paper-insulated cable. The ordering arbitrary adjusting general user demands require considerable computation power [10], complexity of DO is from this perspective negligible.

Notation: Bold upper- and lower-case letters describe matrices and column vectors. $[\mathbf{A}]_{ij} = a_{ij}$ denotes the ijth element of matrix $\mathbf{A}$. Letters $\mathbb{Z}, \mathbb{Z}_j, \mathbb{R}, \mathbb{C}$ refer to integers, complex integers, real and complex numbers, respectively. We denote matrix inversion, transposition and conjugate transposition as $(\star)^{-1}, (\star)^{T}, (\star)^{H}$. Symbol $\triangleq$ denotes equality from definition.

Organization: Section II describes general THP scheme and its properties in which reference method [4] is defined. The proposed scheme is described in Sec. III and some ordering

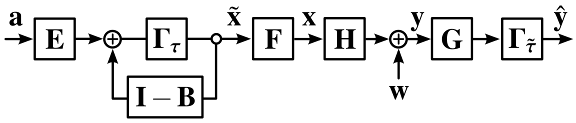

Fig. 2. General Tomlinson-Harashima Precoding scheme.

strategies are in Sec. IV. Comparison to ER-THP, numerical results and conclusions are content of Sec. V, VI and VII.

II. System Model

A. Downstream Channel Model

We assume centralized transmission from CO to non-cooperating CPEs in DS as shown in Fig. 1. DMT is employed to turn the frequency selective channel into a set of frequency flat orthogonal channels. On each tone, the signal received by L CPEs $\mathbf{y} \in \mathbb{C}^L$ is modeled as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}, \tag{1}$$

where $\mathbf{x} = [x_1, \ldots, x_L]^T \in \mathbb{C}^L$ denotes the transmitted signal vector and $\mathbf{w} \in \mathbb{C}^L$ is the AWGN. The cable bundle is assumed to contain only the L lines. We avoid tone indexing to simplify the notation. Main diagonal elements $h_{ii} = [\mathbf{H}]_{ii}$ of channel matrix $\mathbf{H} \in \mathbb{C}^{L\times L}$ characterize insertion loss of direct lines, and off-diagonal elements $h_{ij} = [\mathbf{H}]_{ij}$ with $i \neq j$ characterize FEXT. The channel is static and assumed to be known at the transmitter.

B. General THP Scheme and Basic Properties

We describe considered THP schemes in the common framework in Fig. 2. Linear block $\mathbf{E}$ represents the ordering (or later assumed lattice reduction). The feedback loop consisting of non-linear modulo $\boldsymbol{\Gamma}_{\boldsymbol{\tau}}$ block and linear block given by lower triangular matrix $\mathbf{B}$ with units along the main diagonal implements the inversion of $\mathbf{B}$ while reducing transmitted power by modulo $\boldsymbol{\Gamma}_{\boldsymbol{\tau}}$. $\mathbf{F}$ is a feedforward filter which also ensures transmitted signal to satisfy energy constraints. Diagonal matrix $\mathbf{G}$ describes linear operations performed by non-cooperating receivers. Let the input to the precoding chain be vector $\mathbf{a} = [a_1, \ldots, a_L]^T \in \mathbb{C}^L$ forming data symbols and the output be decision variable vector $\hat{\mathbf{y}} \in \mathbb{C}^L$.

1) Linearized Scheme: Block $\boldsymbol{\Gamma}_{\boldsymbol{\tau}}$ is the modulo function over base τ with origin shifted by $\tau/2$ applied individually along each dimension of the input $\mathbf{x}$. Particularly,

$$\begin{aligned}
\Gamma_\tau[x] &\triangleq (x + \tau/2)_{\mathrm{mod}\,\tau} - \tau/2, \quad x \in \mathbb{R},\\
\Gamma_\tau[x] &\triangleq \Gamma_\tau[\Re\{x\}] + \mathrm{j}\Gamma_\tau[\Im\{x\}], \quad x \in \mathbb{C},\\
\boldsymbol{\Gamma}_{\boldsymbol{\tau}}[\mathbf{x}] &\triangleq \left[\Gamma_{\tau_1}[x_1], \ldots, \Gamma_{\tau_L}[x_2]\right]^T, \quad \mathbf{x} \in \mathbb{C}^L.
\end{aligned}$$

Every modulo reminder equals to the input minus an integer multiple of base τ such that the reminder is lower than τ. Therefore $\boldsymbol{\Gamma}_{\boldsymbol{\tau}}[\mathbf{x}] = \mathbf{x} - \mathbf{d}$, where $\mathbf{d}$ is a vector such that $\mathbf{x} - \mathbf{d} \in [-\tau_1/2, \tau_1/2) \times \cdots \times [-\tau_L/2, \tau_L/2)$. The ith component of $\mathbf{d}$ is $d_i \in \tau_i \mathbb{Z}_\mathrm{j}$ where $\mathbb{Z}_\mathrm{j} \triangleq \mathbb{Z} + \mathrm{j}\mathbb{Z}$ denotes complex integers and $\boldsymbol{\tau} = [\tau_1, \ldots, \tau_L]^T$ is a vector of thresholds. Figure 3 shows the linearized scheme where $\boldsymbol{\Gamma}_{\boldsymbol{\tau}}$ is replaced by additive term $-\mathbf{d}$.

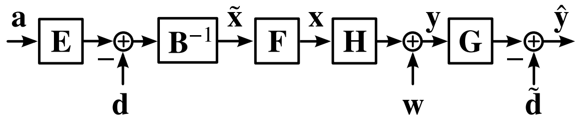

Fig. 3. Linearized general THP scheme.

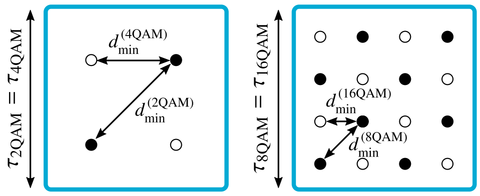

Fig. 4. Threshold τ for several considered square-shape QAM constellations.

2) Zero-Forcing Condition: Zero-Forcing (ZF) precoding inverts the channel by eliminating the crosstalk such that

$$\hat{\mathbf{y}} = \mathbf{GHFB}^{-1}\mathbf{Ea} - \mathbf{GHFB}^{-1}\mathbf{d} + \mathbf{Gw} - \tilde{\mathbf{d}} \tag{2}$$

equals to input data $\mathbf{a}$ plus noise. The whole chain of linear blocks in Fig. 3 needs to fulfill ZF condition

$$\mathbf{GHFB}^{-1}\mathbf{E} = \mathbf{I}, \tag{3}$$

with $\mathbf{I}$ being $L \times L$ identity matrix. Condition (3) implies $\mathbf{GHFB}^{-1} = \mathbf{E}^{-1}$ which leads to $\hat{\mathbf{y}} = \mathbf{a} - \mathbf{E}^{-1}\mathbf{d} + \mathbf{Gw} - \tilde{\mathbf{d}}$. We obtain ZF property $\hat{\mathbf{y}} = \mathbf{a} + \mathbf{Gw}$ when

$$\mathbf{E}^{-1}\mathbf{d} + \tilde{\mathbf{d}} = 0 \tag{4}$$

which is realized by a proper design of thresholds $\boldsymbol{\tau}$ and $\tilde{\boldsymbol{\tau}}$.

3) Modulo Threshold τ: Size of τ is chosen to wrap constellations within $\tau \times \tau$ frame such that the distance from the edge point to the boundary is half of minimal distance $d_{\min}$. Figure 4 shows the frame for several QAM constellations considered in this paper. It is straightforward to verify that square-shaped QAM constellations including odd-bit cardinality variants (black points in Fig. 4) have $\tau = \sqrt{M} d_{\min}$. Table I lists values of τ considered here.

4) Per-Line Power Constraint: Transmitted power on each line is constrained not to overcome a specified limit. Without loss of generality, we use constellations normalized to unit mean symbol energy $E\left[|a_i|^2\right] = 1$ ($E[\star]$ denotes the statistical expectation) for which the energy limit implies

$$E\left[|x_i|^2\right] \leq 1. \tag{5}$$

We need to keep in mind and downscale constellations to pre-compensate energy increase ΔE due to modulo $\boldsymbol{\Gamma}_{\boldsymbol{\tau}}$. Square-shape even-bit QAM constellations have $\Delta E \simeq M/(M-1)$ [4]. The same formula holds for square-shape odd-bit cardinalities if twice higher value of M is used. For example, ΔE is the same for 2QAM and 4QAM as shown in Table I. Odd-bit constellations with a square shape have significantly lower ΔE than popular cross-shaped constellations [5].

TABLE I

Threshold τ and energy increase ΔE due to modulo $\boldsymbol{\Gamma}_{\boldsymbol{\tau}}$ for considered unit-mean square-shape M-QAM constellations.

M	2, 4	8, 16	32, 64	128, 256	512, 1024	2048, 4096
τ	2.83	2.53	2.47	2.45	2.45	2.45
ΔE [dB]	1.25	0.28	0.068	0.017	0.0042	0.0011

C. Reference THP Scheme and Basic Properties

The proposed THP scheme enhances the scheme described in [4] by ordered QR decomposition. Reference scheme [4] is described by definition of the blocks in Fig. 3 as

$$\mathbf{E} = \mathbf{I}, \quad \mathbf{B} = \mathrm{diag}(\mathbf{R})^{-1}\mathbf{R}^H, \quad \mathbf{F} = \mathbf{Q}, \quad \mathbf{G} = \mathrm{diag}(\mathbf{R})^{-1}, \quad (6)$$

where $\mathrm{diag}(\mathbf{R})^{-1} = \mathrm{diag}\left(r_{11}^{-1}, \ldots r_{LL}^{-1}\right)$ with diagonal components $r_{ii} = [\mathbf{R}]_{ii}$. Unitary matrix $\mathbf{Q}$ and upper-triangular $\mathbf{R}$ follow from QR decomposition of transposed channel matrix

$$\mathbf{H}^H = \mathbf{QR}. \quad (7)$$

Let us confirm basic properties introduced in Sec. II-B. The reference scheme fulfills ZF condition (3)

$$\mathbf{GHFB}^{-1}\mathbf{E} = \mathrm{diag}(\mathbf{R})^{-1}\mathbf{R}^H\mathbf{Q}^H\mathbf{QR}^{-H}\mathrm{diag}(\mathbf{R})\mathbf{I} = \mathbf{I}. \quad (8)$$

Transmitted signal meets per-line energy constraint (5)

$$E\left[|x_i|^2\right] = \sum_{j=1}^{L} |q_{ij}|^2 E\left[|\tilde{x}_j|^2\right] = \sum_{j=1}^{L} |q_{ij}|^2 = 1, \quad (9)$$

where $q_{ij} = [\mathbf{Q}]_{ij}$. We use the fact that $\mathbf{x}$ is approximately uncorrelated [6] and the energy increase due to $\Gamma_{\boldsymbol{\tau}}$ has been pre-compensated $E\left[|\tilde{x}_j|^2\right] = 1$. This last equality follows from that unitary $\mathbf{Q}$ has unit-length rows. The decision variable

$$\hat{\mathbf{y}} = \mathbf{a} + \mathrm{diag}(\mathbf{R})^{-1}\mathbf{w} \quad (10)$$

implies that output SNR at the ith line is

$$\gamma_i = \gamma_{base} \cdot r_{ii}^2, \quad (11)$$

where γ_{base} is the baseline input SNR. The main diagonal components $\left\{r_{ii}^2\right\}_{i=1}^{L}$ can attain different values providing different SNR at the each line. In this case, different bit-loading per-line is required as well as different modulo threshold $\tau_i \neq \tau_j$ in $\boldsymbol{\tau} = [\tau_1, \ldots, \tau_L]^T$, see Table I for actual values. There is no integer precoding operation as $\mathbf{E} = \mathbf{I}$ and so selection of $\tilde{\boldsymbol{\tau}} = \boldsymbol{\tau}$ fulfills modulo condition (4).

III. Ordered Tomlinson-Harashima Precoding

A. Ordered QR Decomposition

The ordered THP scheme proposed here incorporates the ordered QR decomposition of transposed channel

$$\mathbf{H}^H = \mathbf{QRP}^T \quad (12)$$

into the reference scheme (Sec. II-C). Permutation matrix $\mathbf{P}$ describes arbitrary permutation $[1, \ldots, L]^T \to [p_1, \ldots, p_L]^T$ as

$$\mathbf{P}\begin{bmatrix} 1 \\ \vdots \\ L \end{bmatrix} = \begin{bmatrix} p_1 \\ \vdots \\ p_L \end{bmatrix}, \quad \text{where} \quad \mathbf{P} = \begin{bmatrix} \mathbf{e}_{p_1} \\ \vdots \\ \mathbf{e}_{p_L} \end{bmatrix} \quad (13)$$

and $\mathbf{e}_i$ denotes a row vector with 1 in the ith position and 0 elsewhere. Note that $\mathbf{PX}$ denotes permutation of rows of $\mathbf{X}$ and $\mathbf{XP}^T$ permutation of columns since $\mathbf{XP}^T = \left(\mathbf{PX}^T\right)^T$.

B. Proposed THP Scheme and Basic Properties

The ordered THP scheme is given by the following matrices

$$\mathbf{E} = \mathbf{P}^T, \mathbf{B} = \mathrm{diag}(\mathbf{R})^{-1}\mathbf{R}^H, \ \mathbf{F} = \mathbf{Q}, \ \mathbf{G} = \mathbf{P}\,\mathrm{diag}(\mathbf{R})^{-1}\mathbf{P}^T. \quad (14)$$

Key observation is that if diagonal matrix has permuted rows and columns by the same permutation (as $\mathbf{G}$ in (14)) then it remains diagonal and can be performed by non-cooperating receivers. Now, we show that ZF condition (3) is satisfied

$$\mathbf{GHFB}^{-1}\mathbf{E} = \mathbf{P}\mathrm{diag}(\mathbf{R})^{-1}\mathbf{P}^T\mathbf{PR}^H\mathbf{Q}^H\mathbf{QR}^{-H}\mathrm{diag}(\mathbf{R})\mathbf{P}^T = \mathbf{I},$$

since $\mathbf{P}^T$ describes inverse permutation and so $\mathbf{PP}^T = \mathbf{I}$. As in the reference scheme in Sec. II-C, feedforward matrix $\mathbf{F}$ is unitary and therefore transmitted signal meets per-line energy constraint (5). The decision variable $\hat{\mathbf{y}} = \mathbf{a} + \mathbf{P}\,\mathrm{diag}(\mathbf{R})^{-1}\mathbf{P}^T\mathbf{w}$ implies output SNR at the ith line to be

$$\gamma_i = \gamma_{base} \cdot r_{p_i p_i}^2, \quad (15)$$

where γ_{base} denotes baseline SNR and p_i is the ith element of permutation output (13). Similarly to the reference THP, different values of main diagonal components $\left\{r_{p_i p_i}^2\right\}_{i=1}^{L}$ require different bit-loading with thresholds $\boldsymbol{\tau} = [\tau_1, \ldots, \tau_L]^T$ where generally $\tau_i \neq \tau_j$. Vector of thresholds $\tilde{\boldsymbol{\tau}}$ needs to be chosen to fulfill condition (4) which means $\tilde{\boldsymbol{\tau}} = \mathbf{P}\boldsymbol{\tau} = [\tau_{p_1}, \ldots, \tau_{p_L}]^T$.

Remark 1. Any type of ordering $\mathbf{P}$ can be concatenated with the proposed ordered THP and it is a degree of freedom to be exploited. The reference scheme is obtained for the ordering $\mathbf{P} = \mathbf{I}$ which means that optimized ordering can only improve the performance of the reference scheme.

IV. Optimized Ordering of THP in G.fast Downstream

There is a rich number of possible orderings (see [11] and references therein) to be concatenated with the scheme proposed here. Generally, different orderings lead to different SNR at each line (15). Optimal selection is a multi-objective optimization problem where utility target considering fairness has significant impact on the result. We mainly focus on max-min fairness by maximizing the minimum rate and thus provide the same quality of service to each CPEs, although we discus sum-rate and simple combination of both as well.

A. V-BLAST (VB) Ordering

The ordering strategy introduced in [8] is the optimal max-min fair ordering maximizing the minimum SNR. The algorithm requires multiple calculations of channel matrix pseudo-inverse and so its complexity is much higher than the complexity of closely-related semi-optimal algorithm [9]. Instead of [8], we pragmatically use [9] since it performs close to the optimum without the computational burdens.

VB ordering [9] is based on Gram-Schmidt (GS) QR decomposition of the transposed channel $\mathbf{H}^H = \tilde{\mathbf{H}} = \mathbf{QR}$. In the ith iteration, the algorithm choses the column vector

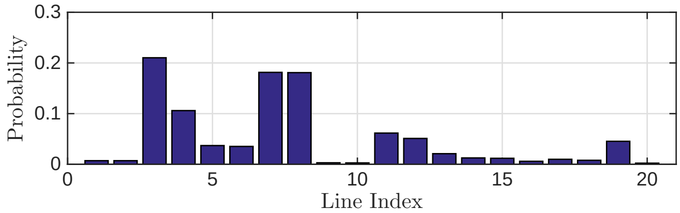

Fig. 5. Empirical probability density function of being weakest (i.e., being selected as the first to enter GS procedure in V-BLAST ordering).

$\tilde{\mathbf{h}}_i$ of $\tilde{\mathbf{H}} = \left[\tilde{\mathbf{h}}_1, \ldots, \tilde{\mathbf{h}}_L\right]$ which minimizes the diagonal element $r_{ii} = [\mathbf{R}]_{ii}$ given as

$$r_{ii} = \left\| \tilde{\mathbf{h}}_i - \sum_{j=1}^{i-1} \left\langle \tilde{\mathbf{h}}_i, \mathbf{q}_j \right\rangle \mathbf{q}_j \right\|, \tag{16}$$

where $\langle \mathbf{h}, \mathbf{q} \rangle = \mathbf{q}^H \mathbf{h}$ denotes an inner product. The order in which $\tilde{\mathbf{h}}$ are chosen forms permutation matrix $\mathbf{P}$ in (12). This strategy ("weakest first") leads to the ordering which maximize the minimum of $\left\{r_{ii}^2\right\}_{i=1}^{L}$ elements and so SNR (15).

B. Inverse V-BLAST (IVB) Ordering

IVB describes ordering with opposite approach than VB. In each GS iteration, always such a column vector $\tilde{\mathbf{h}}_i$ is chosen for which diagonal element (16) is maximal. It is a greedy maximization approach which maximize the sum-rate. We freely interchange sum-rate and mean-rate since the difference is just a scaling factor. The IVB ordering ("strongest first") is also known as QR decomposition with pivoting [12].

C. Dynamic Ordering (DO)

Although VB is the optimal max-min fair ordering, we do not obtain equal rates when aggregated over multiple DMT tones in numerical results in Fig. 11. So, if on a single tone, we selected instead of VB ordering a different ordering in favor of the line with the minimal aggregated rate, we would obtain the higher minimum. It means that VB ordering is optimal on a single tone, but it does not reach the global optimum if applied independently on each tones. The reason behind is that G.fast channel does not have the same statistical properties on each line. Some lines are more often the weakest lines (being selected first by VB) due to asymmetric physical arrangement of twisted-copper pairs within the cable bundle as confirmed by numerical evaluation in Fig. 5.

We propose DO strategy taking into account this statistical asymmetry providing the highest minimum aggregated rate. DO ordering is inspired by VB approach which states that being taken first into GS is an advantage. Instead of VB "weakest first" approach applied independently on each tone, we propose to take first the line with so far minimum aggregated rate ("aggregated minimum first"). DO is ordering with memory deciding the order inductively in sequence. If the ordering on tone index 1 to $i-1$ has been already chosen, then DO orders the lines on the ith tone as the order of bit-loading aggregated over tones from 1 to $i-1$. Figure 6 shows an illustrative example explaining why DO provides higher aggregated minimum than VB. Complexity of DO is

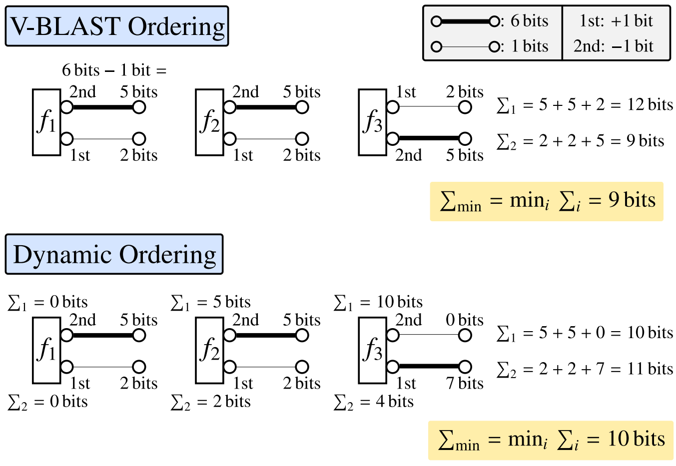

Fig. 6. Let us assume a DMT system with 2 lines and 3 tones $\{f_i\}_{i=1}^{3}$. Thick are strong lines with throughput of 6 bits and thin are weak lines with throughput of 1 bit. Entering GS procedure first is an advantage increasing the throughput by +1 bit and second decreasing the throughput by −1 bit. The minimum aggregated rate $\sum_{\min}$ of V-BLAST ("weakest first") is lower than DO ("aggregated minimum first"). Symbol $\sum_i$ denotes aggregated rate at the ith line. We assume initial order at f_1 of DO to be given by VB.

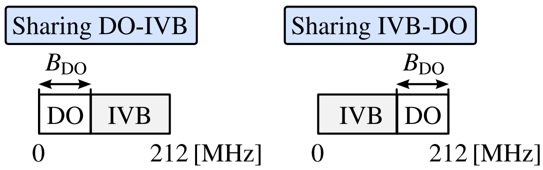

Fig. 7. Frequency sharing between two extrema types of orderings DO (maximizing minimum) and IVB (maximizing sum-rate). Parameter B_{DO} describes bandwidth assigned to DO.

negligible, since the order is given by cumulative summation performed once at the beginning of transmission. The order is computed outside of QR algorithm and can be connected to whatever type of QR implementation, not only the one based on GS as in [9].

D. Frequency-Sharing Between DO and IVB

IVB ordering maximizes sum-rate on a single tone as well as when applied independently on multiple tones, unlike in max-min case of VB and DO ordering. We propose a simple frequency-sharing between two extrema types of ordering IVB and DO to adjust the fairness among CPEs. By frequency-sharing, we mean similar concept as time-sharing, but in the frequency domain. We propose to divide bandwidth on lower and upper parts where we expect different behavior (e.g., diagonal dominant property is present only on lower frequencies as shown in Fig. 10). DO ordering is allocated to lower frequencies in case DO-IVB and to higher frequencies in case IVB-DO as shown in Fig. 7. Numerical evaluation in Fig. 8 shows that DO-IVB sharing achieves better results.

V. Comparison with Ordered Equal Rate THP (ER-THP)

Ordering optimization has been introduced in [6], [7] for THP structure which we denote as ER-THP (term centralized THP is also used [13]). Label 'equal rate' corresponds to the feature that ER-THP provides constant SNR. The proposed THP scheme and ER-THP have essentially different structure.

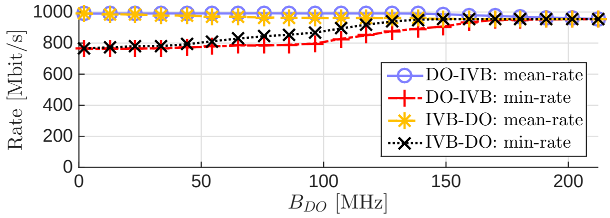

Fig. 8. Frequency sharing between DO and IVB enables to adjust trade off between mean and min-rate. For instance, both min-rate and mean-rate equal to $\sim$ 950 Mbps for DO-IVB sharing with B_{DO} = 212 MHz, but if we permit a slight decrease of minimum, we could have mean-rate $\sim$ 975 Mbps while having minimum still $\sim$ 950 Mbps (here $B_{\mathrm{DO}} \simeq$ 170 MHz). Similarly, when mean-rate is priority, $B_{\mathrm{DO}} \simeq$ 125 MHz increases min-rate from $\sim$ 760 Mbps ($B_{\mathrm{DO}} \simeq$ 0 MHz) to $\sim$ 875 Mbps keeping the same mean-rate.

A. Ordered ER-THP Scheme and Basic Properties

Ordered ER-THP is defined by the following matrices

$$\mathbf{E} = \mathbf{P}^T, \mathbf{B} = \mathbf{R}^H \mathrm{diag}(\mathbf{R})^{-1}, \mathbf{F} = \frac{1}{g}\mathbf{Q}\,\mathrm{diag}(\mathbf{R})^{-1}, \ \mathbf{G} = g\mathbf{I}, \quad (17)$$

where ordered QR decomposition (12) is used. Automatic gain control scaling g establishes power constrain (5) so

$$E\left[|x_i|^2\right] = \frac{1}{g^2}\sum_{j=1}^{L}\left|\tilde{f}_{ij}\right|^2 \le 1, \quad (18)$$

with labeling $\tilde{\mathbf{F}} = \mathbf{Q}\,\mathrm{diag}(\mathbf{R})^{-1}$ and $\left[\tilde{\mathbf{F}}\right]_{ij} = \tilde{f}_{ij}$. The constraint is fulfilled by the following scaling using $(2,\infty)$-mixed norm $\|\star\|_{2,\infty}$ as

$$g^2 = \left\|\tilde{\mathbf{F}}^T\right\|_{2,\infty}^2 \triangleq \max_i \sum_{j=1}^{L}\left|\tilde{f}_{ij}\right|^2 = \max_i \sum_{j=1}^{L} {|q_{ij}|^2}/{r_{jj}^2}. \quad (19)$$

Notice, that average power constrain assumed in [6], [7] leads to the scaling with Frobenius norm $\|\star\|_F$ as $g^2 = {1}/{L}\left\|\tilde{\mathbf{F}}\right\|_F^2 \triangleq {1}/{L}\,\mathrm{tr}\left(\tilde{\mathbf{F}}\tilde{\mathbf{F}}^H\right)$. We confirm that ER-THP meets ZF condition (3)

$$\mathbf{GHFB}^{-1}\mathbf{E} = g\mathbf{IPR}^H\mathbf{Q}^H\frac{1}{g}\mathbf{Q}\,\mathrm{diag}(\mathbf{R})^{-1}\mathrm{diag}(\mathbf{R})\mathbf{R}^{-H}\mathbf{P}^T = \mathbf{I}.$$

The decision variable $\hat{\mathbf{y}} = \mathbf{a} + g\mathbf{w}$ implies output SNR to be

$$\gamma_i = \gamma_{base} \cdot {1}/{g^2}, \quad (20)$$

where γ_{base} denotes baseline SNR. Constant SNR yields the same bit-loading and the same modulo threshold $\boldsymbol{\tau} = [\tau, \ldots, \tau]^T$ on every line, therefore vector of thresholds $\tilde{\boldsymbol{\tau}} = \boldsymbol{\tau}$ fulfills modulo condition (4).

B. VB Ordered and Lattice Reduced (LR) ER-THP

Performance of ER-THP is given by scaling factor g^2. Using inequality ${1}/{r_{ii}^2} \le {1}/{\min_i r_{ii}^2}$, we rephrase (19) as

$$g^2 = \max_i \sum_{j=1}^{L}\frac{|q_{ij}|^2}{r_{jj}^2} \le \max_i \sum_{j=1}^{L}\frac{|q_{ij}|^2}{\min_k r_{kk}^2} = \frac{1}{\min_k r_{kk}^2}. \quad (21)$$

We see that g^2 is minimized when $\min_k r_{kk}^2$ is as large as possible, therefore VB ordering (maximizing the minimum of $\{r_{ii}\}_{i=1}^{L}$) is again preferable. Reference [14] shows that even smaller value of g^2 is obtained with LR QR decomposition

$$\mathbf{H}^H = \mathbf{QRT}^{-1}, \quad (22)$$

where QR decomposes reduced channel as $\tilde{\mathbf{H}} = \mathbf{QR}$ where reduced channel is $\tilde{\mathbf{H}} = \mathbf{H}^H\mathbf{T}$ and $\mathbf{T}$ is a unimodular integer matrix. LR ER-THP is given by (17) using decomposition (22) where $\mathbf{E} = \mathbf{T}^H$. We use familiar LLL implementation of LR with moderate algorithm complexity parameter $\delta = {3}/{4}$ [15].

Unfortunately, LR decomposition (22) cannot be used in the proposed THP scheme (14) with $\mathbf{E} = \mathbf{T}^H$, as matrix $\mathbf{G} = \mathbf{T}^{-H}\,\mathrm{diag}(\mathbf{R})^{-1}\mathbf{T}^H$ (unlike in the case of ordering) is not diagonal anymore and thus cannot be performed by non-cooperating CPEs. For the sake of comparison, we consider ER-THP scheme enhanced by both LR and VB ordering with complexity parameter set to an extreme value $\delta = 1$. The scheme has impractical implementation complexity but gives the highest SNR as confirmed by simulations in Fig. 11.

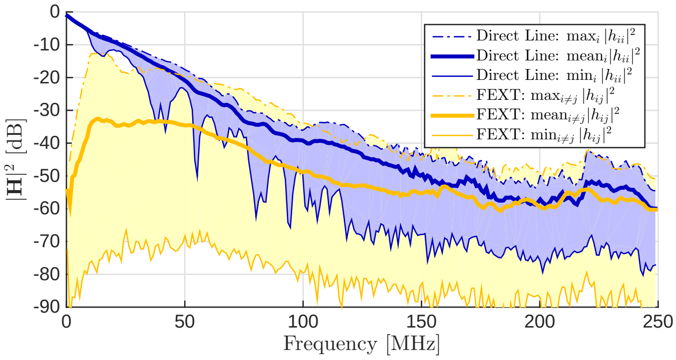

Fig. 9. Direct line and crosstalk power characteristics of tested 100 m long paper-insulated cable. Values are smoothen by average over 1 MHz bin.

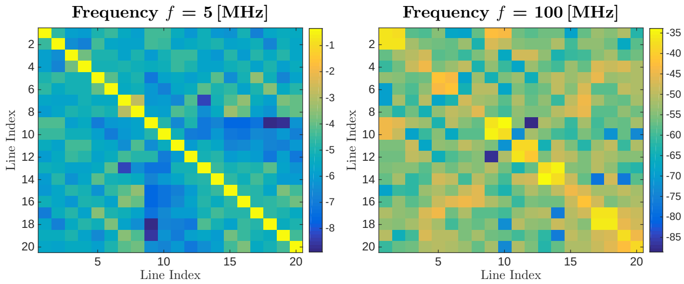

Fig. 10. Diagonal dominant property of channel matrix $|\mathbf{H}|^2$ [dB] at carrier frequency 5 [MHZ] is not present at higher frequency 100 [MHz].

VI. Performance Evaluation over G.fast Channel

A. Tested 100m Long Paper-Insulated Cable

Figures 9 and 10 show strong FEXT of 100m long paper-insulated G.fast cable where diagonal dominant property of VDSL2 is not present any more.

B. Evaluation Procedure and Simulation Parameters

Bit-loading at the ith line is computed according to [16] by insertion of SNR γ_i (11), (15), (20) into the gap formula

$$b_i = \lfloor \log_2\left(1 + {\gamma_i}/{\Gamma}\right)\rfloor, \quad \text{for} \quad b_i \in [2, 12] \quad (23)$$

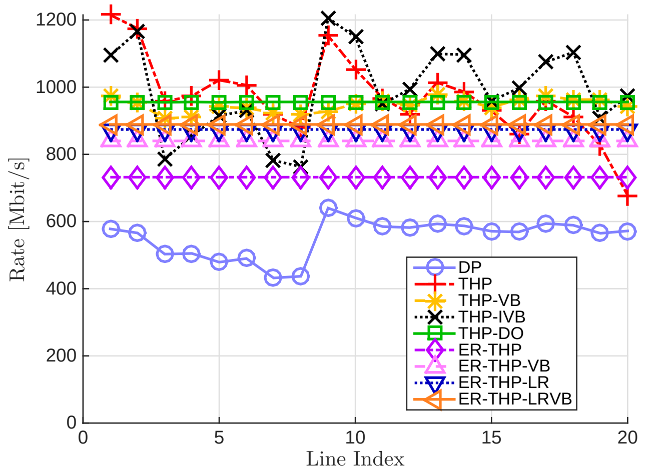

Fig. 11. Aggregated rates of considered precoding schemes over tested 100m long paper-insulated cable. The acronyms and basic rate statistics of considered precoding schemes are summarized in Table III.

TABLE II
SIMULATION PARAMETERS [16]

Transmit PSD	-76 [dBm/Hz]	Coding Gain	5 [dB]
Noise PSD	-140 [dBm/Hz]	Shannon Gap	9.8 [dB]
Band	2.1 – 212 [MHz]	Bit Loading	2 – 12 [bits]
Tone Spacing Δf	51.750 [kHz]	Framing Overhead	12 %
Margin	6 [dB]		

and $b_i = 0$ otherwise, where symbol $\lfloor\star\rfloor$ denotes floor operation and gap Γ = Shannon gap+margin−coding gain [dB]. Aggregated rates are obtained by summation of bit-loading (23) over all DMT tones multiplied by Δf (1 − Framing Overhead), where Δf denotes the tone spacing. Considered parameters are listed in Table II. We use bit-loading modification described in [5] to incorporate energy increases ΔE (shown in Table I) due to modulo $\mathbf{\Gamma}_{\boldsymbol{\tau}}$. The algorithm allocates bits according to (23) and then recomputes SNR corrected by ΔE and update bit-allocation accordingly.

C. Numerical Results

Numerical results in Fig. 11 and Table III compare several FEXT cancellation methods in DS. Non-linear precoding based on THP or ER-THP clearly outperforms linear DP precoding used in VDSL2 [3]. We confirm that ER-THP provides constant aggregated rates to all users, where gains by VB ordering and LR are significant. THP with un-equal rates provides higher sum-rate than ER-THP. As expected, VB ordered THP considerably increase min-rate and IVB ordered THP considerably increase sum-rate. Proposed DO ordered THP provides the highest aggregated minimum rate among all considered methods. The achieved rates are fairly stable vs. line index being close to G.fast target of 1 Gbps.

VII. CONCLUSION

Contribution of this paper is two fold, a new ordered THP scheme and a novel Dynamic Ordering (DO) strategy has been proposed, which together leads to the highest aggregated minimum rate in G.fast downstream. Unlike existing ordered

TABLE III
MEAN AND MINIMUM AGGREGATED RATES OF SEVERAL FEXT CANCELLATION PRECODING SCHEMES DEPICTED IN FIG. 11. THE RATES ARE IN Mbits/s.

Acronym	Precoding Scheme	[mean,min]-rate
DP	Diagonal Precoding	[552, 432]
THP	Tomlinson-Harashima Precoding	[970, 678]
THP-VB	THP using VB ordering	[947, 907]
THP-IVB	THP using Inverse VB ordering	[990, 763]
THP-DO	THP using Dynamic Ordering	[956, 955]
ER-THP	Equal-Rate THP	[732, 732]
ER-THP-VB	ER-THP using VB ordering	[840, 840]
ER-THP-LR	ER-THP using Lattice Reduction	[874, 874]
ER-THP-LRVB	ER-THP-LR using VB ordering	[889, 889]

Equal Rate (ER) THP scheme, the proposed scheme better adapts to asymmetric channel statistics of G.fast channel. Although the results are related to concrete G.fast settings, the proposed scheme has universal application in general multiple-input multiple-output systems including wireless scenario (e.g., paper [13] shows that sum-rate of THP is always higher or equal than sum-rate of ER-THP when the same type of channel matrix decomposition is considered).

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