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Article

On the Rainbow Connection Number of Snowflake Graph

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Abstract. Let *G* be an arbitrary non-trivial connected graph. An edge-colored graph *G* is called a rainbow connected if any two vertices are connected by a path whose edges have distinct colors, such path is called a rainbow path. The smallest number of colors required to make *G* rainbow connected is called the rainbow connection number of *G*, denoted by *rc(G)*. A snowflake graph is a graph obtained by resembling one of the snowflake shapes into vertices and edges so that it forms a simple graph. Let $\mathrm{Snow}(n, a, b, c)$ be a generalized snowflake graph, i.e., a graph with n paths of the stem, a pair of outer leaves, b middle circles, and c pairs of inner leaves. In this paper we determine the rainbow connection number for generalized snowflake graph $\mathrm{Snow}(n, a, b, c)$.

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1. Introduction

The concept of rainbow connection was introduced by Chartrand et al. [1-3] For an arbitrary nontrivial connected graph $G = (V, E)$ and some positive integer k, define $\alpha: E(G) \rightarrow \{1,2, \ldots, k\}$ as an edge-coloring of G, where the adjacent edges can be colored the same. Let u and v be two vertices in G. A (u, v)-path in G is called a rainbow (u, v)-path if all edges are colored differently. Graph G is rainbow connected if for every two vertices $u, v \in V(G)$, there exists a rainbow (u, v)-path. In this case, the coloring α is a rainbow coloring. If α is a rainbow coloring with k colors, then α is a rainbow k-coloring. The rainbow connection number $rc(G)$ is the smallest k such that G has a rainbow k-coloring. Next, graph G is strongly rainbow connected if for every two vertices $u, v \in V(G)$, there exists a rainbow (u, v)-geodesic path. In this case, the coloring β is a strong rainbow coloring. If β is a strong rainbow coloring with k colors, then β is a strong rainbow k-coloring. The strong rainbow connection number $src(G)$ is the smallest k such that G has a strong rainbow k-coloring.

Some previous results related to the strong rainbow and rainbow connection numbers of graph are listed as follows. Chartrand et.al. determined the rainbow connection number of some classes of graphs, such as complete graph, tree, cycle, and wheels. Futhermore, some result of rainbow connection numbers for certain classes of graphs have been obtained, such as triangle-net graph, fan and sun, forest, C_m o P_n and C_m o C_n, triangular snake graph, comb product of graph [4-9]. There are several development obtained such as strong rainbow connection number for some certain graphs [10-11], the rainbow vertex connection [12-14], rainbow disconnection number [15], rainbow vertex-disconnection [16], rainbow k-connectivity [17-18], rainbow antistrong connection [19], rainbow antimagic [20-26], rainbow for spanning tree [27-29], k-rainbow cycle [30], rainbow path in graph[31-32], and 3-Rainbow index [33-34].

A communication network's security measures make use of the rainbow connection number in significant ways. One method for ensuring that any two users of a communication network can communicate securely is to assign unique passwords to each path that connects them (which may involve other users acting as intermediaries) to prevent password repetition. Naturally, it was believed that we would use the fewest feasible passwords. The rainbow connection number represents the bare minimum of these passwords [9,35]. Other applications for rainbow connection numbers are displayed across nine Toronto neighborhoods, analysis of student metacognition skills, and complexity of rainbow coloring problem [36-38].

Constructed the snowflake graph and the generalized snowflake graph, denoted by Snow and $Snow(n, a, b, c)$, respectively [39]. Generalized snow graph is a snow graph with n paths of steam, a pair of outer leaves, b middle circle, and c pair of inner leaves. In this paper we determine the rainbow connection number for generalized snowflake graph $Snow(n, a, b, c)$.

2. Preliminary Notes

As for alternative lower bounds instead of those involving the diameter, Lin Chen et al. [8] note that for any total rainbow connected colouring of G, the colours of the bridges must be pairwise distinct. Similar observations hold for rainbow connected and rainbow vertex-connected colourings, where the colours of the bridges must be pairwise distinct. Hence, the following result holds.

Proposition 2.1. [1], [8] Let G be a connected graph. Suppose that B is the set of all bridges. Then

$$|B| \leq rc(G) \leq src(G).$$

Since every coloring that assigns distinct colors to the edges of a connected graph is both a rainbow coloring and a strong rainbow coloring, every connected graph is rainbow-connected and strongly rainbow-connected with respect to some coloring of the edges of G. Thus the rainbow connection numbers $rc(G)$ and $src(G)$ are defined for every connected graph G.

Proposition 2.2. [1], [33] If G is a nontrivial connected graph of size m whose diameter (the largest distance between two vertices of G) denoted by diam(G). Then,

$$diam(G) \leq rc(G) \leq src(G) \leq m.$$

The definition of a snowflake graph is taken from [10]. a graph obtained by resembling one of the snowflake shapes into vertices and edges so that it forms a simple graph. The generalized snowflake graph is defined and the theorem related to the rainbow connected number of the snowflake graph is given. Stem is a path whose n-2 vertices have degrees greater than two. Inner leaves are pair of vertices have degrees one and adjacent to a vertex that is on the outside of the circle. Middle circle is a cycle subgraph on middle snow graph. Outer leaves are pair of vertices have degrees one and adjacent to a vertex that is on the inside of the circle.

Definition 2.3. [33] A generalized snowflake graph, denoted by $\mathrm{Snow}(n, a, b, c)$, is a graph with n paths of the stem, a pair of inner leaves, b middle circle, and c pairs of outer leaves as shown in Figure 1. The sets of vertices and edges of $\mathrm{Snow}(n, a, b, c)$ are defined as follows.

$V(\mathrm{Snow}) = \{v_0\} \cup \{v_{i,j}, x_{i,k}, y_{i,r}, z_{i,s} | i \in [1, n], j \in [1, a + b + 1], k \in [1, b], r \in [1,2c], s \in [1,2a]\}$

$E(\mathrm{Snow}) = \{v_0, v_{i,1} | i \in [1, n]\} \cup \{v_{i,j}v_{i,j+1} | i \in [1, n]; j \in [1, a + b + c]\} \cup \{v_{i,j}z_{i,2j-1}, v_{i,j}z_{i,2j} | i \in [1, n];$
$j \in [1, a]\} \cup \{v_{i,j}x_{i,j-a}, v_{(i+1)(\mathrm{mod}\ n),j}x_{i,j-a} | i \in [1, n]; j \in [a + 1, b]\} \cup \{v_{i,j}z_{i,2j-2(a+b)-1}$
$v_{i,j}z_{i,2j-2(a+b)} | i \in [1, n]; j \in [a + b + 1, a + b + c]\}$

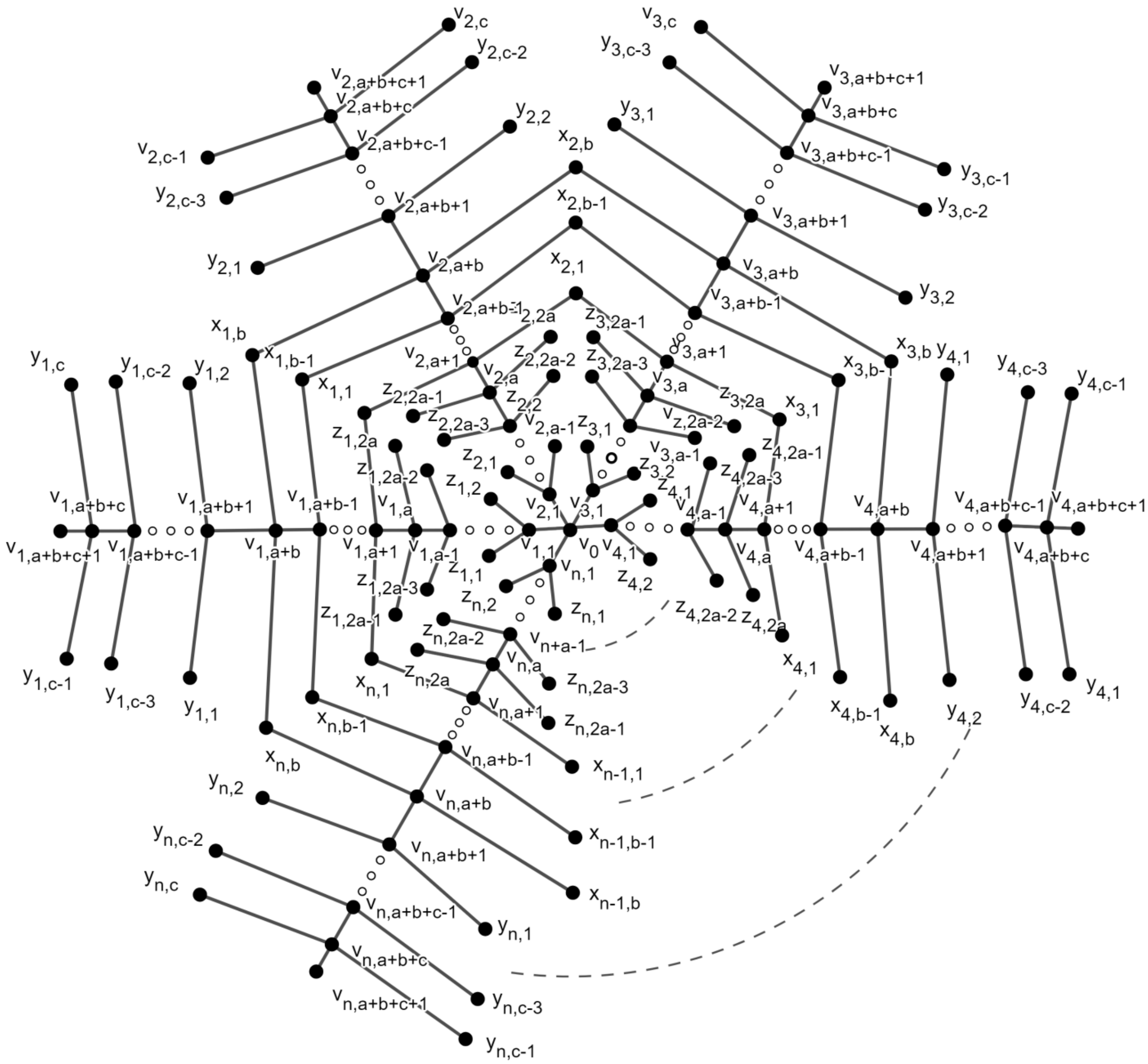

Figure 1. $\mathrm{Snow}(n, a, b, c)$ graph

3. Results and Discussion

In this section we determine the rainbow connection number of generalized snowflake graph for $\mathrm{Snow}(n, a, b, c)$,. To determine such rainbow connected number, we obtain a lemma related to the lower bounds of the rainbow connected number on a graph that contains bridge.

Lemma 3.1. Let $\mathrm{Snow}(n, a, b, c)$ be a graph in Definition 2.3, $B(\mathrm{Snow}(n, a, b, c))$ is the set of all bridges in Snow(n, a,b,c). Then

$$|B(\mathrm{Snow}(n, a, b, c))| = n(2a + 3c + 1).$$

Proof. Let $B = B(\mathrm{Snow}(n, a, b, c))$. We can check easily that the set $B = \{v_{i,j}v_{i,2j-1}, v_{i,j}v_{i,2j} | i\ [1, n]; j \in [1, a]\} \cup \{v_{i,j}v_{i,2(j-a+b)-1}, v_{i,j}v_{i,2(j-a+b)} | i \in [1, n]; j \in [a + b + 1, a + b + c]\} \cup \{v_{i,j}v_{i,j+1} | i \in [1,6]; j \in [a + b, a + b + c]\}$ are bridges. Therefore, $|B| = n(2a + 3c + 1)$.

(Q.E.D)

Theorem 3.2. Let $\mathrm{Snow}(n, a, b, c)$ be a graph in Definition 2.3. Let $n, a, b, c \in \mathbb{N}$ and $n \geq 3$. If $b \leq (3c - 1)(n - 1) + a$, then

$$rc(\mathrm{Snow}(n, a, b, c)) = n(2a + 3c + 1).$$

Proof. From Proposition 2.1 and Lemma 3.1, we have that:

$$rc(\mathrm{Snow}) \geq |B(\mathrm{Snow}(n, a, b, c))| = n(2a + 3c + 1).$$

Next, define an edge-coloring $\alpha: E(\mathrm{Snow}(n, a, b, c)) \to [1, n(2a + 3c + 1)]$ as follows.

$$\alpha(v_0, v_{i,1}) = 2c((i + 1)(\mathrm{mod}\ n) - 1) + 1 \quad ;\ i \in [1, n]$$

$$\alpha(v_{i,j}, v_{i,j+1}) = \begin{cases} 2c((i + 1)(\mathrm{mod}\ n) - 1) + 2j + 1 & ;\ i \in [1, n], j \in [1, a - 1]. \\ 2c((i - 2)(\mathrm{mod}\ n)) + 2j & ;\ i \in [1, n]; i \neq 2, j \in [a, 2a - 1]. \\ 2j & ;\ i = 2, j \in [a, 2a - 1]. \\ 2an + 2c((i + k)(\mathrm{mod}\ n) - 1) & ;\ k \in [1, n - 1]; i \in [1, n], \\ \quad +j - 2a + 3 & \quad j \in [2a + (2c - 2)(k - 1), 2a + (2c - 2)(k) - 1]. \\ 2n(a + c) + (c + 1)((i + 1) & ;\ k \in [1, n - 1]; i \in [1, n]; s = 2a + (2c - 2)(n - 1), \\ \quad (\mathrm{mod}\ n) - 1) + j - s + 1 & \quad j \in [s + (c + 1)(k - 1), s + (c + 1)(k) - 1 < a + b], \\ 2n(a + c) + (c + 1)(i - 1) + & ;\ i \in [1, n], j \in [a + b, a + b + c]. \\ \quad j - (a + b) + 1 & \end{cases}$$

$$\alpha(x_{i,j}, v_{k,j+a}) = \begin{cases} 2an + 2c(i - 1) + 1 & ;\ i \in [1, n], j \in [1, b], k = i. \\ 2an + 2c(i - 1) + 2 & ;\ i \in [1, n], j \in [1, b], k = i + 1. \end{cases}$$

$$\alpha(y_{i,j}, v_{i,s}) = 2an + 2c(i - 1) + j \quad ;\ i \in [1, n], j \in [1,2c], s = \left\lfloor \frac{2(a + b) + j}{2} \right\rfloor.$$

$$\alpha(z_{i,j}, v_{i,\lfloor \frac{j}{2} \rfloor}) = 2a(i - 1) + j \quad ;\ i \in [1, n], j \in [1,2a].$$

Next, we will show that there is a rainbow *(u,v)*-path for every two vertices u and v in $\mathrm{Snow}(n, a, b, c)$. Consider the following cases.

1. If u is adjacent to v, then the rainbow *(u,v)*-path is the edge *uv*.
2. If $d(u, v) = 2$, then the rainbow *(u,v)*-path is the path with length 2.
3. Let $u = v_0$.
   a) If $v = v_{i,j}$, for $i \in [1, n]$ and $j \in [1, a + b + 1]$, then the rainbow *(u,v)*-path is $u, v_{i,1}, v_{i,2}, \ldots, v$.

b) If $v = z_{i,j}$, for $i \in [1, n]$ and $j \in \{2,4,\ldots,2a\}$, then the rainbow *(u,v)*-path is $u, v_{i,1}, v_{i,2}, \ldots, v_{i,\frac{j}{2}}, v$.

c) If $v = z_{i,j}$, for $i \in [1, n]$ and $j \in \{1,3,\ldots,2a-1\}$, then the rainbow *(u,v)*-path is $u, v_{(i+1)(modn),1}, v_{(i+1)(modn),2}, \ldots, v_{(i+1)(modn),a+1}, x_{i,1}, v_{i,a+1}, v_{i,a}, \ldots, v_{i,\frac{j+1}{2}}, v$.

d) if $v = x_{i,j}$, for $i \in [1, n]$ and $j \in [1, b]$, then the rainbow *(u,v)*-path is $u, v_{i,1}, v_{i,2}, \ldots, v_{i,a+j}, v$.

e) If $v = y_{I,j}$, for $i \in [1, n]$ and $j \in [1,2c]$, then the rainbow *(u,v)*-path is $u, v_{I,1}, v_{I,2}, \ldots,$ $v_{I,a+b+\lfloor\frac{j+1}{2}\rfloor}, v$.

4. Let $u = v_{i,j}$.

a) If $v = v_{i,k}$, for $i \in [1, n]$, $j, k \in [1, a + b + c + 1]$, and $j \leq k$, then the rainbow *(u,v)*-path is $u, v_{i,j+1}, v_{i,j+2}, \ldots, v$.

b) If $v = v_{i,k}$, for $i \in [1, n]$, $j, k \in [1, a + b + c + 1]$, and $j > k$, then the rainbow *(u,v)*-path is $u, v_{i,j-1}, v_{i,j-2}, \ldots, v$.

c) If $v = v_{k,l}$, for $I, k \in [1, n]$, $j, l \in [1, a + b + c + 1]$, then the rainbow *(u,v)*-path is $u, v_{I,j-1}$, $v_{i,j-2}, \ldots, v_0, v_{k,1}, v_{k,2}, v$.

d) If $v = z_{i,l}$, for $i \in [1, n]$, $j \in [1, a]$, $j \leq \lfloor\frac{l+1}{2}\rfloor$, and $l \in [1,2a]$, then the rainbow *(u,v)*-path is $u, v_{i,j+1}, v_{i,j+2}, \ldots, v_{\lfloor\frac{l+1}{2}\rfloor}, v$.

e) If $v = z_{I,l}$, for $i \in [1, n]$, $j \in [1, a]$, $j \left\lfloor\frac{l+1}{2}\right\rfloor$, and $l \in [1,2a]$, then the rainbow (u,v)-path is u, $v_{i,j-1}, v_{i,j-2}, \ldots, v_{\lfloor\frac{l+1}{2}\rfloor}, v$.

f) If $v = z_{(i+1)(modn),l}$, for $i \in [1, n]$, $j \in [1, a]$, and $l \in [1,2a]$, then the rainbow (u,v)-path is $u, v_{i,j+1}, v_{i,j+2}, \ldots, v_{i,a+1}, x_{i,1}, v_{(i+1)(modn),a+1}, v_{(i+1)(modn),a}, \ldots, v_{(i+1)(modn),\lfloor\frac{l+1}{2}\rfloor}, v$.

g) If $v = z_{k,l}$, for $I, k \in [1, n]$; $i \neq k$; $k \neq i + 1$, $j \in [1, a]$, and $l \in [1,2a]$, then the rainbow *(u,v)*-path is $u, v_{i,j-1}, v_{i,j-2}, \ldots, v_0, v_{k,1}, v_{k,2}, \ldots, v_{k,\lfloor\frac{l+1}{2}\rfloor}, v$.

h) If $v = x_{k,l}$, for $I, k \in [1, n]$, $j \in [1, a]$, and $l \in [1, b]$, then the rainbow *(u,v)*-path $u, v_{I,j-1}$, $v_{i,j-2}, \ldots, v_0, v_{(k-1)(modn),1}, v_{(k-1)(modn),2}, \ldots, v_{(k-1)(modn),l+a}, v$.

i) If $v = y_{I,k}$, for $i \in [1, n]$, $j \in [1, a]$, and $k \in [1,2c]$, then the rainbow *(u,v)*-path is $u, v_{I,j+1}$, $v_{i,j+2}, \ldots, v_{i,\lfloor\frac{l+2a+2b+1}{2}\rfloor}, v$.

j) If $v = y_{k,l}$, for $I, k \in [1, n]$, $i \neq k$, $j \in [1, a]$, and $l \in [1,2c]$, then the rainbow *(u,v)*-path is u, $v_{i,j-1}, v_{i,j-2}, \ldots, v_0, v_{k,1}, v_{k,2}, \ldots, v_{k,\lfloor\frac{l+2a+2b+1}{2}\rfloor}, v$.

k) If $v = x_{i,k}$, for $i \in [1, n]$, $j \in [a + 1, a + b]$, $k \in [1, b]$, $j + a \leq k$, then the rainbow (u,v)-path is $u, v_{i,j+1}, v_{i,j+2}, \ldots, v_{i,k}, v$

l) If $v = x_{i,k}$, for $i \in [1, n]$, $j \in [a + 1, a + b]$, $k \in [1, b]$, $j + a > k$, then the rainbow (u,v)-path is $u, v_{i,j-1}, v_{i,j-2}, \ldots, v_{i,k}, v$

m) If $v = x_{k,l}$, for $i, k \in [1, n]$, $i \neq k$, $j \in [a + 1, a + b]$, $l \in [1, b]$, $j \leq k$, then the rainbow (u,v)-path is $u, v_{i,j+1}, v_{i,j+2}, \ldots, v_{i,l+a}, x_{i,l}, v_{(i+1)(modn),l+a}, x_{(i+1)(modn),l}, \ldots, v_{k,l+a}, v$

n) If $v = x_{k,l}$, for $i, k \in [1, n]$, $i \neq k$, $j \in [a + 1, a + b]$, $l \in [1, b]$, $j > k$, then the rainbow (u,v)-path is $u, v_{i,j-1}, v_{i,j-2}, \ldots, v_{i,l+a}, x_{i,l}, v_{(i+1)(modn),l+a}, x_{(i+1)(modn),l}, \ldots, v_{k,l+a}, v$

o) If $v = y_{I,k}$, for $i \in [1, n]$, $j \in [a + 1, a + b]$, $k \in [1,2c]$, then the rainbow (u,v)-path is u, $v_{i,j+1}, v_{i,j+2}, \ldots, v_{i,\lfloor\frac{k+2a+2b+1}{2}\rfloor+1}, v$

p) If $v = y_{k,l}$, for $I, k \in [1, n]$, $i \neq k$, $j \in [a+1, a+b]$, $k \in [1,2c]$, then the rainbow (u,v)-path is $u, v_{I,j+1}, v_{I,j+2}, \dots, v_{I,a+b}, x_{I,b+1}, v_{(i+1)(modn),a+b}, x_{(i+1)(modn),b+1}, \dots, v_{k,a+b}, v_{k,a+b+1}, \dots, v_{k,\lfloor\frac{l+2a+2b+1}{2}\rfloor+1}, v$.

q) If $v = v_{k,l}$, for $I, k \in [1, n]$, $i \neq k$, $j, l \in [a+b+1, a+b+c+1]$, then the rainbow (u,v)-path is $u, v_{I,j-1}, v_{I,j-2}, \dots, v_{I,a+b}, x_{(i-1)(modn),b}, v_{(i-1)(modn),a+b}, x_{(i-2)(modn),b}, \dots, v_{k,a+b}, v_{k,a+b+1}, \dots, v$.

r) If $v = y_{i,l}$, for $i \in [1, n]$, $j \in [a+b+1, a+b+c+1]$, $l \in [1,2c]$, and $j \leq \lfloor\frac{l+2a+2c+1}{2}\rfloor$, then the rainbow (u,v)-path is $u, v_{i,j+1}, v_{i,j+2}, \dots, v_{i,\lfloor\frac{l+2a+2c+1}{2}\rfloor}, v$

s) If $v = y_{i,l}$, for $i \in [1, n]$, $j \in [a+b+1, a+b+c+1]$, $l \in [1,2c]$, and $j > \lfloor\frac{l+2a+2c+1}{2}\rfloor$, then the rainbow (u,v)-path is $u, v_{i,j-1}, v_{i,j-2}, \dots, v_{i,\lfloor\frac{l+2a+2c+1}{2}\rfloor}, v$

t) If $v = y_{k,l}$, for $I, k \in [1, n]$, $i \neq k$, $j \in [a+b+1, a+b+c+1]$, and $l \in [1,2c]$, then the rainbow (u,v)-path is $u, v_{I,j-1}, v_{I,j-2}, \dots, v_{I,a+b}, x_{(i-1)(modn),b}, v_{(i-1)(modn),a+b}, x_{(i-2)(modn),b}, \dots, v_{k,a+b}, v_{k,a+b+1}, \dots, v_{k,\lfloor\frac{l+2a+2c+1}{2}\rfloor}, v$.

5. Let $u = z_{i,j}$

a) If $v = v_{i,l}$, for $i \in [1, n]$, $j \in [1,2a]$, and $l \in \{a+1, a+b+c+1\}$, then the rainbow (u,v)-path is $u, v_{i,\lfloor\frac{j+1}{2}\rfloor}, v_{i,\lfloor\frac{j+1}{2}\rfloor+1}, \dots, v$

b) If $v = v_{(i+1)(modn),l}$, for $i \in [1, n]$, $j \in [1,2a]$, and $l \in [a+3, a+b+c+1]$, then the rainbow (u,v)-path is $u, v_{I,\lfloor\frac{j+1}{2}\rfloor}, v_{I,\lfloor\frac{j+1}{2}\rfloor+1}, \dots, v_{I,a+2}, x_{I,2}, v_{(i+1)(modn),a+2}, v_{(i+1)(modn),a+3}, \dots, v$.

c) If $v = v_{(i+1)(modn),l}$, for $i \in [1, n]$, $j \in [1,2a]$, and $l \in [a+1, a+2]$, then the rainbow (u,v)-path is $u, v_{i,\lfloor\frac{j+1}{2}\rfloor}, v_{i,\lfloor\frac{j+1}{2}\rfloor+1}, \dots, v_{i,l}, x_{i,l-a}, v$.

d) If $v = v_{k,l}$, for $I, k \in [1, n]$, $i \neq k$, $k \neq i+1$, $j \in [1,2a]$, and $l \in [a+1, a+b+c+1]$, then the rainbow (u,v)-path is $u, v_{I,\lfloor\frac{j+1}{2}\rfloor}, v_{I,\lfloor\frac{j+1}{2}\rfloor+1}, \dots, v_{I,a+1}, x_{I,1}, v_{(i+1)(modn),a+1}, x_{(i+1)(modn),1}, \dots, v_{k,a+1}, v_{k,a+2}, \dots, v$.

e) If $v = x_{i,l}$, for $i \in [1, n]$, $j \in [1,2a]$, and $l \in [1, b]$, then the rainbow (u,v)-path is $u, v_{i,\lfloor\frac{j+1}{2}\rfloor}, v_{i,\lfloor\frac{j+1}{2}\rfloor+1}, \dots, v_{i,l+a}, v$.

f) If $v = x_{k,l}$, for $i, k \in [1, n], k \leq i$, $j \in [1,2a]$, and $l \in [1, b]$, then the rainbow (u,v)-path is $u, v_{i,\lfloor\frac{j+1}{2}\rfloor}, v_{i,\lfloor\frac{j+1}{2}\rfloor+1}, \dots, v_{i,l+a}, x_{i,l}, v_{(i+1)(modn),l+a}, x_{(i+1)(modn),l}, \dots, v_{k,l+a}, v$.

g) If $v = z_{i,l}$, for $i \in [1, n]$, $j, l \in [1,2a]$, and $j \leq l$, then the rainbow (u,v)-path is $u, v_{i,\lfloor\frac{j+1}{2}\rfloor}, v_{i,\lfloor\frac{j+1}{2}\rfloor+1}, \dots, v_{i,\lfloor\frac{l+1}{2}\rfloor}, v$.

h) If $v = z_{i,l}$, for $i \in [1, n]$, $j, l \in [1,2a]$, and $j > l$, then the rainbow (u,v)-path is $u, v_{i,\lfloor\frac{j+1}{2}\rfloor}, v_{i,\lfloor\frac{j+1}{2}\rfloor-1}, \dots, v_{i,\lfloor\frac{l+1}{2}\rfloor}, v$.

i) If $v = z_{(i+1)(modn),l}$, for $i \in [1, n]$, $l \in [1,2a]$, $j \in \{1,3,2a-1\}$, and $j \leq l$, then the rainbow (u,v)-path is $v_{i,\lfloor\frac{j+1}{2}\rfloor+1}, \dots, v_{i,a+1}, x_{i,1}, v_{(i+1)(modn),a+1}, v_{(i+1)(modn),a}, \dots, v_{i+1,\lfloor\frac{l+1}{2}\rfloor}, v$.

j) If $v = z_{(i+1)(modn),l}$, for $i \in [1, n]$, $l \in [1,2a]$, $j \in \{2,4,6\}$, and $j > l$, then the rainbow (u,v)-path is $u, v_{i,\lfloor\frac{j+1}{2}\rfloor}, v_{i,\lfloor\frac{j+1}{2}\rfloor-1}, \dots, v_0, v_{(i+1)(modn),1}, v_{(i+1)(modn),2}, \dots, v_{(i+1)(modn),\lfloor\frac{l+1}{2}\rfloor}, v$.

k) If $v = z_{(i-1)(mod n),l}$, for $i \in [1,n]$, $l \in [1,2a]$, $j \in \{2,4,6\}$, and $j < l$, then the rainbow (u,v)-path is $u, v_{i,\lfloor\frac{j+1}{2}\rfloor}, v_{i,\lfloor\frac{j+1}{2}\rfloor-1}, \dots, v_0, v_{(i-1)(mod n),1}, v_{(i-1)(mod n),2}, \dots, v_{(i-1)(mod n),\lfloor\frac{l+1}{2}\rfloor}, v$.

l) If $v = z_{(i+1)(mod n),l}$, for $i \in [1,n]$, $l \in [1,2a]$, $j \in \{2,4,6\}$, and $j \geq l$, then the rainbow (u,v)-path is $u, v_{i,\lfloor\frac{j+1}{2}\rfloor}, v_{i,\lfloor\frac{j+1}{2}\rfloor+1}, \dots, v_{i,a+1}, x_{(i-1)(mod n),1}, v_{(i-1)(mod n),a+1}, v_{(i-1)(mod n),a}, \dots,$ $v_{(i-1)(mod n),\lfloor\frac{l+1}{2}\rfloor}, v$.

m) If $v = z_{(i-1)(mod n),l}$, for $i \in [1,n]$, $l \in [1,2a]$, $j \in \{2,4,6\}$, and $j > l$, then the rainbow (u,v)-path is $u, v_{i,\lfloor\frac{j+1}{2}\rfloor}, v_{i,\lfloor\frac{j+1}{2}\rfloor-1}, \dots, v_0, v_{(i-1)(mod n),1}, v_{(i-1)(mod n),2}, \dots, v_{(i-1)(mod n),\lfloor\frac{l+1}{2}\rfloor}, v$.

n) If $v = z_{k,l}$, for $i, k \in [1,n]$, $k \neq \{i-1(mod n), i+1(mod n)\}$, and $i, j \in [1,2a]$, then the rainbow (u,v)-path is $u, v_{i,\lfloor\frac{j+1}{2}\rfloor}, v_{i,\lfloor\frac{j+1}{2}\rfloor-1}, \dots, v_0, v_{k,1}, v_{k,2}, \dots, v_{k,\lfloor\frac{l+1}{2}\rfloor}, v$.

o) If $v = y_{i,l}$, for $i \in [1,n]$, $j \in [1,2a]$, and $l \in [1,2c]$, then the rainbow (u,v)-path is $u, v_{i,\lfloor\frac{j+1}{2}\rfloor}, v_{i,\lfloor\frac{j+1}{2}\rfloor+1}, \dots, v_{i,\lfloor\frac{l+2a+2b+1}{2}\rfloor+1}, v$

p) If $v = y_{k,l}$, for $i, k \in [1,n], k \neq i$, $j \in [1,2a]$, and $l \in [1,2c]$, then the rainbow (u,v)-path is $u, v_{i,\lfloor\frac{j+1}{2}\rfloor}, v_{i,\lfloor\frac{j+1}{2}\rfloor+1}, \dots, v_{i,a+b}, x_{i,b}, v_{(i+1)(mod n),a+b}, x_{(i+1)(mod n),b}, \dots, v_{k,a+b}, v_{k,a+b+1}, \dots,$ $v_{k,\lfloor\frac{l+2a+2b+1}{2}\rfloor+1}, v$.

6. Let $u = x_{i,j}$.

a) If $v_{k,l}$. for $i, k \in [1,n]$, $j \in [1,b]$, and $l \in [a+b+1, a+b+c]$, then the rainbow (u,v)-path is $u, v_{(i+1)(mod n),j+a}, x_{(i+1)(mod n),j}, v_{(i+2)(mod n),j+a}, x_{(i+2)(mod n),j}, \dots, v_{k,j+a}, v_{k,j+a+1}, \dots, v$.

b) If $v = x_{k,l}$, for $i, k \in [1,n]$, $i \neq k$, $j, l \in [a+1, a+b]$, and $j + a \leq k$, then the rainbow (u,v)-path is $u, v_{(i+1)(mod n),j+1+a}, v_{(i+1)(mod n),j+2+a}, \dots, v_{(i+1)(mod n),l+a}, x_{(i+1)(mod n),l},$ $v_{(i+2)(mod n),l+a}, x_{(i+2)(mod n),l}, \dots, v_{k,l+a}, v$

c) If $v = x_{k,l}$, for $i, k \in [1,n]$, $i \neq k$, $j, l \in [a+1, a+b]$, and $j + a > k$, then the rainbow (u,v)-path is $u, v_{(i+1)(mod n),j-1+a}, v_{(i+1)(mod n),j-2+a}, \dots, v_{(i+1)(mod n),l+a}, x_{(i+1)(mod n),l},$ $v_{(i+2)(mod n),l+a}, x_{(i+2)(mod n),l}, \dots, v_{k,l+a}, v$

d) If $v = x_{k,l}$, for $I, k \in [1,n]$, $i \neq k$, $j \in [a+1, a+b]$, and $l \in \{1,3,\dots,2a-1\}$, then the rainbow (u,v)-path is $u, v_{(i-1)(mod n),j+a}, x_{(i-1)(mod n),j}, v_{(i-2)(mod n),j+a}, x_{(i-2)(mod n),j},$ $\dots, v_{k,j+a}, v_{k,j+a+1}, \dots, v_{k,\lfloor\frac{l+2a+2b+1}{2}\rfloor+1}, v$.

e) If $v = x_{k,l}$, for $i, k \in [1,n]$, $i \neq k$, $j \in [a+1, a+b]$, and $l \in \{2,4,\dots,2a\}$, then the rainbow (u,v)-path is $u, v_{(i+1)(mod n),j+a}, x_{(i+1)(mod n),j}, v_{(i+2)(mod n),j+a}, x_{(i+2)(mod n),j}, \dots, v_{k,j+a},$ $v_{k,j+a+1} \dots, v_{k,\lfloor\frac{l+2a+2b+1}{2}\rfloor+1}, v$.

7. Let $u = y_{i,j}$.

a) If $v = y_{i,l}$, for $i \in [1,n]$, $j \in [a+b+1, a+b+c+1]$, $l \in [1,2c]$, and $j \leq l$. , then the rainbow (u,v)-path is $u, v_{i,\lfloor\frac{j+2a+2c+1}{2}\rfloor}, v_{i,\lfloor\frac{j+2a+2c+1}{2}\rfloor-1}, \dots, v_{i,\lfloor\frac{l+2a+2c+1}{2}\rfloor}, v$

b) If $v = y_{i,l}$, for $i \in [1,n]$, $j \in [a+b+1, a+b+c+1]$, $l \in [1,2c]$, and $j > l$. , then the rainbow (u,v)-path is $u, v_{i,\lfloor\frac{j+2a+2c+1}{2}\rfloor}, v_{i,\lfloor\frac{j+2a+2c+1}{2}\rfloor+1}, \dots, v_{i,\lfloor\frac{l+2a+2c+1}{2}\rfloor}, v$

c) If $v = y_{k,l}$, for $i, k \in [1,n]$, $i \neq k$, and $j, l \in [a+b+1, a+b+c+1]$, then the rainbow (u,v)-path is $u, v_{i,\lfloor\frac{j+2a+2c+1}{2}\rfloor}, v_{i,\lfloor\frac{j+2a+2c+1}{2}\rfloor+1}, \dots, v_{i,a+b}, x_{(i-1)(mod n),b}, v_{(i-1)(mod n),a+b},$ $x_{(i-2)(mod n),b}, \dots, v_{k,a+b}, v_{k,a+b+1}, \dots, v_{k,\lfloor\frac{l+2a+2c+1}{2}\rfloor}, v$.

Since every vertex from $V(Snow(n,a,b,c))$ have rainbow-edge path, so we conclude that

$$rc(Snow(n,a,b,c)) \leq n(2a+3c+1).$$

By combining upper bound and lower bound we have

$$rc(Snow(n,a,b,c)) = n(2a+3c+1).$$

(Q.E.D)

Example 3.3. Snowflake graph, donate by Snow, is a graph with 6 paths of the stem, 1 pair of inner leaves, 2 middle circle, and 2 pairs of outer leaves. Such that $Snow = Snow(6,1,2,2)$. Then

$$rv(Snow) = 54$$

Proof. By Theorem 3.2, since $b = 2 < (3c-1)(n-1)+1 = (3(2)-1)(6-1)+1 = 26$, then we have $rv(Snow(6,1,2,2)) = n(2a+3c+1) = 6(2(1)+3(2)+1) = 54$. Therefore,

$$rv(Snow) = 54.$$

In Figure 2 we show the edges coloring of the snow graph.

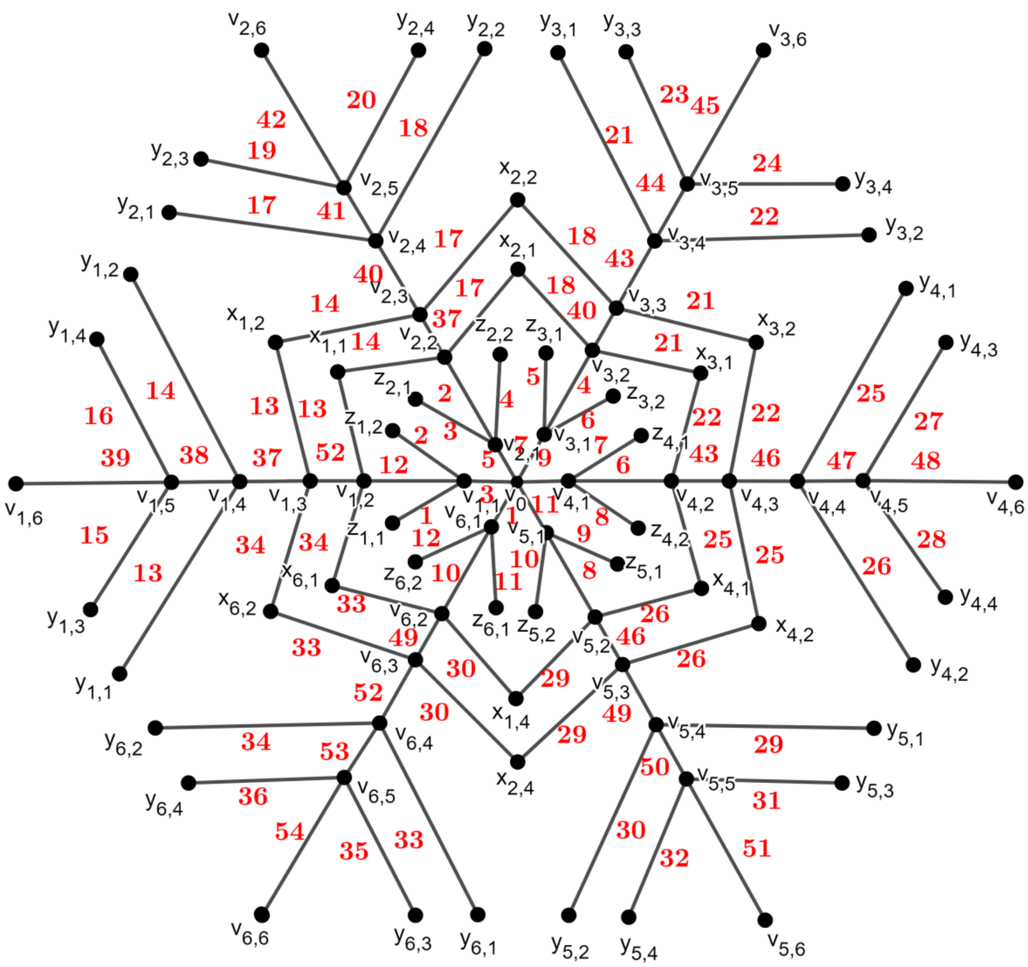

Figure 2. A Rainbow 54-coloring of a snowflake graph

(Q.E.D)

4. Conclusion

In this paper, we have found $rc(Snow(n, a, b, c)) = n(2a + 3c + 1)$ proved in Theorem 3.2 and gave example for $rc(snow) = 54$ in Example 3.3. However, it is interesting to find other measures of these graphs such as edge metric dimension, total metric dimension, partition dimension, strong rainbow connection number, rainbow edge connection number, and chromatic locating number of this graph. In addition, it is worth considering that there are many other graphs that could be formed based on various snowflakes shapes other than the shape we define in this paper.

On the result obtained in this papes, the generalization of rainbow connection number value is based on the assumption of the number of middle circle b, that is determined by the number of outer leaves c and the number of path a. We assume that if there are (3c-1)(n-1)+t middle circle, then the rainbow connection number value is rc(G)+t. The upper bound of this condition could be proven by coloring every 4-degree vertices on t newly added midddle circle. This problem is still an open problem since the proof is not found yet.

References

[1] Chartrand, G., Johns, G. L., McKeon, K. A., & Zhang, P. (2008). Rainbow connection in graphs. *Mathematica Bohemica*, 133(1), 85–98.

[2] Jiang, H., Li, W., Li, X., & Magnant, C. (2021). On proper (strong) rainbow connection of graphs. *Discuss. Math. Graph Theory*, *41*(2), 469-479.

[3] Zhu, X., Wei, M., & Magnant, C. (2021). Generalized Rainbow Connection of Graphs. *Bulletin of the Malaysian Mathematical Sciences Society*, *44*(6), 3991-4002.

[4] Yulianti, L., Nazra, A., Muhardiansyah, & Narwen. (2021). On the rainbow connection number of triangle-net graph. *Journal of Physics: Conference Series*, 012004.1836, 2021.

[5] Sy, S., Medika, G. H., & Yulianti, L. (2013). The Rainbow Connection of Fan and Sun. *Applied Mathematical Sciences*, 64th ed.7, *HIKARI Ltd*: Indonesia, 3155–3159.

[6] Sunil Chandran, L., Issac, D., Lauri, J., & van Leeuwen, E. J. (2022). Upper bounding rainbow connection number by forest number. *Discrete Math*, 345(7), 112829.

[7] Maulani, A., Pradini, S., Setyorini, D., & Sugeng, K. A. (2020). Rainbow connection number of Cm o Pn and Cm o Cn. *Indonesian Journal of Combinatorics*, 3(2), 95–108.

[8] Rocha, A., Almeida, S. M., & Zatesko, L. M. (2020). The Rainbow Connection Number of Triangular Snake Graphs. *Anais do Encontro de Teoria da Computação (ETC)*, 65–68.

[9] Fitriani, D., Salman, A., & Awanis, Z. Y. (2022). Rainbow connection number of comb product of graphs. *Electronic Journal of Graph Theory and Applications (EJGTA)*, 10(2), 461–474.

[10] Chen, L., & Al, E. T. (2018). On various (strong) rainbow connection numbers of graphs. *Australasian Journal Of Combinatorics*, 70(1), 137–156.

[11] Helda Mercy, M., & Annammal Arputhamary, I. (2020). A Study on Strong Rainbow Vertex-Connection in Some Classes of Generalized Petersen Graphs. *Procedia Comput Sci*, 172, 9–15.

[12] Li, W. jing, Jiang, H., & He, J. bei. (2022). Rainbow and Monochromatic Vertex-connection of Random Graphs. *Acta Mathematicae Applicatae Sinica, English Series 2022 38:4*, 38(4), 966–972.

[13] Fauziah, D. A., Dafik, Agustin, I. H., & Alfarisi, R. (2019). The rainbow vertex connection number of edge corona product graphs. *IOP Conf Ser Earth Environ Sci*, 243(1), 012020.

[14] Lima, P. T., van Leeuwen, E. J., & van der Wegen, M. (2021). Algorithms for the rainbow vertex coloring problem on graph classes. *Theor Comput Sci*, 887, 122–142.

[15] Bai, X. Q., Huang, Z., & Li, X. L. (2022). Bounds for the Rainbow Disconnection Numbers of Graphs. *Acta Mathematica Sinica, English Series 2022 38:2*, 38(2), 384–396.

[16] Li, X., & Weng, Y. (2021). Further results on the rainbow vertex-disconnection of graphs. *Bull. Malays. Math. Sci. Soc.*, 44(5), 3445–3460.

[17] Chen, X. lin, Li, X. liang, & Lian, H. shu. (2020). Rainbow k-connectivity of Random Bipartite Graphs. *Acta Mathematicae Applicatae Sinica, English Series 2020 36:4*, 36(4), 879–890.

[18] Bača, M., Salman, A. N. M., Simanjuntak, R., & Susanti, B. H. (2020). Rainbow 2-connectivity of edge-comb product of a cycle and a Hamiltonian graph. *Proceedings - Mathematical Sciences 2020 130:1*, 130(1), 1–12.

[19] Hu, Y., & Wei, Y. (2020). Rainbow Antistrong Connection in Tournaments. *Graphs and Combinatorics 2020 37:1*, 37(1), 167–181.

[20] al Jabbar, Z. L., Dafik, Adawiyah, R., Albirri, E. R., & Agustin, I. H. (2020). On rainbow antimagic coloring of some special graph. *J Phys Conf Ser*, 1465(1), 012030.

[21] B, S., Dafik, S., H, A. I., & R, A. (2020). On Rainbow Antimagic Coloring of Some Graphs. *Journal of Physics: Conference Series ICOPAMBS 2019*, 1465, 1–8.

[22] I, K., I, K. A., Dafik, & R, A. (2019). On The Rainbow Antimagic Connection Number of Some Wheel Related Graphs. *International Journal of Academic and Applied Research (IJAAR)*, 12(3), 60–64.

[23] Budi, H. S., Dafik, Tirta, I. M., Agustin, I. H., & Kristiana, A. I. (2021). On rainbow antimagic coloring of graphs. *J Phys Conf Ser*, 1832(1), 012016.

[24] Joedo, J. C., Dafik, Kristiana, A. I., Agustin, I. H., & Nisviasari, R. (2022). On the rainbow antimagic coloring of vertex amalgamation of graphs. *J Phys Conf Ser*, 2157(1).

[25] Sulistiyono, B., Slamin, Dafik, Agustin, I. H., & Alfarisi, R. (2020). On rainbow antimagic coloring of some graphs. *J Phys Conf Ser*, 1465(1).

[26] Septory, B. J., Utoyo, M. I., Dafik, Sulistiyono, B., & Agustin, I. H. (2021). On rainbow antimagic coloring of special graphs. *J Phys Conf Ser*, 1836(1), 012016.

[27] Rinaldi, G. (2021). Regular 1-factorizations of complete graphs and decompositions into pairwise isomorphic rainbow spanning trees. *Australasian Journal Of Combinatorics*, 80(2), 178–196.

[28] Glock, S., Kühn, D., Montgomery, R., & Osthus, D. (2021). Decompositions into isomorphic rainbow spanning trees. *Journal of Combinatorial Theory, Series B*, *146*, 439-484.

[29] Lu, L., Meier, A., & Wang, Z. (2021). Anti-Ramsey number of edge-disjoint rainbow spanning trees in all graphs. *arXiv preprint arXiv:2104.12978*.

[30] Li, S., Shi, Y., Tu, J., & Zhao, Y. (2019). On the complexity of k-rainbow cycle colouring problems. *Discrete Appl Math (1979)*, 264, 125–133.

[31] Brause, C., Jendrol', S., & Schiermeyer, I. (2021). From Colourful to Rainbow Paths in Graphs: Colouring the Vertices. *Graphs and Combinatorics 2021 37:5*, 37(5), 1823–1839

[32] Babiński, S., & Grzesik, A. (2022). Graphs without a rainbow path of length 3. *arXiv preprint arXiv:2211.02308*.

[33] Awanis, Z. Y., Salman, A., Saputro, S. W., Bača, M., & Semaničová-Feňovčíková, A. (2020). The strong 3-rainbow index of edge-amalgamation of some graphs. *Turkish Journal of Mathematics*, 44(2), 446–462.

[34] Awanis, Z. Y., & Salman, A. N. M. (2019). The 3-rainbow index of amalgamation of some graphs with diameter 2. *J Phys Conf Ser*, 1127(1), 012058.

[35] Hastuti, Y., Agustin, I. H., Prihandini, R. M., & Alfarisi, R. (2019). The total rainbow connection on comb product of cycle and path graphs. *IOP Conference Series: Earth and Environmental Science*, Vol. 243, No. 1, p. 012114.

[36] Bitterman, A. (2021). The Rainbow Connection: A Time-Series Study of Rainbow Flag Display Across Nine Toronto Neighborhoods. *Urban Book Series*, 117–137.

[37] Dafik, Sucianto, B., Irvan, M., & Rohim, M. A. (2019). The Analysis of Student Metacognition Skill in Solving Rainbow Connection Problem under the Implementation of Research-Based Learning Model. *International Journal of Instruction*, 12(4), 593–610.

[38] Eiben, E., Ganian, R., & Lauri, J. (2018). On the complexity of rainbow coloring problems. *Discrete Appl Math (1979)*, 246, 38–48

[39] Fajri, M. R., Fajri, L. H., Ashari, J., Abdurrahman, A., & Arsyad, A. (2022). On the Metric Dimension for Snowflake Graph. *EKSAKTA: Berkala Ilmiah Bidang MIPA*, 23(04), 275–290.