POLITECNICO DI MILANO

Fifth FreeFem workshop on Generic Solver for PDEs:
FreeFem++ and its applications

NUMERICAL INVESTIGATION OF BUOYANCY DRIVEN FLOWS IN TIGHT LATTICE FUEL BUNDLES

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INDEX

INTRODUCTION

NEW SODIUM FAST REACTORS

- Generation-IV International Programme
- breeder or burner reactors
- low pitch-diameter ratio
- natural convection

Reactor	P (mm)	D (mm)	P/D
BN-600	9.82	6.9	1.42
FFTF	7.2644	5.842	1.24
Monju	7.87	6.5	1.21
Phénix	7.8	6.65	1.17
Superphénix	10.5	8.5	1.24
4S	15.1	14	1.08

EXAMPLE: SUPERPHÉNIX

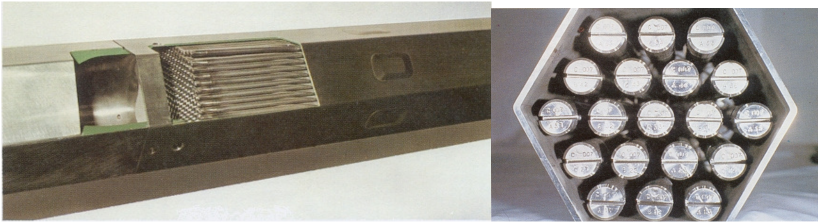


Figure: Detail of fuel assemblies for the Superphénix reactor.

NEW SODIUM FAST REACTORS

- low pitch-diameter ratio
- natural convection

Thermohydraulic consequences:

flow oscillations between subchannels

increased heat, mass and momentum transfer between subchannels

not shown by subchannel analysis codes (`COBRA`, `RELAP`,...)

require modeling

$\hookrightarrow$ can CFD support subchannel analysis codes?

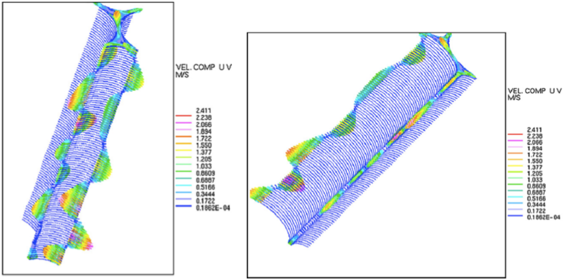
GOAL

can CFD support subchannel analysis codes?

Yes, without thermal coupling (Baglietto, Merzari, Ninokata)

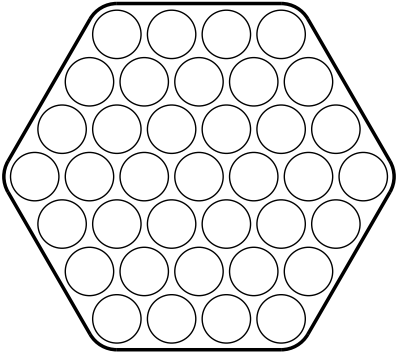
$\hookrightarrow$ How about thermal coupling?

H. Ninokata, E. Merzari and A. Khakim, *Analysis of low Reynolds number turbulent flow phenomena in nuclear fuel pin subassemblies of tight lattice configuration*. Nuclear Engineering and Design 239 (2009)



MATHEMATICAL MODEL

COMPUTATIONAL DOMAIN



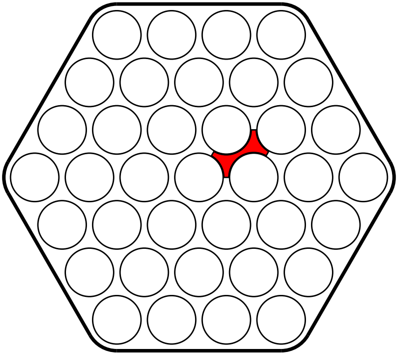
Krauss & Meyer experiment: 37-pin rod bundle

$P/D = 1.06$

too expensive

T. Krauss and L. Meyer, *Experimental investigation of turbulent transport of momentum and energy in a heated rod bundle*. Nuclear Engineering and Design 180 (1998)

COMPUTATIONAL DOMAIN



Krauss & Meyer experiment: 37-pin rod bundle

$P/D = 1.06$

too expensive

$\hookrightarrow$ simulate a small periodic part

T. Krauss and L. Meyer, *Experimental investigation of turbulent transport of momentum and energy in a heated rod bundle*. Nuclear Engineering and Design 180 (1998)

MATHEMATICAL MODEL

The hypotheses

Incompressible flow

Stokesian flow

Boussinesq approximation

The equations

$$
\begin{cases}
\partial_t \boldsymbol{u} - \boldsymbol{u} \times (\nabla \times \boldsymbol{u}) - \nabla \cdot (2\nu \boldsymbol{D}(\boldsymbol{u})) + \nabla p_{\mathrm{T}} = \boldsymbol{g}\beta(\vartheta - \vartheta_0) \\
\nabla \cdot \boldsymbol{u} = 0 \\
\partial_t \vartheta + \boldsymbol{u} \cdot \nabla \vartheta - \alpha \Delta \vartheta = 0 \\
+ \text{ b.c. and i.c.}
\end{cases}
$$

VARIATIONAL FORM

Variational Navier-Stokes

Find $\boldsymbol{u} \in \mathbf{H}^1(\Omega)$, $\boldsymbol{u} = \boldsymbol{t}$ on Γ_D, $p \in \mathrm{L}^2_0(\Omega)$ such that $\forall t > 0$, $\forall \boldsymbol{v} \in \mathbf{H}^1_{0,\Gamma_\mathrm{D}}(\Omega)$, $\forall q \in \mathrm{L}^2(\Omega)$

$$\begin{cases} m(\boldsymbol{u},\boldsymbol{v}) + a(\boldsymbol{u},\boldsymbol{v}) + \widehat{c}(\boldsymbol{u},\boldsymbol{u},\boldsymbol{v}) + b(\boldsymbol{v},p) = F(\boldsymbol{v}) \\ b(\boldsymbol{u},q) = 0 \\ \boldsymbol{u}(t=0,\Omega) = \boldsymbol{u}_0. \end{cases}$$

Variational forms introduced:

$$a(\boldsymbol{u},\boldsymbol{v}) = (\nabla\boldsymbol{v}, \nu\nabla\boldsymbol{u}) \quad b(\boldsymbol{v},p) = -(\nabla\cdot\boldsymbol{v}, p)$$
$$m(\boldsymbol{u},\boldsymbol{v}) = (\boldsymbol{v}, \partial_t\boldsymbol{u}) \quad F(\boldsymbol{v}) = (\boldsymbol{v}, \boldsymbol{f}) + \langle \boldsymbol{v}, \boldsymbol{d}\rangle_{\Gamma_\mathrm{N}}$$
$$\widehat{c}(\boldsymbol{w},\boldsymbol{u},\boldsymbol{v}) = -(\boldsymbol{v}, \boldsymbol{u}\times(\nabla\times\boldsymbol{w}))$$

VARIATIONAL FORM

Variational energy equation

Find $\vartheta \in \mathrm{H}^1(\Omega)$, $\vartheta = \vartheta_\mathrm{D}$ on Γ_D, such that

$$\begin{cases} (\varphi, \mathrm{d}_t \vartheta) + e(\vartheta, \varphi) = \langle \varphi, \alpha \nabla \vartheta \rangle_{\Gamma_\mathrm{N}} & \forall \varphi \in \mathrm{H}^1_{0,\Gamma_\mathrm{D}}(\Omega) \\ \vartheta(t = 0, \Omega) = \vartheta_0 \end{cases}$$

where

$$e(\vartheta, \varphi) = (\nabla \varphi, \alpha \nabla \vartheta)$$

where d_t denotes the total derivative:

$$\mathrm{d}_t \vartheta = \partial_t \vartheta + \boldsymbol{u} \cdot \nabla \vartheta$$

WELL-POSEDNESS

Before trying to solve a problem, see if it is correctly posed

Hadamard definition

A problem is well posed if:

- a solution exists
- the solution is unique
- the solution depends continuously on data

WELL-POSEDNESS

Proved for energy equation, if $\boldsymbol{u}$ is sufficiently regular (Hille-Yosida).

For Navier-Stokes,

Caution

- existence of weak solutions $\rightarrow$ shown
- uniqueness of weak solutions $\rightarrow$ open problem (proved for small times or small data)
- regularity of weak solutions $\rightarrow$ open problem (only partial regularity results)

NUMERICAL METHOD

GALERKIN PROJECTION

Choose two finite dimensional spaces:

$\boldsymbol{V}_h \subset \mathbf{H}^1(\Omega)$ for velocity
$Q_h \subset \mathrm{L}^2(\Omega)$ for pressure

and project the continuous solution onto these spaces.

How to choose $\boldsymbol{V}_h$ and Q_h? Many possibilities:

- Lagrangian elements $\mathbb{P}_0, \ldots, \mathbb{P}_4$
- Discontinuous elements $\mathbb{P}_{0\mathrm{dg}}, \ldots, \mathbb{P}_{4\mathrm{dg}}$
- Boundary elements (implemented using $\mathbb{P}_{0\mathrm{edge}}$)
- Mortar
- ...

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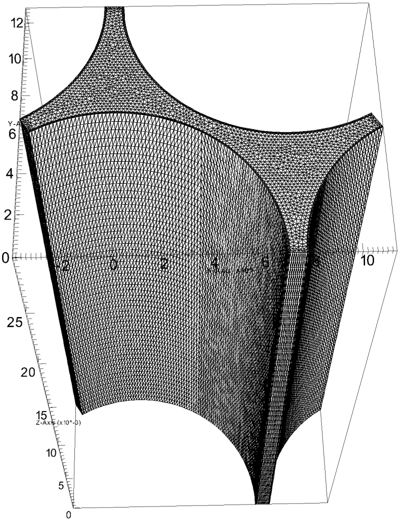
Boundary elements (implemented using $\mathbb{P}_{0\mathrm{edge}}$)
Mortar
...

TRIANGULATION

With **bamg** and **TetGen**:



FINITE ELEMENT METHOD

Write velocity and pressure as linear combination of the basis functions $\{\boldsymbol{\phi}_j\}$ and $\{\psi_k\}$ for each element:

$$\boldsymbol{u}_h = \sum_{j=1}^{N_u} u_j \boldsymbol{\phi}_j \qquad p_h = \sum_{k=1}^{N_p} p_k \psi_k.$$

New unknowns: the nodal values $\{u_j\}$ and $\{p_k\}$.
Substituting into variational Navier-Stokes, and projecting on each dof $(\boldsymbol{v}_h = \boldsymbol{\phi}_j, p_h = \psi_k)$:

$$\begin{cases} m(\boldsymbol{v}_h, \boldsymbol{u}_h) + a(\boldsymbol{v}_h, \boldsymbol{u}_h) + \widehat{c}(\boldsymbol{v}_h, \boldsymbol{u}_h, \boldsymbol{u}_h) + b(\boldsymbol{v}_h, p_h) = F(\boldsymbol{v}_h) \\ b(\boldsymbol{u}_h, q_h) = 0 \qquad \forall \boldsymbol{v}_h \in \boldsymbol{V}_h, \, \forall q_h \in Q_h. \end{cases}$$

FINITE ELEMENT METHOD

Still a nonlinear problem
$\hookrightarrow$ Implicit Euler in time + Picard linearization
For each time step $n+1$, solve the problem:

$$\begin{cases} \dfrac{\boldsymbol{u}^{n+1}}{\Delta t} - \dfrac{\boldsymbol{u}^{n}}{\Delta t} - \boldsymbol{u}^{n} \times (\nabla \times \boldsymbol{u}^{n+1}) - \nabla \cdot (2\nu \boldsymbol{D}(\boldsymbol{u}^{n+1})) - \nabla p_{\mathrm{T}} = \boldsymbol{g}\beta\vartheta^{n} \\ \nabla \cdot \boldsymbol{u}^{n+1} = 0 \\ \dfrac{\vartheta^{n+1}}{\Delta t} - \dfrac{\vartheta^{n}}{\Delta t} + \boldsymbol{u}^{n} \cdot \nabla \vartheta^{n+1} - \alpha \Delta \vartheta^{n+1} = 0. \end{cases}$$

FINITE ELEMENT METHOD

Eventually, a linear algebra problem appeared:

$$\begin{bmatrix} \mathcal{C}^n & \mathcal{B}^T \\ \mathcal{B} & 0 \end{bmatrix} \begin{Bmatrix} U^{n+1} \\ P^{n+1} \end{Bmatrix} = \begin{Bmatrix} G^{n+1} \\ 0 \end{Bmatrix}$$

Main difficulties

- saddle point problem
- pressure locking (incompressibility)
- non symmetric matrix

FINITE ELEMENT METHOD

The algebraic formulation reads:

$$\begin{bmatrix} \mathcal{C}^n & \mathcal{B}^T \\ \mathcal{B} & 0 \end{bmatrix} \begin{Bmatrix} U^{n+1} \\ P^{n+1} \end{Bmatrix} = \begin{Bmatrix} G^{n+1} \\ 0 \end{Bmatrix}$$

Main difficulties

- saddle point problem
  $\hookrightarrow$ *bubble-stabilization on velocity*
- pressure locking (incompressibility)
  $\hookrightarrow$ *add penalization*
- non-symmetric matrix
  $\hookrightarrow$ *GMRES*

TURBULENCE MODEL

cannot resolve all the motions' scales (too expensive)
$\hookrightarrow$ solve only the large eddies, and model the small eddies

Smagorinsky LES

model the unresolved scales with subgrid diffusion:

$$s(\boldsymbol{v}_h, \boldsymbol{u}_h^n)\boldsymbol{u}_h^{n+1} = (\nabla \boldsymbol{v}_h, -2C_{\mathrm{S}}^2\Delta^2|\boldsymbol{D}(\boldsymbol{u}_h^n)|\boldsymbol{D}(\boldsymbol{u}_h^n))\boldsymbol{u}_h^{n+1}$$

to be added to momentum balance equation
similarly for energy balance equation:

$$(\nabla\varphi_h, \boldsymbol{h}) = -(\nabla\varphi_h, \frac{\nu_{\mathrm{T}}}{\mathrm{Pr}_{\mathrm{T}}}\nabla\vartheta_h^{n+1})$$

RESULTS

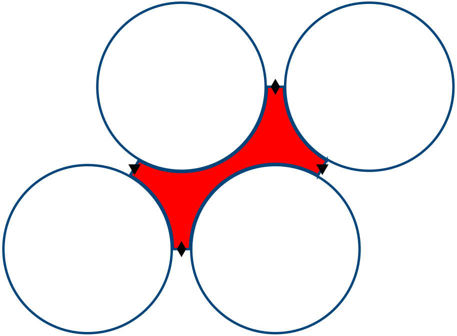
GENERAL PARAMETERS

Main data

- $P/D = 1.06$
- $\mathrm{Re} = 38\,754$
- $\mathrm{Gr} = 1181$
- $q'' = 1.05 \cdot 10^4\ \mathrm{W m^{-2}}$

Software used: `FreeFem++-mpi`

A NOTE ON PERIODICITY



curved sides: no-slip for velocity and imposed heat flux for energy
straight sides: periodic b.c.

A NOTE ON PERIODICITY

velocity: fully periodic
periodic pressure and temperature are not physical
$\hookrightarrow$ decompose pressure and temperature:

$$p_{\mathrm{T}}(\boldsymbol{x},t) = \frac{\Delta p}{H} z + \widetilde{p}_{\mathrm{T}}(\boldsymbol{x},t) \qquad T(\boldsymbol{x},t) = \frac{\Delta T}{H} z + \widetilde{T}(\boldsymbol{x},t)$$

and impose periodic b.c. only on the fluctuating part $\widetilde{p}_T$, $\widetilde{T}$
New equations:

$$\begin{cases} \partial_t \boldsymbol{u} - \boldsymbol{u} \times \nabla \times \boldsymbol{u} - \nabla \cdot (2\nu \boldsymbol{D}(\boldsymbol{u})) + \nabla \widetilde{p}_{\mathrm{T}} = \boldsymbol{g}\beta(\vartheta - \vartheta_0) - \frac{\Delta p}{H} \boldsymbol{\kappa} \\ \nabla \cdot \boldsymbol{u} = 0 \\ \partial_t \widetilde{\vartheta} + \boldsymbol{u} \cdot \nabla \widetilde{\vartheta} - \alpha \Delta \widetilde{\vartheta} = -\frac{\Delta T}{H} u_z \end{cases}$$

WALL TEMPERATURE

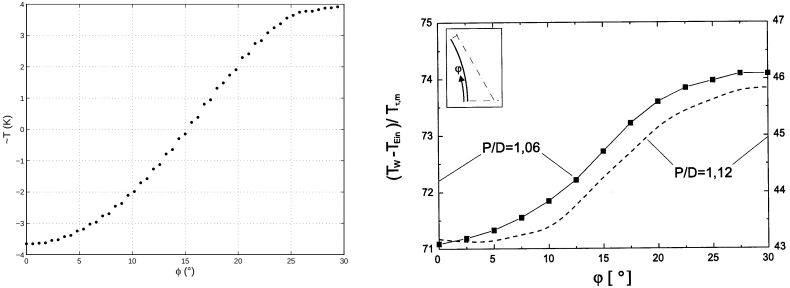
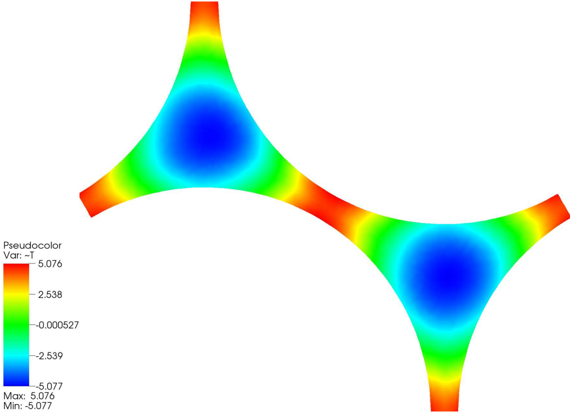


Figure: Profile of time averaged wall temperature as computed (left) and from the results of Krauss and Meyer (right).

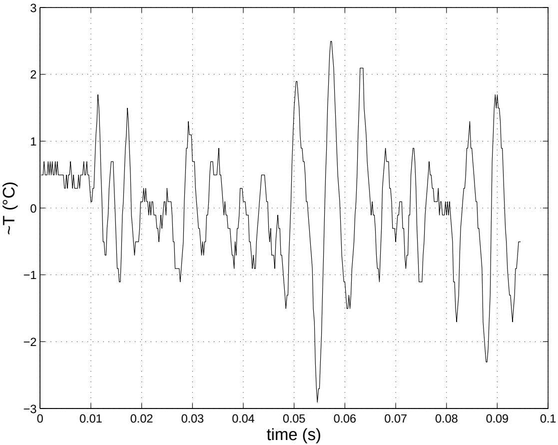
AVERAGED TEMPERATURE



(a) (b)

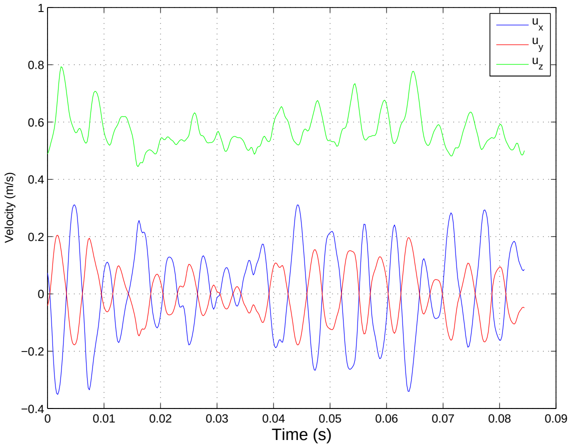
Figure: Time averaged fluctuating temperature along the mid-height slice plane.

TEMPERATURE TIME EVOLUTION



Oscillation frequency: 117 Hz, in line with experiment

VELOCITY TIME EVOLUTION



Oscillation frequency: 117 Hz, in line with experiment

COHERENT STRUCTURES

Defined by Zaman and Hussain as

> Connected, large-scale turbulent fluid mass with a phase correlated velocity over its spatial extent.

or, iso-value for the Q-factor:

$$Q = \mathrm{II}_{\nabla \boldsymbol{u}} = \frac{1}{2}(\boldsymbol{\Omega}\boldsymbol{\Omega} - \boldsymbol{D}\boldsymbol{D})$$

where

$$\boldsymbol{\Omega} = \frac{1}{2}(\nabla \boldsymbol{u} - \nabla \boldsymbol{u}^T)$$
$$\boldsymbol{D} = \frac{1}{2}(\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^T)$$

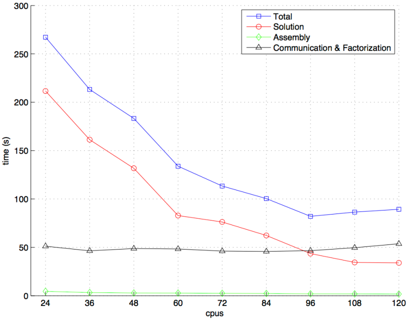
K. B. M. Q. Zaman and A. K. M. F. Hussain, *Taylor hypothesis and large-scale coherent structures.* Journal of Fluid Mechanics 112 (1981)

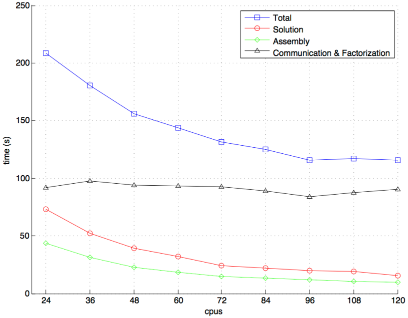
HOW TO TREAT THE CONVECTIVE TERM?

Problem	Matrix	RHS	Factorization	Solution	All
2d str A	11.72	0.595	8.99	20.74	47.36
2d str R	11.11	0.613	10.21	22.38	54.72
2d str L	7.09	9.265	6.268	19.73	49.95
3d str A	5.318	4.062	14.70	10.27	70.32
3d str R	5.570	6.497	15.90	10.59	73.15
3d str L	2.058	25.76	15.61	10.84	70.77
3d ustr A	4.186	0.997	15.93	10.48	41.55
3d ustr R	5.939	0.975	13.41	17.08	50.52
3d ustr L	3.739	28.80	16.88	9.498	78.56

$$\mathrm{A}: \boldsymbol{u}\cdot(\nabla\boldsymbol{u}) \quad \mathrm{R}: \boldsymbol{u}\times(\nabla\times\boldsymbol{u}) \quad \mathrm{L}: \frac{D\boldsymbol{u}}{Dt}$$

SCALABILITY





CONCLUSIONS

CONCLUSIONS

Results agree quite well with experimental data:

- oscillations' frequency
- wall temperature distribution

PERSPECTIVES

- full fuel bundle simulation
- domain decomposition
- improve turbulence modeling (VMS)
- POD in time and Reduced Basis for optimal control

THANK YOU FOR YOUR ATTENTION

QUESTIONS? SUGGESTIONS? NEW IDEAS?

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