Progress In Electromagnetics Research B, Vol. 58, 59–69, 2014

Optimization of a Dual Ring Antenna by Means of Artificial Neural Network

Linh H. Manh*, Francesco Grimaccia, Marco Mussetta, and Riccardo E. Zich

Abstract—In literature, heuristic algorithms have been successfully applied to a number of electromagnetic problems. The associated cost functions are commonly linked to full-wave analysis, leading to complexity and high computational expense. Artificial Neural Network is one of the most effective biological inspired techniques. In this article, an efficient surrogate model is trained to replace the full-wave analysis in optimizing the bandwidth of microstrip antenna. The numerical comparison between ANN substitution model and full-wave characterization shows significant improvements in time convergence and computational cost. To verify the robustness of this approach, all these concepts are integrated into a case study represented by a rectangular ring antenna with proximity-coupled feed antenna.

1. INTRODUCTION

Thanks to the advantage of being small, lightweight and capable of conforming to the shape of the body, microstrip ring resonators have been adopted successfully in a wide range of applications, from medical area [1] to the satellite mobile communications [2]. In antenna design engineers always have to deal with difficult electromagnetic problems as obtaining desired radiation pattern and usable bandwidth. In order to properly understand and solve such problems, every time designer should adjust several degrees of freedom tuning electrical parameters and appropriately choosing the exciting source [3]. The trade-off among all the degrees of freedom becomes quite complex and standard analytics, or direct antenna synthesis are often not applicable. In this context, it is possible to exploit advantages of evolutionary algorithms which allow to effectively and simultaneously manage several parameters to maximize an objective function representing the desired configuration of a certain problem [4]. One of the most popular stochastic techniques is the Particle Swarm Optimization (PSO) algorithm which provides a great capability of exploring solution domain without getting trapped in local minima. By introducing additional term in the equations which control the evolution, a class of variation of conventional PSO called Meta-PSO was recently proposed [5]. Meta PSO was proven to be a robust global optimizer for EM problems in [6, 7].

In this work, we investigate a multi-layer antenna with dual square ring on the top layer, proximity coupled to a feeding microstrip line located in the middle of the two layers, as shown in Figure 1. The ground plane is situated on the bottom layer. The antenna is designed to resonate at ISM band, specifically in the frequency range from 2.4 GHz to 2.48 GHz.

The starting point of the design procedure is the optimization of the antenna bandwidth by adopting a full wave FEM approach and heuristic technique [8]. According to the scheme depicted in Figure 2, return loss is retrieved as a function of geometrical parameters, and the evaluation is based on the interaction between optimizer and EM modelling. However, this approach is computationally expensive since it requires a full wave analysis for each time of assessment. In addition, the storage of data produced by these simulations needs a large amount of dynamic memory. With the aim of

Received 28 November 2013, Accepted 14 January 2014, Scheduled 20 January 2014
* Corresponding author: Linh Ho Manh (linh.ho@polimi.it).
The authors are with the Department of Energy, Politecnico di Milano, Italy.

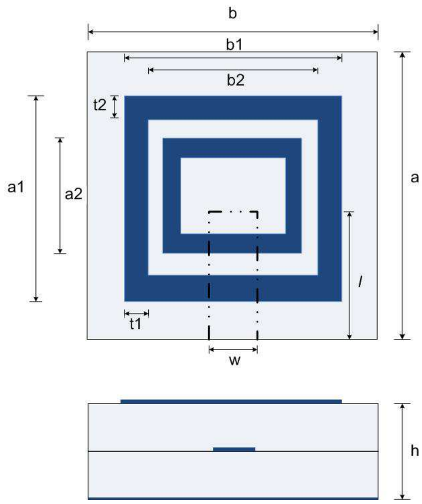

Figure 1. Top view and side view of the antenna.

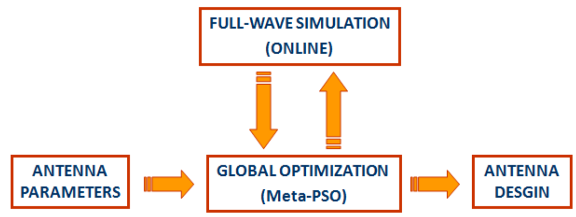

Figure 2. The block diagram of conventional optimization scheme.

reducing these computational efforts and memory consumption, in this paper the authors introduce a simplified equivalent model of the antenna, which will be directly managed by the optimization tool. Artificial Neural Networks (ANNs) have already been successfully used as a powerful tool in EM-based modeling [9] and inverse scattering problem [10]. They are considered as non-linear, general-purpose interpolator systems that once they are trained sufficiently, they can give the output for every input within the training space [11, 12].

In this article, a novel surrogate model of the antenna is introduced, in order to be able to reproduce its behavior with a significant reduction of computational cost with respect to [8]. It is based on the use of Artificial Neural Network to compute the reflection coefficient of the antenna at any certain values of parameters. By implementing separated Neural Networks for different outputs and Levenberg-Marquandt algorithm, the novel approach here proposed is aimed to achieve better performance in terms of time of convergence and accuracy. Afterwards, to check the precision of the proposed method, the resulting configuration will be validated by the full-wave simulator.

2. DIFFERENTIATED META-PSO ALGORITHM

The construction of conventional PSO is based on a model of social interaction between independent individuals (particles) and uses social knowledge (i.e., the experience accumulated during the evolution) in order to search the parameter space by controlling the trajectories of the particles according to a swarm-like set of rules [13]. The position of each particle, which represents a particular solution of the problem, is used to compute the value of the function to be optimized. Individual particles traverse the problem hyperspace and are attracted by both the position of their best past performance and the position of the global best performance of the whole swarm.

A specific class of PSO that has been used in this study is Meta-PSO, which essentially consists in multiple swarms which introduce added terms to the procedure like inter-swarm or "racial knowledge". The basic idea of MPSO using several swarms (denoted by index $j = 1, 2, \ldots, N_s$) instead of one is to reach a better and faster exploration capability, lowering the chances of being trapped in potential local minima. Each particle $i = 1, 2, \ldots, N_p$ in a swarm j is characterized by its position in the n-dimensional parameter space and its velocity $V_{j,i}$.

The effectiveness of MPSO can be further enhanced by keeping swarms apart from each other and in this way widening the global research [14]. This ability can be fulfilled by introducing an inter-swarm repulsion: it allows the best swarm to keep exploring the surroundings of the current best position, whereas other swarms are repelled and obliged to extend the search in other points of the space, hence improving the possibility of escaping from the local minimum.

2.1. Interpretation of Optimization Scheme

Sampling target data and using neural networks are two discrete steps. Regarding this "Regular Sampling Method", the desired outputs are obtained by full-wave analysis from formally chosen geometrical inputs in the region of interest. Each parameter is selected by 5 values, and more variables to be optimized also mean that the needed training set grows exponentially, as denoted in Table 1.

Table 1. Computational cost for different problems.

Assessments	3 inputs	4 inputs	5 inputs
Number of samples	25	125	625
Time consumption	40 mins	3.5 hour	18 hours

Knowledge extracted from physical model is used as the target data for training Artificial Neural Network. After being trained successfully, the satisfied ANN will be employed as an equivalent model to full-wave analysis. Since ANN architecture only deals with binary and simple activation function, this surrogate model saves a critical amount of execution time. The best results ever found by ANN will be validated by full-wave analysis in order to check the accuracy of the simplified model.

3. ARTIFICIAL NEURAL NETWORK

An artificial neural network (ANN) is a well-known computational model that simulates the features and behaviors of human brain neurons. It consists of a pool of simple processing units (neurons or cells) which communicate by sending signals to each other over a large number of weighted connections. It is also known that ANN is a self-adaptive modeling tool that changes its structure on the basis of external or internal information that flows through the network during the learning phase. In many practical terms, neural network are non-linear statistical data modeling tool that can be used to build up the complex relationship between inputs and outputs. In order to estimate the electromagnetic field, ANN has been used rather than full-wave analysis, and the good agreement was shown in [15] between the predictions and measurements.

Thanks to the immense potential of ANN, it has been intensively employed in many applications such as control of non-linear system function of its parameters using Multiple Layer Perceptron Neural Network (MLP) [16]. Moreover, in [17] a neural network-based solution is carried out to predict the phase characterization of reflect waves by varying the size of radiating elements.

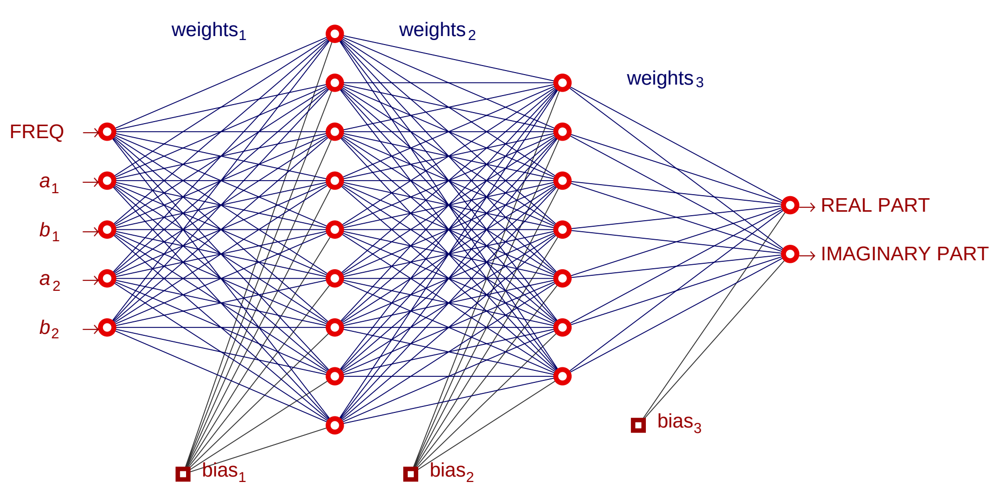

Figure 3. The considered multilayered perceptron structure, with 5 inputs and 2 outputs neurons and 2 hidden layers of 9 and 7 neurons, respectively.

The MLP implements a feed-forward topology, in which the data flow from the input to the output layers is strictly forward, and consists of an input layer, one or more hidden layer, and an output layer. The resulting network structure is that depicted in Figure 3, where the dependencies between variables are represented by the connections among neurons. The input composition in each neuron is made by a nonlinear weighted sum:

$$f(x) = k(x) \sum_i w_i g_i(x) \tag{1}$$

where $k(x)$ is the nonlinear activation function which emulates the activity of biological neurons in the brain. This function must be always normalized and differentiable. The most common function for this purpose is the sigmoid, which is:

$$k(x) = \frac{1}{1 + e^{-x}} \tag{2}$$

3.1. Neural Network Training

There are a number of methods for neural network training, a comparative study was presented in [18]. A neural network works properly when, for any set of inputs, it produces the desired set of outputs. This means that the connections between different nodes in the network are set appropriately, i.e., the weights w_i have been correctly chosen.

The definition of these weights is generally done during a training phase: in the so called supervised learning scheme the neural network is fed with a set of input-output pairs already known, called Training Set (TS): for a given number N of these pairs $(x_i,\, y_i)$ where $x_i \in X$, $y_i \in Y$, it is necessary to find a function:

$$f : X \to Y \tag{3}$$

that matches the examples of the TS. Thus, weights are changed according to a suitable learning rule, until the error on the ANN outputs is minimized [19].

3.1.1. Error Backward Propagation

Among different learning rules, Error Back-Propagation (EBP) is a well-known analytical algorithm used for neural networks training. In literature, there are several forms of back-propagation, all of them requiring different levels of computational efforts. The conventional back-propagation method is, however, the one based on the gradient descent algorithm. EBP propagates error backwards through the network to allow the error derivatives for all network weights to be efficiently computed. In other words, network weights are optimized in order to reach a good and accurate output, and this objective is reached typically minimizing the mean-squared error between the networks output, $f(x_i)$, and the target value y_i over all the N example pairs. Training is time and memory consuming, and it is the most critical phase in the ANN setup, since it must provide continuous feedback on the quality of solutions obtained thus far. To test the ANN generalization capability, a Validation Set (VS) is defined too, containing known $(x_i,\, y_i)$ pairs not used in the TS, in order to check the correct association between unknown input and output data. In general, backpropagation algorithms update weights between layers based on the gradient of error function:

$$E = \frac{1}{2} \| f(x, i) - y(i) \| \tag{4}$$

3.1.2. Levenberg-Marquandt (LM) Method

EBP could be regarded as one the most significant breakthroughs for training neural networks. However, EBP is also considered not efficient because of the slow convergence when dealing with huge validation data set problems. There are two main reasons for this drawback: the step size is always adequate to the gradients, so when the gradient is gentle, the training process is very slow. The second reason is that the curvature of the error surface may not be the same in all directions, so the classic error valley problem may occur and result in slow convergence [20]. This disadvantage can be greatly solved by the use of second-order derivative of total error function for weight updates. Levenberg-Marquandt has been implemented in non-linear and multi-variable optimization in recent years [21]. In this technique, the Hessian matrix gives a proper evaluation of the change of gradient vector. In order to simplify the calculating process, the LM makes the approximation of Hessian matrix by means of Jacobian Matrix

$$H \approx J^T J + \mu I \tag{5}$$

where I is the identity matrix, J the Jacobian Matrix, and μ is always positive, called combination coefficient. The update rule of LM method can be derived as:

$$w_{k+1} = w_k - \left(J_k{}^T J + \mu I\right)^{-1} J_k e_k \tag{6}$$

3.2. Neural Network Use in Antenna Characterization

The relation between the aforementioned inputs and output can be properly modeled by ANN interpolator. Once being sufficiently trained, the ANN can be considered as a black box where the desired output can be forecasted for any arbitrary set of input data. As sketched in Figure 1, the inputs include four geometrical parameters of the antenna and the frequency, while the outputs are real and imaginary parts of reflection coefficient. It is also worth noting that antenna radiation is a lossy process, and return loss is always a complex number. It is separated into two parts: Real and Imaginary before being recombined to produce amplitude which is the main interest in terms of bandwidth optimization problem.

As shown in Figure 4, in order to derive the optimal antenna configuration, ANN is used as an effective interface between antenna designer and global optimization. Initially, the full-wave simulator creates the validation data just for the training phase. The full procedure is guided by Meta-PSO, and the inputs are tuned before entering the ANN which is able to produce a corresponding reflection coefficient with the full-wave simulator. In the next section, the accuracy and time reduction will be presented.

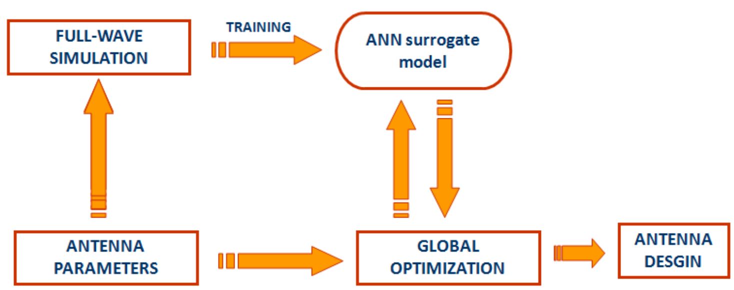

Figure 4. The block diagram of new approach with ANN surrogate model.

4. MODEL DESCRIPTION AND ANALYSIS

The effectiveness of a proper ANN has been observed, considering both its numerical efficiency and the error introduced by the model. Concerning the numerical efficiency, the input consists of 5 patterns, and the first is the frequency band of interest, ranging from 1.5 GHz to 3.5 GHz. The remaining four inputs are length and width of two rectangular rings. The reason for these selections is their obvious influence on the radiating behaviors of the top patch. For each of the four geometrical parameters, we take 5 samples; in total, we have 625 independent cases of reflection coefficient to form the training set data. In order to make ANN model work, suppose a training set (x, y), where x is an input vector, y the desired output for x, and d the output of ANN for x. Both EBP and LM update the weights between layers based on the gradient of the error function:

$$E = \frac{1}{2}\|d(w,x) - y\|^2 \tag{7}$$

where $d(w, x)$ is the output for an input vector. The relation between the presented inputs and outputs can be properly modeled by the use of a multi-layer ANN. This adaptive system consists of two hidden layers with 9 neurons in the first layer and 7 in the second one. Bias is added to the input and hidden layer, and a sigmoid function is deployed as an activation function. Each different set of input geometrical parameters generates a diverse antenna configurations, and all of them will be evaluated by ANN when it experiences an adequate training. The capability of knowledge-based ANN in predicting the reflection coefficient is presented.

Concerning the ANN model numerical efficiency, we take into account the degree of complexity with respect to different set of inputs. Firstly, we have to rebuild the reflection by full-wave analysis approach; then we make the comparisons with ANN surrogate model. After obtaining the two inputs Real and Imaginary parts the return loss, they are recombined to form the absolute value (Amplitude). All the figures show the behaviors of amplitude according to the change of different geometrical parameters and frequencies. Regarding the 3D plot, the axis of frequency remains unchanged since we need to investigate the structure in a fixed frequency range (from 1.5 GHz to 3.5 GHz) with the resolution of 400 steps. The other interval is one of the geometrical parameters that has been discretized with 17 samples each. What attained from the full-wave approach is considered as the target value for the training of Neural Network. The color bar of top figures stands for the change of the amplitude from 0 to 1 while that of the bottom figures (from 0 to 0.2) indicates the error introduced by ANN approximation. As can be seen in the plots, at some certain cases, the largest error introduced by ANN model is approximately 0.1; this difference can be neglected since we exploit ANN as an effective tool to minimize the computation effort. At the end of the optimization process, the best configuration interpreted by ANN model will be validated again by full-wave approach. According to the reported analyses, the following considerations can be drawn up:

(i) The behavior of the proposed antenna changes remarkably by the perturbation of dimensions of external radiators a_1, b_1. On the contrary, this value appears not to be influenced substantially by variables a_2 and b_2, the electrical length of internal ring radiators.

(ii) The errors gained from ANN surrogate model are relatively trivial. The difference between ANN simulation and physical assessment by full-wave approach can be neglected. Hence, the ANN approximation of reflection coefficient can be entrusted and qualified enough to be integrated into the global optimizer.

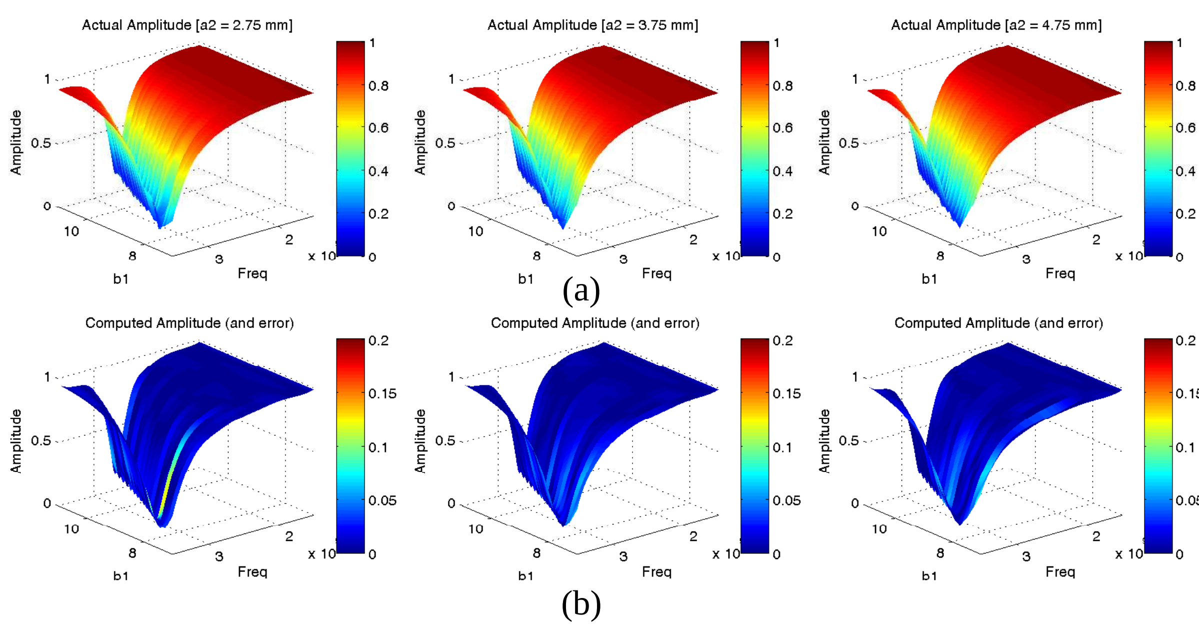

Figure 5. Reflection coefficient amplitude versus b_1 of (a) the proposed antenna configuration, computed with the full wave and (b) reconstructed with the ANN in the certain frequency range.

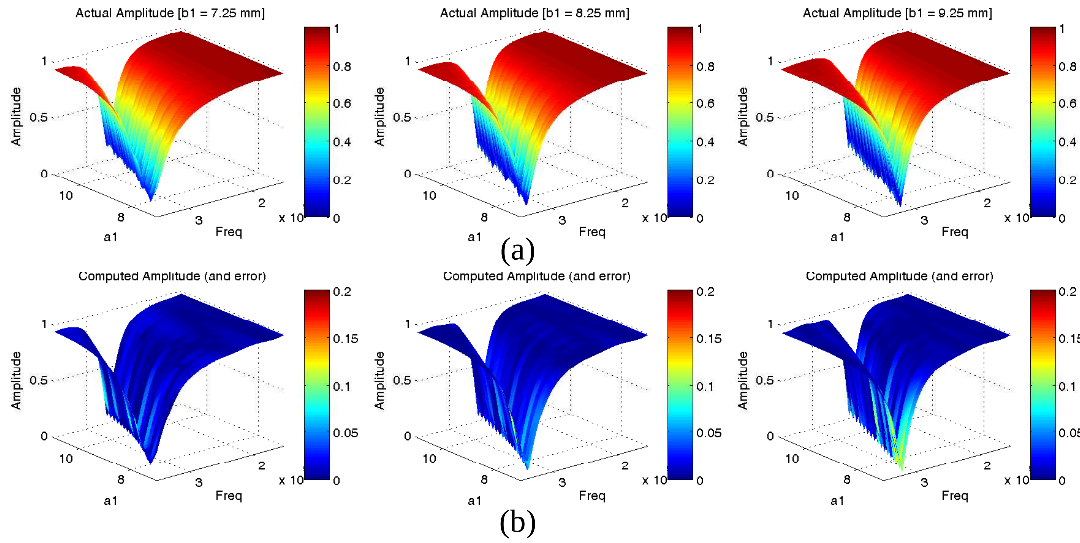

Figure 6. Reflection coefficient amplitude versus a_1 of (a) the proposed antenna configuration, computed with the full wave and (b) reconstructed with the ANN in the certain frequency range.

5. NUMERICAL RESULTS

Figures 5, 6, 7, 8 illustrate the dependence of $|S_{11}|$ on geometrical parameters a_1, b_1, a_2 and b_2, respectively. As can be seen from the graphs, the perturbations of a_1 and b_1 have bigger influence on the structure than a_2 and b_2. In Figure 8, by changing b_1 from 7 to 9 mm with the step of 1 mm, the resonant frequencies shift significantly. Instead, in Figure 5, while varying a_2 from 2.75 to 4.75 with the same step of 1 mm, the center frequency remains almost the same. All the top plots of each figure represent target data for testing and training ANN model. All the comparisons are reported in the bottom parts showing a great accordance between the reconstructed data by ANN estimation and target one by full-wave approach. Since training is the most crucial and expensive phase in the use of ANN, computational effort ought to be taken into account. Regarding the network size, the more complex the problem is, the bigger the designed network is supposed to be. In this context, a "split" NN architecture is proposed to prevent the NN from under-training risk. The original NN is divided into two equal parts, and each one is responsible for one output. The dimension of NN structure is also reduced by a half, leading to the decrease in time convergence.

However, when dealing with large-scale problem with huge amount of data set, EBP algorithm is not adequate to handle that kind of sophisticated problem. In order to tackle this issue, a second-order algorithm namely Levenberg-Marquandt (LM) is adopted. The optimal solution would be using LM training for two separated Neural Networks and then unifying those two distinguished outputs to reform the amplitude of reflection coefficient. Table 2 reports the details of computational time for each kind of network. As can be observed from Table 1 and Table 2, the total optimization time is reduced radically by the use of ANN. The driving reason is quite apparent: ANN only consists of simple processing units, and it treats mainly binary objects. On the other hand, full-wave characterization always has to deal with a huge number of integral equations.

Figure 9 illustrates the robustness of the proposed method: LM training for two separated networks. For what concerns the EBP algorithm, the division of original Neural Network into two separated ones significantly declines mean square error to the minimum value of below 0.001. However, as reported in Figure 9, LM is proved more effective in minimizing the error grade. Both full network approach and separated one demonstrate great improvement in solution accuracy. The best result is achieved by implementing LM-2 outputs. The best configuration of the proposed antenna: $a = 40$ mm; $b = 40$ mm; $a_1 = 19.76$; $b_1 = 21$; $a_2 = 13.14$; $b_2 = 5.5$; $h = 4.8$ mm.

After the optimization run, the resulting geometrical configurations are validated by full-wave analysis. Figure 10 shows the comparisons between the different uses of ANN (by LM training). It

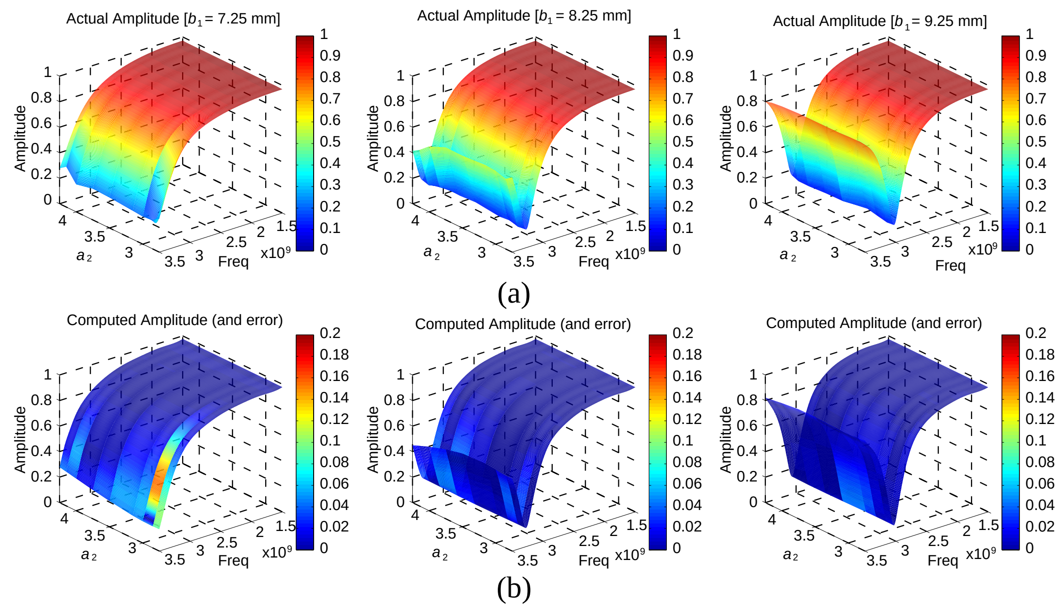

Figure 7. Reflection coefficient amplitude versus a_2 of (a) the proposed antenna configuration, computed with the full wave and (b) reconstructed with the ANN in the certain frequency range.

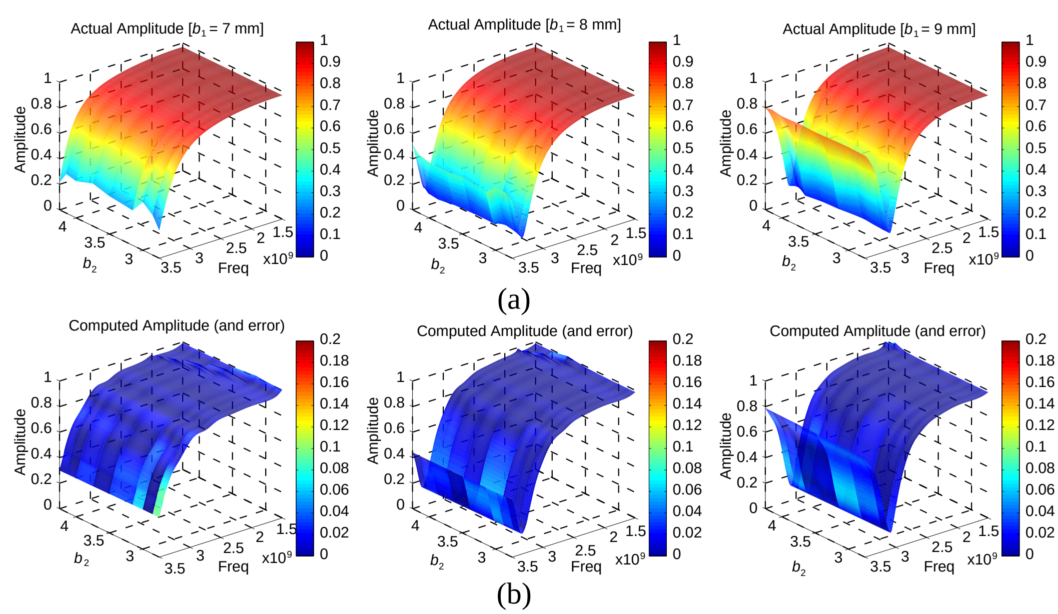

Figure 8. Reflection coefficient amplitude versus b_2 of (a) the proposed antenna configuration, computed with the full wave and (b) reconstructed with the ANN in the certain frequency range.

demonstrates that all proposed methods have a good match with target data. However, the 2-output approach exhibits a better performance since the output data are closer to the validation by full wave analysis when the absolute difference between the target data and ANN outcome is just 0.0005. All the convergence curves of different approaches with different Meta PSO optimization schemes are presented in Figure 11. For the reported simulations, 8 swarms of 10 agents have been considered. In total, the whole population of 80 particles are tested for 10 iterations. With the aim of enlarging the bandwidth over the requirement, the extended bandwidth (BW) is related to cost value according to the formulation:

$$\text{Cost Value} = 100 + \text{BW}/2 \tag{8}$$

where BW is the measure of $-10\,\text{dB}$ bandwidth obtained from $|S_{11}|$ in Figure 10.

Table 2. Comparisons of computational time and numerical efficiency between the training algorithms.

Assessment Categories	EBP full	EBP seperrated	LM full	LM separated
Training Time	20 hours	10 hours	1 hour	30 minutes
Error Committed	0.01	0.001	0.0005	0.0004
Optimization Time (when being integrated)	7 mins	5 mins	5 mins	4 mins

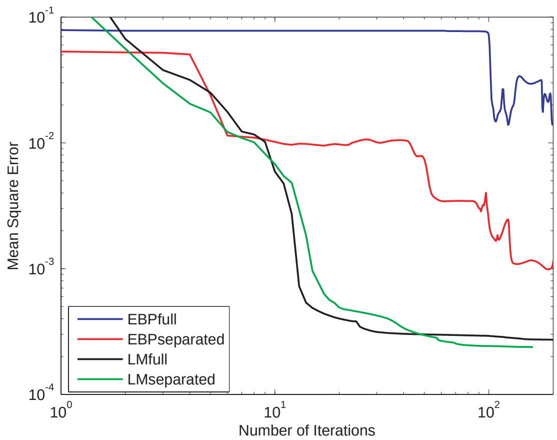

Figure 9. Training error level as a function of number of iterations versus proposed methods.

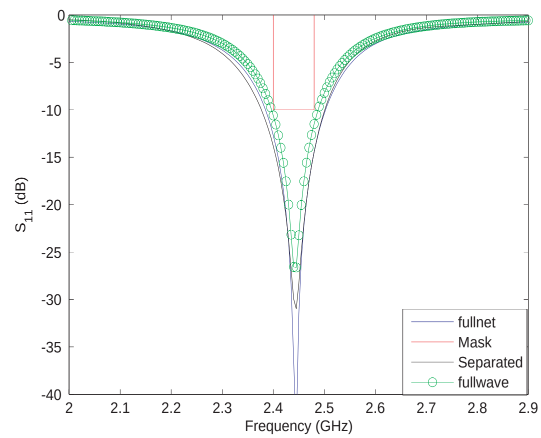

Figure 10. ANN optimization and full-wave analysis validation.

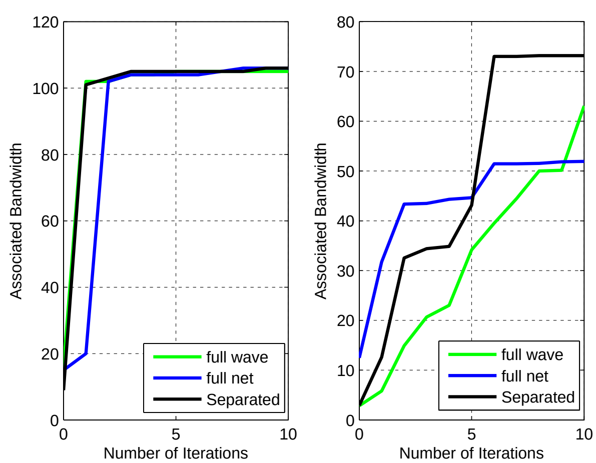

Figure 11. Comparison of ANN model with full-wave analysis model when being integrated with global optimization tool.

Figure 12 shows the radiation pattern of the optimized antenna at the center resonant frequency band, 2.45 GHz. The maximum value of gain recorded is 4 dB, satisfying the constraint for indoor electrical appliances. It is worth noticing that the behaviors of ANN models are close to full-wave one, it again confirms the differences between the approximation and physical characterization are negligible. Figure 13 indicates the gain of the proposed antenna with respect to the change of frequency. As can be observed, radiation pattern peaks in the operating frequency range from 2.4 to 2.5 GHz.

All the simulations were done by the use commercial full-wave analysis, on an Intel(R) Core I-7 2600 CPU, 3.4 GHz, 8 GB RAM system.

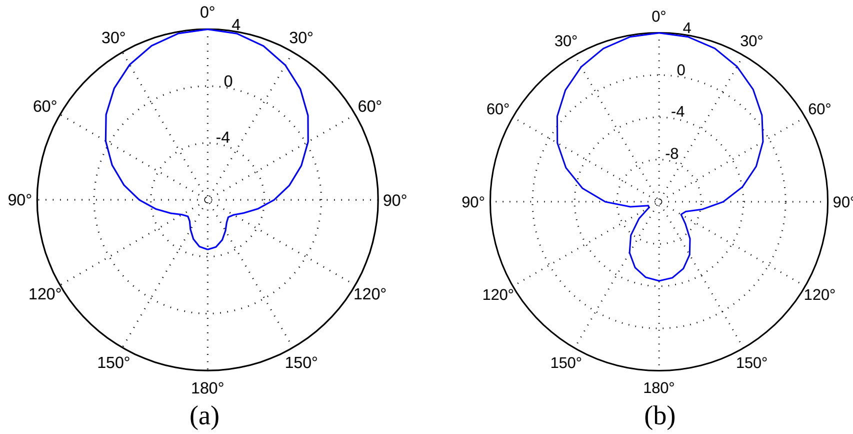

Figure 12. Radiation pattern of the antenna. (a) xOz. (b) yOz.

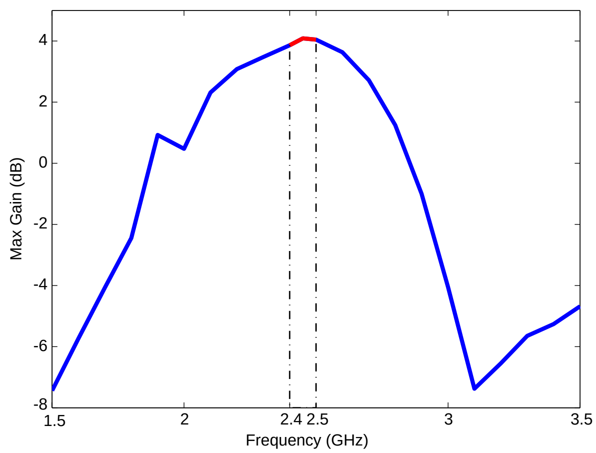

Figure 13. Maximum total gain as a function of frequency in the region of interest.

6. CONCLUSION

In this work, the presented results show the ability of Artificial Neural Network as an important vehicle in frequency EM-modeling. Achieved efficiency demonstrates the robustness of the surrogate model in terms of sensible reduction in computing resource. The accuracy of the solution makes Artificial Neural Network suitable to be implemented as a convenient interface between antenna designers and global optimization tools. This accurate and fast model can also be applied to a number of EM applications as well as non-linear modeling problems.

REFERENCES

1. Bahl, I. J., S. S. Stuchly, and M. A. Stuchly, "A new microstrip radiator for medical applications," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 28, No. 12, 1464–1469, Dec. 1980.
2. Batchelor, J. C. and R. J. Langley, "Microstrip ring antennas operating at higher order modes for mobile communications," *IEE Proceedings Microwaves, Antennas and Propagation*, Vol. 142, No. 2, 151–155, Apr. 1995.
3. Pirinoli, P., G. Vecchi, and M. Orefice, "Full-wave spectral analysis and design of annular patch antenna with electromagnetically coupled microstrip feed line," *IEEE Transactions on Antennas and Propagation*, Vol. 52, No. 9, 2415–2423, Sep. 2004.

4. Gandelli, A., F. Grimaccia, M. Mussetta, P. Pirinoli, and R. E. Zich, "Genetical swarm optimization: An evolutionary algorithm for antenna design," *Journal of Automatika*, Vol. 47, Nos. 3–4, 105–112, 2006.
5. Selleri, S., M. Mussetta, P. Pirinoli, R. E. Zich, and L. Matekovits, "Some insight over new variations of the particle swarm optimization method," *IEEE Antennas and Wireless Propagation Letters*, Vol. 5, No. 1, 235–238, Dec. 2006.
6. Matekovits, L., M. Mussetta, P. Pirinoli, S. Selleri, and R. E. Zich, "Improved PSO algorithms for electromagnetic optimization," *Antennas and Propagation Society International Symposium*, 33–36, 2005.
7. Zich, R. E., M. Mussetta, F. Grimaccia, A. Gandelli, H. M. Linh, G. Agoletti, M. Bertarini, L. Combi, P. F. Scaramuzzino, and A. Serboli, "Comparison of different optimization techniques in microstrip filter design," *2012 Asia-Pacific Symposium on Electromagnetic Compatibility (APEMC)*, 549–552, May 21–24, 2012.
8. Manh, H. L., M. Mussetta, F. Grimaccia, and R. E. Zich, "Differentiated Meta-PSO for rectangular ring antenna with proximity coupled feed," *Antennas and Propagation Society International Symposium*, 640–641, Orlando, FL, 2013.
9. Robustillo, P., J. A. Encinar, and J. Zapata, "ANN element characterization for reflectarray antenna optimization," *Proceedings of the 5th European Conference on Antennas and Propagation (EUCAP)*, 957–960, Apr. 11–15, 2011.
10. Bermani, E., S. Caorsi, and M. Raffetto, "An inverse scattering approach based on a neural network technique for the detection of dielectric cylinders buried in a lossy half-space," *Progress In Electromagnetics Research*, Vol. 26, 67–87, 2000.
11. Caputo, D., A. Pirisi, M. Mussetta, A. Freni, P. Pirinoli, and R. E. Zich, "Neural network characterization of microstrip patches for reflectarray optimization," *3rd European Conference on Antennas and Propagation, EuCAP 2009*, 2520–2522, Mar. 23–27, 2009.
12. Washington, G., "Aperture antenna shape prediction by feedforward neural networks," *IEEE Transactions on Antennas and Propagation*, Vol. 45, No. 4, 683–688, Apr. 1997.
13. Robinson, J. and Y. Rahmat-Samii, "Particle swarm optimization in electromagnetics," *IEEE Transactions on Antennas and Propagation*, Vol. 52, No. 2, 397–407, Feb. 2004.
14. Selleri, S., M. Mussetta, P. Pirinoli, R. E. Zich, and L. Matekovits, "Differentiated Meta-PSO methods for array optimization," *IEEE Transactions on Antennas and Propagation*, Vol. 56, No. 1, 67–75, Jan. 2008.
15. Luo, M. and K.-M. Huang, "Prediction of the electromagnetic field in metallic enclosures using artificial neural networks," *Progress In Electromagnetics Research*, Vol. 116, 171–184, 2011.
16. Zhang, Y. and L. Wu, "Weights optimization of neural network via improved BCO approach," *Progress In Electromagnetics Research*, Vol. 83, 185–198, 2008.
17. Nesil, S., F. Gunes, and U. Ozkaya, "Phase characterization of a reflectarray unit cell with Minkowski shape radiating element using multilayer perceptron neural network," *2011 7th International Conference on Electrical and Electronics Engineering (ELECO)*, II-219–II-222, Dec. 1–4, 2011.
18. Thiruvalar Selvan, P. and S. Raghavan, "Neural model for circular-shaped microshield and conductor-backed coplanar waveguide," *Progress In Electromagnetics Research M*, Vol. 8, 119–129, 2009.
19. De Vita, P., F. De Vita, A. Di Maria, and A. Freni, "An efficient technique for the analysis of large multilayered printed arrays," *IEEE Antennas and Wireless Propagation Letters*, Vol. 8, 104–107, 2009.
20. Yu, H. and B. M. Wilamowski, "Levenberg-Marquardt training," *Industrial Electronics Handbook, Volume 5 — Intelligent Systems,* 2nd edition, Chapter 12, 12-1–12-15, CRC Press, 2011.
21. Cho, S.-W. and J.-H. Lee, "Efficient implementation of the capon beamforming using the Levenberg-Marquardt scheme for two dimensional AOA estimation," *Progress In Electromagnetics Research*, Vol. 137, 19–34, 2013.