School of Industrial and Information Engineering

Campus Leonardo

Department of Electronics, Information and Bioengineering

POLITECNICO DI MILANO

Meeting on Tomography and Applications
Discrete Tomography and Image Reconstruction
Milano, April 20-22, 2015

Exploiting Numerical Features in Computer Tomography Data Processing and Management by CICT

Rodolfo A. Fiorini, http://www.linkedin.com/pub/rodolfo-a-fiorini-ph-d/45/277/498

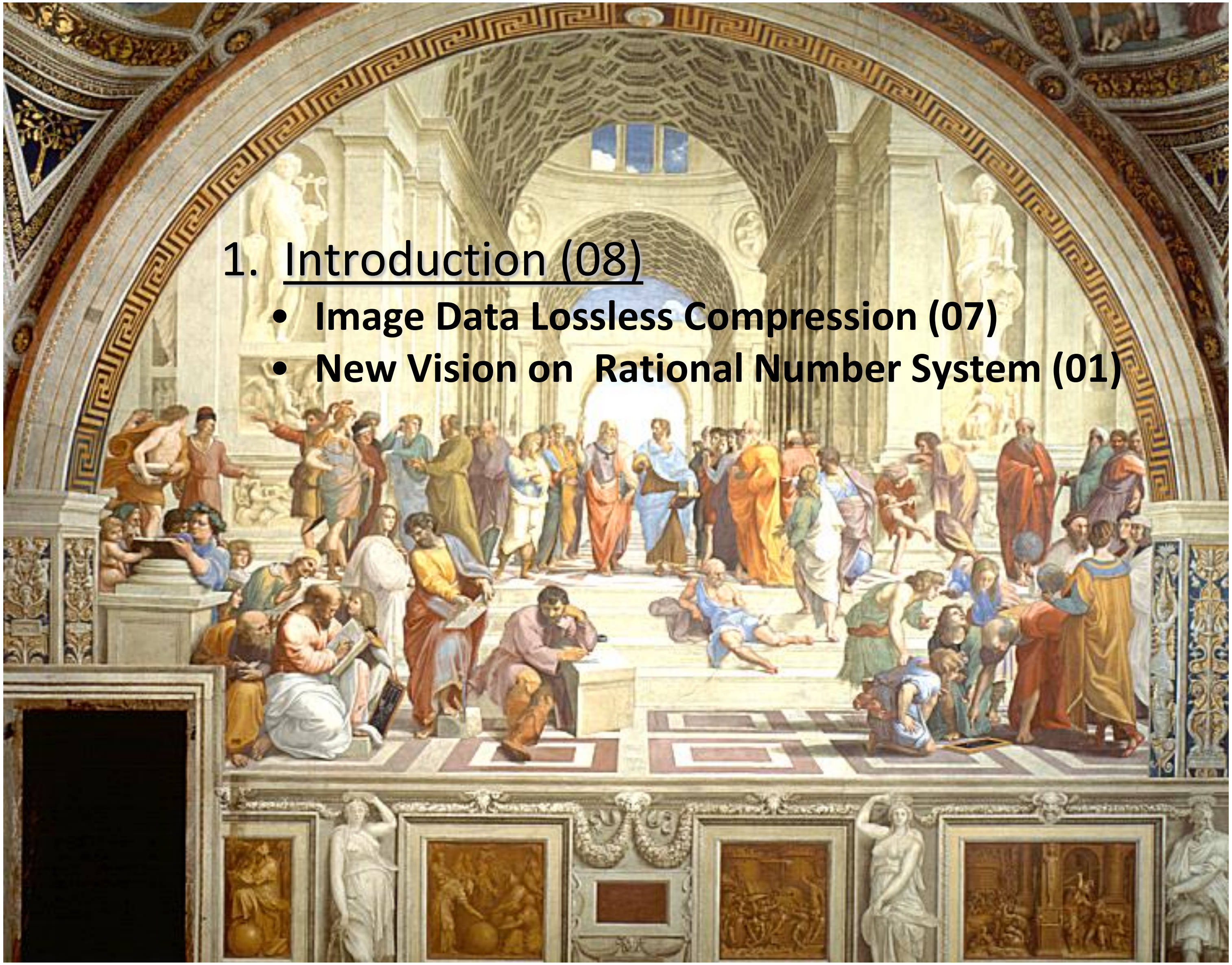
« Le seul véritable voyage ce ne serait pas d'aller vers de nouveaux paysages, mais d'avoir d'autres yeux… »

Valentin Louis Georges Eugène Marcel Proust (1871-1922)
da La Prisonnière (1923).


Announcement
If you survive the end of my presentation and like to know more:
http://www.nature.com/natureevents/science/events/32111
GSI & CICT 2015
International Minisymposium/Workshop on Applied Geometric Science of Information and Computational Information Conservation Theory
16th - 20th July 2015, Vouliagmeni Beach (near Athens), Greece
http://www.cscc.co
Final Paper Deadline: 30th April, 2015

Presentation Outline

1. Introduction (00)



1. Introduction (08)
    - **Image Data Lossless Compression (07)**
    - **New Vision on Rational Number System (01)**

1. Introduction (01)

The discipline of Computed Tomography (**CT**) has been used over the past 50 years. Initially for material analysis and in medical imaging (CAT-scanners), to get minimal invasive images of internal structures.

The impact of new high resolution Computer Tomography (**HRCT**) technology is to generate new challenges associated with the problem of formation, acquisition, compression, transmission, and analysis of enormous amount of data.

In the past, computational information conservation theory (**CICT**) has shown potentiality to provide us with new techniques to face this challenge conveniently.

The first attempt to develop lossless data compression technique by CICT **SN** combinatorial approach was presented at “Meeting on Tomography and Applications”, April 26-27, **2012**, in Milano, Italy.

1. Introduction (02)

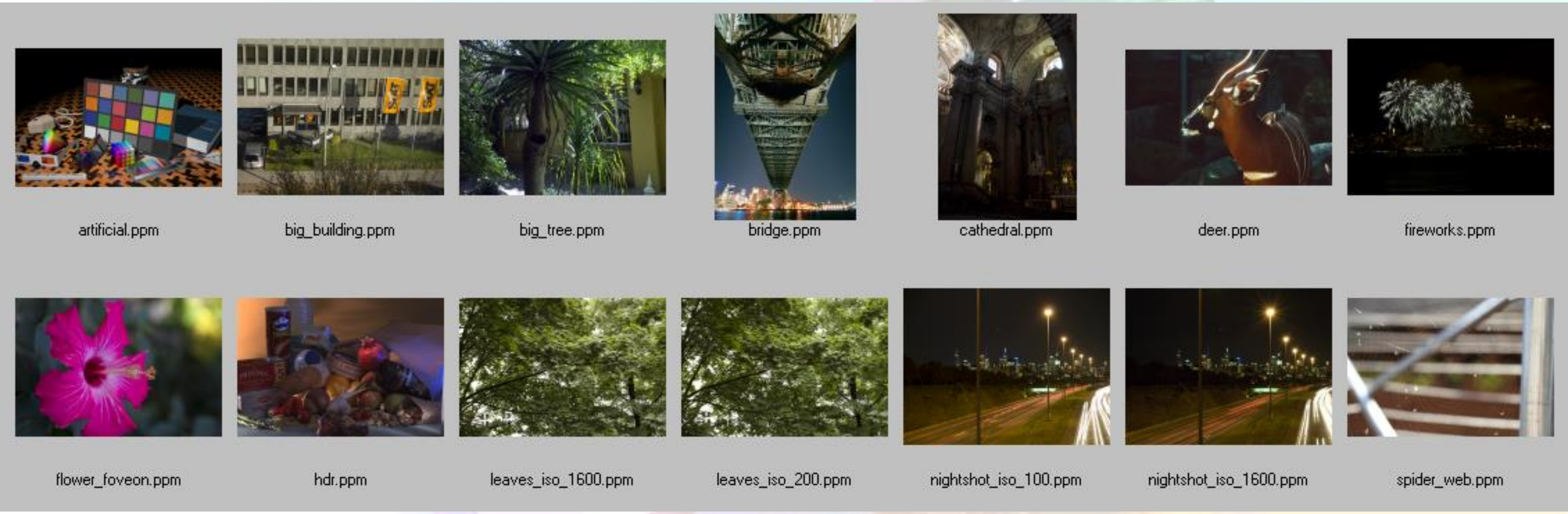
CICT SN (Solid Number) Encoding
True Color Image Example (512 by 768 pixel)

1. Introduction (03)

CICT Natural Compression (NC) Picture Benchmark Dataset

The New Image Compression Test Set - Jan 2008

http://www.imagecompression.info/test_images



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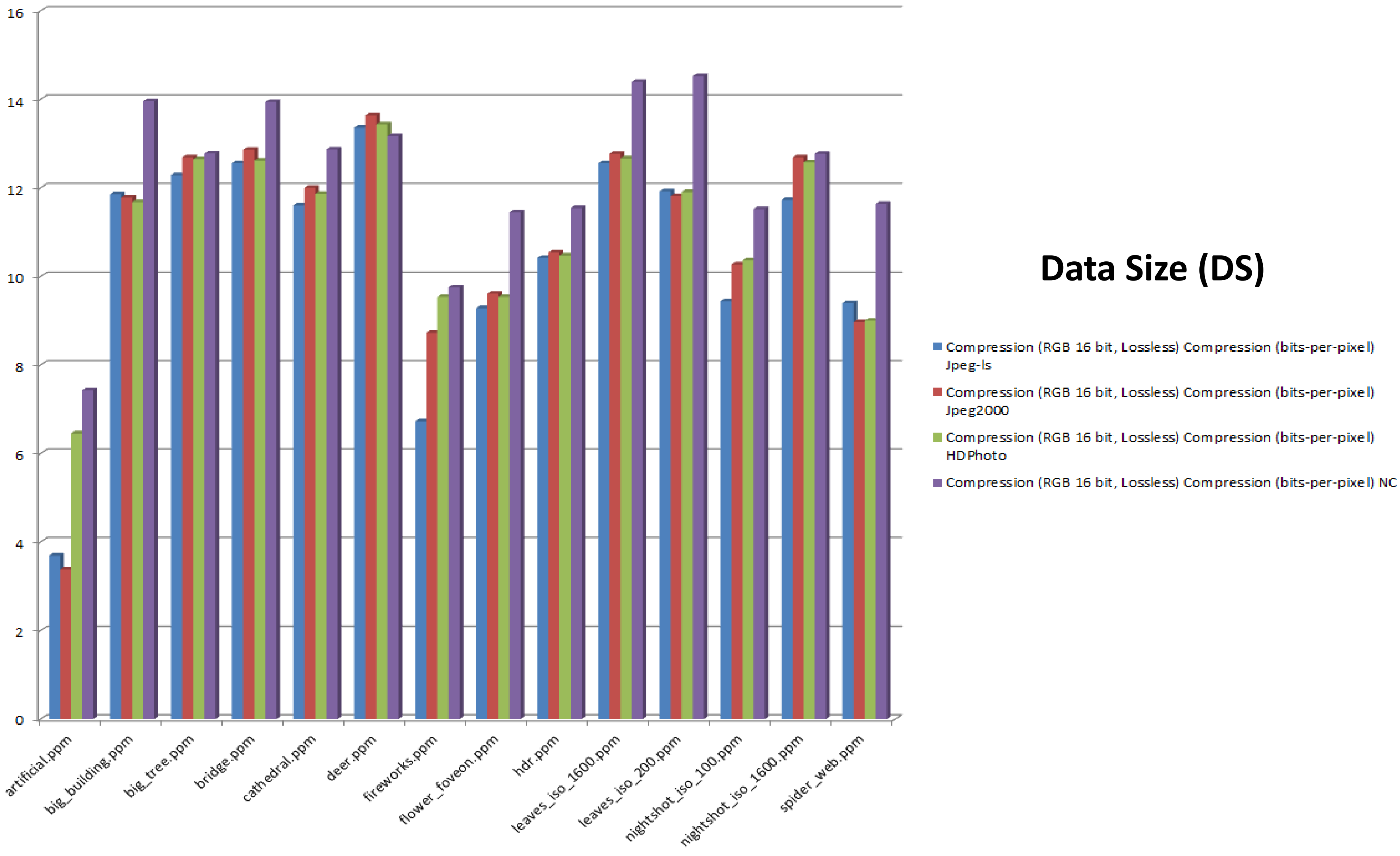
CICT Natural Compression (NC) Picture Benchmark Dataset

The New Image Compression Test Set - Jan 2008
http://www.imagecompression.info/test_images

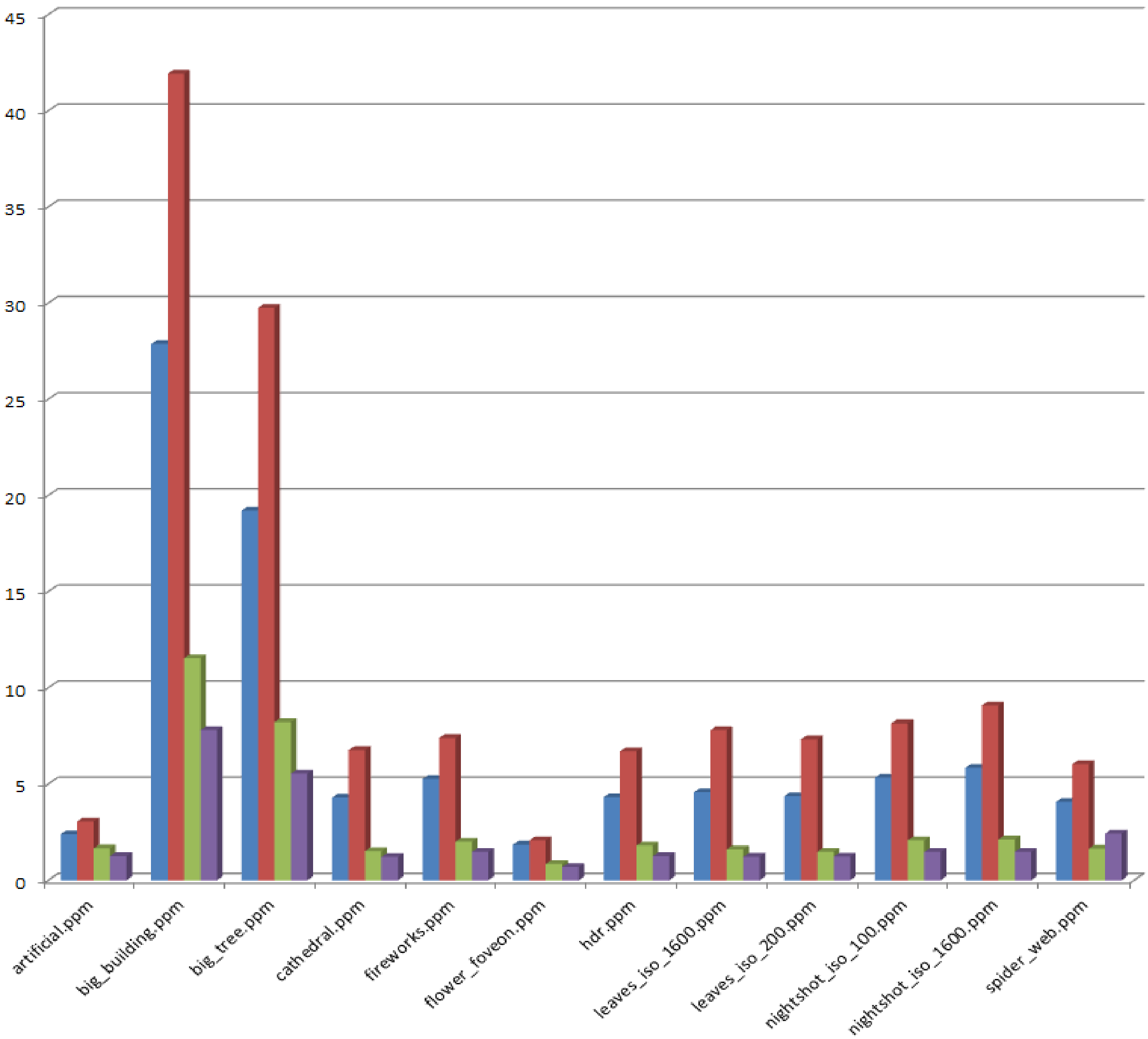
Picture Benchmark Database

NAME	PIXELS	BPP	SIZE (MB)	NOTE
artificial	3072x2048	24	36.80	full
big_building	7216x5412	24	223.46	full
big_tree	6088x4550	24	158.50	full
bridge	2749x4049	24	63.69	full
cathedral	2000x3008	24	34.42	full
deer	4043x2641	24	61.10	full
fireworks	3136x2352	24	42.21	full
flower_foveon	2268x1512	24	19.62	full
hdr	3072x2048	24	36.80	full
leaves_iso200	3008x2000	24	34.42	full
leaves_iso1600	3008x2000	24	34.42	full
nightshot_iso100	3136x2352	24	42.29	full
nightshot_iso1600	3136x2352	24	42.29	full
spider_web	4256x2848	24	69.36	full

1. Introduction (05)



1. Introduction (06)



Compression Time (CoT)

1. Introduction (07)

CICT Natural Compression (NC) Results

Algorithm speed (CoT) outperforms other lossless traditional methods (from 2 to 5 five times faster). Data size (DS) is not so competitive.

But algorithm parameters have not been tuned to optimum compromise DS/CoT, so there is plenty of room for improvement. **NC is high resilient technique and no complex and computationally demanding encoders/decoders are required**.

Modern lossless compressors use classical probabilistic models only, and are unable to match high end application requirements like **NC** "Arbitrary Bit Depth" (**ABD**) resolution and information "Dynamic Upscale Regeneration" (**DUR**).

A combination of **NC** with **generalized RLE-like** or tailored **Huffman coding** can offer the simplest compromise to match best result for all generic content images with other current methods.

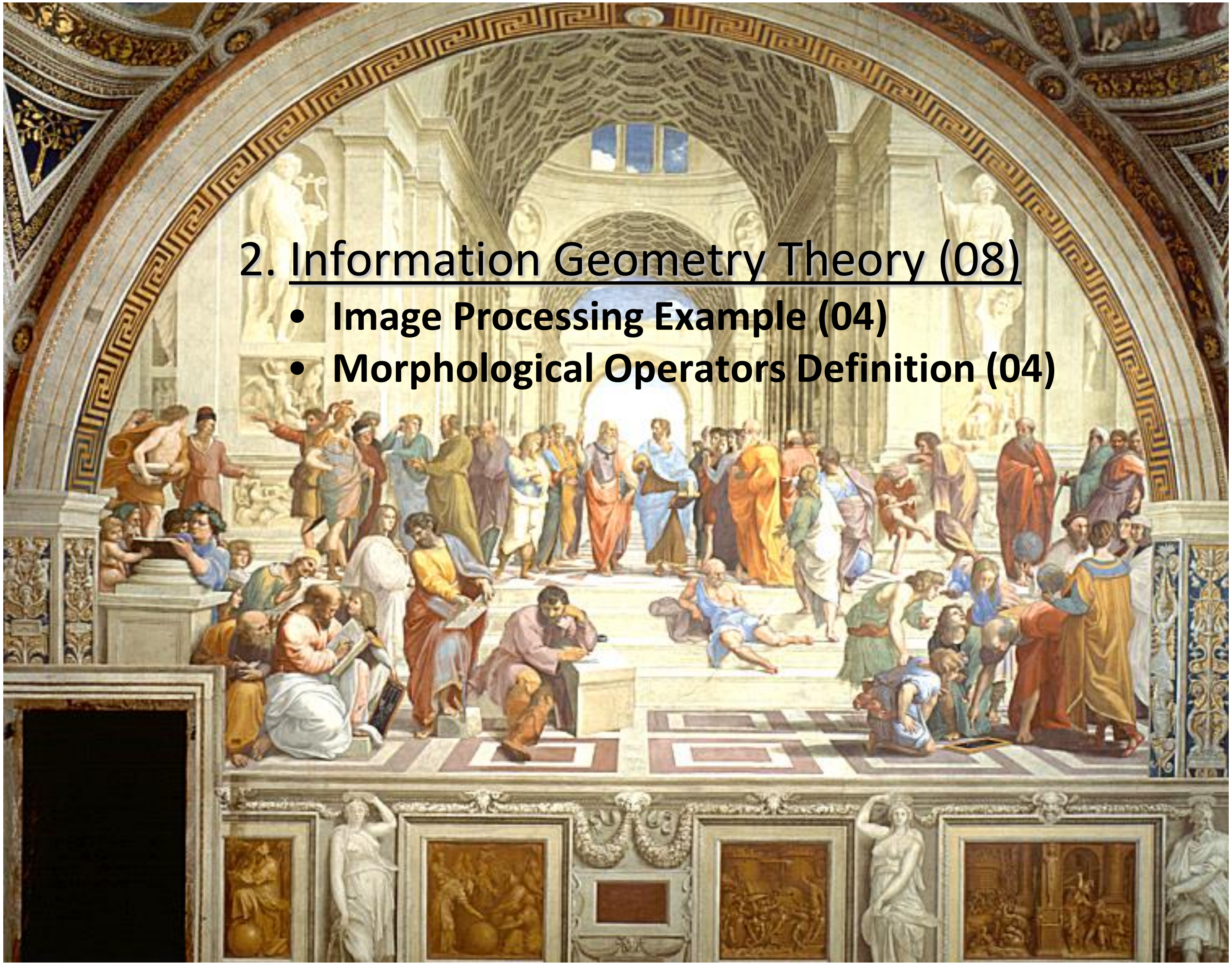
CICT Previous Presented Results on Tomography

The first successful attempt to develop deterministic "random noise source" profiling by **SN** combinatorial approach (i.e. **OECS**, Optimized Exponential Cyclic Sequence) from discrete parameter generator was presented at "Meeting on Tomography and Applications", May 21-23, **2013**, in Milano, Italy.

Elementary arithmetic long **division remainder sequences** can be even interpreted as **combinatorially optimized coding sequences** for **finite fields Galois' geometries (Hyperbolic Geometry)**, **indistinguishable from traditional random noise sources** by classical Shannon entropy and contemporary most advanced instrumentation.

This **new awareness** was presented at "Meeting on Tomography and Applications", May 07-09, **2014**, in Milano, Italy, to guide the development of more convenient and accurate instrumentation system.

2. Information Geometry Theory (00)



2. Information Geometry Theory (08)

- **Image Processing Example (04)**
- **Morphological Operators Definition (04)**

Information Geometry

In **1945**, by considering the space of probability distributions, Indian-born mathematician and statistician **Calyampudi Radhakrishna Rao** (1920-) **suggested the differential geometric approach to statistical inference**. He used **Fisher information matrix** in defining the metric, so it was called **Fisher – Rao metric**.

In **1975**, American statistician **Bradley Efron** (1938-) carried the argument a step forward when he **introduced a new affine connection** on the parameter space manifold, and thus shed light on **the role of the embedding curvature** of the statistical model in the relevant space of probability distributions.

So, **Information Geometry** emerged from the study of the geometrical structure of a manifold of probability distributions **under the criterion of invariance**. It defines a **Riemannian metric** uniquely, which is the **Fisher information metric**. Moreover, **a family of dually coupled affine connections** are introduced.

Mathematically, this is a study of a **triple {M, g, T}**, where **M** is a manifold, **g** is a Riemannian metric, and **T** is a third-order symmetric tensor.

2. Information Geometry Theory (02)

A Practical Example in Image Processing (IP)

In the past, image models ***f*** were thought of having a scalar intensity $\boldsymbol{t} \in \boldsymbol{R}$ at each pixel ***p*** (i.e. ***f***(***p***) = ***t***). By IG approach, we can have an univariate Gaussian probability distribution of intensities $\boldsymbol{n}\,(\boldsymbol{\mu}, \boldsymbol{\sigma}^2) \in \boldsymbol{N}$, i.e. image ***f*** is defined as the function:

$$f : \begin{Bmatrix} \Omega \to N \\ p \mapsto n\left(\mu, \sigma^2\right) \end{Bmatrix} \qquad \textbf{(01)}$$

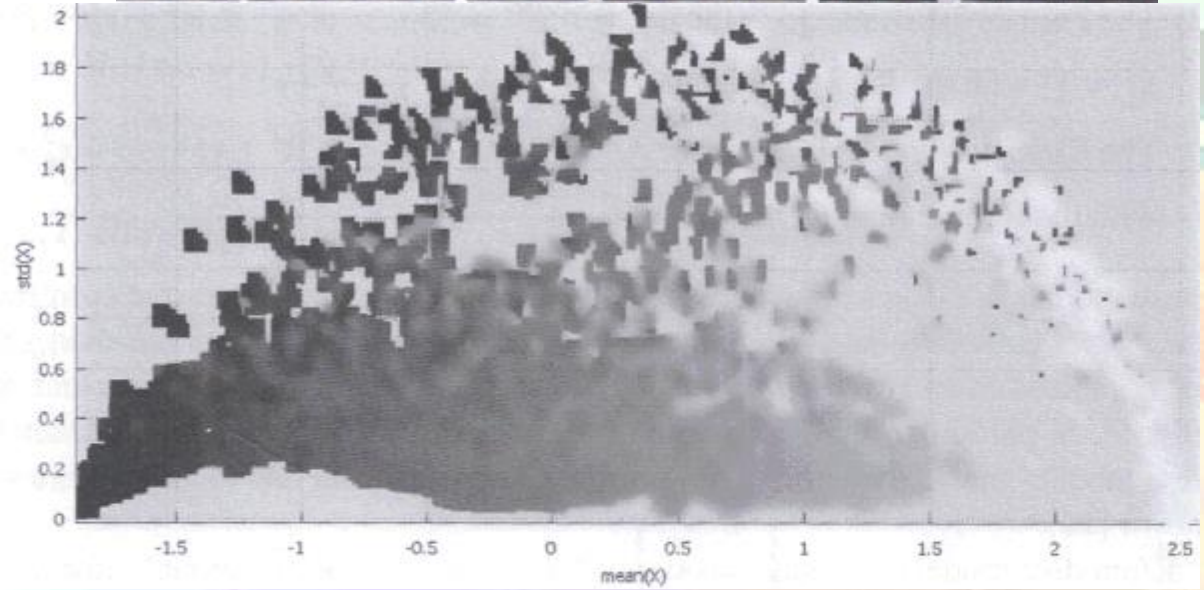
where **Ω** is the support space of pixels ***p*** (e.g. for 2-D images $\boldsymbol{\Omega} \subset \boldsymbol{Z}^2$) and ***N*** denotes the family of univariate Gaussian probability distribution functions (**pdf**).

(J. Angulo, S. Velasco-Forero 2014)

2. Information Geometry Theory (03)

A Practical Example in Image Processing (IP)

On the left, original image. In the center the mean ***μ*** of each patch from a structuring element 5x5 pixel square (moving window). On the right their standard deviation ***σ***.



(J. Angulo, S. Velasco-Forero 2014)

2. Information Geometry Theory (04)

A Practical Example in Image Processing (IP)

In IG, the Fisher information metric is a particular Riemannian metric which can be associated to a smooth manifold whose points are probability measures defined on a common probability space. It can be obtained as the infinitesimal form of the Kullback-Leibler divergence (relative entropy). An alternative formulation is obtained by computing the negative of the Hessian of the Shannon entropy.

Therefore, the Fisher information geometry of univariate normal distribution is essentially **the geometry of the Poincaré upper-half plane (PUHP)** with the following change of variables:

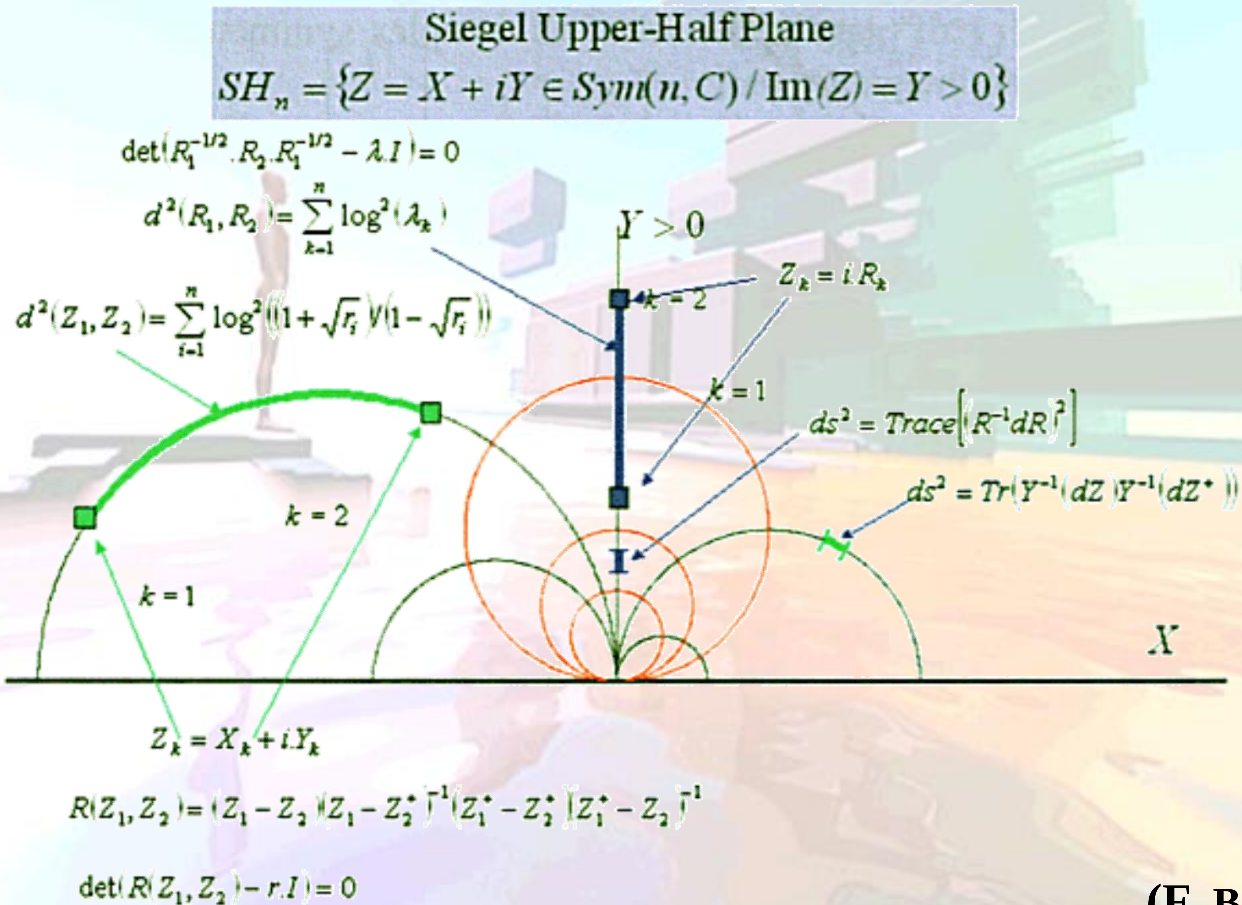
$$x = \mu / \sqrt{2} = \mu^{*}, \qquad y = \sigma. \qquad \textbf{(02)}$$

(J. Angulo, S. Velasco-Forero 2014)

2. Information Geometry Theory (05)

Siegel Upper-Half Space

The Poincaré upper-half plane (**PUHP**) for **2-D problems**, and the Siegel upper-half space (**SUHS**) for **3D problems** (rotational symmetry along Y axis).



(F. Barbaresco, 2014)

2. Information Geometry Theory (06)

A Practical Example in Image Processing (IP)

Mathematical morphology defines nonlinear IP operators based on the computation of **supremum/infimum**-convolution filters (i.e. dilation/erosion operators) in local neighborhoods.

Morphological operators involve that the space of Gaussian distribution ***N*** must be **endowed of a partial ordering** leading to a **complete lattice structure**.

In practice, it means that given a set of Gaussian pdfs, we need to be able to define a Gaussian pdf which corresponds to the infimum of the image patch-set and another one to the supremum.

A possible way to deal with the partial ordering problem of ***N*** is based on considering that **the univariate Gaussian pdfs are points in a Riemannian manifold (hyperbolic space)**, according to the IG approach.

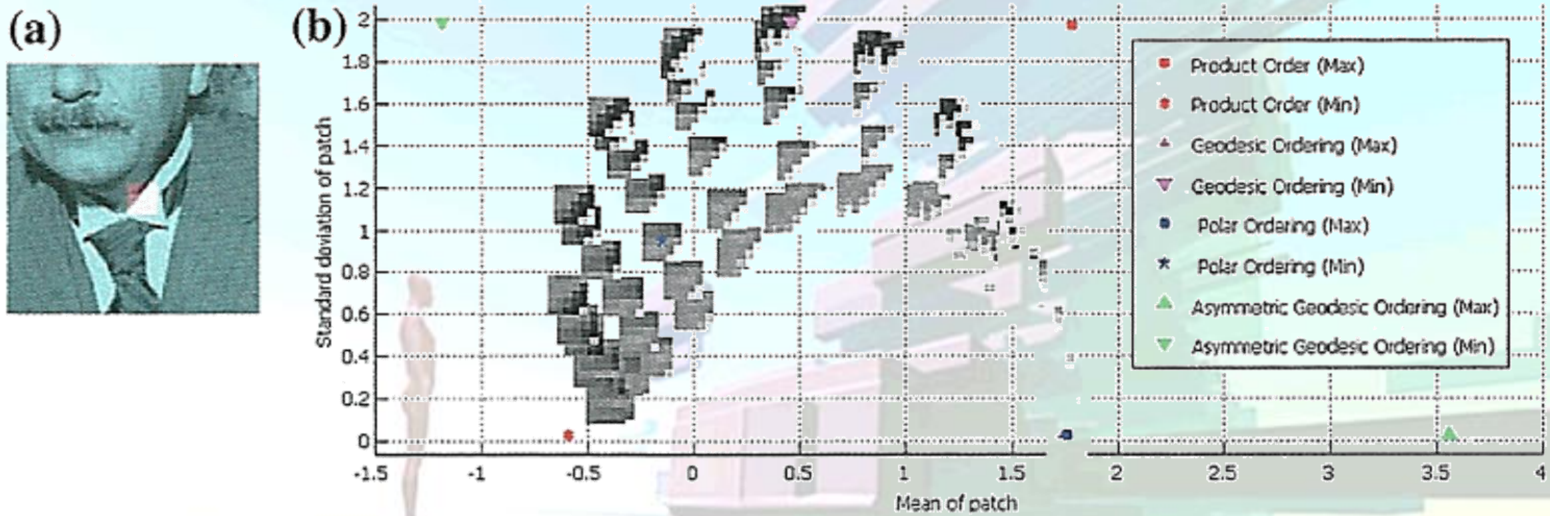
The notion of **ordering invariance** in the **PHUP with respect to** simple transitive group T of **the group of motions** was considered in the Soviet literature by **A.K. Guts** in the 70s, according to the following transformation:

$$z = x + i\,y \mapsto z' = (\lambda x + \alpha) + i\lambda y \,. \quad \textbf{(03)}$$

(J. Angulo, S. Velasco-Forero 2014)

2. Information Geometry Theory (07)

A Practical Example in Image Processing (IP)

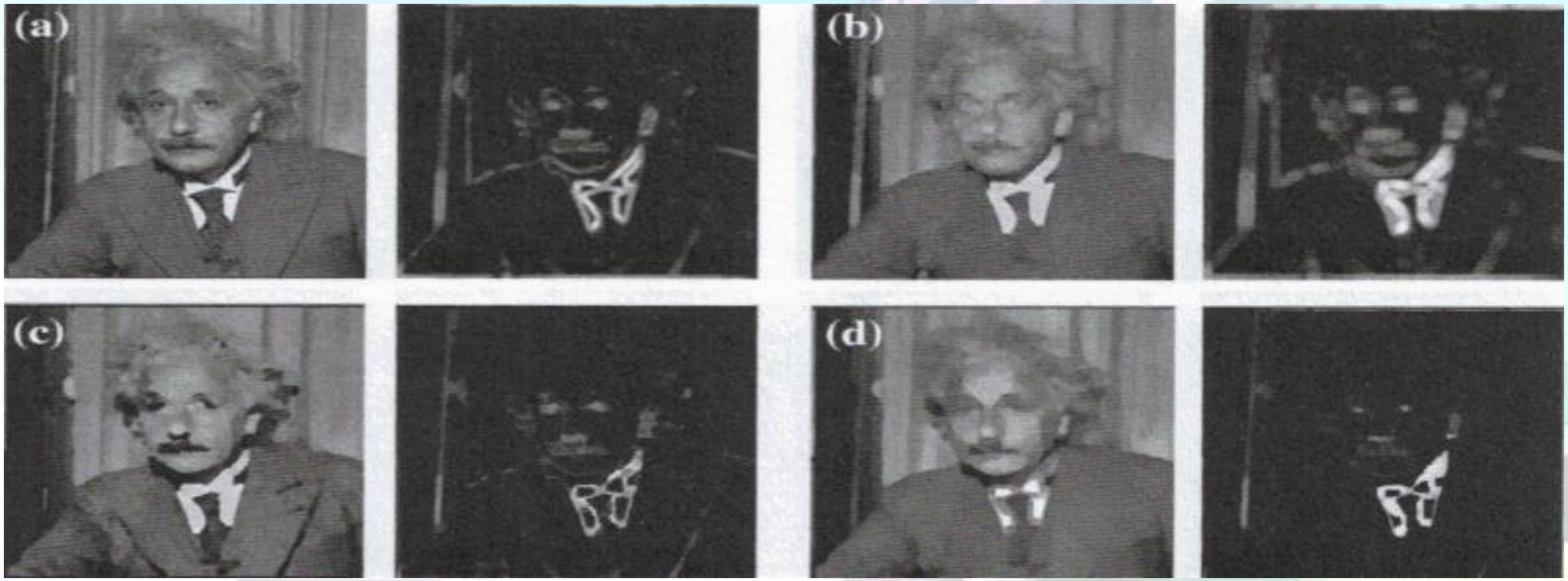


Supremum and infimum of a set of 25 patches parameterized by their mean and standard deviation: **(a)** in *pink* the region where the overlapped patches are taken; **(b)** embedding into the space $\boldsymbol{H^2}$ of the coordinates ($\boldsymbol{\mu^*/,\sigma}$) and corresponding **sup** and **inf** for the different ordering strategies.

(J. Angulo, S. Velasco-Forero 2014)

2. Information Geometry Theory (08)

A Practical Example in Image Processing (IP)



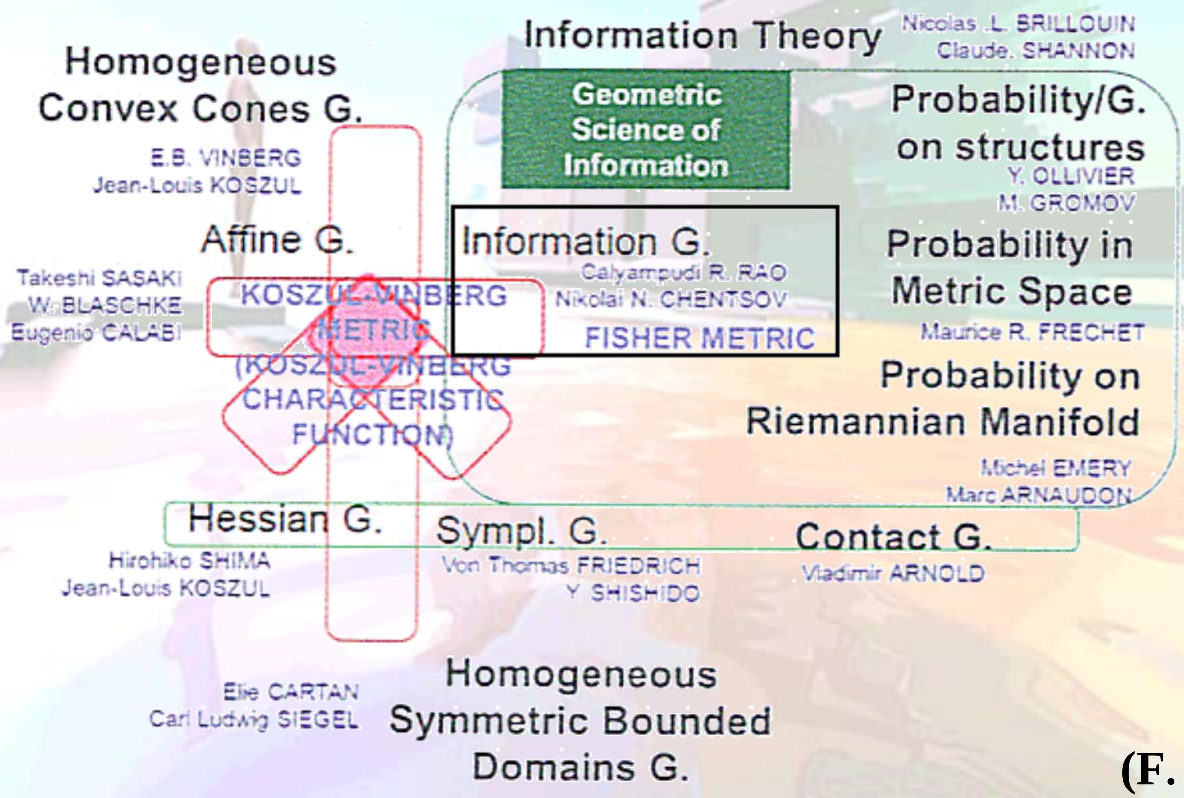
Comparison of dilation of Gaussian distribution-valued image: **(a)** original image, showing both the real and the imaginary components; **(b)** upper half-plane product ordering (equivalent to standard processing); **(c)** upper half-plane polar ordering; **(d)** upper half-plane polar ordering with parameter α=0.01. Again the structuring element is a square of 5x5 pixels.

(J. Angulo, S. Velasco-Forero 2014)

2. Information Geometry Theory (09)

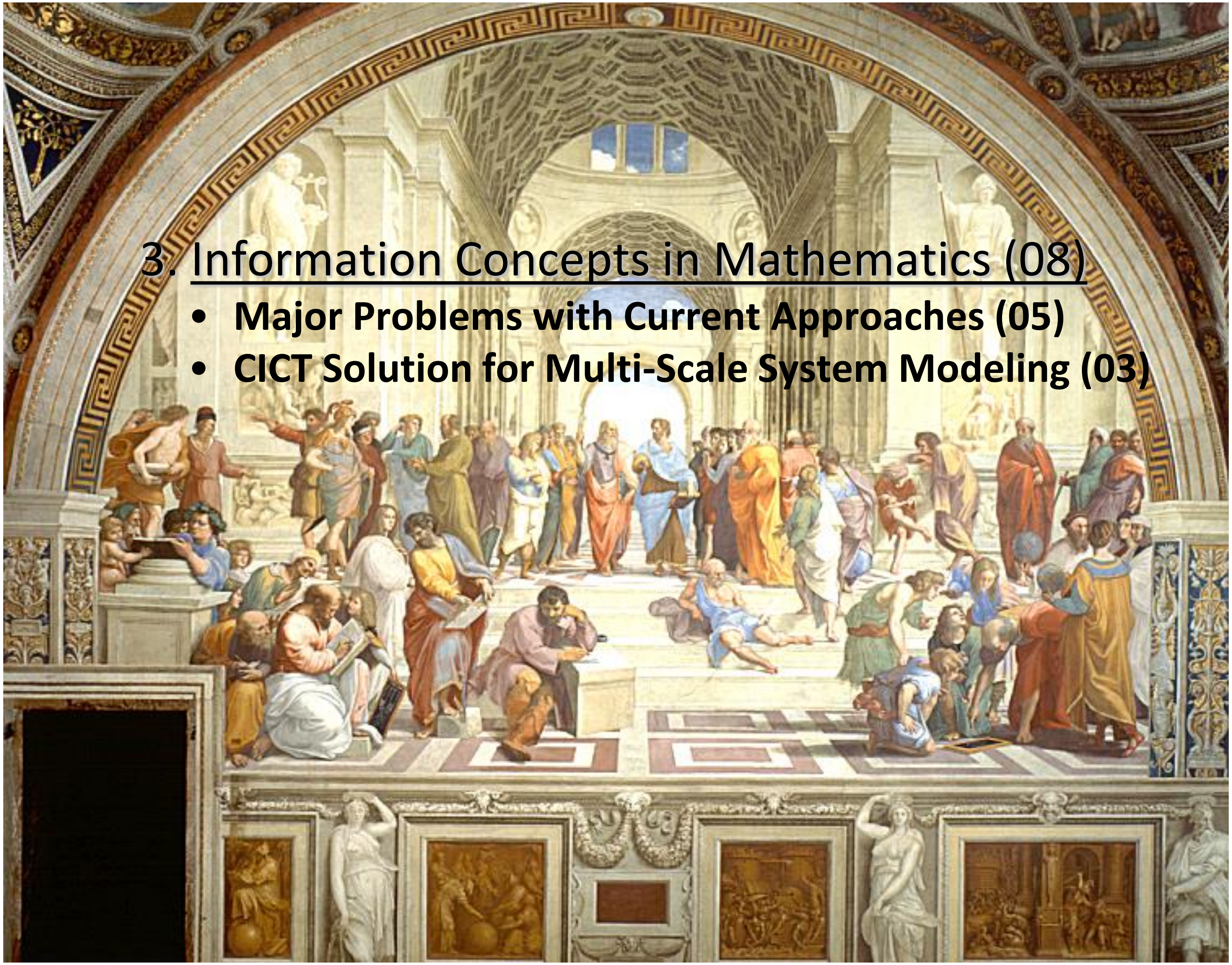
Current Landscape of Geometric Science of Information

Hessian (J.L. Koszul), Homogeneous Convex Cones (E. Vinberg), Homogeneous Symmetric Bounded Domains (E. Cartan, C.L. Siegel), Symplectic (T. von Friedrich, J.M. Souriau), Affine (T. Sasaki, E. Calabi), Information (C. Rao, N. Chentsov). Through Legendre Duality, Contact (V. Arnold) is considered as the odd-dimensional twin of symplectic geometry and could be used to understand Legendre mapping in information geometry.



(F. Barbaresco, 2014)

3. Info Concepts in Mathematics (00)



3. Information Concepts in Mathematics (08)

- **Major Problems with Current Approaches (05)**
- **CICT Solution for Multi-Scale System Modeling (03)**

3. Info Concepts in Mathematics (01)

Information Concept Modeling in Math
has been approached by
Two Large Theoretical and Operative Areas
interlinked by
Irriducible Complementarity.

Continuous Probabilistic Approach (Stochastic Measure)
Well Developed and Applied in all Scientific Areas.
(Infinitesimal Calculus + Stochastic Analysis, e.g. Information Geometry).

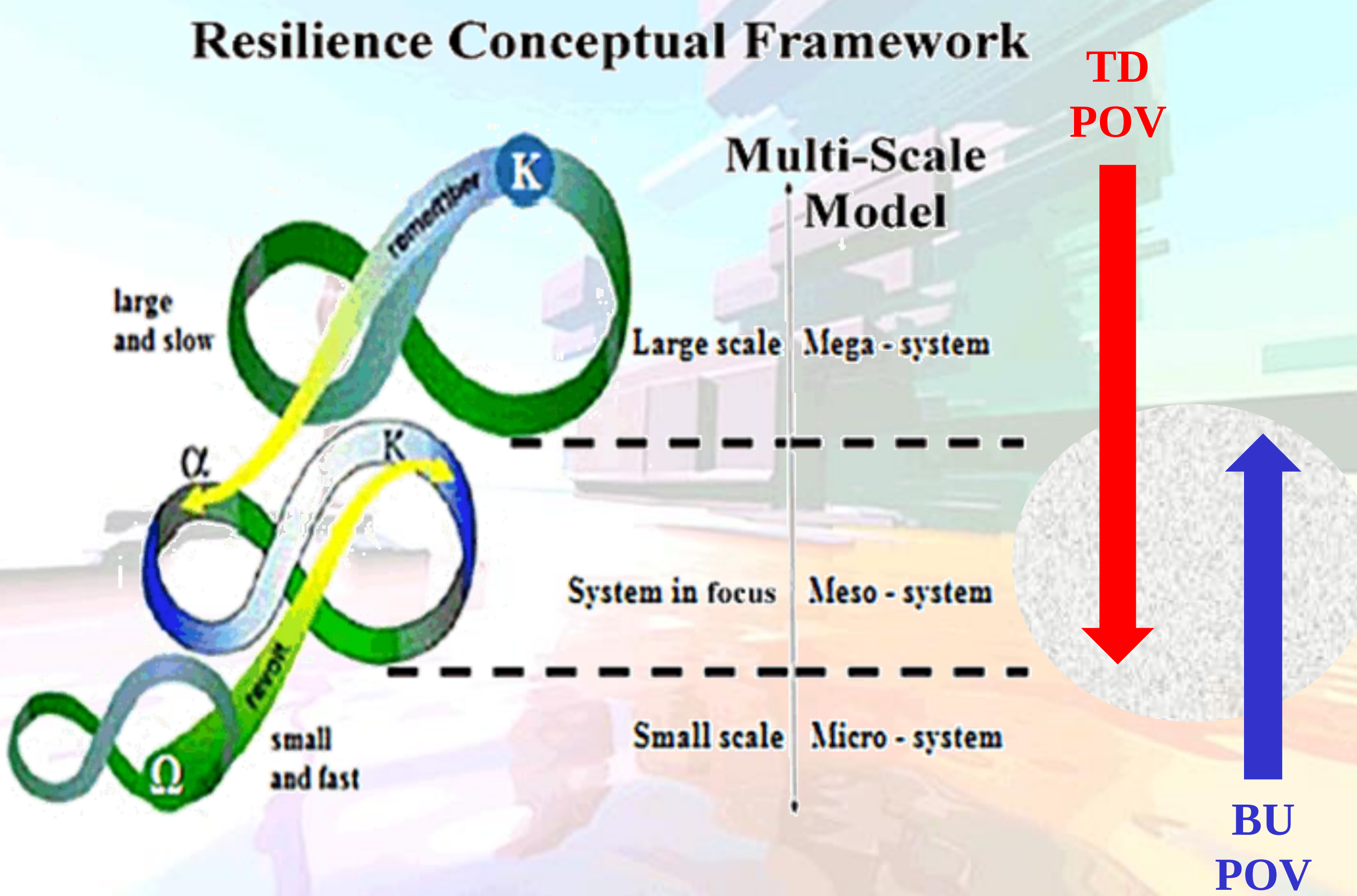
Discrete Deterministic Approach (Combinatorially Based)
Less Developed and Applied in a few quite specific Scientific Areas.
(Finite Difference Calculus + Combinatorial Calculus, e.g. CICT).

Major Problem with Probabilistic Approach

- In **2004**, University of Michigan physicist **Mark Newman**, along with biologist **Michael Lachmann** and computer scientist **Cristopher Moore**, applied Shannon's approach to electromagnetic transmission.
- Specifically, they show that if electromagnetic radiation is used as a transmission medium, **the most information-efficient encoding format** for a given message is **indistinguishable from blackbody radiation**.
- So, paradoxically if you don't know the code used for the message **you can't tell the difference between an information-rich message and a random jumble of letters** (noise as **"unstructured information"** concept).

3. Info Concepts in Mathematics (03)

The Root of the Problem for Multi-Scale System Modeling

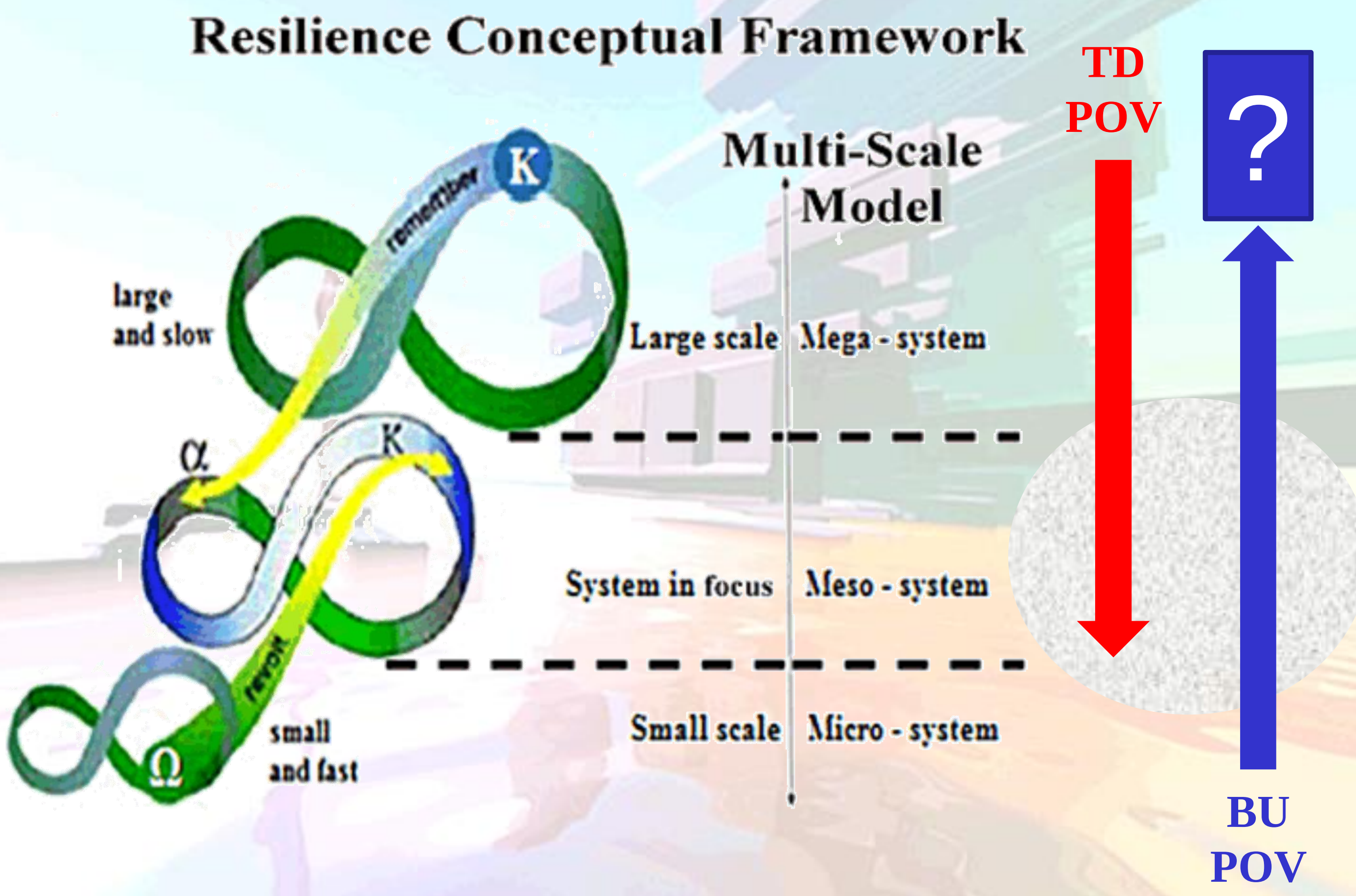


Major Problem with Combinatorial Approach

- In **1951**, Cybernetician **Ross W. Ashby** (1903 –1972) has shown that a few symbolic computational strategies are practically unachievable (**"combinatorial explosion"** concept).
- E.g. A 20 by 20 LED grid (you can turn them on and off) is associated to 2^{400} different patterns, i.e. $2^{400} > 10^{100}$ different combinations (a transcomputational number).
- A **brute force approach strategy** to find a specific pattern **is going to fail**: an "**Earth-sized computer**", **computing since our contemporary estimated Universe creation**, (according to our best measurement of the age of the universe, as of 22 March 2013 (13.798 ± 0.037 billion years ($4.354 \pm 0.012 \times 10^{17}$ seconds) within the Lambda-CDM concordance model), **would be unable to achieve the desired result** (to find our desired pattern).

3. Info Concepts in Mathematics (05)

The Root of the Problem for Multi-Scale System Modeling



3. Info Concepts in Mathematics (06)

Computational Information Conservation Theory

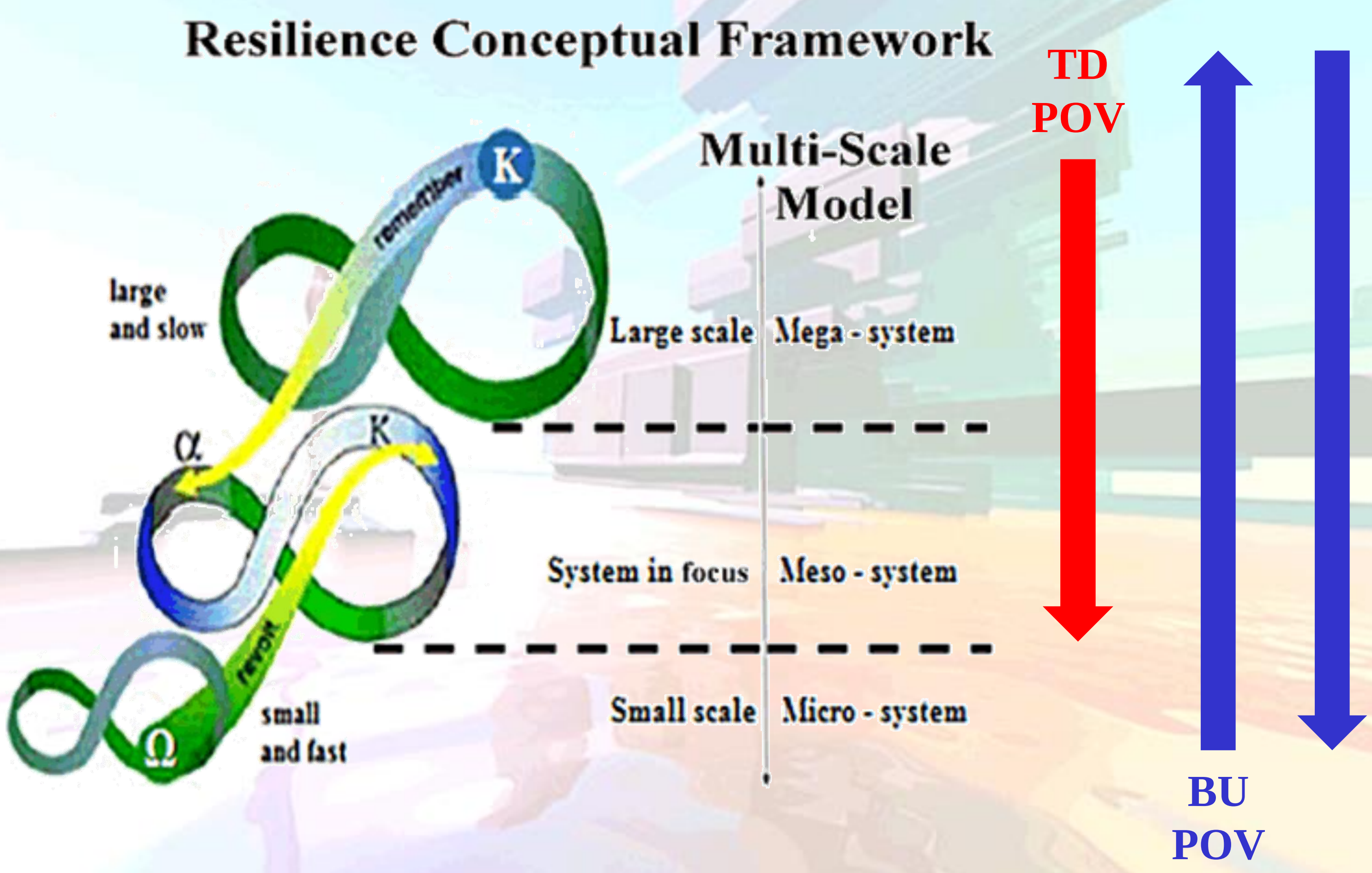
From a computational perspective, all approaches that use a TD POV allow for **starting from an exact global solution** panorama of analytic solution families, which, unfortunately, offers a **shallow local solution computational precision** to real specific needs; in other words, overall system information from global to local POV is not conserved, as **misplaced precision leads to information dissipation**.

On the contrary, to develop resilient and antifragile system, **we need asymptotic exact global solution** panoramas **combined to deep local solution computational precision with information conservation.**

To grasp a more reliable representation of reality, **researchers and scientists need two intelligently articulated hands**: both stochastic and combinatorial approaches synergically articulated by natural coupling.

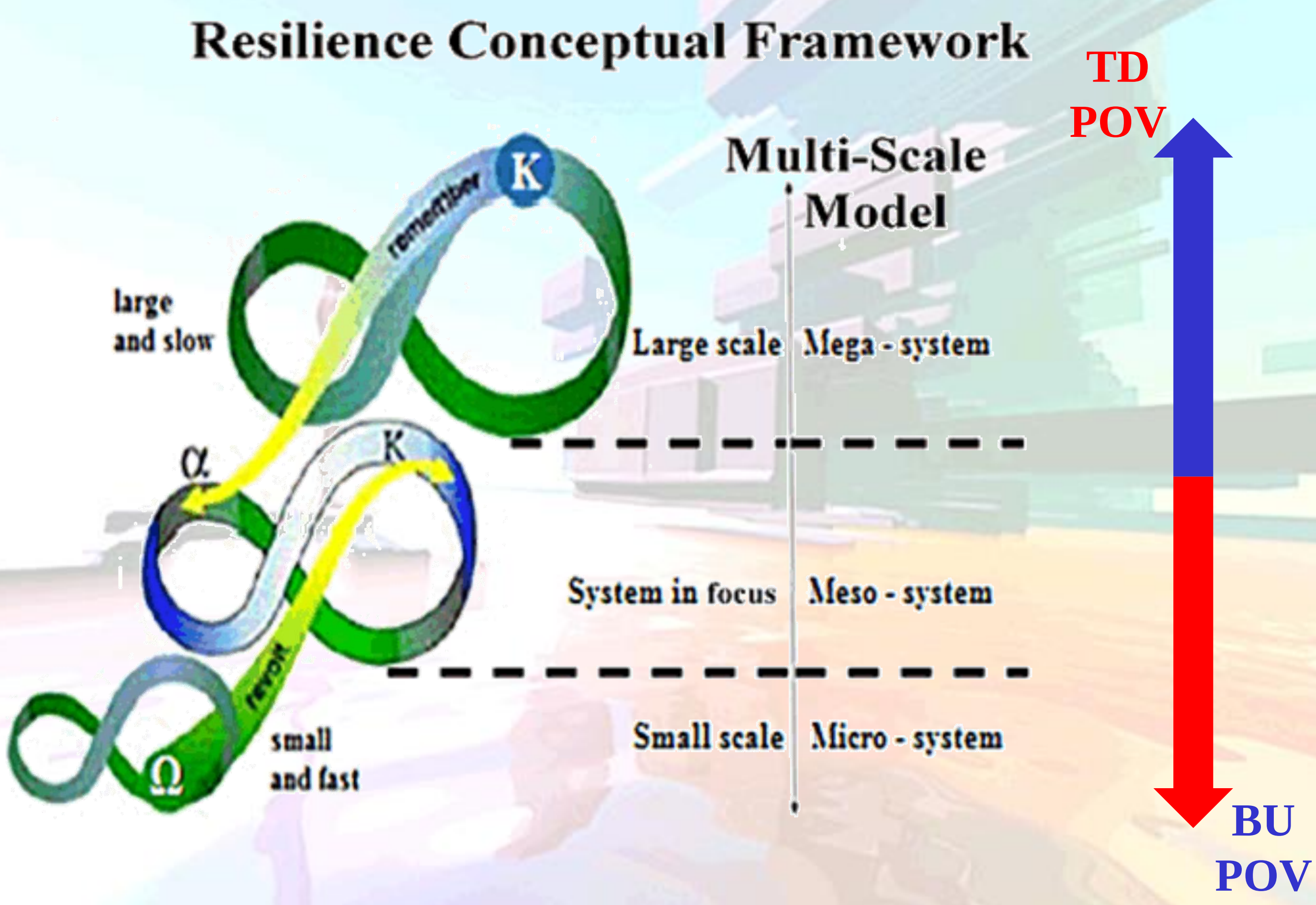
The first attempt to identify basic principles to achieve this goal for scientific application has been developing by the CICT research group at Politecnico di Milano University since the beginning of the 1990s.

CICT Solution to the Problem for Multi-Scale System Modeling



3. Info Concepts in Mathematics (07)

CICT Solution to the Problem for Multi-Scale System Modeling

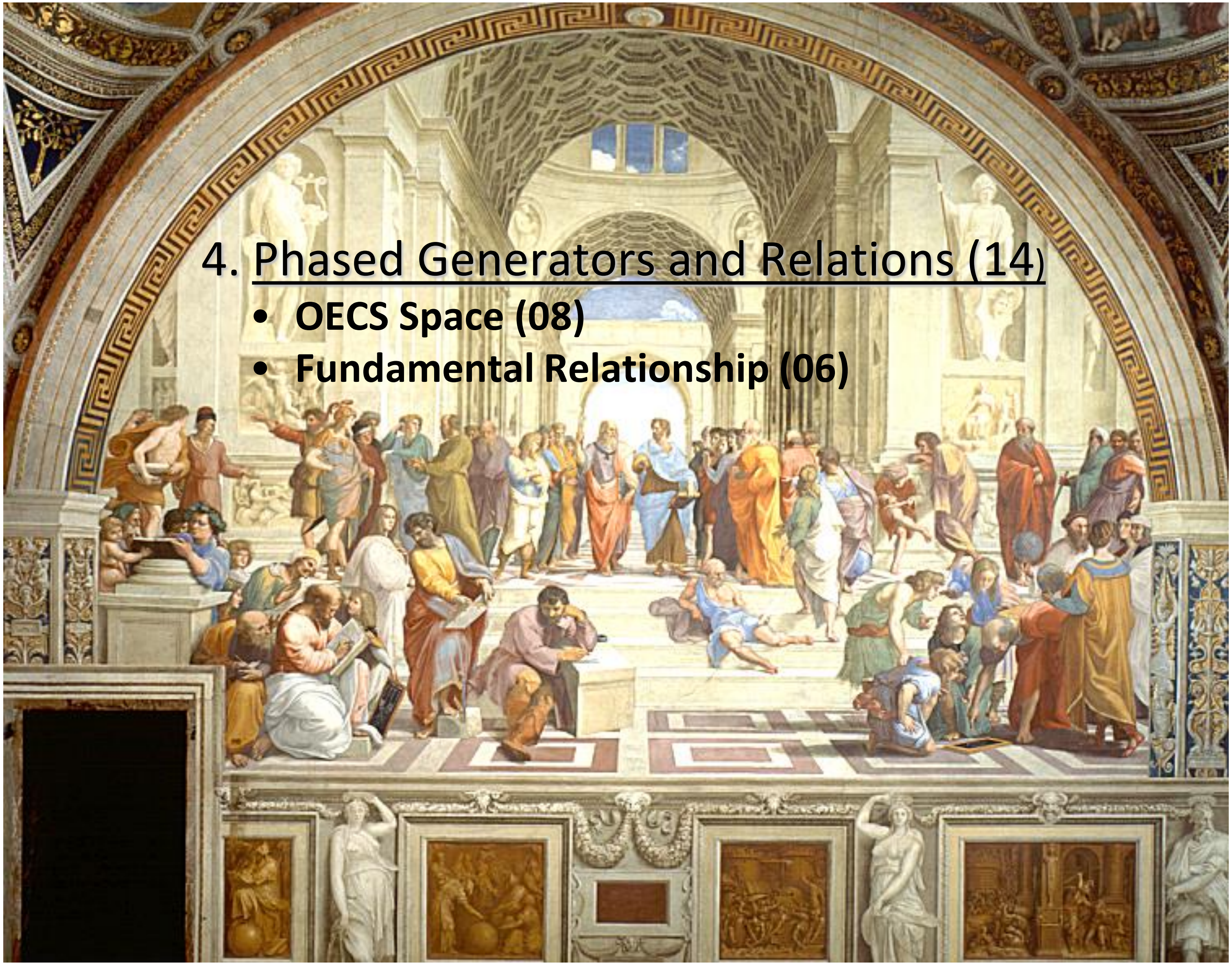


3. Info Concepts in Mathematics (08)

CICT Solution to the Problem for Multi-Scale System Modeling

HOW 

4. Phased Generators and Relations (00)



4. Phased Generators and Relations (14)

- **OECS Space (08)**
- **Fundamental Relationship (06)**

4. Phased Generators and Relations (01)

CICT emerged from the study of the geometrical structure of a discrete manifold of ordered hyperbolic substructures, coded by formal power series, **under the criterion of evolutive structural invariance at arbitrary precision**.

It defines an **arbitrary-scaling discrete Riemannian manifold uniquely**, under HG metric, that, **for arbitrary finite point accuracy level *L* going to infinity** (exact solution theoretically), **is homeomorphic to traditional Information Geometry Riemannian manifold**.

In other words, **HG can describe a projective relativistic geometry directly hardwired into elementary arithmetic long division remainder sequences**, offering many competitive computational advantages over traditional Euclidean approach.

1. Introduction (12)

New Vision on Rational Number System

Elementary Arithmetic long **Division** minority components (**Remainders**, R), for long time, **concealed relational knowledge** to their dominant result (**Quotient**, Q), not only can always allow **quotient regeneration** from their remainder information **to any arbitrary precision**, but even to achieve **information conservation** and **coding minimization,** by combinatorial **OECS** (Optimized Exponential Cyclic Sequences), for dynamical systems.

Then traditional ***Q* Arithmetic** can be even regarded as a highly sophisticated **open logic**, **powerful and flexible LTR and RTL formal numeric language of languages**, with self-defining consistent word and rule, **starting from elementary generator and relation**.

This **new awareness** can guide the development of successful more convenient algorithm, application and powerful computational system.

(Fiorini & Laguteta, 2013)

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SN (Solid Number) Family Group (First Order) Remainder OECS Recursion

1/7	0.	$Q_1 = 1$	$Q_2 = 4$	$Q_3 = 2$	$Q_4 = 8$	$Q_5 = 5$	$Q_6 = 7$
		$R_1 = 3$	$R_2 = 2$	$R_3 = 6$	$R_4 = 4$	$R_5 = 5$	$R_6 = 1$
2/7	0.	$Q_1 = 2$	$Q_2 = 8$	$Q_3 = 5$	$Q_4 = 7$	$Q_5 = 1$	$Q_6 = 4$
		$R_1 = 6$	$R_2 = 4$	$R_3 = 5$	$R_4 = 1$	$R_5 = 3$	$R_6 = 2$
3/7	0.	$Q_1 = 4$	$Q_2 = 2$	$Q_3 = 8$	$Q_4 = 5$	$Q_5 = 7$	$Q_6 = 1$
		$R_1 = 2$	$R_2 = 6$	$R_3 = 4$	$R_4 = 5$	$R_5 = 1$	$R_6 = 3$
4/7	0.	$Q_1 = 5$	$Q_2 = 7$	$Q_3 = 1$	$Q_4 = 4$	$Q_5 = 2$	$Q_6 = 8$
		$R_1 = 5$	$R_2 = 1$	$R_3 = 3$	$R_4 = 2$	$R_5 = 6$	$R_6 = 4$
5/7	0.	$Q_1 = 7$	$Q_2 = 1$	$Q_3 = 4$	$Q_4 = 2$	$Q_5 = 8$	$Q_6 = 5$
		$R_1 = 1$	$R_2 = 3$	$R_3 = 2$	$R_4 = 6$	$R_5 = 4$	$R_6 = 5$
6/7	0.	$Q_1 = 8$	$Q_2 = 5$	$Q_3 = 7$	$Q_4 = 1$	$Q_5 = 4$	$Q_6 = 2$
		$R_1 = 4$	$R_2 = 5$	$R_3 = 1$	$R_4 = 3$	$R_5 = 2$	$R_6 = 6$
7/7	0.	$Q_1 = 9$	$Q_2 = 9$	$Q_3 = 9$	$Q_4 = 9$	$Q_5 = 9$	$Q_6 = 9$
		$R_1 = 7$	$R_2 = 7$	$R_3 = 7$	$R_4 = 7$	$R_5 = 7$	$R_6 = 7$

Solid Number (SN) Family Group (first order) OECS Modular Trajectory Rescaling

(Precision = 10^{-2})

Geometric Series Representation	Compact Representation
$\frac{1}{07} = \sum_{k=0}^{\infty} \frac{1}{10^2} \left(\frac{93}{10^2}\right)^k,$	01(93)
$\frac{1}{07} = \sum_{k=0}^{\infty} \frac{2}{10^2} \left(\frac{86}{10^2}\right)^k,$	02(86)
$\frac{1}{07} = \sum_{k=0}^{\infty} \frac{3}{10^2} \left(\frac{79}{10^2}\right)^k,$	03(79)
$\vdots$	
$\frac{1}{07} = \sum_{k=0}^{\infty} \frac{14}{10^2} \left(\frac{02}{10^2}\right)^k.$	14(02)

4. Phased Generators and Relations (03)

The Discrete Continuum of Egyptian Fractions

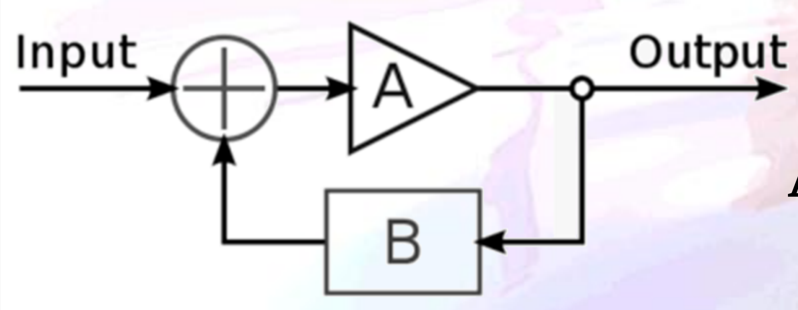
1/(N-1) ⟵ 1/N ⟶ 1/(N+1)

Upscale Contiguity Operator

$$\sum_{k=1}^{\infty}\left(\frac{1}{N}\right)^{k}=\frac{1}{N-1}\quad,\ k=1,2,3,\ldots\in N$$

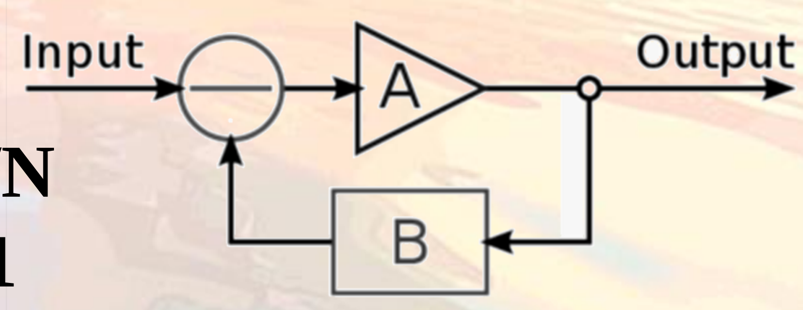
Downscale Contiguity Operator

$$\sum_{k=0}^{\infty}(-1)^{k}\left(\frac{1}{N}\right)^{(k+1)}=\frac{1}{N+1}\quad,\ k=0,1,2,3,\ldots\in N$$



A = 1/N

B = 1



4. Phased Generators and Relations (04)

OECS Word Space Example (Precision = 10^{-2})

1/03	1/04	1/05	1/06	1/07	1/08	1/09	1/10	1/11	1/12	1/13	1/14	1/15	1/16	1/17	1/18	1/19	1/20
37 (11)	29 (16)	24 (20)	20 (20)	18 (26)	16 (28)	15 (35)	14 (40)	13 (43)	12 (44)	11 (43)	11 (54)	10 (50)	10 (60)	09 (53)	09 (62)	09 (71)	09 (80)
36 (08)	28 (12)	23 (15)	19 (14)	17 (19)	15 (20)	14 (26)	13 (30)	12 (32)	11 (32)	10 (30)	10 (40)	09 (35)	09 (44)	08 (36)	08 (44)	08 (52)	08 (60)
35 (05)	27 (08)	22 (10)	18 (08)	16 (12)	14 (12)	13 (17)	12 (20)	11 (21)	10 (20)	09 (17)	09 (26)	08 (20)	08 (28)	07 (19)	07 (26)	07 (33)	07 (40)
34 (02)	26 (04)	21 (05)	17 (02)	15 (05)	13 (04)	12 (08)	11 (10)	10 (10)	09 (08)	08 (04)	08 (12)	07 (05)	07 (12)	06 (02)	06 (08)	06 (14)	06 (20)
33 (01)	25 (00)	20 (00)	16 (04)	14 (02)	12 (04)	11 (01)	10 (00)	09 (01)	08 (04)	07 (09)	07 (02)	06 (10)	06 (04)	05 (15)	05 (10)	05 (05)	05 (00)
32 (04)	24 (04)	19 (05)	15 (10)	13 (09)	11 (12)	10 (10)	09 (10)	08 (12)	07 (16)	06 (22)	06 (16)	05 (25)	05 (20)	04 (32)	04 (28)	04 (24)	04 (20)
31 (07)	23 (08)	18 (10)	14 (16)	12 (16)	10 (20)	09 (19)	08 (20)	07 (23)	06 (28)	05 (35)	05 (30)	04 (40)	04 (36)	03 (49)	03 (46)	03 (43)	03 (40)
30 (10)	22 (12)	17 (15)	13 (22)	11 (23)	09 (28)	08 (28)	07 (30)	06 (34)	05 (40)	04 (48)	04 (44)	03 (55)	03 (52)	02 (66)	02 (64)	02 (62)	02 (60)
29 (13)	21 (16)	16 (20)	12 (28)	10 (30)	08 (36)	07 (37)	06 (40)	05 (45)	04 (52)	03 (61)	03 (58)	02 (70)	02 (68)	01 (83)	01 (82)	01 (81)	01 (80)

4. Phased Generators and Relations (05)

Two Basic Manifold Reflection Types

Incidence (∩)

vs.

Correspondence (∪)

4. Phased Generators and Relations (06)

Incidence (∩)

1/03	1/04	1/05	1/06	1/07	1/08	1/09	1/10	1/11	1/12	1/13	1/14	1/15	1/16	1/17	1/18	1/19	1/20
37 (11)	29 (16)	24 (20)	20 (20)	18 (26)	16 (28)	15 (35)	14 (40)	13 (43)	12 (44)	11 (43)	11 (54)	10 (50)	10 (60)	09 (53)	09 (62)	09 (71)	09 (80)
36 (08)	28 (12)	23 (15)	19 (14)	17 (19)	15 (20)	14 (26)	13 (30)	12 (32)	11 (32)	10 (30)	10 (40)	09 (35)	09 (44)	08 (36)	08 (44)	08 (52)	08 (60)
35 (05)	27 (08)	22 (10)	18 (08)	16 (12)	14 (12)	13 (17)	12 (20)	11 (21)	10 (20)	09 (17)	09 (26)	08 (20)	08 (28)	07 (19)	07 (26)	07 (33)	07 (40)
34 (02)	26 (04)	21 (05)	17 (02)	15 (05)	13 (04)	12 (08)	11 (10)	10 (10)	09 (08)	08 (04)	08 (12)	07 (05)	07 (12)	06 (02)	06 (08)	06 (14)	06 (20)
33 (01)	25 (00)	20 (00)	16 (04)	14 (02)	12 (04)	11 (01)	10 (00)	09 (01)	08 (04)	07 (09)	07 (02)	06 (10)	06 (04)	05 (15)	05 (10)	05 (05)	05 (00)
32 (04)	24 (04)	19 (05)	15 (10)	13 (09)	11 (12)	10 (10)	09 (10)	08 (12)	07 (16)	06 (22)	06 (16)	05 (25)	05 (20)	04 (32)	04 (28)	04 (24)	04 (20)
31 (07)	23 (08)	18 (10)	14 (16)	12 (16)	10 (20)	09 (19)	08 (20)	07 (23)	06 (28)	05 (35)	05 (30)	04 (40)	04 (36)	03 (49)	03 (46)	03 (43)	03 (40)
30 (10)	22 (12)	17 (15)	13 (22)	11 (23)	09 (28)	08 (28)	07 (30)	06 (34)	05 (40)	04 (48)	04 (44)	03 (55)	03 (52)	02 (66)	02 (64)	02 (62)	02 (60)
29 (13)	21 (16)	16 (20)	12 (28)	10 (30)	08 (36)	07 (37)	06 (40)	05 (45)	04 (52)	03 (61)	03 (58)	02 (70)	02 (68)	01 (83)	01 (82)	01 (81)	01 (80)

Correspondence (∪)

1/03	1/04	1/05	1/06	1/07	1/08	1/09	1/10	1/11	1/12	1/13	1/14	1/15	1/16	1/17	1/18	1/19	1/20
37 (11)	29 (16)	24 (20)	20 (20)	18 (26)	16 (28)	15 (35)	14 (40)	13 (43)	12 (44)	11 (43)	11 (54)	10 (50)	10 (60)	09 (53)	09 (62)	09 (71)	09 (80)
36 (08)	28 (12)	23 (15)	19 (14)	17 (19)	15 (20)	14 (26)	13 (30)	12 (32)	11 (32)	10 (30)	10 (40)	09 (35)	09 (44)	08 (36)	08 (44)	08 (52)	08 (60)
35 (05)	27 (08)	22 (10)	18 (08)	16 (12)	14 (12)	13 (17)	12 (20)	11 (21)	10 (20)	09 (17)	09 (26)	08 (20)	08 (28)	07 (19)	07 (26)	07 (33)	07 (40)
34 (02)	26 (04)	21 (05)	17 (02)	15 (05)	13 (04)	12 (08)	11 (10)	10 (10)	09 (08)	08 (04)	08 (12)	07 (05)	07 (12)	06 (02)	06 (08)	06 (14)	06 (20)
33 (01)	25 (00)	20 (00)	16 (04)	14 (02)	12 (04)	11 (01)	10 (00)	09 (01)	08 (04)	07 (09)	07 (02)	06 (10)	06 (04)	05 (15)	05 (10)	05 (05)	05 (00)
32 (04)	24 (04)	19 (05)	15 (10)	13 (09)	11 (12)	10 (10)	09 (10)	08 (12)	07 (16)	06 (22)	06 (16)	05 (25)	05 (20)	04 (32)	04 (28)	04 (24)	04 (20)
31 (07)	23 (08)	18 (10)	14 (16)	12 (16)	10 (20)	09 (19)	08 (20)	07 (23)	06 (28)	05 (35)	05 (30)	04 (40)	04 (36)	03 (49)	03 (46)	03 (43)	03 (40)
30 (10)	22 (12)	17 (15)	13 (22)	11 (23)	09 (28)	08 (28)	07 (30)	06 (34)	05 (40)	04 (48)	04 (44)	03 (55)	03 (52)	02 (66)	02 (64)	02 (62)	02 (60)
29 (13)	21 (16)	16 (20)	12 (28)	10 (30)	08 (36)	07 (37)	06 (40)	05 (45)	04 (52)	03 (61)	03 (58)	02 (70)	02 (68)	01 (83)	01 (82)	01 (81)	01 (80)

Incidence-Correspondence Alternation

1/03	1/04	1/05	1/06	1/07	1/08	1/09	1/10	1/11	1/12	1/13	1/14	1/15	1/16	1/17	1/18	1/19	1/20
37 (11)	29 (16)	24 (20)	20 (20)	18 (26)	16 (28)	15 (35)	14 (40)	13 (43)	12 (44)	11 (43)	11 (54)	10 (50)	10 (60)	09 (53)	09 (62)	09 (71)	09 (80)
36 (08)	28 (12)	23 (15)	19 (14)	17 (19)	15 (20)	14 (26)	13 (30)	12 (32)	11 (32)	10 (30)	10 (40)	09 (35)	09 (44)	08 (36)	08 (44)	08 (52)	08 (60)
35 (05)	27 (08)	22 (10)	18 (08)	16 (12)	14 (12)	13 (17)	12 (20)	11 (21)	10 (20)	09 (17)	09 (26)	08 (20)	08 (28)	07 (19)	07 (26)	07 (33)	07 (40)
34 (02)	26 (04)	21 (05)	17 (02)	15 (05)	13 (04)	12 (08)	11 (10)	10 (10)	09 (08)	08 (04)	08 (12)	07 (05)	07 (12)	06 (02)	06 (08)	06 (14)	06 (20)
33 (01)	25 (00)	20 (00)	16 (04)	14 (02)	12 (04)	11 (01)	10 (00)	09 (01)	08 (04)	07 (09)	07 (02)	06 (10)	06 (04)	05 (15)	05 (10)	05 (05)	05 (00)
32 (04)	24 (04)	19 (05)	15 (10)	13 (09)	11 (12)	10 (10)	09 (10)	08 (12)	07 (16)	06 (22)	06 (16)	05 (25)	05 (20)	04 (32)	04 (28)	04 (24)	04 (20)
31 (07)	23 (08)	18 (10)	14 (16)	12 (16)	10 (20)	09 (19)	08 (20)	07 (23)	06 (28)	05 (35)	05 (30)	04 (40)	04 (36)	03 (49)	03 (46)	03 (43)	03 (40)
30 (10)	22 (12)	17 (15)	13 (22)	11 (23)	09 (28)	08 (28)	07 (30)	06 (34)	05 (40)	04 (48)	04 (44)	03 (55)	03 (52)	02 (66)	02 (64)	02 (62)	02 (60)
29 (13)	21 (16)	16 (20)	12 (28)	10 (30)	08 (36)	07 (37)	06 (40)	05 (45)	04 (52)	03 (61)	03 (58)	02 (70)	02 (68)	01 (83)	01 (82)	01 (81)	01 (80)

4. Phased Generators and Relations (09)

The **CICT fundamental relationship** that ties together numeric body information of divergent and convergent monotonic power series in any base (in this case decimal, with no loss of generality), with ***D* ending by digit 9**, is given by the following CICT fundamental LTR-RTL correspondence equation:

$$\frac{1}{D} = \sum_{k=0}^{\infty} \frac{1}{10^W}\left(\frac{\overline{D}}{10^W}\right)^k \Rightarrow \ldots \Leftarrow Div\left(\frac{1}{D}\right) = \sum_{k=0}^{\infty}(D+1)^k \qquad \text{Eq.(7)}$$

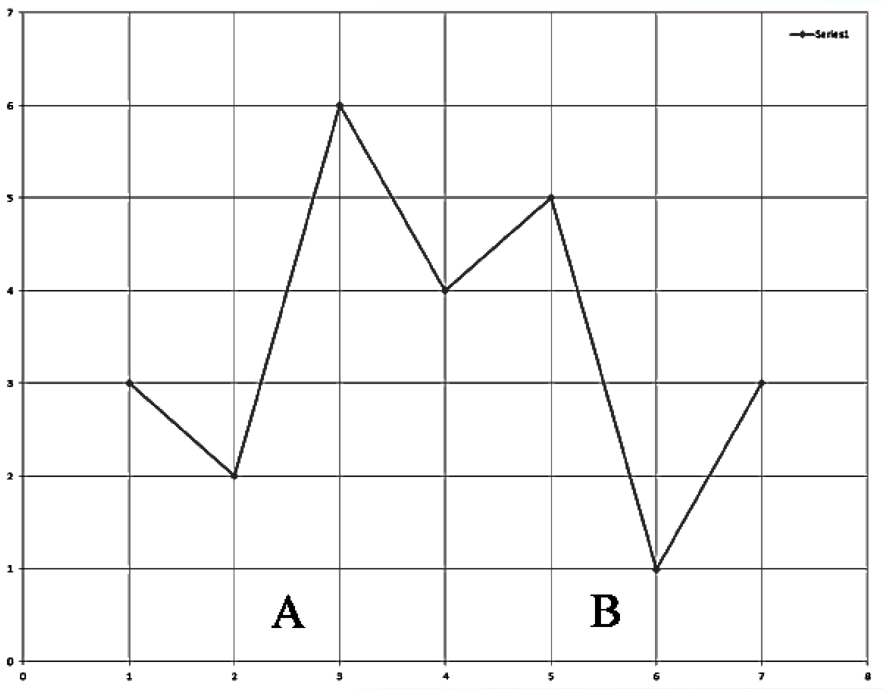
where $\overline{D}$ is the additive 10^W complement of D, i.e. $\overline{D} = (10^W - D)$, W is the word representation precision length of the denominator D and "Div" means "Divergence of".

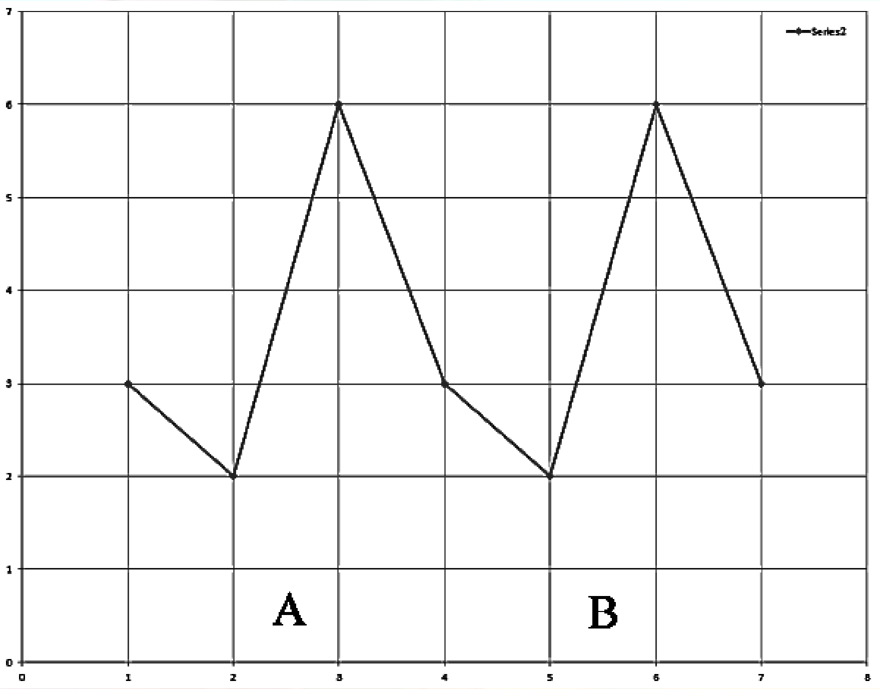
Further generalizations related to ***D* ending by digit 1 or 3 or 7 are straightforward**.

Increasing the level of representation accuracy, the total number of allowed convergent paths to 1/*D*, as monotonic power series (as allowed conservative paths), increases accordingly and can be counted exactly, till maximum machine word length and beyond: like **discrete quantum paths denser and denser to one another, towards a never ending "blending quantum continuum,"** by a Top-Down system perspective.

4. Phased Generators and Relations (10)

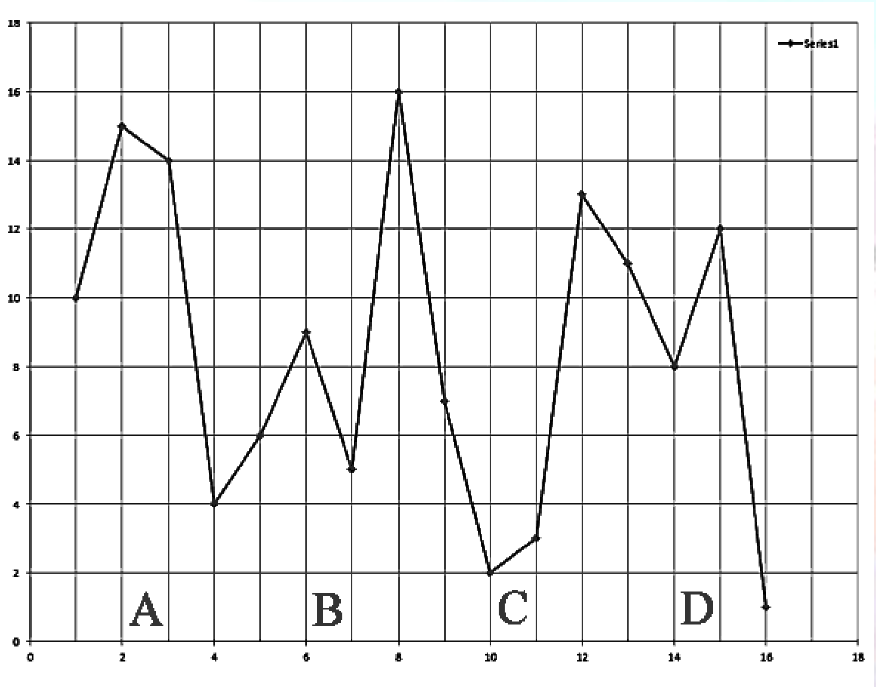
1/7 RFD R_L LTR Oriented
Inner Linear Coordinate Reference (OILCR)

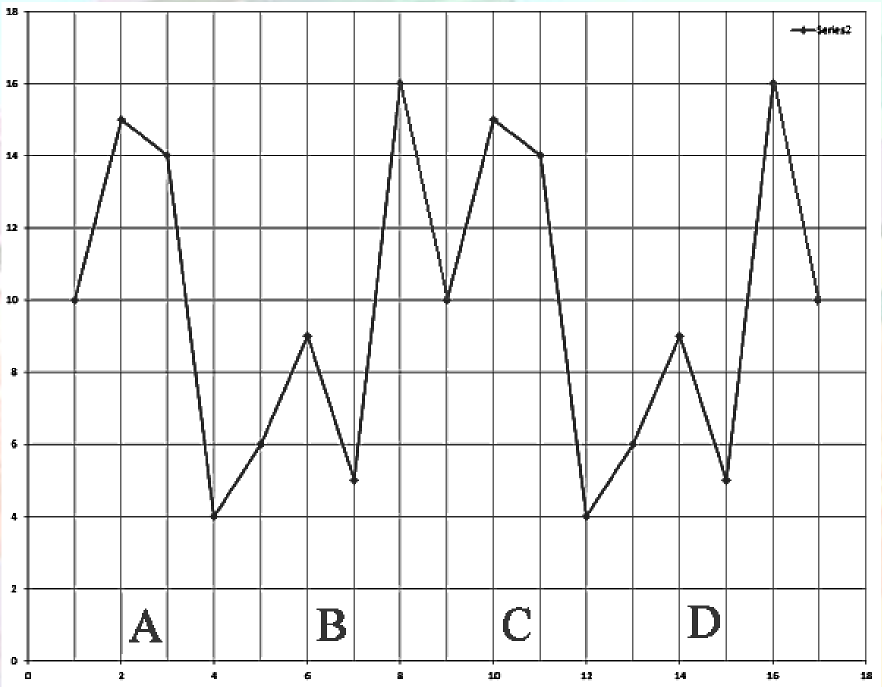




4. Phased Generators and Relations (11)

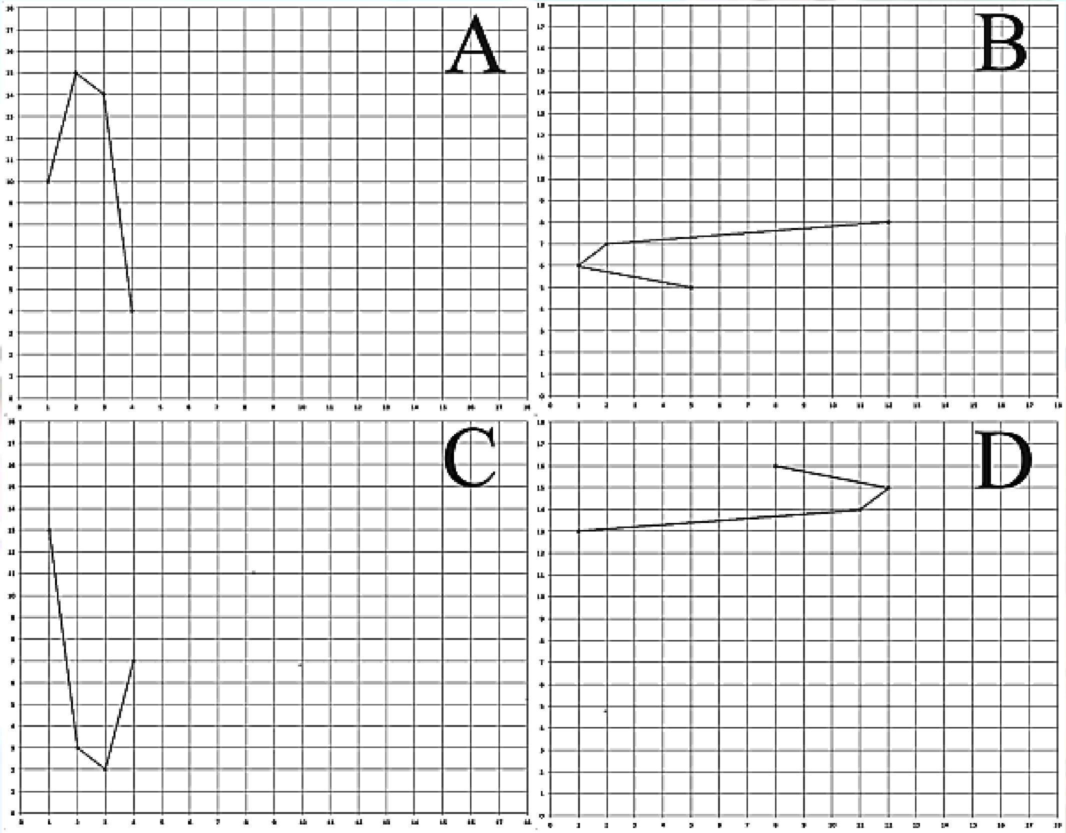
1/17 RFD R_L LTR Oriented Inner Linear Coordinate Reference (OILCR)





4. Phased Generators and Relations (12)

1/17 RFD R_L OILCR
FFT(16) Radix 2 Coordinate Mapping



4. Phased Generators and Relations (13)

Our knowledge of RFD Q_L and corresponding RFD R_L can allow reversing LTR numeric power convergent sequence to its corresponding RTL numeric power divergent sequence uniquely.

Reversing a convergent sequence into a divergent one and vice-versa is the fundamental property to reach information conservation, i.e. information reversibility.

Eventually, **OECS have strong connection even to classic DFT algorithmic structure for discrete data**, Number-Theoretic Transform (NTT), Laplace and Mellin Transforms.

Coherent precision correspondence leads to transparency, ordering, reversibility, kosmos, simplicity, clarity, and **to algorithmic quantum incomputability on real macroscopic machines**.

4. Phased Generators and Relations (14)

POLITECNICO DI MILANO

Rational representations are able to capture two different type of information at the same time, **modulus** (usual quotient information) and associated **outer phase or intrinsic cycle information** which an **inner phase** can be computed from.

So, **rational information** can be better thought to be **isomorphic to vector information** rather than to usual scalar one, at least.

CICT results have been presented in term of classical power series to show the **close relationships to classical and modern control theory** approaches for **causal continuous-time and discrete-time linear systems.**

Traditional rational number system ***Q*** properties allow **to compute evolutive irreducible co-domain** for every computational operative domain used.

Then, all computational information usually lost by using traditional computational approach can be captured and recovered by a corresponding complementary co-domain, step-by-step. Then **co-domain information can be used to correct any computed result, achieving computational information conservation.**

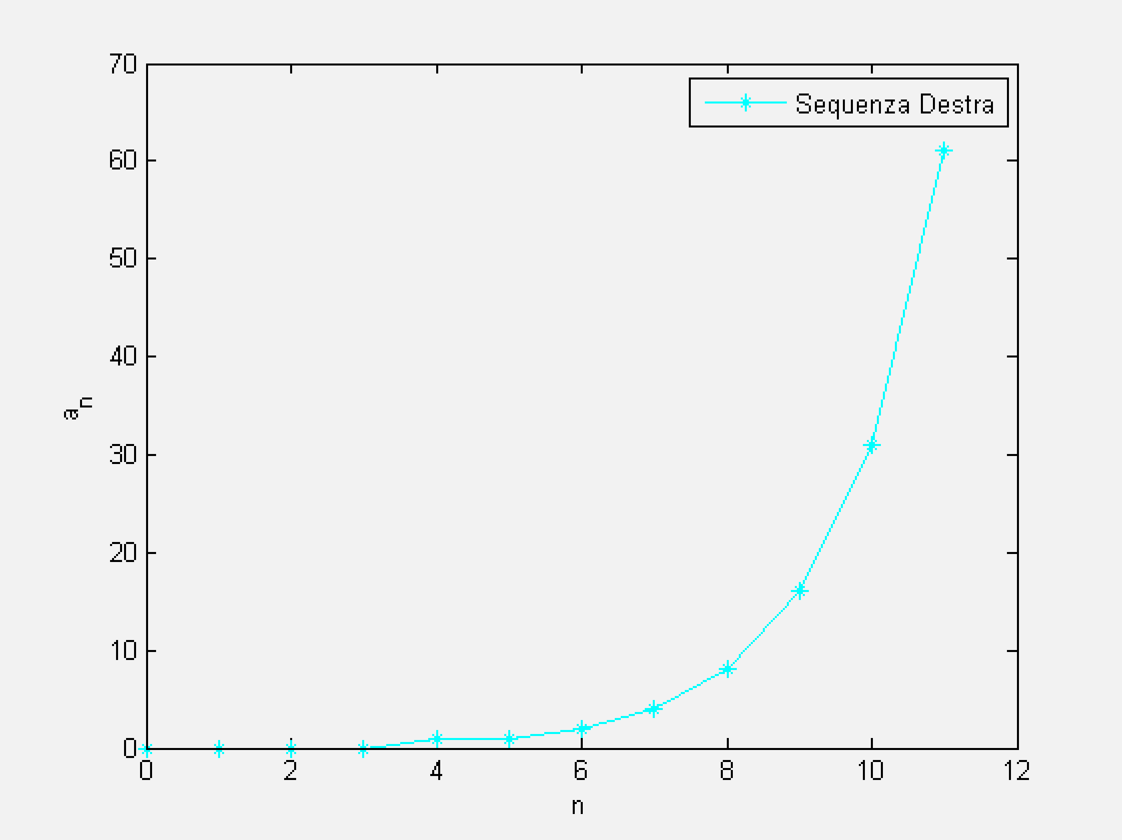
5. Results (00)

5. Results (07)

- **Operative Example (04)**
- **Fundamental CICT OECS Properties (03)**

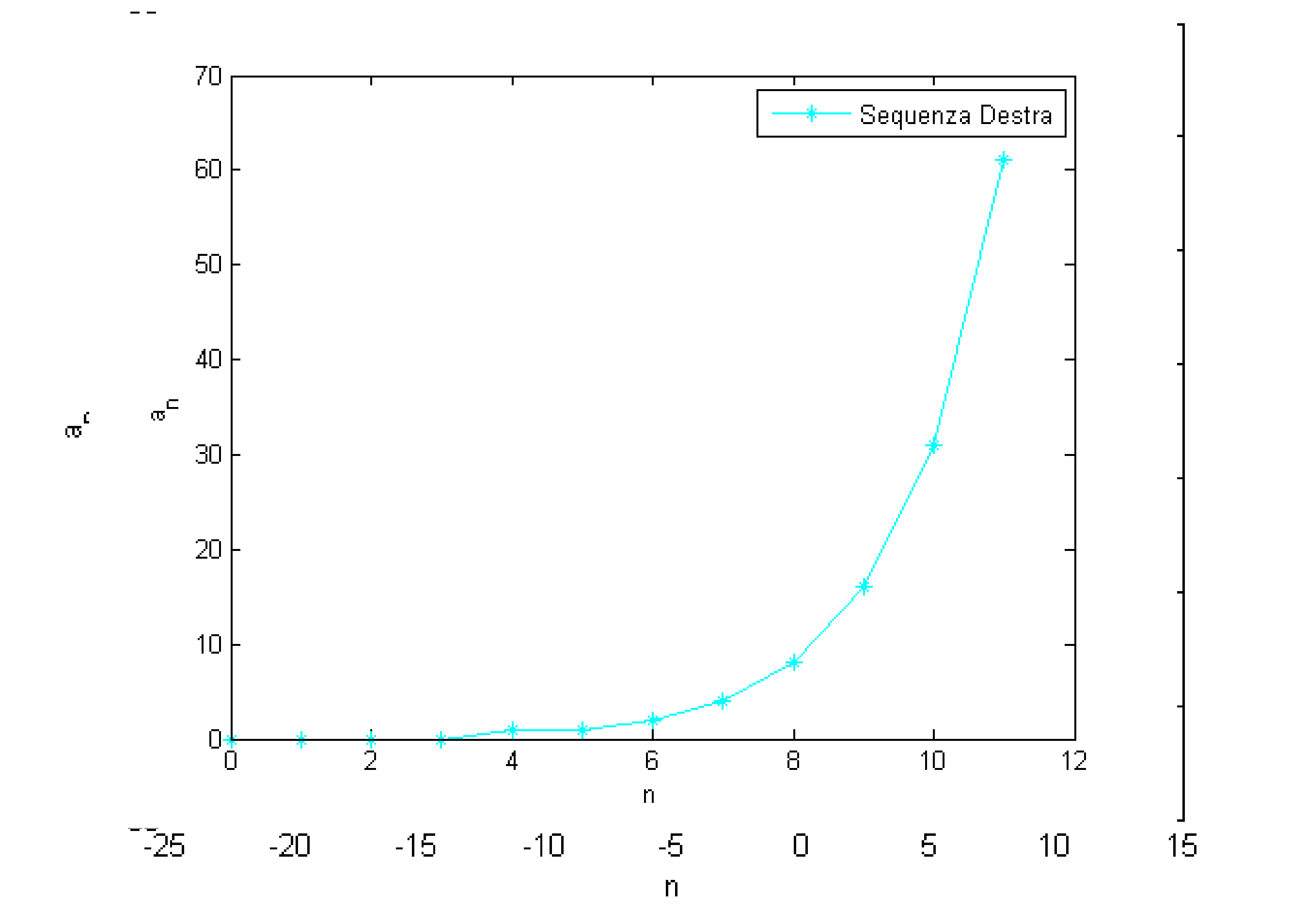
5. Results (01)

As an example, let us take into consideration the simple result of a fifth order **LTR** recurrence relation (for n = 0, 1, 2, …, ∞) as a dynamical system trajectory, depicted in the following figure:



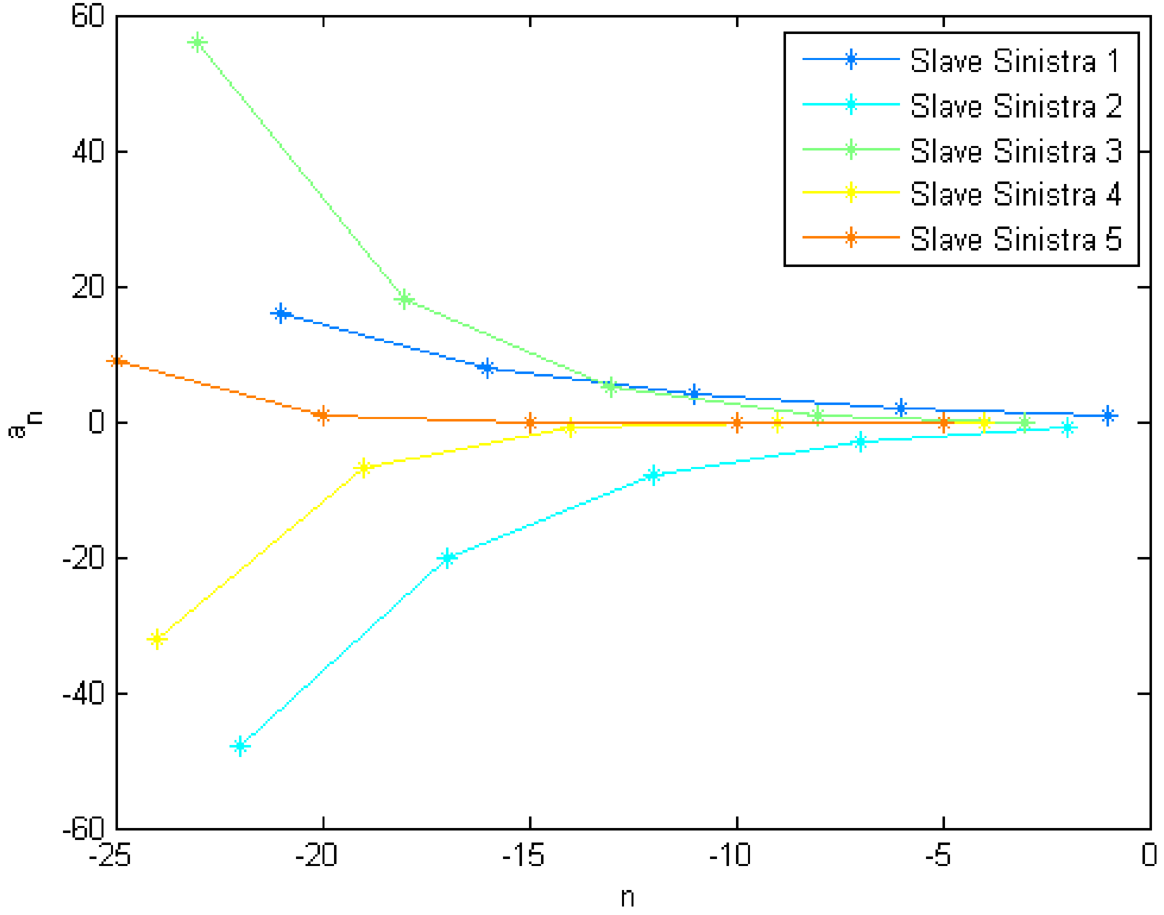
5. Results (02)

Now, **according to CICT**, it is possible to get a **unique RTL function extension**, as reported below, offering a divergent oscillating function, apparently difficult to get any immediate interpretation.

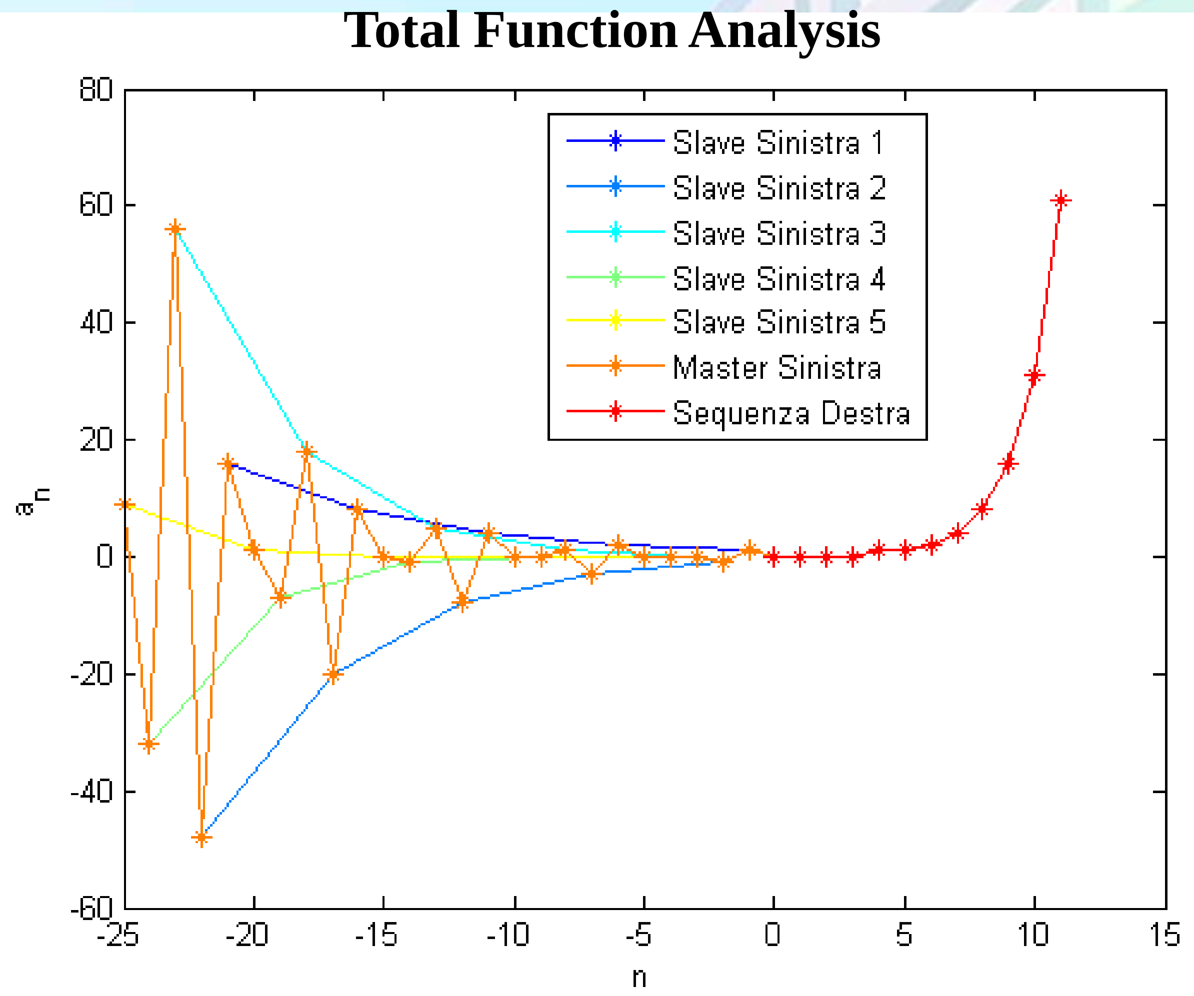


5. Results (03)

As a matter of fact, the divergent oscillating function is the apparently uncoherent overall result of five concealed coherently arranged exponential subprocesses.



5. Results (04)



So, even previously assumed simple function should be more carefully studied by a system dynamics point of view.

5. Results (05)

Fundamental CICT OECS Properties

We got rich new knowledge about fundamental number concepts and properties by **Optimized Exponential Cyclic Sequences** (OECS):

a) Symbolic vs. **OpeRational** Number Representation;
b) Prime vs. **SN Family Group Order** properties;
c) Arbitrary Precision **Exact Rational Number Representation**;
d) Incidence vs. **Correspondence** in **OECS Word Space**;
e) OECS phased generators **Fixed Point** vs. **Pairing** properties;
f) etc… etc…

More specifically, **OECS Family Group of any order** can play a fundamental role by capturing and optimally encoding deterministic information to be lossless recovered at any arbitrary precision.

Combinatorially **OECS** are totally indistinguishable from computer generated pseudo-random sequences or traditional "system noise" to an external Observer.

5. Results (06)

Half-Plane Space vs. OECS Space
Two Irreducible Complementary Operative Spaces

Half-Plane Space

- Inert matter best operational representation compromise.
- A Representation Space endowed with full Flexibility (mapping complexity to simplicity to give space to Imagination).
- Simplified system dynamics framework (Newtonian Approach).
- To model any geometrical space and monitor system dynamics behavior only.
- A Spectator can become a system innatural perturbation.

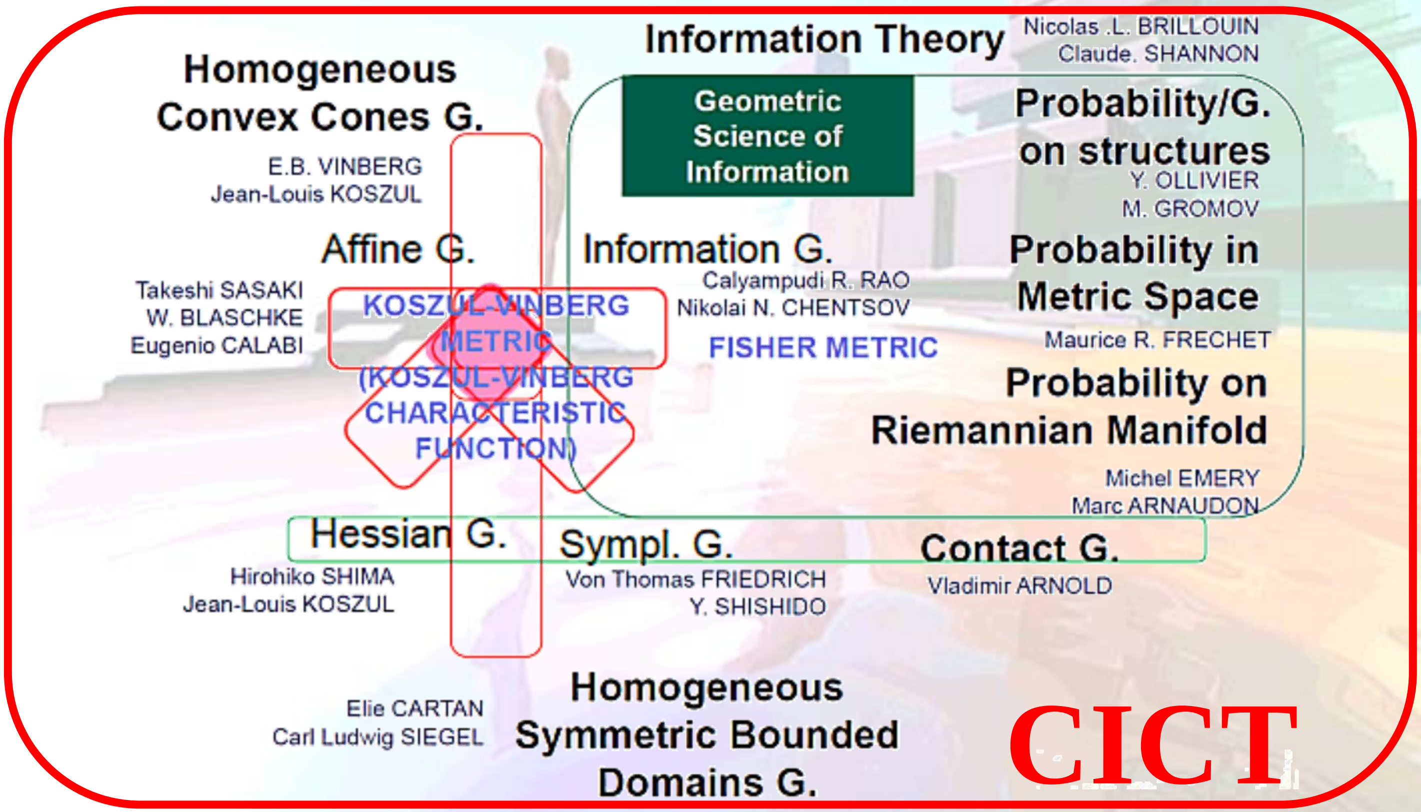
OECS Space

- Livig matter best representation operational compromise.
- An Outer Representation Space one-to-one linked to its Inner Representation Space.
- Natural system dynamics framework (Quantum Physics Approach).
- To model projective relativistic geometry and to anticipate emergent system dynamics.
- An Observer can become a system natural co-artifex.

5. Results (07)

Current Landscape of Geometric Science of Information

Hessian (J.L. Koszul), Homogeneous Convex Cones (E. Vinberg), Homogeneous Symmetric Bounded Domains (E. Cartan, C.L. Siegel), Symplectic (T. von Friedrich, J.M. Souriau), Affine (T. Sasaki, E. Calabi), Information (C. Rao, N. Chentsov). Through Legendre Duality, Contact (V. Arnold) is considered as the odd-dimensional twin of symplectic geometry and could be used to understand Legendre mapping in information geometry.



(F. Barbaresco, 2014)

(R.A. Fiorini, 2014)

6. Summary and Conclusions (00)

6. Summary and Conclusions (04)

- **Quick Recap (03)**
- **Main References (01)**

6. Summary and Conclusions (01)

- To get the right focus to our presentation, we reviewed CICT main results achieved in the past three years by a few operative examples.
- We presented a brief example of Information Geometry Theory applied to Image Processing.
- Then, we discussed the classic information concept modeling in Mathematics by two large theoretical and operative areas interlinked by irreducible complementarity. Their operative compromise and limitation were presented.
- A new vision approach by Inversive Geometry has been applied. To the contrary of traditional approach, inside space forward-backward mapping becomes always possible deterministically.

6. Summary and Conclusions (02)

- New awareness about traditional rational number system ***Q*** numerical properties can guide the development of new competitive algorithm and application.
- Thanks to SN concept, fundamental OECS properties have been reviewed and a few examples discussed. OECS can even be thought as coding sequence for finite fields Galois' geometries (Hyperbolic Geometry), indistinguishable from those generated by traditional random noise sources.
- OECS space is endowed with many properties. Major ones are arbitrary precision Support Quadratic Function natural emergence and two core reflection types: incidence and correspondence.

6. Summary and Conclusions (03)

OECS Space
Core Property Dichotomies Recap

REPRESENTATION LEVEL	CORE DICHOTOMY
DIGIT	Oddness-Evenness
OECS WORD	Fixed Point-Pairing
OECS WORD SPACE	Incidence-Correspondence
INNER-SPACE	Left-Right (KT)
INNER-SPACE	Front-Back (KT)
INNER-SPACE	Top-Bottom (KT)

Main References (01)

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Thank You for Your Attention