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THE HYPOTHETICAL LEARNING TRAJECTORY OF ENUMERATION RULES WITH ISLAMIC VALUES

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Abstract. A context with Islamic values lies in can be used as a starting point in learning mathematics. The challenge is how teachers can deliver, formulate, and connect the mathematical content and the Islamic values in the learning activities. One of the learning approaches that can solve that challenge is Realistic Mathematics Education (RME) by implementing context as one of the activities. This study aims to design hypothetical learning trajectories (HLT) for supporting students in learning the rules of enumeration with the RME approach based on Islamic values. This study uses the first phase of design research (DR) methods. The first phase, namely preliminary design. The HLT is developed in this phase and will then be tested on students in experimental design, and the implemented results are analyzed in the retrospective analysis. Researchers began the study with a literature review, then obtained data about students' difficulties in the enumeration (counting) rule. Based on the review, researchers formulated activities into learning procedures and expected students and teachers' responses, called HLT. The HLT of enumeration rules on data content with RME based on Islamic seen from the perspective of the four emerging modeling levels. At the first level – the 'situational level' – students explore the realistic context of determining the possible route to the mosque. The second level is the 'referential level' where the selection rules are used as a starting point for learning the concept of probability. At the third level – the 'general level' – students use diagrams to generalize the possible outcomes of an experiment and develop an understanding of multiplication rules. Finally, students develop their informal knowledge at the 'formal level' into formal concepts of multiplication rules.

Keywords: Design Research, Realistic Mathematics Education, Multiplication Rules

I. INTRODUCTION

The practice of enumeration (counting) rule is essential for human activities in everyday life. By implementing the concept, humans can mention the number of events that may occur. In Islam, it is also stated that "So that He may know that the Messengers have conveyed the messages of their Lord, while (actually) His knowledge covers what is in them, and He counts everything one by one." Jin 72:28". Even Abdussakir stated that in the Qur'an there are lessons about 38 different numbers. It cannot be denied that the Qur'an talks about mathematics, mostly about numbers.

It is time for all teachers to start designing each lesson and integrate it with Islamic values (Nihayati, Suningsih, &

Abdullah, 2019). By doing this, the researcher hopes to develop good character in students. Teachers can apply it by conveying teaching methods based on noble character. Researchers call it the education that combines faith, shari'a, and morals in learning.

Education in schools must be able to develop student values through religious education, teachers can do this by aligning the progress of science and technology (Muslimin, Putri, & Aisyah, 2020). Abdussakir (2018) stated that Islam does not separate science and religion. The decline in morals and the lack of religious values urge the need for reform in education (Rahman, Abdurrahman, Kadaryanto, & Rusminto, 2015; Maryati & Priatna, 2017). However, this dream has not been realized. Mathematics learning is usually carried

out as a single subject, excluding Islamic values. As a result, mathematics learning becomes rigid, seems complicated, alienated from the realities of life. In the end, it tends to become a subject that students avoid. Mathematics is less able to provide the cultivation of both character and Islamic values. On the other hand, mathematics learning achievement tends to be low (Salafudin, 2015).

The results of the Third International Mathematics and Science Study (TIMSS) and the International Program for Students Assessment (PISA) shows that Indonesian students have a low mathematics achievement. Indonesia's second-grade junior high school students are ranked 34th out of 38 countries, while in PISA 2015, we are ranked 63rd out of 70 countries (Hadi, 2017). Students' ability to solve and interpret problems in various situations is still relatively low. This fact is a significant concern for all parties. Therefore, we need to work on improving the learning quality.

Johnson (2002) and Hadi (2017) confirmed that when students find the meaning of learning mathematics in school, they understand and remember what they have learned. Contextual learning allows students to connect learning in school with real contexts in everyday life. Contextual knowledge extends their context. Providing new experiences for students can encourage their minds to make new relationships. As a result, students can find and construct meaning on their own.

Appropriate learning strategies and methods are needed to develop students' thinking skills. It is based on an orientation to the reform of technical and mathematical skills education (Irawan & Kencanawaty, 2017; (Sembiring, Hadi, & Dolk, 2008). The reform is based on developing problem-solving skills in everyday life. This might be achieved through Realistic Mathematics Education (RME) learning which underlies the learning activities (Wijaya, 2012). Realistic Mathematics Education (RME) is a theory of mathematics learning developed by Dutch education practitioner, Hans Freudenthal, in 1973. The rationale or idea is the assumption that mathematics is a human activity (Wijaya, 2012). It means that through mathematics students do not just sit idly by accepting concepts from teachers. However, students rediscover concepts from learning activities that students do themselves. In addition, this RME learning focuses on applying students' informal solutions and their interpretation through real problems (van Galen, et al., 2008; van den Heuvel-Panhuizen & Drijvers, 2020). The five principles of realistic mathematics education defined by Treffers (2012), namely: Using Context (Phenomenological exploration), Using Models for Progressive Mathematization (Using models and symbols for progressive mathematization), Utilization of Student Construction Results (Using students' construction), Interactivity (Interactivity), and Intertwined.

One of the contexts used in RME in Indonesia is culture (Wahyudi, Zulkardi, & Darmawijoyo, 2016). Kuntowijoyo (2001) states that religion and culture are two things that interact and influence each other to form symbols, and content/values. Previously, it was explained that religion and culture gave meaning to certain mathematical symbols. An Islamic value-based context that students often do in

everyday life (Abdussakir, 2017). It means that it can be integrated with the principle of real context. Even real does not mean concrete, physical, and tangible, but it also cover what students can imagine.

According to Basya (2007) Islamic mathematics is mathematics that uses the Qur'an and the Sunnah of the prophet as postulates. Basya also added that in Islamic mathematics we do not need to prove data that comes from Allah and His messenger even though later in the course of Islamic mathematics it seems to prove the truth. Therefore, the Islamic values associated in mathematics learning are values that are believed and there is no doubt in them.

Mathematics learning integrated with Islamic nuances is mathematics learning which in the learning process is associated with Islamic values. Salafudin (2015) stated that learning mathematics with Islamic values is providing Islamic values in each lesson, both in the form of content and its assessments. Salafudin also added that the values integrated into mathematics learning include aqidah, shari'ah, and moral values. Aqidah values related to matters that the heart must believe in truth, reassure the soul, and become beliefs that are not mixed with doubt. Shari'ah values related to a way of life determined by Allah SWT as a guide in carrying out life in the world to lead to the afterlife, including worship, mu'amalah, manaat, jinayat, and siyasa. Moral values related to the state of a person's soul that encourages him to take actions without first going through thoughts and considerations, including morals towards God, fellow humans, the environment, and animals. Based on the explanation above, this study aims to design students' hypothetical learning trajectories about counting rules through the Realistic Mathematics Education (RME) approach that contains Islamic values.

II. METHODS

This study uses design research as a research method. The research design consists of five characteristics, namely interventionist, process-oriented, reflective components, cyclic characters, and theory-oriented (van den Akker, Gravemeijer, McKenney, & Nieveen, 2006; Prahmana, 2017)). There are three stages in Design Research, namely: preliminary design, experimental design, and retrospective analysis (Prahmana, 2017). This study is the first stage of the design research, the preliminary design. During the preliminary design phase, researchers prepare the learning activity through a literature review. From the literature, researchers received data about students' difficulties in learning probability including enumeration. Based on the review, researchers formulated activities into learning procedures and expected students and teacher's responses, called hypothetical learning trajectory.

Researchers designed learning in the form of HLT to see the implementation of learning using the RME approach. In HLT there are learning steps that must be carried out by teachers in carrying out learning. Through HLT, researchers can make guesses about the answers that may be given by students. Bakker (2004) suggests that the HLT is the relationship between instructional theory and concrete

teaching and learning. The concrete form of HLT consists of three components, namely learning objectives for students, a series of learning activities to encourage student learning, and presumed student learning in which the teacher anticipates student development and thinking.

III. RESULTS AND DISCUSSION

This research aims to design a hypothetical learning trajectory (HLT) of enumeration rules using an Islamic values-based RME approach. This process happened in the preliminary stage of the design research through a literature review.

Enumeration rules consist of several concepts, namely multiplication, permutation, and combination rules. By learning that, students are asked to determine the number of ways an object can be arranged. Students have difficulty determining which enumeration concepts applied to solve the problem (Mawarni, 2016). The students' difficulties are categorized into three types, namely ontogenic, didactical, and epistemological obstacle (Jatmiko, Herman, & Dahlan, 2021). These indicates that some students had difficulties understanding the enumeration rule material.

Based on those facts, researchers formulate a hypothetical learning trajectory which includes learning objectives, a series of activities, and the alleged thinking of students in understanding the rules of enumeration. This initial HLT is useful for researchers and teachers as a guide in designing teaching materials. An overview of HLT in this study is presented in Table I.

TABLE I
THE INITIAL HLT

Type of Activities	Learning Activities	Learning Objectives	Enumeration Concept
Model of	Activity1: Exploring A Map for Going to Mosque	Students are able to understand the introduction of enumeration of an event	Enumeration
	1. Seeking the virtue/mes sage of ayah about enumeration in Muslim life and hadist about going to mosque		
Model of	2. exploring the map to determine routes for going to mosque		
	Activity2: Finding the	Students are able to	Listing the possible way

Route for Friday Prayer	1. Seeking the virtue of hadist shahih of going and returning from praying in the mosque with different paths	determine the possible routes of two consecutive events. We still hope that students find the enumeration in informal ways by showing and listing the possible route on the map.	
	2. Finding the possible routes by exploring the map to determine routes for Friday prayer		
Model For	Activity 3: Designing Your Own Map	Students are able to determine the possible routes and expressing them more formally through the use of diagram and multiplication rules.	The use of diagram
	Sub-Activity 1 Students find the solution in more formal way. They draw a diagram and the roads are represented as connecting line		
Formal Knowledge	Sub-Activity 2 Students grasp the conclusion: To find the number of ways of doing something, multiply the number of choices available at each stage (multiple rule)		
	Activity 4: Students find the number of ways of doing something by multiplying the number of choices available at each stage (multiple rule)	Students are able to solve problem of determining the possible routes by employing the multiplication rules.	Formal form of multiplication rule "The number of possible outcomes from experiment A 1 followed by experiment A 2 until experiment Ak

is $n_1 \times n_2 \times \dots$
 $\times n_k$ "

A. Exploring a Map for Going to Mosque

In the first activity, student activities begin with seeking the virtue/message of ayah about enumeration in Muslim life and hadist about going to mosque. Then, students read a worksheet to explore a map. The students then give their solution on their worksheet.

Informal knowledge: Students are asked to count possible route for going to mosque. Learning Objectives: Students are able to understand the introduction of enumeration of an event. Initially, students are asked to seek the virtue/message of ayah and hadist below,

لَيَعْلَمَنَّ أَنَّ قَدَّ أَتْلُغُوا رَسَلَتِ رَبِّهِمْ وَأَحَاطَ بِمَا لَدَيْهِمْ

وَأَحْصَى كُلَّ شَيْءٍ عَدَدًا

"to ensure that the messengers fully deliver the messages of their Lord—though He "already" knows all about them, and He counts everything one by one." (Surat al-Jinn: 28)" – "sedangkan (sebenarnya) ilmu-Nya meliputi apa yang ada pada mereka, dan Dia menghitung segala sesuatu satu per satu" (in Bahasa).

From Abu Hurairah radhiyallahu 'anhu, the Prophet sallallahu 'alaihi wa sallam said,

وَكُلُّ خَطْوَةٍ تَمْشِيهَا إِلَى الصَّلَاةِ صَدَقَةٌ

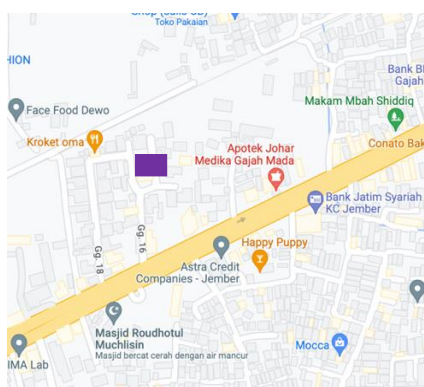
"Every step you take to pray is a charity." (HR. Muslim, no. 1009).

Students are asked a question below to seek the Islamic messages on the hadits provided.

1. According to the hadith above, what is the virtue of going to the mosque?
2. If you go to the mosque closest to your house with different routes, do you think you will get the same virtue?

The class activities constitute enumeration carried out, namely:

Every Idul Fitri, Tio and his family carry out the sunnah of Idul Fitri prayer at Masjid Roudlotul Muhlisin. Pay attention to the location of the mosque from Tio's house (purple square) on the following map.



To be able to pray Idul Fitri at the Roudlotul Muhlisin Mosque, how many different routes can Tio take?

The conjecture of activities contains students' thinking and teacher's responses will be described in Table II.

TABLE III
CONJECTURE OF ACTIVITY 1

No	Activity	Prediction of students' responses	Teacher's responses
1	Seeking the virtue of ayah QS Al Jin: 28	Students know the virtue of the ayah	Teacher gives reinforcement about the tafsir/interpretation that in Muslim life, there are many activities or events related to enumeration. Teacher tells students the interpretation of the ayah "and He counts everything one by one", it can be interpreted that every event has its own calculation. But keep in mind that "while (actually) His knowledge covers what is in them (humans)", then the calculations carried out by humans are entirely God's will in determining the incident.
2	Seeking the virtue of hadist shahih Muslim No. 1009	Students mention the virtue of going to the mosque	Teacher gives elaboration about the good messages on the hadist and gives another hadist HR Muslim No 6666
3	Students exploring the map to determine routes for going to mosque	Students do not mention the virtue of going to the mosque Students draw the possible routes on map Students list the possible routes one by one	Teacher guides students to find out the good messages on the hadist Teacher asks students may they have different way to find the route. Teacher asks questions and discuss other case: How do you think you can find out the number of possible routes can Rika take to pray Idul Fitri? (Rika's house: green circle). Teacher invites
		Students cannot	

finding the route students to explore their difficulties

B. Activity 2: Finding the Route for Friday Prayer

In the second activity, students explore a map again to find the different possible routes for two consecutive events. Unlike the previous activity only one event happens. Students' activities begin with seeking the message of hadist shahih about going to mosque and return home with different path. Then, students get a worksheet and give their solution. Finally, students are asked to present their work in front of the class for discussion.

The second activity aims to support students' understanding of determining the possible routes of two consecutive events. We still hope that students find the enumeration in informal ways by showing and listing the possible route on the map.

The seeking messages of hadits is provided below.

عَنْ أَبِي بِنِ كَعْبٍ قَالَ كَانَ رَجُلٌ لَا أَعْلَمُ رَجُلًا أَبْعَدَ مِنَ الْمَسْجِدِ مِنْهُ وَكَانَ لَا تُحِطُّهُ صَلَاةٌ - قَالَ - فَقِيلَ لَهُ أَوْ قُلْتَ لَهُ لَوْ اسْتَرَيْتَ جَمَارًا تَرَكْتَهُ فِي الظُّلْمَاءِ وَفِي الرَّمْضَاءِ . قَالَ مَا يَسْرُنِي أَنْ مَنَزَلِي إِلَى جَنْبِ الْمَسْجِدِ إِنِّي أُرِيدُ أَنْ يُكْتَبَ لِي مَمَشَائِي إِلَى الْمَسْجِدِ وَرُجُوعِي إِذَا رَجَعْتُ إِلَى أَهْلِي . فَقَالَ رَسُولُ اللَّهِ صَلَّى اللَّهُ عَلَيْهِ وَسَلَّمَ - « قَدْ جَمَعَ اللَّهُ لَكَ ذَلِكَ كُلَّهُ »

"There used to be a person whom I didn't know anyone who was far from the mosque apart from him. But he never missed the prayer. Then someone said to him or I said to him myself, "How about you buy a donkey to ride when it's dark and when the ground is hot." The man then replied, "I am not happy if my house is next to the mosque. I want to record for me my steps towards the mosque and my steps when I return to my family." The Prophet sallallahu 'alaihi wa sallam said, "Verily Allah has recorded for you all." (HR. Muslim, no. 663)

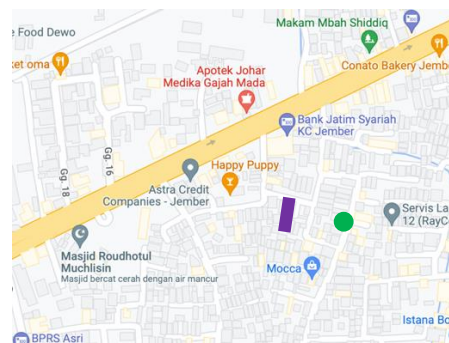
Students are asked:

1. How did the companions of Rasulullah perform their prayers even though his house was far from the mosque?
2. What are the advantages of going and returning from praying in the mosque with different paths?

The main activity for the 2nd activity is:

One day, Samir and Andre made an appointment to go to Friday prayers together. On the map, Samir's house is represented by a green circle while Andre's house by a purple rectangle. Based on the virtues of going to the mosque they know; Samir plans to pick up Andre first and walk together to the mosque.

Show the route Samir can take from his house to Andre's house and then to the mosque!



The conjecture of students' thinking and teacher's reactions can be seen on Table III.

TABLE III
CONJECTURE OF ACTIVITY II

No	Activity	Prediction of students' responses	Teacher's responses
1	Seeking the virtue of hadist shahih Muslim No. 663	Students mention the rewards (pahala) going and returning from praying in the mosque with different paths Students do not mention the reward (pahala) going and returning from praying in the mosque with different paths	Teacher gives elaboration about the good messages on the hadist Teacher guides students to find out the good messages on the hadist
2	Students finding the possible routes by exploring the map to determine routes for Friday prayer	Students draw the possible routes on map with two consecutive events Students list the possible routes of two consecutive events one by one into Students cannot find the routes	Teacher asks students to explain their drawing. Teacher asks students may they have different way to find the route. Teacher invites students to explore their difficulties by listing possible routes and asking further question: "How many Samir travel routes do you get?"

C. Activity 3: Designing Your Own Map

In the third activity, students will make their route for finding the possible ways to the mosque with two or more consecutive events happening. Finally, students present their work in front of the class for discussion. This activity aims to develop students' understanding of determining the possible routes and expressing them more formally through the use of diagram and multiplication rules. Besides, the

Islamic value and messages are integrated through the discussion.

1) *Sub-Activity 1*

Next Friday, Samir and Andre will pray Jum'at together at Masjid Al-Huda. The location is farther than the previous masjid so they will ride Samir's motorcycle since Andre doesn't have one. There are 2 ways to go from Samir to Andre's house, while from Andre's house to Masjid Al-Huda there are 4 roads to choose from.

- a. What good deeds did Samir do to reflect a good Muslim? Remember QS Al-Maidah:2
وَتَعَاوَنُوا عَلَى الْبِرِّ وَالتَّقْوَىٰ وَلَا تَعَاوَنُوا عَلَى الْإِثْمِ وَالْعُدْوَانِ وَاتَّقُوا اللَّهَ إِنَّ اللَّهَ شَدِيدُ الْعِقَابِ
“...And cooperate in righteousness and piety, but do not cooperate in sin and aggression. And fear Allah; indeed, Allah is severe in penalty”.
- b. Determine the possible route by drawing your own map! (Hint: draw the roads as the connecting lines)
- c. How can you find the number of possible routes Samir take from home to Andre's house and then to Masjid An-Nur? Describe your solution!

2) *Sub-Activity 2*

Based on Sub-Activity 1(b)

- a. Please number the roads!
- b. How many Samir travel routes do you get?
- c. Explain the conclusion you get from roads' number choices on (a) and the total travel routes you get on (b)!

The conjecture of students' thinking and teacher's responses can be seen on Table IV.

TABLE IVV
CONJECTURE OF ACTIVITY III

No	Activity	Prediction of students' responses	Teachers' responses
1	Sub-Activity 1 Students find the solution in more formal way. They draw a diagram and the roads are represented as connecting line	Students can mention a practice for being a good muslim by helping each other Students can draw a diagram by connecting each point (houses and mosque) with lines (roads) Students draw a	Teachers elaborate the ayat that helping is a noble habit that we must build as a form of concern for fellow human beings. Several ayat in Qur'an also signal that we should make this mutual help a part of our daily life. Teacher asks students to explain their drawing and gives a praise to students Teacher guides

		diagram with random lines	students by giving a clue such as symboling a house as a square, etc. Teacher gives a praise to students and gives other problem with different numbers. Teacher invites students to explore their difficulties by giving other problems and seek the pattern about the number.
2	Sub-Activity 2 Students grasp the conclusion: To find the number of ways of doing something, multiply the number of choices available at each stage (multiple rule)	Students put numbers on the lines they draw and grasp the conclusion of multiplying each number to find the travel routes Students put numbers on the lines they draw but they cannot explain any conclusion about multiplying each number to find the travel routes	

D. *Activity 4: Do Good Deeds*

In the fourth activity, students will solve some problems related to how do good deeds as a muslim. This activity aims to solve problem of determining the possible routes by employing the multiplication rules. Besides, the Islamic value and messages are integrated through the discussion.

1) *Sub-Activity 1*

The takmir of the Masjid Baiturrahman will make a schedule for the composition of the congregational prayer officers for each week. There are 3 names to become priests, namely Mr. Amir, Ikhsan, and Fadli. Then, among the other takmir members, there are two names assigned to be muazzins, namely Mr. Sulaiman and Rizky. How many ways to choose a pair of imam and muazzin for congregational prayers that can be determined by the takmir?

2) *Sub Activity 2*

One day, Mother told Joni to shop for rice and flour. If Joni was told not only to shop for rice at shop A and flour at shop B, but also to shop C to buy vegetables and to shop D to buy salt. There are 5 roads from home to store A, from store A to store B there are 3 roads, from store B to store C there are 3 roads, and from store C to store D there are 2 roads. How many possible routes of travel can Joni take from home to store A then to store B, store C, and to store D?

In this problem there are several events such as buying rice, buying flour, buying vegetables, and buying salt in one activity (shopping). Although in shopping there are many routes that can be passed, Joni does not need to go through all routes. Just choose one route, Joni will be able to shop for all your needs. This means that although there are many ways that can be done to help parents in an activity, it does not mean that we have to use all of these methods. However, there is only one way, sometimes we feel burdened, like to refuse, and argue with parents' orders for various reasons.

Whereas, students as a child are commanded to do good to their parents as God commands in His word.

وَقَضَىٰ رَبُّكَ أَلَّا تَعْبُدُوا إِلَّا إِيَّاهُ وَبِآلِ وَٰلِدَيْكَ إِحْسَانًا ۗ إِنَّمَا

يَتَّبِعَنَّ عِنْدَكَ الْكِبَرَ أَحَدُهُمَا أَوْ كِلَاهُمَا فَلَا تَقُلْ

هُمَا أَفِي وَلَا تَنْهَهُمَا وَقُل لَّهُمَا قَوْلًا كَرِيمًا ﴿١٧٣﴾

“And your Lord has commanded that you should not worship other than Him and should do good to your parents as well as possible. if one of them or both of them reach old age in your care, then never say to them the word "Ah" and do not yell at them and say to them a noble word. (Surat al-Isra' 17:23).

The conjecture of students' thinking and teacher's responses can be seen on Table V.

TABLE V
CONJECTURE OF ACTIVITY IV

No	Activity	Prediction of students' responses	Teachers' responses
1	Sub-Activity 1 Students find the number of ways of doing something by multiplying the number of choices available at each stage (multiple rule)	Students multiply the number of choices available at each stage (multiple rule) Students do not multiply the number of choices available at each stage	Teacher gives a praise to students and gives other problem with different numbers. Teacher invites students to explore their difficulties by giving other problems and seek the pattern about the number.
2	Sub-Activity 2 Students find the number of ways of doing something by multiplying the number of choices available at each stage (multiple rule)	Students multiply the number of choices available at each stage (multiple rule) Students do not multiply the number of choices available at each stage	Teacher gives a praise to students and gives other problem with different numbers. Teacher invites students to explore their difficulties by giving other problems and seek the pattern about the number.

In this study, the instructional design employs Jum'at prayers in an Islamic-realistic context. The context was utilized as the framework for this study because students expressed a positive to learn mathematics using an Islamic

context (Muslimin, Putri, & Aisyah, 2020). Additionally, a previous study depicted that learning mathematics in based Islamic context could effectively improve students' mathematical concept understanding (Ulpah & Novikasari, 2020). Students are unable to understand mathematical concepts because they are typically taught practical formulas in schools rather than being fully informed about the idea of enumeration and how they are used in daily life. In contrast, as it is a human activity, mathematics must be connected to human life (van Galen, et al., 2008). Therefore, it is crucial to employ contexts in the mathematics learning process, especially in Islamic schools (Mariana, 2017).

In this design, real context is used as a starting point for learning which is also successfully applied to this design. Students see a map of the route to the prayer Jum'at in near mosque story to introduce the activity in the set learning design that uses the context of the Jum'u'ah prayer of Islamic context. Based on the learning activities used in this design, the RME approach was implemented (Hadi, 2017). In the first activity, student activities begin with seeking the virtue/message of ayah about enumeration in Muslim life and hadith about going to the mosque. The activity aimed to impose Islamic values in students' life. Then, students were engaged to use their informal knowledge by exploring a map for counting possible routes to go to a certain mosque. In this activity, students used concrete things such as a map to find a route. And, in the second activity, namely model of, they still elaborate the listing of the possible way of finding the route of Friday prayer. Next, in the third activity, students enter the mode for which they employ a diagram or model to determine the possible routes of Friday prayer and express them more formally by designing their map. In the last activity, students were able to solve other Islamic-context-based problems by employing the formal form of multiplication rules. As students grasped the idea of enumeration rules formally through a series of learning activities, then the hypothetical learning trajectory of the learning could be carried out (Hadi, 2017).

The trajectory of the enumeration rule design using the Islamic context is adjusted to the student learning sequences and the competency standards included in the Indonesian education curriculum (As'ari, et al., 2018). According to the curriculum, students must comprehend the idea of using the enumeration rule to determine the probability. It means that students should understand the concepts of addition and multiplication rules, and list the possible way by using the formal way of multiplication rule.

To support earlier studies employing the RME approach and integrating with Islamic values in learning mathematics, this research includes an extra reference in mathematics education. For instance, the context of congregational prayer to learn power and root, ratio and scale, and plane figure (Novikasari & Ulpah, 2022), the context of Eid Fitri, Jama prayer, and pilgrimage of hajj to learn operation in integers, equality and speed comparison, and theoretical and empirical chances consecutively (Kusaeri, Sadieda, Indayati, & Faizeien, 2018), doing good deed guided by Qur'an and hadith to learn subtraction, multiplication, and division of

integers (Nihayati, Suningsih, & Abdullah, 2019). As a result, this study adds context studies to be used as a foundation for studying mathematics.

IV. CONCLUSIONS

The realistic-Islamic context generated in this study is a designated series of learning activities that are expected can support the development of students' conceptual understanding of enumerations, especially multiplication rules. The HLT consists of four activities; exploring a map for going to a mosque, finding the route for Friday prayer, designing your own map, and doing good deeds. This HLT is designed to reconstruct students' understanding of multiplication rules to determine the possible way of events. And most importantly, the Islamic context can instill good values and deeds that can be applied in everyday life. Hopefully, this hypothetical learning trajectory is a base for further research to implement it in a teaching experiment, analyze the results using retrospective analysis, and build it into a local teaching theory.

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