A FUZZY-BASED BUSINESS DECISION MAKING SYSTEM: FROM A MULTI-OBJECTIVE PERSPECTIVE

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Abstract

In order to provide essential managerial services for an organization, there is need for making accurate critical business-biased decisions. A business activity hinged on an effective administrative course of action will not only portray the manager of the business as adept but also help advance the financial interests of the organization, while minimizing its losses in this respect. In this paper, a decision making model for controlling business activities is developed, using a fusion of linear programming methods and a set of fuzzy membership functions. In the research conducted, it is revealed that: to improve the effectiveness of a model used for making multiple objective decisions for business related activities, the use of a fuzzy method is more effective than the use of a non-fuzzy method in minimizing the objective functions. It was also discovered that when computing the objective functions of a problem with fuzzy-like outcomes, a more accurate result can be obtained by using a linear programming model, fortified with a technique for managing imprecise data.

Keywords: Business Activities, Decision Making, Fuzzy Method, Linear Programming, Objective Functions.

1. INTRODUCTION

Prior to the development of mathematically tailored systems of planning and conducting business related activities, most business decisions were made out of personal instinct and experience. Many of these decisions were often erroneous as they were usually based on vague information. In the modern day, some organizations still use intuitive methods in making business decisions, which often makes the decision inadequate and not well suited to solving serious problems (Kreß & Metternich, 2022).

A multiple objective decision process is crucial to planning a business activity. A decision can be represented by fuzzy numbers, since it is often imprecise. Models developed, using fuzzy programming methods should be regarded as new conventional decision making methods rather than as a new contribution to multiple objective decision making methods (Dyson, 1980). This paper aims at developing a decision model for organizing business activities (using a cocoa processing plant as a case study), by examining, which is better of applying the fuzzy and non-fuzzy multi objective decision model (under fuzzy constraints), using crisp and fuzzy objective functions.

This remaining part of this paper provides a literature review of related works on different types of multi objective decision models and relevant fuzzy set theory definition. It also describes the Non-Fuzzy Multiple Objective Decision Model (MODM) and the Fuzzy Multiple Objective Decision Model (FMODM). The outcomes of the two models are portrayed in the development of a decision making model for a cocoa processing company, which produces, transports and delivers cocoa liquid at different cocoa product manufacturing sites.

2. LITERATURE REVIEW

One of the most important subjects in modern day decision making methods for businesses is the theory of decision making. This theory adopts the use of optimization methods linked with concepts of single and multiple criteria. Decision making models that deal with multiple criteria are more difficult to model. This is because; they have to do with human conviction or judgment. The points of preferences indicated by the human decision maker are what brings about or is referred to as human judgment (Holden & Ellner, 2017). The idea of goal programming emanated from the efforts made in order to a model decision processes for a business, using the multiple criteria technique (Pal & Chakraborti, 2013). The method required the decision maker to pitch each of the objectives involved in the decision making, to a certain number of goals that need to be met (Buckley, Feuring, & Hayashi, 2001). Meeting these goals entails, providing a solution to a multi criterion problem. In the end, the “ideal solution” confirms that the best solution to the problem has been established. This solution has to be that which optimizes all the criteria concurrently, however it is considered to be unattainable. As such, the decision maker deliberates on workable solutions which are very close to the ideal solution. Generally, in goal programming, the preferences of the decision maker are represented with objectives, weights, mutual benefit and stages of the goals, in order to resolve a problem. Researchers in Cheng, Yang and Hwang (2000) proposed a fuzzy multicriteria model, which consists of linear mathematical programming and a comparison with stochastic programming. The advantages and disadvantages of the reviewed fuzzy mathematical programming techniques were illustrated using an optimal portfolio selection problem. In Das, Mandal and Edalatpanah (2016) a method for solving whole fuzzy linear programming problems was developed. They carried out numerical experimentations, showing the preference of the proposed method over the current ones. The transitional step towards fuzzy multi criteria models is using models that consider some fuzzy values. Some of these models are linear mathematical formulation of multiple objective decision making processes presented by mainly crisp and few fuzzy values. Many authors have studied such models (Cheng, Yang & Hwang, 2000; Sadjadi, Seyedhosseini, & Hassanlou, 2011; Haimes, & Chankong, 2012; Zimmermann, 1978); Sangaiah, Tirkolaee, & Goli, 2020). Interactive multiple objective system technique contributed to the improvement of flexibility and robustness of multiple objective decision making methodology. Fuzzy systems are designed for modelling available knowledge (information) and thinking process (Akinrotimi & Oladele, 2018). Research carried out by Ren, Xu, & Gou, (2016), in developing an approach to solve multi criteria problems with Pythagorean fuzzy information supports this assertion as the researchers conducted simulation tests to analyze how the risk attitudes of the decision makers exert influence on the results of multi criteria decision making, under uncertainty. They researchers applied the developed technique in selecting a governor for the Asian Infrastructure Investment Bank to show its real-world applicability. The approach was found to scale well in solving the multi criteria problem.

3. METHODOLOGY

3.1 The Fuzzy Logic Approach

Fuzzy set theory uses linguistic variables rather than quantitative variables to represent imprecise concepts. Linguistic variables analyze the vagueness of human language (Yager, Reformat & To, 2019).

3.1.1 Fuzzy Set

Let X be a universe of discourse. $\mathring{A}$ is a fuzzy subset of X if for all $x \epsilon X$, there is a number $\mu\mathring{A}(x)$ ϵ /0, 1/assigned to represent the membership of x to $\mathring{A}$, and $\mu\mathring{A}(x)$ is called the membership function of Å.

3.1.2 Fuzzy number

A fuzzy number Å is a normal and convex subset of X. normally implies

$\exists X \epsilon R \ v \ \mu\mathring{A}(x) = 1$

Convexity implies:

$\forall x_1 \epsilon X, \quad x_2 \epsilon X, \ \forall \alpha \epsilon /0.1/$

$\mu\mathring{A}(\alpha x_1 + (1-\alpha) x_2) \geq \min \mu\mathring{A}(x_1), \min \mu\mathring{A}(x_2)$

3.1.3 Fuzzy Decision

The fuzzy set of alternatives resulting from the intersection of the fuzzy constraints and fuzzy objective functions Bellman and Zadeh (1970) A fuzzy decision defined in an analogy to non-fuzzy environments 'as the selection of activities which simultaneously satisfy objective functions and constraints". Fuzzy objective function is characterized by its membership functions. In fuzzy set theory the intersection of sets normally corresponds to the logical "and'. The 'decision' in a fuzzy environment can therefore be viewed as the intersection of fuzzy constraints and fuzzy objective functions. The relationship between constraints and objective functions in a fuzzy environment is fully symmetric (Zimmermann, 1978).

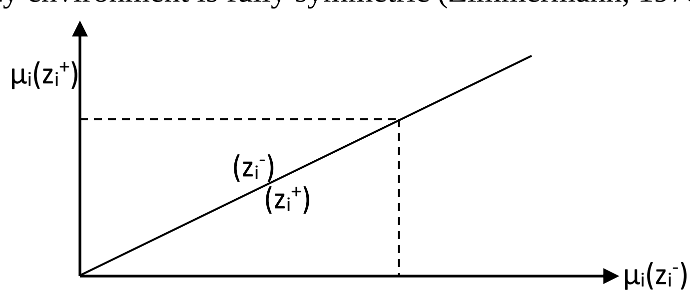

Figure 1: Objective Function as a Fuzzy Number

3.2 Non-Fuzzy Multi-Objective Problem

A general linear multiple criteria decision making model can be presents as a vector x written in the transformed form:

$x^T = [\, x_1, x_2 \ldots\ldots x_n \,]$

This maximizes objective functions:

$$\max z_i = \sum_{j=1}^{n} c_i \tag{1}$$

With constraints,

$$\max z_i = \sum_{j=1}^{n} a_i \leq b_i$$

$$i = 1, 2 \ldots m,\ x \geq 0 \tag{2}$$

where c_{ij}, a_{ij} and b_i are crisp (non) values. This problem has been studied and solved by many authors. Zimmermann has solved this problem by using the fuzzy linear programming (Zimmermann, 1978). He formulated the fuzzy linear program by separating every objective function z_i its maximum z_i^+ value by solving.

$$z_i^+ = \max z_i = \textstyle\sum_j c_{ij}x_j \text{ , and } z_i = \min z_i = \textstyle\sum_j c_{ij}x_j \tag{3}$$

with the constraints in (2) solutions z_i^+ and z_i are known as individual best and worst solutions respectively. Since for every objective function z_i . Its value changes linearly from z_i^- to z_i^+ it may be considered as a fuzzy number with the membership function $\mu_i\,(z_i)$ as

shown in figure 1:

$$\mu_j(z_j) = \begin{cases} 0 & \text{for } z_i \leq z_i^- \\ \dfrac{(z_i - z_i^-)}{(z_i^+ - z_i^-)} & \text{for } z_i^- \leq z_i^+,\ 1,2,\ldots n \\ 1 & \text{for } (z_i \geq z_i^+) \end{cases} \tag{4}$$

According to Bellman-Zadeh's principle of decision making in the fuzzy environment the grade of membership of a decision j. specified by objective z_i is obtained by:

$$\alpha = \min \mu_i(z_i), \quad j=1, 2, \ldots k \tag{5}$$

Maxmin j

Subject to

$$\alpha \leq \mu_i\,(z_i),\ j=1,2,\ldots k \quad 0 \leq \alpha \leq 1 \tag{6}$$

According to this principle the optimal values of multicriteria optimization correspond to maximum of j. The auxiliary linear program is obtained by:

$$z_i^{-} = \max \alpha \quad (7)$$

With constraints (4, 6) taking into account (1) and (4)

$$\sum_{j=1}^{n} c_i + (z_i + - z_i -)\ \alpha \leq (z_i - z_i^{-}) / (z_i^{+} - z_i^{-}) \qquad i = 1,2,\ldots k \quad (8)$$

$$0 \leq \alpha \leq 1,\ xj \geq 0 \qquad j = 1,2\ldots n$$

The original linear constraints in (2) are added to these constraints. So, we find a vector x subject to

$$z_i (x_i) \geq\sim z_i^0 \quad \forall_I,\ x \epsilon X \quad (9)$$

Where z_i^0, $\forall_I$ are corresponding goals, and $\geq \sim$ is a soft or quasi inequality. The objective functions are assured to be maximized

$$\max/\min\ [z_i (x) \ldots .\ z_i (x)] \quad (10)$$

$$X = \{x | g_s (x) \{\geq = \leq\} 0,\ s=1 \ldots .m\}$$

where z_i (x), j∈J are maximization objectives, z(x) i∈I are the minimization objectives, *IUJ* = { 1,2,….*n*} are considered as fuzzy constraints. All functions $z_j(x)$, $g_s(x)$, (i = l,..n; s = l,…m) can be linear and nonlinear. With the tolerances of fuzzy constraints given, the membership functions μ_i (x), $\forall_i$ could be established. The feasible set solution obtained through min-operator is defined by interaction of the fuzzy objective set. The feasible set is presented by its membership

$$\mu_i (x) = \min(\mu_i(x) \ldots .\ \mu_i(x),) \quad (11)$$

If a decision maker deals with a maximum μ_D (x) in the feasible set then the solution procedure is max (mini μ_i (x)) x∈X, suppose the overall satisfactory level of compromise is a = $\min_i \mu_i$ (x) then the problem can be explained as:

Find max a subject to:

$$\alpha \leq \mu_i (x)\ \forall_I \quad x \in X,$$

Assuming that membership functions, based on preference or satisfaction, are linear and non-decreasing between z_i^+ (x) and z_i^-(x) for $\forall_i$

$$\mu_j (z_j) = \begin{cases} 0 & \text{for } z_i \leq z_i^{-} \\ \dfrac{(z_i - z_i^{-})}{(z_i^{+} - z_i^{-})} & \text{for } z_i^{-} \leq z_i^{+},\ 1,2,\ldots n \\ 1 & \text{for } (z_i \geq z_i^{+}) \end{cases} \quad (12)$$

The only feasible solution region is the area $\{x|z_i\ (x) \leq z_i\ (x) \leq z_i+\}\forall_i$ and $x \in X$, hence we can write:

Find max a subject to

$$\mu_k(x) = [z_i(x) - z^-_i] / [z^+_i - z^-_i] \geq \alpha\ x \in X \quad (13)$$

This problem can be solved by using two-phase approach. The first phase relates to the search for the optimal value of α^0 in order to find a possible solution (x^0). If the possible solution is unique, x^0 is an optimal non-dominated solution. Otherwise, the second phase is introduced to search for the maximum arithmetic mean value of all membership restricted by original constraints and $\alpha \geq \alpha^0 \forall$. That is,

$$\text{Max } (\textstyle\sum_i \alpha_i) / I \quad (14)$$

$$\alpha' \leq \alpha_i \leq \mu_i\ (x).\ \forall_i,\ x \in X,$$

for i objective functions and α' solution (7). The objective functions (10) could be written

$$\text{Max } [\textstyle\sum_i \mu_i\ (x)] / I$$

$$\alpha' \leq \mu_i\ (x),\ \forall_i,\ x \in X, \quad (15)$$

By unifying both objective (7) and (11) the second step can be automatically solved after the first step following the solution procedure of the simplex method

$$\text{Max } \alpha + \delta\ [\textstyle\sum_i \mu_i\ (x)],\ / I \qquad \alpha \leq \mu_i\ (x),\ \forall_i\ x \in X, \quad (16)$$

Where δ is sufficiently small positive number. Since the weights between objectives are not equal we can write.

$$\max\ \alpha + \delta \textstyle\sum_i w_i \mu_i\ (x) \qquad \alpha \leq \mu_i\ (x),\ \forall_i\ x \in X, \quad (17)$$

For w_i as the relative importance of the i^{th} objective and $\sum_i \mu_i = 1$. The coefficient α represents the degree of acceptability or degree of possibility for the optimal solution. For manufacturing industry activities the minimal value of the coefficient αn can be prescribed. Hence two new constraints are added in this linear program:

$$\alpha \geq \alpha_1 \quad \alpha \leq \alpha_n \qquad \text{, where } 0 \leq \alpha \leq I \quad 0 \leq \alpha, \quad (18)$$

Coefficient of satisfaction (φI) in relation to the best individual solutions zi+ are:

$$(\varphi i) = \max\ z_i / z_i^+ \qquad i = 1, 2, \ldots n$$

From the aspect of fuzzy set theory the augmented max-min approach allows for compensation among objectives. Firstly one reaches the solutions at a large unit, and then by re-evaluating these solutions the compromise solutions at a smaller unit are obtained.

3.3 Fuzzy Multi-Objective Problem

The Fuzzy Multiple objective Decision Model (FMODM) studied by Xu, & Zhou (2011). Fuzzy-like multiple objective decision making (Vol. 263). Berlin: Springer states that the effectiveness of a decision makers' performance in a decision process can be improved as a result of the high quality of analytic information supplied by a computer. They propose an interactive Fuzzy Multiple objective Decision Model (IFMODM) to solve a specific domain of Multiple Objective Decision Model (MODM).

$$\text{Max}\ (z_i(x)\ldots., z_n(x)) \quad (19)$$

Subject to

$$g_j(x) \leq \sim b_j \quad j=1,\ldots,m \quad x\geq 0$$

where b_j $\forall_j$ are fuzzy resources available with corresponding maximal tolerances t_i. Their membership functions are assumed to be non-increasing linear functions between b_j and $b_j + t_j$

The objective functions (5.1) are redefined into:

$$\text{Max}\ z_i\ (C_i, x) \quad i=1, 2\ldots.I \quad (20)$$

Subject to:

$$g_j\,(A, x)\ \{\leq\ =\ \geq\}\ b_i \quad j=1,2,\ldots..m,\ x \geq 0$$

We present the model (20) limitations as fuzzy inequalities since the limitations prevent the objective functions from reaching their individual optimum.

Find x, subject to:

$$z_i\,(C_i, x) \geq\sim z_i^0, \forall i \qquad g_j(A_j, x)\leq b_j, \forall_j\ x \geq 0 \quad (21)$$

where z_i^0, $\forall i$ are the goals of the objectives and $\geq\sim$ is a soft or fuzzy inequality. With the known tolerances of fuzzy constraints the membership functions $\mu_i\,(z_i)$, $\forall i$ to measure satisfaction levels of fuzzy objective constraint could be established. It is supposed that membership functions are based on a preference concept. The membership functions can be any non-decreasing functions for maximization objectives and non-increasing functions for maximization objectives such as linear, exponential, and hyperbolic. In Xu, & Zhou (2011), fuzzy-like multiple objective decision making (Vol. 263). Berlin: Springer., the researchers assume linear membership functions since the other types of membership functions can be transferred into equivalent linear forms.

Each objective of equation (20) should have an individual best (z_i^+) and individual worst solution (z_i^-)

$$Z_i^+ = \max z_i\,(C_{i,}x),\ x\in X,$$

$Z_i^- = \min z_i (C_i, x), x \in X,$ (22)

The linear membership function can be defined as in (8). According to (17) and (18) the following augmented problem can be defined.

$$\max \alpha + \delta \sum_i w_i \mu i (x) \quad (23)$$

$\alpha \leq \mu_i (x), \forall i, x \varepsilon X, \alpha \varepsilon [0,1]$

where δ is a sufficiently small positive number, and w_i ($\sum_i w_i = 1$) is of relative importance or weight. If a decision maker wants to provide his/her goals z_i^0 and corresponding tolerances t_i for objectives, than for $z_i^0 \leq z_i^+$ and $(z_i^0 - t_i)b \geq z_i^-$ the problem will become:

Find x, subject to

$$z_i (C_i, x)) \geq\sim z_i, \forall i \text{ and } x \in X, \quad (24)$$

where $z_i^0, \forall i$ as well as tolerances t_i are given. Then

$$\text{Max } \alpha + \delta \in_i w_i \mu_i (z_i)$$

$$\mu_i (z_i) = 1 - [z_i^0 - z_i (C_i, x)] / t_i \geq \alpha \; x \in X, \alpha \in [0,1] \quad (25)$$

The problem can be further considered as:

$$\text{Max } \alpha + \delta[\sum_i w_i \mu_i (z_i) + \delta \sum_i q_i \mu_i (g_i)] \quad (26)$$

Subject to:

$\mu_i (z_i) = [z_i (C_i, x) - z_i] / [z_I^+ - z_I^-] \geq \alpha \; \forall i$

$\mu_i (g_i) = 1 - [g_i (A_j, x) - b_j] / t_I \geq \alpha \; \forall_j$ $\quad x > \alpha \in [0,1]$

where w_i and g_i, $\forall i$, j are of relative importance and $\sum_i w_i + \sum_j g_j = 1$

The computer program was written using MATLAB 2015. Input data are: number of objectives *k,* number of constraints *m,* number of unknowns *n,* goals z_i (i=1,2,…k), elements c_{ij} (i=1,2,…k; j =1,2,…n), a_{ij} (i=1,2,…n), b_i (i=1,2,…m), tolerances t_i (i= 1,2,…k) and d_i (i=1,2,…m). The program determines the individual best z_i^+ solution and the individual worst solution z_i^- for every objective i_i (i=1, 2,…k). The objective functions are (3) and the constraints are (2). The obtained values z_i^+ and zj, based on the modified Zimmermann's procedure, are used to solve the linear program with the objective function (17) and constraints (2) (8) and (18). For the non-fuzzy problem, this program gives the values of unknown x_j (j =1,2,…n) maximal values of objective function z_i (i=1,2,…k),, coefficient of acceptability α and coefficients of satisfaction φ_i (i=1,2,…k). For the fuzzy problem, the linear program with the objective function (5,3) and the constraints (5,6) gives: the optimal value of unknown x_i (i=1,2,…n), objective function z_i coefficients of satisfaction φ_i(i=1,2,…k) and coefficient of acceptability α.

3.4 Case Study Analysis and Modeling

Liquid cocoa is a raw material for producing cocoa products such as chocolates and cocoa butter, which are both in high demand, for local consumption and export to foreign countries. These two commodities are derived from cocoa beans after they have undergone the process of fermentation, drying, roasting and separation from their outer covering (skins).Usually, cocoa beans are then grounded, to produce "cocoa mass". This mass is melted to obtain liquid cocoa, which can be separated into cocoa solids (from which cocoa powder can be obtained) and cocoa butter. It can also be cooled and molded into blocks of raw chocolate. Data obtained from a cocoa processing plant has been analyzed for building our proposed decision making model. Cocoa liquids is shipped in barrels over a distance ranging 1500m-3000m to four cocoa processing sits , including, two cocoa butter cream manufacturing sites and two cocoa beverage producing sites (Sites A-D) . Three pumps and eleven interior vibrators are used for delivering the cocoa liquid at each manufacturing sites. Table 1, illustrates the manufacturing capacities of the plant, the operational capacity of the pumps and the labor requirements at the four sites. The analysis carried out, demonstrates the complexity of the variable and constraints of this liquid cocoa production plant and delivery system. The liquid cocoa producing company manager's task shall be, to increase the profit, by using the maximum capacity of the cocoa processing plant while meeting the requirement of the four manufacturing sites for liquid cocoa, through a feasible schedule.

Table 1: Cocoa Processing Plant Capacity and Manufacturing Site's Resource Demands

	Cocoa Processing Plant	Site A	Site B	Site C	Site D	Remark
Plant Capacity	70m^3/h 2640m^3 weekly					300m^3 (tolerance)
Transit mixers (total =8)		9.25m^3/h	10.15m^3/h	8.25m^3/h	12.57m^3/h	Operated by 7 Workers
Cocoa pumps (total =4)	-	18m^3/h	25m^3/h	32m^3/h	40m^3/h	Operated by 6 Workers
Interior vibrators (total =12)		50m^3/h				
Worker requirement	5	6	7	9	11	
Minimal Cocoa requirement (tolerance)		15.0m^3/h 600 m^3/ week (49m^3)	230m^3/h 788 m^3/ week (65m^3)	24.7m^3/h 950 m^3/ week (75m^3)	28.5m^3/h 1026 m^3/ week (92m^3)	-
Weekly values are based on 48 working hours/week						

3.5 Objective Formulation

Success of any decision model will directly depend on the formulation of the objective function, taking into account all the influential factors (Ghanbari, Ghorbani-Moghadam, Mahdavi-Amiri, 2020). The final objective function was modeled taking into the account, independent factors, profit expressed as ₦/m^3, index of work quality (performance) and worker satisfaction.

3.5.1 Profit

The expected profit as related to the volume of Cocoa to be manufactured is modeled as the first objective and is shown in Table 2. The minimal expected weekly profit as a fuzzy value is Z^0 = ₦9, 000 per week.

Table 2: Modelling Profit as an Objective

Site Name	Site A	Site B	Site C	Site D
Expected profit (₦/week)	15,000	9,000	10,000	12,000

3.5.2 Index of quality

Quality is often valued above quantity in most organizations. As such, the index of quality at the manufacturing site is modeled as the second objective. The index is of range 9 points/m^3 (poor) quality to 13 points/m^3 (excellent) quality and the assigned values are shown in Table 3.

Table 3: Modelling Index of Quality as an Objective

Site Name	Site A	Site B	Site C	Site D
Index of Quality	9	10	7.5	13

3.5.3 Worker Satisfaction Index

As shown in table 3, the index of worker satisfaction was modeled as the third objective and is from range 9 to 14 points per m^3 of liquid cocoa produced, transported and deposited.

Table 4: Modelling Worker Satisfaction as an Objective

Site Name	Site A	Site B	Site C	Site D
Worker Satisfaction Index	8	7	9	14

The fuzzy solution that gives higher profit with possibility of realization α= 0.852

3.6 Variables that Optimize the Objective Function

After knowing the objective function the next task, is to determine the variables that optimize the objective function (Das, Mandal, & Edalatpanah, 2016). In our experiment, the problem is to find: the optimal value of unknowns xi (i=1, 2, 3,4) that represent quantities of Cocoa which have to be delivered to Site A, B, C and D respectively and corresponding optimal values of the objective functions z_1, z_2 and z_3. According to problem requirements and available data (Table 1, 2, 3 and 4) the objective functions can be modeled as follows:

- *max* z_1=15x_1+9x_2+10x_3 + 12x_4*(>,~)* 25000 with tolerance, t_1=2200 (profit)
- *max* z_2=8x_1+7x_2+ 9x_3 + 14x_4*(>,~)* 21500 with tolerance, t_2=1800 (index of quality)
- *max* z_3= 7x_1+5x_2+4x_3+16x_4*(>,~)* 19000 with tolerance, t_3=1500(worker satisfaction index)
- x_1*+x2+x3+*x_4*(<,~)* 2640, tolerance d_1=200h (weekly capacity of the Cocoa plant)
- 0.119x_1 +0.108x_2 +0.139x_3 +0.126x_4*(≤,~)* 8x48=384h, tolerance d_2=23h (weekly use of 8 transit mixers, taking into account of their working capacities)
- 0.063*x1*+0.045*x2*+0.038*x3*+0.0267x_4*(<,~)* 4x48=168h, tolerance *d3*=10h (weekly engagement of 4 Cocoa pumps)
- 0.100x_1+0.117x_2+0.150x_3+0.198x_4*(<,~)* 24x48=1152h, tolerance d_4=94h (weekly engagement of 24 workers for interior delivering, placing and consolidating Cocoa at sites A, B, C and D).
- Minimal weekly requests for Cocoa from the four manufacturing sites:

Site A, *x1≥ 590* m^3, tolerance *d5*=50m^3
Site B, *x2 ≥ 760* m^3, tolerance *d6*=70 m^3
Site C, *x3≥ 760* m^3, tolerance *d7*=75 m^3
Site D, *x4≥ 780* m^3, tolerance *d8* = 84 m^3

- The minimal value of the degree of acceptability is $\alpha_1 \geq 0.80$.These constraints written in full are as follows:
- *x1+x2+x3+*x_4*(<,~)2640*
- *0.119x1+0.108x2+0.139x3+ 0.126* x_4*(<,~) 384*
- *0.063x1+0.045x2+0.038x3+ 0.0267* x_4*(<,~) 168*
- *0.100x1+0.117x2+0.150x3 + 0.198* x_4*(<,~)1152*

The individual best and worst non-fuzzy solution for constraints (b) and individual objective functions (a) is then obtained using the linear programming method to solve the above equations. The obtained results are summarized in Table 5.

4. RESULTS AND DISSCUSSION

The solution to the multiple objective functions using the results obtained for z_i^+ and z_i^- as shown in Table 5 and using the modified Zimmermann's procedure as discussed in Section 4, were implemented in MATLAB 2015 and executed on a Windows 2010, Core i5 Computer System. The results obtained are summarized in Table 6. The simulations were repeated three times and found that the results are stable. Coefficient of acceptability of this solution was found to be $\alpha=0.956$. When the objective functions were modeled using the described fuzzy approach the obtained solutions are as summarized in Table 7. Coefficient of acceptability of this solution $\alpha=0.892$.As depicted in Figures 2 and 3, the obtained results clearly shows the superiority of fuzzy approach. However it is also interesting to note that there is not much difference between fuzzy and non-fuzzy solutions for the three objective functions. The difference is being less that *2* percent. The coefficients of acceptability of the solutions α, indicating the possibility of realizing these solutions, are very high. According to this, the decision maker could accept:

- the non-fuzzy solution that gives smaller profit with possibility of realization $\alpha=0.956$
- the fuzzy solution that gives higher profit with possibility of realization $\alpha=0.892$

Table 5: Individual Best and Worst Non-Fuzzy Solution

Objective	X_1 (m^3/week)	X_2 (m^3/week)	X_3 (m^3/week)	X_4 (m^3/week)	Z_1^+(₦)	Z_1^-(₦)
1	745.03	758.00	912.00	985.00	27402.49	0
2	593.00	925.15	912.00	985.00	21357.00	0
3	758.03	756.00	912.00	985.00	194681.00	0

Table 6: Optimal results using non-fuzzy procedure

X_1 (m^3/week)	X_2 (m^3/week)	X_3 (m^3/week)	X_4 (m^3/week)	Max (Z_1)	Max (Z_2)	Max (Z_3)	Max (Z_4)
744.93	956.57	912.0	1054.0	φ	φ	φ	φ
				27,291	25,140	19,259	20,762
				1.355	1.355	1.357	1.357

•φ is the coefficient of satisfaction

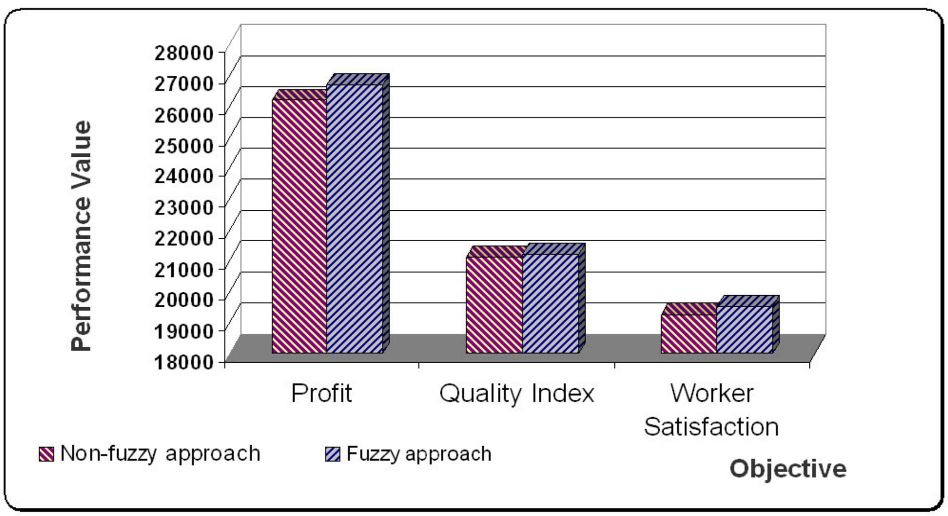

Figure 1: Comparison of fuzzy and non-fuzzy approach showing the performance value of objective functions.

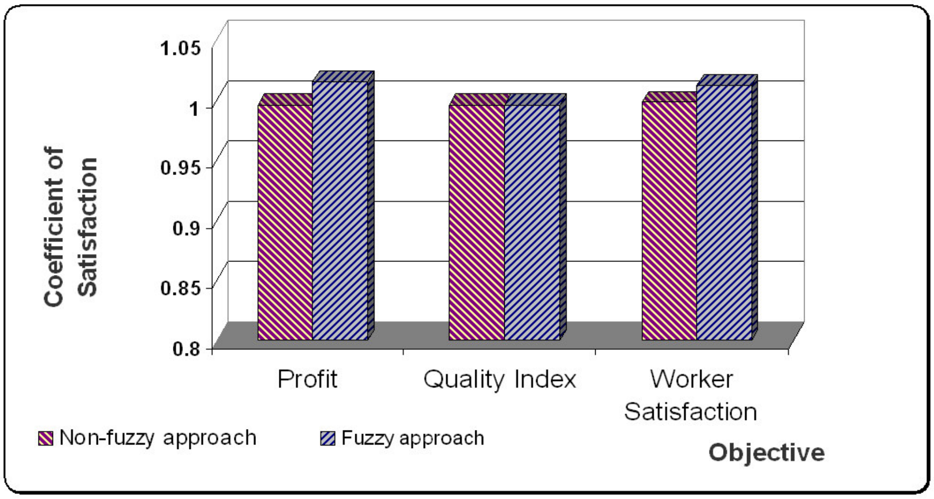

Figure 2: Comparison of fuzzy and non-fuzzy approach showing the coefficient of satisfaction for the different objective functions.

It can be observed form Figures 1 and 2 that except for the quality index bar-chart result in the coefficient of satisfaction graph, the fuzzy approach outperforms the non-fuzzy approach in determining the optimal values for profit, quality index (on performance value) and worker satisfaction. In addition, with the developed system the decision maker (e.g manager) can vary the values of the coefficient α in the interval *0, 1*and to receive the corresponding optimal values of production and profit with corresponding values of possibility. Considering a future expansion of this work, a careful study of the optimal values of the objective functions and the various constraints, suggest that an expert knowledge problem domain can be utilized in providing a deeper understanding of the achieved results.

5. CONCLUSION

Organizing business activities involve making cogent decisions that can either bring profit or loss to the business. Decision making procedures that are capable of bolstering up the business activities that involve imprecise data can be analyzed using the multi-objective criteria fuzzy models. The modeling of the cocoa processing plant problem presented in this paper involves the combination of fuzzy linear objective functions and constraints. The results show how that the fuzzy method used, in terms of individual solution for the four objective functions and coefficients of satisfaction more efficient. It also shows that the difference between fuzzy and non-fuzzy objective functions for the individual best solutions. There is however less than 20% possibility of realizing optimal profit. The software developed for the purpose of implementing the developed model, is capable of calculating the optimal profit for a given possibility of realization coefficient. This research work has shown that a similar or better level of satisfaction for the obtained results can be achieved when membership functions are introduced into a linear programming model, either in constraints, or both as objective functions and constraints.

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