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The Impact of a Five-day Number Sense Intervention on High School Student's Quantitative Reasoning Skills and Self-efficacy

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THE IMPACT OF A FIVE-DAY NUMBER SENSE INTERVENTION ON HIGH SCHOOL STUDENT'S QUANTITIVATE REASONING SKILLS AND SELF-EFFICACY

by

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ABSTRACT

Understanding numerical quantities and applying this knowledge in practical applications is essential throughout life. A well-developed number sense comes from learning foundational skills and continuing to rely on these skills and concepts in higher mathematical education as well as in adulthood. Prior research shows that K-8 students lack a conceptual understanding of fraction, decimal, and percentage concepts (NCTM, 2009). While there is literature that identifies a deficit in these mathematical areas, there is a need to examine possible activities and interventions that can be performed throughout secondary education courses that support growth in students' conceptual understanding of rational number concepts. The purpose of this research study is to investigate how a five-day rational number sense intervention can affect students' number sense in the subtopics of fractions, decimals, and percentages as well as their self-efficacy.

For this study, 63 students from three different math periods at the same school and taught by the same instructor participated. These 63 students were divided into three groups: a control and two intervention groups. Both intervention groups received five days of instructional activities revolving around various rational number concepts and practical applications. To collect data, a pre-assessment consisting of ten mathematical computation questions, five contextualized mathematical questions, and five self-efficacy questions was used. After the intervention was conducted, an identical post-assessment was administered. Student follow-up interviews (N=4) were conducted to gain additional insight into the effects of the intervention.

A dependent t-test compared pre-assessment results to post-assessment results for the *computational items*. Both intervention groups earned significantly higher scores on the post-test than on the pre-test. The control group did not display any significant score differences between pre- and post-assessment. To examine the *contextualized math items*, similar dependent t-tests were conducted to compare pre-and post-assessment results. There was a significant improvement in one of the two intervention classes. Analysis of the *self-efficacy items* showed that students had a significant increase in self-efficacy post-intervention. Implications for improvements, future research, and expanded interventions to support advancing students' number sense are discussed.

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DEDICATION

I would like to thank everyone that offered support and guidance throughout this journey. To my one and only best friend, G: Thank you for always offering to help and listening to my thoughts and ideas. You continue to support me through actions and words more than you know. To my parents: Thank you for continuing to support me and push me to achieve all my goals. To my class fellow, soon to be Dr. Jennifer Crissey: Thank you for always being a text away and understanding my frustrations and struggles throughout this process. To my students: I could not have done this without each and every one of you so thank you for participating in the study. I sincerely hope that you were able to gain a deeper understanding on number sense topics and find an application to fractions, decimals, and percentages in your own life.

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CHAPTER I - INTRODUCTION

Introduction

Number sense, or the ability to understand numerical quantities and relationships (Way, 2005), is an essential skill necessary from elementary school through adulthood. Agustin (2012) states that a certain level of competence in quantitative reasoning is critical for becoming a productive citizen because it allows one to interpret and represent quantities in work and in life. Number sense supports long-term mathematical understanding and investigation (NCTM, 2009), but is also a daily tool with infinitely many quantitative applications, from mileage to time to budgeting.

Quantitative reasoning, or communicating and applying numerical quantities (Agustin et al., 2012), builds from a well-developed number sense and being able to logically reason and problem-solve. Basic skills such as calculating percentages in order to find tax, a tip, or a discount, working with decimals to estimate or calculate with money, and using fractions to make measurements, conversions when cooking, or partition quantities can be learned through essential procedures and algorithms that students are taught in upper-level math courses (Agustin et al., 2012).

Procedural and Conceptual Understanding

Unfortunately, in the last several decades, instruction in high school mathematics classrooms has made few advancements from a focus on procedural understanding to conceptual understanding (Hiebert, 1997; Moss, 1999; Star, 2005; Morales, 2014; Nahdi, 2020; Borji, 2021). This can result in students going through their formal mathematical education not developing an understanding of the computations they perform, and therefore unable to apply what they have learned outside of the classroom. With 85% of jobs now classified as “skilled” (Rosen, 2003) and the mathematical problems presented at these jobs requiring application of basic arithmetic, students need to develop an understanding of basic number sense tasks and be able to transfer their knowledge on these tasks (Rosen, 2003; Stone, Alfeld, & Pearson, 2008; James, 2013). A change can be made in how students are exposed to topics such as fractions, decimals, percentages, ratios, and basic arithmetic in school in order to strengthen their number sense knowledge. A more developed number sense will benefit students in their education and their role as a citizen (Agustin, 2012). Number sense, or understanding numerical quantities, is essential when handling money (e.g., grocery shopping, taxes, tipping at a restaurant) or completing daily activities (e.g., cooking, driving). Regardless of career, there are applications of number sense that are relevant to everyday life.

In high school Algebra, students are expected to know the slope formula, the quadratic formula, growth and decay models, and more. The emphasis placed on obtaining correct answers from procedural approaches eliminates exploration of the reasoning behind mathematical processes (Borji, 2019; Hurrell, 2021). As a result, there is a general assumption that mathematics consists of formulas and rules that are to be memorized (Boaler, 2016). Hiebert (1997) argued that when students merely memorize rules and symbols, they may be learning, but what they are learning is not mathematics. In order to really know mathematics, one must understand the relationship between mathematical representations and quantities (Hiebert, 1997). Because there is a lack of understanding behind the numerical relationships that support these formulas and rules, students fall into a trap of never advancing their conceptual understanding (Rudolph, 2011). Conceptual understanding revolves around knowing not only isolated mathematical facts but also how to connect and integrate these facts for functional and transferable use (Schoenfeld, 1992).

Several arguments have been made for ways to increase conceptual understanding in the classroom, including improving mental calculation skills, using word problem applications, and having students ask for the information needed to solve a problem (Hope & Sherrill, 1987; Trushkowsky, 2015; Meyer, 2015). Building from a strong foundation of mental calculations allows students to understand numeric relationships and then apply their knowledge. Word problems have been identified as a way to help students relate school-based mathematics to real-world applications. However, the current structure of word problems and how they are presented creates an aversion toward them and an over eagerness to use mathematical formulas and step-by-step procedures instead (Trushkowsky, 2015). This boils down to students not fully conceptualizing the meaning of what they are being asked to do. Meyer (2015) argues that students are often taught procedurally, where they are given all the information they need in order to solve a problem, and aren't involved in the process of formulating the question or understanding the significance of the answer. These habits continue to produce students who are incompetent at basic mathematical skills (Meyer, 2015). Research (Stone, Alfeld, & Pearson, 2008) has shown an influx of high school graduates who lack the basic math knowledge required for entry level workforce positions and for postsecondary education. Basic math knowledge includes but is not limited to number computations, working with fractions and decimals, estimation, and use of ratios (NCTM, 2009). The root of the problem that Stone, Alfeld, & Pearson (2008) identify in the workforce can be classified as students' deficiency in number sense (Rudolph, 2021).

Number sense includes solving mathematical problems that aren't restricted to using a specific formula or algorithm (Way, 2005). As mentioned above, procedural teaching methods contribute to students practicing mathematics through formulas and algorithms. For example, students are generally taught two-digit multiplication by lining the numbers up and regrouping. If asked to multiply 45 and 12, a student that has a strong number sense may forgo the procedure and use knowledge about place value to take 45x10 and add that to 45x2. Seeing mathematical relationships and thinking flexibly are part of a well-developed number sense.

Number sense itself offers a wide range of definitions and categories. As identified by Way (2005), five components of number sense are number meaning, number relationships, number magnitude, operations with numbers, and number references. Additional elements of number sense often include identifying numerical patterns, estimating, number transformations, calculating error, efficient procedures, and interpreting results (Jordan et al., 2006; Hope, 1989; Reys, 1994; Reys & Yang, 1998). Being able to make practical and reasonable decisions with numerical problems comes from having a developed number sense.

Developing Number Sense

While number sense primarily develops throughout a student's elementary education, the elements are foundational for building habits of mind that can be applied in high school mathematics courses (NCTM, 2009). Number sense often bridges over to quantitative reasoning in secondary or post-secondary education. For example, if asked to calculate 15% of $32, estimation and number relationships could be used in order to first find 10% ($3.20) and then 5% ($1.60) which is half of the value for 10%. Strong quantitative reasoning also contributes to analyzing results and making sure they make sense. In the situation above, 15% is $4.80 which is close to $5. $5 would be less than ⅙ of $32 and ⅙ as a percentage is between 16% and 17%. This connected way of thinking shows that the answer is reasonable. Quantitative reasoning skills reside in applications of basic mathematics, interpreting quantities in context, and being able to draw reasonable and relevant conclusions (Agustin et al., 2012). It is necessary that students develop foundational number sense skills in order to find success in quantitative reasoning. A curriculum with an intentional focus on number sense has been seen to increase mathematical development in elementary-aged students (Shumway, 2019). However, the need for number sense continues throughout students' educational careers. *For this reason, I posit there is a need to revisit number sense topics relevant to everyday mathematics as students progress in their learning throughout secondary education.*

Number sense focuses on a student's ability to understand numerical concepts and relationships, yet the current curricula are not always built with the intent of meeting this goal (Moomaw, 2010). With no set national curriculum, students in different states, cities, and schools learn Algebra in varying ways. Algebra standards must be taught, but the way teachers approach instruction is inconsistent, even within school districts. In a school setting, the teacher must value number sense to take precedence over merely mastering mathematical formulas and algorithms (Reys, 1994). The way mathematics is taught dictates how much attention is brought to number sense and how well students develop ways of thinking consistent in fluency with numbers. It has been shown that number sense develops through meaningful classroom experiences and activities that focus on calculations, measurement, and estimation (Hope, 1989; Reys, 1994; Kieren, 1996; Moss & Case 1999; Irwin, 2001; Moore, 2014). When a student performs a calculation, a measurement, or an estimation, they should be able to explain the purpose behind the process. A curriculum that emphasizes open-ended questions, problem-solving, and interpretation is key in developing number sense.

Not only do teacher-student interactions play a role in developing number sense, but peer interaction has been found beneficial as well (Irwin, 2001). A curriculum that promotes the development of number sense encourages student collaboration, creativity, investigative reasoning, and multiple solution pathways (Reys, 1994). These key factors can be present in informal education opportunities and aid in a deeper understanding of numerical relationships. Number sense development has been linked to students' informal education with numbers (Jordan et al., 2006). Previous research (Irwin, 2001) has shown that students interact with numbers differently based on their ability level. Higher-level students interact flexibly in order to problem-solve, whereas lower-level students use methods of recall in order to apply a specific procedure or formula (Boaler, 2016). Students that tend to have a more difficult time with number combinations and number sense in general lack mental manipulation of quantities and a basic understanding of counting principles (Jordan et al., 2006). A foundation with numbers, operations, and quantity relationships is necessary to successfully expand more advanced mathematics principles. Giving students time and experience in varying problem-solving settings can develop flexible ways of thinking and build self-efficacy.

Self-efficacy in Mathematics

Self-efficacy can be defined as an individual's belief in their ability to execute a certain task or perform to a set ability level (Merriam-Webster, n.d.). In mathematics, many students lack self-efficacy and believe they cannot perform well (Boaler, 2016). Students tend to lack confidence in their mathematical ability partially because of the inconsistencies between everyday mathematics and school-focused algorithms (Case & Sowder, 1990). These discrepancies make it difficult for students to apply what they learn in a classroom setting to real-life situations. The disconnect causes students to be unsure in many mathematical situations, such as calculating a discount, finding a tip, or equating fractions. There are ways that formal education can help students build self-efficacy and develop a positive mathematical self-image. Students' attitudes and self-image about mathematics are partially a product of how well they understand mathematics (Kloosterman, Raymond, & Emenaker, 1996). A curriculum that is relevant to everyday mathematics, highlights numerical relationships, and encourages students to try new things and learn from their mistakes can help contribute to understanding content and increase students' self-efficacy and self-image in math (Boaler, 2016).

Developing a mathematical curriculum focusing less on procedural approaches and more on applied mathematical processes and making connections might help high school students bridge the gap between formal and informal mathematics applications. Boaler (2016) found that habits, such as being open to different experiences or feeling comfortable being wrong, are demonstrated by successful people. Students are often reluctant to try a new way of thinking or uncomfortable sharing answers for fear of making mistakes (Boaler, 2016; Usher, 2009). Specific habits of the mind can be developed in mathematics classrooms to shift students away from being fearful of trying new things. Students’ mindsets and self-efficacy can have a major impact on their ability to learn. One of the biggest influences on self-efficacy in math is a student’s past performance (Usher, 2009). When students lack self-efficacy, their negativity can feed into a fixed mindset where they believe they cannot change their mathematical ability. This mindset hampers developmental progress, negatively impacting their ability to advance number sense concepts (Boaler, 2016; Usher, 2009).

Statement of the Problem

Research has shown that elementary and middle school students struggle with rational number concepts (Moss, 2015). Rational numbers include integers, fractions, decimals, and their applications. The lack of developing strong number sense carries over and can be seen even at the college level by students struggling with sophisticated reasoning using elementary math (Agustin, 2012). Furthermore, into adulthood, we are expected to develop computational skills, strong logical reasoning, and problem-solving skills (Rosen, 2003). However, Rosen (2003) found that over one-third of job applicants lack the necessary basic math skills to be deemed qualified for entry-level positions. Stone, Alfeld, & Pearson (2008) also reported on high school graduates' lack of basic mathematical skills needed to enter the workforce. Underdeveloped number sense can have a lasting impact.

Prior research has shown specifically that number and measurement skills developed in grades K-8 are essential for success in high school mathematics (NCTM, 2009). Foundational components of number sense are important in higher-order mathematical thinking (Jordan, 2006). Although several research studies have been conducted evaluating students' number sense ability, these have taken place with elementary or middle school students (Morais & Serrazina, 2017; Moss & Case, 1999; Irwin, 2001; Moss, 2005). There have also been several research studies that indicate the lack of quantitative reasoning skills in college students or adults (Rosen, 2003; Agustin, 2012; Moore, 2014). While the current research communicates the problem of underdeveloped number sense in K-8 students, there is a lack of research targeted at the high school level and, more specifically, how we can help correct this situation in secondary education. *Specifically, how can a number sense intervention at the high school level help correct the misconceptions from early formal education and bridge the gap to strengthen quantitative reasoning skills in adults?* This gap in the literature exposes a need to examine possible activities to support conceptual understanding of rational number tasks, specifically in practical applications that high school students could carry over to real-life adult situations.

Purpose of the Study

This study investigated how a five-day rational number sense intervention can affect high school students number sense in the subtopics of fractions, decimals, and percentages. The goal was to investigate how students respond to an intervention focused on understanding elementary rational number concepts and applications. As a by-product of this intervention, I also looked at how students' mathematical self-efficacy might be affected. A mixed-methods design was used to compare two intervention groups of students that underwent a five-day rational number sense intervention to a control group that did not participate in the intervention. The intervention took place with one Geometry class and one Algebra II class during class time. The control group consisted of a single Geometry class that was not exposed to any material in the intervention and continued with traditional math learning. Collection of quantitative data occurred via pre- and post-assessments targeting both mathematical computation and contextualized mathematical thinking. Quantitative data on self-efficacy was collected through Likert scale items on pre- and post-assessments. Follow-up qualitative data was collected by selected student interviews. Students who demonstrated major growth from the pre-assessment to the post-assessment, specifically on the contextual questions were selected for brief semi-structured interviews.

The independent variable of interest was a five-day rational number sense intervention. The dependent variables included students' performance on rational number computation and context questions, as well as student self-efficacy levels measured by self-reported values. All participants in the research study were high school Geometry or Algebra II students taught by the researcher at Purvis High School in the Spring of 2022.

Theoretical Framework

A constructivist approach to learning and development was used as the basis for this work with connection to neo-Piagetian theory. Constructivism, or the idea that students construct or build knowledge through experience, focuses heavily on student-centered learning activities (Olusegun, 2015). Constructivism supports a deeper level of understanding in that students are discovering and transforming information (Olusegun, 2015; Villanueva, 2015). As an example, classroom activities/lessons in the intervention classes were presented in a way that allowed students to make their own meaning by drawing on culturally and contextually relevant examples. In the intervention, students worked to apply fraction, decimal, and percentage concepts to find taxes taken out of their paychecks and to find the amount of money they spend on gas getting to school and work in a week. Constructivism also supports the idea of a short-term intervention to build connections because this theory states that students learn by fitting new information together with what they already know (Olusegun, 2015; Villanueva, 2015). Figure 1 shows the basis of constructivism in relation to mathematics, which acts as a foundation for how a constructivist approach was used in the five-day number sense intervention to benefit students.

Figure 1. Constructivism- Social Cognitive Theory (Barker, 2011)

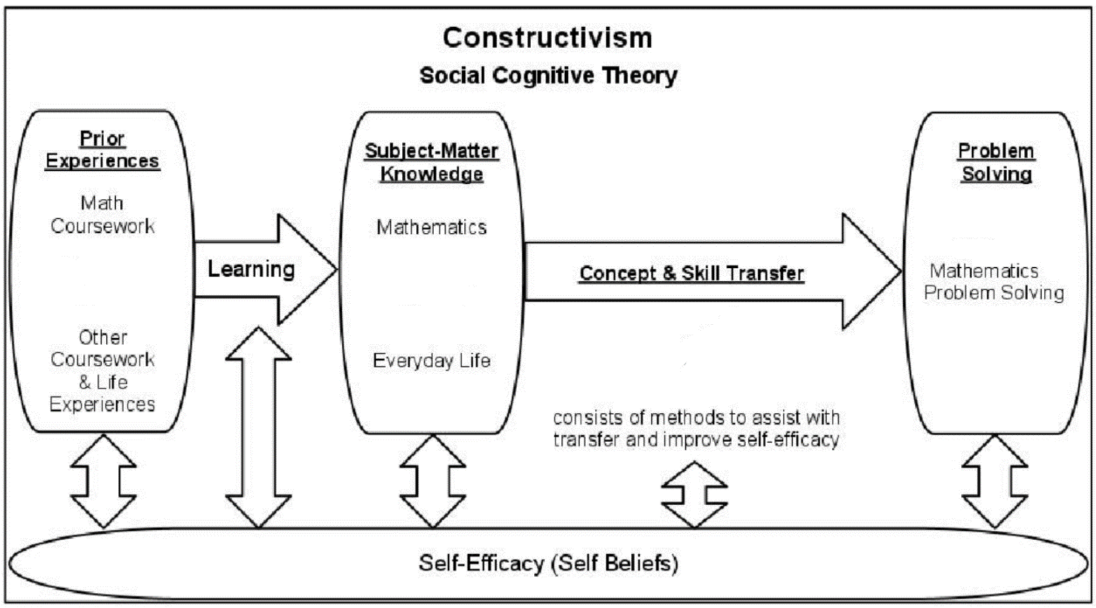


Figure 1 shows that students use prior experiences and knowledge to build upon in learning new concepts. Students then take subject-matter knowledge, that is often taught in class, and use this in combination with their prior experience to build an understanding of a concept. In relation to the five-day number sense intervention, the tasks and activities that students were asked to engage and participate in were related to their probable life experiences and practical involvements. The intention of the intervention was to build a foundation of number sense that was deeply rooted in student experiences and everyday life. A constructivist approach states that students are better able to transfer knowledge that is meaningful to them in order to problem solve (Barker, 2011). Lastly, Figure 1 emphasizes how self-efficacy plays an important role throughout a student’s entire learning experience. Consistent with Figure 1, intervention methods were applied to improve self-efficacy and allow students a sense of ownership in their learning efforts and understanding.

Neo-Piagetian theory posits that cognitive development progresses as students’ experiences and learning increase coincident with biological maturation (Case & Sowder, 1990). The neo-Piagetian theory states that students progress through stages of cognitive development in a stair-step-like fashion. The sensorimotor and interrelational stages occur before the age of 4. The dimensional and vectoral stages mature as students go through primary and secondary education. In the dimensional stage, students construct cognitive representations that are mutually related (Sevinc, 2019). To advance to the fourth stage, the vectoral stage, students must be able to relate different dimensions from stage three (Sevinc, 2019). The neo-Piagetian theory supports the idea that without a strong foundation of number sense, students cannot continue to develop and construct meaning between quantities. In connection to the stages, the concepts of fractions, decimals, and percentages are all interrelated. Still, to understand their relationships, students must pass into the vectoral stage, where they can create mental representations of each quantity and relate these topics to one another. An example of this would be organizing numbers given in several different representations (fractions with unlike denominators and decimals) from least to greatest by converting them all to a common form.

Number sense development is a process that can be progressed and matured with growing experiences and increased knowledge (Reys, 1994). For this reason, it is necessary to continue emphasizing number sense throughout a student’s entire mathematical career. Constantly creating opportunities in the classroom for students to expand their knowledge of numerical relationships and meaningful mathematics will help increase their conceptual understanding (Hurrell, 2021). Parallel to developing strong mathematical skills, a strong mathematical mindset, or when students take an active approach in making sense of mathematical concepts, is essential in learning new ideas, making connections, and distinguishing relationships. Boaler (2016, p.36) specifically notes the interaction between number sense and a mathematical mindset by stating, “number sense reflects a deep understanding of mathematics, but it comes about through a mathematical mindset that is focused on making sense of numbers and quantities.”

Research Questions

The following questions were investigated in this research study to examine students’ current number sense ability and address the overarching inquiry into ways to improve students' number sense at the high school level.

RQ1: In what ways does a five-day number sense-focused intervention impact students' overall number sense and understanding of rational numbers?

RQ2: What types of tasks and activities do high school students report are most beneficial for improving quantitative reasoning, specifically with fractions, decimals, and percentages?

RQ3: How are high school students' mathematical self-efficacy levels impacted by a five-day rational number-focused intervention?

RQ1 and RQ3 were investigated through pre- and post-assessments with statistical data analysis. RQ2 was investigated through a post-test survey of students.

Research Hypotheses

Research Hypothesis 1: Students participating in the five-day rational number sense intervention will experience a significant increase from their pre-assessment score to their post-assessment score.

Research Hypothesis 2: Students that participate in the five-day rational number sense intervention will experience a significant increase in scores from their pre-assessment to their post-assessment on both pure mathematical questions and mathematical context questions compared to students in the control group that do not participate in the five-day rational number sense intervention.

Research Hypothesis 3: The activities that rely on real-world applications of fractions, decimals, and percentages will be reported on the post-assessment reflection (Appendix B, Section 4) as the activities that are most beneficial at improving quantitative reasoning.

Research Hypothesis 4: Students participating in the five-day rational number sense intervention will experience an increase in reported self-efficacy in basic number sense computations and applications.

Limitations and Delimitations

This research was conducted with the knowledge of the following limitations and delimitations:

- The intervention was limited to only one teacher for the three class periods. This research included two class periods of Geometry and one class period of Algebra II. One class period of Geometry and the Algebra II class period took part in the five-day intervention. The other Geometry class served as the control group. The results of the study may not generalize to other teachers or teaching styles.

● The research study was limited to only students enrolled in Geometry or Algebra II at Purvis High School in the Spring of 2022. Students in other math classes, at other high schools, during the Spring of 2022 were not eligible to participate in this study due to scheduling needs and classes taught by the researcher. The results of the study may not generalize to students enrolled in other math classes at other schools.

● The research study was limited to primarily students in 10th or 11th grade as these are the typical grades of students in Geometry or Algebra II. The results of the study may not generalize to the entire high school population including 9th and 12th-grade students.

● The intervention was limited by a short time frame and took place over five days.

● The research study was limited by the effort and participation of students in the five-day intervention and the pre- and post-assessment responses. Since the researcher is also the teacher, that may influence student responses favorably or unfavorably when self-reporting self-efficacy or answering reflection questions about the intervention.

CHAPTER II – LITERATURE REVIEW

History of Number Sense/QR/Math Literacy

The term number sense evolved in the 1990s from the term quantitative intuition (Sowder, 1992). Quantitative intuition can be drawn from the varying meanings and contexts that numbers are used for. Developing intuition about numbers allows students to develop a network of well-organized numerical relationships that can be applied in flexible and creative ways when problem-solving (Sowder, 1992). Quantitative intuition is self-evident, meaning that students who secure this skill are aware of their abilities. Number sense developed from this idea of quantitative intuition and covers a very broad spectrum of mathematical skills. The term number sense is so broad that its operational definition is not concrete among researchers. However, many researchers agree that number sense includes students developing the ability to flexibly relate numbers and computations and assess the reasonableness of their results or answer (Way, 2005; Jordan et al., 2006; Hope, 1989; Reys, 1994; Reys & Yang, 1998).

Number sense is a broad term, with subdisciplines such as number meaning, number relationships, number magnitude, operations with numbers, identifying numerical patterns, estimating, number transformations, calculating error, efficient procedures, and interpreting results (Way, 2005). Number sense can come in the form of estimating what ¼ cup looks like without measuring, using partial products to calculate a tip at a restaurant, or correctly adding fractions to calculate a batting average after a weekend tournament of several games. Sowder (1992) suggests that all of the subdisciplines of number sense include a set of characteristics. The major characteristics of number sense are it's complex and requires abstract thinking, there can be multiple solution pathways, it often involves uncertainty in that not everything required to complete a task is explicitly known, the thinking process to obtain a solution is effortful and does not just require a mindless procedure, and interpretation is required to make meaningful assumptions or conclusions (Sowder, 1992). Way (2005) notes that developing number sense and understanding the relationships between numerical values supports building conceptual understanding. Focusing on individual subdisciplines of number sense and being aware of the set of characteristics that they possess will help to develop a better understanding of how they all fit together as a whole.

Developing quantitative reasoning skills is so important in young adults to help prepare them for everyday applications. Over the past decade, several of the fastest-growing careers have been computer engineers, systems analysts, and database administrators all of which require both technological skills and strong mathematical and problem-solving skills (Rosen, 2003; Stone, Alfeld, & Pearson, 2008). Teaching quantitative reasoning would greatly benefit students looking to go into these growing careers as well as other possible pathways. Stone, Alfred, and Pearson (2008) assert that high school students lack the mathematical skills necessary to enter the workforce directly out of high school or meet the requirements of college entrance exams. Hope and Sherrill (1987) explain that even for seemingly straightforward calculations, such as multiplying 90 and 70 without a calculator, 45% of a sample of 17-year-olds were not able to correctly compute. It is not only the fact that students are unable to perform these computations but that they lack the quantitative reasoning skills to problem solve and devise a strategy to figure out the solution without access to technology. Yet, students believe that the math they learn in school will not be relevant after high school. While students may not use every piece of algebra or geometry in their future careers, they will use the basic math and numerical skills applied in these courses. “Most mathematics problems in the workplace involve applications of what is typically referred to as “basic arithmetic” (Rosen, 2003, p.46). This foundational math is a stepping stone for high math education, but also includes the math knowledge that is transferrable out of the school environment. Current highly valued workplace skills are developed computational skills, strong critical thinking skills, problem-solving, and logical reasoning (Rosen, 2003). Students who have developed strong quantitative reasoning and number sense have the upper hand in the valued skills that workplaces are looking for.

The benefits of developing a strong number sense and being quantitatively literate will follow students well into adulthood. Quantitative literacy allows students to understand and make sense and judgments of real-world situations based on data (Rosen, 2003). Rational number concepts are prevalent in everyday life; they are used to follow recipes, calculate discounts, find fuel efficiency, exchange money, make shopping decisions, understand financial statements and investments, and interpret scaled maps or drawings (Moss, 2005). The need for mental calculations deepens an understanding of number concepts and meets a practical necessity (Hope & Sherrill, 1987). The inability to manipulate numbers mentally reflects a weakness in number sense (Jordan et al., 2006). The need to understand rational number representations and interpretations does not vanish once students reach adulthood. Arguably, it is even more essential for students to have developed a strong number sense of these specific topics in everyday contexts. This is because of the practical applications and frequency that they will use rational numbers in estimation and reasoning in their career choice, finances, and budgeting.

Now that there is an understanding of the importance of students developing a strong number sense, it is necessary to look at how informal and formal education play a part in shaping students' number sense. Number sense develops at a very young age through informal learning experiences outside of school. Interviews have shown that even before students start formal instruction about fractions, they have impressive intuitive reasoning skills (Lamon, 2007). Children as young as six months have been observed for acuity on the approximate number system. A previous research study demonstrated that children’s approximate number sense before one year of age was a predictor for mathematical achievement even years later (Starr, Libertus, & Brannon, 2013). As children mature and are introduced to fractional concepts in a classroom setting, rules and algorithms begin replacing these intuitive thoughts. Trushkowsky (2015) identifies teachers' overeagerness in using procedures and formulas. When teachers take this approach, they are taking a teaching-centered approach verses a student-centered approach. By telling students the formulas that they should use, teachers are inhibiting growth in student learning that comes from discovery, making connections, and creating learning pathways. Procedures and formulas have their place in mathematics but understanding the why behind their use is equally important for student’s conceptual understanding. In some cases, it has been seen that students who replace their reasoning strategies with more formal algorithms are hindered and may perform worse on fractional instruction tasks (Lamon, 2007). “Curriculum should provide school experiences to help children construct intuitive knowledge” (Behr, 1992). Mathematics instruction in school can target developing number sense and quantitative reasoning skills by observing students’ informal knowledge and building on this intuition instead of stripping it away and replacing it with algorithms or formulas.

Deficiencies in elementary school curriculum, such as computing with fractions and converting between fractions and decimals, have been identified when looking at rational numbers (Morales, 2014). The importance of students developing number sense skills at a young age is that these skills can predict mathematical ability in adolescence and understanding of the number system as a whole (Steffe, 2011). One very specific deficiency identified is the lack of experience students have with qualitative reasoning about number size, relations, and numerical operations (Morales, 2014). Shumway (2019) performed an experimental research study to help increase students' number sense in elementary school by comparing two groups of students. One group received three weeks of counting-focused instructional treatment, and one group received nine weeks of the same type of counting-focused instructional treatment. The research study consisted of sixty elementary-aged students from three separate classrooms in one school in the western United States. Both groups' scores increased from pre-test to post-test, but the more extended nine-week treatment group outperformed the group that received only three weeks. The implications of this would need to be studied further in a longitudinal study. Still, it can be expected that students who develop stronger number sense in elementary school will continue to build and understand quantitative topics better in higher-level math courses.

In addition to the time spent teaching number sense, how instruction is tailored can also affect how students develop quantitative reasoning skills. As mentioned, current mathematics teaching tends to favor a procedural approach with less focus on students’ conceptual understanding (Hurrell, 2021; Borji, 2019). Students are trained to look for certain problems and apply the correct formula or method. Each problem type has a mold or a singular taught approach. Students are not necessarily expected to think critically or apply their mathematical knowledge. A shift in focus needs to be made to emphasize quality problems over the quantity of problems that students complete (Reys, 1994). Quality problem-solving can be modeled by teachers encouraging students to invent their own methods to solve, internal questioning to judge the reasonableness of their answers, and using writing assignments to have students summarize their thought processes (Reys, 1994).

In a study done on elementary school-aged children, 39 students were given similar mathematical problems in a classroom setting in the form of a worksheet and then using a context outside of the classroom. It was found that when solving the problems in the classroom setting 44% of students used traditional arithmetic methods that were taught in class. However, in an out-of-classroom setting, only 9% of students used the traditionally taught methods (Schubauer-Leoni, 1997). There is a disconnect between mathematics learning in school and mathematics learning out of school. This is partially due to the setting that students are placed in and the practices taught (Abreu & Crafter, 2015). Problem-solving strategies that may be applied in classroom mathematics are not always practical or feasible in out-of-school settings. Different situations warrant adapting mathematical knowledge to fit the circumstances or problem-solving flexibility. Students who are taught through a series of rules or procedures that lack meaning can easily forget or misapply the rule. On the other hand, creative thinking that is not dictated by rules and algorithms allows students to develop a mind focused on reasoning and sensemaking (Lamon, 2007). When students have developed this flexibility, it does not matter the numbers or quantities in a problem because they instead use the context of the problem to solve it.

Schubauer-Leoni's (1997) research considers if students understand the arithmetic they are performing and, more importantly, why they are using a certain method. It was found that a control group, where students worked individually and were not given feedback or guidance, were unlikely able to explain the reasoning behind the arithmetic they were performing. Lamon (2007) argues that children lack this underlying awareness of why they are solving in a way that they are, but instead over-depend on textbook formulas and representations, copying a model to solve their own problem. Many students complete 12 years of mathematics in public education and complete thousands of problems, all of which can be solved in a matter of minutes (Schoenfeld, 1992; Borji, 2019). Students tend to rely on algorithms and procedures that they learn in school even when they do not make sense to use in the context of a problem set in the real world (McNeil, 2009). This shows that there is a lack of understanding in applications. Once habits of procedure have developed, breaking this way of thinking and computing can be difficult. McNeil (2009) states that once students have constructed a representation of a concept or been taught how to solve a problem one way, it can be very challenging to let go of that representation and create new meaning or develop a new way of thinking about a problem. Number sense is a developed skill, and without number sense, students continue to practice conventional methods when solving problems without exploring other possibilities that may be more sensible. A foundational understanding of math reasoning and number sense allows students to use algorithms with a thorough understanding to solve problems (Moss, 2015). Even long after an algorithm or formula has been forgotten, students who can rely on their number sense background are able to analyze and use intuition to reach a conclusion.

The lack of instruction that focuses directly on number sense may also be because of how well-trained teachers are in this domain. Content knowledge of the teacher has been shown to be a factor in student learning (Guerriero, 2014). Moore (2014) asserts that one reason students do not develop quantitative reasoning skills is that teachers, specifically preservice teachers, lack these skills and therefore do not provide instruction that is dictated by developing quantitative reasoning. When students are taught procedurally, they rely on this type of understanding instead of focusing on individual quantity meanings and their relationships (Hurrell, 2021). Moore (2014) interviewed nine preservice teachers to better understand their experience with quantitative reasoning. Preservice teacher interviews showed that most interviewees were unable to justify their answers to given problems but could merely recite a rule or concept. They lacked a deeper understanding of the process and its mathematical workings (Ma, 2010).

Preservice teachers are not the only adults struggling with the concept of number sense. One of the most challenging and complex topics that are encountered in mathematics is rational numbers. Because of the demand to conceptually understand representations of rational numbers to be able to complete mathematical problems at all levels, this is a task that even adults struggle with (Morais & Serrazina, 2017; Moss, 2005). In a previous study, master's students enrolled in an elementary school teacher training program were tested on their understanding of basic rational number concepts, including fractions, decimals, and percentages. It was found that a majority found the concepts challenging and struggled to recall specific rules they had once learned (Moss, 2005).

In another research study on adults' development of number sense, Lave (1988) aimed to show that schools strongly contribute to school-based performance as opposed to practical, situational tasks. Lave’s (1988) research found that school-based practices did not generalize beyond the classroom tasks that students were asked to perform. This study acted as a starting point for researchers to examine the discontinuity between school practices and everyday usage. Research in the field of mathematics has shown that performance on classroom-based problems and solving everyday math problems do not equate. Lave (1988) reported that when adults were given an arithmetic school test and then asked similar questions in a grocery store, their performance levels were drastically different, with adults scoring 98% in the grocery store setting but only 59% on the school-based test. Even when the school curriculum is made to model everyday mathematical situations, the simulation is not offering a fluid connection. The debate is not whether students will benefit from a curriculum focused on number sense and simple mathematics, but rather on how to construct a curriculum that bridges contexts of everyday mathematics and develops skills such as problem-solving and quantitative reasoning. Two recent high school research studies advocate for an increase in quantitative reasoning curriculum at the secondary level to help better enhance number sense skills and prepare students for the future. Both of these research studies offer support for implementing an intervention similar to the one designed in this research study for high school classrooms to promote quantitative reasoning skills and overall number sense.

First, in a recent research study by Stone, Alfred, and Pearson (2008), 595 high school students across 203 classrooms were split into two groups, a control group and an intervention group that received a contextual math-enhanced Career and Technical Education (CTE) curriculum. The math-enhanced CTE curriculum included more rigorous and relevant mathematics taught by instructors who attended professional development workshops on a seven-element pedagogic framework to embed foundational mathematics in the CTE curriculum by application and relevant contexts. The classroom tasks and materials focused on number relations and numerical estimation about construction and culinary skills. They also covered measurement and spatial sense in relation to students' specific career path interests. Stone, Alfred, and Pearson (2008) found that after students were instructed using the experimental curriculum for a year, they were able to perform significantly better than control students on standardized math tests without a decline in their technical skills ability. This experimental curriculum effectively enhanced high school students' basic math skills through career and technical education classes.

Agustin (2012) argues for a separate quantitative reasoning course that goes beyond the CTE content to help increase students' mathematical literacy. In Agustin's (2012) research study, 564 first-year college students primarily enrolled at Southern Illinois University were given a quantitative reasoning test that required only elementary math skills but also more complex thinking skills. It was found that students had the most difficulty in numerical and algebraic relations and drawing logical conclusions from numerical information. Both of these are subtopics of number sense. With the average score of students from all first-year courses being 55% and the average score for students in Algebra being 35%, the results of this study show the need to develop quantitative reasoning skills further. It is believed that one reason students lack these quantitative reasoning skills is traditional mathematics courses tend to only go over alike application problems. What this means is that teachers may go over an application problem in class and show students how to set it up. Then, the homework or test question that students are given is worded the exact same, with the only change being the numerical values. Contradictory to this, to strengthen conceptual understanding, research supports students being asked to apply quantitative ideas in new or unfamiliar situations (Agustin, 2012). Quantitative understanding is a skill that students use well into adulthood to make informed decisions in all aspects of their personal and professional lives.

How content is structured, and instruction is delivered can be traced back to how mathematics is defined. The misconception that mathematics is a set of rules and procedures only to be applied results in students learning mathematics in the same way. When the idea of mathematics is expanded to seeking solutions, exploring patterns, and formulating conjectures, as opposed to memorizing procedures, formulas, or routine exercises, this puts emphasis on sense-making in mathematics (Schoenfeld, 1992; Hurrell, 2021). Mathematically capable students are quantitatively literate, meaning they can interpret the numerical data they experience in everyday life and make judgments or decisions based on their intuition. Schoenfeld (1992) examines how many mathematics courses are heavily textbook-focused, with example problems following a common pattern where the same formula is to be applied for each problem (Borji, 2019). Number sense cannot be developed when students do not have the time or opportunity to reason independently. Reasoning is typically not associated with specific rules or mechanized procedures but instead favors habits of the mind that are flexible in order to analyze relationships and quantities (Lamon, 2007). When students are presented with a set technique, given a guided example, and then asked to complete sample problems modeled after the same style, this compromises a student's developmental understanding and reasoning behind mathematics (Schoenfeld, 1992). Students often think that mathematics is a set of rules and procedures that are to be memorized and do not understand that in the number sense, the numerical relationships are the reason these rules work (Rudolph, 2011).

"Teaching definitions, algorithms, and applications of rational number knowledge has not facilitated the development of rational number sense and the ability to reason" (Lamon, 2007, p. 647). It is evident that current mathematics instruction that favors cookie-cutter textbook examples does not help students develop a strong number sense. Routine problems that follow the same format essentially make students memorize a process. Because the topic of rational numbers is so complex, to competently problem solve, students must be able to actively make sense of what they are learning (Moss, 2015). Unfortunately, most middle school students rely on memorized rules instead of creating meaning for rational numbers (Moss, 2005). Taking one or more math courses does not guarantee the development of quantitative reasoning (Agustin, 2012). Quantitative reasoning and numerical literacy must be intentionally taught throughout students' mathematical careers because they are processes that develop and mature as students gain experiences and knowledge (Reys, 1994). "Number sense theory indicates that number sense cannot be taught as a lesson or unit of study, rather number sense development is ongoing and requires multiple connected experiences with number sense ideas” (Shumway, 2019, p.309). My five-day number sense intervention is not a permanent solution to students’ incompetence in this area. Instead, the intervention aims to offer students a chance to better understand the topics of fractions, decimals, and percentages, make relevant connections to their own lives, and revisit topics they might not exhibit confidence in.

Curriculum that Emphasizes Everyday Mathematics

What types of activities are most effective in helping high school students develop quantitative reasoning skills?

The need for this research can be seen from previous research on students’ performance on rational number tasks. Lamon’s (2007) literature review concludes that there are common areas in which students show incompetence in mathematics. Specifically, students struggle with the topics of qualitative reasoning that include expressing conceptual knowledge about number size, relations, and numerical operations (Behr, 1992). “Students need to be exposed to problem situations that give rise to the need for mathematics” (Meyer, 2001). This section will explore previous mathematics curricula that have effectively increased students’ number sense and conceptual understanding. Specifically, curricula that target fractions, decimals, and percentages, as well as their relationships are of interest.

A widely known curriculum, Mathematics in Context (Wisconsin Center for Education Research, 2001), developed with support from the National Science Foundation, was designed to emphasize connections between mathematical topics and meaningful problems in the real world (Meyer, 2001). This curriculum is structured differently from a standard approach to learning a process or procedure and then applying that to world problems. Mathematics in Context introduces concepts to promote discussion and stimulate mathematical thinking. This program is geared towards middle school students but takes elementary math concepts and expands on them (Meyer, 2001). The curriculum is split into four separate strands with one entire strand dedicated to numbers, fractions, decimals, ratios, percentages and their relationships to one another (Meyer, 2001). The traditional curriculum covers rational number topics separately and superficially. However, Mathematics in Context aims to connect these ideas through a series of mathematical tasks that use problem-solving and reasoning strategies (Moss, 2005).

Math in Context is just one example of a curricular movement to support students' development in number sense. The Rational Number Project (National Science Foundation, 1997) was a second experimental curriculum focused on mathematical concept development over achievement on tests. This project mainly focused on student interviews to get detailed information on how students acquire new mathematical concepts. Both curricula emphasized mathematical understanding by focusing on the primary goal of number sense: deepening understanding of numerical concepts and relationships (Moomaw et al., 2010). With this goal in mind, a curriculum using problems that require students to qualitatively reason before applying numerical values to a problem can help students better develop an intuitive understanding that can be more widely applied across various situations (Meyer, 2001). A curriculum that emphasizes creativity and investigation as well as allows students to see the connections between mathematics and the real world promotes number sense (Reys, 1994). Specifically, curricula should focus on students constructing principles and applying qualitative reasoning to rational number problems (Behr, 1992). Curricula that help students develop qualitative reasoning skills first can then use this knowledge to guide quantitative thinking, particularly with rational numbers.

Hope (1989) asserts that number sense for students can be developed through a curriculum and in-class activities that are meaningful and purposeful and that include three components: calculating, measuring, and estimating. When students are presented with practical problems where calculations must be done for a purpose, they tend to be very accurate (Hope, 1989). I will specifically use calculating during the intervention in activities that involve students finding prices after discounts are applied. Measuring will occur during the intervention as students rearrange fractional pieces to create flag patterns. Lastly, estimation involves comparing quantities (Hope, 1989), which will be supported during the intervention by activities that allow students to arrange rational numbers in order on a number line. Hope (1989) expresses that number sense best develops when students are introduced to messy aspects of everyday problem-solving. It is essential for students to think through practical applications and not always be given cookie-cutter problems with all the information they need for solving (Trushkowsky, 2015).

A previous research intervention by Behr (1985) aimed to increase students' performance on fractional concepts by increasing instructional time on rational number concepts prior to implementing a fraction curriculum. This intervention was performed in elementary school classrooms with a large focus on using manipulatives to teach rational number concepts. The results did not provide clear evidence that this intervention was successful, as roughly half of the students showed little or no advance in understanding computing with fractions or interpreting rational number size (Behr et al., 1985; & Lamon, 2007). However, this is just one research study that does not have convincing data to support further instruction on rational number concepts. Researchers note that the primary reason for the lack of results was the age of the students and the length of intervention. Behr (1985) states that there are no quick fixes to educating students on rational number concepts. Although my intervention will be short in length, with the sample of students being much older, the hope is that they have previously been exposed to more rational number concepts and can make connections quicker. Previous interventions and research on elementary school-aged students support that creating knowledge on the topics of fractions, decimals, percentages, ratios, and proportions involves a long-term learning process (Jordan, 2006; Moss, 1999). At a young age, students may not possess enough background knowledge on the individual components of fractions, decimals, and percentages to begin constructing new knowledge and creating meaningful relationships between them. Number sense is not a concept that can be taught in a single class or at a single grade level. It is a developed skill that should continue to be taught over the course of a student's mathematical journey.

As mentioned above, curriculum should be framed to build on a student's pre-instructional strengths, such as the intuitive strategies and reasoning skills that young children informally learn before attending school (Lamon, 2007). Connecting these flexible and intuitive thoughts and interweaving them with more formal knowledge about fractions, decimals, and percentages is important. Moss (2005) suggests that instruction should begin with percentages since students have often seen percentages in everyday life, and they can be viewed in terms of 100 as a whole. Bridging context allows students to use their informal prior knowledge and develop and integrate it to further their understanding to create a network of connections. The first time that students learn the individual concepts of fractions, decimals, and percentages, they are typically around ten years of age. They therefore fall into the elaborated bidimensional thought category as identified by Case and Sowder (1990). In this stage of thought, students should be able to move back and forth between two number formats (Case & Sowder, 1990). However, it isn't until students are in their late teens that they reach the end of the vectorial stage. It is not until well into the vectorial stage that students develop a deeper understanding of connections between rational numbers (Case & Sowder, 1990). This alone presents the need to continue addressing rational number topics throughout math courses as students continue into middle school and high school.

Morais and Serrazina (2017) performed a research study on teaching elementary-aged students decimal representations based on their prior knowledge of fractions. Through the course of instruction, the teacher connected fractional and decimal representations by using number lines, 10 and 100 grids, and money to relate the topics. In addition to the physical representations used, the teacher used money as an example to talk about the relationship between decimals and fractions (as well as percentages of a dollar). Student interviews showed that students could relate these multiple representations in a way that they were effectively comprehending rational number relationships. Instruction and class discussion with a focal point on the connection between different representations can empower the development of rational number comprehension (Morais & Serrazina, 2017). In my intervention, I will use several tasks that Morias and Serrazina (2017) did to show the relationship between fractions, decimals, and percentages. I will use number line visuals to compare rational numbers in different representations. I will also use money to connect decimals (cents) to fractional parts of a dollar.

A second study by Moss (2005) implemented a new way of teaching fractions, decimals, and percentages to students in a connected manner. The instruction targeted middle school students (N= 68) and used a series of hands-on activities where students explored concepts and began connecting their already informal knowledge on the three topics. Students used pipes and tubes of varying lengths to identify percentages, they used beakers of liquid to compute percentages, string to guess percentages of unknown object lengths, stopwatches to connect percentages and decimals, and cards with varying representations of rational numbers. Visual displays allowed students to interact with one another and create their meaning without formal instruction. Activities were targeted at correcting students' most common misconceptions with ordering and comparing rational numbers. Students were even asked to compose their own word problems and create challenging problems for one another to solve. The results showed that students who participated in this new instruction made sense of new representations and provided flexible approaches when solving new problems (Moss, 2015). The biggest takeaway from this four-week instruction was that students who completed it often performed better than students who were several years older but that had not been given this specific instruction (Moss, 2015).

Not only did students who received the instruction perform better, but they were also more inclined to justify their answers and give quality reasons for their solutions. These are characteristics of improved self-efficacy. One specific concept developed by this instruction was students reasoning on percentages. Prior to instruction, students were unable to calculate 65% of a total by hand. Upon completion of the treatment curriculum, when students were asked this same question, they were able to break 65% into more common percentages in order to calculate. Most commonly, students found 50% and 25% of the number first, totaling 75%, and then subtracted 10% out in order to calculate the needed 65%. Teaching percentages in a flexible manner where students used physical objects to demonstrate parts of a whole led them to understand better how to piece together different percentages in order to generate the necessary amount. It is evident that an overall understanding of the number system was gained throughout this process. In my intervention, I used a number line activity and student construction of word problems from Moss (2005) to challenge and engage students in connecting different representations of rational numbers.

I propose to use similar curricular contexts, problems, and methods for a high school intervention with the aim of (1) correcting prior misconceptions commonly observed in high school math classrooms, (2) helping connect rational number topics to real-life applications now that students have a more advanced base knowledge, and (3) building students' self-efficacy in their mathematical abilities. Students at the high school level have more advanced knowledge of fraction, decimal, and percentage concepts from being taught them over the course of several years. Additionally, at the high school level, students have developed a natural maturity making contextualized problems more relevant to their personal experiences. Though prior research has been done at the elementary and middle school level, there is a gap in research for an explanation about how to help improve students' quantitative reasoning skills throughout secondary education. This highlights a need to continue to explore number sense at the high school level. Lamon (2007) supports the premise that this intervention is based on by alluding to the fact that students need a starting place when interpreting and making sense of rational numbers. Given sufficient base knowledge, as well as time to view different interpretations and representations of numbers and explore connections without a set of rules, helps students develop a fraction sense and a way of thinking about forms of rational numbers in a flexible manner (Lamon, 2007). Even problems that are structured in a way meant to apply to real-life situations are often artificial and negate the practical utilization of math. When students cannot connect to the problems provided and see the relevance to everyday applications, they are more likely to give up on trying to make sense of the mathematical processes and resort to a procedural approach (Schoenfeld, 1992). To address this in my own five-day intervention, I will have students create the context in which they want specific problems to be set in. Students will be able to pick examples of meaningful scenarios such as calculating the different percentages of their work paycheck, calculating distances to and from school with the cost of gas, comparing sports teams through percentages, and other personally and culturally relevant contexts. Cultural relevance, specifically in mathematics, benefits students in several ways. These include facilitating brain processes, motivating students, cultivating problem-solving skills, and promoting a sense of belonging (Muniz, 2019). For students to make informed decisions and perform efficiently and effectively when problem-solving with rational numbers, a curriculum that focuses on the interconnections of fractions, decimals, and percentages is necessary (Moss, 2005).

Fractions/Decimals/Percentages

One reason students lack a deeper understanding of fractions, decimals, and percentages is that they do not learn the intertwining relationships between the three (Steffe & Kieren, 1994; Moss, 2015). There is a lack of time in the current curriculum to relate fractions, decimals, and percentages to one another and convert between their various representations (Moss, 2015). Lamon (2007) suggests that for students to really grasp rational number concepts (fractions, decimals, and percentages), they must have experience with multiple representations. This is where the former curriculum has been lacking and not preparing students with an adequate foundation. Decimals should not be taught as an isolated concept but must be connected to other rational number representations (Morais & Serrazina, 2017). Behr (1992) supports this idea by pointing out that various constructs, including fractions, decimals, and equivalence classes, must be brought to students' attention in a connected manner for them to understand rational numbers and their applications fully. Using a technique called bridging, physical manipulations are used to represent quantities and help students see different forms or representations of rational numbers. Providing instruction through bridging helps with interactions of formation and how real-world problem situations could be modeled (Behr, 1992). Additionally, transforming between different representations allows students to alternate between forms and pick the most appropriate or efficient form in the context of problems (Morais & Serrazina, 2017). A more powerful and deeper understanding can be gained when students develop multiple representations and connections between those representations.

Moss and Case's (1999) research study looked at whether an experimental curriculum could increase students' understanding of rational numbers. Experimental curriculum included introducing vocabulary, visual experiments, number lines, decimal board games, lessons on halving strategies, and challenge problems. A devised Rational Number Test was used to test the difference in performance between the control group and the intervention group on their conceptual understanding of fractions, decimals, percentages, and their relationships to one another. The intervention group, who had 20 class lessons of the experimental curriculum, scored significantly higher than the control group did on the post-test. It was evident through assessment interviews that students in the intervention group reasoned through problems in qualitatively different ways than those in the control group. The intervention group demonstrated a deeper understanding of the relationships between different representations of rational numbers (Moss & Case, 1999). Mastery of number sense in the realm of fractions, decimals, and percentages requires students to be able to convert between the various forms and use a flexible approach when considering possible representations (Moss, 2005).

Irwin (2001) performed a three-day intervention aimed at improving students' understanding of decimals by relating decimals to everyday contexts. Through a pre-test and post-test comparison of two groups, a control group that did not work on contextualized problems and an intervention group that did work on contextualized problems, it was found that the intervention group made significant progress in their knowledge of decimals. The research question that Irwin (2001) aimed to investigate was if the understanding of decimals could be improved by asking them to solve problems in everyday contexts. Working on contextualized problems may help to increase the retention of specific concepts, including decimals and fractions. In the short term, it could be seen that students who worked on contextualized problems with decimals were more competent two months after the unit than those who did not work on contextualized problems (Irwin, 2001).

In addition to Irwin (2001), Suh et al. (2008) investigated students' decimal number sense. In this study, teachers planned an intervention in a single fifth-grade class that started with students creating a math knowledge map that related decimals and fractions. This intervention focused on representational fluency, or a student's ability to use multiple representations and translate between them. By promoting drawing different representations, incorporating place value charts, and introducing the concept of money to teach decimal and fraction relationships, it was found that these activities allowed students to work more flexibly between rational number relationships, promoting the development of decimal number sense (Suh et al., 2008).

Students' success in understanding fractions comes from having a fundamental knowledge base about rational numbers (Lamon, 2007). For students to understand rational numbers as a whole, they must be able to differentiate between fractions, order fractions, and find equivalent fractions (Behr, 1992). Rational number sense can be defined as students having insight about relative sizes of numbers and being comfortable dealing with different interpretations and representations of quantities to compute, problem solve, or make a judgment call (Lamon, 2007). Lamon's (2007) research study on students' knowledge of rational numbers shows a major gap in student understanding of equivalent fractions. The research found that when students were given two fractions and asked if they were equivalent, a majority could identify them as equivalent if the denominator of one fraction was a multiple of the other. However, when students were asked if the fractions 4/6 and 6/9 were equivalent, they arrived at the conclusion that they were not, confirming that they do not comprehend what equivalence really means at a deeper level. Instead, students are recalling procedural notions that have been taught about how to find equivalent fractions by multiplying one entire fraction to get a new fraction.

The lack of basic rational number concepts makes it necessary for later learning in intermediate and higher-level grades (Behr, 1992). More complex fraction and proportion concepts are typically taught in middle school, however, developing an understanding of the relationship between the two and their relationship to decimals and percentages is part of the curriculum that spans from elementary to high school (Lamon, 2007). Fractions alone have been deemed one of the most difficult and cognitively demanding concepts to teach, yet also one of the most crucial topics that contributes to success in higher mathematics courses (Lamon, 2007). Algebra heavily favors an understanding of rational number concepts, and students who do not have a strong foundation in this area will struggle to succeed (Moss, 2005). Agustin (2012) states that basic concepts such as percentages, ratios, decimals, and estimation are all essential skills that students need to be proficient in, opposed to polynomials and derivatives, which may not be as applicable to all students.

Self-efficacy

Dan Meyer (2015) identifies a three-act task (engaging students, seeking information needed to problem solve, and discussing a solution) in solving mathematical problems that can be used to help students of all mathematical levels engage in problem-solving and gain confidence in their mathematical abilities. The first act is pure discussion without numbers, with the intent of making the material accessible to all student levels and increasing student interest and curiosity. Introducing mathematics in a way that is inviting for students who may not think they are "good at math" or may not enjoy math can help build self-efficacy and create a shift in students' attitudes. Opening up a class discussion allows students to give their own perspective of the problem without focusing on the skills needed to mathematically compute an answer. The focus is taken away from computation and more emphases is put on understanding what the question is asking. Lamon (2007) supports this idea of not necessarily quantifying objects when introducing problems to students. Lamon (2007) states that it's a foundational skill to be able to relate quantities that are not quantified. For example, this type of reasoning helps children visualize that a half is larger than a third without putting fractional numbers on individual pieces. Seeing fractions and decimals represented in recognizable contexts without numerical values is an accessible and relatable way to begin to develop number sense at an early age. Recognizing quantities helps to bridge the gap and transition to classroom mathematics. Students who are deemed skilled problem solvers possess the skill of reasoning not just quantitatively but qualitatively about the components of problems before beginning to use the numerical values in solving (Behr, 1992). Setting up a solution pathway or a solving strategy can be done before numerical values are given in context. Student interest comes from setting up these solution pathways and reasoning through why a strategy works. Skilled problem solvers can recognize this and reason through problems based on scientific principles and known numerical relationships. Teachers can help guide student curiosity by setting up problem solving opportunities and challenging the ways that students think about numerical quantities.

A student's self-identity can be altered by how mathematics is presented and defined. Schoenfeld (1992) discusses how mathematics is commonly presented as a topic of certainty, where doing mathematics means applying certain rules or algorithms, and knowing mathematics means remembering when to apply each of these rules to get a correct answer. Students construct their own mathematical knowledge (Behr, chapter 14). *Instead, classroom instruction should be centered around providing students with ways to grow in mathematical skills and as mathematical thinkers.* In a survey where students were asked to identify what math really is, nine out of ten students agreed with the statement that doing math requires lots of practice and following rules (Schoenfeld, 1992). When students identify math as a subject of following select rules, where their answer is either correct or incorrect, it correlates to them viewing themselves as either good or bad at math, purely depending on their answers.

Some common beliefs about mathematics from students are that problems only have one correct answer and only one way to get to that answer, understanding a mathematical procedure will allow you to solve all of the practice problems for that lesson, and mathematics learned in school has little to do with the real world (Schoenfeld, 1992). Students with more fluent number sense have broken this mold of thinking and expanded their beliefs about math to look at connections and relationships. Nebesniak and Heaton (2010) have identified students who are more confident in their mathematical abilities as those that are willing to try new problems, learn from their mistakes, and help other students. These students are more interested in understanding how a problem is set up and how to arrive at an answer as opposed to just getting a solution. Viewing math as a connected discipline and understanding the interworking relationships, as opposed to viewing it as a series of rules that correspond to problem sets can shift students' concept of mathematics and their self-identity. Schommer-Aikins' (2005) research study of 1,269 middle school students supports this by showing a correlation between student confidence and their concept of mathematics. It was found that students who believe in just finding a quick solution often do not view mathematics as useful and are less likely to problem solve correctly, therefore decreasing their overall confidence level.

As identified by Robinson (2017), some methods can be used to increase student self-efficacy in the classroom, including emphasizing effort and understanding of concepts as opposed to correct answers, making math relevant, teaching based on student interest, and making personal connections. Additionally, Nebesniak and Heaton's (2010) research shows that an increase in cooperative learning and student engagement in the classroom also boosts self-efficacy. Generally, students are more confident attempting new problems in a group setting (Nebesniak & Heaton, 2010). During the five-day intervention, students are asked to work in groups and collaborate to expand their thinking, learn from each other, and support each other to increase self-efficacy. "The confidence created when a student's mathematical reasoning is secure bodes well for future mathematics learning" (Moss, 2015, p. 343).

CHAPTER III - METHOD

Research Design

Based on the research questions, a mixed methods research design was the most effective in gathering data and providing the best evidence for understanding student growth in number sense topics. The first research question examined how student knowledge changed after a five-day remedial intervention on number sense. Quantitative data was collected through pre- and post-assessment scores. Comparisons between pre- and post-assessment scores were warranted to inspect if a five-day number sense intervention made a difference on students' computational practices and/or conceptual understanding.

Qualitative data was collected and analyzed to gain insight on the second research question: what activities, or activity types, during the intervention best-supported learning number sense topics. Through means of written responses on students' post-assessment and informal interviews, this question was best suited to be supported through qualitative data.

Lastly, the third research question examined students' self-efficacy through self-reported values. A Likert scale was used to gather quantitative data prior to the intervention and after the intervention. Likert scale values were compared to examine if there was a difference in students' self-efficacy throughout the research study.

Participants

Participants were 63 students enrolled at Purvis High School in Spring 2022 from three separate class periods, all taught by the teacher researcher. The intervention was performed in two classes: one Geometry class consisting of 21 students and one Algebra

II class consisting of 22 students. Each intervention group had a different class dynamic. Intervention group one was a very quiet and attentive class that preferred independent work. Intervention group two was a very active class that engaged in classroom discussion easily. It should be noted that there was an observable difference in the class dynamic and participation level between the two intervention groups. A control class of 20 students enrolled in Geometry also completed a pre- and post-assessment. The class dynamic for the control group was similar to intervention group one. Participants ranged from 14 to 18 years of age, with 96.8% of students in 10th or 11th grade. Of the participants in both the intervention and control group, 47.6% are male and 52.4% are female, 17.5% are African American and 82.5% are Caucasian.

Prior to participating in the intervention, all students were given a contract to sign (see Appendix C). This contract outlines the expectations for participation, effort, and contributions throughout all activities. In return for full participation in the five-day intervention and completion of both the pre- and post-assessment, students received a 100 minor grade that was used to replace their lowest minor grade over the course of the semester.

Five-Day Intervention Curriculum

Over the course of five days, with 90 minutes each day, students engaged in a curriculum that revisited and redefined rational number concepts, emphasizing fractions, decimals, percentages, and their relationships. A five-day curriculum was constructed using tasks and challenges from the work of Kieren, Davis, and Mason (1996), NCTM (2013), NCTM (2015), Irwin (2001), and Brown and Avila (2014), all reviewed above. These tasks were adapted and assembled in the following order (see Table 1) to provide a five-day rational number sense intervention. Methods of intervention instruction and practice have been developed from research from Moss and Case (1999) and Meyer (2001) on best practices for teaching students rational number concepts.

Table 1 *Five-day Number Sense Intervention Curriculum*

	Intervention Curriculum	Instructional Approach
Day 1: Cartoon Corner: Percentages	From NCTM (2013), the activity of comparing percentages in drink options like milk and juice will be used. Students will answer the following questions in a class discussion (1) what does it mean for juice to be 100% or a different percent? (2) what do percentages mean in relation to milk? (3) what other food/drink items use percentages? Different drink options will be available for students to preview and try. A powerpoint will be presented to the class with the questions from NCTM (2013) (see Appendix A). In small groups students will discuss each question. Groups will share and contribute to the whole class discussion in order to answer each question.	Moss and Case (1999) identify that one of the current flaws in teaching and understanding rational number concepts is that too much time is dedicated to a procedure of manipulating rational numbers opposed to teaching conceptual

Table 1 Continued

	Students will work in partners to complete the questions from NCTM (2015) (see Appendix A). After group work, students will create a daily log of what they eat. They will use MyFitnessPal to look up the fat, carbohydrates, protein, and calories in each food item. They will compile a table with eat food item and its corresponding macros. Students will be asked to find the percentage of fat, carbohydrates, and protein that their diet contains. Students will compare their personal percentages to recommended daily values.	meaning. The Day 1 intervention draws on something students can relate to and understand (percentages in drinks) and then aims to build on that component through analyzing different problems in context.

Table 1 Continued

Day 2: Fraction Flags	To introduce the day, students will be given a fraction kit that consists of pieces ranging from halves to twelfths. There will be time for students to manipulate pieces and compare pieces of different sizes. Class discussion will center around the following questions: (1) what are possible equivalent fractions that could be made with the pieces? (2) how can we add fractional pieces of differing sizes? (3) how can we solve for an area once pieces are overlapped? Modeled after Kieran, Davis, and Mason's (1996) activity, students will design unique flags, sports uniforms, or school memorabilia composed of fractional pieces. In partners students will make up fractional questions about their design to answer and respond to. Students will swap partners and engage in more discussion about the components of their design.	Mathematics teaching is not telling students what to do, but providing them with tools and opportunity to advance their knowledge (Kieran, Davis, & Mason, 1996). The fraction flag intervention uses this mindset in order to engage students in a hands-on activity where they must construct their own meaning and representations of

Table 1 Continued

	Several designs will be displayed to the entire class for class discussion to emerge on fractional components and fraction operations.	fractions. Meyer (2001) suggests presenting students with a problem before they know how to solve it. In this case, students are developing their own problem through designing a flag of fractional components and then must come up with a procedure to solve.

Table 1 Continued

Day 3: Decimal Development	Moss and Case's (1999) number line activity will be used to show the relationship between percentages and decimals. Replicating from Moss and Case (1999), students will be asked to step a certain percentage of a number line laid out on the floor with increments from 0 to 1. Students will relate the percentage they move to a distance. Next, the number line will be changed from 0 to 2. Students will complete the same activity, given a certain percentage to move along the number line they must find the corresponding decimal of their location. On a larger scale, students will then look at the distance from their house to school or work. Students will map a route, stopping at different locations and determine and compare distances using decimal and fractional representations.	Moss and Case (1999) recognize that one difficulty students have with rational numbers is notation. The intervention is structured to provide students with a guide on converting rational numbers in one representation to another and comparing these numbers as well. Using measurement and distances in the first part of the intervention

Table 1 Continued

	Secondly, solving decimal problems set in context from NCTM (2001) will be used. Students will work in partners to answer questions involving ordering decimals, estimating decimal operations, and converting rational number representations to decimals. A whole class discussion will take place to clear up misconceptions and answer all solving decimal partner problems.	teaches percentages and decimals in context which is what Meyer (2001) suggests. Additionally, algorithms and formulas are not discussed, but instead estimation, measurement, and calculation are done with a purpose which supports Hope's (1989) philosophy on promoting number sense.

Table 1 Continued

Day 4: Discounts on Discounts	Through class discussion, students will brainstorm ways they can be asked to find a discount. For example, in a sales ad it may say take 25% off a total purchase. When shopping on a clearance rack, items may be 50% off and in addition there is a student discount of 15% off a total purchase. A class chart will be made that poses different questions about finding discounts. Students will explain the necessary information they would need in order to answer each question. Brown and Avila's (2014) publication on discounts will be used as a guide to ask students follow up questions. Students will read over a given scenario from Brown and Avila (2014) and be asked to find different percentages, prices, and explain their computations. Students will work in pairs to answer additional questions provided by Brown and Avila (2014). Class discussion will review each group's answers	The intervention follows Meyer's (2010) idea on reforming math classes to start with students constructing problems without numbers. Meyer (2010) then supports the idea of students analyzing the problem and deciding what information they need before they are given it. Meyer (2001) advocates for math taught in the context of real

Table 1 Continued

	and provide clarification on any calculations. Lastly, students will go online to their favorite shopping website and add at least three items to their cart. The teacher will give them a scenario where each item is a certain percent off, and they have to calculate tax and shipping to find their total order cost.	problems where the situation warrants the use of mathematics. This intervention follows instructional procedures consistent with Meyer (2001).

Table 1 Continued

Day 5: Rational Number Relationships	Students will be given task cards that have various representations of rational numbers (fractions, decimals, and percentages of a number) and must arrange them from least to greatest. Students will work in pairs for this activity. They will be asked to verbally explain why each card is placed in a position. Students will create a video to illustrate their comprehension of the rational number topics covered over the course of the intervention. Zakrzeski (2015) outlines the advantages to teaching fractions, decimals, and percentages through an iBook. Similar to Zakrzewski's (2015) approach students will be asked to create a short iBook/video to teach their classmates about fractions, decimals, and percentages. Students will be required to give at least two example problems, cover fractions, decimals, and percentages, and	Lamon (2007) supports the idea that in order for students to deeply understand rational number concepts (fractions, decimals, and percentages) they must have experience with multiple representations. The intervention exposes students to different representations and asks them to familiarize

Table 1 Continued

	use at least one contextualized problem. Students will record and share their videos with the class.	themselves with converting between representations to compare quantities. Meyer (2001), Trushkowsky (2015), and Zakrzeski (2015) all support teaching rational numbers by including technology.

Instrumentation

All three classes of students were assessed Friday before the school week intervention and the Friday the following week at the intervention's conclusion. On both assessments, students did not use a calculator, and there was a 30-minute time limit. The pre-assessment (see Appendix B), given before the intervention, was administered to students on paper. The post-assessment (see Appendix B) included all the same questions as the pre-assessment with two additional reflection questions about the intervention. The pre- and post-assessments were broken down into three sections: self-efficacy questions (5), mathematical computation questions (10), and mathematical context questions (5). The first section consisted of five self-efficacy questions, answered on a five-point Likert Scale with options from strongly disagree to strongly agree. The ten mathematical computation questions and five mathematical context questions were given in a free response format with space for students to show their thought processes and solving strategies. The following table demonstrates the reason for using each item on the pre- and post-assessment.

Table 2 *Self-efficacy Questions (5)*

Question	Reference	Rationale for Using Question
(1) I am confident in my ability to find percentages without the use of a calculator.	This question is not borrowed from previous research.	Hope and Sherrill (1987) argue that mental calculation is a way to develop and deepen understanding of numbers and their properties. Hope and Sherrill (1987) also express that students who rely on calculators become unskilled mental calculators and when asked to do basic mental operations they perform unnecessary substeps and take excessive time.

Table 2 Continued

(2) I am confident in my ability to convert between fractions and decimals without the use of a calculator.	This question is not borrowed from previous research.	Reys and Yang (1998) state that most students do not connect their understanding of fractions with decimal representations. Understanding how students' rate their self-efficacy in this skill is relevant in relation to their performance on both the mathematical computation questions and the mathematical context questions.

Table 2 Continued

(3) I believe that I can be good at math.	This question is not borrowed from previous research.	Both Robinson (2017) and Boaler (2016) discuss the necessity of developing a growth mindset in students. A growth mindset emphasizes understanding content over answers (Robinson, 2017). Boaler (2016) states that a fixed mindset can be damaging to students' self-efficacy and as a by-product, effect achievement.

Table 2 Continued

(4) I can apply what I have learned in math class to my everyday life.	This question is adapted from Robinson (2017).	In Robinson's (2017) article, she identifies that students' math self-efficacy contributes to their performance on basic math questions. Additionally, Robinson (2005) notes that students may lack self-efficacy specifically in cases where they must connect mathematical formulas to real life applications as they are unsure how to do this.
(5) I am confident in my ability to calculate a tip at a restaurant.	This question is not borrowed from previous research.	This question draws on Robinson's (2017) claim that students' specifically lack self-efficacy in applying mathematics to real life applications. Calculating a tip is a necessary mathematical skill that can be developed by strong number sense.

Table 3 *Mathematical Computation Questions (10)*

Question	Reference	Rationale for Using Question
(1) What is 65% of 160?	This question is borrowed from Moss & Case (1999).	Moss & Case (1999) found that a majority of high school students cannot answer this question or offer an answer that is off by more than an order of magnitude. With Moss & Case's (1999) research being over 20 years old it's necessary to revisit this question in a present high school setting.

Table 3 Continued

(2) Is there a number between 0.35 and 0.36? If so, can you name one?	This question is adopted from Moss & Case (1999).	Moss & Case (1999) asked upper elementary aged students for a number that lies between .3 and .4. Some students correctly identified numbers in the range such as .35 and .309. Other students stated there was not a number. This question will look at whether a similar misconception about comparing numbers and infinite numbers carries over into high school.

Table 3 Continued

(3) 15 is 75% of what number?	This question is borrowed from Moss & Case (1999).	In Moss and Cases' (1999) research study they found that 50% of upper elementary aged students answered this question incorrectly before exposing them to an experimental curriculum. Upon completion of the experimental curriculum 88% of students were able to correctly answer this question.
(4) Name a fraction between 0 and 1/10 whose numerator is not 1.	This question is borrowed from Reys (1994).	This question is designed to help clarify students' understanding of fractions between 0 and 1 (Reys, 1994). Activities during the five day intervention, including ordering fractions and placing rational numbers on a number line target the conceptual thinking behind this question.

Table 3 Continued

(5) Is 3/8 or 7/13 closer to .5? Why?	This question is borrowed from Reys & Yang (1998).	In a sample of 6th and 8th grade students who were asked this question, it was found that 10% of 6th grade students and 28% of 8th grade students answered correctly with correct reasoning (Reys & Yang, 1998). This question requires students to compare rational numbers in different representations as well as reason with fractions who have unlike denominators. Students had to answer why they picked the appropriate fraction to receive the point for correctly answering. Students could justify their answer choice through a picture, mathematical computations, or words. No partial credit was awarded.

Table 3 Continued

(6) Are the fractions 12/14 and 30/35 equivalent? Explain.	This question is adapted from Lamon (2007).	Lamon (2007) asked students if 3/5 and 7/11 were equal. She found that students held a misconception that for fractions to be equal you had to multiply both the numerator and denominator by the same whole number. Additionally, Moss (2005) asserts that students hold the misconception that the missing pieces in a fraction dictate the relative size of a fraction. When students are asked to compare 12/14 and 30/35, Moss (2005) suggests that students may look at what is missing out of each fraction. Since 12/14 is missing 2 pieces and 30/35 is missing 5 pieces, students might wrongfully assume that 12/14 is larger since

Table 3 Continued

		it is missing less. Students had to answer why they picked the appropriate fraction to receive the point for correctly answering. Students could justify their answer choice through a picture, mathematical computation, or words. No partial credit was awarded.
(7) What is 1/8 written as a decimal?	This question is borrowed from Moss (2005)	In Moss's (2005) research study she identified that students have a difficult time understanding the quantities of rational numbers. A common answer that Moss (2005) received to this question when she asked 5th grade students was .08 or .8. Students tend not to realize the unreasonableness of their answers (Moss, 2005).

Table 3 Continued

(8) What is 93.04 written as a mixed number in simplest form?	This question is adapted from Irwin (2001).	In her research study, Irwin (2001) identified prevalent student misconceptions about when zero is important and when it can be omitted when converting decimals to fractions. This question additionally gauges students' ability to reduce fractions.
(9) What is 23 ¼ written as a decimal?	This question is adapted from Irwin (2001).	Irwin (2001) asked the question of what 93 ¼ was written as a decimal in a research study of 11- and 12-year-old students. Some students were able to relate 1/4 to a quarter and stated that it was 93.25. However, other students incorrectly used the fraction to conclude the answer was 93.04 or 93.4.

Table 3 Continued

(10) Order the following list of numbers from least to greatest: 2/3, 0.5, 9/20, 3/4, 0.53, 0.7.	This question is adapted from Moss (2005).	In Moss's (2005) study she had students order three rational numbers: 2/3, 0.5, and 3/4. She found that comparing numbers of mixed representations was difficult for students. A misconception was students believed fractions were small parts, so they placed them as less than the decimal representations.

Table 4 *Mathematical Context Questions (5)*

Question	Reference	Rationale for Using Question
(1) On a 25-question multiple choice test, Wendy answered 80% of the questions correctly. Of those that she answered correctly, she guessed at 20% of them. How many problems did she guess correctly?	This question is borrowed from NCTM (2015).	Questions from NCTM (2015) will be used on day one of the five-day intervention. This question replicates the type of questions that will be discussed in class and in small groups during the intervention based on the context it is set in and the idea of using a multi-step process with percentages.
(2) A store discounts an item by 60%. Then it discounts the discounted price by another 40%. What is the total percentage discount?	This question is borrowed from NCTM (2015).	Similar to the first question in this section, this question comes from NCTM (2015). Students will be exposed to similar questions about calculating discounts during the five-day intervention.

Table 4 Continued

(3) A book is marked down from \$8.00 to \$6.80. What is the discount as a percentage of the original price?	This question is adapted from Moss & Case (1999).	The original question that Moss & Case (1999) used in their research study asked for a discount as a percentage of the original price when the price was changed from \$8.00 to \$7.20. Only 6% of an experimental group of upper elementary aged students were able to correctly answer this question. After participating in the experimental curriculum, 56% of students answered this question correctly.

Table 4 Continued

(4) Lawson, Jameson, and Young are three candidates for the mayor’s office in a city election in which the candidate with the most votes wins. Jameson is the most disliked candidate in that 60% of voters surveyed indicated that they would not vote for Jameson. Is it still possible for Jameson to win the election for mayor? Explain.	This question is borrowed from Agustin (2012).	In Agustin’s (2012) research study of college freshmen, nearly 60% of students missed this question. In fact, even of the students who were in the highest mathematics course, Calculus I, approximately 50% still answered this incorrectly. Explaining the correct solution for this question requires students to use algebraic relations and draw logical conclusions. They must be able to reason with percentages and think of the candidates and votes in fractional thirds. Students had to answer why they picked the appropriate fraction to receive the point for correctly

Table 4 Continued

		answering. Students could justify their answer choice through a picture, mathematical computation, or words. No partial credit was awarded.
(5) Erik ate 1/3 of his Hershey's chocolate bar and wanted to share the remaining portion of the bar with two friends. Amy took 1/4 of it and then, Isabella took half of what was left. The 3 of them decided to split the final portion equally. What fraction of Erik's original candy bar did Amy eat?	This question is not borrowed from previous research.	This question incorporates partitioning and comparing fractional parts dealing with different original amounts. To correctly answer this question students must work through a process of portioning the chocolate bar by how much is given to start and how much ends up being left over. Moss (2005) states that multi step rational number problems continue to cause trouble for adults.

Procedures

Approval for research and to collect data was granted by The University of Southern Mississippi's Instructional Review Board (IRB) (Appendix E) and the Purvis High School principal (Appendix F). The teacher researcher conducted the study with all three of her classes in the Spring of 2022. A verbal description of the study was explained to students. An informed consent letter was sent home with students to be signed by their parents/legal guardians, and a student assent form was signed by individual students. Students also received a student contract (Appendix C) stating they would be able to withdraw from the study at any time. It also stated that completion of the study would benefit students by awarding them a 100% minor replacement grade. Students who returned both signed letters and a student contract were assigned a student ID number. Demographic data was collected from each student who participated, including grade level, age, race, and gender. Student ID numbers were used to replace student names on all data collected. Once student data was collected, it was transferred to SPSS, where only student ID numbers were attached to assessment scores and demographic information.

Student confidentiality was a major priority prior to, during, and after the research. To employ strict confidentiality, precautions and safeguards were put in place. All hard data were gathered only by the researcher. Student pre- and post-assessments on paper were immediately coded with a student ID number and recorded in SPSS. All hard data, parent/guardian consent forms, student consent forms, and student contracts were kept in a locked filing cabinet. All data in SPSS was unidentifiable and did not retain student names. Even so, all electronic data was saved on a password-protected laptop, personal to the researcher.

Data Analysis

Pre- and post-assessments consisted of 15 mathematics items, ten computation questions, and five context questions, all in a free-response format. To perform quantitative analysis, all questions were graded based on the arrival of the correct answer. No partial credit was awarded. A student could score one point for a correct answer to each question resulting in a total possible score of 15 points. A dependent t-test compared the control and intervention groups' pre-assessment and post-assessment scores within each group. Since each of the three sample groups were of size less than 30, a t-test was appropriate over a z-test. This statistical test compared the mean pre-assessment scores within a group to the mean post-assessment scores of the same group to assess the five-day number sense intervention addressing RQ1.

In addition to quantitative analysis of the mathematical assessments, qualitative analysis was used to look for error patterns in students' work. For any question where the same incorrect answer was provided more than once, a follow-up analysis was done to investigate the student's reasoning behind the incorrect answer. Comparisons were made qualitatively within the intervention group from the pre-assessment to the post-assessment. Additionally, four semi-structured student interviews were conducted to probe students' commonly missed questions and develop more insight into self-efficacy responses and students' problem-solving strategies. All interview data were transcribed and analyzed using grounded theory with emergent themes (Strauss & Corbin, 1998; Tan, 2010). The themes were initially organized by the questions in the semi-structured interview protocol (see Appendix D), focusing on self-efficacy, problem-solving word problems, extracting numbers from a context, and building understanding across rational number representations such as fractions, decimals, and percentages. The researcher monitored other emergent themes during the coding and analysis. Additionally, the two additional free-response questions and responses on the post-test helped provide student perspective and feedback on the most effective activities during the intervention (see Appendix B). Along with student interviews, those two questions were analyzed for common themes and patterns to best provide evidence addressing RQ2: the types of tasks and activities that high school students report are most beneficial at improving quantitative reasoning, specifically with fractions, decimals, and percentages.

The pre- and post-assessment also contained five self-efficacy questions with answer choices on a five-point Likert scale. Each interval on the Likert scale corresponded with a numerical value as follows; strongly disagree=1, disagree=2, neither agree nor disagree=3, agree=4, strongly agree=5. Quantitative analysis for each question was performed by finding the mean score for each question on the pre-test and comparing it to the mean score on the post-test. The five self-efficacy questions ranged from specific to broad as far as student's confidence in working with rational numbers generally and situationally. Analysis of the self-efficacy questions only captures an overview of student's self-evaluation about their belief to perform in mathematics and their comfortability working with rational numbers. Further analysis would be needed to look more specifically at student's self-efficacy in a classroom setting, outside the classroom, working with a specific rational number, etc. A dependent t-test was used to explore differences in student responses, i.e., their self-efficacy level, within the intervention group before the intervention (pre-assessment) and after the intervention (post-assessment) to provide data for RQ3. Again, because the sample size of each group was less than 30, a t-test was appropriate over a z-test.

Researcher Bias

As mentioned previously, I served as a teacher-researcher in this study for all three classes. The results of this research may not be generalizable to other instructors. For future research, the methods and lesson plans for the five-day intervention are detailed enough to be replicated by other instructors. Replication of the study could help eliminate researcher bias.

Since the researcher was the only teacher leading the intervention, selection bias was present based on the sample of students used as participants. To participate in the study, students had to meet the criteria of being in the researcher's math class during the Spring semester of 2022. Students were randomly assigned teachers, so the students taking part in the intervention are representative of the average high school student at Purvis High School. This inclusion criterion did not consider all students at Purvis High School. Precautions were taken to prevent further researcher bias. To avoid design bias, the research methods were reviewed with committee members. Student data were coded upon collection of the pre- and post-assessments, and anonymity was maintained. Follow-up interview questions were reviewed and checked for leading and loaded questions.

CHAPTER IV – RESULTS

Description of Sample

There were 63 students in the teacher-researcher spring 2022 classes that participated in the study by completing both the pre- and post-assessment. Table 5 below shows the frequency and percentages of students who participated that were male (47.6%), female (52.4%), in 9th grade (1.6%), in 10th grade (60.3%), in 11th grade (36.5%), in 12th grade (1.6%), enrolled in geometry in the spring of 2022 (65.1%), and enrolled in algebra II in the spring of 2022 (34.9%).

Table 5 *Descriptive Statistics of All Participants*

0-Male 1-Female

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Male	30	47.6	47.6	47.6
	Female	33	52.4	52.4	100.0
	Total	63	100.0	100.0	

0-9th, 1-10th, 2-11th, 3-12th

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	9	1	1.6	1.6	1.6
	10	38	60.3	60.3	61.9
	11	23	36.5	36.5	98.4
	12	1	1.6	1.6	100.0
	Total	63	100.0	100.0	

0-Geometry 1-Algebra II

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	0	41	65.1	65.1	65.1
	1	22	34.9	34.9	100.0
	Total	63	100.0	100.0	

Students were separated into three groups; a control group (*N*=20), intervention group one composed of students taking Geometry in the spring of 2022 (*N*=21), and intervention group two composed of students taking Algebra II in the spring of 2022 (*N*=22). The descriptive statistics for gender, grade, class, and age of each of these groups are presented in Tables 6, 7, and 8 below.

Table 6 *Descriptive Statistics of Control Group*

0-Male 1-Female

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Male	10	50.0	50.0	50.0
	Female	10	50.0	50.0	100.0
	Total	20	100.0	100.0	

0-9th, 1-10th, 2-11th, 3-12th

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	10	13	65.0	65.0	65.0
	11	7	35.0	35.0	100.0
	Total	20	100.0	100.0	

0-Geometry 1-Algebra II

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	0	20	100.0	100.0	100.0

Age 14-18

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	15	3	15.0	15.0	15.0
	16	11	55.0	55.0	70.0
	17	5	25.0	25.0	95.0
	18	1	5.0	5.0	100.0
	Total	20	100.0	100.0	

Table 7 *Descriptive Statistics of Intervention Group One (Geometry)*

0-Male 1-Female

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Male	11	52.4	52.4	52.4
	Female	10	47.6	47.6	100.0
	Total	21	100.0	100.0	

0-9th, 1-10th, 2-11th, 3-12th

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	9	1	4.8	4.8	4.8
	10	15	71.4	71.4	76.2
	11	4	19.0	19.0	95.2
	12	1	4.8	4.8	100.0
	Total	21	100.0	100.0	

0-Geometry 1-Algebra II

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	0	21	100.0	100.0	100.0

Age 14-18

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	15	7	33.3	33.3	33.3
	16	11	52.4	52.4	85.7
	17	2	9.5	9.5	95.2
	18	1	4.8	4.8	100.0
	Total	21	100.0	100.0	

Table 8 *Descriptive Statistics of Intervention Group Two (Algebra II)*

0-Male 1-Female

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	Male	9	40.9	40.9	40.9
	Female	13	59.1	59.1	100.0
	Total	22	100.0	100.0	

0-9th, 1-10th, 2-11th, 3-12th

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	10	10	45.5	45.5	45.5
	11	12	54.5	54.5	100.0
	Total	22	100.0	100.0	

0-Geometry 1-Algebra II

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	1	22	100.0	100.0	100.0

Age 14-18

		Frequency	Percent	Valid Percent	Cumulative Percent
Valid	15	3	13.6	13.6	13.6
	16	6	27.3	27.3	40.9
	17	13	59.1	59.1	100.0
	Total	22	100.0	100.0	

Results of Research Question One

Research Question One stated: In what ways does a five-day number sense-focused intervention impact students' overall number sense and understanding of rational numbers? The hypothesis was that students participating in the five-day rational number sense intervention would experience a significant increase from their pre-assessment score to their post-assessment score. More specifically, students that participate in the five-day rational number sense intervention will experience a significant increase in scores from their pre-assessment to their post-assessment on both pure mathematical (computational) questions and mathematical context questions, compared to students in the control group that do not participate in the five-day rational number sense intervention. SPSS was used to conduct a t-test comparing the mean scores from the pre-assessment to the mean scores on the post-assessment, separately for computational questions and questions in context, for each of the three groups: control, intervention group one (geometry), and intervention group two (algebra II).

When an dependent t-test was performed on the control group to compare the difference in pre-assessment computational scores ($M = 2.95$, $SD = 3.170$) to post-assessment computational scores ($M = 3.00$, $SD = 3.195$) the results of the t-test supported that there was not a significant difference between mean scores, $t(19) = -.160$, $p = .874$, $d=.036$. When an dependent t-test was performed on the control group to compare the difference in pre-assessment context scores ($M = 1.00$, $SD = .795$) to post-assessment context scores ($M = .85$, $SD = .745$) the results of the t-test supported that there was not a significant difference between mean scores, $t(19) = .767$, $p = .453$, $d=.171$. The results of these two t-tests support the hypothesis that there was no significant difference between the pre- and post-assessment for either context questions or computational questions for the control group.

When a dependent t-test was performed on intervention group one, students in the researcher's geometry class, to compare the difference in pre-assessment computational scores ($M = 3.48$, $SD = 2.620$) to post-assessment computational scores ($M = 5.81$, $SD = 2.337$) the results of the t-test supported that there was a significant difference between mean scores, $t(20) =-4.427$, $p <.001$, $d=.966$. When an dependent t-test was performed on intervention group one to compare the difference in pre-assessment context scores ($M = .71$, $SD = .784$) to post-assessment context scores ($M = .95$, $SD = .740$) the results of the t-test supported that there was not a significant difference between mean scores, $t(20) = -1.156$, $p = .261$, $d=.252$. The results of these two t-tests partially support the hypothesis that there is a significant difference between the pre- and post-assessment for the intervention group. There is a statistically significant difference between pre- and post-assessment scores for computational questions favoring an increase in scores post-intervention, but not a significant difference for context questions.

When an dependent t-test was performed on intervention group two, students in the researcher's algebra II class, to compare the difference in pre-assessment computational scores ($M = 5.32$, $SD = 2.950$) to post-assessment computational scores ($M = 7.27$, $SD = 2.394$) the results of the t-test supported that there was a significant difference between mean scores, $t(21) =-4.101$, $p <.001$, $d=.874$. When an dependent t-test was performed on intervention group two to compare the difference in pre-assessment context scores ($M = .73$, $SD = .827$) to post-assessment context scores ($M = 1.73$, $SD = 1.032$) the results of the t-test supported that there was a significant difference between mean scores, $t(21) = -5.374$, $p <.001$, $d=1.146$. The results of these two t-tests support the hypothesis that there is a significant difference between the pre- and post-assessment for the intervention group for both computational questions and questions in context, favoring an increase in scores post-intervention.

Results of Research Question Two

Research Question Two stated: What types of tasks and activities do high school students report are most beneficial at improving quantitative reasoning, specifically with fractions, decimals, and percentages? The research hypothesis was that the activities from the intervention that rely on real-world applications of fractions, decimals, and percentages would be reported on the post-assessment reflection (Appendix B, Section 4) as the activities that are most beneficial at improving quantitative reasoning. Qualitative data was gathered from students based on their written self-reported reflections. The reflection did not gather specific student feedback because of their vague responses. In addition to the reflection, four students were selected to be interviewed by the researcher. Two students were selected from intervention group one (geometry), and two students were selected from intervention group two (algebra II). The researcher selected these students based on their significant increase in pre- to post-assessment scores on the contextualized math questions. Interviews were very conversational, and the researcher asked follow-up probing questions about the intervention tasks and how the students may have benefitted from specific tasks. The questions in Appendix D guided these conversations. The following sections will attempt to answer research question two, as stated above, by first looking at common themes from both intervention groups as a whole and then looking specifically at the four selected interviewed students.

Entire Intervention Group

To perform qualitative analysis and look for emerging themes in students' post-reflection questions, common and repetitive words or phrases were sorted based on frequency. From this, two common themes emerged from students' responses. Those two themes were *shopping discounts* and *representations.*

In the post-reflection question that asked students about the activity they felt most benefited them, there was an overwhelming response about the online shopping simulation. Some student responses to this question included “online shopping,” “finding percents off,” “clearance shopping,” “budgeting when shopping,” “discounts,” and “taxes.” All of these responses were associated with the activity performed during the intervention, where students could go to a website of their choosing and add several items to their cart, each that was a specific percentage off, and then calculate tax and shipping costs.

The second emerging theme dealt with representing rational numbers in different forms. Students were asked to reflect on what they learned over the course of the intervention in the post-assessment. Again, student responses were vague, but common wording indicated that students understood the relationship between fraction, decimal, and percentage representations. Some student responses include, “1/8 is the same as half of 1/4”, “decimals can be fractions,” “50% is half”, “adding fractions and decimals,” and “finding the percent off of something.” Each of these responses showed that students were connecting the different representations of rational numbers and were thinking of them as coherent, not separate ideas.

Student Interviews

Student interviews were conducted to better understand why specific activities may have been more beneficial than others. In addition to having a whole group qualitative analysis on emerging themes from the post-assessment, the individual student interviews offered a deeper insight into how students viewed the intervention, what they found applicable to their own life, and what could be improved upon. The following paragraphs will highlight the conversation with these four selected students.

When students were asked to expand on their favorite activity in the intervention and if it was practical or applicable to their life, the responses were as follows.

> “I really like going shopping online at Academy’s and then finding the new price of fishing rods and lures after a percent was taken off. I wish they really were on sale. I’m not sure that I would ever really find percentage off with out a calculator, but it did give me an idea of how much cheaper things are when they’re on sale.”
>
> “I think my favorite thing we did during the week was the online shopping. I like this because I got to pick out whatever I wanted, like things I would actually buy. Then I had to think about extra costs like tax and shipping and how that can change the overall cost.”
>
> “I liked when we looked at the food we eat and the percent of fat and stuff in it. This let me see the types of foods that I’m putting in my body and will help me lead to better food choices.”
>
> “The best part of the week was when we did the shopping thing. I like this because I was able to pick where I wanted to shop. I could find the sale price and it made me think about how big of a sale things would have to be so that I would buy them. I think that not everything we did during the week was realistic because I’ll always have my phone for a calculator, but it did let me see better what percents look like.”

These student responses were consistent with the themes identified by the whole group post-assessment reflection responses. Student interviews helped expand on the themes *shopping discounts* and *representations.* The students that were interviewed predominately agreed that the online shopping activity was their favorite. As they explained, this was because it was tailored to them, and they could make personal decisions about where to shop and what they were shopping for. This supports Meyer's (2001) and Shumway's (2019) research that activities that are student-centered and draw on their personal experiences can be influential and overall create meaning by connecting the content to an experience. Additionally, students reflected on the concept of understanding what a specific percentage off looked like or what a specific tax percentage would do to an overall total amount. These comments showcased that students were applying quantitative reasoning skills and taking it a step further than merely computing but comprehending the values they were working with. Lastly, one downfall in relevance that students mentioned was the idea of typically having a calculator present and not finding it necessary to perform computations in their heads. Although there may be some truth to this, the argument supported by Moss's (2015) research is that while a calculator may allow for quick computations, it doesn't make sense of the numbers. Number sense was really the overarching target of this activity and intervention. As mentioned above, students attribute the online shopping activity to helping with understanding specific discounts and price increases and decreases. Through student interviews, it became more apparent that students not only enjoyed but benefited most from the activities that allowed them to make personal choices and connections with the rational number content.

Results of Research Question Three

Research Question Three stated: How are high school students' mathematical self-efficacy levels impacted by a five-day rational number-focused intervention? The hypothesis was that students participating in the five-day rational number sense intervention would experience an increase in self-reported self-efficacy in basic number sense computations and applications. A dependent t-test was used to compare self-reported self-efficacy values on a students' pre-assessment to self-reported self-efficacy values on the students' post-assessment. Students answered a series of questions in Likert Scale format that evaluated their self-efficacy. These Likert Scale values were averaged, and the mean of their reported self-efficacy on the pre-assessment was compared to the mean of their reported self-efficacy on the post-assessment. A t-test was conducted in SPSS for intervention group one and intervention group two separately.

When an dependent t-test was performed on intervention group one to compare the difference in reported pre-assessment self-efficacy values ($M = 2.857$, $SD = .566$) to reported post-assessment self-efficacy values ($M = 3.333$, $SD = .664$) the results of the t-test supported that there was a significant difference between mean scores, $t(20) = -3.531$, $p = .002$, $d = .771$. The results of this t-test support the hypothesis that there is a significant difference between pre- and post-assessment values in self-efficacy for the intervention group. Furthermore, it can be seen that students had a statistically significant increase in self-efficacy post-intervention.

When an dependent t-test was performed on intervention group two to compare the difference in reported pre-assessment self-efficacy values ($M = 2.736$, $SD = .801$) to reported post-assessment self-efficacy values ($M = 3.445$, $SD = .819$) the results of the t-test supported that there was a significant difference between mean scores, $t(21) = -3.766$, $p = .001$, $d = .803$. The results of this t-test support the hypothesis that there is a significant difference between pre- and post-assessment values in self-efficacy for the intervention group. Furthermore, this data reveals that students had a statistically significant increase in self-efficacy post-intervention.

Summary

In this chapter, the research questions were answered by dependent t-tests as well as qualitative analysis and student interviews. To measure student understanding the pre- and post-assessment means were compared and analyzed to look for a statistical difference. As can be seen in Figure 2 below, both intervention groups significantly increased their computational scores from the pre-assessment to the post-assessment. Additionally, Figure 3 below shows mixed results. Intervention group one did not significantly increase mean context scores between the pre- and post-assessment, but intervention group two did. From these results hypothesis one was partially supported.

Figure 2. Mean Computational Scores Between Pre- and Post-Assessment by Group

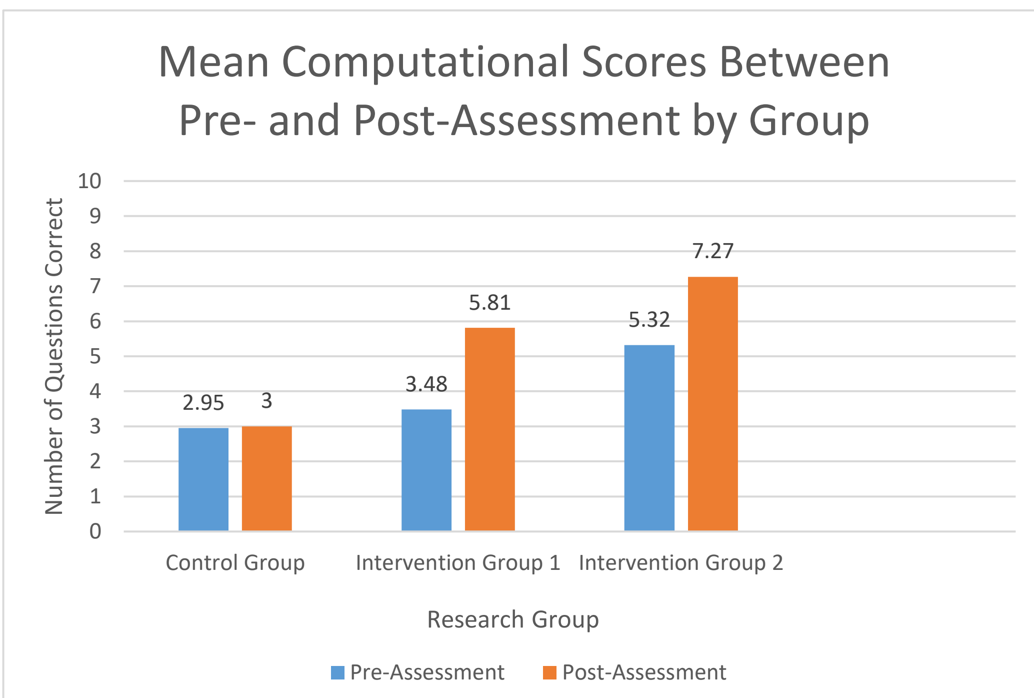
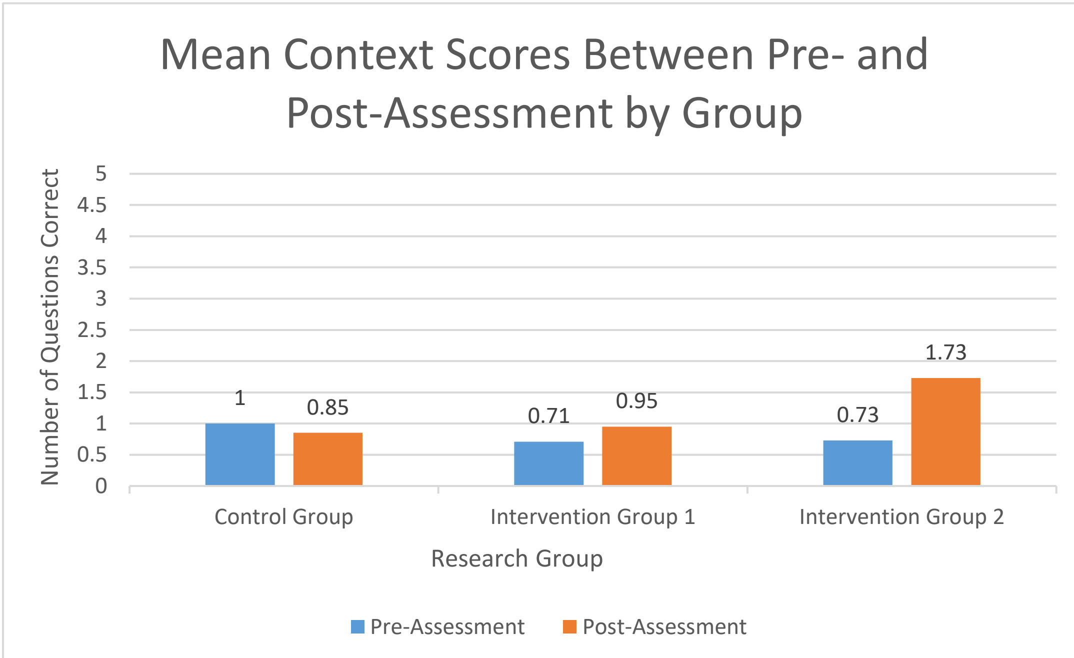
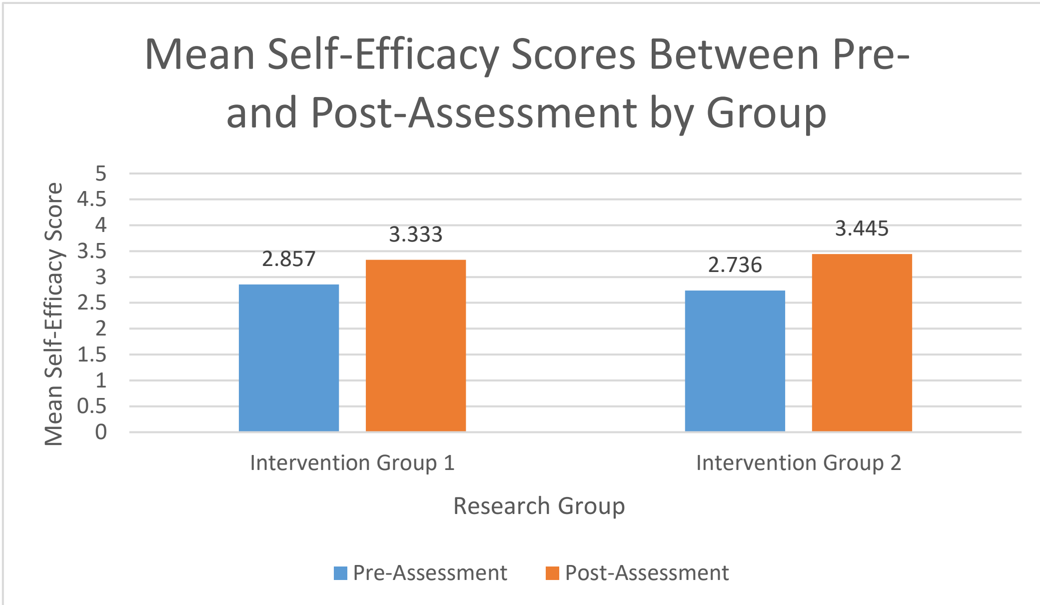


Figure 3. Mean Context Scores Between Pre- and Post-Assessment by Group



To answer research question two, qualitative data was analyzed for themes. Based on the intervention groups responses on the post-assessment reflection and four individual student interviews, the students reported that the most beneficial activity dealt with calculating discounts, taxes, and costs by online shopping. This supported research hypothesis two in that practical and realistic activities would be favored. Lastly, when looking at self-efficacy, a dependent t-test did show a significant increase in mean scores between the pre-assessment and post-assessment for both intervention group one and intervention group two. This supported research hypothesis three. A comparison of mean self-efficacy scores can be seen in Figure 4 below.

Figure 4. Mean Self-Efficacy Scores Between Pre- and Post-Assessment by Group



CHAPTER V – DISCUSSION

Introduction

The purpose of this study was to investigate how a five-day rational number sense intervention can affect high school students' number sense in the subtopics of fractions, decimals, and percentages. Additionally, students answered a written questionnaire to measure their self-efficacy level pre- and post-intervention. This study more specifically aimed to see how students responded to an intervention geared toward practical life applications dealing with rational number concepts. A mixed-methods design, including dependent t-tests and theme analysis, was used to compare an intervention group of students that underwent a five-day rational number sense intervention to a control group that did not participate in the intervention. The intervention students consisted of two class periods, one Geometry class and one Algebra II class. A control group, a second Geometry class, did not receive the intervention. Pre- and post-assessment data was compared within groups to see how the intervention affected each group of students. This final chapter will discuss the conclusions drawn from the research, limitations of the study, and recommendations for practice and future research to expand upon what has been found in this study.

Analysis of Research Questions

Regardless of age, rational number concepts can be difficult for students to learn, and there are deficiencies in this topic, particularly in grade K-8 students (Morales, 2014). More so, being able to make sense and meaning of these concepts and apply them to practical situations is expected but not often obtained (NCTM, 2009). Students' self-efficacy can be influenced by the content and success of applying number sense through practice. When high school students are expected to know how to solve a standard task, work a particular problem, or know a concept because it is “easy” or an “elementary standard,” it can be harmful to their self-efficacy when they can’t. These statements drove the research questions and study to see how high school students would be affected by a five-day number sense intervention. Three categories were examined to better define how students would be affected: rational number computational problems, rational number contextual problems, and self-efficacy. In addition to these categories, students were also asked to reflect on the intervention to shed some light on activities that they found beneficial and applicable to their everyday life. Descriptive statistics showed a significant increase in students’ rational number computational scores from pre-intervention to post-intervention. Descriptive statistics also showed mixed results in students’ rational number contextual scores from pre-intervention to post-intervention. Lastly, descriptive statistics showed a significant increase in students’ self-efficacy from pre-intervention to post-intervention. Each of these results will be analyzed and discussed further below, with qualitative analysis to follow.

Research Question One asked: in what ways does a five-day number sense focused intervention impact students' overall number sense and understanding of rational numbers? To address this question, three groups were given a pre-assessment consisting of 10 rational number computational problems. One group acted as a control; as stated in Chapter IV, their mean results for questions answered correctly did not vary after one week with no intervention. Intervention group one started with a lower mean score on the pre-assessment (3.48 questions answered correctly) compared to intervention group two (5.32 questions answered correctly). This is likely because the two groups were made from different math courses, geometry and algebra II. Students in intervention group one were all in geometry, and 76.2% were in 10th grade or younger. Students in intervention group two were all in algebra II, where only 45.4% were in 10th grade or younger. The grade and corresponding age difference could have been the reason for pre-assessment scores differing between the groups. It is possible that as high school students mature and take higher math courses, they become more fluent with rational number topics. Shumway (2019) indicates that number sense is developed through ongoing learning and continuous related experiences connecting a number sense idea. More exposure to rational number concepts in continued math courses could benefit student's conceptual understanding. However, it is important to recognize that the higher mean score for the pre-assessment for intervention group two was a 5.32/10 = 53.2%, which shows there is still a significant need for improvement. To account for the difference in groups, intervention group one was not compared to intervention group two. Each group was kept separate to see if there was improvement in the number of questions they correctly responded to on the post-assessment. From the results of pre- and post-assessment in the computational section only, both intervention groups improved their scores significantly. To answer research question one in part, the five-day number sense intervention significantly and positively impacted students' computational understanding short-term.

Students' context scores were also compared from the pre-assessment to the pos-assessment to help answer research question one. For this section of the assessment, there were only five questions. Still, each question was more involved and required students to connect rational number concepts and apply their knowledge to correctly solve the question. It is interesting to note that both intervention group's pre-assessment scores were considerably low on this section. Both groups had an average score of less than one out of five, equating to less than 20% of the questions being answered correctly. By analyzing students computational scores separately from their context scores, it became evident that questions set in context were where students struggled. This supports the rationale behind the study and the call to action that students lack an understanding of working with rational numbers, applications, and drawing conclusions based on their results. It is necessary that students have a foundation of working with fractions, decimals, and percentages in order to apply them in a context setting. While a cause-and-effect relationship cannot be assumed there may be something to say about building a strong computational background to be able to successfully perform on contextual questions.

Again, intervention group one's mean pre-assessment score on the contextualized items (.71 questions correct) was lower than intervention group two's mean pre-assessment score (.73 questions correct), but not by a significant amount. It was found that intervention group one did not significantly increase the mean number of questions correct from the pre-assessment to the post-assessment. However, intervention group two significantly increased the number of correct questions from the pre-assessment to the post-assessment, providing mixed results. There are a couple of factors that may have contributed to these results. First, as mentioned above the difference in the group was the course they were taking, which directly corresponded with their grade and age. The older students were in intervention group two, which may have influenced their performance. These students were much closer to graduation and may have taken the application activities and questions more seriously. The students in intervention group one were preparing for an upcoming ACT test offered by the school, and therefore, were likely more focused on computational-style questions. Additionally, the classroom dynamic for each group was very different. Intervention group one was a very quiet and attentive class. They listened and tried examples and activities on their own, but it was difficult to get whole group discussion and responses from them. On the other hand, intervention group two was a very active class and had no issues with voicing their responses or opinions when asked or even when not asked. Although it was not directly measured, there was an observable difference in the class dynamic and participation level between the two intervention groups. This could have also contributed to the mixed results for the contextual questions.

In connection to research question one, more specifically when analyzing the contextual component of rational numbers, the trend in data does show that students were impacted and more so had an increase in understanding post-intervention on rational number contextual problems. Still, there is not enough significant evidence to support this. Further research would need to be conducted in this area. A future intervention could be restructured in a way to better target applications of rational numbers. As stated above, students must have a solid foundational understanding of rational number concepts before being able to solve problems set in context. A future intervention could take a week to first focus on building computational skills and understanding the basics of fractions, decimals, and percentages. It is once students are proficient in this area that the intervention could move to a second week of modeling rational numbers in context and looking at applications. Isolating each component of the intervention could potentially be a way to gather more appropriate data on specific aspects of student’s number sense.

As mentioned above, descriptive statistics were also used to address research question three: how are high school students' mathematical self-efficacy levels impacted by a five-day rational number-focused intervention? Students were asked to answer a series of five questions that would measure their self-efficacy prior to participating in the intervention. All questions were on a five-point Likert scale, and it was found that intervention group one’s mean self-efficacy score was a 2.857, and intervention group two’s mean self-efficacy score was a 2.736 (out of 5). These scores were similar and show consistency across groups even though mean content scores differed between groups. It is possible that students had similar self-efficacy scores because of the similar school environment and culture they have grown up in. Self-efficacy is largely influenced by how math is taught and defined (Schoenfeld, 1992). The participants in this research largely share this common background of schooling. When asked to complete the five question self-efficacy survey post-intervention, both groups had a significant increase in mean self-efficacy score when analyzed by a dependent t-test. To better answer research question three, high school students’ mathematical self-efficacy levels were significantly increased by a five-day rational number intervention. Some contributing factors to this increase, as observed by the researcher, follow. Students were asked to sign a contract prior to completing the intervention that explicitly stated that they were not expected to know the content to come and that the activities would not be graded. However, they could earn a replacement (minor) grade for full participation. The idea behind the contract was to take any pressure off the students and let them freely explore topics and concepts they might feel expected to know but don’t. The low expectations of prior knowledge, the non-graded activities, and the positive learning environment throughout the week may have all contributed to an increase in familiarity and comfortability with the material and, in return, higher reported self-efficacy levels.

Qualitative analysis was used to help answer research question two: what types of tasks and activities do high school students report are most beneficial at improving quantitative reasoning, specifically with fractions, decimals, and percentages? From student post-reflection surveys and student interviews, general themes emerged, and it became clear that students preferred tasks where they had control over customizing and personalizing the assignment. Students heavily favored an online shopping activity where they selected the website they wanted to purchase items from and then selected the items they wanted to purchase. The freedom for students to choose items made the assignment meaningful and practical. Some students went on a shopping spree and purchased wants, such as clothes, shoes, video games, etc., whereas others purchased needs, such as groceries and household items. The variety in purchases showed how each student could make this activity unique to them.

Student responses on the post-reflection stated that not only was this their favorite activity, but it was the one they found to be most beneficial in learning fraction, decimal, and percentage concepts because of the practicality. Through observation and student conversation, it was clear that many students argued for typically having a calculator at their convenience and stated that they wouldn't actually need to calculate a percentage of a cost by hand. Through student interviews, this idea became increasingly apparent. However, when pressed further, the students that were interviewed agreed that doing the mental calculations made them more aware of an estimate of a cost after a certain percentage was taken off. It also made them more aware of tax and additional costs added to a purchase. In response to research question two, through self-reported student reflections and student interviews, the type of tasks that were most beneficial at improving quantitative reasoning, specifically with fractions, decimals, and percentages, were those that were meaningful and practical to students and where students were able to personalize parts of the activity to fit their interests and future plans.

Limitations

There were several limitations of this study that are outlined below. First, the study only included high school students from one school in south-central Mississippi. Additionally, there was only one instructor, the researcher, for all three classes. The results of the study may not be generalizable to groups of students at other high schools or other instructors. Student assignments to class periods were not random since the school placed students in particular math courses. All students who participated in the study were enrolled in Geometry or Algebra II and the results of the study may not generalize to students enrolled in other high school math courses.

As previously noted, the researcher was also the instructor and observer for students in this study. The researcher had developed a positive relationship with a majority of the students by the point the research was conducted. This positive relationship could have influenced student responses favorably when participating in the self-efficacy assessment or when answering the post-assessment reflection questions. Students self-responded in the assessments, and a limitation of this study is that students could have felt an obligation to respond in a certain way.

The study is also limited by the short time frame. All results were gathered within a month’s timeframe, and the intervention that students underwent only lasted one week.

Since all data collected was based on student responses, the research is limited by the honesty and integrity of the students. During student interviews, again, responses relied on students truthfully answering follow-up questions and providing an accurate representation of their thoughts and experiences throughout the intervention process.

Lastly, the research study was conducted post-COVID-19 pandemic. Precautionary measures were still taken to limit student contact and exposure. However, because of the aftermath of the pandemic and these precautionary measures, it may have limited student interaction in groups and participation in the intervention activities. The lack of student interaction and collaboration may have affected the results.

Recommendations for Practice

Although the five-day number sense intervention did not significantly affect all students' ability to increase their knowledge on contextualized rational number math problems, the intervention did have a positive effect on students' short-term ability to correctly compute rational number pure computational math problems. These findings support the work of Moss and Case (1999) and Irwin (2001). Based on this research, it is recommended that high school teachers integrate rational number activities into their curriculum. Incorporating rational number activities that give students the opportunity to work with fractional, decimal, and percentage quantities without a calculator will allow them to better develop numerical reasoning and gain a deeper understanding of how quantities in differing formats relate to one another and how to manipulate and compute rational numbers mentally. It is recommended that teachers create interactive activities that involve rational number concepts so that students can continue to practice and build on their prior knowledge. It was seen that although rational number concepts, including computing with fractions and decimals, are part of many elementary school curriculums (Meyer, 2001), by the time students reach high school, they have forgotten these concepts and lack strategies and an understanding of quantitatively reasoning their answers.

As a byproduct, the five-day number sense intervention significantly increased students' reported self-efficacy levels. For this reason, it is recommended that teachers create an open line of communication with students and do not set the standard that students should automatically know or remember all prior math skills, especially those involving rational numbers. It is recommended that teachers promote the idea of exploration in math and relearning fundamental topics through relevant activities to increase self-efficacy levels.

A final recommendation is to include instructional activities where students can personalize the assignment to their own interests. Through student interviews, it was seen that the activity that students felt they learned the most from was an online shopping inventory where they were asked to find discounts, taxes, and other price changes on items of their choosing. Based on student feedback, it is recommended that teachers include highly engaging, student-centered activities that allow students to tailor an assignment to their interests while still developing a deeper conceptual understanding of mathematical concepts.

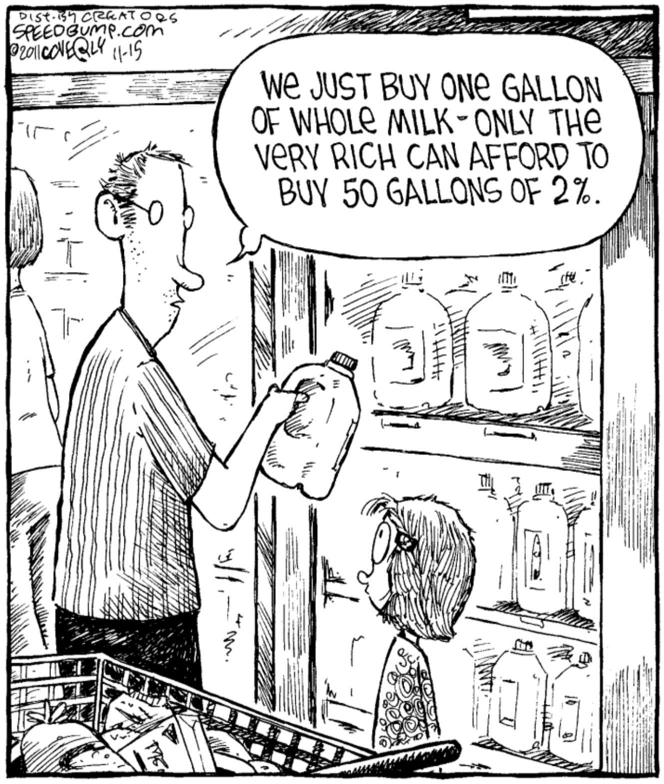
Recommendations for Future Research

Additional research is needed with a larger group of high school students across the state to generalize the study results. It would be recommended to randomly sample a set of high schools in Mississippi to get a more diverse and larger sample. It would also be recommended to have an intervention group and control group for each high school course; Algebra, Geometry, and Algebra II. Because of the limitations of this study, it was not possible to compare a control group of students enrolled in Algebra II to the intervention group of students in Algebra II. Having the opportunity to keep more variables constant would allow a better comparison between groups.

A second recommendation for future research would be to look at the long-term implications of a five-day number sense intervention. Does this intervention make a difference a month from now or six months from now? If students do not revisit these concepts, do they relapse back to where they were prior to the intervention? A longitudinal study would be needed to gather a second or third round of post-assessment data to answer these questions and consider the lasting implications of a short intervention.

Lastly, the data gathered showed that students in intervention group one did not significantly increase in score from the pre-assessment to the post-assessment for contextualized questions. Does the limited time of the intervention play a factor in this? Future research could consist of a semester-long intervention period where activities on rational numbers are integrated into the course curriculum daily to see if this would increase students' conceptual knowledge of rational numbers. Previous research from Shumway (2019) states that it takes time and continuous opportunities to develop conceptual knowledge and build connections to remember and apply content. With more available resources and a less time-restrictive environment, a rational number intervention could be tested over the course of a semester.

Discussion questions presented to the whole class (NCTM, 2013).



"SPEED BUMP" © DAVE COVERLY/WWW.SPEEDBUMP.COM

WHAT ABOUT THE OTHER 98%?

1. Why is the father comparing his 1 gallon of whole milk with 50 gallons of 2% milk?

2. In the United States, *2% milk* means that 2% of the weight of the milk is fat. For whole milk, 3.25% of the weight is fat.

a. How many times as much fat is in 1 gallon of whole milk as in 1 gallon of 2% milk?

b. How many gallons of 2% milk have the same amount of fat as 1 gallon of whole milk?

c. How many quarts of 2% milk have the same amount of fat as 1 gallon of whole milk?

3. *Half and half,* which is 1/2 milk and 1/2 light cream, is about 12.5% fat. It is often sold in pints or quarts.

a. How many gallons of 2% milk have the same amount of fat as 1 quart of half and half?

b. About how many gallons of whole milk have the same amount of fat as 1 quart of half and half?

4. Whipping cream has a fat content of about 36% and is often sold in half pints.

a. How many gallons of 2% milk have the same amount of fat as 1 half pint of whipping cream?

b. About how many quarts of whole milk have the same amount of fat as 1 half pint of whipping cream?

c. How many quarts of half and half have the same amount of fat as 1 half pint of whipping cream?

5. One gallon of whole milk contains about 128 grams of fat. Suppose you pour yourself an 8-fluid-ounce glass of whole milk and then accidently spill half the milk out of the glass. How many grams of fat are in the milk that remains in the glass?

CHALLENGE

6. A quart of whole milk has 31.7 grams of fat. A quart of 2% milk has 19.5 grams of fat. Suppose milk were sold in 3 liter containers. How many grams of fat would be in a 3 liter container of—

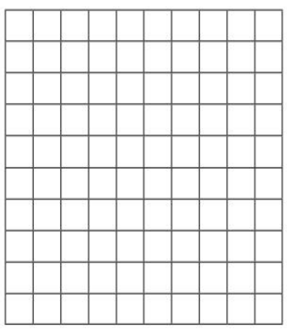
a. whole milk?

b. 2% milk?

Discussion questions presented to partners (NCTM, 2015).

PERCENTAGE RIGHT; PERCENTAGE WRONG

1. a. Shade 75% of the large square below.



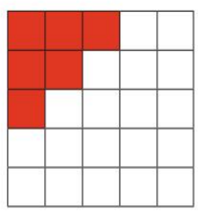
b. What percentage of the large square is not shaded?

2. a. What is Baldo's mathematical error in the cartoon? Explain why this may be humorous.

b. Suppose that there were 20 problems on Baldo's math test. How many problems did he answer correctly? How many did he answer incorrectly?

3. Suppose that Baldo took a science test and answered 12 problems correctly. This number represented 40% of the total problems on the test. How many problems were on the test?

4. What percentage of the large square is shaded? Explain your reasoning.



5. On a 25 problem multiple-choice test, Baldo answered 80% of the problems correctly. Of those he answered correctly, he guessed at 20% of them. How many problems did he guess correctly?

CHALLENGE

6. A store discounts an item by 60%. Then it discounts the discounted price by another 40%. What is the total percentage discount?

7. A store marks up an item from \$50 to \$100.

a. What is the percentage increase?

b. \$100 is what percentage of \$50?

APPENDIX B – Pre- and Post-Assessment

Section 1: Please circle the choice that best reflects your agreement with each statement.

1. I am confident in my ability to find percentages without the use of a calculator.

   Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

2. I am confident in my ability to convert between fractions and decimals without the use of a calculator.

   Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

3. I believe that I can be good at math.

   Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

4. I can apply what I have learned in math class to my everyday life.

   Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

5. I am confident in my ability to calculate a tip at a restaurant.

   Strongly Disagree   Disagree   Neutral   Agree   Strongly Agree

Section 2: Be sure to show your work and circle your final answer.

1. What is 65% of 160?
2. Is there a number between 0.35 and 0.36? If so, can you name one?
3. 15 is 75% of what number?
4. Name a fraction between 0 and 1/10 whose numerator is not 1.
5. Is $3/8$ or 7/13 closer to .5? Why?
6. Are the fractions 12/14 and 30/35 equivalent? Explain.
7. What is ⅛ written as a decimal?
8. What is 93.04 written as a mixed number in simplest form?
9. What is 23 ¼ written as a decimal?

10. Order the following list of numbers from least to greatest: $2/3$, 0.5, 9/20, $3/4$, 0.53, 0.7.

Section 3: Be sure to show your work and explain your thinking. Circle your final answer.

1. On a 25 question multiple choice test, Wendy answered 80% of the questions correctly. Of those that she answered correctly, she guessed at 20% of them. How many problems did she guess correctly?
2. A store discounts an item by 60%. Then it discounts the discounted price by another 40%. What is the total percentage discount?
3. A book is marked down from $8.00 to $6.80. What is the discount as a percentage of the original price?
4. Lawson, Jameson, and Young are three candidates for the mayor's office in a city election in which the candidate with the most votes wins. Jameson is the most disliked candidate in that 60% of voters surveyed indicated that they would not vote for Jameson. Is it still possible for Jameson to win the election for mayor? Explain.

5. Erik ate 1/3 of his Hershey's chocolate bar and wanted to share the remaining portion of the bar with two friends. Amy took 1/4 of it and then, Isabella took half of what was left. The 3 of them decided to split the final portion equally. What fraction of Erik's original candy bar did Amy eat?

(Post Test Only)

Section 4: Please answer the following questions about the previous 5 days of activities.

1. What is something you learned from the activities over the past 5 days?

2. Was there an activity that you felt was most effective? If so, which one?

APPENDIX C – Five Day Number Sense Student Contract

Over the next five days we will revisit topics that you have learned in previous math classes. We will mainly be focusing on fractions, decimals, and percentages and how they can be applicable to use outside of the classroom. The only goal for these next five days is for you to better understand these topics. I understand that these may be topics that you are often expected to know, but haven't had the opportunity to practice lately or aren't comfortable with. The next five days will act as a clean slate to revisit and relearn each of these topics and their relationships to one another. There are no silly questions. There is no penalty for wrong answers. In fact, I encourage you to explore, try new things, and ask questions when you don't understand. Pre and post assessments will NOT be for a grade. Furthermore, none of the activities over the next five days will be graded. However, full participation and effort is expected. By signing this contract, to fully participate in all activities, and upholding this by contributing with full effort, and completing both the pre and post assessment you will earn a 100 for a minor grade that can replace your lowest minor grade for the semester.

I, ______________________________, have read and understand the above criteria for participating in the upcoming activities and assessments.

_________________________________ _____________

Signature Date

APPENDIX D – Semi-structured Student Interview Questions

1. Explain one of the activities that you took part in over the course of the intervention. What did you like about the activity? What did you not like?

2. How can calculating with decimals, fractions, and percentages be used in real life?

3. What is one misconception that you had about fractions, decimals, or percentages prior to participating in this intervention?

4. What's the biggest thing you are taking away from this intervention?

5. How has your mathematical image shifted throughout the past week? What are some specific times you felt confident? Were there times that you felt less confident? Explain.

APPENDIX E – Purvis High School Principal Approval Letter

LAMAR COUNTY
SCHOOL DISTRICT
empowering learners

P.O. Box 609
424 Martin Luther King Drive
Purvis, MS 39475
Phone: 601-794-1030 Fax: 601-794-1012
www.lamarcountyschools.org

I hereby give Rebecca Steele-Mackey permission to conduct her research study titled The Impact of a Five Day Number Sense Intervention on High School Student's Quantitative Reasoning Skills and Number Sense at Purvis High School.

Purvis High Principal
3/25/22

Purvis High School - P. O. Box 1089 - 220 School Street, Purvis, MS 39475

APPENDIX F – The University of Southern Mississippi's Instructional Review Board (IRB) Approval Letter

Office *of* Research Integrity

THE UNIVERSITY OF SOUTHERN MISSISSIPPI.

118 COLLEGE DRIVE #5116 • HATTIESBURG, MS | 601.266.6756 | WWW.USM.EDU/ORI

NOTICE OF INSTITUTIONAL REVIEW BOARD ACTION

The project below has been reviewed by The University of Southern Mississippi Institutional Review Board in accordance with Federal Drug Administration regulations (21 CFR 26, 111), Department of Health and Human Services regulations (45 CFR Part 46), and University Policy to ensure:

- The risks to subjects are minimized and reasonable in relation to the anticipated benefits.
- The selection of subjects is equitable.
- Informed consent is adequate and appropriately documented.
- Where appropriate, the research plan makes adequate provisions for monitoring the data collected to ensure the safety of the subjects.
- Where appropriate, there are adequate provisions to protect the privacy of subjects and to maintain the confidentiality of all data.
- Appropriate additional safeguards have been included to protect vulnerable subjects.
- Any unanticipated, serious, or continuing problems encountered involving risks to subjects must be reported immediately. Problems should be reported to ORI via the Incident submission on InfoEd IRB.
- The period of approval is twelve months. An application for renewal must be submitted for projects exceeding twelve months.

PROTOCOL NUMBER:	21-412
PROJECT TITLE:	The Impact of a Five Day Number Sense Intervention on High School Student's Quantitative Reasoning Skills and Mathematical Confidence
SCHOOL/PROGRAM	Center for STEM Education
RESEARCHERS:	PI: Rebecca Steele-Mackey Investigators: Steele-Mackey, Rebecca~Cwikla, Julie~
IRB COMMITTEE ACTION:	Approved
CATEGORY:	Expedited Category
PERIOD OF APPROVAL:	12-Apr-2022 to 11-Apr-2023

Donald Sacco Jr.

Donald Sacco, Ph.D.
Institutional Review Board Chairperson

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