



Magneto hydrodynamics flow of Ag-TiO₂ hybrid nanofluid over a permeable wedge with thermal radiation and viscous dissipation

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ABSTRACT

Hybrid nanofluids, which are made by suspending non-identical nanoparticles, have been a prominent research area because of their high efficiency in heat transfer. The analysis of the magneto hydrodynamics flow of Ag-TiO₂ hybrid nanofluid over a permeable wedge with heat radiation and viscous dissipation is mathematically examined in this paper. Ordinary differential equations are deduced by applying the corresponding similarity transformations to the mathematical modelling of the governing partial differential equations. The dimensionless governing equations are solved using the built-in bvp4c function in the MATLAB package to compute the dual solutions and the stability analysis. A respectable degree of agreement has been obtained after comparing the current results with the earlier study. Prandtl number, magnetic parameter, radiation parameter, Eckert number, and other governing factors have all been studied, along with their physical impacts on fluid flow. The graphical results have been demonstrated and described in relation to the profiles of temperature and velocity distribution, skin friction as well as the Nusselt number. It has been established that the higher volume percentage of titania nanoparticles has the potential to improve thermal conductivity, and the first solution has been found to be stable in this flow.

1. Introduction

Scientists and researchers are constantly striving to increase the thermal conductivity of materials in order to get the greatest possible outcome in boosting the heat transfer rate. Formerly, conventional base fluid was utilized as the heat transfer fluid. However, due to the low thermal conductivity, the process is inefficient. Therefore, solid particles had been dispersed into the conventional fluid to boost the thermal conductivity. Maxwell [1] pioneered this notion by determining the possibility of increasing the heat transfer rate of fluids containing a large volume fraction of solid particles. Nevertheless, the oversized particles produce sedimentation and inhibit heat transfer reactivity. As a result, nanoparticles of a smaller scale have been developed to improve both suspension stability and thermal conductivity, which is known as nanofluid. Choi and Eastman [2] began the investigation to overcome the problem of low thermal conductivity and reinvent the heat transfer rate of fluids that can be coordinated by floating metallic nanoparticles. Therefore, the nanofluid has gained enormous attraction and has been

explored with different aspects like thermal physical properties, viscosity, magnetic, chemical reaction and stability. The reviews from the researchers mostly agreed that the nanofluid tremendously improves the performance of thermal conductivity in conventional fluids [3–5]. The study was then expanded by dispersing two different types of nanoscale-sized particles simultaneously, leading to the coining of the phrase “hybrid nanofluid”.

The uniqueness of hybrid nanofluid is attributed to the composition of two dissimilar metallic nanoparticles that dissolve in the based fluid. By selecting the two ideal nanoparticles, thermal conductivity will be improved and produce a better rate of heat transfer [6]. The combination’s goal is to either strengthen each component’s ability to function alone or to compensate for any weaknesses to generate an optimum heat transfer rate. As a result, many scientists are curious to find out how hybrid nanofluids can increase thermal conductivity. They used a wide range of different aspects in their research to determine the efficacy of enhancing heat transmission, including magnetic field, porous media, diverse plate surface properties, thermal radiation, viscosity, chemical reaction, and others. A spectacular thermal network and rheological

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Nomenclature*Roman letters*

a	positive strength constant
Ag	argentum/silver nanoparticles
b	stretching/shrinking constant
B	magnetic field
Ec	Eckert number
$f(\eta), F(\eta)$	velocity function
H_2O	water
k	thermal conductivity
k^*	mean absorption coefficient
m	the power-law parameter of Falkner-Skan
M	magnetic parameter
Nu_x	local Nusselt number
Pr	Prandtl number
q_w	surface heat flux
q_r	radiative heat flux
Rd	thermal radiation parameter
Re_x	local Reynolds number
S	suction parameter
T	temperature
TiO_2	titania/titanium dioxide nanoparticles
u, v	velocity components along the x and y directions, respectively
u_e	external fluid flow
u_w	surface velocity
v_w	mass flux velocity
x, y	Cartesian coordinates

Greek symbols

α	thermal diffusivity
β	Hartree pressure gradient
Ω	total wedge angle
ψ	stream function
γ_1	smallest eigenvalue
σ^*	Stefan-Boltzman constant
μ	dynamic viscosity
ρ	density
σ	electrical conductivity
γ	eigenvalue
η	similarity variable
θ	dimensionless temperature
ϕ_1	Argentum nanoparticles volume fraction
ϕ_2	Titania nanoparticles volume fraction
λ	stretching/shrinking parameter
ν	kinematic viscosity

Subscripts

w	condition at the surface
∞	condition at the ambient
nf	nanofluid
f	base fluid
hnf	hybrid nanofluid

Superscript

'	differentiation with respect to η
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features that were influenced by the synergistic interaction of dispersion composite nanoparticles were also documented in numerous reviews of hybrid nanofluids. The anomalous rise in the heat transfer rate of hybrid nanofluid will be a major feature in thermal physical roles where it can function as a cooler or warmer, making it advantageous and effective in the heat exchanger and solar energy system applications [7,8]. Bhatti et al. [9] analysed the behaviours of diamond and Silica nanoparticles in water-based hybrid nanofluid through the solar collector application and noticed that the vigorous doping for both nanoparticles accomplished a preferable thermal elevation, especially close to the wall.

Additionally, the fluid flows through wedge-shaped surfaces with a variety of effects also drawn extensive attention in the past few decades. The problem on wedge-shaped surfaces had been considered and pioneered by Falkner and Skan [10], who proposed the solutions of boundary layer equations for streaming flows over static wedges with arbitrary angles. They developed a similarity transformation by which the differential boundary layer equations can be reduced to a nonlinear normal differential equation of third order, and subsequently described it arithmetically. Almost any situation involving general fluid flow, such as in factories, research facilities, or even in the building of prototypes for future aerospace or defence technologies, will involve boundary layer flow past a wedge. Flows past a wedge are used in a wide variety of fields, including nuclear power plants, the flow of molten metals over ramped surfaces, polymer processing, the development of flaps on aircraft wings to improve lift, manoeuvrability and drag, the launching of chilled air through AC panels, crude oil extraction, the modelling of warships and submarines, liquid metal flows in heat exchangers, and many more. Transonic flows over airfoils and wings, are also one of the primary topics of research involving wedge angle [11]. Gorla et al. [12] studied the impact of Brownian motion and thermophoresis parameter on the mixed convection of nanofluid past a vertical wedge. Khan et al. [13] explored the influence of heat generation/absorption on Falkner-Skan flow of Carreau nanofluid over a wedge and found that the

temperature gradient increased as the wedge angle parameter increased. A year later, Awaludin et al. [14] noticed that increasing the angle of the wedge improved the range of solutions. In subsequent years, several researchers did studies on Falkner-Skan flow using combinations of other physical characteristics to ascertain the impacts of the flow [15–17].

Furthermore, hybrid nanofluids subjected to magnetic field have a big potential for applications in industrial sectors such as geothermal energy extractors, liquid-metal cooling of nuclear reactors, electro-magnetic casting, generators and flow meters as well as medical sectors like cancer therapy, asthma treatment and drug release. The study of electrically conducting fluids under the influence of magnetic fields is known as magnetohydrodynamics (MHD). A moving conductive fluid can experience currents from magnetic fields, which polarises the fluid and changes the magnetic field in a reciprocal manner. Researchers consider MHD in their model since it is believed to be one of the decisive variables to increase thermal conductivity in the fluid flow [18]. The formation of electrically conducting fluid is due to the coupling between the magnetic field and velocity field, inducing the currents into such a hybrid nanofluid flow and releasing forces acting on the fluid and remoulding the magnetic field itself. The presence of magnetic field with Lorentz force interacts with the buoyancy force in the governing flow and temperature fields, hence, the phenomenon of MHD is described completely through the momentum and energy differential equations which comprise Navier-Stokes equations and Maxwell's equations. As mentioned by Wakif et al. [19], the increasing resistive Lorentz forces along with higher suction parameter tend to slow down the fluid velocity. Nowadays, numerous researchers worked on the stimulation of MHD and heat transfer of hybrid nanofluid flow to determine the viability and practicality of the effects of MHD on hybrid nanofluids [20–22]. Pordanjani et al. [23] examined the effect of magnetic field on the convective heat transfer of hybrid nanofluid in a square diagonal cavity. They claimed that the entropy generation hardly changes when

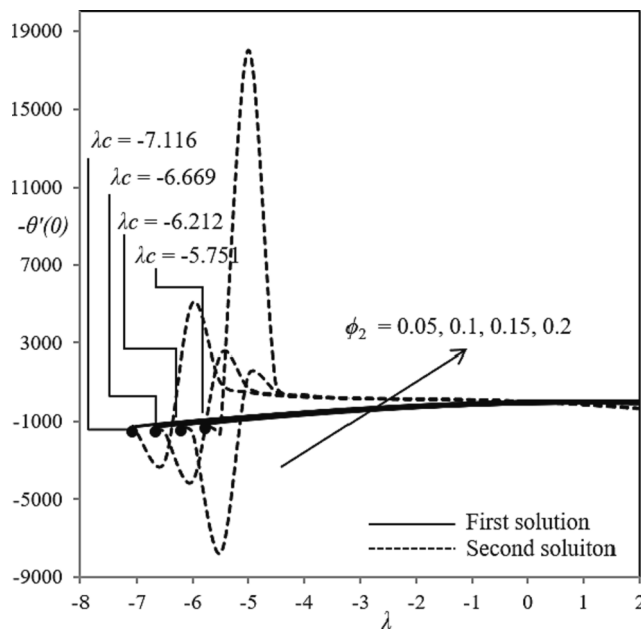


Fig. 20. Impact of TiO₂ nanoparticles concentration on the Nusselt number.

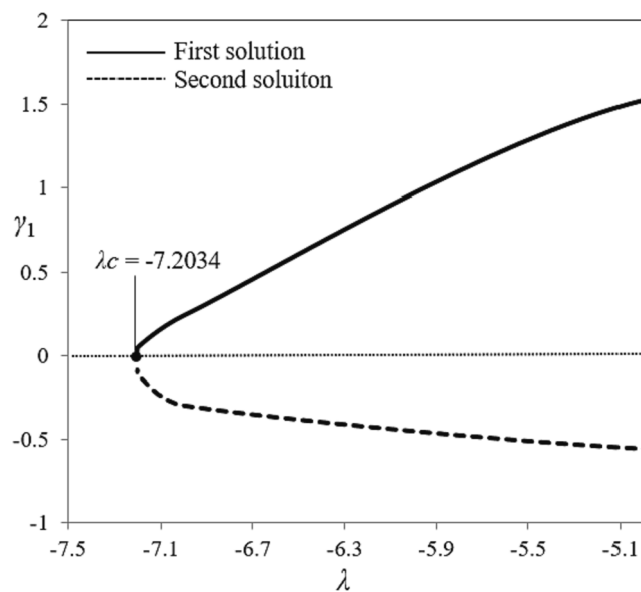


Fig. 21. Smallest eigenvalue γ_1 on λ for dual solutions.

- The temperature distribution profile as well as heat transfer rate enhances when the thermal radiation parameter grows in number.
- An increase in the suction parameter upsurges the velocity gradient and shear stresses but reduces the temperature distribution profile.
- Higher volume fraction of argentine nanoparticles increases the skin friction coefficient but reduces the local Nusselt number.
- Titania nanoparticle's higher concentration causes the diminution of the skin friction coefficients and enhances the thermal conductivity of the hybrid nanofluid.
- The first solution for this study is feasible and stable while the second solution is unstable according to the computed smallest eigenvalue.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence

the work reported in this paper.

Data availability

No data was used for the research described in the article.

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