UNCERTAINTY EVALUATION FOR CONSTRAINED STATE ESTIMATION IN WATER DISTRIBUTION SYSTEMS

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ABSTRACT

In this paper an alternative uncertainty treatment for the traditional unconstrained weighted-least squares (WLS) method is presented. This treatment enables hydraulic constraints (i.e., null demands at transit nodes, null flows at closed pipes, pumps or valves, etc.), high precision measurements and upper and lower variable bounds (i.e., head levels at tanks) to be included within the *state estimation* (SE) problem for water distribution systems. With this approach there is no need to choose appropriate weights associated with these types of measurements in order to correctly assess uncertainty for the SE problem. The method set out herein tackles these as constraints and works with the linear system of equations derived from imposing first order optimality conditions for the constrained SE problem. This approach enables general quantification of the SE uncertainty for all the hydraulic variables within the water system by applying the first order second moment (FOSM) method. Moreover, it enables standard computation of the covariance residual matrix associated with it, which is necessary to detect erroneous measurements. An illustrative example

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and a case study are shown to bring out the fact that the SE uncertainty results are more accurate and to show how the numerical conditioning of the system is affected, which may be crucial when dealing with large-scale water networks.

Keywords: weighted least squares, exact measurements, residuals treatment, uncertainty analysis

INTRODUCTION

At present the tendency in improving water distribution systems management is to install Supervisory Control and Data Acquisition (SCADA) systems, whose implementation has been boosted recently with the aim of making the *smart city* challenge a reality. These platforms enable certain hydraulic variables to be monitored continuously where measurement devices are located, but do not, by themselves, enable the hydraulic state of the network to be inferred. State estimation techniques were developed in the 1970s in the power supply industry with that purpose in mind (Schweppe and Wildes, 1970), in order to turn the information provided by a monitored set of metering devices into real information about the state of the system. Unlike power supply systems, water networks are usually characterized by having a low degree of instrumentation, which is one reason why the application of SE algorithms to water systems is still a topic of ongoing research (Kang and Lansey, 2009).

There have been several approaches to incorporating SE into water systems (see Andersen et al. (2001) for references). Among all of these, the WLS method stands out in the water distribution sector for solving both the state estimation (Bargiela, 1984; Powell et al., 1988; Brdys and Ulanicki, 2002; Kang and Lansey, 2009) and parameter estimation problems (Datta and Sridharan, 1994; Piller, 1995; Reddy et al., 1996; Kapelan et al., 2003). With this approach the solution is typically found via the so called Gauss Newton or normal equation method, which fundamentally transforms the unconstrained WLS problem into a linear system of equations that must be solved iteratively. With this approach, weights must be assigned to the different available measurements in order to show how accurate these are. This constitutes a numerical problem when there are hydraulic constraints or high precision measurements, as the weight associated with these must in theory be infinite or very large, respectively. This could be true for null demands at transit nodes, null flows at closed pipes, pumps or valves, etc., which should act as *exact measurements* for the SE problem. To overcome this problem, these measurements are typically considered to be highly accurate, but alternative constrained WLS methods have been developed in the power supply field (Korres, 2002; Abur and Expósito, 2004; Gómez-Quiles et al., 2013), which lessen the risk associated with working with ill-conditioned systems. These approaches have proved to be computationally efficient, hence application of similar techniques to water management systems would help to control ill-conditioning, inherent to the matrices involved in the normal equation approach as reported by Bargiela (1984). Additionally, a constrained WLS approach would enable upper and lower bounds for the SE of some variables, such as head levels at tanks, to be set.

In this context, the aim of this paper is to set out an alternative treatment with respect to the WLS problem to determine the uncertainty related to SE in water distribution systems including: i) hydraulic constraints or high precision measurements and ii) upper and lower limits for state estimation . For this purpose, we use the first order optimality conditions of the constrained WLS. It must be stressed that we do not focus on the solution for the constrained SE problem as this can be solved either using standard mathematical programming solvers or ad hoc algorithms which have already been developed in the literature (Caro et al., 2008; Caro and Conejo, 2012), where extensive comparisons in terms of SE performance can be found. Rather, we focus on quantifying the uncertainty associated with it because i) it is a novel contribution and ii) it is essential for evaluating how accurate the SE results are, especially when *pseudomeasurements* (i.e., not readings taken from a meter, but predictions expected for hydraulic variables associated with greater uncertainty) are taken into account to guarantee the system observability (Bargiela and Hainsworth, 1989). Additionally, this method will enable the residual covariance matrix (which is required to compute normalized residuals and to detect erroneous measurements (Caro et al., 2011)) to be computed.

The rest of this paper is set out as follows: in the first section, the traditional unconstrained WLS version (normal equation method) used to quantify SE uncertainty and compute the residual covariance matrix is presented. Then, the constrained approach is set out as a method for tackling the same problems. Subsequently, an illustrative example and a case study are presented to show the differences between both methods when analyzing SE uncertainty and the impact hydraulic constraints or high precision variables have in the overall numerical conditioning of the problem. Finally, some important conclusions are drawn.

TRADITIONAL UNCONSTRAINED WLS STATE ESTIMATION FORMULATION

Generally speaking, an algorithm for SE enables the most likely state of a system to be computed by combining the information provided by a monitored measurement set and the system of governing equations. Specifically, the SE for water distribution systems at a selected time (i.e., *pseudo-static* state estimation) is based on the following non-linear model:

$$\boldsymbol{z} = \boldsymbol{g}(\boldsymbol{x}) + \boldsymbol{\epsilon}, \tag{1}$$

where $\boldsymbol{z} \in \mathbb{R}^m$ is the measurement vector (which may include piezometric heads at nodes, tank levels, pipe flows or consumptions at nodes), $\boldsymbol{x} \in \mathbb{R}^n$ is the state variable vector (constituting nodal heads as in Díaz et al. (2015)), $\boldsymbol{g} : \mathbb{R}^n \to \mathbb{R}^m$ is the nonlinear relationship between measurements and state variables (derived from applying mass and energy conservation equations) and $\boldsymbol{\epsilon}$ is the measurement error vector (typically assumed to be unbiased $E[\boldsymbol{\epsilon}] = \boldsymbol{0}$ and with the variance-covariance matrix $\boldsymbol{C}_z$).

Traditional SE techniques consist in finding the most likely values for the state variables $\boldsymbol{x}$ by solving the following unconstrained WLS problem:

$$\underset{\boldsymbol{x}}{\text{Minimize}} \quad f_{obj}(\boldsymbol{x}) = \tfrac{1}{2}\boldsymbol{\epsilon}^T \boldsymbol{W} \boldsymbol{\epsilon} = \tfrac{1}{2}\left[\boldsymbol{z} - \boldsymbol{g}(\boldsymbol{x})\right]^T \boldsymbol{W} \left[\boldsymbol{z} - \boldsymbol{g}(\boldsymbol{x})\right] \ , \tag{2}$$

whose optimal solution corresponds to $\hat{\boldsymbol{x}}$ and where $\boldsymbol{W} = \boldsymbol{C}_z^{-1}$ is the $m \times m$ diagonal matrix for the measurement weights. Note that one condition that is required but not sufficient for the SE problem to have a sole solution is $m \geq n$.

As mentioned before, Equation (2) has traditionally been solved using the well-known normal equation method (Expósito and Abur, 1998). According to this approach, the SE uncertainty can be quantified by calculating the variance-covariance matrix for the state variables ($\boldsymbol{C}_x$) as:

$$\boldsymbol{C}_x = [\boldsymbol{J}^T \boldsymbol{W} \boldsymbol{J}]^{-1}, \tag{3}$$

where $\boldsymbol{J} \in \mathbb{R}^{m \times n}$ is the measurement Jacobian matrix evaluated at the optimal solution obtained from solving problem (2). Note that a theoretical and sufficient condition for quantifying the SE uncertainty is for matrix $\boldsymbol{J}$ to have full rank n, i.e., for the system to be observable (Díaz et al., 2015).

Once $\boldsymbol{C}_x$ has been computed, the variance-covariance matrix for the remaining hydraulic variables (pipe flows $\boldsymbol{Q}$ and nodal demands $\boldsymbol{q}$) can be inferred by applying the FOSM method again as follows:

$$\boldsymbol{C}_{Q,q} = \boldsymbol{J}_{Q,q} \boldsymbol{C}_x \boldsymbol{J}_{Q,q}{}^T, \tag{4}$$

where $\boldsymbol{J}_{Q,q}$ refers to the part of the measurement Jacobian matrix that relates pipe flows and nodal demands to nodal heads, respectively.

Concurrently, the residual covariance matrix $\boldsymbol{\Omega}$ can be obtained with this approach according to the expression (Expósito and Abur, 1998):

$$\boldsymbol{\Omega} = [\boldsymbol{I} - \boldsymbol{J}(\boldsymbol{J}^T \boldsymbol{W} \boldsymbol{J})^{-1}(\boldsymbol{J}^T \boldsymbol{W})] \boldsymbol{W}^{-1} [\boldsymbol{I} - \boldsymbol{J}(\boldsymbol{J}^T \boldsymbol{W} \boldsymbol{J})^{-1}(\boldsymbol{J}^T \boldsymbol{W})]^T. \tag{5}$$

CONTRAINED WLS STATE ESTIMATION FORMULATION

Considering there are hydraulic constraints, high precision measurements and lower and upper bounds for variables, the SE problem presented in Eq. (2) has been amended as follows:

$$\underset{\boldsymbol{x}}{\text{Minimize}} \quad f_{obj}(\boldsymbol{x}) = \tfrac{1}{2} \boldsymbol{\epsilon}^T \boldsymbol{W} \boldsymbol{\epsilon} = \tfrac{1}{2} \left[\boldsymbol{z} - \boldsymbol{g}(\boldsymbol{x})\right]^T \boldsymbol{W} \left[\boldsymbol{z} - \boldsymbol{g}(\boldsymbol{x})\right] \tag{6}$$

subject to

$$\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{0}. \tag{7}$$

$$\boldsymbol{g}(\boldsymbol{x}) \leq \boldsymbol{0}, \tag{8}$$

where equality constraints (7) represent hydraulic constraints and/or high precision measurements, and inequality constraints (8) represent the upper and lower bounds for the state variables. As mentioned previously, problem (6)-(8) can be solved directly using mathematical programming techniques by means of a nonlinear solver, because the mathematical programming solvers which are available at present can work with sparsity, are robust and computationally efficient and provide highly accurate results. This is true for solvers such as CONOPT (Drud, 1996) or MINOS (Murtagh and Saunders, 1998). Furthermore, these solvers enable inequality constraints representing physical limits to be incorporated with ease (Caro et al., 2008). However, since we have focused on assessing uncertainty, we have assumed that the optimal solution to problem (6)-(8) is known and equal to $\hat{\boldsymbol{x}}$.

Quantifying uncertainty means carrying out a local analysis at the optimal solution, thus, once an optimal solution for the SE problem is known, the binding inequality constraints are considered to be equality constraints and non-binding ones are disregarded (Caro et al., 2008), i.e., vector $\boldsymbol{f}(\boldsymbol{x})$ includes p equality constraints and q_Λ active inequality constraints, where Λ is the set of active inequality constraints. Therefore, the first order optimality conditions for problem (6)-(8) at the optimum $\hat{\boldsymbol{x}}$ correspond to:

$$\begin{aligned} &\sum_{i=1}^{m} \nabla_x [\frac{1}{2}\omega_i (z_i - g_i(\hat{\boldsymbol{x}}))^2] + \sum_{i=1}^{p+q_\Lambda} \boldsymbol{\lambda}_i \nabla_x f_i(\hat{\boldsymbol{x}}) = 0 \\ &f_i(\hat{\boldsymbol{x}}) = 0, \; i = 1, \ldots, p + q_\Lambda, \end{aligned} \tag{9}$$

where $\boldsymbol{F} = \nabla_x f(\hat{\boldsymbol{x}})$ is the $(p + q_\Lambda) \times n$ equality constraint Jacobian and $\boldsymbol{\lambda}$ is the $(p + q_\Lambda) \times 1$ Lagrangian multiplier vector associated with the equality constraints in (7)-(8).

If we differentiate the optimality conditions (9) in such a way that the KKT conditions hold (Caro et al., 2011), the following linear system of equations is obtained:

$$\begin{bmatrix} \boldsymbol{J}^T\boldsymbol{W}\boldsymbol{J} & \boldsymbol{F}^T \\ \boldsymbol{F} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \dfrac{\partial \boldsymbol{x}}{\partial \boldsymbol{z}} \\ \dfrac{\partial \boldsymbol{\lambda}}{\partial \boldsymbol{z}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{J}^T\boldsymbol{W} \\ \boldsymbol{0} \end{bmatrix}, \tag{10}$$

where the coefficient matrix of the system is $\boldsymbol{U}$.

As for quantifying the SE uncertainty , note that when i) there are hydraulic constraints or high precision measurements and/or ii) binding upper or lower bounds, these are considered to be part of $\boldsymbol{F}$, and hence $\boldsymbol{W}$ and therefore $\boldsymbol{J}^T\boldsymbol{W}\boldsymbol{J}$ does not necessarily have full rank even if the system is observable. For this reason, the inverse of matrix $\boldsymbol{U}$ must be computed so that the part that establishes the partial derivatives of $\boldsymbol{x}$ with respect to $\boldsymbol{z}$ can be selected. According to Caro et al. (2011), that part would be $\boldsymbol{E}_1$, which is the upper-left quadrant of matrix $\boldsymbol{U}^{-1}$ as shown below:

$$\begin{bmatrix} \dfrac{\partial \boldsymbol{x}}{\partial \boldsymbol{z}} \\ \dfrac{\partial \boldsymbol{\lambda}}{\partial \boldsymbol{z}} \end{bmatrix} = \boldsymbol{U}^{-1} \begin{bmatrix} \boldsymbol{J}^T\boldsymbol{W} \\ \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} \boldsymbol{E}_1 & \boldsymbol{E}_2^T \\ \boldsymbol{E}_2 & \boldsymbol{E}_3 \end{bmatrix} \begin{bmatrix} \boldsymbol{J}^T\boldsymbol{W} \\ \boldsymbol{0} \end{bmatrix}. \tag{11}$$

Therefore, the linear relationship among the differentials becomes:

$$\boldsymbol{dx} = \boldsymbol{E}_1\boldsymbol{J}^T\boldsymbol{W}\boldsymbol{dz} = \boldsymbol{S}_{xz}\boldsymbol{dz}, \tag{12}$$

where $\boldsymbol{S}_{xz}$ represents the sensitivity matrix for the state variables $\boldsymbol{x}$ with respect to the measurements $\boldsymbol{z}$. Hence, the variance-covariance matrix for the state variables $\boldsymbol{C}_x$ can be derived using the FOSM method as follows:

$$\boldsymbol{C}_x = \boldsymbol{S}_{xz}\boldsymbol{W}^{-1}\boldsymbol{S}_{xz}{}^T. \tag{13}$$

With this approach the variance-covariance matrix $\boldsymbol{C}_x$ (Eq. (13)) is equal to that obtained with the traditional WLS method (Eq. (3)) if there are no hydraulic constraints, high precision measurements and binding upper and lower bounds.

Once these computations have been made, the variance-covariance matrix for the other hydraulic variables within the water system ($\boldsymbol{C}_{Q,q}$) could be inferred from $\boldsymbol{C}_x$ using Eq. (4). Concurrently, the residual covariance matrix $\boldsymbol{\Omega}$ can be obtained with this approach according to the general expression set out by Caro et al. (2011):

$$\boldsymbol{\Omega} = (\boldsymbol{I} - \boldsymbol{J}\boldsymbol{S}_{xz})\boldsymbol{W}^{-1}(\boldsymbol{I} - \boldsymbol{J}\boldsymbol{S}_{xz})^T. \tag{14}$$

ILLUSTRATIVE EXAMPLE

The purpose of this illustrative example is to show the difference between quantifying SE uncertainty using the methodology set out herein and the traditional method based on normal equations using just weights. For this reason, the small water network set out by Díaz et al. (2015) has been amended (see Fig. 1) in order to transform nodes 2 and 4 into transit nodes, where water demand is known to be equal to zero ($q_2 = q_4 = 0$) as long as there is no leakage. Additionally, with this example demand pseudomeasurements are considered to be available at nodes 3 and 5 (with the coefficient of variation associated with it being CV$= 0.2$ for both of them), water level readings are available at tanks 1 and 6 (with a measurement accuracy of $\sigma_h = 0.1$ m) and flow meters are available at pipes 1-2, 2-3, 2-5 and 3-4 (with a measurement accuracy of $\sigma_Q = 0.25$ m^3/h). This results in an observable water network, in which the SE uncertainty and system conditioning are to be analyzed. Note that we do not solve the SE problem itself, but we use the network state solution assuming the measurements are error-free. This is because we focus on the effects both approaches have on uncertainty evaluation, as the impact of state estimation has been previously studied by other authors (Caro and Conejo, 2012).

By applying the methodology set out in section 3 to the illustrative example, the results summarized in the first row of Table 1 are obtained, where SE uncertainty has been quantified for both heads (σ_{SE_h}) as well as demands (σ_{SE_q}) at every node. Also, the reciprocal of the condition number estimate of $\boldsymbol{U}$ has been calculated in order to evaluate how sensitive the solution to a system of linear equations is to data errors (0 corresponds to an ill-conditioned system and 1 to a well-conditioned system). These results display consistent uncertainty when compared to the accuracy of the measurement devices and accurately display the demand uncertainty at transit nodes, which is zero as they are specifically considered to be hydraulic constraints. Note that the $\boldsymbol{W}$, $\boldsymbol{J}$, $\boldsymbol{F}$, $\boldsymbol{U}$ and $\boldsymbol{S_{xz}}$ matrices for this example have been collated in Appendix S1.

Furthermore, the traditional unconstrained WLS approach has been implemented by means of the normal equation method used in SE uncertainty quantification. Here, a weight has to be assigned even to hydraulic constraints, as these are not given any special treatment. For a high degree of accuracy to be displayed, their standard deviation ($\sigma_{transit}$) is considered to be a number of orders of magnitude lower than the minimum standard deviation there is assumed to be for the remaining measurements ($\frac{\sigma_{min}}{10^n}$). In this paper, we consider $n = 2, 4, 6$ and 8 to test the sensitivity of the method to the weight assumption for exact measurements, whose results have been collated in Table 1 together with the reciprocal of the condition number estimate of $\boldsymbol{J}^T\boldsymbol{W}\boldsymbol{J}$. The results show that for $n = 2$, the numerical condition of the matrix to be inverted is even better than that of the matrix to be inverted with the constrained WLS method set out, but this comes at the cost of a loss of accuracy in the SE of demand at the transit nodes, whose uncertainty is now 0.0025 m^3/h instead of 0 m^3/h. Note that the accumulative effect of these deviations can be significant when dealing with large network systems. If the weight of error-free measurements is increased by considering $n = 4$, the SE of demand uncertainty associated with it is consequently reduced, but there is a deterioration in the $\boldsymbol{J}^T\boldsymbol{W}\boldsymbol{J}$ condition number. In fact, results show that for $n = 6$ the system is ill-conditioned, which leads to SE uncertainties different from the values obtained with lower weights and to the constrained WLS approach. Finally, if $n = 8$, the condition number attains a value of 0, with which it is not possible to invert $\boldsymbol{J}^T\boldsymbol{W}\boldsymbol{J}$, i.e., it is impossible to quantify SE uncertainty.

These results prove that the normal equation approach is sensitive to the selection of the weights associated with the hydraulic constraints or the high precision measurements of the variables, whereas with the methodology set out herein, this problem can be overcome. Note that the network topology determines conditioning of the system, but the constrained WLS formulation ensures the SE results yielded as well as the subsequent process of quantifying uncertainty is independent.

HANOI NETWORK CASE STUDY

In order to show that this method is still beneficial even with an increase in complexity in topology, Table 2 provides results for the same analysis when applied to the Hanoi network (Fujiwara and Khang, 1990), an example which has been widely used in other works. Note that this network originally has 1 tank, 31 demand nodes and 34 pipes, but demand nodes 3, 16, 23 and 25 have been turned into transit nodes in this study to demonstrate these measurements are exact ones. Regarding the measurement set, it has been assumed that only the tank level has been metered ($\sigma_h = 0.1$ m) and demand is pseudomeasured (CV $= 0.2$). The results given in Table 2 are analogous to those obtained in the illustrative example and bring out the fact that use of the traditional unconstrained WLS approach could lead to non-quantifiable SE uncertainty scenarios for $n \geq 6$. Moreover, it shows how the numerical problem increases with the size of the water distribution system and thereby proves how useful the constrained approach is.

CONCLUSIONS

An alternative treatment to the unconstrained WLS approach for SE in water distribution systems is set out in this paper, in which hydraulic constraints (i.e., null demands at transit nodes, null flows at closed pipes, pumps or valves, etc.), high precision measurements and upper and lower bounds for variables (i.e., head levels at tanks) are consistently included. The method set out herein uses the linear system of equations derived from imposing first order optimality conditions for the constrained SE problem and enables the SE uncertainty of the hydraulic variables and the associated residual covariance matrix to be calculated. These are both useful when assessing the results yielded by the SE problem. Both the illustrative example and the case study given in this paper prove that with the traditional normal equation method, the SE results are sensitive to the weight selected for said hydraulic constraints or high-precision measurements, which on varying could lead to an ill-conditioned system. Therefore, the constrained WLS method set out herein ensures more accurate results for SE uncertainty, without sacrificing the precious information yielded by the hydraulic constraints, high precision measurements or upper and lower bounds within the setting of typically non-redundant or low redundancy water distribution systems.

SUPPLEMENTAL DATA

Appendix S1 is available online in the ASCE Library (www.ascelibrary.org).

REFERENCES

Abur, A. and Expósito, A. G. (2004). *Power System State Estimation: Theory and Implementation*. Marcel Dekker, New York, USA.

Andersen, J. H., Powell, R. S., and Marsh, J. F. (2001). "Constrained state estimation with applications in water distribution network monitoring." *Int. J. of Syst. Sci.*, 32(6), 807–816.

Bargiela, A. and Hainsworth, G. (1989). "Pressure and flow uncertainty in water systems." *J. Water Resour. Plann. Manage.*, 115(2), 212–229.

Bargiela, A. B. (1984). "On-line monitoring of water distribution networks." Ph.D. thesis, University of Durham, United Kingdom.

Brdys, M. A. and Ulanicki, B. (2002). *Operational control of water systems: structures, algorithms and applications*. London, United Kingdom.

Caro, E. and Conejo, A. J. (2012). "State estimation via mathematical programming: a comparison of different estimation algorithms." *IET Generation, Transmission & Distribution*, 6(6), 545–553.

Caro, E., Conejo, A. J., and Mínguez, R. (2008). *Optimization Advances in Electric Power Systems*. Nova Science Publishers, Inc., Chapter A mathematical programming approach to state estimation, 1–26.

Caro, E., Conejo, A. J., and Mínguez, R. (2011). "A sensitivity analysis method to compute the residual covariance matrix." *Electr. Power Syst. Res.*, 81(5), 1071–1078.

Datta, R. S. N. and Sridharan, K. (1994). "Parameter estimation in water-distribution systems by least squares." *J. Water Resour. Plann. Manage.*, 120(4), 405–422.

Díaz, S., González, J., and Mínguez, R. (2015). Observability analysis in water transport networks: algebraic approach. *J. Water Resour. Plann. Manage.*, 142(4), doi: 10.1061/(ASCE)WR.1943-5452.0000621.

Drud, A. (1996). "CONOPT: A system for large scale nonlinear optimization." *Reference manual for CONOPT subroutine library*, ARKI Consulting and Development A/S, Bagsvaerd, Denmark.

Expósito, A. G. and Abur, A. (1998). "Generalized observability analysis and measurement classification." *IEEE Trans. Power Syst.*, 13(3), 1090–1095.

Fujiwara, O. and Khang, D. B. (1990). "A two-phase decomposition method for optimal design of looped water distribution networks." *Water Resour Res*, 26(4), 539–549.

Gómez-Quiles, C., Gil, H. A., De la Villa Jaén, A., and Gómez-Expósito, A. (2013). "Equality-constrained bilinear state estimation." *IEEE Trans. Power Syst.*, 28(2), 902–910.

Kang, D. and Lansey, K. (2009). "Real-time demand estimation and confidence limit analysis for water distribution systems." *J. Hydraul. Eng.*, 135(10), 825–837.

Kapelan, Z. S., Savic, D. A., and Walters, G. A. (2003). "Multiobjective sampling design for water distribution model calibration." *J. Water Resour. Plann. Manage.*, 129(6), 466–479.

Korres, G. N. (2002). "A robust method for equality constrained state estimation." *IEEE Trans. Power Syst.*, 17(2), 305–314.

Murtagh, B. A. and Saunders, M. A. (1998). "MINOS 5.5 User‘s Guide." *Report SOL 83-20R SOL 83-20R*, Department of Operations Research, Stanford University, Stanford, California.

Piller, O. (1995). "Modeling the behavior of a network - hydraulic analysis and sampling procedures for parameter estimation." Ph.D. thesis, University of Bordeaux, France.

Powell, R. S., Irving, M. R., and Sterling, M. J. H. (1988). "A comparison of three real-time state-estimation methods for on-line monitoring of water distribution systems." *Computer Applications in Water Supply*, Vol. 1, 333–348.

Reddy, P. V. N., Sridharan, K., and Rao, P. V. (1996). “´WLS method for parameter estimation in water distribution networks.” *J. Water Resour. Plann. Manage.*, 122(3), 157–164.

Schweppe, F. C. and Wildes, J. (1970). “Power system static state estimation, part I: Exact model.” *IEEE Trans. Power App. Syst.*, PAS-89(1), 120–125.

List of Tables

TABLE 1. SE uncertainty and system conditioning for different formulation scenarios in the illustrative example

Formulation	Node number	σ_{SE_h} (m)	σ_{SE_q} (m^3/h)	Condition of the matrix to be inverted ($\boldsymbol{U}$ or $\boldsymbol{J}^T\boldsymbol{W}\boldsymbol{J}$)
Proposed WLS method	1	0.0726	-	4.0291×10^{-10}
	2	0.0725	0	
	3	0.0717	0.6804	
	4	0.0709	0	
	5	0.0745	1.1586	
	6	0.0726	-	
Traditional WLS $\sigma_{transit} = \frac{\min(\sigma)}{10^2}$	1	0.0726	-	1.0962×10^{-8}
	2	0.0725	0.0025	
	3	0.0717	0.6804	
	4	0.0709	0.0025	
	5	0.0745	1.1586	
	6	0.0726	-	
Traditional WLS $\sigma_{transit} = \frac{\min(\sigma)}{10^4}$	1	0.0726	-	1.0963×10^{-12}
	2	0.0725	0.0000	
	3	0.0717	0.6804	
	4	0.0709	0.0000	
	5	0.0745	1.1586	
	6	0.0726	-	
Traditional WLS $\sigma_{transit} = \frac{\min(\sigma)}{10^6}$	1	0.0769	-	9.7573×10^{-17}
	2	0.0768	0.0000	
	3	0.0758	0.6810	
	4	0.0750	0.0000	
	5	0.0788	1.1618	
	6	0.0762	-	
Traditional WLS $\sigma_{transit} = \frac{\min(\sigma)}{10^8}$	1	-	-	0
	2	-	-	
	3	-	-	
	4	-	-	
	5	-	-	
	6	-	-	

TABLE 2. SE uncertainty and system conditioning for different formulation scenarios in the Hanoi network case study

Formulation	Node number	σ_{SE_h} (m)	σ_{SE_q} (m^3/h)	Condition of the matrix to be inverted ($\boldsymbol{U}$ or $\boldsymbol{J}^T\boldsymbol{W}\boldsymbol{J}$)
Proposed WLS method	3	2.4885	0	7.8871×10^{-8}
	16	4.0557	0	
	23	3.5678	0	
	25	4.2736	0	
Traditional WLS $\sigma_{transit} = \frac{\min(\sigma)}{10^2}$	3	2.4885	0.0600	4.2583×10^{-13}
	16	4.0557	0.0600	
	23	3.5678	0.0600	
	25	4.2736	0.0600	
Traditional WLS $\sigma_{transit} = \frac{\min(\sigma)}{10^4}$	3	2.4730	0.0006	4.3108×10^{-17}
	16	4.0282	0.0006	
	23	3.5437	0.0006	
	25	4.2439	0.0006	
Traditional WLS $\sigma_{transit} = \frac{\min(\sigma)}{10^6}$	3	-	-	0
	16	-	-	
	23	-	-	
	25	-	-	
Traditional WLS $\sigma_{transit} = \frac{\min(\sigma)}{10^8}$	3	-	-	0
	16	-	-	
	23	-	-	
	25	-	-	

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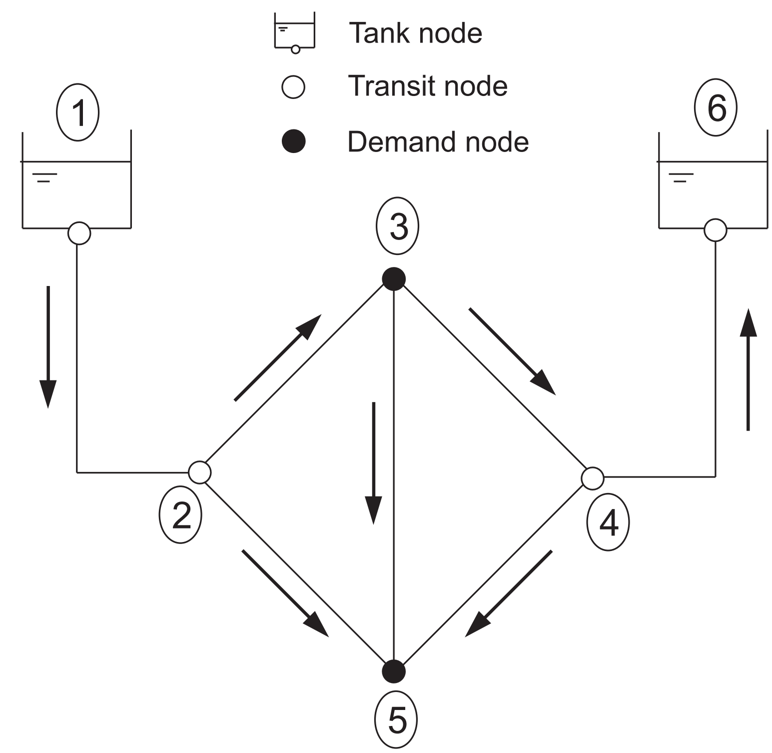

FIG. 1. Illustrative example network (modified from Díaz et al. (2015))