TOPOLOGICAL OBSERVABILITY ANALYSIS IN WATER DISTRIBUTION SYSTEMS

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ABSTRACT

The aim of *observability analysis* (OA) is to determine if a given measurement setting is sufficient to compute the current status of a water distribution network. There are several approaches in the technical literature to making such an analysis. With all of them there is an assumption that the lie of the land of the network in terms of the statuses of its pumps and/or valves is known. In this paper we omit this assumption and introduce the concept of *topological observability analysis* (TOA), which aims to determine not only if it would be possible to compute the hydraulic state of a network from the available measurement set (ordinary OA), but also if the statuses of pumps and valves would be observable as well. Additionally, we propose a method that modifies the standard measurement Jacobian matrix by incorporating either equations and/or unknowns depending on the available information for each specific pump or valve. The rest of the analysis can be undertaken using any of the existing methods for OA in the literature. An illustrative example

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is presented by way of illustration to show the potential TOA has which would certainly enhance on-line monitoring systems.

Keywords: state estimation, topological analysis, optimal meter placement, network monitoring

INTRODUCTION

Nowadays, large water distribution networks are being modernised with the installation of supervisory control and data acquisition (SCADA) systems. This has focused attention on the need for using automatic processing tools that enable the information provided by the available real-time measurements distributed throughout the network to be interpreted. This is what occurs with state estimation techniques, which have been applied on an academic and scientific level in the water industry since the late 70s (Sterling and Bargiela, 1984; Bargiela, 1984; Powell et al., 1988; Andersen et al., 2001; Kumar et al., 2008), but have hardly been implemented in real-life networks on an operational level (Carpentier and Cohen, 1991; Powell, 1992; Preis et al., 2011; Cheng et al., 2014). Essentially, a state estimator is an algorithm that enables flow conditions to be inferred from the hydraulic network equations and the available measurement set (flow meters, head level meters, pressure meters, among others) at any time and location.

In order to guarantee full applicability and effectiveness of state estimation techniques, *observability analysis* (OA) needs to be undertaken first. OA is a strategy that evaluates if the measurement set available is sufficient to compute the current state of the network, enabling the observable variables to be identified, i.e. variables that could be effectively computed based on existing telemetry data (Carpentier and Cohen, 1991). In this respect, there have been several approaches to implementing OA in water distribution systems (Bargiela, 1985; Nagar and Powell, 2004; Díaz et al., 2016a; Díaz et al., 2016c), but in all of them there is an assumption that the lie of the land of the network in terms of the statuses of its pumps and/or valves is known.

In this paper, we omit this assumption and introduce the concept of *topological observability analysis* (TOA), which aims not only to identify if the current status of the network could be computed from the available measurement set in a subsequent state estimation process (ordinary OA), but also if the statuses of its pumps and valves could be inferred as well. The novelty of evaluating observability of pumps and valves answers a necessity in the real operation of water systems: even if the network topology is likely to be known under normal operating conditions, unexpected practical issues (such as changing demand patterns or the occurrence of sudden ruptures) may require the statuses of the pumps and/or valves to be changed in order to maintain serviceability. Therefore, there is a risk of these quick response operations not being recorded in the water system model, which should be conveniently updated once the changes have been identified. Note that the importance of taking into account how changes in the statuses of the pumps and valves can modify the network topology has been discussed before in the context of solving (Giustolisi et al., 2008; Laucelli et al., 2015) and calibrating (Laucelli et al., 2011) flow networks, but this has yet to be integrated into the OA problem. In this regard, TOA would enable assessment of possible locations for additional metering devices for topological purposes, i.e., to ensure that the statuses of some pumps and valves can be inferred from the existing measurements. Note that in TOA, like in OA, only relationships among variables are considered, disregarding the uncertainty effect of the associated measurements.

Therefore, the aim of this paper is twofold: firstly, to introduce the concept of TOA in water distribution networks, and secondly, to present a method that allows the measurement Jacobian matrix to be amended in order to assess observability of how the land lies. More specifically, in this paper observability of pumps and valves is analysed by including equations and/or unknowns in the Jacobian matrix, depending on the information available for each specific pump or valve. The rest of this paper is organised as follows: in the first section, the construction of the standard measurement Jacobian matrix is outlined. Secondly, the amendments required to undertake TOA are presented. Then, an illustrative example is discussed to show the possibilities of using TOA for water distribution networks, as well as its potential for optimal meter placement. Finally, conclusions are duly drawn.

OBSERVABILITY ANALYSIS IN WATER DISTRIBUTION SYSTEMS

The state estimation problem is normally approached by means of the normal equations method (Expósito and Abur, 1998), which enables iterative calculation of the optimal solution ($\hat{\boldsymbol{x}}$) of the original unconstrained weighted least squares approach as:

$$\boldsymbol{\Delta}\hat{\boldsymbol{x}}_{(\nu+1)} = [\mathbf{J}_{(\nu)}^T \boldsymbol{C}_z{}^{-1}\mathbf{J}_{(\nu)}]^{-1}[\mathbf{J}_{(\nu)}^T \boldsymbol{C}_z{}^{-1}](\boldsymbol{z} - \boldsymbol{g}(\hat{\boldsymbol{x}}_{(\nu)})), \tag{1}$$

with $\hat{\boldsymbol{x}}_{(\nu+1)} = \hat{\boldsymbol{x}}_{(\nu)} + \boldsymbol{\Delta}\hat{\boldsymbol{x}}_{(\nu+1)}$. Note that ν is an iteration counter and $\mathbf{J}_{(\nu)} \in \mathbb{R}^{m\times n}$ is the Jacobian measurement matrix at point $\hat{\boldsymbol{x}}_{(\nu)}$ (see Díaz et al. (2016a) for details). This matrix represents the sensitivity of the available measurements $\boldsymbol{z} \in \mathbb{R}^m$ with respect to the state variables vector $\boldsymbol{x} \in \mathbb{R}^n$, which is the minimum set of variables that enables the hydraulic state of the network (considered as nodal heads in this paper) to be calculated, according to the non-linear model $\boldsymbol{g} : \mathbb{R}^n \to \mathbb{R}^m$. $\boldsymbol{C}_z$ is the measurement error vector ($\boldsymbol{\epsilon} = \boldsymbol{z} - \boldsymbol{g}(\boldsymbol{x})$) variance-covariance matrix, typically assumed to be unbiased.

According to Equation 1, a theoretical and sufficient condition for there to be a unique solution to the state estimation problem is for the matrix $\mathbf{J}$ to remain full rank, in which case the system would be observable. Therefore, it is first mandatory to construct the measurement Jacobian matrix adequately, which gathers the first-order partial derivatives of all the variables that could be measured within a water distribution system (in rows) with respect to the state variables (in columns), taken as head levels in this paper. Note that in standard OA, possible measurements within a water network are: 1) head levels, 2) pipe flows and 3) water demands. Therefore, the potential measurements vector is:

$$\boldsymbol{z} = \left(h_i;\ \forall i \in \mathcal{V}, Q_{ij};\ \forall ij \in \mathcal{L}; q_i;\ \forall i \in (\mathcal{V}^{\mathrm{Q}} \cup \mathcal{V}^{\mathrm{T}})\right)^T, \tag{2}$$

where $\mathcal{V}$ and $\mathcal{L}$ represent the set of nodes and pipes that exist in the system respectively, and $\mathcal{V}^{\mathrm{Q}}$ and $\mathcal{V}^{\mathrm{T}}$ specifically refer to demand and transit nodes, as tank nodes can be characterised only by their head level. Note that actual measurements are specifically represented with a tilde and their corresponding rows in the measurement Jacobian matrix are shadowed, as shown in the illustrative example.

The structure of the resulting Jacobian matrix [$\mathbf{J}$] is shown in Fig. 1, where n_{pi} and n_q in

J represent the number of pipes where flow could be metered and the number of nodes where demand can be measured, respectively, and **I** is the identity matrix. Note that the derivatives associated could be calculated according to the expressions in Díaz et al. (2016a). Moreover, the matrix should be normalised to improve its condition number for observability purposes, although its original version must be used if state estimation uncertainty is to be evaluated as in Bargiela and Hainsworth (1989) or Díaz et al. (2016b).

Then, any existing method for OA could be applied to evaluate observability of the system, either by evaluating the rank of the available measurement Jacobian matrix, which can be built by just selecting the rows of metered variables and all columns, or the full Jacobian matrix. Note that in the first type of methods, like the null space method presented in Castillo et al. (2005), only observability of the state variables is evaluated, whereas if the relationships contained in the full matrix are considered (like the algebraic approach in (Díaz et al., 2016a)), the observability of every single variable can be assessed.

TOPOLOGICAL OBSERVABILITY ANALYSIS IN WATER DISTRIBUTION SYSTEMS

The statuses of pumps and valves in water distribution systems change over time in order to adapt the network performance to the varying patterns in demand. For this reason, OA must move forward so as to assess observability of the network topology in terms of the statuses of its pumps and/or valves. Thus, TOA would constitute a powerful tool to identify unnotified changes in operation or abnormal operating conditions.

To account for the existence of pumps and valves in OA, the structure of the Jacobian matrix presented in Fig. 1 must be amended. In this respect, three scenarios must be differentiated depending on the information available for each specific pump or valve, which are here treated as link elements:

- **The pump or valve characteristic curve or setting is known, and the device is known to be working**. In this case, the standard measurement Jacobian matrix is not amended, and observability of the whole system (including pump and/or valve elements) could be evaluated using any existing OA technique.

However, if, as mentioned before, a more sophisticated technique is used to assess observability of each of the variables, an equation should be added to represent the existing relationship between the flow through the pump/valve and its head levels at both end nodes, i.e. its characteristic curve. In this case, the derivative $\frac{\partial Q}{\partial h_i}$ could be computed easily, as all the parameters that define the pump or valve operations have been characterised. For example, if there is a generic pump with characteristic curve $h_j - h_i = \Delta h = AQp_{ij}^2 + B|Qp_{ij}| + C$, where Qp_{ij} refers to the flow through the pump from node i to node j, the associated derivatives could be written as:

$$\frac{\partial Qp_{ij}}{\partial h_i} = \frac{-1}{2A|Qp_{ij}| + B} \tag{3}$$

$$\frac{\partial Qp_{ij}}{\partial h_j} = \frac{1}{2A|Qp_{ij}| + B}, \tag{4}$$

where i and j represent the initial and final nodes, respectively. Note that these expressions would become -1 and 1 when the Jacobian matrix is to be normalised. Similarly, additional expressions can be derived for particular flow controlling devices, such as pressure reducing or sustaining valves, general purpose valves or ordinary gate valves.

- **The pump or valve is known to be closed**. In this case, there is an additional state variable that refers to the flow through the pump or valve (Qp_{ij} or Qv_{ij}), but there is also direct measurement of the water flow through the device: $\tilde{Q}p_{ij} = 0$ or $\tilde{Q}v_{ij} = 0$. Therefore, a flow state variable column has to be added for each closed element, together with a flow measurement row, which should be incorporated into the measurement set available. Note that this row and column show zero values for all their positions except when they cross, where a value of 1 must be placed (identity matrix).
- **The pump or valve status is unknown**. Also in this case, an additional state variable column that refers to the flow through the pump or valve (Qp_{ij} or Qv_{ij}) must be added. However, in this scenario there is no information about how the pump or valve is operating,

thus no additional measurement is possible. Therefore, unknowns are added to the measurement Jacobian matrix but equations are not, and more metering devices are necessary to obtain observability for the system.

It must be emphasized that it is only when a pump or valve is known to be closed that information is added to the available measurement set. In any other case, the measurement set available corresponds exactly to the metering devices there are within the water distribution system, and the number of additional state variables will depend on whether there is information available for each device or not. This idea is summarised in Fig. 2, where n_{pu} pumps (Qp_{ij}) and n_v valves (Qv_{ij}) have been incorporated into the standard measurement Jacobian matrix presented in Fig. 1, and measurements available thanks to the network topology, i.e. closed pumps and/or valves, are shaded in light grey. Note that in this amended matrix there are several empty sets ($\mathbf{\Omega}$) in those positions where there is no relationship between the measurement and either head levels or flow state variables, depending on the information available for each pump and/or valve. For this reason, water demand measurements q can be related to either the h or Q state variables depending on the information available.

ILLUSTRATIVE EXAMPLE

An example is presented below to illustrate the potential TOA has. For this purpose, the illustrative network proposed by Díaz et al. (2016a) has been amended (see Fig. 3) to incorporate the presence of a gate valve (link 2-7) and a pump (link 6-8) at strategic locations. For this to happen, elevation at node 6 has been reduced to zero and two transit nodes (7 and 8) have been added to include both devices as link elements (Appendix S1 contains detailed characteristics of this example).

In this network, two scenarios are considered to explain the potential TOA has. All water demands and tank levels are metered in both of them, so achieving full observability actually depends on the information available for the pump and valve and if there are any extra meters. The following cases are analysed: 1) the pump is known to be working with a characteristic curve given by parameters $A = -2.2204 \cdot 10^{-16}$, $B = -0.3126$ and $C = 125.2806$, the gate valve is known to be open and the energy loss condition at the valve is assumed to be known, and 2) there is no information about the statuses of the pumps and valves. Subsequently, the associated modified Jacobian matrix will be constructed for both scenarios, after which TOA for the system will be undertaken according to the null space method (Castillo et al., 2005) and the algebraic approach proposed by Díaz et al. (2016a), respectively.

Case 1: Known pump and valve statuses

For this first scenario the null space approach proposed by Castillo et al. (2005) is used. Therefore, only the Jacobian matrix associated with the measurements available is built, as shown in Fig. 4. Note that all rows are shaded in light grey because the matrix is only made up of available measurements (also marked with a tilde), which in this case correspond to water levels at tank nodes and water demands for the remaining nodes. As can be seen in this figure, eight state variables exist, as there are eight nodes in the illustrative network and the pump characteristic curve and gate valve status are known. Note that the signs correspond to a sign criteria in which flow is considered to be positive whenever the water moves from the low numbering node i, to the high numbering node j.

In order to carry out TOA, the null space of the measurement Jacobian matrix available must be computed. In this case, the associated null space is an empty set, i.e., the system is fully observable.

Case 2: Unknown pump and valve statuses

In this scenario, the algebraic approach proposed by Díaz et al. (2016a) is used, so the full measurement Jacobian matrix is computed. The normalised Jacobian matrix for this scenario is shown in Fig. 5, where measurement rows available are shaded in grey and there are two additional flow state variable columns due to the lack of information about the pump and valve. In this case, the resulting system is unobservable, as there are ten unknowns and only eight measurements available. Note that all the measurements in the transformed Jacobian shown in Fig. 6 have been transferred to columns by pivoting their corresponding rows, however, there are still two columns associated with valve and pump flows (Qv_{2-7}, Qp_{6-8}) which could not be replaced by additional measurements. Therefore, all row variables containing non-null elements in these two columns are unobservable variables. The advantage of using the aforementioned algebraic procedure used here is that the resulting transformed matrix can be useful to identify locations where additional measurement devices could be added in order to attain observability for the whole system.

This matrix shows how all measurements available have been pivoted throughout the process from rows to columns, but still more information is required to estimate the statuses of the pumps and valves. Contrary to the null space approach, with the transformed matrix it can be seen that in order to guarantee observability of the pump status, it would be necessary to incorporate a flow meter at any location except pipe 3-7, where a null element exists in the corresponding position, or an additional pressure meter at any node. However, if both pump and valve observability must be achieved, two additional metering devices should be added. More specifically, these instruments would need to have an associated invertible matrix in columns Qp_{6-8} and Qv_{2-7}. For example, the addition of flow meters Q_{1-2} and Q_{3-5} would lead to a fully observable system, and so would the incorporation of Q_{2-5} and Q_{3-4}, but not Q_{1-2} and Q_{4-8}. Note that if, for example, new information was received by the telemetry system about pump 6-8 being closed, row Qp_{6-8} would become an available measurement, thus the transformed matrix could be amended by pivoting this row additionally .

Therefore, TOA has the potential to, at any time, consider other assumptions apart from knowing the statuses of the pumps or valves, as is likely to be the case in large real on-line monitored water distribution systems. Nevertheless, it must be stressed that TOA (as well as OA), is only capable of evaluating if there are enough relationships to calculate the state variables of the system from the measurements available, but as data obtained by telemetry data is prone to errors, this could eventually lead to an incorrect status determination within any state estimation algorithm. This is a subject for further research.

CONCLUSIONS

In this paper the concept of TOA in water distribution networks is presented, which enables not just observability of the hydraulic state of the network to be analysed, but also assessment of whether the network topology in terms of the statuses of its pump and/or valves would be observable. Therefore, this approach constitutes a breakthrough with respect to existing OA techniques, where the network topology is always assumed to be known. The method presented herein is based on a slight amendment of the standard measurement Jacobian matrix to account for an analysis of the statuses of the pumps and/or valves.

The illustrative example presented in this paper shows how simple it is to include topological considerations in the measurement Jacobian matrix. Moreover, the strategy set out enables use of any of the numerical methods there are for observability analysis based on the manipulation of this matrix. The methodology shown here enables identification of in which locations unnotified changes in operating conditions could be potentially detected.

It is worth stressing that although this method only enables observability to be checked with the hypothesis of error-free measurements, this is a necessary step before attempting to make any topological state estimation with noisy measurements because: (1) unobservable elements with error-free measurements would remain unobservable with noisy measurements; and (2) observable elements with error-free measurements would or would not remain observable depending on the measurement distribution and uncertainty. This is a subject for further research.

SUPPLEMENTAL DATA

Appendix S1 is available online in the ASCE Library (www.ascelibrary.org).

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