OBSERVABILITY ANALYSIS IN WATER TRANSPORT NETWORKS: AN ALGEBRAIC APPROACH

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ABSTRACT

Nowadays, *state estimation* (SE) techniques are applied to different network systems in order to convert system measurements into real information about the network state. SE applications to water systems are relatively novel, but these techniques have been implemented in other fields for decades. In those applications, *observability analysis* (OA) is required prior to application of SE techniques with different purposes: i) to identify redundant information, ii) to detect elements that make no contribution in the subsequent SE process or iii) to identify observable islands. However, no discussion has been found in the pertinent literature as regards any interest in applying OA to water networks, with there being only a few basic applications. The aim of this paper is twofold: firstly, to present the implementation of a novel algebraic OA approach to water networks, which is based on the application of a Gauss elimination technique to the measurement Jacobian matrix, and to discuss and justify the interesting aspects of implementing an OA in Water Transport Networks (WTN) prior to using SE whilst also presenting the issues that this technique may resolve. The

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results obtained highlight the algorithm potential for real supply systems, improving the knowledge of what information provided by SCADA systems is really worth compiling.

Keywords: State estimation, network monitoring, optimal meter placement, observable islands

INTRODUCTION

Water supply is nowadays moving forward as there is an attempt to improve serviceability. The first step to reach this goal is computing service quality indicators (Abdelbaki et al., 2014) and assessing how the network performs (Cabrera et al., 1999; Chae, 2012). These tasks require adequate knowledge of how the network behaves and its hydraulic status under different flow circumstances. With this in mind, and also with the purpose of supporting the decision-making process in water systems, there is an actual trend to merge comprehensive Information Communication Technology (ICT) programs, usually made up of SCADA systems, Geographic Information Systems (GIS), and Hydraulic Modelling Systems (HMS). This integrated platform is intended to improve the efficiency of network operations and asset maintenance, for which SE techniques are adopted as an effective way to process the information gathered by SCADA systems.

SE techniques were conceived in the 70s with the aim of characterizing the electric state of complex power systems (Schweppe and Wildes, 1970) and were implemented in the water industry shortly afterwards (Coulbeck, 1977). Generally speaking, a state estimator is an algorithm that computes the current state of a system through the combination of the information provided by on-line measurements and network flow equations. However, for any state estimator to function correctly, the measurement set should at least provide estimation of the *state variables*, which is the minimal set of variables that allows the status of the network to be fully characterized. In this regard, the first issue is: is any configuration of measurement devices valid to fully characterize the hydraulic state of the network? The answer is **no**: the measurement set must ensure that all variables within the system can be infered from the system equations, i.e., the system must be observable. This explains, in general, the necessity of carrying out OA before using SE.

However, the necessity of OA in water systems has been overlooked over the years. This is because telemetry data has been typically complemented by predictions of consumption, which are referred to as *pseudo-measurements*, to make up for the lack of measurement devices. In this respect, both measurements and pseudo-measurements are plagued with uncertainty that may lead to deviations in the SE process, but this is particularly important for pseudo-measurements, which can vary largely since they are just estimations based on existing data (Walski, 1983). To tackle this problem, great effort has been made to characterize the uncertainty associated with these estimations and their effect in the overall SE process (Bargiela and Hainsworth, 1989), as well as to implement online estimation of demand so as to carry out the subsequent SE efficiently (Kang and Lansey, 2009; Preis et al., 2011; Okeya et al., 2014). Note that if pseudo-measurements together with tank levels are considered to be the available measurements, the system of equations to solve the water flow through the network is a compatible system and determined with a unique solution (the number of equations is equal to the number of unknowns), i.e. the water system would always be observable. Nevertheless, the use of pseudo-measurements as a substitute for real water demand increases the uncertainty of SE (Nagar and Powell, 2004), thereby reducing the possibility of detecting changes in the network behavior and effectively monitoring the system.

In this paper, we drop this classical assumption by initially removing pseudo-measurements and focusing on Water Transport Networks (WTN). WTN have a low number of demand points related to District Metered Area (DMA) consumption, which are typically measured to control the flow into each sector. This is crucial for the management of large systems (Tzatchkov et al., 2006) and makes it possible to avoid the use of pseudo-measurements by installing metering devices in appropriate locations. In this regard, OA permits information to be obtained about the minimum number and location of alternative measurement devices to achieve or, at least, enhance observability without making use of pseudo-measurements. Therefore, this strategy reduces the uncertainty factor for SE and permits testing of how the possible loss of one or several measurements (due to sensor failure, communication failure, etc.) affects observability of the WTN.

There are additional reasons to make use of OA. SE procedures use the relationships among variables due to the network topology and the flow equations governing the water movement throughout the network, hence they permit estimates of variables to be obtained which are not directly measured. OA is a previous analysis of which variables are observable from the available measurement set which is monitored by the telemetry system, thereby enabling those regions of the system where SE would provide reliable results to be identified. Moreover, OA is especially required if iterative methods based on least-squares are used, because those methods only work for observable systems, i.e. if any of the state variables are not observable according to the measurement configuration, then it is not possible to obtain the estimate of the system (Abur and Expósito, 2004). The problem is even more critical if mathematical programming or heuristic techniques, such as genetic algorithms, are used for minimizing the SE errors, because those procedures provide a solution for the SE problem even when the system might be unobservable and this might go unnoticed. For this reason, OA is quite established in power systems, where sensor placement problems are to be dealt with while conceiving and operating the network.

Another important issue discussed in the pertinent technical literature is uncertainty does not just depend on the number and accuracy of the meters installed, but also on their distribution throughout a network (Bargiela and Hainsworth, 1989; Kang and Lansey, 2009, 2010). This research led to several studies that presented optimal meter placement schemes in water systems (Yu and Powell, 1994; Kang and Lansey, 2010), which followed the same lines of research as in electric power networks (Clements, 1990; Ramesh et al., 2007). Starting from the work by Walski (1983), who was amongst the first to directly address the issues of the sampling design in the context of model calibration for water distribution systems (Kapelan et al., 2003), different criteria have been tested, such as those based on the quantification of calibration uncertainty (variance reduction methods) such as alphabetic optimality criteria (D-optimality, A-optimality, V-optimality) as discussed by Kiefer and Wolfowitz (1959) or Savic et al. (2009), among others. These criteria would be directly applicable for the optimal location of sensors for state estimation. However, in this paper we present OA as a tool that provides information for the selection of sensor locations based on the increased resilience of the system in the face of the loss of one or several measurements, i.e., ensuring that the system is robust enough to remain fully or highly observable regardless of the loss of any measurement. Note that there is another research trend for optimal location of sensors associated with detection of contamination events for which this method is not directly applicable. Its application would require the equations and variables governing the evolution of contamination within the system to be adapted, which is beyond the scope of this paper.

In summary, implementing OA as a previous and complementary step to SE in WTN answers the following questions: i) whether any set of measurements is enough to appropriately carry out SE, ii) how robust is that measurement set in the face of the potential loss of measurements, iii) which variables are observable and unobservable, iv) which pseudo-measurements are required to fulfill the observability condition , and v) how to locate new sensors in order to increase resilience against the loss of one or several assets.

Regarding OA techniques, these have been deeply explored in power systems but the only condition studied in order for the system to be observable in water networks is that the measurement Jacobian matrix is full rank (Nagar and Powell, 2004; Vale and Schenzer, 2014). This approach provides a yes or no answer for observability checking, and should be applied to every possible subset of measurements to be considered within the system. It is the most basic method, but unsuitable for medium-large networks. Therefore, it is worth exploring how other but more efficient existing OA methodologies can be of application to water supply networks. In this regard, there have been three different approaches, essentially, for addressing observability problems in power systems: graphical (or topological) methods, numerical (or algebraic) methods and hybrid combinations. Topological methods (Krumpholz et al., 1980; Clements et al., 1982; Quintana et al., 1982; Nucera and Gilles, 1991) are associated with topological algorithms based on building a spanning tree of full rank and generally involve combinatorial computational complexity. They have been applied to water systems by Carpentier and Cohen (1991). Algebraic alternatives have not been applied to water systems in the available literature so far, but they have been applied systematically in the power supply field. They make use of either the gain matrix (Monticelli and Wu, 1985a,b; Gou and Abur, 2000, 2001) or the measurement Jacobian matrix of the system (Exposito and Abur, 1998; Gou, 2006; Castillo et al., 2005, 2006, 2007; Solares et al., 2009; Pruneda et al., 2010), which they factorize or transform to extract observability information. Some authors have adopted hybrid techniques (Contaxis and Korres, 1988; Korres and Katsikas, 2003), and other alternative approaches based on mathematical programming techniques have been briefly explored (Habiballah and Irving, 2001; Caro et al., 2013). Of all the available contributions in the technical literature, the algebraic proposal by Pruneda et al. (2010) is especially suitable for water networks due to the possibility of simultaneously analyzing the observability of a set of available measurements and the remaining potential measurements in the system. This approach starts from the full Jacobian matrix of possible measurements within the network and transfers columns to rows using a Gauss-based elimination technique to progressively express state variables as functions of available measurements. It basically analyzes how the incorporation of any measurement affects the observability of both state and network variables. Therefore, the algorithm allows to check observability for the given subset, but also to identify critical and redundant measurements, thereby enabling identification of observable variables and islands if the system is not fully observable.

For the aforementioned reasons, the aim of this paper is twofold. Firstly, to adapt and implement the algebraic OA procedure previously developed by Pruneda et al. (2010) to water systems and, secondly, to evaluate and discuss the advantages and usefulness of OA when applied to WTN. This approach permits observability information of the full system to be extracted by analyzing any subset of available measurements, thereby avoiding repetitive calculations. Moreover, this proposal provides information about the existing control points and other potential measurements that might substitute or reinforce them and so helps to identify optimum locations for the installation of future devices. Furthermore, a robust methodology is put forward in order to prioritize sensor investment within the network management policy, either in cities with existing but poorly metered SCADA systems or where these platforms are to be installed from scratch.

The rest of the paper is organized as follows: in the first section an overview of the SE and OA problems is set out. Then, the structure of the measurement Jacobian matrix of the system for water networks is explored. Note that this matrix is the starting point for application of the OA method. The algorithm for OA is outlined in the following section, including the process for detection of observable islands. Subsequently, an illustrative example is presented to explore in detail what possible applications the methodology offers, followed by a discussion on how the developed methodology could be applied to real WTN. Finally, relevant conclusions are duly drawn.

STATE ESTIMATION AND OBSERVABILITY ANALYSIS: A GENERAL OVERVIEW

As previously mentioned, in general, an algorithm for SE must provide the most likely state of the network given a series of available measurements at a given instant in time. It is like taking an instantaneous snapshot of the network status, i.e. *pseudo-static* state estimation, allowing to calculate the state variables from the measurement set. Let us consider the vector of measurements $\boldsymbol{z} \in \mathbb{R}^m$ including pressures at nodes, tank levels, pipe flows and DMA consumptions, the vector of state variables $\boldsymbol{x} \in \mathbb{R}^n$ including nodal heads and the nonlinear relationship $\boldsymbol{g} : \mathbb{R}^n \rightarrow \mathbb{R}^m$ between measurements and state variables for a certain system, which results from the application of the mass and energy conservation equations. Thus, this relationship can be mathematically written as:

$$\boldsymbol{z} = \boldsymbol{g}(\boldsymbol{x}) + \boldsymbol{\epsilon}, \tag{1}$$

which represents a system of nonlinear equations, where $\boldsymbol{\epsilon}$ are the errors associated with measurements. These errors are traditionally assumed to be gaussian with zero mean, i.e. unbiased $E[\boldsymbol{\epsilon}] = \mathbf{0}$, and variance-covariance matrix $\boldsymbol{R}$.

SE consists in finding the most likely values of the state variables $\boldsymbol{x}$ by solving the following Weighted Least Squares (WLS) problem:

$$\underset{\boldsymbol{x}}{\text{Minimum}} \quad F(\boldsymbol{x}) = \boldsymbol{\epsilon}^T \mathbf{R}^{-1} \boldsymbol{\epsilon} = \left[\boldsymbol{z} - \boldsymbol{g}(\boldsymbol{x})\right]^T \mathbf{R}^{-1} \left[\boldsymbol{z} - \boldsymbol{g}(\boldsymbol{x})\right] \ , \tag{2}$$

where $\hat{\boldsymbol{x}}$ corresponds to the optimal solution of problem (2). Note that errors are multiplied by the inverse of the variance-covariance matrix associated with error measurements, and since they are usually independent, it is a diagonal matrix. Therefore, the objective function attempts to minimize the sum of square errors defined by equation (1), giving more credibility to those measurements with lower standard deviation errors.

Problem (2) can be solved using the normal equations method (Exposito and Abur, 1998), which allows calculating the optimal solution of state variables at iteration $\nu + 1$ by iteratively solving the following linear system of equations:

$$[\mathbf{J}_{(\nu)}^T \mathbf{R}^{-1} \mathbf{J}_{(\nu)}] \hat{\boldsymbol{x}}_{(\nu+1)} = [\mathbf{J}_{(\nu)}^T \mathbf{R}^{-1}] (\boldsymbol{z} - \boldsymbol{g}(\hat{\boldsymbol{x}}_{(\nu)})), \tag{3}$$

where $\mathbf{J}_{(\nu)} \in \mathbb{R}^{m \times n}$ is the Jacobian measurement matrix at point $\hat{\boldsymbol{x}}_{(\nu)}$, and ν is an iteration counter.

According to (3), a theoretical and sufficient condition for the existence of a unique solution for the SE problem (2) is that the system is determined compatible, i.e. $\mathbf{J}$ matrix has full rank n. As mentioned before, this minimum condition is usually satisfied in water systems at the expense of considering estimations as actual measurements to overcome the scarcity of measurement devices. However, this strategy is often very poor and may lead to unrealistic results if uncertainties in both network parameters and measurements are taken into account (Nagar and Powell, 2004). Note that in this paper we refer to the SE problem as the one considering uncertainties in measurements, while the problem including also network parameter uncertainties is called *calibration* and is out of the scope of this work.

The full rank jacobian condition, which makes matrix $[\mathbf{J}_{(\nu)}^T \mathbf{R}^{-1} \mathbf{J}_{(\nu)}]$ invertible, identifies the system as observable or unobservable. However, besides this condition, OA has received very little attention in water systems. The measurement Jacobian matrix plays a crucial role for the system to be observable. Besides, the matrix maintains the structural relationships among measurements and state variables even if the equation (1) is linearized around any point $\boldsymbol{x}_0$:

$$\Delta \boldsymbol{z} = \mathbf{J}_0 \Delta \boldsymbol{x} + \Delta \epsilon \tag{4}$$

where $\Delta \boldsymbol{z} = \boldsymbol{z} - g(\boldsymbol{x}_0)$ is the measurement residual vector, $\Delta \boldsymbol{x}$ is the incremental change in the system state and $\Delta \epsilon$ corresponds to the incremental change in errors.

The information about the relationships among measurements and other variables for OA purposes is gathered in the measurement Jacobian matrix at any given flow state. Thus, this analysis is independent with respect to the uncertainty associated with measurements, demands and network parameters, since it is only based on the relationships among variables due to the network topology. For this reason, it is customary to now define the state variables and how the measurement Jacobian matrix can be calculated for any given flow status of the network $\boldsymbol{x}_0$.

MEASUREMENT JACOBIAN MATRIX IN WATER NETWORKS

In general, the Jacobian matrix of a system is the matrix of all first-order partial derivatives of a vector-valued function. Therefore, for the particular case of water networks, the Jacobian matrix is a way of rewriting the system's governing flow equations and grouping them according to its state variables, with respect to which partial derivatives are computed.

Any water network can be represented as a network $\mathcal{N} = (\mathcal{V}, \mathcal{L})$, formed by a set of vertex or nodes ($\mathcal{V}$) interconnected by a group of links ($\mathcal{L}$). Particularly, nodes can be divided in demand/source nodes ($\mathcal{V}^{\mathrm{Q}}$, where water is either subtracted or introduced in the system), transit nodes ($\mathcal{V}^{\mathrm{T}}$, where flow neither leaves nor enters the system), tank nodes ($\mathcal{V}^{\mathrm{R}}$, tanks or reservoirs where change in volume is significant) or reservoir nodes ($\mathcal{V}^{\mathrm{R}\infty}$, where change in volume is negligible and the volume can be considered as infinite). Distinction between different types of nodes is important to model their behavior through the convenient equations, as shown later. Flow direction within pipes is assumed positive whenever water moves from lower to higher numbering node. Therefore, two subsets Ω_i^{O} and Ω_i^{I} are defined for each node i corresponding, respectively, to water outflows from node i to the rest of nodes with numeration $j > i$ and connected to i through a pipe, and water inflows to node i from the rest of nodes with numeration $j < i$ and connected to i through a pipe.

Once the basic definitions of the hydraulic model network have been set up, the Jacobian matrix computation process can be explained. It requires the selection of the set of state variables, the specific network model definition and the organization of this information within a matrix which considers all possible measurements in the system. It is important to point out, as previously mentioned, that in this work we propose a pseudo-static approach that considers flow as steady, hence subsequent times can be analyzed as if they were independent. For this reason we consider equations independent of time t.

Selection of the state variables

In general, the hydraulic variables involved in water networks at a given instant in time are the water flow through each pipe ($Q_{ij};\ \forall ij \in \mathcal{L}$), the pressure at each node ($p_i;\ \forall i \in \mathcal{V}$), the head level associated with each of the nodes ($h_i;\ \forall i \in \mathcal{V}$), and the water demand and/or provision to the system in each node ($q_i;\ \forall i \in \mathcal{V}^{\mathrm{Q}}$), which is positive for source, negative for demand and null for transit nodes. The rest of parameters required to define the status of the system, such as node elevations ($e_i;\ \forall i \in \mathcal{V}$), pipe lengths ($L_{ij};\ \forall ij \in \mathcal{L}$) and diameters ($D_{ij};\ \forall ij \in \mathcal{L}$), and roughness coefficients ($r_{ij};\ \forall ij \in \mathcal{L}$), are assumed to be known within SE and OA problems.

According to Brdys and Ulanicki (2002), a set of state variables is a minimal set of variables whose values are sufficient to compute, by using the network model, the value of any other network variable. Therefore, selection of the state variables is not unique (Andersen and Powell, 2000). For instance, Brdys and Ulanicki (2002) select as state variables all nodal heads $h_i;\ \forall i \in \mathcal{V}$. In contrast, Nagar and Powell (2004) select as state variables nodal heads at non reservoir nodes $h_i;\ \forall i \in \left(\mathcal{V}^{\mathrm{Q}} \cup \mathcal{V}^{\mathrm{T}}\right)$. We have selected as state variables all nodal heads $h_i;\ \forall i \in \mathcal{V}$ (including reservoir heads) for three reasons: i) any combination of nodal heads leads to a certain and credible flow solution (which may not happen if considering pipe flows as state variables), ii) it also facilitates observable islands detection, as shown later, and iii) it allows the consideration of error measurements in reservoir or tank water levels. According to this selection, there are as many state variables as the number of nodes in the network (n).

The network model: relationships among measurements and state variables

Equation (1) states that there is a functional relationship among measurements and state variables. This nonlinear relationship is derived from the network's hydraulic model, i.e. from the application of mass conservation to all non-reservoir junctions ($\forall i \notin \mathcal{V}^{\mathrm{R}} \cup \mathcal{V}^{\mathrm{R}\infty}$) and branch flow-head characteristics within pipes (Brdys and Ulanicki, 2002).

In this work, we consider that there are three type of measurements: i) nodal heads (assuming that the node elevation is known and pressure or water levels can be measured, respectively, by means of piezometers and water level sensors), ii) pipe flows , and iii) demands, thus the vector including all possible measurements in the network corresponds to:

$$\boldsymbol{z} = \left(\tilde{h}_i;\ \forall i \in \mathcal{V}, \tilde{Q}_{ij};\ \forall ij \in \mathcal{L}, \tilde{q}_i;\ \forall i \in (\mathcal{V}^{\mathrm{Q}} \cup \mathcal{V}^{\mathrm{T}})\right)^T. \tag{5}$$

Note that the tilde refers to measurements, which can be either associated with readings from a metering device or pseudo-measurements, as those different types of measurements are equivalent from the OA perspective. The relationship between measurements and state variables based on the network model is explicitly gathered in Supplemental Data, section A.

The measurement Jacobian matrix

The measurement Jacobian matrix includes the first-order partial derivatives of all the variables that can be measured in the system with respect to the nodal heads, i.e. state variables. Therefore, the Jacobian matrix contains as many columns as the number of nodes n, each of which is associated with a nodal head $h_i;\ \forall i \in \mathcal{V}$, and as many rows as the total number of measurements (m) that can be metered within the system, represented by the vector given in (5). The structure of the

Jacobian matrix for a generic water network is as follows:

$$
\mathbf{J} = \begin{array}{c} \\ \tilde{h}_1 \\ \vdots \\ \tilde{h}_n \\ \hdashline \tilde{Q}_1 \\ \vdots \\ \tilde{Q}_{n_p} \\ \hdashline \tilde{q}_1 \\ \vdots \\ \tilde{q}_{n_q} \end{array}
\begin{array}{c} \begin{array}{ccccc} h_1 & \dots & h_i & ... & h_n \end{array} \\
\left[\begin{array}{ccccc}
\frac{\partial \tilde{h}_1}{\partial h_1} & \dots & \frac{\partial \tilde{h}_1}{\partial h_i} & \dots & \frac{\partial \tilde{h}_1}{\partial h_n} \\
\vdots & \dots & \vdots & \dots & \vdots \\
\frac{\partial \tilde{h}_n}{\partial h_1} & \dots & \frac{\partial \tilde{h}_n}{\partial h_i} & \dots & \frac{\partial \tilde{h}_n}{\partial h_n} \\
\hdashline
\frac{\partial \tilde{Q}_1}{\partial h_1} & \dots & \frac{\partial \tilde{Q}_1}{\partial h_i} & \dots & \frac{\partial \tilde{Q}_1}{\partial h_n} \\
\vdots & \dots & \vdots & \dots & \vdots \\
\frac{\partial \tilde{Q}_{n_p}}{\partial h_1} & \dots & \frac{\partial \tilde{Q}_{n_p}}{\partial h_i} & \dots & \frac{\partial \tilde{Q}_{n_p}}{\partial h_n} \\
\hdashline
\frac{\partial \tilde{q}_1}{\partial h_1} & \dots & \frac{\partial \tilde{q}_1}{\partial h_i} & \dots & \frac{\partial \tilde{q}_1}{\partial h_n} \\
\vdots & \dots & \vdots & \dots & \vdots \\
\frac{\partial \tilde{q}_{n_q}}{\partial h_1} & \dots & \frac{\partial \tilde{q}_{n_q}}{\partial h_i} & \dots & \frac{\partial \tilde{q}_{n_q}}{\partial h_n}
\end{array}\right] \end{array} \tag{6}
$$

where n_p and n_q represent, respectively, the number of pipes where flow can be measured and the number of nodes where demands can be metered and/or estimated. Explicit expressions required to build the Jacobian matrix can be found in Supplemental Data, section B.

Let us remind the reader that in order to apply the proposed technique, a numerical instance of the Jacobian matrix $\mathbf{J}_0$ is required to particularize (6) for any likely and realistic physical status of the system $\boldsymbol{x}_0$, with the additional condition of avoiding null flows within pipes. This situation induces the mathematical indetermination $\frac{1}{0}$ in expressions related to $\dfrac{\partial \tilde{Q}_{ij}}{\partial h_k}$ (see Supplemental Data, section B), which produces numerical ill-conditioning of the Jacobian matrix. Besides, since the aim of this work is to focus on OA, it is also possible to perform the normalization of each row by dividing all its elements by its corresponding maximum absolute value. This strategy reduces numerical errors derived of the application of the observability algorithm. It should be noted that the use of an algebraic method analyzes not only topological but also numerical observability. Nevertheless, provided that the pipe parameters and the reference network status is realistic, it is unlikely to detect unobservable numerical systems that are, at the same time, topologically observable.

ALGEBRAIC OBSERVABILITY ANALYSIS

The algorithm used in this paper to undertake water networks observability analysis is an adaptation of the one proposed by Pruneda et al. (2010) for power systems. In the next subsections, we focus in both the OA algorithm itself and the method for island identification.

Algorithm for observability analysis

The proposed methodology starts from the computed and normalized measurement Jacobian matrix $\mathbf{J}_0$. However, this proposal requires the reorganization of the Jacobian matrix rows to place in first position those associated with the subset of available measurements in the network ($\mathbf{J}_\mathrm{a}$ of size $m_a \times n$), which are those available for observability purposes. Then, the rest of candidate measurements within the system are included, i.e. ($\mathbf{J}_\mathrm{c}$ of size $m_c \times n$), which are those not available but accessible at a certain cost. Therefore, the structure of the reorganized Jacobian matrix becomes $\mathbf{W} = \begin{bmatrix} \mathbf{J}_\mathrm{a} \\ \mathbf{J}_\mathrm{c} \end{bmatrix}$. Note that the total number of possible measurements is equal to $m = m_a + m_c$.

The fundament of the algorithm is to transform this original Jacobian matrix $\mathbf{W}$ into a matrix $\mathbf{W}^*$ through a Gauss-based elimination technique. In order to facilitate the method understanding, the following vectors $\boldsymbol{U}_w$, $\boldsymbol{V}_w$, $\boldsymbol{I}_{U_w}$ and $\boldsymbol{I}_{V_w}$ are defined and updated throughout the transformation process, which is as follows:

Input: Matrix $\mathbf{W}$ and the sets of available and candidate measurements.

Step 1: Initialization. Set the iteration counter to $\nu = 1$. Note that the counter indicates the row within $\mathbf{W}$ where the pivot element (see **step 2**) is looked for. Set null the binary vectors $\boldsymbol{I}_{U_w}$ and $\boldsymbol{I}_{V_w}$ associated with the state variables (columns of matrix $\mathbf{W}$) and measurements (rows of matrix $\mathbf{W}$), respectively, as shown in Table 1. Additionally, initialize vectors $\boldsymbol{U}_w$ and $\boldsymbol{V}_w$ containing, respectively, the state variables and the list of available and candidate measurements corresponding to the rows of matrix $\mathbf{W}$ (dimension $m \times 1$). Note that available measurements have been marked with superindex "a" and candidate measurements with superindex "c". Continue in **step 2**.

Step 2: Maximum absolute value. Locate the largest absolute value and non-null component of matrix $\mathbf{J}_\mathrm{a}$ associated with a null element in vector $\boldsymbol{I}_{U_w}$. Let us assume that it corresponds to component $w_{k,j}$ in Table 1 at iteration $\nu = k$. This element is selected as pivot, which means that the corresponding j-th and k-th elements in vectors $\boldsymbol{U}_w$ and $\boldsymbol{V}_w$, respectively, are going to be exchanged as illustrated in Table 2. In addition, the corresponding column j-component of vector $\boldsymbol{I}_{U_w}$ is set to 1. If there is not such a component, go to **step 4**, otherwise, continue in **step 3**.

Step 3: Matrix update. Once the pivoting element located in row k and column j is selected, the actual matrix has to be updated using a Gauss elimination strategy as follows:

1. Replace the pivoting element $w_{k,j}$ by $\frac{1}{w_{k,j}}$.
2. Update the rest of elements associated with the pivot j-th column dividing by the pivoting element as shown in Table 2.
3. Transform the rest of elements related to the k-th row multiplying them by $-\frac{1}{w_{k,j}}$.
4. The remaining elements of the matrix $w_{f,e}$ not belonging to the k-th row and j-th column, i.e. not boldfaced in Table 2, are transformed: $w_{f,e} \leftarrow w_{f,e} - \frac{w_{f,j} w_{k,e}}{w_{k,j}}$.

Step 4: Observability checking. Once the matrix is transformed, check if any of the elements of vector $\boldsymbol{V}_w$ is observable so far. Note that any element of $\boldsymbol{V}_w$ associated with row f is observable if all the elements in the row f associated with null column components in vector $\boldsymbol{I}_{U_w}$ are equal to zero. If that it is the case, set the corresponding component in $\boldsymbol{I}_{V_w}$ equal to 1. It must be noticed that in Table 2 a nodal head measurement has been pivoted, thus it is for sure observable (it directly provides the value of a state variable) and position k within vector $\boldsymbol{I}_{V_w}$ is set to one. If $\nu = m_a$ continue with **step 5**, otherwise, update the iteration counter $\nu \leftarrow \nu + 1$ and continue with **step 2**.

Step 5: Output. The process has finished. If all elements in $\boldsymbol{I}_{U_w}$ are equal to 1, the state is observable; otherwise, it is not. Return vectors $\boldsymbol{U}_w$, $\boldsymbol{I}_{U_w}$, $\boldsymbol{V}_w$, and $\boldsymbol{I}_{V_w}$ and the transformed matrix $\mathbf{W}^*$.

According to the functioning of the observability algorithm explained above, if the system is observable, n rows associated with available measurements in $\mathbf{W}$ are transferred to columns in matrix $\mathbf{W}^*$, and conversely, all columns related to state variables in $\mathbf{W}$ are transferred to rows in matrix $\mathbf{W}^*$. This fact indicates that the system is observable because all the state variables can be now determined from the available measurements. However, if full transfer is not achieved, some of the existing measurements allow the observability of certain variables. This information is contained in vector $\boldsymbol{I}_{V_w}$.

Regarding the classification of measurements by type, available measurements transferred to columns are called essential, because they are needed to characterize the hydraulic state of the system. Besides, if their loss makes the state unobservable, essential measurements are also called critical. In contrast, available or candidate measurements that only depend on essential measurements are called redundant, while if they are related to essential measurements and state variables, they are called non-redundant.

Note that vector $\boldsymbol{U}_w$ starts the algorithm containing the state variables and finishes the process containing the essential measurements and those non-observable state variables that cannot be transferred to rows, if any. Similarly, vector $\boldsymbol{V}_w$ contains the available and candidate measurements before the application of the algorithm and finishes with state variables and non-essential measurements. Binary vector $\boldsymbol{I}_{U_w}$ indicates if the corresponding components in vector $\boldsymbol{U}_w$ are essential or not, while binary vector $\boldsymbol{I}_{V_w}$ indicates if the corresponding state variables and measurements related to $\boldsymbol{V}_w$ are observable or redundant, respectively. Therefore, apart from providing observability information of the system, the transformed matrix $\mathbf{W}^*$ also allows to identify critical and redundant measurements of the analyzed network. Hence, if the state is observable, the matrix can be used to determine the set of redundant measurements that can replace a set of essential measurements so that the state of the system remains observable. On the contrary, if the system is unobservable, the matrix helps to identify the variables that can be observed with the available measurements, which is the required information for island identification.

An additional feature of the adopted algorithm is that the transformation to exchange rows and columns is reversible, as pointed out by Pruneda et al. (2010). Therefore, in case the transformed matrix has been initially computed and one measurement is lost, the method could be applied backwards without the need to start the process from scratch.

Before continuing with the island identification algorithm, it is important to highlight some peculiarities of the OA algorithm for water networks:

1. Full observability requires at least one available nodal head measurement. Note that in power systems this condition is equivalent to the requirement of setting a reference bus (Pruneda et al., 2010).
2. Demands at transit nodes ($\mathcal{V}^{\mathrm{T}}$) are equal to zero and are treated as measurements. Thus, the minimum number of available demand measurements ($\tilde{q}_i; \forall i \in (\mathcal{V}^{\mathrm{Q}} \cup \mathcal{V}^{\mathrm{T}})$ in m_a) is equal to the number of transit nodes within the system.

Method for island identification

If the state of the system is unobservable for a given set of available measurements, it is of interest to identify observable islands. In water supply networks, we can define observable islands as regions of the system where all state variables are known regardless of the lack of full observability of the network. The procedure to detect them consists on grouping the observable variables that can be identified from matrix $\mathbf{W}^*$, including either state variables or other hydraulic variables, such as flows.

The method for island identification starts by assuming that each of the nodes associated with observable state variables constitute islands themselves. Thus, the set of islands $\mathbf{I} = [\{1\}$, ..., $\{i\}$, $\{j\}$, ... $\{n\}]$ can be defined if state variables h_1, ..., h_i, h_j, ... h_n are observable, which is guaranteed if the corresponding element in the resulting vector $\boldsymbol{I}_{V_w}$ is equal to one. Next step is to extend island coverage, thus it is required to analyze observability in their surroundings. With this purpose, *observable branches* are evaluated. We define as observable branches those lines (pipes) that can be observed with the available measurements according to the resulting transformed matrix

$\mathbf{W}^*$. Therefore, if the flow through a pipe that goes from node i to junction j is observable, the observable island associated with i can be extended to j or vice versa. To avoid duplication, only j is added to the observable island related to i, thus $\mathbf{I} = [\{1\}$, ..., $\{i,j\}$,..., $\{n\}]$. Repeating this procedure step by step through all the observable branches, junctions are grouped in observable islands.

The interest of this method for island identification is that it enables us to show graphically those areas where nodal heads and flows are observable. However, information about demands is not given explicitly. Note that demands at nodes within observable islands are only observable if all the flows that enter or leave the junction are included in the island. This idea is represented by drawing the limit of the observable island through the middle of the node, as shown in the following example.

ILLUSTRATIVE EXAMPLE

A small water network proposed by Wurbs and James (2002) is used to illustrate the OA methodology. Figure 1 shows the layout of the system, including the initial assumed flow directions. The system is formed by two elevated reservoirs at nodes 1 and 6 $\in \mathcal{V}^{\mathrm{R}_\infty}$, and four intermediate junctions, each of which is subjected to constant demands ($\in \mathcal{V}^{\mathrm{Q}}$) and connected to each other through seven pipes. Network parameters are given in Supplemental Data, section C.

Measurement Jacobian matrix

The measurement Jacobian matrix of the network can be derived following the previously presented methodology. The resulting normalized matrix particularized for the state given in Supplemental Data, section C, results in:

$$
\mathbf{J_0} = \begin{array}{c} \\ \tilde{h}_1 \\ \tilde{h}_2 \\ \tilde{h}_3 \\ \tilde{h}_4 \\ \tilde{h}_5 \\ \tilde{h}_6 \\ \tilde{Q}_{1,2} \\ \tilde{Q}_{2,3} \\ \tilde{Q}_{2,5} \\ \tilde{Q}_{3,5} \\ \tilde{Q}_{3,4} \\ \tilde{Q}_{4,5} \\ \tilde{Q}_{4,6} \\ \tilde{q}_2 \\ \tilde{q}_3 \\ \tilde{q}_4 \\ \tilde{q}_5 \end{array}
\begin{array}{c} h_1 \quad h_2 \quad h_3 \quad h_4 \quad h_5 \quad h_6 \\
\left[\begin{array}{cccccc}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\hdashline
1 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 & 0 \\
0 & 0 & 1 & 0 & -1 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 1 & 0 & -1 \\
\hdashline
-0.51 & 1 & -0.23 & 0 & -0.26 & 0 \\
0 & -0.26 & 1 & -0.40 & -0.34 & 0 \\
0 & 0 & -0.34 & 1 & -0.33 & -0.33 \\
0 & -0.29 & -0.34 & -0.38 & 1 & 0
\end{array}\right]
\end{array}
\tag{7}
$$

Nodal head measurements can be easily identified, as they are associated with the 6×6 identity matrix. Similarly, flow measurements are represented by those rows where there is a 1 at the initial node and a -1 at the final node thanks to the proposed normalization. Finally, the four last rows correspond to nodal demands at non-reservoir nodes ($\forall i \notin \mathcal{V}^{\mathrm{R}_\infty}$) expressed in terms of the state variables involved through the expressions presented in Supplemental Data, section B.

Observability analysis

We analyze two different initial measurement configurations, resulting in observable and unobservable states, respectively.

Observable case

Before applying the algorithm for observability analysis, the measurement Jacobian matrix must be reorganized to place in first position the rows corresponding to the available measurements within the system ($\mathbf{J}_{\mathrm{a}}$), and then the group of candidate measurements ($\mathbf{J}_{\mathrm{c}}$), which are available at a certain cost. The first example assumes the following measurements are available: nodal heads at reservoirs ($\tilde{h}_1$ and $\tilde{h}_6$) and water flow in pipes 1-2 ($\tilde{Q}_{1,2}$), 2-3 ($\tilde{Q}_{2,3}$), 2-5 ($\tilde{Q}_{2,5}$) and 3-4 ($\tilde{Q}_{3,4}$). Note that nodal demands are not included as available information because only readings from metering devices are taken as available measurements in this theoretical WTN. Table 3 provides the corresponding measurement Jacobian matrix and Figure 2 shows the measurement layout.

The application of the described algorithm leads to the resulting transformed matrix $\mathbf{W}^*$ given in Table 4. Since all elements in the resulting vector $\boldsymbol{I}_{U_w}$ are equal to one, we can conclude that the system state is observable. Besides, since all state variables are observable, all candidate measurements are redundant, i.e. all elements in $\boldsymbol{I}_{V_w}$ are equal to 1. Candidate measurements also represent the rest of hydraulic variables within the network and thus we can conclude that all those variables can also be calculated. Measurements $\tilde{h}_1$, $\tilde{h}_6$, $\tilde{Q}_{1,2}$, $\tilde{Q}_{2,3}$, $\tilde{Q}_{2,5}$ and $\tilde{Q}_{3,4}$, belonging to vector $\boldsymbol{U}_w$ are essential, because they have been transferred from vector $\boldsymbol{V}_w$ to $\boldsymbol{U}_w$. Moreover, they are critical, because if any of them is lost, the system would become unobservable.

As mentioned before, the transformed matrix also provides information about the set of redundant measurements that can replace a set of essential measurements so that the state of the system remains observable. In this respect, all non-null elements in matrix $\mathbf{W}^*$ allow replacements preserving observability. Therefore, as the structure of the matrix results more determinant for observability analysis than the values themselves, non-null elements have been shaded in light grey. For example, element $w^*_{13,1} = 1 \neq 0$ indicates that redundant measurement $\tilde{Q}_{4,6}$ (corresponding row element in $\boldsymbol{V}_w$) can replace the essential measurement $\tilde{h}_1$ (corresponding column in $\boldsymbol{U}_w$). For the same reason, nodal head measurements $\tilde{h}_2$, $\tilde{h}_3$, $\tilde{h}_4$, $\tilde{h}_5$ and demand measurement $\tilde{q}_4$ can also replace the essential measurement $\tilde{h}_1$. This application has potential, because it permits to detect how a nodal head measurement can be substituted by a different kind of measurement, being possible to maintain observability if flow measurement devices are installed at certain locations. Similarly, if the set of essential measurements $\tilde{Q}_{2,3}$ and $\tilde{Q}_{2,5}$ is lost, they can be replaced by the set of redundant measurements $\tilde{Q}_{3,5}$ and $\tilde{Q}_{4,6}$, because they have an associated invertible matrix within $\mathbf{W}^*$, i.e.:

$$\begin{vmatrix} -1 & 1 \\ -1 & 0 \end{vmatrix} = 1 \neq 0.$$

Alternatively, the same set of essential measurements could be replaced by nodal head measurements $\tilde{h}_3$ and $\tilde{h}_5$, or by any combination of two demand measurements.

Let us assume that we want information about the observability of the system if only measurements $\tilde{Q}_{2,3}$ and $\tilde{Q}_{2,5}$ are available. Note that without further calculations, it is possible to ensure that no state variables are observable because they all depend on other essential measurements, which we are assuming as no longer available. Besides, it is also possible to conclude that measurement $\tilde{Q}_{3,5}$ would result redundant, because it only presents non-null elements in columns associated with available measurements $\tilde{Q}_{2,3}$ and $\tilde{Q}_{2,5}$. Similarly, if only $\tilde{h}_1$ and $\tilde{Q}_{1,2}$ were available, state variables h_1 and h_2 would be observable, and measurement $\tilde{h}_2$ would be redundant.

Finally, this approach also helps to identify locations where measurement devices should be placed to improve resilience against the loss of measurements. As commented before, SE calculates the most likely hydraulic state of the network from the available measurements. However, as the use of instrumentation is associated with measurement errors or even instrumentation might fail to deliver its measurement, redundancy is required to correct those deviations and ensure reliable results even in case one or several assets are damaged. The algorithm for observability analysis confirms that including nodal demand measurements is very convenient, because as shown by matrix $\mathbf{W}^*$ for the illustrative example, those rows representing demand measurements ($\tilde{q}_2$, $\tilde{q}_3$, $\tilde{q}_4$ and $\tilde{q}_5$) present non-null elements in many of the essential measurements involved. Thus, they provide a very complete overview of the system, reason why pseudo-measurements have been traditionally taken into account at the cost of increasing the uncertainty for the later SE process. Moreover, this approach permits to identify those measurements which provide the highest redundancy. In this particular case, measuring the demand at node 4 ($\tilde{q}_4$) would be interesting to enhance the resilience of the system, as it would make all the essential measurements non-critical, i.e. it would keep the system observable even if any other measurement is lost. Matrix $\mathbf{W}^*$ also permits to identify how other non-demand redundant measurements, such as $\tilde{Q}_{4,6}$ or $\tilde{h}_4$, would enhance redundancy for SE. This justifies why this methodology helps to consider where to locate the minimum measurement devices required to achieve observability (if it is not attained) and where to place additional control points (once observability is guaranteed) to enhance robustness in the subsequent SE process.

Unobservable case

In this case, we consider that the set of available measurements includes both nodal heads at the reservoir nodes ($\tilde{h}_1$ and $\tilde{h}_6$) and flow measurements in pipes 1-2 ($\tilde{Q}_{1,2}$), 2-3 ($\tilde{Q}_{2,3}$), 2-5 ($\tilde{Q}_{2,5}$) and 3-5 ($\tilde{Q}_{3,5}$), as shown in Figure 3. The only difference with respect to the observable case previously analyzed is that measurement $\tilde{Q}_{3,4}$ is replaced by $\tilde{Q}_{3,5}$.

Once again, this scenario can be analyzed implementing the algorithm to the newly organized Jacobian matrix, where $\tilde{Q}_{3,5}$ is part of the available measurement subset instead of $\tilde{Q}_{3,4}$ as in Table 3. Execution of the proposed methodology leads to the transformed matrix shown in Table 5, where non-null elements have been shaded in light grey.

From Table 5 the following observations are pertinent:

1. State variable h_4 has not been pivoted, because all its $\mathbf{J}_\mathrm{a}$ column components are null. Thus the system is unobservable and includes one redundant measurement $\tilde{Q}_{3,5}$.
2. Matrix $\mathbf{W}^*$ provides information of how to achieve observability, i.e. including in the available measurement set any of the measurements $\tilde{h}_4$, $\tilde{Q}_{3,4}$, $\tilde{Q}_{4,5}$, $\tilde{Q}_{4,6}$, $\tilde{q}_3$, $\tilde{q}_4$ or $\tilde{q}_5$. Note that their corresponding row elements associated with the unobservable state variable h_4 present non-null components. This information could also have been extracted from Table 4, because the essential and critical measurement $\tilde{Q}_{3,4}$ could be replaced by $\tilde{h}_4$, $\tilde{Q}_{4,5}$, $\tilde{Q}_{4,6}$, $\tilde{q}_3$, $\tilde{q}_4$ or $\tilde{q}_5$, but no others. This fact proves that the application of the algorithm to one subset of measurements provides the observability information of the entire network regardless of

the subset of available measurements being considered, and without the need to start the process from scratch.

3. Vector $\boldsymbol{I}_{V_w}$ allows to know which other variables are observable. For instance, q_2 can be calculated because its corresponding element on that vector is equal to 1. Note that the element in its row associated with the column related to the unobservable state variable h_4 is null, i.e. its information can be extracted from the remaining measurements.

4. If, for instance, measurement $\tilde{Q}_{3,4}$ becomes available, the system would become observable and the essential measurements $\tilde{h}_1$, $\tilde{Q}_{1,2}$, $\tilde{Q}_{3,4}$ and $\tilde{h}_6$ would become critical, because all of them present null elements in the row related to the redundant measurement $\tilde{Q}_{3,5}$.

If we focus on the detection of observable islands, we start having 5 initial islands which correspond to the nodes where state variables are observable in first place: $\{1\}$, $\{2\}$, $\{3\}$, $\{5\}$ and $\{6\}$. Also, we can identify the observable branches in the network for this case scenario: $\tilde{Q}_{1,2}$, $\tilde{Q}_{2,3}$ and $\tilde{Q}_{2,5}$ as essential measurements, and $\tilde{Q}_{3,5}$ as redundant measurement. Therefore, we can undertake the presented algorithm to group the observable information, as shown in Table 6. At the end of the process the observable islands are $\{1,2,3,5\}$ and $\{6\}$, as shown in Figure 3. It must be noticed that this procedure allows to guarantee that nodal heads and flows are observable, but not demands. For example, nodal demand 2 (q_2) is observable because the corresponding measurement $\tilde{q}_2$ is redundant according to matrix $\mathbf{W}^*$, but nodal demands 3 and 5 are not, because they depend on the flow in pipes 3-4 and 4-5, which cannot be observed.

DISCUSSION: APPLICABILITY TO REAL WATER TRANSPORT NETWORKS

The previous illustrative example shows the potential of the methodology presented to analyze the topology of a simple network, but its applicability to real systems needs to be discussed. To begin with, note that the computational complexity of the method is analogous to that of the Gauss elimination method for solving linear systems of equations, which can be efficiently solved even for large scale systems. Moreover, the Jacobian matrix required for the analysis has been normalized to decrease the probability of numerical errors due to ill-conditioning. Thus, from the computational point of view, it is highly suitable for its practical implementation in large water supply systems.

From a practical point of view, we have restricted the application of the OA algorithm for real networks to the case of Water Transport Networks that supply sectorized areas. Hence, OA could be applied to this primary network, and would enable an approximate picture of the real system observability. Moreover demand management within DMA would be improved even for the worst case scenario of unexpected individual or simultaneous failure of essential meters. For example, let us suppose we run the OA algorithm just for the primary network, obtaining the transformed matrix $\boldsymbol{W}^*$. Note that this calculation can be done off-line without computational time limitations to store the matrix in the system for its posterior on-line use. If we wish to carry out OA prior to SE and factor in the specific loss of measurements at a specific instant in time, we could run the algorithm in reverse order by removing measurements and updating matrix $\boldsymbol{W}^*$ consequently. Thus, it is possible to quickly answer the question of whether the system remains observable or not, and if the latter is the case, which measurements and/or pseudo-measurements could be included to recover observability.

CONCLUSIONS

In this paper the necessity of applying OA before any SE method has been justified, especially for its automatic application in real-time. From the practical perspective, its application is especially relevant for WTN, where the use of pseudo-measurements is not required. This makes WTN suitable for the implementation of OA techniques, which enables the set of metering devices that makes the network observable to be identified without relying on pseudo-measurements. This strategy would permit any state of the network in the subsequent SE process to be characterized, even when unexpected changes occur, which is a present and future requirement in order to obtain the maximum benefit from modern ICT systems within water supply systems.

In particular, the novel implementation for WTN of an algebraic OA method adapted from power systems (which allows extraction of the maximum amount of information possible as regards observability issues) is presented in this paper. The methodology presented herein allows observability checking, identification of critical and redundant measurements and detection of observable islands. Moreover, it has the following additional features: i) it informs if any measurement set makes the system observable, and if it is not, which variables would be observable and unobservable, thus if we run an SE algorithm based on optimization techniques, it is possible to know what information is really trustworthy and what information must be discarded, ii) it shows how observability changes if any of the measurements disappears from the meter set (sensor failure, communication failure, etc.), and so constitutes a useful tool for carrying out a rapid analysis of observability of the resulting system in case the SCADA system fails to deliver any measurement, iii) it provides a criteria for the location of measurements with special emphasis on observability issues, and iv) it provides criteria to install new sensors if the operator wants to increase resilience against loss of sensors and/or measurements, thereby ensuring that the most important variables in the system remain observable.

The method has potential for use in large networks, due to its simplicity in terms of computational performance. Additionally, the possibility of running the algorithm in reverse order would reduce the number of iterations required to update matrix $\boldsymbol{W}^*$ thereby reducing the number of measurements lost , which is considerably faster than starting the process from scratch and suitable for its real-time application. Therefore, this approach is a robust and practical method for analyzing specific measurement configurations within WTN and supports the installation of measurement devices in modern networks provided with SCADA systems.

On a final note, it must be pointed out that work is underway to prove how observability issues affect SE results within water transport networks. Also, note that how to tackle unknown valve and tank statuses (so called topological observability in electric systems) is a topic of ongoing research.

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SUPPLEMENTAL DATA

Supplemental Data is available online in the ASCE Library (ascelibrary.org).

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List of Tables

TABLE 1. First step of the algorithm for observability analysis at iteration $\nu = k$

$\boldsymbol{I}_{V_w}$	$\boldsymbol{V}_w$	0	$\dots$	0	$\dots$	0	$\leftarrow \boldsymbol{I}_{U_w}$
$\downarrow$	$\downarrow$	h_1	$\dots$	$\boldsymbol{h_j}$	$\dots$	h_n	$\leftarrow \boldsymbol{U}_w$
0	$\tilde{h}_1^{\mathrm{a}}$	$w_{1,1}$	$\dots$	$\boldsymbol{w_{1,j}}$	$\dots$	$w_{1,n}$	
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
0	$\tilde{\boldsymbol{h}}_{\boldsymbol{k}}^{\mathbf{a}}$	$\boldsymbol{w_{k,1}}$	$\dots$	$\boldsymbol{w_{k,j}}$	$\dots$	$\boldsymbol{w_{k,n}}$	
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
0	$\tilde{h}_i^{\mathrm{a}}$	$w_{f,1}$	$\dots$	$\boldsymbol{w_{f,j}}$	$\dots$	$w_{f,n}$	
0	$\tilde{Q}_1^{\mathrm{a}}$	$w_{(f+1),1}$	$\dots$	$\boldsymbol{w_{(f+1),j}}$	$\dots$	$w_{(f+1),n}$	
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
0	$\tilde{Q}_{ij}^{\mathrm{a}}$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
0	$\tilde{q}_1^{\mathrm{a}}$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
0	$\tilde{q}_i^{\mathrm{a}}$	$w_{m_a,1}$	$\dots$	$\boldsymbol{w_{m_a,j}}$	$\dots$	$w_{m_a,n}$	
0	$\tilde{h}_1^{\mathrm{c}}$	$w_{(m_a+1),1}$	$\dots$	$\boldsymbol{w_{(m_a+1),j}}$	$\dots$	$w_{(m_a+1),n}$	
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
0	$\tilde{h}_i^{\mathrm{c}}$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
0	$\tilde{Q}_1^{\mathrm{c}}$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
0	$\tilde{Q}_{ij}^{\mathrm{c}}$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
0	$\tilde{q}_1^{\mathrm{c}}$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
0	$\tilde{q}_i^{\mathrm{c}}$	$w_{m,1}$	$\dots$	$\boldsymbol{w_{m,j}}$	$\dots$	$w_{m,n}$	

TABLE 2. Second, third and fourth step of the algorithm for observability analysis at iteration $\nu = k$

$\boldsymbol{I}_{V_w}$	$\boldsymbol{V}_w$	0	$\dots$	$\mathbf{1}$	$\dots$	0	$\leftarrow \boldsymbol{I}_{U_w}$
$\downarrow$	$\downarrow$	h_1	$\dots$	$\boldsymbol{\tilde{h}^{\mathbf{a}}_k}$	$\dots$	h_n	$\leftarrow \boldsymbol{U}_w$
0	$\tilde{h}^{\mathrm{a}}_1$	$w_{1,1} - w_{k,1}\frac{w_{1,j}}{w_{k,j}}$	$\dots$	$\boldsymbol{\frac{w_{1,j}}{w_{k,j}}}$	$\dots$	$w_{1,n} - w_{k,n}\frac{w_{1,j}}{w_{k,j}}$	
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
1	$\boldsymbol{h_j}$	$\boldsymbol{-\frac{w_{k,1}}{w_{k,j}}}$	$\dots$	$\boldsymbol{\frac{1}{w_{k,j}}}$	$\dots$	$\boldsymbol{-\frac{w_{k,n}}{w_{k,j}}}$	
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
0	$\tilde{h}^{\mathrm{a}}_i$	$w_{f,1} - w_{k,1}\frac{w_{f,j}}{w_{k,j}}$	$\dots$	$\boldsymbol{\frac{w_{f,j}}{w_{k,j}}}$	$\dots$	$w_{f,n} - w_{k,n}\frac{w_{f,j}}{w_{k,j}}$	
0	$\tilde{Q}^{\mathrm{a}}_1$	$w_{(f+1),1} - w_{k,1}\frac{w_{(f+1),j}}{w_{k,j}}$	$\dots$	$\boldsymbol{\frac{w_{(f+1),j}}{w_{k,j}}}$	$\dots$	$w_{(f+1),n} - w_{k,n}\frac{w_{(f+1),j}}{w_{k,j}}$	
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
0	$\tilde{Q}^{\mathrm{a}}_{ij}$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
0	$\tilde{q}^{\mathrm{a}}_1$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
0	$\tilde{q}^{\mathrm{a}}_i$	$w_{m_a,1} - w_{k,1}\frac{w_{m_a,j}}{w_{k,j}}$	$\dots$	$\boldsymbol{\frac{w_{m_a,j}}{w_{k,j}}}$	$\dots$	$w_{m_a,n} - w_{k,n}\frac{w_{m_a,j}}{w_{k,j}}$	
0	$\tilde{h}^{\mathrm{c}}_1$	$w_{(m_a+1),1} - w_{k,1}\frac{w_{(m_a+1),j}}{w_{k,j}}$	$\dots$	$\boldsymbol{\frac{w_{(m_a+1),j}}{w_{k,j}}}$	$\dots$	$w_{(m_a+1),n} - w_{k,n}\frac{w_{(m_a+1),j}}{w_{k,j}}$	
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
0	$\tilde{h}^{\mathrm{c}}_i$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
0	$\tilde{Q}^{\mathrm{c}}_1$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
0	$\tilde{Q}^{\mathrm{c}}_{ij}$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
0	$\tilde{q}^{\mathrm{c}}_1$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$	$\dots$	$\vdots$	
0	$\tilde{q}^{\mathrm{c}}_i$	$w_{m,1} - w_{k,1}\frac{w_{m,j}}{w_{k,j}}$	$\dots$	$\boldsymbol{\frac{w_{m,j}}{w_{k,j}}}$	$\dots$	$w_{m,n} - w_{k,n}\frac{w_{m,j}}{w_{k,j}}$	

TABLE 3. Illustrative example W matrix for the observable case.

W		0	0	0	0	0	0
$\nu = 0$		h_1	h_2	h_3	h_4	h_5	h_6
0	$\tilde{h}_1$	1	0	0	0	0	0
0	$\tilde{h}_6$	0	0	0	0	0	1
0	$\tilde{Q}_{1,2}$	1	-1	0	0	0	0
0	$\tilde{Q}_{2,3}$	0	1	-1	0	0	0
0	$\tilde{Q}_{2,5}$	0	1	0	0	-1	0
0	$\tilde{Q}_{3,4}$	0	0	1	-1	0	0
0	$\tilde{h}_2$	0	1	0	0	0	0
0	$\tilde{h}_3$	0	0	1	0	0	0
0	$\tilde{h}_4$	0	0	0	1	0	0
0	$\tilde{h}_5$	0	0	0	0	1	0
0	$\tilde{Q}_{3,5}$	0	0	1	0	-1	0
0	$\tilde{Q}_{4,5}$	0	0	0	1	-1	0
0	$\tilde{Q}_{4,6}$	0	0	0	1	0	-1
0	$\tilde{q}_2$	-0.51	1	-0.23	0	-0.26	0
0	$\tilde{q}_3$	0	-0.26	1	-0.40	-0.34	0
0	$\tilde{q}_4$	0	0	-0.34	1	-0.33	-0.33
0	$\tilde{q}_5$	0	-0.29	-0.34	-0.38	1	0

TABLE 4. Illustrative example transformed matrix W* for the observable case

$\mathbf{W}^*$		1	1	1	1	1	1
	$\nu = 6$	$\tilde{h}_1$	$\tilde{Q}_{1,2}$	$\tilde{Q}_{2,3}$	$\tilde{Q}_{3,4}$	$\tilde{Q}_{2,5}$	$\tilde{h}_6$
1	h_1	1	0	0	0	0	0
1	h_6	0	0	0	0	0	1
1	h_2	1	-1	0	0	0	0
1	h_3	1	-1	-1	0	0	0
1	h_5	1	-1	0	0	-1	0
1	h_4	1	-1	-1	-1	0	0
1	$\tilde{h}_2$	1	-1	0	0	0	0
1	$\tilde{h}_3$	1	-1	-1	0	0	0
1	$\tilde{h}_4$	1	-1	-1	-1	0	0
1	$\tilde{h}_5$	1	-1	0	0	-1	0
1	$\tilde{Q}_{3,5}$	0	0	-1	0	1	0
1	$\tilde{Q}_{4,5}$	0	0	-1	-1	1	0
1	$\tilde{Q}_{4,6}$	1	-1	-1	-1	0	-1
1	$\tilde{q}_2$	0	-0.51	0.23	0	0.26	0
1	$\tilde{q}_3$	0	0	-0.60	0.40	0.34	0
1	$\tilde{q}_4$	0.33	-0.33	-0.66	-1	0.33	-0.33
1	$\tilde{q}_5$	0	0	0.71	0.38	-1	0

TABLE 5. Illustrative example transformed matrix W* for the unobservable case

$\mathbf{W}^*$		1	1	1	0	1	1
$\nu = 6$		$\tilde{h}_1$	$\tilde{Q}_{1,2}$	$\tilde{Q}_{2,3}$	h_4	$\tilde{Q}_{2,5}$	$\tilde{h}_6$
1	h_1	1	0	0	0	0	0
1	h_6	0	0	0	0	0	1
1	h_2	1	-1	0	0	0	0
1	h_3	1	-1	-1	0	0	0
1	h_5	1	-1	0	0	-1	0
1	$\tilde{Q}_{3,5}$	0	0	-1	0	1	0
1	$\tilde{h}_2$	1	-1	0	0	0	0
1	$\tilde{h}_3$	1	-1	-1	0	0	0
0	$\tilde{h}_4$	0	0	0	1	0	0
1	$\tilde{h}_5$	1	-1	0	0	-1	0
0	$\tilde{Q}_{3,4}$	1	-1	-1	-1	0	0
0	$\tilde{Q}_{4,5}$	-1	1	0	1	1	0
0	$\tilde{Q}_{4,6}$	0	0	0	1	0	-1
1	$\tilde{q}_2$	0	-0.51	0.23	0	0.26	0
0	$\tilde{q}_3$	0.40	-0.40	-1	-0.40	0.34	0
0	$\tilde{q}_4$	-0.67	0.67	0.34	1	0.33	-0.33
0	$\tilde{q}_5$	0.38	-0.38	0.34	-0.38	-1	0

TABLE 6. Identification of observable islands for the illustrative example unobservable case

Observable branch	Observable islands				
	$\{1\}$	$\{2\}$	$\{3\}$	$\{5\}$	$\{6\}$
$Q_{1,2}$	$\{1,2\}$		$\{3\}$	$\{5\}$	$\{6\}$
$Q_{2,3}$	$\{1,2,3\}$			$\{5\}$	$\{6\}$
$Q_{2,5}$	$\{1,2,3,5\}$				$\{6\}$
$Q_{3,5}$	$\{1,2,3,5\}$				$\{6\}$

List of Figures

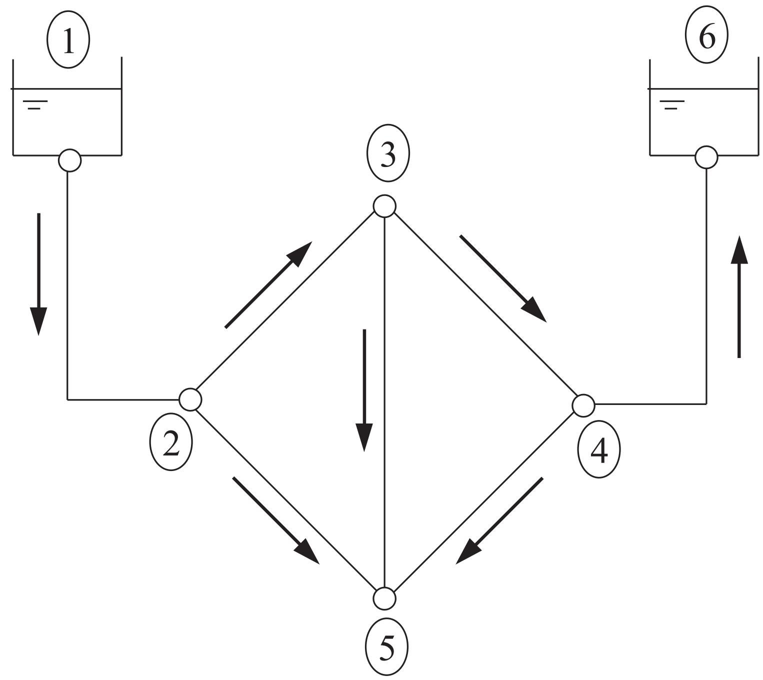

FIG. 1. Network layout of the illustrative example. Modified from Wurbs and James (2002)

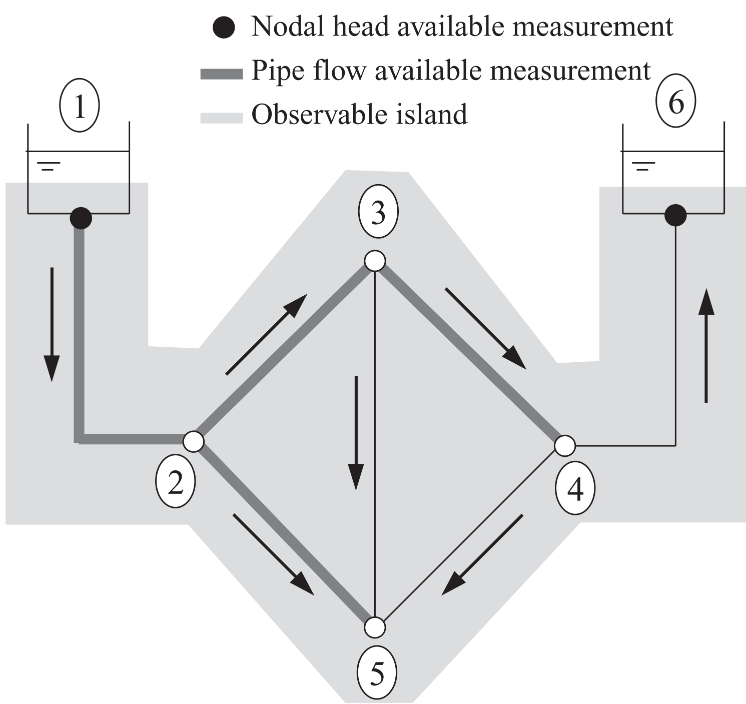

FIG. 2. Layout of the observable case for the illustrative example

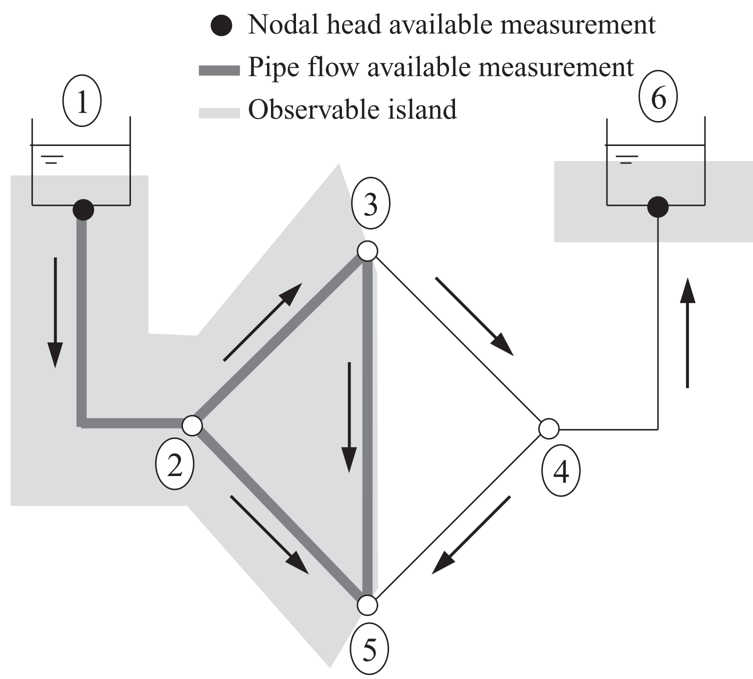

FIG. 3. Layout of the unobservable case for the illustrative example