ANALYTICAL STOCHASTIC MICROCOMPONENT MODELLING APPROACH TO ASSESS NETWORK SPATIAL SCALE EFFECTS IN WATER SUPPLY SYSTEMS

Sarai Díaz[1] and Javier González[2]

[1]Corresponding author: Dr. Eng, Dept. of Civil Eng., Univ. of Castilla-La Mancha, Av. Camilo José Cela s/n, 13071 Ciudad Real (Spain). E-mail: Sarai.Diaz@uclm.es

[2]Dr. Eng, Dept. of Civil Eng., Univ. of Castilla-La Mancha, Av. Camilo José Cela s/n, 13071 Ciudad Real (Spain). // HIDRALAB INGENIERÍA Y DESARROLLOS, S.L., Spin-Off UCLM, Hydraulics Laboratory Univ. of Castilla-La Mancha, Av. Pedriza, Camino Moledores s/n, 13071 Ciudad Real (Spain). E-mail: Javier.Gonzalez@uclm.es

The final publication is available at
https://ascelibrary.org/doi/abs/10.1061/%28ASCE%29WR.1943-5452.0001237

ABSTRACT

End-uses at water supply systems typically follow a random pulse behaviour, which blurs as consumptions are aggregated upstream, affecting flow rate variability along the spatial scale. Instantaneous variability has impact on the capacity of a hydraulic model to represent rapidly changing flow network scenarios, but traditional models only simulate average conditions. This paper analyses the spatial scale effect in instantaneous flow variability by making use of a novel analytical approach to SIMDEUM microcomponent-based stochastic demand model. Analytical results show good correspondence to previous results at Benthuizen case study and demonstrate the potential use of the approach to assess the effect of network size in a realistic system. Results prove that demand coefficients of variation increase in the periphery of water systems according to power

laws, highlighting the necessity of considering real variability rather than average conditions in these areas where real water flows never correspond to average flows. This is of utmost importance when dealing with real measurements and water quality applications.

INTRODUCTION

Water demand has traditionally been considered as deterministic for the purpose of drinking water supply systems modelling, as average scenarios have been typically assumed to simplify hydraulic simulations. The conventional "top-down" demand allocation process consists of assigning a demand multiplier pattern to the average or base demand on each node (Blokker et al. 2011a). Such an approach implies that water demand patterns are strongly correlated among all nodes. This is reasonable for water transport networks, which are the main arteries that supply water to sectorized urban areas, but it is not so appropriate for water distribution systems that ensure water provision on a more local scale (Filion et al. 2008). In such part of the network, the spatial and temporal variability of water demand is significant, and it is necessary to consider the stochastic nature of demands in order to make sure that sufficient (e.g. water supply at peak hours) and good-quality water (e.g. residence time and/or water quality) is provided to all users at all times. Otherwise, real flow and simulated average scenarios may differ significantly, and this can lead to a host of network issues (Buchberger and Wu 1995). The top-down simplification has worked well for several decades, but the limited availability and increased variability of water resources as a result of climate change (Zhang et al. 2019) and the growing requirements of a concerned society in terms of service quality (Mahmoud et al. 2018) have required to begin to focus on instantaneous stochastic demands (Vertommen et al. 2012; Pérez-Sánchez et al. 2017). This has led to the development of so-called "bottom-up" hydraulic models, which have been used to simulate the stochastic complexity of water demands since the end of the 90s (Creaco et al. 2017a).

According to the literature review presented by Creaco et al. (2017a), the available stochastic demand models at high temporal and spatial resolutions when implementing a "bottom-up" approach can be classified in two different groups: (1) models that use stochastic processes to simulate the overall water demand at a household, without differentiating the contribution of each inhabitant or appliance, and (2) models that construct the overall water demand at a household by adding demand microcomponents of each end-user at a fixture level (e.g. tap, washing machine, dishwasher). Buchberger and Wu (1995) presented the first type of models, which used Poisson Rectangular Pulses (PRP) to simulate the intensity, duration and frequency of water demands at a residence. According to this approach, parameters and probability functions that constitute a PRP can be obtained from flow measurements at monitored households (Buchberger and Wells 1996). This set up a basis for the analysis, over which different alternative pulse models have been presented (e.g. Alvisi et al. 2003, Creaco et al. 2015). These methods are relatively straightforward to build, as they depend on few parameters, but they require a significant amount of measurements, and parameter extrapolation to other populated areas is complicated (Creaco et al. 2017a). The second family of models follows the same idea of rectangular pulses, but they compute water consumption for each microcomponent from statistical information obtained from surveys. According to the aforementioned literature review, there is only one method within this category: the SIMDEUM model (Blokker et al. 2009; Blokker et al. 2010; Blokker et al. 2011a). SIMDEUM uses Monte Carlo simulations to compute demand patterns for each end-use, so it requires many parameters that are on the other hand easier to obtain.

Experimental campaigns are still required around the globe to better characterize water consumption at different locations, but both types of models are nowadays considered feasible approaches to estimate high-resolution water demands (Creaco et al. 2017a). It must be highlighted that bottom-up stochastic demand models were originally conceived to improve the knowledge about local flow fields and their consequences on water quality modelling (Buchberger and Wu 1995), especially in the periphery of water distribution systems, where velocities and head losses are low and thus water quality assessment is crucial (Blokker et al. 2008). However, their application field has extended ever since, at least at a scientific level. For example, SIMDEUM model has been applied for hydraulic network modelling (including some first attempts on leakage and transient simulations) and water quality assessment, but it has also been used for the design of drinking water supply systems and installations, prediction of future demands and so on (Blokker et al. 2017).

These applications prove that demand pulse modelling has potential and will eventually impact on different aspects of the practical analysis and design of water supply systems. But they also show that such tools have evolved from their original micro-scale perspective, and it is not straightforward to operationally extend them to full-scale real distribution systems (Creaco et al. 2017b).

The aim of this work is to present an analysis of stochastic demand behaviour, using a novel analytical approach in order to characterize network spatial scale effects in flow (i.e. aggregated demand) variability. The proposed approach belongs to the second group of stochastic demand methods, which (like SIMDEUM) generate residential water demand by adding up microcomponent consumption. The original SIMDEUM model is fed with survey-based parameters to provide plausible instantaneous water flows associated with Monte Carlo simulations, so it could be used to characterize the statistical behaviour of water demands. However, multiple simulations would be required, leading to excessive computational times for large urban areas or biased results if the number of simulations is insufficient (Blokker et al. 2011a). The new proposal directly provides statistical characterization (mean and variance values) of instantaneous demands. Such an approach is possible by assuming that end-uses and end-users are independent among each other, hence mean and variance values can be progressively added up (like in the PRP approach from Buchberger and Wu 1995) to assess the effect of spatial aggregation on demand uncertainty. This makes the change from the fixture or household level to the full-scale network and vice versa easier and it avoids the inconvenients associated with Monte Carlo simulations. As the new approach provides variance estimation, it can also be used to better characterize nonlinear trends in water quality modelling (Morton and Henderson 2008), which are far more complex than the simplified approaches typically used in engineering practice (Blokker et al. 2008). Moreover, as the model computes instantaneous water demand on a per second basis, results are potentially useful for real-time applications. Such is the case of state estimation techniques, which provide the most likely hydraulic state of the system based on the available noisy measurements gathered by a telemetry system (e.g. Kumar et al. 2008; Díaz et al. 2016; Díaz et al. 2018a; Díaz et al. 2018b), or other similar on-line tools (e.g. Wright et al. 2015; Sanz et al. 2016).

The rest of the paper is organised as follows. Firstly, the analytical microcomponent-based stochastic demand approach is presented. This includes a description of the underlying model hypotheses and the mathematical formulation that enables to analytically compute mean and variance of instantaneous water demands. Then, the method is applied to Benthuizen case study and compared to previous results from the literature (Blokker et al. 2011a). Once it has been validated, analytical results are further discussed to analyse network spatial scale effects. Finally, relevant conclusions are duly drawn.

METHODOLOGY

Hypotheses

The analytical stochastic demand approach presented in this paper is based on the following assumptions:

- It is inspired on the original SIMDEUM model (Blokker et al. 2010), as this end-use approach is nowadays a recognized methodology to simulate microcomponent-based stochastic water demand. Therefore, it uses the same information and it has the same expansion possibilities that SIMDEUM. As it will be explained later on, the present approach comes to combine the original PRP analytical approach to stochastic demand modelling (Buchberger and Wu 1995) with SIMDEUM's survey-based focus.
- Only residential water demand is here considered. However, the approach here presented could be extended to non-residential buildings (e.g. offices, schools, hospitals), like it has been done before with other models (Blokker et al. 2011b).
- Bathroom tap (for washing/shaving and brushing teeth uses), outside tap (for garden use and others, e.g. cleaning), WC (6L/9L with or without water saving configuration), bathtub, shower (with or without water saving configuration), dishwasher, washing machine and kitchen tap (for consumption, doing dishes, washing hands and others) end-uses have been considered in this work, as originally proposed for the SIMDEUM model (Blokker et al. 2010).

- The activation of an end-use is independent from the rest of openings associated with that end-use. This means that each inhabitant can start an end-use regardless of when he/she has previously used it.
- The average number of openings (i.e. frequency) for each end-use is independent from the number of devices that enable that end-use. This means that, for example, the number of WC flushes of a user is independent from the number of WC devices in the household.
- End-uses are independent among each other for a specific inhabitant: the same person can start several end-uses at the same time. This is reasonable for some long-duration uses (e.g. washing machine, dishwasher), but it may be questionable for some other cases of overlap (e.g. shower-kitchen tap).
- Each inhabitant acts independently from the rest of inhabitants in the house, as each person is related to a specific type of pattern (like in Blokker et al. 2010). This assumption is reasonable for some households, but it impedes the simulation of family-coordinated activities.
- Each house is independent from the rest of houses in the neighbourhood. This assumption is compromised for some specific events or festivities, but it works fine in ordinary conditions.

The first three hypotheses are posed to delimit the scope of the model presented in this paper, whereas the rest focus on establishing independence among end-uses, inhabitants and households. The proximity of these hypotheses to the real behaviour has been validated, as it will be shown in the Case study and spatial scale effect analysis section.

Mean instantaneous demand

Water demands are random variables, so the expected value of any group of demands (Z) can be computed by adding the expected or mean value of each of the water demands (e.g. X and Y) (Haan 1977):

$$Z = X + Y \tag{1}$$

$$\mathrm{E}(Z) = \mathrm{E}(X + Y) = \mathrm{E}(X) + \mathrm{E}(Y) \tag{2}$$

This elementary property enables to compute the mean instantaneous demand at a specific time t and level of spatial aggregation s ($\mu_{t,s}$) by adding mean values of water consumption for all end-uses and inhabitants within the household or households included in an area:

$$\mu_{t,s} = \sum_{i=1}^{n_{hou}} \mu_{hou_i} = \sum_{i=1}^{n_{hou}} \left(\sum_{j=1}^{n_{hab_i}} \mu_{hab_j} + \sum_{k=1}^{4} \mu_{ktap_k} \right); \tag{3}$$

where n_{hou} is the total number of houses, and μ_{hou_i} is the instantaneous demand at household i. At the same time, household demand is composed by the addition of the total number of inhabitants in the house (n_{hab_i}), who each (j) have an individual demand μ_{hab_j}, as shown in Figure 1. It must be highlighted that the mean instantaneous demand for the kitchen tap (μ_{ktap_k}) is not considered individually but on an overall household-basis, as suggested by Blokker et al. (2010). Sub-index k refers to the four possible uses mentioned earlier on for the kitchen tap (consumption, doing dishes, washing hands and others). Note that $\mu_{t,s}$ refers to a specific level of spatial aggregation s. This may refer to a network node that represents one individual household, a full multi-story building, a neighbourhood, a District Metered Area (DMA) or a full town. The overall mean value can be easily scaled up by aggregating as many individual demands as required.

Instantaneous demand for each inhabitant (μ_{hab_j}) must take into account the rest of end-uses stated in the Hypotheses section:

$$\mu_{hab_j} = \sum_{l=1}^{2} \mu_{btap_l} + \sum_{m=1}^{2} \mu_{otap_m} + \mu_{wc_n} + \mu_{bath} + \mu_{shower_o} + \mu_{dishw} + \mu_{washm}; \tag{4}$$

where μ_{btap_l} and μ_{otap_m} refer to the aforementioned bathroom and outside tap uses, respectively. On the other hand, μ_{wc_n} and μ_{shower_o} represent the type of WC and shower that exist in the household, and μ_{bath}, μ_{dishw} and μ_{washm} refer to a prototype bathtub, dishwasher and washing machine typically used (Blokker et al. 2010). Terms could be added if additional fixtures, types of end-use or types of devices were to be considered. In general, Eq. (4) represents the addition of end-use terms, meaning that each inhabitant activates several times different end-uses along the day. In order to simplify notation, Eq. (4) will be from now on written as the addition of several end-uses (u) mean demand (μ_u):

$$\mu_{hab_j} = \sum_{u=1}^{n_{use}} \mu_u, \tag{5}$$

out of the n_{use} uses that are included in this work.

In order to analytically compute the mean instantaneous demand of an end-use at time t, two exclusionary probability density functions must be considered: one that refers to the unitary probability of one opening of an end-use u being on/open at time t ($P_{ou}(t)$, with o for open) and another one that refers to the unitary probability of one opening of the end-use u being off/closed at that time t ($P_{cu}(t)$, with c for closed):

$$P_{ou}(t) + P_{cu}(t) = 1 \tag{6}$$

Such probabilities refer to the probability of an end-use u being open/closed at time t when one opening occurs along a day. Note that duration of each end-use (see Table 1) is considerably smaller than the discretization time step typically assumed for demand patterns (i.e. every hour), and even smaller than 24 h, so $P_{ou}(t) \ll 1$ (typically smaller than 1%) and $P_{cu}(t) \approx 1$ for all end uses here considered.

In reality, not only one opening takes place for each end-use u during a day, but a number of openings may occur, whose mean value is here called μ_{N_u} (i.e. mean frequency of use). Considering previous conditions, it can be demonstrated that the daily (rather than unitary) probability of the end-use being on along a full day is equal to $\mu_{N_u} \cdot P_{ou}(t)$. Considering this, and the fact that mean instantaneous demand must account the probability of the end-use being on and off, it can be written that:

$$\mu_u = \mu_{N_u} \cdot P_{ou}(t) \cdot \mu_{io_u} + (1 - \mu_{N_u} \cdot P_{ou}(t)) \cdot \mu_{ic_u}, \tag{7}$$

where μ_{io_u} is the mean intensity of the end-use u when it is open (with o for open) and μ_{ic_u} its mean intensity when it is closed (with c for closed). In this equation, the off (i.e. right-hand side term of the addition) is zero because the mean intensity when the end-use is not working is assumed to be null ($\mu_{ic_u} = 0$), i.e. there is watertight closure for all end-uses. In order to further simplify notation, the intensity when open can be rewritten as $\mu_{io_u} = \mu_{i_u}$ so that Eq. (7) can be expressed as:

$$\mu_u = \mu_{N_u} \cdot P_{ou}(t) \cdot \mu_{i_u}. \tag{8}$$

Buchberger and Wu (1995) already highlighted the additive nature of mean (and even variance) water demands from homogeneous and nonhomogeneous PRP processes, but their work did not explain how mean intensity and duration values could be computed from available survey-based information. SIMDEUM development years later led to the tabulation of both μ_{N_u} and μ_{i_u} as survey-based parameters in countries like The Netherlands (Blokker et al. 2010). Therefore, the emphasis of this analytical approach must be put in characterizing $P_{ou}(t)$. $P_{ou}(t)$ represents the unitary probability of one opening of an end-use u being open at time t, hence it must consider all the previous instants such that the opening occurs and the end-use duration (i.e. time open) is long enough to keep acting at the evaluated time t. Considering typical end-use duration Cumulative Distribution Functions (CDF), $P_{ou}(t)$ can be computed as:

$$P_{ou}(t) = \begin{cases} P_{ou_d}(t) = \int_{-\infty}^{t} f_j(x) \cdot \big(1 - F_{d_u}(t-x)\big) \cdot dx & \text{if duration follows a lognormal CDF} \\ P_{ou_f}(t) = \int_{t-\mu_{d_u}}^{t} f_j(x) \cdot dx & \text{if duration is a fixed value} \end{cases} \tag{9}$$

According to SIMDEUM model, kitchen taps, bathroom taps, outside taps and shower fixtures follow a lognormal CDF F_{d_u} ($P_{ou_d}(t)$), whereas WCs, bathtubs, dishwashers and washing machines usually discharge water over a fixed-duration ($P_{ou_f}(t)$). $P_{ou_f}(t)$ can be easily computed by integrating $f_j(x)$, which is the per hour slope of the CDF of each type of resident's demand pattern, i.e. it is the probability density function value of the end-use opening instant for each type of inhabitant. CDF and hence $f_j(x)$ computation for different types of end-users is carefully explained based on Table 4 and Figure 1 at Blokker et al. (2010). In that work, five different types of resident exist (adults with job away from home, adults without job away from home, seniors, teenagers and children), and the same are assumed here (i.e. there are five $f_j(x)$ patterns, one for each type of inhabitant). Patterns are here discretized every hour, so 24 $f_j(x)$ values must be considered for each type of end-user along a full day to identify the duration over which the fixed-value end-use is on, but this discretization could be refined if additional reliable pattern information was available. In order to compute $P_{ou_d}(t)$, we can simplify the daily pattern part of the integral in Eq. (9) as a summation for each hour:

$$\begin{aligned} P_{ou_d}(t) &= \int_{-\infty}^{t} f_j(x) \cdot \left(1 - F_{d_u}(t-x)\right) \cdot dx \\ &= \sum_{h=\left[\frac{t-max(d_u)}{3600}\right]+1}^{\left[\frac{t}{3600}\right]+1} f_j(h) \cdot \left(min(t, 3600 \cdot h) - (h-1) \cdot 3600 - \int_{x=(h-1)\cdot 3600}^{x=min(t,3600\cdot h)} F_{d_u}(t-x) \cdot dx \right), \end{aligned} \tag{10}$$

where [] refers to the integer part of its argument, $h \in \mathbb{Z}$, t is expressed in seconds since the beginning of the day (e.g. $t = 36000\ s$ equivalent to 10:00 in the morning) and $max(d_u)$ is the maximum end-use duration, e.g. such associated with a CDF $F_{d_u} = 0.999$. Eq. (10) shows that it is only necessary to solve the defined integral of a lognormal CDF $F_{d_u}(t-x)$ from $a = (h-1) \cdot 3600$ to $b = min(t, 3600 \cdot h)$ in order to calculate $P_{ou_d}(t)$ and thus μ_u. It can be analytically derived that the integral of any lognormal duration CDF is equivalent to:

$$\int_a^b F_{d_u}(t-x) \cdot dx = -\left\{ \left[\Phi\left(\frac{ln(t-x) - \mu}{\sigma} \right) \cdot (t-x) \right]_a^b - e^{\frac{\sigma^2}{2}+\mu} \left[\Phi\left(\frac{ln(t-x) - (\mu + \sigma^2)}{\sigma} \right) \right]_a^b \right\}, \tag{11}$$

where Φ represents a standard normal CDF. Note that μ and σ are the mean and standard deviation of the associated normal distribution. As survey-based parameters in this case (Table 1) correspond to lognormal distributions $(\mu_{d_u}, \sigma_{d_u})$, in this work μ and σ at Eq. (11) have to be computed accordingly.

Instantaneous demand variance

The variance of the sum (Z) of two random variables (X and Y), like in Eq. (1), can be generally written as (Haan 1977):

$$\mathrm{Var}(Z) = \mathrm{Var}(X) + \mathrm{Var}(Y) + 2 \cdot \mathrm{Cov}(X, Y) \tag{12}$$

Independence hypotheses assumed in this work ensure that there is null covariance $\mathrm{Cov}(X, Y)$ among water consumption values, so demand variance at a specific time and level of spatial aggregation ($\sigma^2_{t,s}$) can be computed by adding demand variances of all involved end-uses and inhabitants (see Figure 1):

$$\sigma^2_{t,s} = \sum_{i=1}^{n_{hou}} \sigma^2_{hou_i} = \sum_{i=1}^{n_{hou}} \left(\sum_{j=1}^{n_{hab_i}} \sigma^2_{hab_j} + \sum_{k=1}^{4} \sigma^2_{ktap_k} \right). \tag{13}$$

Eq. (13) is analogous to Eq. (3), so $\sigma^2_{hou_i}$, $\sigma^2_{hab_j}$ and $\sigma^2_{ktap_k}$ represent the water consumption variance for each household, inhabitant and kitchen tap end-use involved, respectively. Note that all variances are referred to previously obtained mean values for the s and t level of aggregation being considered. This means that the variance of each end-use needs to be translated in order to consider mean instantaneous demand as described later on (see Eq. 16).

Accordingly, each occupant's water demand variance can be computed from the end-uses assumed to be available at the household:

$$\sigma^2_{hab_j} = \sum_{l=1}^{2} \sigma^2_{btap_l} + \sum_{m=1}^{2} \sigma^2_{otap_m} + \sigma^2_{wc_n} + \sigma^2_{bath} + \sigma^2_{shower_o} + \sigma^2_{dishw} + \sigma^2_{washm}, \tag{14}$$

whose notation can in turn be simplified by expressing the variance of each inhabitant's water consumption as a summation of demand variances for each end-use:

$$\sigma^2_{hab_j} = \sum_{u=1}^{n_{use}} \sigma^2_u. \tag{15}$$

Note that the variance of each end-use must be computed with respect to the average consumption of each end-use, as obtained before with Eq. (7) or (8). This is due to the fact that the second moment of a variable with respect to a position that is displaced from the origin must take into account the distance between such points (Haan 1977). Similarly to Eq. (7), demand variance for each-end use must be obtained considering two terms that refer to the daily probability of end-use u being on at time t and the associated intensity variance, and the probability of it being closed and the corresponding intensity variance:

$$\sigma_u^2 = \mu_{N_u} \cdot P_{ou}(t) \cdot \left(\sigma_{io_u}^2 + (\mu_{io_u} - \mu_u)^2\right) + (1 - \mu_{N_u} \cdot P_{ou}(t)) \cdot \left(\sigma_{ic_u}^2 + (\mu_{ic_u} - \mu_u)^2\right). \qquad (16)$$

The main difference between this expression and Eq. (7) is that here variance needs to be translated in order to take into account the mean instantaneous demand (μ_u) previously computed with Eq. (8). In order to simplify notation, parameters that characterize water intensity when the end-use is open can be written as $\mu_{io_u} = \mu_{i_u}$ and $\sigma_{io_u}^2 = \sigma_{i_u}^2$, as it can be assumed that $\mu_{ic_u} = \sigma_{ic_u}^2 = 0$. Eq. (16) can then be written as:

$$\sigma_u^2 = \mu_{N_u} \cdot P_{ou}(t) \cdot \left(\sigma_{i_u}^2 + (\mu_{i_u} - \mu_u)^2\right) + (1 - \mu_{N_u} \cdot P_{ou}(t)) \cdot \mu_u^2, \qquad (17)$$

where $P_{ou}(t)$ can be calculated according to Eqs. (9)-(11).

Note that this analytical approach helps to characterize the traditional SIMDEUM model, as it requires exactly the same input parameters, but it enables to explicitly compute mean instantaneous demand and variance values (statistical properties). Such a simplified approach could be used for many applications where based on a stochastic model, water demand statistics are required, like generating mean values, designing rules for maximum flows (Buchberger et al. 2012) or considering the probability of flow stagnation (Blokker et al. 2008).

CASE STUDY AND SPATIAL SCALE EFFECT ANALYSIS

The analytical approach for stochastic demand modelling is here applied to Benthuizen case study, presented in the literature before by Blokker et al. (2011a). The selected network is a test area (circa 140 homes, 300 inhabitants) located within the town of Benthuizen (The Netherlands), which is convenient given that SIMDEUM was conceived over ten years ago for Dutch cases of application (Blokker and Vreeburg 2005) and thus most of the available survey-based parameters (needed to run both SIMDEUM and the analytical approach here presented) are associated with this country. According to the original publication, water flow had been measured in this town in 2004 and 2006 (Beuken et al. 2008) to prove that the network had no leaks, but it was not until July 2007 that flow was measured and a tracer was dosed at the entrance of the test area with the aim of comparing top-down and bottom-up demand allocation models. More specifically, Blokker et al. (2011a) used these measurements to compare the effect of stochastic demand modelling in terms of demand multiplier patterns and residence times for a branched and looped network configuration.

In this work, the analytical approach is validated by comparing it to previous Benthuizen results. The model is run with Benthuizen's population characteristics (Blokker et al. 2011a) and Dutch residential survey-based parameters (Blokker et al. 2010), which are here gathered in Table 1 to illustrate the variability included in the model. Note that variances in Table 1 have been computed according to Blokker et al. (2010) guidelines. This publication indicates that intensity variance can be assumed null for appliances with fixed intensity (WC, bathtub, shower, dishwasher and washing machine), but it can be computed assuming uniform distributions with null minimum intensity and twice the mean maximum intensity for taps. In what regards duration variances, the same authors recommend to consider the variance equal to 130% the mean value for taps, and 50% the mean value for showers when assuming a lognormal duration CDF. Therefore, parameters in Table 1 only need to be complemented with $f_j(x)$ values from Blokker et al. (2010) (i.e. slopes of daily pattern CDF), which represent the average daily pattern for each type of end-user, in order to run the analytical approach here presented.

The analytical solution obtained with 2011's parameters is then compared to every-minute water flow measurements at the entrance to the test area, which equates to the aggregated demand for the full neighbourhood thanks to the fact that the area is free of leaks. In Blokker et al. (2011a) measurements were made over 7 days, but only the associated weekdays are here considered in order to avoid significant changes in daily patterns and/or model parameters. Note that Blokker et al. (2010) provide specific patterns for weekends, but duration, intensity and frequency parameters are kept the same regardless of the day of the week. As this assumption may lead to deviations in results, only weekdays (24th-27th and 30th July 2007) are considered in this paper. Systematic measurements start in the afternoon of 24th July and finish in the morning on 30th July. As indicated by Blokker et al. (2011a), all measurements are subtracted 8.67 l/min (i.e. 520 l/h) to account for the additional demand that was induced in the experimental campaign to maintain the minimum flow required by an electrical conductivity monitor system.

Before presenting results, it must be highlighted that in this work the same SIMDEUM parameters that were fitted by Blokker et al. (2011a) have been assumed. This paper's goals do not include model calibration, which is a relevant issue given the high number of survey-based input parameters. The present work focuses on analytically computing main statistics from a stochastic urban water consumption model, characterizing the statistical properties of such instantaneous consumptions. The advance of these results is that they are an improved approach compared to using just a single Monte Carlo Simulation or the average of a reduced number of Monte Carlo simulations (as in Blokker et al. 2010).

Analytical model validation

Fig. 2 gathers results at the entrance of Benthuizen case study on the days here considered. In terms of measurements, graphs show each day's flow-meter signal at the entrance of the test area (location 1 in Blokker et al. 2011a). In what regards the analytical model, each figure includes the aggregated mean demand-value and its associated 95% confidence interval for the whole neighbourhood under study, which can be computed with the corresponding variance. Note that thanks to the analytical approach, mean values and confidence intervals have only been computed once (i.e. parameters are the same for all weekdays) and they are only repeated in all figures for representation purposes. Also, it must be highlighted that the analytical model has been run only once every minute (at the central second of each minute) to accelerate the process. This implies that every second within each minute is assumed to have the same behaviour, which is reasonable given that $f_j(x)$ values (i.e. the daily pattern) have been discretized every hour.

In order to quantify agreement among records and the present model, run with Blokker et al. (2011a) parameters summarized in Table 1, percentages of exceedance are computed with respect to the confidence interval. Percentages of the number of times that metered signals surpass confidence interval thresholds have been computed, with values of 2.22%, 2.29%, 3.13%, 4.62% and 4.01% for each day, respectively. As these values are all below 5%, they are the first indication that the analytical approach represents well Benthuizen reality considering a 95% confidence level. Note that a distribution function comparison test is not possible here beacuse there are only a few days of measurements available, but statistical tests could be carried out if the sample size was greater. In any case, such an analysis would be recommended after convenient calibration.

The average time required for the proposed methodology to compute mean and variance values for the whole neighbourhood in an Intel Core i7-6700 CPU 3.40 GHz 16 GB RAM desktop computer (using Matlab R2016a) is 0.6 seconds. This value is negligible with respect to the computational cost of a Monte Carlo simulation and its required sensitivity analysis to ensure that the effect of the number of simulations on mean and variance values can be disregarded (Blokker et al. 2011a), which would further increase if the network extended. This makes the approach affordable for real-time applications in similarly sized networks.

Upstream network aggregation spatial scale effect in instantaneous flow variability

Now that the methodology has been validated for Benthuizen case study, analytical results will be further explored in order to illustrate how the approach can contribute to make the transition from the household level (micro-scale) to the network dimension (macro-scale) and vice versa easier. In order to analyse spatial scale effects in instantaneous flow variability, theoretical conditions must first be stated. Suppose the case that there is an entity that includes N independent elements with the same mean (μ), variance (σ^2) and coefficient of variation ($CV = \sigma/\mu$). It can be assumed that:

$$\mu_{tot} = N \cdot \mu \tag{18}$$

$$\sigma_{tot}^2 = N \cdot \sigma^2 \quad (19)$$

$$CV_{tot} = \frac{\sigma_{tot}}{\mu_{tot}} = \frac{1}{\sqrt{N}} \cdot CV, \quad (20)$$

where μ_{tot}, σ_{tot}^2 and CV_{tot} represent the resulting mean, variance and coefficient of variation values for the whole entity. This means that CV_{tot} is affected by the problem scale, following a law in which CV is inverse to the square root of the problem dimension. Henceforth the representation of cumulative coefficients of variation versus number of entities in a double logarithmic scale will always be associated with a straight line with a slope of -0.5. This tendency implies that as the number of elements (i.e. the level of aggregation) increases, the associated CV reduces. In terms of demand within water supply systems, this means that demand uncertainty relatively diminishes when zooming out of individual households. This behaviour of the coefficient of variation decreasing as the problem dimension increases is well-known at present (e.g. Blokker et al. 2008; Vertommen et al. 2012) and highlights the importance of monitoring the intrinsic stochastic uncertainty associated with demand values at hydraulic models. However, the power of this law has not been studied yet. It is -0.5 (fractal dimension) only under the aforementioned assumptions. The real exponent might differ since real end-uses do not all have equal mean and variance.

Table 1 shows that very different end-uses in terms of frequency, duration and intensity co-exist in a real network. Moreover, Tables 2 and 3 provide the relative contribution of each end-use to the overall mean and variance Benthuizen demand (i.e. flow at the entrance), respectively. Note that mean (Eq. 3) and variance (Eq. 13) values are computed in the analytical approach as a summation. Tables 2 and 3 show the percentage of each end-use contribution for the whole neighbourhood at three different instants of day (night - at 02:00 -, morning - at 08:00 - and evening - at 20:00 -) and the daily average. These results show that shower, WC, kitchen tap and washing machine end-uses dominate both mean and variance water consumption at this case study, no matter the time. Because these end-uses are quite different among them, the assumption of adding similar terms, based on which Eq. (20) is obtained, cannot be made. The approach here presented can be used to analyse the scale effect in coefficients of variation for real water distribution networks, where heterogeneous uses coexist. Note that, from now on, spatial scaling effect analysis will focus on CV because this variable represents the ratio between standard deviation and mean values, i.e. the relative deviation of water demands. This choice is also justified by the fact that the engineering community usually works with average demand scenarios, and having an estimation of CV could help to compute standard deviations (i.e. assess variability) from values typically used in practice.

Fig. 3 summarises how the demand coefficient of variation changes with the number of inhabitants, the number of households and mean demand values according to analytical results for Benthuizen case study at three different instants within the day (night - at 02:00 -, morning - at 08:00 - and evening - at 20:00 -). First and second rows of the figures show the computed cumulative CV values versus the number of inhabitants and households, respectively, in the test area and their associated line of fit in a double logarithmic scale. The adjusted lines have a slope of approximately -0.5 no matter the time. This implies that even when real systems are heterogeneous, the scale power law keeps approximately equal to -0.5. The slight deviation with respect to the theoretical -0.5 value is because different types of end-uses and occupants (with different hourly patterns) coexist at Benthuizen, but the tendency clearly remains and the deviation from -0.5 is not high. The third row in Fig. 3 shows demand CV values versus mean demand values according to the analytical model here presented, and their associated lines of fit. These graphs show that dispersion has increased with respect to previous rows, but they also prove that it can still be roughly assumed that demand coefficients of variation keep a relationship with mean demands to the power of -0.5. In order to show that values in Fig. 3 are not the result of chance, Fig. 4 provides relative frequency histograms for the aforementioned slopes (i.e. exponents of power laws) every minute. These graphs illustrate that even though there are slight variations around the theoretical value, a -0.5 value can be assumed to compute demand uncertainty in absence of better data.

Assuming power laws can speed up the extension of scope from individual elements (e.g. fixtures or households) to whole areas within the system. Note that traditional pulse models, like first family's PRP household-based models or second family's microcomponent-based SIMDEUM method, can also be repeatedly applied to individual households (i.e. micro level) in order to progressively extend demand uncertainty analysis to the full network reality or macro level (bottom-up approach). This process would be expensive, as it would require massive experimental campaigns or computational cost, respectively, but it can be done. However, the change from the macro to the micro spatial level cannot be afforded with existing methodologies, as they are based on individual parameters for specific households and end-uses, respectively. This means that up until now there was no straightforward manner of estimating high resolution demand uncertainty from average demand scenarios (top-down approach). In other words, it was not possible to complement top-down demand allocation strategies with bottom-up-based demand uncertainty estimations. Results from the analytical model here presented have shown that power laws could potentially be assumed to determine demand coefficients of variation (i.e. variance values) from average scenarios, bridging the gap between bottom-up and top-down approaches. Such a result can have interesting applications in real water networks. For example, if a similar branched water supply system (e.g. another part of Benthuizen's supply network - out of the test area -) with a flow meter at the entrance (i.e. known μ_{FM}, σ^2_{FM} and CV_{FM}, with FM for flow meter) was under study, an equation to obtain water flows coefficients of variation (CV) from mean values (μ) at any point of the system could be derived:

$$\log(CV) \approx \log(CV_{FM}) - 0.5 \cdot \log\left(\frac{\mu}{\mu_{FM}}\right), \tag{21}$$

which is the same that

$$CV \approx CV_{FM} \cdot \left(\frac{\mu}{\mu_{FM}}\right)^{-0.5} \tag{22}$$

These equations are only valid for branched systems, where flows at any point are equal to the aggregated downstream water demand assuming that there are no leaks. The model could even be improved by including a new end-use to simulate leakage, as proposed by Blokker et al. (2010). Using Eq. (21) or (22) implies assuming a uniform (i.e. uniformly heterogeneous) behaviour throughout the system, which may be a sufficient approximation at some locations (e.g. residential neighbourhoods) but not good enough at other areas (e.g. areas with important industry or public buildings). If the region where the method was to be applied was clearly non-uniform, the analytical model would have to be run according to realistic survey-based duration, intensity and frequency parameters. However, if the area can be assumed uniform, analytical equations could be used to better understand and quickly estimate demand uncertainty throughout the system. Note that Eqs. (21)-(22) are consistent with Fig. 3, as they imply that flow or aggregated demand coefficients of variation increase at water supply systems endpoints (i.e. low demands, low flows). This implies that in the periphery of the network, real water flows never correspond to average flows and thus, it is essential to assess not only mean values but also associated variances. The approach is useful to understand how variance statistical property of water consumption changes spatially, enabling spatial disaggregation of such properties under the assumption of similar water use patterns in the disaggregated area. Moreover, in case no better data are available, Benthuizen results (Figure 3) can be used to estimate consumption variability in areas with similar water end-users distribution and patterns. Nevertheless, it is always preferred to develop experimental campaigns for systematic validation.

This example only aims to show the reader one possible use of analytically based stochastic demand models, but alternative applications could be developed based on the approach and results here presented. For example, a simplified method to estimate flow uncertainty from mean demands at looped networks could be developed, or a methodology to systematically calibrate the analytical stochastic demand model based on historical billing information could be posed. Also, changing population habits (e.g. as a result of climate change) or seasonal variations of demand could be easily incorporated in the analytical approach to assess the long-term performance of water supply systems. These applications are only mentioned for the purpose of motivation, but they are out of the scope of this paper and they are a subject for further research. Note that the present approach provides mean and variance of instantaneous water demands, but it is not straightforward to temporally aggregate that information, because time series coming from the model are affected by correlation (Creaco et al. 2019), and this has to be taken into account when developing the analytical formulation of temporally aggregated consumption. Therefore, temporal aggregation is out of the scope of this paper.

CONCLUSIONS

This paper analyses network spatial scale effects in water supply system demands thanks to a novel analytical approach to stochastic demand modelling. The proposed methodology belongs to a family of pulse models that computes residential water demand on a household basis by aggregating microcomponents or end-uses at each home (kitchen tap, bathroom tap, outside tap, WC, bathtub, shower, dishwasher and washing machine). The approach is presented to complement the use of the recognized SIMDEUM model, which runs Monte Carlo simulations to estimate stochastic demand patterns based on easy-to-obtain survey-based parameters. As Monte Carlo simulations may be time-consuming for large urban areas, a set of analytical expressions is derived in this work to compute the mean and variance of instantaneous demands produced by the stochastic model. This is possible by assuming independence in terms of water consumption among end-uses, inhabitants and households. The method is essentially posed as a summation of individual and independent uses of water, which can be easily extended to consider different levels of spatial aggregation.

The analytical approach has been here applied to Benthuizen case study by making use of a SIMDEUM model fitting previously obtained in the literature. Analytical results are further explored, empirically demonstrating that a power law can be used to relate water demand flow variations at different scales. The scale power law has been found approximately constant for different scale variables and times of day, and even when reality is set by heterogeneous end-uses, the power keeps equal to the theoretical case where all end-uses are similar. This simplifies the application of the method to cases where specific scale analyses have not been performed yet, and it highlights the necessity of considering real variability rather than only average conditions at network endpoints. This conclusion can in turn be used to derive analytical equations that facilitate the micro to macro spatial change of scale (and vice versa) for coefficient of variation characterization, always under the assumption of homogeneous water end-user distribution and patterns. The approach here presented can, like other existing stochastic models, be utilized to progressively aggregate downstream water demands (i.e. micro to macro level) in a more efficient way. According to results after fitting the coefficient of variation spatial scale law, instantaneous demand variance can be computed from mean values, so high resolution spatial flow variability can be determined from average flow network scenarios. This makes the novel bottom-up approach a valuable asset to assist the traditional top-down demand allocation process, which could then consider demand variability and thus better simulate local pressures or water quality in the periphery of water supply systems, among other uses.

DATA AVAILABILITY STATEMENT

Some or all data, models, or code used during the study were provided by a third party (Benthuizen measurements). Direct requests for these materials may be made to the provider as indicated in the Acknowledgements.

ACKNOWLEDGEMENTS

The authors want to thank Dr. Mirjam Blokker for providing the Benthuizen case study measurements.

REFERENCES

Alvisi, S., Franchini, M., and Marinelli, A. (2003). "A stochastic model for representing drinking water demand at residential level." *Water Resour. Manage.*, 17(3), 197–222.

Beuken, R., Lavooij, C., Bosch, A., and Schaap, P. (2008). "Low leakage in the Netherlands confirmed." *Water Distribution Systems Analysis Symposium 2006*, S. G. Buchberger, R. Clark, W. Grayman, and J. Uber, eds., American Society of Civil Engineers.

Blokker, E., Beverloo, H., Vogelaar, J., Vreeburg, J., and van Dijk, J. (2011a). "A bottom-up approach of stochastic demand allocation in a hydraulic network model: a sensitivity study of model parameters." *J. Hydroinf.*, 13(4), 714–728.

Blokker, E., Buchberger, S., Vreeburg, J., and van Dijk, J. (2009). "Comparison of water demand models: PRP and SIMDEUM applied to Milford, Ohio, data." *Water Distribution Systems Analysis 2008*, K. Van Zyl, ed., American Society of Civil Engineers.

Blokker, E., Pieterse-Quirijns, E., Vreeburg, J., and van Dijk, J. (2011b). "Simulating nonresidential water demand with a stochastic end-use model." *J. Water Resour. Plann. Manage.*, 137(6), 511–520.

Blokker, E. and Vreeburg, J. (2005). "Monte Carlo simulation of residential water demand: A stochastic end-use model." *World Water and Environmental Resources Congress, Alaska, USA*.

Blokker, E., Vreeburg, J., Buchberger, S., and van Dijk, J. (2008). "Importance of demand modelling in network water quality models: A review." *Drink. Water Eng. Sci.*, 1, 27–38.

Blokker, E., Vreeburg, J., and van Dijk, J. (2010). "Simulating residential water demand with a stochastic end-use model." *J. Water Resour. Plann. Manage.*, 136(1), 19–26.

Blokker, M., Agudelo-Vera, C., Moerman, A., van Thienen, P., and Pieterse-Quirijns, I. (2017). "Review of applications of SIMDEUM, a stochastic drinking water demand model with small temporal and spatial scale." *Drink. Water Eng. Sci.*, 10, 1–12.

Buchberger, S., Blokker, M., and Cole, D. (2012). "Estimating peak water demands in hydraulic systems I - Current practice." *Proceedings of 14th Water Distribution Systems Analysis Symposium, Adelaide, Australia*.

Buchberger, S. and Wells, G. (1996). "Intensity, duration and frequency of residential water demands." *J. Water Resour. Plann. Manage.*, 122(1), 11–19.

Buchberger, S. and Wu, L. (1995). "Model for instantaneous residential water demands." *J. Hydraul. Eng.*, 121(3), 232–246.

Creaco, E., Blokker, M., and Buchberger, S. (2017a). "Models for generating household water demand pulses: Literature review and comparison." *J. Water Resour. Plann. Manage.*, 143(6), 04017013.

Creaco, E., De Paola, F., Fiorillo, D., and Giugni, M. (2019). "Bottom-up generation of water demands to preserve basic statistics and rank cross-correlations of measured time series." *J. Water Resour. Plann. Manage.*, 146(1).

Creaco, E., Farmani, R., Kapelan, Z., Vamvakeridou-Lyroudia, L., and Savic, D. (2015). "Considering the mutual dependence of pulse duration and intensity in models for generating residential water demand." *J. Water Resour. Plann. Manage.*, 141(11), 04015031.

Creaco, E., Pezzinga, G., and Savic, D. (2017b). "On the choice of the demand and hydraulic modeling approach to WDN real-time simulation." *Water Resour. Res.*, 53(7), 6159–6177.

Díaz, S., Mínguez, R., and González, J. (2016). "Stochastic approach to observability analysis in water networks." *Ingeniería del Agua*, 20(3), 139–152.

Díaz, S., Mínguez, R., and González, J. (2018a). "Topological state estimation in water distribution systems: Mixed-integer quadratic programming approach." *J. Water Resour. Plann. Manage.*, 144(7), 04018026.

Díaz, S., Mínguez, R., González, J., and Savic, D. (2018b). "Explicit expressions for state estimation sensitivity analysis in water systems." *J. Water Resour. Plann. Manage.*, 144(4), 06018001.

Filion, Y., Karney, B., Moughton, L., Buchberger, S., and Adams, B. (2008). "Cross correlation analysis of residential demand in the city of Milford, Ohio." *Water Distribution Systems Analysis Symposium 2006*, S. G. Buchberger, R. Clark, W. Grayman, and J. Uber, eds., American Society of Civil Engineers.

Haan, C. (1977). *Statistical methods in hydrology*. The Iowa State University Press / Ames, Iowa, USA.

Kumar, S. M., Narasimhan, S., and Bhallamudi, S. M. (2008). "State estimation in water distribution networks using graph-theoretic reduction strategy." *J. Water Resour. Plann. Manage.*, 134(5), 395–403.

Mahmoud, H., Kapelan, Z., and Savic, D. (2018). "Real-time operational response methodology for reducing failure impacts in water distribution systems." *J. Water Resour. Plann. Manage.*, 144(7).

Morton, R. and Henderson, B. (2008). "Estimation of nonlinear trends in water quality: An improved approach using generalized additive models." *Water Resour. Res.*, 44(7), W07420.

Pérez-Sánchez, M., Sánchez-Romero, F., and López-Jiménez, P. (2017). "Energy footprint of water depending on consumption patterns in pressurized water networks." *Ingeniería del Agua*, 21(3), 197–212.

Sanz, G., Pérez, R., Kapelan, Z., and Savic, D. (2016). "Leak detection and localization through demand components calibration." *J. Water Resour. Plann. Manage.*, 142(2), 04015057.

Vertommen, I., Magini, R., Cunha, M., and Guercio, R. (2012). "Water demand uncertainty: the scaling laws approach." *Water supply systems analysis: selected topics*, A. Ostfeld, ed., IntechOpen, 105–129.

Wright, R., Abraham, E., Parpas, P., and Stoianov, I. (2015). "Control of water distribution networks with dynamic DMA topology using strictly feasible sequential convex programming." *Water Resour. Res.*, 51(12), 9925–9941.

Zhang, H., Wang, B., Liu, D., Zhang, M., Feng, P., Cheng, L., Yu, Q., and Eamus, D. (2019). "Impacts of future climate change on water resource availability of eastern Australia: A case study of the Manning river basin." *J. Hydro.*, 573, 49–59.

List of Tables

TABLE 1. Frequency, duration and intensity parameters for Benthuizen case study (Blokker et al. 2010)

End-use type / subtype		Frequency: Mean number of openings per day and inhab. μ_{N_u}	Duration: Distrib.	Duration: Mean (s) μ_{d_u}	Duration: Var (s^2) $\sigma^2_{d_u}$	Duration: Others	Intensity: Distrib.	Intensity: Mean (l/s) μ_{i_u}	Intensity: Var (l^2/s^2) $\sigma^2_{i_u}$
Kitchen tap	Consumption	4.73	Lognormal	16	20.8		Uniform	0.083	0.0023
	Doing dishes	3.15	Lognormal	48	62.4		Uniform	0.125	0.0052
	Washing hands	3.15	Lognormal	15	19.5		Uniform	0.083	0.0023
	Others	1.58	Lognormal	37	48.1		Uniform	0.083	0.0023
Bathroom tap	Washing and shaving	1.35	Lognormal	40	52		Uniform	0.042	0.0006
	Brushing teet	2.75	Lognormal	15	19.5		Uniform	0.042	0.0006
Outside tap	Garden	0.33	Lognormal	300	390		Uniform	0.1	0.0033
	Other	0.11	Lognormal	15	19.5		Uniform	0.1	0.0033
WC	9L	6	Fixed	216			Fixed	0.042	
	9L with water saving	6	Fixed	108			Fixed	0.042	
	6L	6	Fixed	144			Fixed	0.042	
	6L with water saving	6	Fixed	72			Fixed	0.042	
Bathtub		0.044	Fixed	600			Fixed	0.2	
Shower	No water saving	0.7	Lognormal	510	255		Fixed	0.142	
	With water saving	0.7	Lognormal	510	255		Fixed	0.123	
Dishwasher		0.3	Fixed	21/cycle		4 cycles over 7200 s	Fixed	0.167	
Washing machine		0.3	Fixed	75/cycle		4 cycles over 7200 s	Fixed	0.167	

TABLE 2. Instantaneous relative contribution of each end-use to Benthuizen demand mean values according to analytical model results.

	Instantaneous relative contribution (%)			
End-use	02:00	08:00	20:00	Daily average
Kitchen tap	11.83	12.05	12.1	11.97
Bathroom tap	3.17	3.32	3.26	3.20
Outside tap	4.79	5.03	4.93	4.83
WC	26.20	27.49	27.00	26.44
Bath	1.18	1.24	1.24	1.18
Shower	37.21	39.03	38.31	37.55
Dishwasher	2.21	1.26	1.57	1.80
Washing machine	13.41	10.57	11.59	13.03

TABLE 3. Instantaneous relative contribution of each end-use to Benthuizen demand variance values according to analytical model results.

	Instantaneous relative contribution (%)			
End-use	02:00	08:00	20:00	Daily average
Kitchen tap	14.77	15.44	15.38	15.09
Bathroom tap	1.56	1.68	1.64	1.59
Outside tap	5.63	6.06	5.90	5.73
WC	9.66	10.28	9.98	9.76
Bath	2.08	2.24	2.22	2.10
Shower	43.33	46.45	45.17	44.02
Dishwasher	3.25	1.91	2.35	2.64
Washing machine	19.72	15.95	17.36	19.06

List of Figures

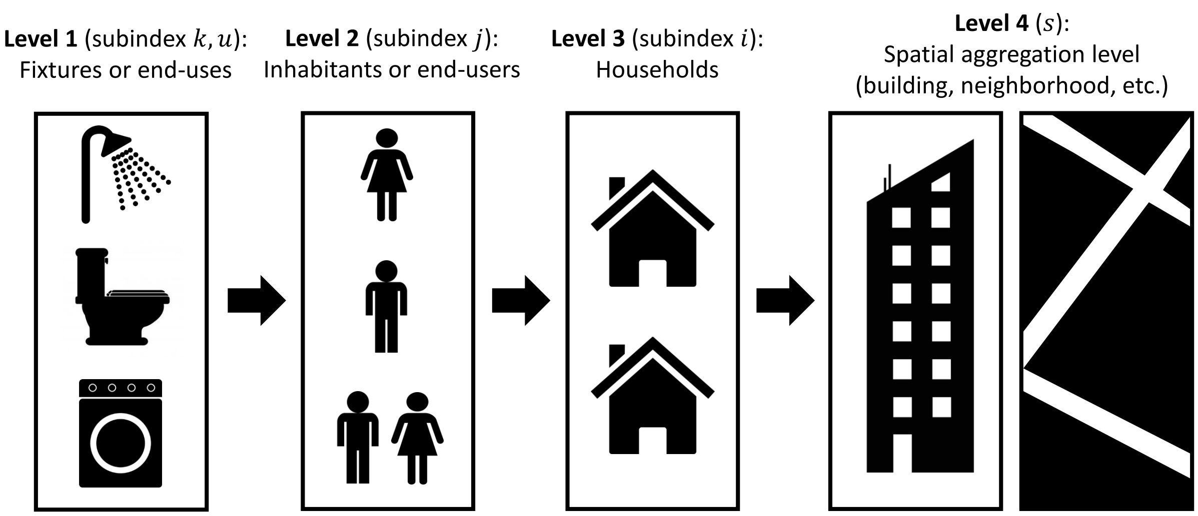

Fig. 1. Spatial hierarchy of elements and agents used to compute mean and variance of instantaneous residential water demands. Subindices k and u refer to the possible uses of the kitchen tap (end-use per household) and the end-uses per inhabitant (respectively), j to inhabitants, i to households and s to the spatial aggregation level being considered.

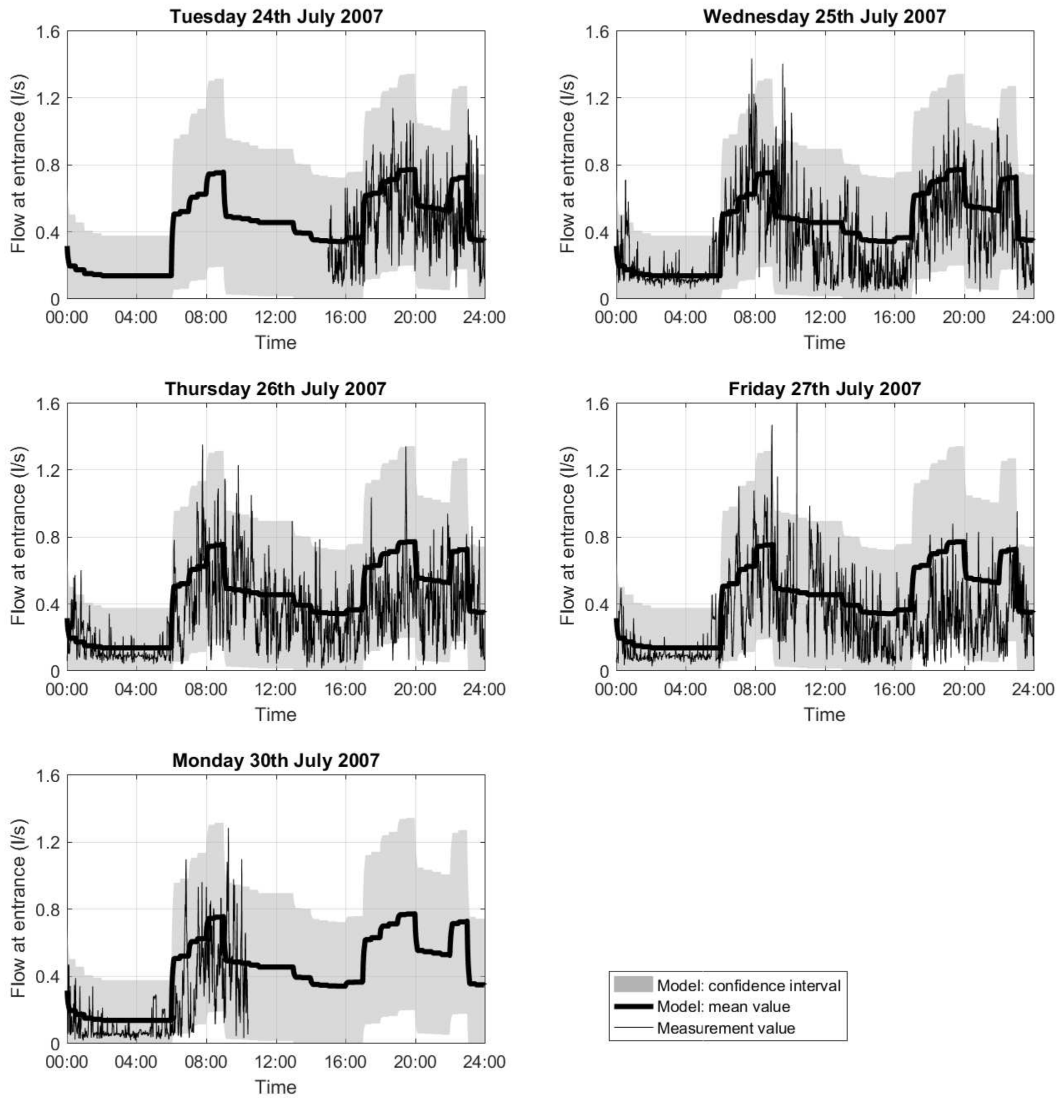

Fig. 2. Daily evolution of water flow at the entrance of Benthuizen test area: analytical model results (mean and 95% confidence interval) vs flow measurements on 24th-27th and 30th July 2007.

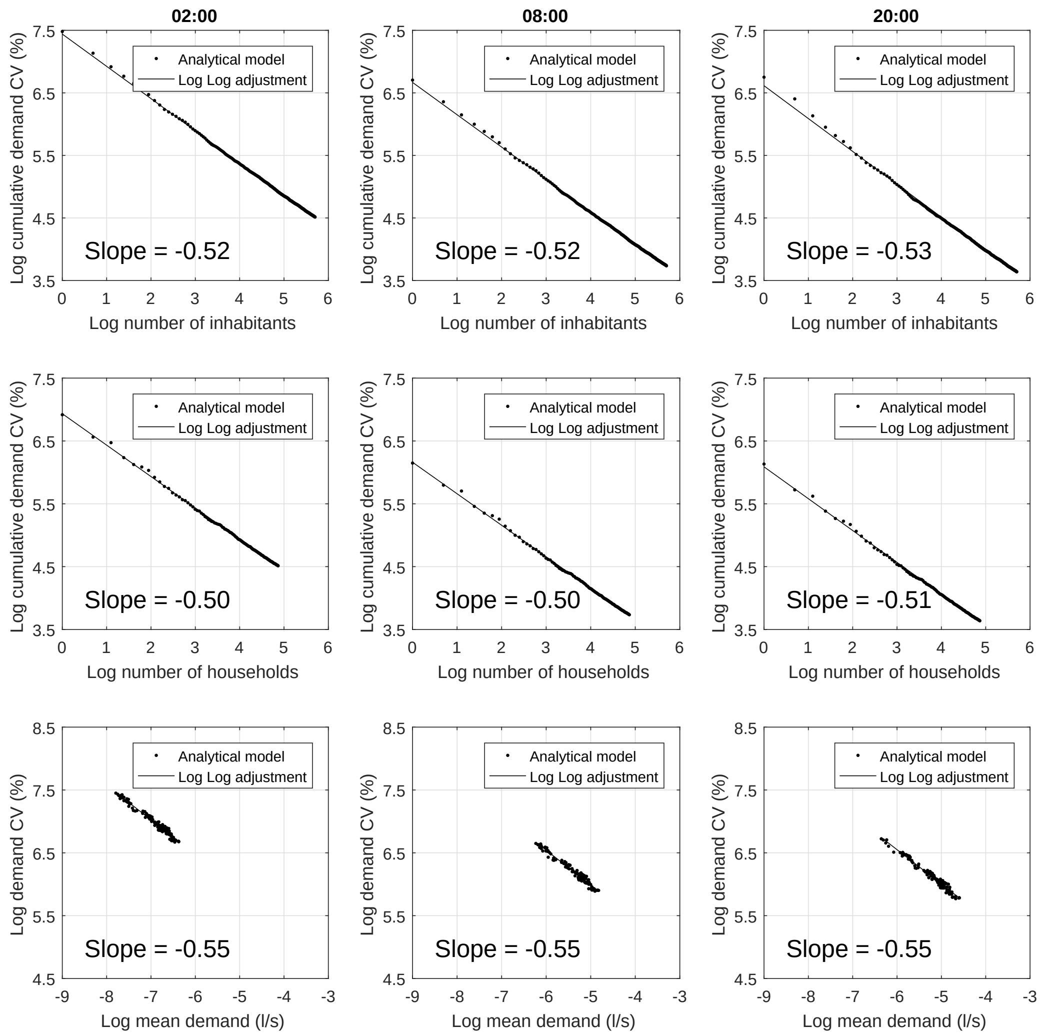

Fig. 3. Logarithmic representation of analytical model results for: (a) cumulative demand CV vs. number of inhabitants (first row), (b) cumulative demand CV vs. number of households (second row), and (c) demand CV vs. mean demand (third row) at Benthuizen case study.

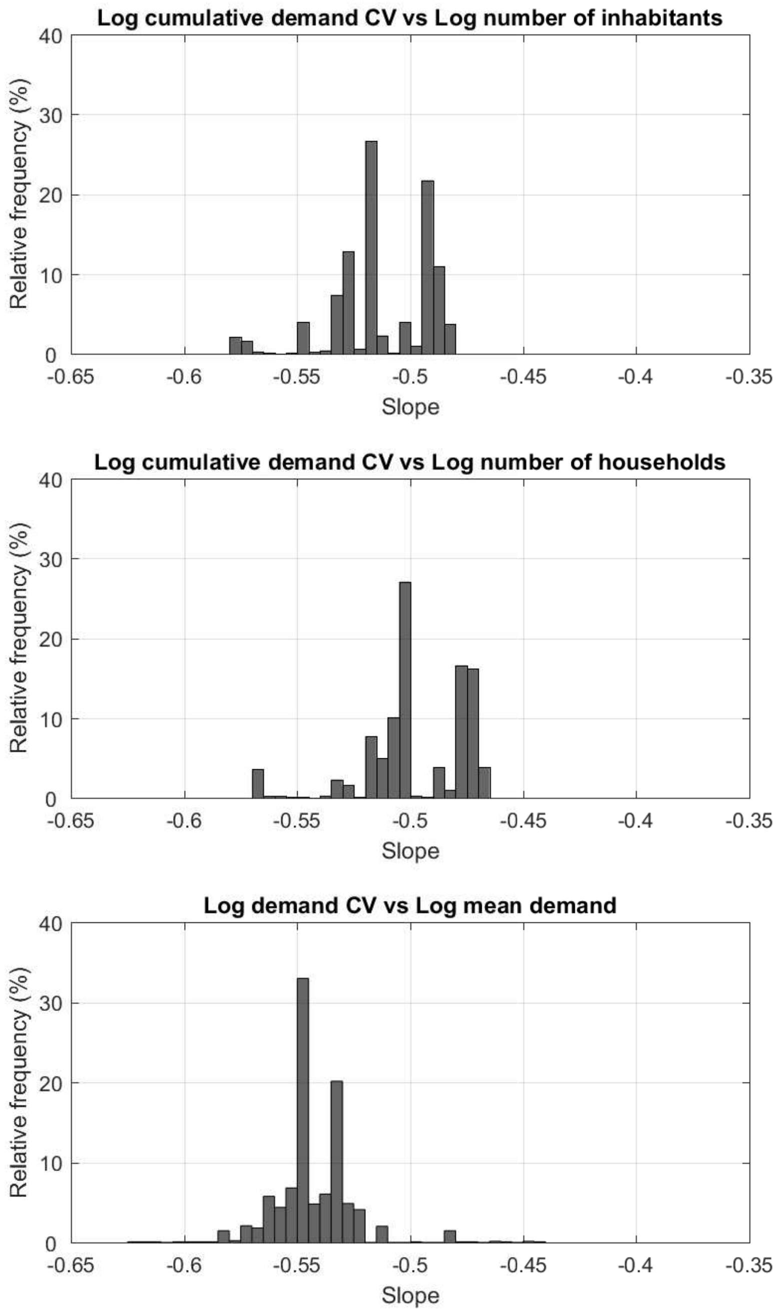

Fig. 4. Relative frequency histograms for logarithmic slopes (i.e. exponents of power laws) resulting from the analytical model for: (a) cumulative demand *CV* vs. number of inhabitants (first graph), (b) cumulative demand *CV* vs. number of households (second graph), and (c) demand *CV* vs. mean demand (third graph) at Benthuizen case study.