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*Author:*
**John, Harriet R**

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*exploring networking strategies for the work of Bakhtin and Vygotsky through teacher and student perspectives on mathematical methods*

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# Connecting Theories: Exploring networking strategies for the work of Bakhtin and Vygotsky through teacher and student perspectives on mathematical methods

Harriet John

A dissertation submitted to the University of Bristol in accordance with the requirements for award of the degree of Doctor of Philosophy (PhD) in the Faculty of Social Sciences and Law.

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**Abstract**

My thesis began as a practical problem addressing the undervaluing of informal mathematical language and methods used by low prior attaining students. I wanted to gather teacher and student perspectives on mathematical methods. I began by exploring the dialogic theory of Bakhtin but discovered a debate in the field about whether Bakhtin's work could be used as an extension of Vygotsky's dialectic theory. As a result, I used Radford's connecting theories framework (2008) to shape an investigation which explores principles, methodology and research questions as points of connection between the theories of Vygotsky and Bakhtin. Linking the networking approaches of Prediger et al. (2008), to Radford's connecting theories allowed me to analyse the work of other authors in the field and develop my own analytical framework based on Vygotsky and Bakhtin. I used this framework both to analyse transcriptions of teacher group discussion and student group discussion based around examples of student work, and to explore networking approaches. Initially, I used "comparing"/ "contrasting" approaches to extend my understanding of Bakhtin's and Vygotsky's theories before adopting a "combining" networking approach to further investigate the perspectives of students on mathematical methods. I used a dialectic approach to represent the significance of the curriculum in discussion around mathematical methods, and a dialogic approach to analyse the detail of how the context and socio-cultural background shapes impacts on discussion. I concluded that a connecting theories approach allowed for analysis of more data and a deeper level of analysis than using a single theory. Through connecting theories, I also investigated the possibility of analysing mathematical methods as utterances using Bakhtin's work. I suggest that Bakhtin and Vygotsky's theories can be effectively networked to provide analysis and suggest a number of future steps to either apply the networked theories to practical problems or further theoretical issues.

**Dedication**

For Grandma

The wisest woman I know

**Acknowledgements**

I would like to thank the following people without whom my studying and this thesis would not have been possible:

My examiners, for taking the time to read my work and for your feedback.

My supervisors, for your unfailing patience and always asking the right questions. It has been a privilege to work with you and I cannot express how grateful I am for all of your help and support.

My colleagues and students, for your contribution to and interest in this project. Your support has been amazing.

My Mum, for being my rock. You have shown me that anything is possible if you are determined enough and prepared to work hard.

My Dad, for always backing me and for reading every word.

My family and friends, for putting up with cancelled plans, always listening, and sending many, many "are you OK?" messages.

My English teacher, for teaching me I could write.

Lastly, my Grandparents. The importance they placed on education has shaped the lives and learning of the whole family. Also, for asking me if it was finished every time I called for the last two years!

## COVID Impact Statement

In accordance with University of Bristol guidelines, I hereby include a statement on the effects of COVID-19 on my research.

As you will see from my initial plan (Chapter Five), this project was initially intended to have three main stages of research. The first was the creation of artefacts by students, the second was a series of student and teacher groups and the third was the researcher going into lessons and researching in the classroom. Stages one and two were carried out successfully. Stage three depended on the analysis of stage two. Once the analysis of stage two was completed, I could see if a third stage would be necessary in order to conclude my study. Unfortunately, the restrictions introduced by COVID-19 meant that this third stage was removed as a possibility.

Initially, schools closed so students were not present in classroom situations during the term in which I would have been collecting this data. Once schools returned, restrictions such as the need for staff to socially distance from students, limitations on what work could be collected and the additional pressures for staff of teaching in a COVID-safe environment continued to make any in class data collection unrealistic. These restrictions remained in place for the final eighteen months of my PhD programme and limited my research insofar as they removed the option of collecting data that would have firmly rooted my study in the classroom. However, an element of the teacher-student exchange was visible during analysis of the student groups and, as such, that area of the analysis was not completely closed to this report.

In addition to the practical constraints imposed by the pandemic, there was a significant personal impact. Lockdown meant living in isolation for months whilst supporting struggling students and their families remotely. My already demanding job teaching and working in a Sixth Form pastoral role expanded to fill weekends and holidays. Cancelled exams, results day chaos due to government U-turns and, once we returned to school, the difficulties of operating a COVID-safe environment have made the last eighteen months some of the most difficult of my teaching career. This had a knock-on impact on my studying.

I submit this statement to summarise the practical and personal impact of the pandemic on my PhD project.

**Author's Declaration**

I declare that the work in this dissertation was carried out in accordance with the requirements of the University's *Regulations and Code of Practice for Research Degree Programmes* and that it has not been submitted for any other academic award. Except where indicated by specific reference in the text, the work is the candidate's own work. Work done in collaboration with, or with the assistance of, others, is indicated as such. Any views expressed in the dissertation are those of the author.

SIGNED: H John DATE: 16/8/21

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## How To Read This Thesis

I wish to offer a few points and additional information which might help the reader navigate this work:

1. The study is progressive, exploring theory and methodology throughout. As such, it follows a slightly unusual format. There are three subsections – Principles, Methodology and Research Questions, each of which is explained in Chapter One. The title of each chapter indicates which subsection it is part of.

2. Due to the unusual format, research questions do not feature until close to the end of the study. This is intentional and, again, explained in more detail in Chapter One.

3. For certain key terms e.g., internalization, I have retained the American spelling used in the literature for consistency.

4. Certain authors appear in translation from Cyrillic e.g., Vygotsky, Bakhtin, Vološinov. For these authors, I have used the spellings from the printed works I have, which means there is occasional variation in spelling between works e.g., some authors use Voloshinov rather than Vološinov.

5. As this thesis represents a culmination of several years of research, previous work of mine has influenced the writing. I have referenced these pieces where I have revisited an idea they raised e.g., John, 2106 in section 1.2.

# Chapter One – Introduction

> To develop connectivity of theories means to reduce isolated approaches and gain more connected knowledge.
>
> (Prediger et al., 2008, p. 17)

> I know how to set it out it just doesn't work.
>
> (Student Group One, Appendix Nine, Contribution 23)

This thesis is an attempt at connecting two theories using a variety of networking strategies (Prediger et al., 2008) in order to build a conceptual framework for more empirical research moving forward. The project as a whole is a detailed exploration of the theories to be networked and uses data collected from a series of student and teacher discussion groups to help in this exploration. In this introduction, I am going to provide an overview of my study, beginning with the context and initial practical issue from which it arose. I am then going to outline the key theoretical and structural points that will explain the thesis design and what to expect whilst reading it.

## 1.1 Context

I have been a secondary school mathematics teacher for eleven years and involved in some form of educational research for ten of those, studying alongside a full-time teaching job. My research and classroom practice are not separate, but complement one another. In order to give a sense of the context from which this study emerged, I wish to present four vignettes taken from both the research and teaching aspects of my time in education.

### 1.1.1 The "Bottom Set"

My experiences in my first school, where I conducted my Masters research, led to an interest in the mathematics of low prior attaining (LPA) students. LPA students are those who have performed below the cohort average in previous national assessments. The school was a large state comprehensive and students were placed in groups (known as sets) for mathematics based on prior attainment. Sixteen-year-old students in what is sometimes referred to in England as the "bottom set" (set 11 of 11), preparing for national examinations at the end of the academic year found mathematics demoralising. There was

a stigma around being in the bottom set and they saw little point investing effort in a subject where they were predicted to achieve low grades in their final examinations if they were predicted a grade at all. A grade of that level opened no doors for them so they saw lessons as an exercise in futility.

In an attempt to bring more focus to these students and develop my pedagogy, my Masters research looked at Realistic Mathematics Education (RME) as a method of instruction for LPA mathematics students. It involved working with this group in lessons, presenting them with a problem and allowing them to share their initial ideas, before helping them to develop these ideas through a process of "vertical mathematizing" (Treffers, 1993, p. 94) to improve the efficiency of their methods or make the methods more suitable for tackling challenging problems. This research raised the issue of what constitutes a more sophisticated method and how we as teachers support students to progress beyond their initial methods.

### 1.1.2 Italy

After completing my Masters, I left the UK to teach in a private international school in Italy. In this school, at least 90% of the students I taught had English as an additional language (EAL). I taught a small class of four Higher Level International Baccalaureate students. These are 16 and 17-year-old students who are engaging with the most challenging mathematics available in a secondary school environment. I would teach them in English, introducing concepts and tasks. They would converse with me in English, asking questions and taking part in class discussion. Then, when they turned to the task (with any written instruction presented in English), they would chat about it in Italian amongst themselves. I would follow as best I could (my Italian is notoriously and, occasionally, hilariously terrible) and guide in English when necessary. They, however, would continue in Italian even though they took on board whatever advice I had offered.

My experiences in this school led to an interest in the work of Barwell and his research in EAL classrooms (e.g., 2015), which introduced me to the ideas of Bakhtin, leading me to look at a more complex idea of language beyond that of variation in national language. This interest in language and its connections to mathematical methods led to my current interest in how language shapes and creates meaning in mathematics, both as part of our methods as well as how we talk about these methods.

### 1.1.3 Counting on Your Fingers

My current school is another large comprehensive school. A conversation with one 11-year-old student who was struggling to keep track of where she was in a multiplication table demonstrated why my interest in mathematical methods is relevant. I suggested she use her fingers to keep track of where she was in reciting the table and her response was "my primary school teacher said we're not allowed to use our fingers". I was surprised that her previous teacher had considered undesirable a technique the student would benefit from using. My approach has always been that if students need something to support their methods, such as using their fingers, sketching a number line, or drawing a picture, then they should not be embarrassed or discouraged from using it. I became interested in why a method with an unreliable outcome was preferred by this student's previous teacher to a method that was more reliable for that student.

### 1.1.4 Chunking

Whilst at the British Society for Research into Learning Mathematics (BSRLM) conference in 2015, I attended a talk that looked at why chunking was considered by the Primary National Curriculum for mathematics as preferable to a method such as the widely used bus stop or short division method.

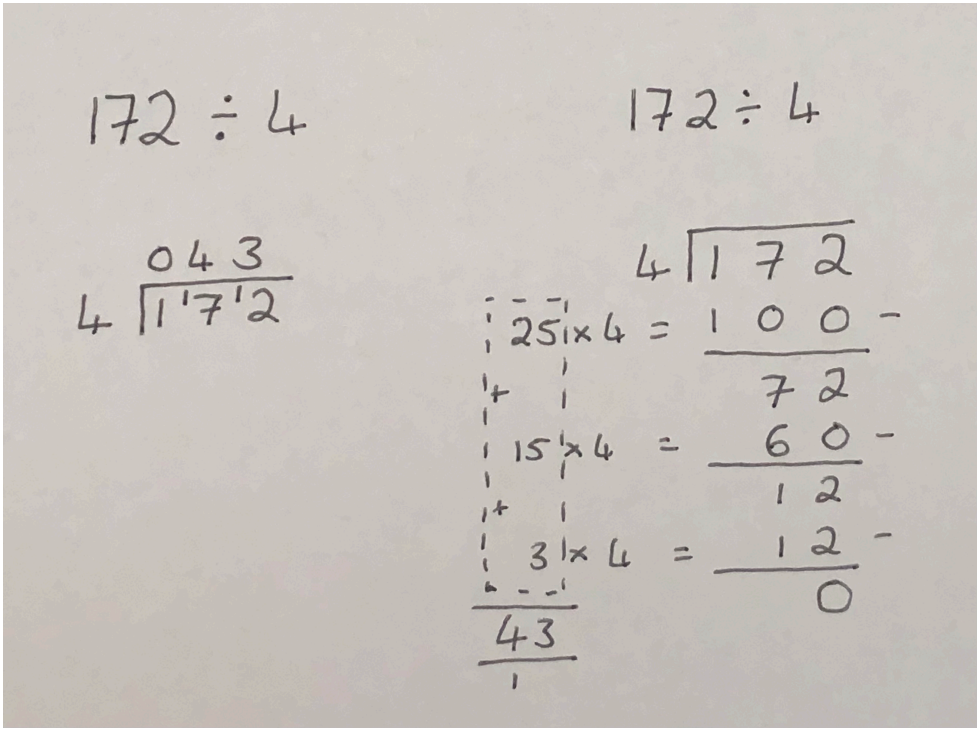


Figure 1.1 – Author's own examples of the bus stop method (left) and chunking method (right) for division

The discussion was around the fact that method marks are awarded for chunking in the external examinations sat by 10- and 11-year-old students at the end of their primary school education, whereas bus stop carries no such method marks. The reasoning seems to be that using chunking implies greater conceptual understanding of division on the part of the student than if they had used the bus stop method, considered as having been learned by rote. The mark scheme even went so far as to dictate how many steps students should take in their chunking to solve a problem (Tutcher, 2015). In the question-and-answer part of the talk one attendee asked "What is to stop students learning chunking by rote? What made chunking a better way of demonstrating conceptual understanding?"

I had seen how students, particularly LPA students, are at risk of having methods dictated to them in a way that undervalues any method considered less sophisticated (third vignette, section 1.1.3, use of fingers). The way the curriculum and examination mark schemes reinforce the use of specific methods may mean students are told not to use methods that work for them, leading to a possible loss in confidence and a perception that they "can't do" mathematics. Judgements are made about these students and their mathematics based on a set of examinations and a series of written solutions to a set of standard questions.

The vignettes I have presented here give an idea of the context from which this project has emerged and the basis for my interest in student mathematical methods along with the role that language, including the mathematical methods themselves, plays not just in the classroom but in the judgement of students and their mathematical attainment. The practical aim of my research is to explore student and teacher perspectives of mathematical methods. I want to work with teachers as they are in the position of motivating students struggling with the demands of the curriculum. They also regularly mark assessments so have a clear idea of the examination requirements for students. My colleagues possess a huge amount of professional knowledge but, at the time I began my study, there were limited opportunities for us to get together and discuss mathematics as a department due to time constraints and staffing difficulties. My hope is that this study gives back to my department by providing the opportunity for my colleagues to share their ideas, experience and expertise whilst allowing me to collect a range of perspectives helping to move beyond solely my ideas about mathematical methods.

I also want to include the perspectives and voices of LPA students to give them an opportunity to share their ideas and thoughts around mathematical methods. This group of students is at risk of not having their voices heard in a subject where not attaining an arbitrary grade at sixteen years old closes many doors and particular prescribed methods impact most upon students who struggle to have any method for approaching a problem, let alone a pre-set one. I want to give these students the opportunity to talk about their mathematics in a way that they do not often get to.

### 1.2 The Practical Research Problem

I identified the practical research problem as part of an unpublished assignment I wrote in the formative stages of my PhD research (John, 2016). I paraphrase my findings here.

The Mathematics National Curriculum documents were updated in the UK in 2013 and 2014. The aims of the curriculum were interpreted by the National Centre for Excellence in Teaching Mathematics (NCETM) as the chance to promote "confidence and competence – 'mastery'," (NCETM, 2014, p. 1) for students. However, alongside the development of "conceptual understanding" (NCETM, 2014, p. 2), this mastery curriculum promotes rote recall with the curricula for certain age groups specifying areas that need to be memorised by students. In Key Stage One and Two, students are expected to memorise their multiplication tables "up to 12 x 12" (Department for Education, 2013, p. 25) and for Key Stage Four, they are expected to memorise the value of certain trigonometric ratios (Department for Education, 2014, p. 10). For teachers, the contrast between conceptual understanding and rote memorisation has raised questions about what we prioritise in the classroom. Is being able to calculate multiplication tables using a reliable method enough? Even if that method is keeping track of our tables on our fingers? Or is memorisation and immediate recall the aim, even if this does not lead to a method that can be expanded to other similar problems?

The ambiguity over aims is echoed by the research into support for LPA students. One of the National Curriculum aims is that most students "will move through the programme of study at broadly the same pace" (Department for Education, 2014, p. 3). Some authors have supported rote memorisation and recall as a key way to support LPA students' progress (Kroesbergen & Van Luit, 2005). However, my previous research with LPA students (John,

2013) focused on Hans Freudenthal's Realistic Mathematics Education (RME) Programme (van den Heuvel-Panhuizen & Drijvers, 2014) as a way of using students' informal methods. These methods were suggested by students in response to "realistic" prompts (Dickinson and Eade, 2005, p. 3) and, through a process of questioning and discussion with the teacher, students were guided through a process of "vertical mathematizing" (Treffers, 1993, p. 94). I wanted to see what the progression from informal to more formal methods looked like in the classroom and what teachers could do to facilitate it. My research focused on the teacher's prompting during "vertical mathematizing" (Treffers, 1993, p. 94) and I was left with questions such as: When should a teacher prompt a student? Is it right to prompt students to move to a more sophisticated method when they have a secure method already? Can prompting students to move to a more sophisticated method mean they are less secure and may be completing the method by rote rather than due to conceptual understanding?

As I developed my interest in mathematical methods at the beginning of my doctoral research, I was faced with a key dilemma: What does it mean for something to be formal or informal? Understanding what is happening as students develop their conceptual understanding became more complex than a simple development from informal to formal methods. I wanted to further explore the ideas of learning, development and meaning for LPA mathematics students but also look at teacher perspectives of mathematical methods and the impact that the constraints of the curriculum have had on the discussion around mathematical methods. However, to investigate how teachers and students view different mathematical methods I first need to explore what is meant by formal and informal methods and the underlying ideas around learning in a way that allows for practical research, which led to my theoretical research aim.

### 1.3 The Theoretical Research Problem

As I have mentioned previously (section 1.1.2), I came across the work of Barwell during my time working in a school with a significant proportion of EAL students. His focus on tensions in language use and interpreting the ideas of Bakhtin for the classroom linked to my interest in formal and informal methods. Some authors have used Bakhtin's dialogics to extend or complement Vygotsky's work on dialectics (e.g., Wells, 1999; Wertsch, 1991, 1998), whilst others have clearly argued that the two have significant differences in their underlying ontology (e.g., Wegerif, 2008; Matusov, 2011).

Originally, I intended to discuss the relationship between Bakhtin and Vygotsky's work in the initial stages of my research and then move on. As the research at the start of my doctoral studies progressed, it became clear that one chapter of discussion on this would not be enough and, in fact, both theories had a lot to offer in the context of my study. As a result, the focus of this study has shifted to assess whether, despite their differences, using both theories could enhance an analysis of mathematical methods through a process of connecting theories (Radford, 2008).

### 1.3.1 Connecting Theories

In order to explore the possibility of connection between these two theories, I am going to use the work of Luis Radford, particularly his 2008 article *Connecting theories in mathematics education*. Radford outlines three areas in which connections can be made between theories:

> - A system, P, of *basic principles*, which includes implicit views and explicit statements that delineate the frontier of what will be the universe of discourse and the adopted research perspective.
> - A *methodology*, M, which includes techniques of data collection and data-interpretation as supported by P.
> - A set, Q, of paradigmatic *research questions* (templates or schemas that generate specific questions as new interpretations arise or as the principles are deepened, expanded or modified).
>
> (Radford, 2008, p. 320)

Within my study, the chapters are grouped into three subsections, one for each of the three areas for connecting theories listed above. I will take each of these aspects in turn, exploring the possible connections between the work of Bakhtin and Vygotsky. My aims here are two-fold. Firstly, by working this closely with the two theories, I aim to develop a deeper understanding of the individual theories themselves. Secondly, I wish to establish if connecting the theories will allow me to better analyse the discussion of mathematical methods on the part of teachers and students.

### 1.3.2 Networking Strategies

In order to investigate points of connection between the two theories, I am going to test a variety of networking strategies from Prediger et al. (2008). These are summarised in the following diagram:

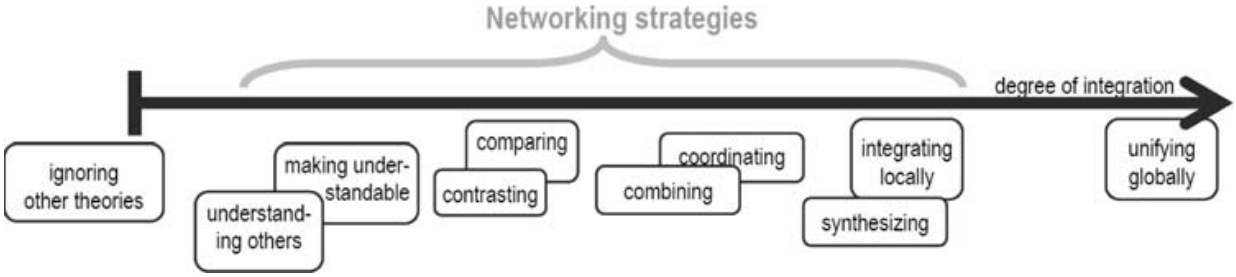


Figure 1.2 – Networking Strategies (Prediger et al. 2008, p. 8)

The extremes of this scale "ignoring other theories" and "unifying globally" are not considered to be networking strategies as they indicate either a position where other theories are not viable at all or a position where all theories represent part of the whole. As such, neither position considers/requires networking strategies. The networking strategies are presented as pairs on a spectrum. These pairs can be summarised as follows:

- "Understanding others" and "making own theories understandable": The minimum requirement for networking theories is "the hard work of understanding others and reciprocally, with making the own theory understandable" (Prediger et al. 2008, p. 9).
- "Comparing" and "contrasting": Two theories are set next to one another to consider variations between the two and to increase understanding of both. The approaches are similar but "comparing refers to similarities and differences in a more neutral way of perceiving theoretical components, contrasting is more focused on stressing differences" (Prediger et al. 2008, p. 9).
- "Coordinating" and "combining": Particularly useful when analysing something specific, allowing for "triangulation […] looking at the same phenomenon from different theoretical perspectives as a method for deepening insights on the phenomenon" (Prediger et al. 2008, p. 10). Coordinating theories "should include a careful analysis of the mutual relationship between the different elements and can only be done by theories with compatible cores" (Prediger et al. 2008, p. 11). On the other hand, "[e]ven theories with conflicting basic assumptions can be combined in

order to get a multi-faceted insight into the empirical phenomenon in view" (Prediger et al. 2008, p. 11).

- "Synthesizing" and "integrating locally": When carefully carried out, taking distinct original theories and developing new frameworks and theory from them. "Synthesizing is used when two (or more) equally stable theories are taken and connected in such a way that a new theory evolves" (Prediger et al. 2008, p. 12). Synthesizing is dependent on a "coherent philosophical base" (Prediger et al. 2008, p. 12) for the two theories. On the other hand, integrating (locally) is used when "the theories' scope and degree of development is not symmetric, and there are only some concepts or aspects of one theory integrated into an already more elaborate dominant theory" (Prediger et al. 2008, p. 12).

My study is split into three subsections to address each of Radford's three areas of connectivity. I will begin by summarising theoretical positions to establish the "principles" of Bakhtin and Vygotsky's theories (Chapter Two) and position these within a review of current writing in the field (Chapters Three and Four) with an eye to the networking strategies employed by these authors. Having collected data (Chapters Five and Six), I will use a series of networking strategies to explore the "methodology" (Chapters Seven and Eight). I will then address "research questions" (Chapter Nine), again using the networking strategies, and will lay out a clear path towards future research based on the outcomes of this study. My hope is that this exploration will act as a precursor to further study. Chapter Ten will discuss and evaluate my approach.

In the next chapter, I will begin to explore principles as the first of Radford's points of possible connection. Using the "understanding others" and "making own theories understandable" approaches of Prediger et al. (2008, p. 9) I am going to look at key aspects of the work of Bakhtin and Vygotsky.

## Chapter Two – Principles: Theoretical Perspectives

The first of Radford's points of possible connection between theories is that of basic principles, which are "implicit views and explicit statements that delineate the frontier of what will be the universe of discourse" (2008, p. 320). In my introduction, I spoke of my interest in the theories of Mikhail Mikhailovich Bakhtin (1895-1975) and, through that, my introduction to the discussion around the use of his work in conjunction with that of Lev Semyonovich Vygotsky (1896-1934). In this chapter, I adopt the networking strategies "understanding others"/ "making own theories understandable" (see Figure 1.2) to explain the main points of the two theories that apply to my context.

I begin by discussing the work of Bakhtin; his theories on discourse, interaction, and meaning in language, the so-called "dialogic orientation of a word" (Bakhtin, 1981, p. 275). I will then discuss the key points of Vygotsky's work on mediational means and development, "[t]he internalization of socially rooted and historically developed activities" (Vygotsky, 1978, p. 57). The organisation of Bakhtin's ideas before Vygotsky's has been chosen to reflect the order in which I encountered and began to explore their work. I chose Bakhtin's and Vygotsky's theories to focus on as they both offer insight into key aspects of my context in mathematics education. There have been several previous attempts to use the work of these two authors together. However, there are aspects of these previous attempts that are unsatisfactory (as I will discuss in Chapters Three and Four). As such, I wanted to explore the possible benefits of using a connecting theories approach for the works of Bakhtin and Vygotsky in a way I believe has not been attempted previously.

I am not expecting to finish this chapter with a complete summary of the work of these two writers. Subsequent chapters and, indeed, the project as a whole will continue to explore these theories. My aim here is to map the territory (after Korzybski, 1951) of the theories and for this to act as a guide through future chapters. At all times I intend to keep in mind the three parts of Korzybski's map analogy:

1. A map is not the territory […]
2. A map covers not all the territory […]
3. A map is self-reflexive […]

(Korzybski, 1951, p. 13)

## 2.1 Bakhtin and Dialogics

Bakhtin (1895-1975) was a Russian literary theorist writing in the early 20th century during a time of extreme turbulence for Russia (part of the Soviet Union between 1922-1991). Despite much of his work (e.g., 1981, 1986) being written as literary analysis, his ideas and theories about language have implications beyond the field of literature. Sections of his work have been gradually translated into English over the latter part of the 20th century (the dates given in references are for the publication of the translations, following the convention of others writing in the field). A variety of researchers (e.g., Wells, 1999; Wertsch, 1991, 1998; Barwell, 2015; Matusov, 2011; Wegerif, 2008; White, 2014a) have used Bakhtin's ideas to explore meaning-making in the classroom.

Bakhtin had a difficult relationship with the then Soviet Union. He was exiled to Kazakhstan at one point and much of his work struggled to find publication. Some of it was lost as a result, including one piece of work he famously used as cigarette papers (Holquist, 1981a, p. xxiv). It is worth noting here that there is a question over authorship of some of the work around Bakhtin (Holquist, 1981a, p. xxvi). Bakhtin worked as part of something known as the Bakhtinian Circle. This group studied similar themes in collaboration, which has led to questions over the authors of certain works. For example, *Marxism and the Philosophy of Language* (1973) is credited to Vološinov. Some Bakhtin experts suggest this is, in fact, Bakhtin's work or at least partially his (Matejka & Titunik, 1986, p. ix). As this attribution is still disputed, I will credit the publications I have, so will credit Vološinov rather than speculate as to authorship as this is beyond the range of my study.

Instead of studying language as a set of sounds, symbols and words with an abstract, set meaning, "Bakhtin was interested in the way language is used at particular moments and the differences in how language is used from one time and place to another; that is, in contemporary terms, he was interested in discourse" (Barwell, 2015, p. 5). In fact, Bakhtin's "extraordinary sensitivity to the immense plurality of experience more than anything else distinguishes Bakhtin from other moderns who have been obsessed with language" (Holquist, 1981a, p. xx). This belief in the situatedness of language meant that, for Bakhtin, "[i]n order to assess and divine the real meaning of others' words in everyday life, the following are surely of decisive significance: who precisely is speaking, and under what concrete circumstances?" (Bakhtin, 1981, p. 340). These seem relatively simple questions to answer but, digging deeper, it becomes clear just how complex a task this is. I

am going to detail several of Bakhtin's ideas about language and meaning-making. These form the basis of his work on dialogics and will be key to my project.

### 2.1.1 The Utterance

Bakhtin's dialogics differed from previous systems as he proposed the utterance as the unit of analysis. An utterance can be a single word, or combination of words, "[o]ne exchanges utterances that are constructed from language units: words, phrases, and sentences" (Bakhtin, 1986, p. 75). The significance of taking the utterance as the basic unit of analysis as opposed to, say, phonics, or individual words (although an utterance may be made up of a single word) is that an utterance cannot be defined as an independent or neutral entity. It is always simultaneously defined by the ways in which the speaker has experienced the utterance previously, the intention of the speaker, the context of the discourse, and the anticipated response of the intended audience: "[t]he word in living conversation is directly […] oriented toward a future answer-word" (Bakhtin, 1981, p. 280). Bakhtin describes an utterance as "living" as in "having taken meaning and shape at a particular historical moment in a socially specific environment" (Bakhtin, 1981, p. 276). So, when he asks, "who precisely is speaking" (Bakhtin, 1981, p. 340), this has far more complex implications than just who is producing words at this moment. I will return to defining the boundaries of utterances in section 2.1.3.

The overlapping influences present in any utterance represent what Bakhtin termed the "heteroglossia" of language. Holquist explains

> at any given time, in any given place, there will be a set of conditions - social, historical, meteorological, physiological - that will insure that a word uttered in that place and at that time will have a meaning different than it would have under any other conditions.
>
> (Holquist, 1981b, p. 428).

The voices of those who have utilised the word or utterance before and of those who are expected to respond to it in the future have all shaped its use in that moment. This "heteroglossia […] a multiplicity of social voices" (Bakhtin, 1981, p. 263) makes an utterance unique and unable to be recreated in the same way in the future.

Bakhtin's framing of how meaning is made in discourse is based around "living utterance" and its dependence on the meanings that have been before and will come after:

> [I]n the makeup of every utterance spoken by a social person […] a significant number of words can be identified that are implicitly or explicitly admitted as someone else's, and that are transmitted by a variety of different means. Within the arena of almost every utterance an intense interaction and struggle between one's own and another's word is being waged, a process in which they oppose or dialogically interanimate each other.
>
> (Bakhtin, 1981, p. 354)

The struggle that occurs in the dialogic space between these interacting utterances is where meaning-making occurs.

The conceptualisation of words as being either one's own or belonging to some other with whom we are interacting is a key feature of Bakhtin's work. Bakhtin does not see this self/other delineation as a static border, instead, "the boundaries between them can change, and a tense dialogic struggle takes place on the boundaries" (Bakhtin, 1986, p. 143). Bakhtin was interested in this struggle and the ways in which the novel allowed the reader to take part in "the process of coming to know one's own language as it is perceived in someone else's language, coming to know one's own horizon within someone else's horizon" (Bakhtin, 1981, p. 365). However, this leaves someone looking to work with and analyse utterances in a uniquely complex position. If an utterance, taking "meaning and shape at a particular historical moment in a socially specific environment, cannot fail to brush up against thousands of living dialogic threads" (Bakhtin, 1981, p. 276) then how do we look at the outcome of this unseen web of influences and try to trace those threads? If every utterance depends on what has come before and what will come after, including influences from people and situations that, as researchers, we have no way of accessing e.g., family influences or meanings that come with unique emotional attachments, analysis using a dialogic framework is challenging.

### 2.1.2 Centripetal and Centrifugal Forces

Bakhtin theorised that every utterance has a force pulling it towards some sort of specific language type, for example, the language of an institution such as a school or a courtroom. Simultaneously, many other sociocultural forces, for example, specific dialects or slang terms, are acting that lead to variations in language. As he says, "[e]very utterance participates in the "unitary language" (in its centripetal forces […]) and at the same time

partakes of social and historical heteroglossia (the centrifugal […] forces)" (Bakhtin, 1981, p. 272).

The tensions between heteroglossia and unitary language forces are present in and shape every utterance. Indeed, Vološinov described discourse as "a little arena for the clash and criss-crossing of differently oriented social accents" (Vološinov, 1973, p. 41). The centrifugal forces are diverse, intricate and will be specific to the voices involved, "it is even possible to have a family jargon define the societal limits of a language" (Bakhtin, 1981, pp. 290–291). Every individual will have their own combination of influences that act on their utterances. The centripetal forces are important in that they act "as a force for overcoming this heteroglossia, imposing specific limits to it, guaranteeing a certain maximum of mutual understanding and crystalizing into a real, although still relative, unity" (Bakhtin, 1981, p. 270). Barwell studied centripetal and centrifugal language forces in the context of discussion in the mathematics classroom, interpreting formal mathematical language as unitary language and informal mathematical language as an example of heteroglossia (Barwell, 2014, p. 914).

It is important to recognise that this push and pull is ongoing: "Bakhtin sees each utterance as being shaped (and constantly reshaped) by this tension between a tendency to uniformity and the necessity of variation" (Barwell, 2015, p. 7). At no point is one tension considered to have won and no tension will disappear to manifest some sort of final form of an utterance, since, "[l]anguage cannot be said to be handed down – it endures, but it endures as a continuous process of becoming" (Vološinov, 1973, p. 81). This lack of a final fixed form is going to be important in the debate in subsequent chapters.

### 2.1.3 Speech Genres

I have discussed the utterance and the centripetal and centrifugal forces at work on our utterances, but how do we define the boundaries of an utterance? If I were considering other units of analysis, such as sentences or individual words, this process would be a relatively simple task, a full stop or the final letter respectively. To establish the boundaries of an utterance, Bakhtin required a "change of speaking subjects" and the "finalisation of the utterance" (Bakhtin, 1986, p. 76). The change of speaking subjects is fairly self-explanatory, but the "finalisation of the utterance" requires a little more unpacking.

The “finalisation of the utterance” requires the utterance to be in a form where it is “guaranteeing the possibility of a response” (Bakhtin, 1986, p. 76). This response can be verbal or otherwise but, in turn, is dependent on three things:

1. “Semantic exhaustiveness of the theme” (Bakhtin, 1986, p. 76): Not an objective completion of the topic, but the “relative finalization” (Bakhtin, 1986, p. 77) of the speaker getting across what they want to say.
2. “The speaker’s plan or speech will” (Bakhtin, 1986, p. 77): What is it the speaker wants to say? Although this cannot be exactly determined by the listener, we can put ourselves in the position of the speaker and “imagine to ourselves what the speaker wishes to say” (Bakhtin, 1986, p. 77). This sense of the speaker’s plan is based on preceding utterances and the context in which the exchange is taking place.
3. “Typical compositional and generic forms of finalization” (Bakhtin, 1986, p. 77): Based on the speech genres present in the exchange, Bakhtin says “[w]e speak only in definite speech genres, that is, all our utterances have definite and relatively stable typical forms of construction of the whole” (Bakhtin, 1986, p. 78).

Speech genres are key to how Bakhtin thought utterances were shaped. Bakhtin was clear that each utterance carries with it the meanings and experiences of those who have used the word before. He also believed that these utterances took on a particular form or pattern depending on the situation they were being used in. For example, language in a religious setting has different characteristics and potential meanings for words than in, say, a corporate setting, or in a conversation between family members around the dinner table: “[E]ach sphere in which language is used develops its own relatively stable types of these utterances. These we may call speech genres” (Bakhtin, 1986, p. 60). Speech genres are flexible and people move between them without noticing, but they offer an extra layer of meaning and purpose to an utterance if those involved in discourse are all familiar with the meanings specific to the speech genre. However, for those on the outside, “the intentions permeating these languages become things, limited in their meaning and expression” (Bakhtin, 1981, p. 289).

When an utterance is formed, it not only revoices the utterances that have come before, but is also shaped for the context it is going to be voiced in. It is shaped with a particular intention in order to generate a response. This is sometimes known as responsivity (e.g., Matusov, 2011, p. 100). A speech genre acts as a centripetal force that provides some

common ground to shape utterances for a specific context, involving "specific forms for manifesting intentions, forms for making conceptualization and evaluation concrete" (Bakhtin, 1981, p. 289). For example, if you were sending an email, the utterances formed to do this would be different if the email was to your line manager or to your mother. The ways in which the email is worded is guided by the speech genre you are working in. As such, "the single utterance, with all its individuality and creativity, can in no way be regarded as a completely free combination of forms of language" (Bakhtin, 1986, p. 81).

### 2.1.4 Assimilation/Appropriation

I have discussed some of the forces acting on the utterance as it comes into being through interaction with the other and how, for the speaker, "his orientation toward the listener is an orientation toward a specific conceptual horizon, toward the specific world of the listener" (Bakhtin, 1981, p. 282). The speaker produces utterances with the aim of getting a response (verbal or otherwise) and it is this orientation towards response that "creates the ground for understanding" (Bakhtin, 1981, p. 282). However, as I have previously discussed, if every utterance is based on unique prior experiences, how is someone without the same set of prior understandings to make sense of the utterance? Speech genres is one of the ways in which creating a common ground for discourse can establish a base level of understanding. In addition, Bakhtin uses assimilation to describe the meaning made around the utterances of others becoming part of a new utterance for the person encountering them.

Bakhtin describes that the dialogic process involves an "active understanding, one that assimilates the word under consideration into a new conceptual system, that of the one striving to understand" (Bakhtin, 1981, p. 282). This is not as straightforward as picking up new meanings or words from others:

> Language is not a neutral medium that passes freely and easily into the private property of the speaker's intentions; it is populated - overpopulated - with the intentions of others. Expropriating it, forcing it to submit to one's own intentions and accents, is a difficult and complicated process.
>
> (Bakhtin, 1981, p. 294)

Assimilation is particularly important in an educational context where students meet new concepts, words and meanings around words as a matter of course and are then expected to use them in future discourse.

In addition to the importance of assimilation for meaning making, Bakhtin goes a step further. Not only is assimilation important for learning new terms or becoming familiar with speech genres, but "[t]he ideological becoming of a human being […] is the process of selectively assimilating the words of others" (Bakhtin, 1981, p. 341), explaining more about Bakhtin's ontological position (how he perceives being). When you engage in discourse and meaning making, you have the potential to affect not just the words of others, but also who they are: "[e]verything that pertains to me enters my consciousness […] from the external world through the mouths of others […] I realize myself initially through others" (Bakhtin, 1986, p. 138). As assimilation is discussed further in this project it is worth noting that it is also referred to as appropriation (e.g., Wertsch in section 3.1.2). I am going to be consistent with the translation of Bakhtin I have but other authors in Chapters Three and Four may use appropriation synonymously with assimilation.

### 2.1.5 Authoritative Word

The mutual meaning making I have previously discussed underpins Bakhtin's dialogics and gives rise to the "internally persuasive discourse [which] is, as it is affirmed through assimilation, tightly interwoven with "one's own word"" (Bakhtin, 1981, p. 345). In contrast, Bakhtin also talks about authoritative discourse, where:

> It is not a free appropriation and assimilation of the word itself that authoritative discourse seeks to elicit from us; rather, it demands our unconditional allegiance […] It is indissolubly fused with its authority - with political power, an institution, a person.
>
> (Bakhtin, 1981, p. 343)

Authoritative discourse or the authoritative word is important as it begins to give us an idea of how Bakhtin views power relations in discourse. Due to the inherent authority, the authoritative word leads to an imbalance in power between those engaged in discourse. This concept of power is relevant in the school context with the presence of the institution of the school, examination board and, behind them, government education policy, with the "authoritative word […] located in a distanced zone, organically connected with a past that is felt to be hierarchically higher" (Bakhtin, 1981, p. 342) in stark contrast to the familiarity of the internally persuasive word.

Through section 2.1, I have discussed five key concepts of Bakhtin's. I have emphasised how "[a] Bakhtinian, dialogic perspective […] emphasises the ephemerality of discourse, its situated, shifting, heteroglossic nature and the relationality through which meaning emerges" (Barwell, 2015, p. 13). In the next section, I am going to look at the work of Vygotsky.

### 2.2 Vygotsky and Dialectics

Vygotsky (1896-1934) was a psychologist who, although born in Belarus, was educated and lived most of his life in Russia (both part of the Soviet Union during the later part of Vygotsky's life). He was writing at approximately the same time as Bakhtin but, as far as we know, the two were not acquainted and their theories were formed with no specific knowledge of the other. Vygotsky was unimpressed with the tendency of psychological scholars of the time to take a wide variety of studies based on differing methodologies and theoretical positions and develop their results into a sweeping general theory. Vygotsky worked to develop a "new methodology that would make psychology scientific, but not at the cost of the naturalization of cultural phenomena, and that would make use of the Marxist method" (Kozulin, 1986, p. xxiii).

Vygotsky developed a concept of mediated action to explain the interrelationship between the development of children's thinking and social interaction:

> The conception of word meaning as a unit of both generalizing thought and social interchange is of incalculable value for the study of thought and language. It permits true causal-genetic analysis, systematic study of the relations between the growth of the child's thinking ability and his social development.
>
> (Vygotsky, 1986, p. 9)

The idea of development was central to his work and is going to form a key part of my discussion.

Vygotsky's ideas developed from the dialectics of Hegel and Marx. His work echoes Hegel's "critical transformation of logic as a science" (Ilyenkov, 2008, p. 194), by "bringing it […] into correspondence with its real object, i.e. with real thought" (Ilyenkov, 2008, p. 194). In their afterword to the collection of Vygotsky's lectures and essays published as *Mind in Society* (1978), John-Steiner and Souberman highlight "[a]n ever-present theme in this volume is the Marxian concept of a historically determined human psychology" (p. 126). I

am going to return to the philosophical origins of Vygotsky in Chapter Three (section 3.5.1).

John-Steiner and Souberman (1978) highlight the concept of development, the educational implications and a historical-cultural approach as some of the most significant ideas of Vygotsky's work. Wertsch (1991) highlights genetic analysis, social origins of mental functioning in the individual and mediation as the key features of Vygotsky's work. Vygotsky himself explains that "[o]ur leading idea throughout the work will be that of development" (Vygotsky, 1986, p. 11). I am going to begin by discussing mediated action, which underpins his ideas around development and the implications for education.

### 2.2.1 Mediation: Tools and Signs

In his theory of mediated action, Vygotsky adapts the stimulus-response model of traditional psychological thinking towards behaviour by adding a mediational means: "In this new process the direct impulse to react is inhibited, and an auxiliary stimulus that facilitates the completion of the operation by indirect means is incorporated" (Vygotsky, 1978, p. 40). He represents this using an oft quoted and adapted triangle.

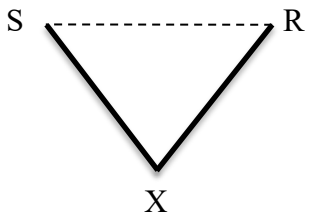


Figure 2.1 – Mediated Action Triangle (Vygotsky, 1978, p. 40)

In this diagram, the S represents the stimulus, the R the response and the X the mediational means. Vygotsky explains that the introduction of mediational means, "permits humans, by the aid of extrinsic stimuli, to control their behavior from the outside" (1978, p. 40). This is a significant change, allowing a formulation of human behaviour, which "breaks away from biological development and creates new forms of a culturally-based psychological process" (Vygotsky, 1978, p. 40).

Mediational means can be split into signs and tools. In their afterward to Vygotsky's *Mind in Society*, John-Steiner and Souberman explain:

> The use of tools and signs share some important properties; both involve mediated activity. But they also diverge from each other: signs are internally oriented, according to Vygotsky, a means of psychological influence aimed at mastering oneself; tools, on the other hand, are externally oriented, aimed at mastering and triumphing over nature.
>
> (1978, p. 127)

Vygotsky is clear that signs and tools are different and any attempt at equating them "loses the specific characteristics of each type of activity and leaves us with one general psychological form of determination" (Vygotsky, 1978, p. 53). One key feature of a sign is that it has a "reverse action" meaning "it operates on the individual, not the environment" (Vygotsky, 1978, p. 39). I explore this further with specific examples in section 3.1.2.

According to Vygotsky, the use of signs and tools forms the basis of all "complex human behaviour" (Vygotsky, 1978, p. 24). His work "accords symbolic activity a specific organizing function that penetrates the process of tool use and produces fundamentally new forms of behaviors" (Vygotsky, 1978, p. 24). The classic, oft referenced, example of sign development from Vygotsky is that of the child pointing. Vygotsky describes a child's initial reaching for an object being interpreted by the mother as a gesture of the child pointing to something it wants. Once the child establishes this and understands the mother's reaction to his gesture:

> [T]here occurs a change in that movement's function: from an object-oriented movement it becomes a movement aimed at another person, a means of establishing relations. The grasping movement changes to the act of pointing […] Its meaning and functions are created at first by an objective situation and then by people who surround the child.
>
> (Vygotsky, 1978, p. 56)

The pointing example illustrates "internalization", described by Vygotsky as the "internal reconstruction of an external operation" (1978, p. 56) as the child has internalized the social influence to make meaning. Internalization forms the basis of Vygotsky's ideas around development. I am going to discuss this next.

### 2.2.2 Development of Concepts

Vygotsky is clear that development is at the heart of his work. I have discussed mediated action and the use of tools and signs and so, in this section, I will lay out the process of development according to Vygotsky and how the use of signs and tools is part of this.

Vygotsky's development is based on the idea that signs begin as external, signifying something between people, but then become internalized to represent something within for the individual.

The process of internalization consists of a series of transformations:

(a) *An operation that initially represents an external activity is reconstructed and begins to occur internally*. Of particular importance to the development of higher mental processes is the transformation of sign-using activity, the history and characteristics of which are illustrated by the development of practical intelligence, voluntary attention, and memory.

(b) *An interpersonal process is transformed into an intrapersonal one*. Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first between people (interpsychological), and then inside the child (intrapsychological) […] All the higher functions originate as actual relations between human individuals.

(c) *The transformation of an interpersonal process into an intrapersonal one is the result of a long series of developmental events*. The process being transformed continues to exist and to change as an external form of activity for a long time before definitively turning inward. For many functions, the stage of external signs lasts forever, that is, it is their final stage of development.

(Vygotsky, 1978, pp. 56–57)

Internalizing signs has been developed from the wider dialectic ideas I will explore in Chapter Four, however, Vygotsky was the first to apply it to the individual developing higher mental functions. It is worth noting that "interpsychological" and "intrapsychological" from the quote above are sometimes translated as intermental and intramental respectively (Wertsch, 1991, p. 26).

In the context of education, Vygotsky talks about the development of higher concepts as the process of developing from preconceptions to higher concepts through "generalizing the generalizations of the earlier level" (1986, p. 202). The process of generalisation draws

together all of the earlier concepts the student has developed and these earlier stages are seen as important and necessary steps towards this higher understanding (1986, p. 203). Vygotsky uses the development of algebraic concepts as an example:

> At the earlier stage certain aspects of objects had been abstracted and generalized into ideas of numbers. Algebraic concepts represent abstractions and generalizations of certain aspects of number, not objects, and thus signify a new departure - a new, higher plane of thought. The new, higher concepts, in turn, transform the meaning of the lower. The adolescent who has mastered algebraic concepts has gained a vantage point from which he sees concepts of arithmetic in a broader perspective.
>
> (Vygotsky, 1986, p. 202)

One key term to highlight from this example is that of mastery. I explained in Chapter One that the 2013/14 National Curriculum for Mathematics is sometimes described as the mastery curriculum (NCETM, 2014). Here, Vygotsky uses mastery to describe a pinnacle of understanding for students to aim for that provides them with a view backward over the preceding concepts.

Vygotsky's idea of development for language says that, "[a]spects of external or communicative speech as well as egocentric speech turn "inward" to become the basis of inner speech" (1978, p. 57). More specifically, he studies word meaning and its internalization from the point at which a child comes across a new concept and explains the "long and complex" process by which "the concept and the corresponding word are fully appropriated by the child" (Vygotsky, 1986, p. 152). Vygotsky "made a distinction between word meaning […] which reflects a generalized concept, and word sense […] which depends on the context of speech" (Kozulin, 1986, p. xxxvii). The internalization process of both word meaning and word sense, along with the ongoing power of this inner speech to affect complex inner processes:

> alters the traditional view that at the moment a child assimilates the meaning of a word, or masters an operation such as addition or written language, her developmental processes are basically completed. In fact, they have only just begun at that moment.
>
> (Vygotsky, 1978, p. 90)

The argument that mastering a mathematical operation is not the end of the developmental process has implications for students who are assessed based on the idea that if they can

use a particular mathematical operation they have shown signs of having completed a developmental stage.

Vygotsky talks about spontaneous and scientific concepts and posits that these two concepts emerge from different situations. Spontaneous concepts come from our everyday experience whereas scientific concepts come from a more formal instruction (1986, p. 158). Vygotsky theorises that these two different types of concepts "develop in reverse directions: Starting far apart, they move to meet each other" (Vygotsky, 1986, p. 192). Scientific concepts are introduced in the setting of formal schooling, particularly through the use and internalization of scientific language (Wegerif, 2008, p. 352).

In the afterward to *Mind in Society*, John-Steiner and Souberman offer a summary of Vygotsky's idea of development:

> For Vygotsky, one of the essential aspects of development is the increasing ability of children to control and direct their own behaviour, a mastery made possible by the development of new psychological forms and functions and by the use of signs and tools in this process. At a later age children extend the boundaries of their understanding by integrating socially elaborated symbols (such as social values and beliefs, the cumulative knowledge of their culture, and the scientifically expanded concepts of reality) into their own consciousness.
>
> (John-Steiner & Souberman, 1978, p. 126)

This description of how the process of internalization affects consciousness is an important aspect of Vygotsky's idea that dialectical processes "lead to the development of autonomous, rational, individual selves" (Wegerif, 2008, p. 350).

Having explained Vygotsky's view of development in terms of internalization and scientific concepts, his Zone of Proximal Development (ZPD) illustrates how learning and development are related. I am going to focus on the ZPD in the next section.

### 2.2.3 The Zone of Proximal Development (ZPD)

Vygotsky defines his Zone of Proximal Development (ZPD) as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in

collaboration with more capable peers" (1978, p. 86). The idea of the ZPD has significant implications for teaching and assessing students.

There is a clear difference between learning and development in Vygotsky's writing. He says that learning, correctly organised, creates the ZPD and "awakens a variety of internal developmental processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers" (Vygotsky, 1978, p. 90). In the classroom, teachers can facilitate the creation of the ZPD as a way to "lead the child to what he could not yet do" (Vygotsky, 1986, p. 189). In essence, Vygotsky is suggesting that what is being taught is ahead of the level of development the student has demonstrated, which ensures the ZPD is ever present, allowing the students to continue in their developmental process. The notion of leading children or those less developmentally advanced has implications for power in educational settings that I will discuss further in section 4.2.

In terms of assessment, the current system assesses the developmental level the student has achieved at this point. However, "[t]he state of a child's mental development can be determined only by clarifying its two levels: the actual developmental level and the zone of proximal development" (Vygotsky, 1978, p. 87). Examinations would establish the former, but not the latter. Vygotsky claims that planning learning around the results of this type of assessment means it would be based on development that has already happened. Instead, he suggests that basing learning ahead of this makes the most of the fact that learning is working ahead of development.

### 2.3 Connecting Theories

With this thesis, I intend to employ a connecting theories approach to explore how Bakhtin's and Vygotsky's theories can, together, build an analysis of teacher and student perspectives on mathematical methods. Through the thesis I will test the different levels of connection (section 1.3) that are possible, which will allow me not only to extend debate in the field as to the benefits a connected theories approach might offer but also how differing approaches to connecting the theories of Bakhtin and Vygotsky can affect an analysis.

The two theories I have outlined in this chapter each link to a specific aspect of the context of my study. Vygotsky's psychological approach offers a framing of development which

echoes that of the National Curriculum, proving valuable when considering teacher and student views of mathematical methods. Bakhtin's literary theory focuses on the intricate links between past use and present context leading to a framing of meaning-making that is deeply inter-subjective and context-based. The impact of previous experience through exploration of tensions or centripetal and centrifugal forces acting on student discourse and methods is not considered in either the National Curriculum documents or examination mark schemes.

In terms of connecting theories, this chapter considered the philosophical principles behind the two theories. I have used the paired "understanding others"/ "making own theories understandable" networking strategies (Prediger et al., 2008, p. 9) to set out the positions of Bakhtin and Vygotsky towards learning. On the surface there are several similarities between the two theories. The focus of both Bakhtin and Vygotsky on the sociohistorical background was a change from the norm of the time in their respective fields: literature for Bakhtin and psychology for Vygotsky. Rather than looking at the individual in isolation, the work of Bakhtin and Vygotsky emphasises the role of others on the individual and their use of language. There is also an apparent similarity between Bakhtin's assimilation and Vygotsky's internalization. However, these concepts are not as close as they initially seem. Internalizing focuses on gaining a particular meaning and aims for something static as someone internalizes a concept, as opposed to word sense that is ever shifting. For Bakhtin, assimilation is that play between our utterance meaning and that of the other and we are working to make meaning in this difference so we can begin to use the word for ourselves, with our own intentions. It is never static. These similarities require further verification.

At this stage, my reading of Bakhtin's work about language and meaning making has focused on speech and written language through his focus on literature. One idea I want to explore in more depth in later chapters is how the theory is related to mathematics. Does his work only apply to those speaking about mathematics, for example in classroom discussion? Or can mathematical methods be construed as a language? Bakhtin writes that in mathematics "[a]cquiring knowledge here is not connected with receiving and interpreting words or signs from the object itself under consideration" (1981, p. 351). This would seem to limit my application of his theory to discussions of mathematics by those in the classroom. However, I am interested in his argument that for other disciplines:

> even the driest and flattest positivism in these disciplines cannot treat the word neutrally, as if it were a thing, but is obliged to initiate talk not only about words but in words, in order to penetrate their ideological meanings – which can only be grasped dialogically, and which include evaluation and response.
>
> (Bakhtin, 1981, p. 352)

If even the most positivist situations require a dialogic approach, why does this not extend to mathematics? Is mathematics in some way unreachable and unchangeable if it cannot be engaged with dialogically? Is the learning of mathematics only possible if accessed through some intermediary by making dialogical meaning about it but not with it? Could mathematics be considered a form of language to better explain how students engage with it?

In the following two chapters, I am going to look at how key authors working in the field have treated Bakhtin's approach to mathematics and some of the arguments they have put forth about connections they see between the work of Bakhtin and Vygotsky. Analysis of previous work in the field will allow me to continue to explore connecting the theories at the level of principles by looking at how others have connected the principles. The authors in the following chapter have used the theories of Bakhtin and Vygotsky in a style similar to that of the middle and left of the networking strategies spectrum (Figure 1.2), although they have not specifically described the links using networking strategies. Those in Chapter Four have used strategies to the middle and to the right of the spectrum. I am going to adopt a meta-level "comparing"/ "contrasting" position to look at the networking strategies of the other authors in an attempt to improve my understanding of the two theories under consideration.

## Chapter Three – Principles: Vygotsky and Bakhtin Together

In Chapter Two, I gave an overview of Bakhtin's and Vygotsky's theories using "understanding others"/ "making understandable" networking strategies (Figure 1.2) as a start to connecting the theories at the level of principles. In this chapter, I expand this overview by considering the work of others in the field. In Chapter One (section 1.1.2), I explained that my introduction to Bakhtin's work came through Barwell's writing. As I began to extend my reading into Bakhtin and his dialogic approach, I found that the further back I went, the more authors paired Bakhtin's work with that of Vygotsky, a contrast to more recent work where time was spent illustrating the differences between their approaches. This chapter and the next set out the main works of authors who have connected the theories of Bakhtin and Vygotsky, highlights important features of their interpretations and tries to situate the way the authors have connected theories using the networking strategies of Prediger et al. (2008). In this chapter, I consider the works of Wertsch, Wells, and Radford, who use Vygotsky's and Bakhtin's theories as extensions of one another or as sharing the same roots, before considering the philosophical basis of the theories in preparation for the next chapter. For each author, I consider how they have connected the two theories and what connecting the theories has brought to their positions. By considering the benefits and limitations of the connections, I can start to consider how I might go about connecting the two theories myself. In Chapter Four, I will move on to look at the works of Wegerif, Matusov, White, and Barwell, who disagree with some of the authors in this chapter, before returning again to the discussion of philosophical positions. I have selected authors due to the new ways they have interpreted Vygotsky and Bakhtin, the detail of their contribution to the debate around connecting the two, or because of the networking strategies their work represents. In many cases (e.g., Wertsch) they are referenced extensively in the wider literature. I will give more information about my choices in each section. I cannot cover the work of all authors writing in the field and there were many I could not include. For example, Mercer has collaborated with Wegerif in the past (1997) and is also widely referenced by others, but his focus on categorising talk and then analysing only certain types introduces a hierarchy to language that would require rigorous investigation beyond the scope of my project.

This chapter and Chapter Four are organised by individual author, considered one after another, whilst simultaneously attempting to capture a sense of the chronology of the

development of ideas in the field. This is not meant in any way to imply that authors worked in isolation and indeed it will quickly become clear that the works of several of these authors overlap and heavily influence one another. My intention is that considering the authors individually will demonstrate issues or changes raised by individuals in key works whilst simultaneously giving a sense of where each of these changes or issues occurred in the wider fabric of discussions, highlighting the historical situatedness and sense of context that is so important to the approaches of Bakhtin and Vygotsky.

### 3.1 James Wertsch

To begin with, I am going to look at the work of James Wertsch. Writing in the late eighties and early nineties, his work is some of the earliest I will focus on in depth. I have included Wertsch as, in addition to writing extensive commentary on Vygotsky individually, he is perhaps one of the best known of the authors who used the work of Vygotsky and Bakhtin to form a single basis of theory. Wertsch's work is widely referenced by other authors (e.g., section 3.3.1 and section 4.1) and formed the basis of much of the debate that follows. His writing ties the theories of Vygotsky and Bakhtin together, addressing some of the issues that arise in doing this, namely the unit of analysis and the issue of identity.

Wertsch said that "the task of a sociocultural approach is to explicate the relationships between human action, on the one hand, and the cultural, institutional, and historical contexts in which this action occurs, on the other" (1998, p. 24). His interest in sociocultural theory has a psychological slant: "I often focus on the psychological moment of action, but my effort throughout will be to formulate psychological claims in such a way that their relationship to sociocultural context is always apparent" (Wertsch, 1998, p. 23). Wertsch explains here how he extends a psychological basis to focus on the importance of context, tying his work to that of Vygotsky who was rooted in psychology (see section 2.2). Wertsch uses Bakhtin's theories to fill gaps he identifies in Vygotsky's work and significantly develops Vygotsky's concept of meditational means by incorporating Bakhtin's ideas.

In my summary of his key ideas, I will first look at Wertsch's justification for joining Vygotsky and Bakhtin in the way that he has, then his framing of meditational means and finally discuss the impact connecting the theories has had.

### 3.1.1 Using Bakhtin and Vygotsky

Wertsch's work is based on a tangible appreciation of the work of Vygotsky. He sees the significance of Vygotsky's work in being one of the first to appreciate how important a consideration of sociocultural factors was in development and learning of the individual. However, he argues that "in certain essential respects [Vygotsky] did not succeed in providing a genuinely sociocultural approach to mind. In particular, he did little to spell out how specific historical, cultural, and institutional settings are tied to various forms of mediated action" (1991, p. 46). He also identified an "ethnocentric bias" (Wertsch, 1991, p. 31) in Vygotsky's work that I will return to later (section 3.3.2). Wertsch explains his view that, "to formulate a more comprehensive sociocultural approach to mental functioning one should identify historically, culturally and institutionally situated forms of mediated action and specify how their mastery leads to particular forms of mediated action on the intramental plane" (1991, p. 48). It is in identifying these specific forms and looking closely at how they affect mental functioning that Bakhtin becomes significant, extending the ideas of Vygotsky discussed in section 2.2.2.

Wertsch explains his plan: "to examine a general sociocultural approach to mind through Vygotsky's writings and then to incorporate some of Bakhtin's ideas, in particular, utterance, voice, social language, and dialogue, to extend Vygotsky's claims about the mediation of human activity by signs" (1991, p. 17). Wertsch explains that Vygotsky's account of development focused on cultural development but also included a "natural line" (Wertsch & Tulviste, 1992, p. 554) of development as well. Wertsch suggests this natural line of development focused on, among other things "the emergence of sensory abilities to motor skills to neurological development" (Wertsch & Tulviste, 1992, p. 554), but that Vygotsky was not specific and that this element of his account of development "minimizes the contributions made by the active individual. Among other things, it raises the question of how individuals are capable of introducing innovation and creativity into the system" (Wertsch & Tulviste, 1992, p. 554). Wertsch argues that Vygotsky's references to a natural line of development is indicative of a lack of research on the topic and prevailing ideas of the time. Wertsch, by introducing the ideas of Bakhtin, returns focus to the individual: "speaking always involves a concrete individual in a unique setting using language tools provided by others to create utterances" (Wertsch & Tulviste, 1992, p. 555), going on to adopt "individual(s)-operating-with-mediational-means" (Wertsch & Tulviste, 1992, p. 555) as the focus for his theory. For my context, Wertsch was one of the earliest authors

whose work I am analysing as an example of a way of connecting the Vygotskian framing of the curriculum with the Bakhtin context-based framing of meaning making.

### 3.1.2 Mediated Action

One of Wertsch's most significant contributions to this field is his focus on mediated action as a unit of analysis. Bakhtin has a clear unit of analysis in his utterance (section 2.1.1), whereas Wertsch sees Vygotsky's focus as word meaning, which Wertsch then goes on to develop into his own view of the unit of analysis, namely mediated action. Wertsch ties Bakhtin's ideas into Vygotsky's work arguing that "in Bakhtin's hands the utterance […] is a form of mediated action" (Wertsch, 1994, p. 205). Wertsch describes mediated action as:

> a natural candidate for a unit of analysis in sociocultural research. It provides a kind of natural link between action, including mental action, and the cultural, institutional, and historical contexts in which such action occurs. This is so because the mediational means, or cultural tools, are inherently situated culturally, institutionally, and historically.
>
> (Wertsch, 1998, p. 24)

The shift in focus for the unit of analysis is important as how Wertsch applies it is a key development in the field. It is also one of the major points of contention in the discussion and debate that came later (see, for example, section 4.1). As such, it is important to unpick what Wertsch means by unit of analysis.

In developing his unit of analysis, Wertsch starts with Vygotsky's idea that "a criterial feature of human action is that it is mediated by tools ("technical tools") and signs ("psychological tools") His primary concern was with the latter (what we are here calling "cultural tools")" (Wertsch & Tulviste, 1992, p. 551). Wertsch (1998) offers two clear examples of tools in use, one technical and one psychological (previously referred to in Vygotsky's work as tools and signs respectively). Wertsch's example of a technical tool is that of a pole vault. Wertsch explains that "unlike many cases of mediated action, especially those involving spoken language, where the mediational means appear to be ephemeral, the materiality of the mediational means in this case is obvious and easy to grasp" (Wertsch, 1998, p. 27). Wertsch characterises the vaulter as the agent and the pole as the mediational means. As the materials making up the pole have varied over the years, so too have the methods used by the vaulters to utilise the tool in maximising the height of their

vaults. The historical development of the tools directly impacts on the mediational means and the agent:

> Pole vaulting considered as a form of mediated action provides a clear illustration of the irreducibility of this unit of analysis. For example, it is futile, if not ridiculous, to try to understand the action of pole vaulting in terms of the mediational means—the pole—or the agent in isolation. The pole by itself does not magically propel vaulters over a cross bar; it must be used skilfully by the agent. At the same time, an agent without a pole or with an inappropriate pole is incapable of participating in the event.
>
> (Wertsch, 1998, p. 27)

The example Wertsch gives of a psychological tool (sign) is specifically related to mathematical methods. He talks about the layout of a multiplication in columns (known colloquially as the column method for multiplication as opposed to the grid method, say, or Chinese multiplication). The column method is a cultural tool, "a specific mediational means […] that make solving the problem possible for us. Without the affordances provided by this cultural tool, it would be quite difficult to carry out complex multiplication problems" (Wertsch, 1998, p. 29). He sees the cultural tool as simplifying the process of multiplication of large numbers from a challenging problem requiring significant conceptual understanding to smaller steps that make it more straightforward to complete.

Wertsch interprets Vygotsky's conception of mediational means as too general and instead "would suggest that mediational means be viewed not as some kind of single, undifferentiated whole but rather, in terms of […] a toolkit" (Wertsch, 1991, p. 93). The toolkit analogy allows for discussion not just about which mediational means or tools are being used or how these tools are useful, but also, why that tool has been selected for use in this context (1991, p. 96). My project focuses on perceptions of mathematical methods chosen by students so the toolkit analogy allows for discussion about how the context influences student choice.

Wertsch considers Bakhtin's speech genres (section 2.1.3) to be an example of mediational means:

> By focusing on speech genres as mediational means, one is constantly reminded that mediated action is inextricably linked to historical, cultural, and institutional settings, and that the social origins of individual mental functioning extend beyond

> the level of intermental functioning. Because utterances inevitably invoke a speech genre, it is no longer possible to view dialogue in terms of two localised voices.
>
> (Wertsch, 1991, p. 144)

He argues that, whilst Bakhtin "provided relatively little detail about what defines and differentiates specific genres" (Wertsch, 1998, pp. 119–120), incorporating speech genres into mediational means allows a better analysis of how specific sociocultural settings affect mediated action, addressing one of the weaknesses Wertsch sees in Vygotsky's theory (see section 3.1.1). In developing his ideas about mediated action, the overlap of the ideas of Vygotsky and Bakhtin are in evidence in the language used by Wertsch. He describes mediational means, Vygotsky's concept, as "defined in terms of an irreducible tension between cultural tools and active agents" (1998, Wertsch, pp. 179–180), utilising the idea of tension from Bakhtin.

Wertsch argues, by focusing on mediated action as the unit of analysis, that we look to the social context in which these actions occur instead of looking to the individual to try and analyse their mental action i.e., we begin our analysis "outside the individual" (Wertsch & Tulviste, 1992, p. 548), which avoids the "individualistic reductionism" in analysis that he describes as "severely limited, if not misguided" (Wertsch, 1998, p. 21). In this way, we have the individual taking part in mediated action but analysis of this individual would miss the sociocultural elements of the environment affecting any mediated action. The change in unit of analysis follows the ideas of Bakhtin and Vygotsky (explained in Chapter Two) in capturing the sociocultural context of an event: "when one focuses primarily on the individual agent's role in mediated action, the fact that cultural tools are involved means that the sociocultural embeddedness of the action is always built into one's analysis" (Wertsch, 1998, pp. 24–25). The unit of analysis also overcomes one of Wertsch's key issues with Vygotsky's work, namely that he "said relatively little about how the active employment of these means generates and transforms meanings and cultural tools and how it gives rise to new ones" (Wertsch, 1994, p. 204).

One key link Wertsch makes between Bakhtin and Vygotsky, that makes his ideas about mediational means possible, is between Vygotsky's concept of internalization (section 2.2.1) and Bakhtin's concept of assimilation (section 2.1.4, referred to by Wertsch as appropriation). Wertsch points out a problem with analysing internalization. When we try and follow the process of internalization to look at what is happening on the intramental

level for an individual, something that goes against the concept of sociocultural research, it also "entails a kind of opposition, between external and internal processes, that all too easily leads to the kind of mind-body dualism that has plagued philosophy and psychology for centuries" (Wertsch, 1998, p. 48). Wertsch argues "that there is actually no need to invoke the terms "internal" and "internalization" [...] Instead, less loaded and less misleading terms such as "mastery" and "knowing how" would seem to be preferable" (Wertsch, 1998, p. 53). He then ties together Bakhtin's idea of assimilation/appropriation with that of mastery:

> it is worth noting that in many instances higher levels of mastery are positively correlated with appropriation. However, this need not be the case. Indeed, some very interesting forms of mediated action are characterized by the mastery, but not by the appropriation of, a cultural tool.
>
> (Wertsch, 1998, p. 56)

This overlapping of mastery (an interpretation of Vygotsky) and appropriation (Bakhtin's assimilation) is an intertwining of the two theories in a way that had not been seen before. This is what later authors took objection to, as we shall see in Chapter Four.

Wertsch also rejects the idea that internalization is always the end goal of intermental functioning, however, he finds the idea that all concepts are internalized unrealistic:

> most forms of mediated action never "progress" toward being carried out on an internal plane. This is not to say that there are not important internal dimensions or changes in internal dimensions in those carrying out these external processes, but it is to say that the metaphor of internalization is too strong in that it implies something that often does not happen.
>
> (Wertsch, 1998, p. 50)

Wertsch taking an interest in the middle ground of what happens before the end point of internalization is reached is a reflection of how he is "maintaining a focus on the irreducible tension between agent and mediational means" (Wertsch, 1998, p. 141). Moving away from the extremes of the theories of Bakhtin and Vygotsky is something I examine in more detail in section 4.5.2.

### 3.1.3 The Impact

Wertsch takes the theory of Vygotsky and, by incorporating ideas from Bakhtin, draws new conclusions and develops the existing theory. As such, his work is an example of the

"synthesizing"/ "integrating locally" networking strategies (Figure 1.2). Wertsch sees Vygotsky as the basis for sociocultural theory and uses Bakhtin's theories to address areas where Vygotsky falls short, which indicates Wertsch's approach is closer to an "integrating locally" approach as one theory is more dominant than the other.

In Chapter Two (section 2.3), I explained how I was interested in the work of Vygotsky due to the way it echoed the curriculum and that of Bakhtin as it suggested a framing of a wider interpretation of meaning. The networking of Vygotsky and Bakhtin in Wertsch's toolkit of mediational means "makes it possible to provide a more adequate response to the question, Who is carrying out the action? Or in the case of speech, Who is doing the talking?" (Wertsch, 1998, p. 26). Wertsch has suggested a move away from Vygotsky's definitive internalization to suggesting mastery as an alternative that avoids extremes and makes the point that internalization does not always take place. This interpretation of appropriation and mastery means that:

> [o]n the one hand […] agents must appropriate the words of others whenever they wish to speak […] On the other hand, agents have in their power a range of possibilities for how these words will be appropriated, a range extending from actively embracing to strongly resisting them.
>
> (Wertsch, 1998, p. 56).

The choice implied here has a strong idea of agency that allows for a sense of self, creativity and the option to question or resist the authoritative word (section 2.1.5) of Bakhtin or the leading of students (section 2.2.3) from Vygotsky. Wertsch's linking of the theories in order to extend them to look at mastery allows him to explore a less linear model of Vygotsky using Bakhtin's work, which is closely linked to my interest in analysing methods, the perceptions of methods, and the possibility of an analytical approach that does not solely focus on the curriculum aims or the wide variety of ways in which those interpreting methods could make meaning.

I am particularly interested in the perspectives of student methods in the context of assessment. Wertsch's work on the toolkit of mediational means has repercussions for an understanding of what assessment is aiming to do or is capable of doing: "when asking about someone's ability level, we are usually asking about someone's skill in functioning with a particular cultural tool" (Wertsch, 1998, p. 45). This way of perceiving assessment "raise[s] general questions about who decides which cultural tools are to be used as means

for assessing our skills and abilities" (p. 45). As a result, from Wertsch's perspective, an examination is reframed from an assessment of the mathematics one knows, to become a test of how well one can apply particular mediational means.

By linking Bakhtin's work with Vygotsky's, Wertsch has networked the theories to address a specific shortcoming he sees in Vygotsky's approach – certainly one benefit the networking approach he has used. However, this linking has allowed Wertsch to not only fill this gap, but also extend beyond the original scope of Vygotsky and Bakhtin's work by redefining the unit of analysis. This extension is one of the ways in which a connecting theories approach can be beneficial.

Despite his use for linking the two theories, Wertsch does not specifically examine the consequences of connecting theories, resulting in blended terminology and unclear philosophical positions, which other authors take issue with (see Chapter Four), among other potential downsides to Wertsch's approach. An "integrating locally" approach does not require a perfect alignment in the "principles" of the two theories, but is based on their having a compatible baseline. Wertsch does recognise that Bakhtin and Vygotsky "clearly differ over many issues" (Wertsch, 1998, p. 19), but sees using Bakhtin's work as an important step towards deriving a working theoretical model that can be used by researchers and maintains that "their ideas are […] compatible on several counts" (1991, p. 17). However, Wertsch does not specifically trace the philosophical basis of the two theories before networking them beyond reasoning that they "lived and worked in the same general intellectual milieu" (1991, p. 17). As such, this philosophical underpinning is something that needs further analysis. I will move onto this at the end of this chapter (section 3.5) and into the next (section 4.6.1).

### 3.2 Luis Radford

Radford is interested in the nature of meaning and his work often includes the use of multiple theories to facilitate his exploration. His interest in connecting theories gave rise to the framework I am using for this study (see section 1.3.1). I have chosen to include Radford as one of the key authors writing about Vygotsky and Bakhtin as, in addition to his theoretical interests, Radford's work has a specific focus on the applications of Vygotsky's and Bakhtin's theories to teaching and learning.

Radford's focus on the history of mathematics and the development of meaning echoes the ideas of Bakhtin and Vygotsky in their steps away from the notion of objective knowledge: "Even the most titanic effort of putting away all our modern knowledge in order to see the historical event in its purity will not succeed: we are damned to bring our modern socio-cultural conceptions of the past with us" (Radford, 1997, p. 27). He goes on to explain that "dialogicality is not restricted to the single objects under consideration: dialogicality is embedded in the dialoguing cultures and goes beyond any single object. Isolation, as an analytical methodological practice, is impossible" (Radford, 1997, p. 27). Radford explains his linking of Bakhtin and Vygotsky saying:

> While we retain the idea of signs as the concrete components of mentation, as suggested by cultural and Vygotskian studies, we borrow from Bakhtin (1986) and Voloshinov (1973) important elements of their theory of dialogue to understand the role of students' discursive actions and interactions.
>
> (Radford, 2000, p. 242)

One of the ways in which he illustrates the combining of the two theories is through using specific examples. Radford uses the example of the emergence of the concept of negative numbers in different cultures to demonstrate his point that:

> the configuration and the content of mathematical knowledge is properly and intimately defined by the culture in which it develops and in which it is subsumed. Consequently, any attempt to study it might take into account the composite extra-mathematical cultural structure in which mathematical knowledge is embedded.
>
> (Radford, 1997, p. 32)

He explains that, historically, in Western culture, positive numbers served as an obstacle to the development of negative numbers (and points out this is still a problem for students today), whereas in Chinese culture, mathematicians used a system of coloured rods to overcome this, demonstrating the way context affects emergence of concepts.

Radford's position that "knowledge is a process whose product is obtained through negotiation of meaning which results from the social activity of individuals and is encompassed by the cultural framework in which the individuals are embedded" (Radford, 1997, p. 32) is a useful definition of the sociocultural framework he links with the work of both Vygotsky and Bakhtin. In his article, *Signs and Meanings in Students' Emergent Algebraic Thinking: A Semiotic Analysis* (2000), Radford uses a case study of students exploring the generalisation involved in engaging with a task algebraically (as opposed to

arithmetically), to explain his theoretical position as based on two principles: "the Vygotskian idea according to which our cognitive functioning is intimately linked, and affected by, the use of signs" (Radford, 2000, p. 240) and "the fact that the signs with which the individual acts and in which the individual thinks belong to cultural symbolic systems which transcend the individual qua individual" (Radford, 2000, p. 241). As such, he clearly links cognitive function with social interaction through the meaning of sign and symbols (semiotics). Despite his use of Vygotsky, Radford rejects Vygotsky's mediated action triangle (Figure 2.1) saying it "cannot suitably account for the conceptual relations of signs and the aspects related to their meaning. Such semiotic triangles often isolate the subject, the object and the act of symbolizing from the other individuals and their contextual activities" (Radford, 2000, p. 241). Instead, Radford considers these elements to be tightly linked, demonstrated using his algebraic case study:

> the appropriation of a new and specific mathematical way of acting and thinking which is dialectically interwoven with a novel use and production of signs whose meanings are acquired by the students as a result of their social immersion into mathematical activities.
>
> (Radford, 2000, p. 241)

This appropriation (again, Bakhtin's assimilation) is seen as being "achieved through the tension between the students' subjectivity and the social means of semiotic objectification" (Radford, 2000, p. 241). Here, Radford has illustrated a tension between the heteroglossia of students' subjective prior experiences and the socially encountered signs and tools.

Based on the combination of Vygotsky's and Bakhtin's ideas, Radford sees the learning that takes place in schools as: "the process of actively and creatively transforming these cultural concepts embodied in texts, artefacts, language, and beliefs into objects of consciousness" (Radford, 2006, p. 60). Transformation into objects of consciousness becomes key in his application of sociocultural theory to the school context. Radford also makes the point that "mathematical enculturation achieved through classroom activities is a straitjacket for the mind. Rather we take classroom activities as a world of possibilities in which our intimate mathematical experiences occur" (Radford, 2000, pp. 259–260). Framing classroom activity as based on opening a range of possibilities is a step away from the Vygotskian position of leading students towards higher level concepts, and closer to the detailed meaning-making framing of Bakhtin.

Radford's collaboration with Roth (2011) focuses on education, specifically on "understanding cognition generally and teaching-learning particularly as they occur in real life" (Roth & Radford, 2011, pp. 9–10). Roth and Radford relate the concept of mastery to the classroom environment, specifically to the relation between teacher and learner or learner and more sophisticated other, explaining that due to "the way Vygotsky understands teaching-learning unit, we need to focus on the relation" (Roth & Radford, 2011, p. 79), following the ideas Wertsch has discussed with his mediated action in that analysis cannot focus solely on the learner (section 3.1.2). However, Roth and Radford go on to explain what the relation means in more detail: "it is not as if the learner constructs the function on his/her own following joint activity […] anything the learner will eventually do, s/he has already done as part of the relation" (Roth & Radford, 2011, p. 79). They suggest that, by focusing on the relation, only then can the role of others on the social origins of objects of consciousness be revealed.

Roth and Radford were clear on the origin of Vygotsky's ideas saying:

> Vygotsky did not just dream up this way of thinking about human development, the 'principle driving force' of which is historically constituted societal interaction. Rather, his ideas fundamentally derive from the way in which Marx/Engels conceive not only of human beings but also about the relationship between individual and collective (society).
>
> (Roth & Radford, 2011, p. 79)

Focusing on the relation between self and other leads to a problem when considering the process of internalization, so key to Vygotsky's theory. Roth and Radford argue that Vygotsky "writes about the transfer to the individual" as part of the process of internalization, but argue that this creates ambiguity as "[i]ndividual and collective consciousness are mutually constitutive so that there can be nothing available to the consciousness of the individual that is not already a possibility configured in collective consciousness" (Roth & Radford, 2011, p. 18). Roth and Radford liken consciousness, being both individual and collective, as being an example of an "inner contradiction" (Roth & Radford, 2011, p. 24), explaining that "if we think of any individual person, we are confronted with the fact that s/he is both (a) a concrete realization of the human species, that is, the general in its concreteness, and (b) a particular human being" (Roth & Radford, 2011, p. 25), arguing that this contradiction "cannot be removed" (Roth & Radford, 2011, p. 24). As a result, they see consciousness as ever-changing or "flow-like" (Roth & Radford,

2011, p. 23) and representing this irreducible contradiction. Roth and Radford's example of the individual person is one they base on Marx/Engels and, indeed, their inclusion of Marx/Engels is a key link made by authors in the field between Vygotsky's ideas and the dialectics of Hegel and, later, Marx (discussed in more detail in section 3.5), but this link is going to be key to the debate in Chapter Four, as the authors in that chapter argue that a dialectic approach represents a focus on overcoming rather than this irreducible contradiction and that Bakhtin has a different basis to his ideas.

Borrowing from the work of multiple authors is common to Radford and his linking of multiple theories without examining the underlying principles makes it challenging to identify a networking strategy to describe his position. This does not invalidate his methods, however it makes it difficult to identify exactly what each theory adds to the analysis. Despite this, I suggest that his use of Bakhtin's tensions and assimilation/ appropriation with Vygotsky's dialectics indicates a "synthesizing"/ "integrating locally" approach. Radford's use of Bakhtin to highlight the key importance of language within the semiotic triangle and the associated movement away from any isolationism suggested by previous conceptions of the semiotic triangle is another example of how connecting the two theories can result in extensions to existing theories. Radford's work linking Bakhtin and Vygotsky has given a specific suggestion as to how the historical and cultural situatedness of knowledge might be analysed. For my analysis of student and teacher perceptions of mathematical methods, this suggestion is particularly useful as it might allow me to analyse how specific interpretations of methods vary from one individual to another, or between cohorts such as variations between interpretations of teachers and students. Radford's connections also suggest a specific way of utilising the literary theory expertise of Bakhtin within the psychological focus of Vygotsky.

In addition to suggesting a specific path for analysing the historical and cultural aspects of interaction, Radford raises a key point about overcoming contradictions that addresses some of the difficulties that come from linking Vygotsky and Bakhtin's positions (discussed further in Chapter Four). Radford's suggestion that not all contradictions can or must be overcome is not only a useful point in the linking of Vygotsky and Bakhtin, but is reflected in his position on connecting theories (2008).

Radford does not specifically detail the philosophical grounds on which he links these authors, rather, emphasising the basis of Vygotsky's theories in Marx/Engels and then finding links with Bakhtin through their work on similar areas. The next author I am going to discuss, Wells, again extends Vygotsky's and Bakhtin's works, but takes a similar approach to Radford in that he does not extensively analyse philosophical compatibility but, with his ambiguous use of language, seems to take a networking strategy that closely connects the two theories.

### 3.3 Gordon Wells

I have chosen to include Wells's work, with a specific focus on his book *Dialogic Inquiry: Toward a Sociocultural Practice and Theory of Education* (1999), which connects the theories of Bakhtin and Vygotsky whilst looking at the implications for the classroom and, new for my study, the role of the teacher. Wells has an unusual approach to some of the technical terminology, redefining, extending or mixing terms traditionally used by either Vygotsky or Bakhtin. I am going to focus here on how Wells combines the work of Vygotsky and Bakhtin, the associated changes he makes to technical definitions as a result, and the specific implications of his work for Vygotsky's Zone of Proximal Development.

#### 3.3.1 Combining Vygotsky and Bakhtin

In a similar way to that of Wertsch (section 3.1), Wells begins with Vygotsky's work before bringing in the ideas of others to help with areas of Vygotsky's work he finds lacking:

> Vygotsky provides a firm basis for a language-based theory of learning and development that is of central importance for education. What his theory does not provide, however, is explicit guidance on the kinds of language use that would best facilitate this developmental process in the classroom.
>
> (Wells, 1999, p. 102)

Wells speculates that "[t]his may well be because, in his [Vygotsky's] brief working life, he did not have the opportunity to gain first-hand experience of working with students and teachers in the classroom" (Wells, 1999, p. 102). The implications of Vygotsky's work for the classroom has been looked at in terms of interaction in the classroom environment but Wells goes one stage further and looks at the implications for teachers directly (section 3.3.2).

Wells highlights how social context is significant to those involved in language use:

> All instances of language use occur – or [...] all texts are created – in particular social contexts […] for the participants to be able to co-construct the text, they have to interpret the context as an instance of a recognizable "situation-type" and to make their interpretation recognizable to their coparticipants.
>
> (Wells, 1999, p. 9)

The specifics of each context mean that "the participants' interpretation of the situation […] predisposes them to make certain types of choices from their meaning potential in co-constructing their text" (Wells, 1999, p. 9). In a classroom, the discourse of students is shaped by the context of the classroom. When responding to teacher questions, students have to interpret the particular meaning relative to the context and then fit their response to that in order to contribute to discussion. In a section titled *Learning Language: Appropriating Culture,* Wells explains, "it is abundantly clear that […] Vygotsky see[s] the use of […] language, as the means whereby, in the course of everyday activity and interaction, the culture is simultaneously enacted and socially 'transmitted' to succeeding generations" (Wells, 1999, p. 21). The approach to learning as being the appropriation of culture is going to become significant in the next chapter (section 4.1).

Wells recognises three areas of Bakhtin's work as key contributions to the discussion around discourse. Firstly, he identifies speech genres, explaining that "utterances do not occur as isolated acts, but are always contextualized by both the specific goals and conditions of the activity in which they occur and by the utterances that both precede and follow" (Wells, 1999, p. 103). He also highlights the role of responsivity in shaping utterance, explaining that "no utterance is final […] all utterances should be treated as no more - and no less - than contributions to the ongoing dialogue, and therefore open to further response" (Wells, 1999, p. 105). Wells also picks up on Bakhtin's inclusion of written texts which "mediate "activity" and […] are also dialogic" (Wells, 1999, p. 236). He incorporates the three aspects of Bakhtin's dialogics to Vygotsky's dialectics, arguing that through these, "Bakhtin offers a valuable pointer as to how participation in discourse allows the child to appropriate or, as Vygotsky puts it, to internalize the mental functions encountered in particular instances of interactions with others" (Wells, 1999, p. 104), reinforcing the importance of Wertsch's toolkit analogy (see section 3.1.2).

Based on his reading of Vygotsky with Bakhtin, Wells offers extended or modified definitions of key concepts such as understanding:

> Understanding, I would now suggest, is the sense of coherence achieved in the act of saying – the impression one has of the elements of the problem or puzzle fitting together in a meaningful pattern […] In order to contribute in a "progressive" manner to the ongoing dialogue, one has to interpret the preceding contribution in terms of the information it introduces as well as of the speaker's stance to that information, compare that with one's own current understanding of the issue under discussion, and then formulate a contribution that will, in some relevant way, add to the common understanding achieved in the discourse so far, by extending, questioning or qualifying what has already been said.
>
> (Wells, 1999, pp. 107–108)

Thus, "by contributing to the joint meaning making with and for others, one also makes meaning for oneself and, in the process, extends one's own understanding" (Wells, 1999, p. 108), Wells demonstrates his view of understanding as a social practice. One ramification of how dependent understanding is on the interpretation of preceding contributions and preparation for responsivity is that understanding becomes "not something one has – not a permanent state or an object in some mental filing cabinet" (Wells, 1999, p. 108). Instead, a change in situation or activity means understanding "must be reachieved in another utterance that is responsive to whatever demands the new activity, or later phase of the same activity, makes. And since these occasions are always different to some degree, so also is the understanding" (Wells, 1999, p. 108). This is a shift away from the more static sense of development of dialectics. Wells points out a subtlety here about how he has interpreted the utterance:

> Although the term is most frequently used to refer to instances of speech or, as in the works of Vygotsky and Bakhtin, writing as well as speech, it can be applied by a straightforward analogy to all instances of semiotically mediated activity and, without straining the analogy very far, to creative problem solving of all kinds.
>
> (Wells, 1999, p. 109)

In this formulation, Wells uses utterance "to refer to both […] the saying/making a solution, on the one hand, and what is said/the solution that is made, on the other" (1999, p. 109), raising two key points: "First, that understanding can be achieved in doing and in comprehending what is done as well as in saying and in comprehending what is said; and second, that knowledge artifacts do not always take the form of linguistic texts" (Wells, 1999, p. 109). Wells connects the two theories in order to introduce a sense of progression from Vygotsky's theories to the making of meaning through the use of language from

Bakhtin. For the context in which I am working, this suggests a way of framing meaning making which allows for the variation from Bakhtin's theories, but one that includes a sense of progression to allow for the learning context as required by the curriculum. He then takes this a step further, extending Bakhtin's theory to interpreting the utterance in a way that goes beyond linguistic texts offers the options of considering mathematical methods themselves as utterances. I began to discuss this in section 2.3 and will return to it again in my methodology (e.g., section 6.3).

### 3.3.2 The Zone of Proximal Development

The Zone of Proximal Development (ZPD) is a significant part of Vygotsky's theory (see section 2.2.3) but Wells continues his pattern of extending theories and makes some important alterations by extending the idea of development away from a linear model. Wells interprets Vygotsky's views of learning as a process of development, one where the higher mental concepts "connote a superior mode of functioning" (Wells, 1999, p. 324). Wells takes Vygotsky's position to mean that societies that engaged in scientific thinking were seen as advanced as opposed to primitive ones that did not (Wells, 1999, p. 324), which ties in with previous critique of Vygotsky for being ethnocentric/Western orientated (see Wertsch, section 3.1.1). Wells raised three issues with Vygotsky's linear development. Firstly, Wells agrees that Vygotsky's terminology and focus on Western society "really do[es] lay him open to the charge of "Eurocentrism,"" (Wells, 1999, p. 325). Secondly, Wells raises issues with "the primacy given to cognition in much of the Vygotskyan-inspired study of human development, and the consequent neglect of the social, affective and motivational dimensions" (Wells, 1999, p. 326). Thirdly, Wells queries "the assumption of inevitable progress" inherent in Vygotsky's theories, explaining that "the development of the individual is dependent on the tools and practices that are made available for appropriation in the activities in which he or she participates" (Wells, 1999, p. 326). As he explains, if a child has had a traumatic experience in childhood or has missed out on socialisation it could limit their development. In addition, "some have argued too, that the coercion that is a pervasive characteristic of formal schooling in almost every culture constitutes an unrecognized but systematic limitation of the creativity and originality of which all human beings are capable" (Wells, 1999, pp. 326–327).

As a result of his objections to the linear model of learning, Wells concludes that:

> it is now no longer possible to accept a conception of learning in the zpd that assumes either a single end in view or a developmental trajectory that is free of contradiction and conflict. Decontextualized rational thinking is not the inevitable apogee of intellectual development, nor is it necessarily optimal in all situations.
>
> (Wells, 1999, p. 327)

Moving away from this end point has implications for the ZPD. Wells explains that "instead of viewing development as progress towards some ideal, therefore, there is an increasing tendency to focus on the transformative nature of learning in the zpd, with an emphasis on diversity rather than on improvement" (Wells, 1999, p. 327). Wells alters Vygotsky's idea that the ZPD is predicated on the development of individuals when coming into contact with more capable others, making the point that,

> it is not necessary for there to be a group member who is in all respects more capable than the others. This is partly because most activities involve a variety of component tasks such that students who are expert in one task, and therefore able to offer assistance to their peers, may themselves need assistance on another task. But it can also happen that in tackling a difficult task as a group, although no member has expertise beyond his or her peers, the group as a whole, by working at the problem together, is able to construct a solution that none could have achieved alone.
>
> (Wells, 1999, pp. 323–324)

As a result, Wells's framing of the learning environment that makes use of the ZPD is not one with hierarchical arrangements based on knowledge, but as "collaborative communities of practice" (Wells, 1999, p. 330).

Significantly, Wells raises the idea that learning in the ZPD is not limited to interacting with those actually present. He claims that the written word can also act in the ZPD, which has echoes of Bakhtin's work with literature and the voices inherent in the written word. He also highlights the role of collaborative communities of practice and the impact the ZPD has on the identity of the student as learning in the ZPD "involves all aspects of the learner – acting, thinking, and feeling; it not only changes the possibilities for participation but also transforms the learner's identity" (Wells, 1999, p. 331). I discussed the individual and collective consciousness in section 3.2 and Wells echoes those ideas, arguing that "because individuals and the social world are mutually constitutive of each other, transformation of the learner also involves transformation of the communities of which he or she is a

member and of the joint activities in which they engage" (Wells, 1999, p. 331), extending the focus from not just what effect the community has on the learner but also the effect the learner has on the community.

Along with a discussion of the ZPD and its significance for the identity of the learner, Wells talks about the significance of the role of the teacher in developing the ZPD for students. Vygotsky focused on the psychological background of the ZPD and, despite talking about the importance of instruction, did not give a lot more detail. Wells, in a significant expansion of Vygotsky's work, explains that "teaching […] involves the ongoing co-construction of each student's zpd and on-the-spot judgements about how best to facilitate his or her learning in the specific activity setting in which he or she is engaged" (Wells, 1999, pp. 328–329). As Wells thinks that the ZPD has a transformative effect on students, so teachers have a significant responsibility in constructing this ZPD, not just to facilitate the teaching of content, but also to communicate morals and values (Wells, 1999, p. 329).

In addition to this reformulation of the ZPD for students and the emphasis on the role of teachers, Wells picks up on the point that learning is not limited to children learning, but is ongoing throughout life (Wells, 1999, p. 104). As part of his study, his team noticed "we teachers were also receiving assistance in our zones of proximal development – not only from each other, but also from the students" (Wells, 1999, p. xix). This is the first time the learning of teachers was considered in the literature and this "agentive view of development: teachers learning in their zones of proximal development, constructing their understanding of the art of teaching through reflective practice" (Wells, 1999, p. 329) is something that I will pick up on in Chapter Five (section 5.2).

Wells's work makes significant adaptations to Vygotsky's ideas and theories. He uses Bakhtin's ideas in order to do this, particularly to expand the concepts of understanding and the impact this has on the ZPD. Wells argues that this change and development of ideas is reflective of:

> a central tenet of Vygotsky's theory that theories, like all other artifacts, are the products of the particular conditions in which they are created; if they are to be useful in other times and places, therefore, they must be treated, not as repositories of truth that are fixed and immutable but as helpful tools for thinking with, which can themselves be improved in the process.

(Wells, 1999, p. 334)

His work has highlighted some key issues for teaching in a way previous authors have not. Most significantly for this study, the idea of responsivity. Firstly, the necessity of responsivity inherent in an utterance means "particular utterances cannot be taken as the expression of the speaker's stable, underlying beliefs and attitudes, but rather must be understood as strategic moves tailored to the speaker's assessment of the exigencies of the immediate discursive situation" (Wells, 1999, p. 105). Responsivity has implications for using the utterance as indicative of identity but also:

> the consequential effects of the particular discursive context on what is said – or written – should also lead us to think seriously about the way in which students' "utterances" are evaluated as evidence of what they have learned and come to understand, whether these be answers to teachers' questions, oral contributions to discussion, or sustained written responses under examination conditions. Here, too, it is important to recognize that no test can tell us what children really think or understand.
>
> (Wells, 1999, pp. 105–106)

If utterances are called into question as a way of assessing what students have learned, then this becomes significant in the examining of mathematics if I take mathematics to be something that can be expressed as an utterance.

Wells's book, *Dialogic Inquiry*, is unusual since, despite being mainly based on Vygotsky's work, it makes no mention of Vygotsky's background in dialectics or any reference to Marx or Hegel (a common link I will discuss in section 3.5.1 and later). In fact, Wells's book is particularly notable as it links Vygotsky to dialogics, a theory usually credited to Bakhtin. Wegerif notes how unusual Wells's interpretation of Vygotsky is, explaining "[d]ialogic is often included as part of a sociocultural position and even sometimes sourced to Vygotsky as well as to Bakhtin (for example, see Wells, 1999, p. 104 and throughout)" (Wegerif, 2008, p. 349). I return to this discussion in section 3.5.1 and Chapter Four to explore arguments against linking Vygotsky and dialogics so closely.

Wells's specific extension to the work of Vygotsky and Bakhtin, particularly in the area of the ZPD, comes about due to his work linking the two theories. His ideas around how a ZPD is constructed, particularly in the classroom and between peer groups of teachers, expand upon the traditional model of a ZPD I have discussed previously (section 2.2.3) and will go

on to be something I continue to explore (sections 7.3.2 and 8.5). In addition, Wells's extension of Bakhtin's utterance (although Wells uses utterance referring to Vygotsky's ideas as well see 3.3.1) to beyond spoken and written language is compelling and something I am going to continue to extend in my own analysis to look at whether written mathematical methods can be analysed in a similar way.

However, a merging of the two theories to the extent that Wells has carried out makes it difficult, as with Radford in section 3.2, to identify what each individual theory brings. This merging also causes confusion when technical terms from one theory are used in a manner different to that which they were originally intended without specific clarification. As I will go on to discuss (section 3.5), with the original theories having disparate philosophical groundings, terminology taken from one or the other and then adapted can lead to confusion, as in Wells's use of utterance in a comment about Vygotsky (section 3.3.1) and potentially undermine the meaning of the originals. The ambiguity could be considered a drawback of these closer networking strategies – either each modification has to be made explicit, or new terminology would have to be employed in order that meaning from the original theories is not associated with what are now extended understandings of the same terms.

As exemplified by this ambiguity in language, Wells's approach closely networks the theories of Vygotsky and Bakhtin. The use of terms from Bakhtin's work being attributed to Vygotsky (or vice versa) or used ambiguously is something that occurs in other places in the book: "Like speech, writing is very much a social mode of communicating and thinking (Bakhtin, 1981), and the activities it typically mediates are collaborative endeavours, even though the participants may not be co-present in time and space" (Wells, 1999, p. 270). Here we see the use of mediate, a term traditionally associated with Vygotsky's mediational means but here used in the context of a discussion around Bakhtin and dialogics. Wells connects the two theories closely; his networking approach is part of the "synthesizing"/ "integrating locally" pairing due to his focus on developing new theory. I would argue that Wells's conception is closer to a synthesising approach due to the level of connection indicated by the way he uses technical terms. His specific approach to linking the two theories, extending definitions of existing technical terms, or developing his own technical terminology means the terminology can then be applied to analysis in a way that allows for the articulation of context-specific meaning making that still adheres to an overall linear

progress, which is particularly useful in for my interest in reconciling different interpretations of mathematical methods with a sense of the linear progression necessitated by the National Curriculum.

### 3.4 Discussion

The authors, whose work I have covered in this chapter, are all examples of those who have connected the work of Bakhtin and Vygotsky. From the work of the three authors, I have been able to identify interpretations of the principles or "implicit views and explicit statements that delineate the frontier of what will be the universe of discourse and the adopted research perspective" (Radford, 2008, p. 320) of the two theories. The starting point of connection used by the authors I have discussed here is the sociocultural background shared by both Bakhtin and Vygotsky, the focus on "cultural, institutional, and historical contexts" (Wertsch, 1998, p. 24). The movement away from the historical standpoint of knowledge as objective is something both Bakhtin and Vygotsky share. The most common way these authors have connected Bakhtin and Vygotsky is by identifying issues or gaps in Vygotsky's work and using Bakhtin's ideas to address them. I have interpreted their networking strategies as "synthesizing"/ "integrating locally" due to their close affiliation between the two theories and the way that the authors focus on "the development of theories" (Prediger et al., 2008, p. 12).

The development and extension of the existing theories offered by the authors in this chapter are benefits of linking theories. By networking the work of Bakhtin and Vygotsky, each of the authors has gone on to offer something new. Wertsch has redefined what he sees as a suitable unit of analysis for Vygotsky as mediated action, Radford has suggested an alternative to the overcoming stage of a dialectic approach which not only extends a traditional dialectic model but also allows for connections between two theories which are based on different philosophical bases as I will go on to explore in section 3.5. Finally, Wells has offered specific ideas for extending the ZPD introducing a flexibility reminiscent of Bakhtin. These adaptations have significant implications for the ideas of identity for the student and the role of the other, particularly, the other as part of collaborative communities of practice. The clear parallels drawn between concepts such as the internalization of Vygotsky and the assimilation/appropriation of Bakhtin are also explored by these writers in the context of education, specifically mathematics education,

which shows clearly how the theories can be applied to the situation in which I am researching.

The benefits of linking the theories of Bakhtin and Vygotsky I have discussed here are what I wish to assess through this project. A linked approach could reflects the structure of the curriculum whilst simultaneously allowing for a contextual-specific meaning making and offers the possibility of extending theories beyond the parameters of their individual groundings. The authors in this chapter have, broadly speaking, adopted similar networking strategies, but it is important to note they have not considered the possible connections at the level of principles, methodology or research questions as in a connecting theories approach. I wish to test what a connecting theories approach can bring to an analysis of student and teacher perspectives of mathematical methods. As I mentioned in Chapter Two (section 2.3), I am also interested in investigating whether or not mathematical methods can be analysed using Vygotsky's and Bakhtin's theories. The extent of the networking demonstrated by authors in this chapter suggests that this is possible. Wertsch has clearly demonstrated the use of the column method for multiplication as an example of a sign in his toolkit of mediational means (section 3.1.2), which incorporates Bakhtin's speech genres. In my exploration of mathematical methods as a language in a Bakhtinian sense, I also see it as subject to centripetal and centrifugal forces and so link Bakhtin's literary theories directly with the teaching and learning of mathematics. Wells goes a step further (section 3.3.1) and expands the definition of the utterance "to all instances of semiotically mediated activity" (Wells, 1999, p. 109). As such, I am going to continue to explore the possibility of framing mathematical methods as a type of sign/utterance in future chapters. My choice to connect Bakhtin's and Vygotsky's theories offers a way of testing this theory in a number of ways. The networking strategies that require less integration may allow the development of an analysis of mathematical methods in a dialogic sense alongside a dialectic analysis such as in a "comparing"/ "contrasting" approach. However, it may also be possible to network the two theories more closely and thus provide an opportunity to develop my idea using a "synthesizing"/ "integrating locally" approach.

At each stage of my study I am going to test a connecting theories approach and critically evaluate the benefits of possible networking strategies. This chapter has focused on authors who have closely linked the theories of Bakhtin and Vygotsky. In the following

chapter, I am going to consider the work of authors who argue against the theories of Bakhtin and Vygotsky being networked in this way in order to understand where the potential pitfalls of a connecting theories at the level of principles are and will, in turn, lead to me understanding to what degree the two theories can be networked and what can be gained from this approach in my context within mathematics education. When identifying networking strategies for the authors in this chapter, one of the difficulties I found was in establishing their interpretations of the philosophical basis of Vygotsky and Bakhtin. This is important as, without "a coherent philosophical base" (Prediger et al., 2008, p. 12), the more tightly interwoven networking strategies cannot be adopted. As such, in both the next section, and the next chapter, I am going to examine the way in which authors interpret the philosophical positions of Vygotsky and Bakhtin.

### 3.5 Philosophical Positions

Before moving on to look at the work of other authors who dispute some of the networking strategies employed by those I have discussed in this chapter, I am going consider the point that arose in section 3.3.2 about influences on Vygotsky and give a short overview of the philosophical backgrounds of Vygotsky and Bakhtin.

#### 3.5.1 The Philosophical Background of Vygotsky

Of the two theories I am focusing on, Vygotsky's influences are more widely agreed upon and, as a result, easier to trace. The authors in this chapter list Hegel and Marx as key influences on Vygotsky's work (see sections 3.2 and 3.3.2). Vygotsky's sense of the historical development not only of the individual, but also of society is based on Hegel's dialectics: "[w]hat distinguished Hegel's mode of thought from that of all other philosophers was the tremendous sense of the historical upon which it was based" (Engels, 1955, p. 372). Dafermos suggests there are different types of dialectics including; Hegel's, which focuses on "a method of thought that included the process of expounding contradictions and their resolution" (Dafermos, 2018, p. A6) and Marx's materialistic dialectics, which adapts Hegel's approach to focus on a representation of "the capitalist mode of production" (Dafermos, 2018, p. A6). Hegel's attempts to move away from the prevailing position where "the existing logical theories did not correspond to the real practice of thought, and thinking about thought" (Ilyenkov, 2008, p. 171) led him to where he "formulated a programme for the critical transformation of logic as a science, he posed

the task of bringing it […] into correspondence with its real object i.e. with real thought" (Ilyenkov, 2008, p. 194).

Hegel's work is based on the dialectic method, which follows the cycle of thesis, antithesis and synthesis: Starting with an initial idea, the thesis, find an opposing idea that comes into conflict with the initial idea, the antithesis, and then there is a process of synthesis where the thesis and antithesis are "brought together, unified in a manner that preserves them, and avoids their different forms of one-sidedness" (Singer, 2001, p. 102). This synthesis stage marks the end of that particular cycle, but not the end of the process, which continues in perpetuity. Hegel applies this dialectic method to the individual's development of self-knowledge and to the wider historical development of society. Hegel's theory of knowledge involves an individual going through cycles of this dialectic method and, with each synthesis, coming closer to self-consciousness, freedom and what he terms the "Universal Reason" (Hegel, 2018, p. 204). The aim of achieving the predetermined end point or "telos" of absolute knowledge "is reached when mind realizes that what it seeks to know is itself" (Singer, 2001, p. 92). Gardiner (2000) explains that some interpretations take this idea of Hegel's to mean that we should be aiming for an overcoming of this difference and a final self-awareness. Others, that difference is necessary and any final overcoming would indicate the "end of history" (Gardiner, 2000, p. 138). Dafermos (2018, p. A6) argues that looking just at this cycle as a summary of all dialectics is an oversimplification, so I am going to look at other key concepts of Hegel and Marx there are echoes of in Vygotsky's work, including the link between consciousness and knowledge, and the master-slave dynamic with its associated themes of identity, otherness and power dynamics.

For Hegel, the sense of self is based on a process of development brought about by the relationship with, and the perceptions of, others. If consciousness relies on recognition of another who has the "authority to bestow that recognition" (Pinkard, 2018, p. xxiii,) the resulting struggle for authority between the two consciousnesses represents the conflict of the thesis, antithesis stage of the dialectic method. However, this cannot proceed to a synthesis stage as we require the other for our own self-consciousness so, as a result, the two consciousnesses settle for an unequal master-slave relationship. This is not a sustainable state, as the recognition of one consciousness by another is imbalanced and uni-directional. Through creating work for the master, the consciousness in the slave role sees the effect they are having on the physical world and achieves self-consciousness as

"the individual cannot know what he is prior to having brought himself to actuality through action" (Hegel, 2018, p. 230). I have used the commonly cited master-slave description of this relationship (Pinkard, 2018, p. xxiii) in order to be consistent with later references by authors I cite (e.g., Matusov, 2011, p. 104), however, it is translated elsewhere as lord and bondsman (Hegel, 2014, p. 182) and master and servant (Hegel, 2018, p. 113).

A sense of history and development led to "the orthodox Marxist tendency to speak of the inevitable breakdown of capitalism and the proletariat ultimately fulfilling its predestined 'historical mission'" (Gardiner, 2000, p. 128). From Hegel's idea about impact on the physical world allowing us to realise our own consciousness, Marxism saw capitalism taking these products and selling them as a removal of consciousness. As part of his work on self-consciousness, Hegel focused on a critique of religion framing God as something other and beyond humans. Marx was part of the school of young Hegelians who moved Hegel's work away from a critique of religion to a critique of a scientific method that aims to abstract, rather than studying "real people in their actual lives" (Singer, 2001, p. 111). They attempted to "rewrite the *Phenomenology* in terms of the path to human liberation. The saga of Mind then becomes the saga of the human spirit" (Singer, 2000, p. 20).

If my aim through this exploration is to identify the principles of Vygotsky's dialectics (as per Radford, 2008), then these philosophical influences indicate underlying principles that are important to take forward to the next steps of my connecting theories. From my analysis here, I have identified principles that include the idea that absolute knowledge comes from humans knowing themselves; development comes from both the opposition and synthesis of the dialectical method; and that power relationships play an important part for realising consciousness.

### 3.5.2 The Philosophical Background of Bakhtin

Exploring the philosophical background for Bakhtin is a lot more complex than tracing the influences on Vygotsky. The authors I have looked at in this chapter have predominantly linked Bakhtin's work to Vygotsky's based on a common sociocultural trend in their writing. The authors have either implied (Radford or Wertsch) or stated (Wells) that Bakhtin and Vygotsky share a similar philosophical grounding. As I move into the next chapter, I will explain how the matter is far from this simple and, in fact, raises a key argument against the compatibility of Bakhtin's and Vygotsky's work.

In Chapter Four, I explore how lack of clarity around the philosophical grounding of Bakhtin leads to a wide variety of theories being linked to Bakhtin, from Wegerif linking Bakhtin's dialogics to Derrida's difference and Merleau-Ponty's boundaries (section 4.1), to White rejecting links between Bakhtin and Marx in favour of "Kantian ethics, Russian formalism, Dostoevskian polyphony and Rabelaisan carnivalesque" (White, 2014a, p. 224, see section 4.3). From classifying Bakhtin's work as phenomenology, hermeneutics, sociocultural theory or none of the above, the influences on Bakhtin are a challenge to trace. To make statements about the underlying principles of a theory with this level of ambiguity surrounding its origins is difficult.

In Chapter Three, I have examined examples of researchers adopting networking strategies for the theories of Bakhtin and Vygotsky. In Chapter Four I continue to consider connecting theories at the level of principles and how other authors have employed alternative networking strategies to the ones considered here.

## Chapter Four - Principles: Bakhtin and Vygotsky Apart

In Chapter Three, I identified networking strategies adapted by authors who have closely connected Bakhtin and Vygotsky to fill perceived gaps in existing theories. In this chapter, I am going to consider the networking strategies (Prediger et al., 2008) of authors who take issue with the close networking of Bakhtin and Vygotsky from a theoretical and philosophical standpoint. I wish to explore the limitations of certain networking strategies and consider the idea that, due to the issues raised by the authors in this chapter, using some networking strategies with Bakhtin's and Vygotsky's theories is not viable. In section 3.5, I began to specifically explore the philosophical positions of Vygotsky and Bakhtin according to those writing in the field and highlighted a particular ambiguity in the origins of Bakhtin's work. I aim to explore this ambiguity further in this chapter. Having a clear origin for Vygotsky's theories but no similar one for Bakhtin means there are statements I can make about dialectics that I cannot about dialogics in the same way. As a result, this chapter is going to continue to explore connecting theories at the level of principles by looking at authors in the field who disagree with using Bakhtin and Vygotsky together, hoping to use their arguments to understand the origins of Bakhtin's theories better and hence allow me to clarify more of the key principles of dialogics.

My aim in Chapter Four is to continue the exploration of the key principles, the first of Radford's possible areas of connection, whilst remembering that "[a]lthough connections are always possible […] there is nonetheless a limit to what can be connected" (2008, p. 323). By exploring the key principles of both Bakhtin's and Vygotsky's work and attempting to shed light on those that influenced Bakhtin, I am hoping to delineate "what a theory can legitimately predicate about its object of discourse" (Radford, 2008, p. 323) so that I can move, in future chapters, to explore methodology. In order to move forward, I need to establish a "dialogue between theories […] with an emphasis on the possible connections between them" (Radford, 2008, p. 324).

In a similar way to Chapter Three, I have focused on key writers in the field choosing to present them one at a time. The authors in this chapter overlap with one another, in some cases co-authoring papers. Taking each author in turn helped me to focus and bring clarity, whilst retaining a sense of the chronology.

### 4.1 Rupert Wegerif

I have chosen to include Wegerif as a key author due to his work identifying ontological differences between Bakhtin and Vygotsky that make the positioning of Bakhtin's dialogics as an extension of Vygotsky's sociocultural theory problematic. Wegerif will appear again in section 4.2 in collaboration with the next author I discuss but, in this section, I am going to focus on his 2008 article, *Dialogic or dialectic? The Significance of Ontological Assumptions in Research on Educational Dialogue*, where Wegerif positions the sociocultural theory of Vygotsky as dialectic and therefore distinct to the dialogics of Bakhtin's work. Wegerif directly addresses some of the points that Wertsch uses in his synthesis of Vygotsky and Bakhtin and discusses some of the underlying philosophical ideas that dialectics and dialogic are based on to illustrate incompatibilities.

Wegerif explains dialogic as "the idea that meaning always implies at least two voices" (2008, p. 348), suggesting "that meaning cannot be grounded upon any fixed or stable identities but is the product of difference" (2008, p. 349). He draws on Sidorkin's work to go further and say that this opening of dialogue is, from a Bakhtinian perspective, the "basis of being human" (2008, p. 350). Wegerif directly contrasts this with Vygotsky's idea that being human is based on self-identity and the overcoming of these differences: "as people get closer to intersubjective understanding in a dialogue, their need for explicit articulation becomes less, words and phrases become abbreviated, and they retreat towards the silence of a single consciousness" (2008, p. 349). How dialectics and dialogics address difference is key to the arguments put forward by the circle of authors writing in this field. The idea of an ever-present difference versus overcoming difference is a theme that is going to become familiar in this chapter. Having explained what he sees as a fundamental distinction between dialogic and dialectic, Wegerif examines aspects of the theories in more detail, particularly the mediated action used by Vygotsky to explain the development of higher mental faculties, the implications of this for identity, and goes into more detail about the ontological concept of difference underpinning dialogics.

Wegerif draws direct links from Vygotsky's position on identity to the dialectics of Marx and Hegel. He suggests that Hegel was the more significant influence of the two by specifically linking Hegel's idea that mind is mediated to Vygotsky's work on tool-mediated action (Wegerif, 2008, p. 351) and the mediational triangle. Wegerif links Vygotsky's account of children learning to point (section 2.2.1) to Wertsch's interpretation of this idea

as “paradigmatic of the teaching and learning of cultural tools in general” (Wegerif, 2008, p. 352). However, he argues that, in order for children to develop their “declarative pointing”, a dialogic relationship must exist first, something Wertsch does not take into account. Wegerif suggests that the necessity of a dialogic space implying mediation by others is a more accurate framing, rather than the mediation by tools offered by Wertsch. Wegerif argues that this framing “makes mediation by others look similar to, and perhaps compatible with, mediation by tools. However, as Bakhtin points out, relationships between things are very different from relationships between voices” (Wegerif, 2008, p. 353), which is significantly different from Wertsch’s synthesis of Bakhtin and Vygotsky on this point, maintaining the distinction between sign and tool from Vygotsky’s original writing (see section 2.2.1). Wegerif argues the distinction between sign and tool needs to be more clearly maintained than in some linked theories interpretations, which has significance for my intention of interpreting mathematical methods as language and suggests a careful consideration of the difference should be maintained through analysis.

Wegerif explains that these relationships are dependent on how one interprets the ontological concept of difference underpinning dialogics. Wegerif positions Bakhtin’s dialogics with, among others, “Derrida’s différance and Merleau-Ponty’s chiasm as a variation on the theme” (Wegerif, 2008, p. 354). For example, Wegerif explains, “Derrida argues that meaning is a product of an, always prior, act of differentiating that includes the differing of space and the deferring of time” (Wegerif, 2008, p. 353), and Merleau-Ponty’s chiasm is the “idea that bounded things or objects stand out from and are defined against an implicit background” (Wegerif, 2008, p. 354). Wegerif claims that while these ideas are not identical, they do “share the claim that meaning for us is not groundable on any kind of thing or identity but arises out of a relation of difference or differentiating” (Wegerif, 2008, p. 354). The key here is not just that difference is necessary for meaning making, but that the aim cannot ever be to overcome this difference. Wegerif recognises Bakhtin’s view that “this attempt to ‘erase the divisions between voices’ would close down the infinite potential for meaning of dialogue” (Wegerif, 2008, p. 358). Wegerif’s argument is contrary to the position of dialectics, which he sees as requiring an overcoming as part of the process of development towards an ideal end point. In the previous chapter (section 3.3.2), Wells also suggested that Vygotsky’s ideal end point was untenable.

Leading on from this analysis of the importance of difference, Wegerif takes issue with Wertsch's use of the ventriloquation of voices, or the process "whereby learners begin by speaking the voices of others without integrating them and then gradually, the initially foreign voices become indistinguishable from their own voice" (Wegerif, 2008, p. 355). In section 3.1.2, Wertsch linked Bakhtin's assimilation/appropriation with Vygotsky's internalization to justify his argument that "learning involves the appropriation of cultural voices" (Wegerif, 2008, p. 355). However, Wegerif argues that "a 'voice' is not a tool, but an answer to the question 'who is speaking?' This raises a conceptual problem for Wertsch's synthesis of Bakhtin and Vygotsky: are we appropriating cultural voices or are they appropriating us?" (Wegerif, 2008, p. 355). For Vygotsky, we internalize cultural tools and so appropriate those cultural voices, whereas for Bakhtin, voice is shaped by the necessity of responsivity and other forces of speech genres. As a result, the cultural voices are appropriating us.

One of the ways Wegerif highlights Bakhtin's issue with dialectics is using a quote that is significant due to the prevalence of its use among those disagreeing with using Bakhtin and Vygotsky's work together:

> Take a dialogue and remove the voices (the partitioning of voices), remove the intonations (emotional and individualizing ones), carve out abstract concepts and judgments from living words and responses, cram everything into one abstract consciousness – and that's how you get dialectics.
>
> (Bakhtin, 1986, p. 147)

This quote is also used by White (see section 4.3) as part of her argument about the incompatibility of Bakhtin and Vygotsky. It is worth noting that the quote is found in *From Notes Made in 1970-71* (Bakhtin, 1986). There is something in the fact that it is exactly that, a note. It does not come with any more detail or the context of a more organised argument or body of work. Whilst I am not suggesting the idea is discounted, this is something to bear in mind since we do have other quotes from earlier work that speak more favourably of dialectics. One example is Bakhtin's explanation that "[u]nderstanding and response are dialectically merged and mutually condition each other" (Bakhtin, 1981, p. 282), implying that Bakhtin sees a use for dialectics in a way not implied by the oft used quote under discussion.

To summarise, by highlighting issues between the theories of Bakhtin and Vygotsky, Wegerif counters the close ties of the "synthesizing"/ "integrating locally" networking strategies (Figure 1.2) of the authors in the previous chapter. Wegerif argues that underlying ontological issues lead to inconsistencies in any schema that takes the two author's work as part of the same overarching theory. Key questions about how "dialogic and dialectic imply incompatible assumptions about meaning: dialogic presupposes that meaning arises only in the context of difference, whereas dialectic presupposes that differences are contradictions leading to a moment of overcoming" (Wegerif, 2008, p. 359). He reinforces the origins of Vygotsky's work in the dialectics of Hegel and Marx and has made links for Bakhtin to the work of Heidegger, Derrida, and Merleau-Ponty to account for Bakhtin's particular use of difference. Some of Wegerif's work does rely on Bakhtin's fragmented later work, a detail that does have to be taken into account, but his writing has given a clear account of the distinction between how Bakhtin and Vygotsky perceive difference. Due to the fundamental disparities Wegerif sees between the two theories, his approach to networking the two can be, at most, one of combining theories, an approach that allows for "theories with conflicting basic assumptions" (Prediger et el., 2008, p. 11). Combining theories is usually used for analysing specific data. In Wegerif's case, he is focusing on improving understanding of the two theories being connected and, as such, I suggest that his article represents a "comparing"/ "contrasting" approach in order to develop a "better understanding of the foreign and the own theories" (Prediger et al., 2008, p. 9).

Wegerif is clear in his writing that using Bakhtin and Vygotsky's theories together as though they share a common theoretical grounding is problematic, which suggests that there are limits to the extent to which the two theories can be networked. More specifically, Wegerif takes issue with the way that Wertsch has extended Vygotsky's mediational means to include the Bakhtinian concept of voice as a tool. Wegerif argues that the overcoming inherent in Vygotsky's approach is incompatible with the difference so fundamental to Bakhtin's dialogic approach. Thus, it is no longer possible to link the theories as Bakhtin's theory is fundamentally undermined. I suggest that linking to the extent suggested by authors in Chapter Three is only possible if the concept of overcoming prevalent in Vygotsky's dialectic approach is reconceptualised as in Radford's interpretation (section 3.2). Following Wegerif's arguments, care must be taken when networking Bakhtin's and Vygotsky's ideas to avoid an inconsistent analysis. I will return to Wegerif's work in the

following section as he has collaborated with Matusov, the next author I have chosen to discuss.

### 4.2 Eugene Matusov

I have chosen to include Matusov as he significantly extended the implications of dialectics for identity by looking specifically at the topic of power relations. Matusov is also significant because of his work with and influence on other writers in the field, being referenced by White (2014a, see section 4.3), Barwell (2015, see section 4.4) and has exchanged ideas with Wegerif (Matusov & Wegerif, 2014). I have chosen to focus on several of his articles that lay out his position on the use of Vygotsky and Bakhtin. The first, *Irreconcilable Differences in Vygotsky's and Bakhtin's Approaches to the Social and the Individual: An Educational Perspective* (2011), lays out his issues with other authors' use of Bakhtin as "compatible with and an extension of Vygotsky's cultural-historical approach" (Matusov, 2011, p. 99). His work maintains a strong ethical focus on the rights of the learner.

Matusov (2011) begins by explaining that, based on where and when Bakhtin and Vygotsky were working, there are "historical, conceptual, academic, paradigmatic, and political (if not also geographic) reasons to see these scholars together as similar and complimentary" (Matusov, 2011, p. 100). Matusov explains how scholars working in the Soviet era were beginning to emphasise the social and historical context of events in a way that the previous positivistic scientific approach had not, echoing the authors from Chapter Three who noted similar connections (e.g., Wertsch, 3.3.1; Radford, 3.2). However, the rest of the article makes it clear that, beyond this basic connection, Matusov finds that "Vygotsky's and Bakhtin's conceptualizations are not only different but also irreconcilable" (Matusov, 2011, p. 100), suggesting that "[f]rom Vygotsky's perspective, Bakhtin's dialogic approach is anti-developmental […] From Bakhtin's perspective, Vygotsky's instrumental approach is monologic and inhuman" (Matusov, 2011, p. 104).

Matusov says that "[f]or both Vygotsky and Bakhtin, human consciousness was the central issue of their investigation" (Matusov, 2011, p. 101), but goes on to highlight differences between how Vygotsky and Bakhtin actually saw human consciousness. He explains that "Vygotsky's sociohistorical approach was deeply instrumental, defining consciousness through activity mediation, while Bakhtin's dialogic approach was essentially ontological,

defining consciousness through bodily experience, responsibility, addressivity, responsivity, respect, human dignity, and relationship with the other" (Matusov, 2011, p. 100). This difference leads to Bakhtin and Vygotsky using consciousness in different ways when addressing the idea of how meaning is made. For Vygotsky, "meaning-making is based on tool and sign mediation" (Matusov, 2011, p. 101), whereas for Bakhtin, meaning-making required the inherent difference between two consciousnesses where the "orientation to the gap in mutual understanding is both a precursor and an outcome of dialogue and dialogic meaning making" (Matusov, 2011, p. 103).

Having established the bases of meaning making for Vygotsky and Bakhtin, Matusov goes on to discuss the implications for the two authors' ideas about development and education. Vygotsky's approach "involves growing mutual understanding and growing intersubjectivity th[r]ough agreement – overlapping meaning – between the educated adult in modern Western society and a child" (Matusov, 2011, p. 102). Matusov interprets Vygotsky as seeing "the educated adult" as an ideal all children are aiming for. He links Vygotsky to Hegel saying, "[f]or Vygotsky, the Absolute Spirit or the Universal Reason was the inner speech of the educated Western (middle-class) adult equipped with the scientific concept. From this perspective, education (and development) involves achieving such absolute mono-consciousness" (Matusov, 2011, p. 115). Matusov explains that "[t]he deficit gap between the ideal scientific thinking and child's actual thinking sets a developmental goal ('the zone of proximal development' [ZPD]) and learning curriculum" (Matusov, 2011, p. 102). Matusov takes issue with what, in his interpretation of Vygotsky, happens: "[i]n the extreme, the perfectly developed person does not need other people at all" (Matusov, 2011, p. 102). The idea of what is happening at the extreme is something that is going to become significant not only in Matusov's writing, but to later authors writing in the same area (see, section 4.5.2).

Matusov's analysis that "[t]here is not a true meeting of two consciousnesses in Vygotsky's developmental paradigm" (Matusov, 2011, p. 103), given that one is always lesser than the other in an interaction, is contrasted with his view of Bakhtin's work as centred around the relationship between consciousnesses. His interpretation is of the importance that Bakhtin placed on the suggestion that the "gap in the mutual understanding between people is a necessary condition for dialogic, humane communication, and for the entire human relationship" (Matusov, 2011, p. 103). Matusov argues that this gap (or difference as

Wegerif framed it in section 4.1) is maintained throughout the interaction between participants and that to close this gap would be disastrous:

> Dialogic interaddressivity implies that people cannot, and even must not, fully know each other. Calculability of the other person (what this person knows, how he or she feels, what exactly he or she will do and why) is not only impossible (on a full scale) but immoral, exploitative, inhumane, and a killer of dialogue.
>
> (Matusov, 2011, p. 103)

Matusov perceives the "relationship between the teacher and the student in a conventional classroom" (Matusov, 2011, p. 103) as an example of an interaction that kills dialogue due to calculability. Matusov does then maintain that an element of dialogue is still alive in a school environment, but likens it to Hegel's analysis of master-slave relations with its "dialogism of two consciousnesses [being] preserved […] in a very distorted form" (Matusov, 2011, p. 104). Likening a teacher-student relationship to that of the master-slave relationship is incredibly strong, but a theme that Matusov develops further in his discussions with Wegerif and Miyazaki (Matusov & Wegerif, 2014; Matusov & Miyazaki, 2014).

In Matusov & Wegerif (2014), the two authors swap emails in a discussion intended to develop their understanding of one another's positions on education and learning. Wegerif explains that "I do not oppose knowledge and the teaching of knowledge in a knowledge based curriculum" (Matusov & Wegerif, 2014, p. E5). Wegerif explains that teaching knowledge already discovered by previous practitioners as "the product of long-term and large scale collective scientific dialogues" (Matusov & Wegerif, 2014, p. E5) and then using a dialogic approach to develop "individual understanding of that knowledge" (Matusov & Wegerif, 2014, p. E5) is his approach. Matusov counters by describing Wegerif's approach as one where "authorship, creativity, and dialogue are allowed for and, thus, limited to only achievement of the Authorities' preset goals" (Matusov & Wegerif, 2014, p. E6). Matusov has a clear sense that any sort of lesson entered into where the teacher has an end point in mind is not dialogic: "[t]hrough skillfully deceptive scaffolding the teacher (and the problem institutionally preset) marched the students' lovely dialogue to the preset math truth by making all other possible alternative truths cognitively impossible and socially/politically dangerous for the participants" (Matusov & Wegerif, 2014, p. E7). He then goes on to liken the direction by the teacher to "enjoying a Jewish prisoner orchestra

playing classical music, while other Jews were marched to the oven in concentration camps" (Matusov & Wegerif, 2014, p. E7).

The extremely strong analogy Matusov has chosen highlights the strong moral objection he holds towards a dialectic approach based on his experiences growing up in the Soviet Union (Matusov, 2011, p. 119). Indeed, in his and Wegerif's dialogue (2014), Wegerif states "[y]ou have mentioned to me […] the importance of your experience under communism in the Soviet Union for forming your educational and political positions" (Matusov & Wegerif, 2014, p. E3). Wegerif recounts Matusov accusing Wegerif of having "'gone over to the dark side' impl[ying] that I had been corrupted by an evil monologic system" (Matusov & Wegerif, 2014, p. E3). Matusov describes this "system" as "conventional monologic education" (Matusov & Wegerif, 2014, p. E4). Wegerif suggests that he (Wegerif), is attempting to affect change from within, but Matusov likens this to family and acquaintances of his (Matusov) that joined the Communist party. They "claimed that they would corrupt the Communist Party from within […] My observation on them was that the Party corrupted them more than they corrupted it" (Matusov & Wegerif, 2014, p. E8), which gives an echo of Wegerif's question: "Are we appropriating cultural voices or are they appropriating us?" (Wegerif, 2008, p. 355). Matusov's recounting of his experiences living in the Soviet Union are important firstly because the theories under discussion emphasise the social, cultural, and historical background of the voices involved and secondly because both Bakhtin and Vygotsky lived under the regime of the Soviet Union themselves. Growing up in the Soviet Union would contribute significantly to the view formed of the State and the potential implications of taking as given the information being fed to you by schools and teachers as agents thereof. Matusov's strong views, on issues such as the monologic education system, drawing parallels between the "distorted, oppressive, and painful" form of dialogue present in classrooms and master-slave relations (Matusov, 2011, p. 104) and, equally disturbingly, between a teacher leading students towards a preset goal and Jewish prisoners being marched to their deaths in concentration camps, are partially a result of his experiences under communism.

Matusov's strong ethical stance against so called monologic teaching is based on one of the conditions he sees as key for a dialogic relationship: "it can be dialogical when the other consciousness is treated as having equal rights with one's own" (Matusov, 2011, p. 103). Here he is referencing Bakhtin (1984a):

> Monologism, at its extreme, denies the existence outside itself of another consciousness with equal rights and equal responsibilities, another I with equal rights (thou). With a monologic approach (in its extreme or pure form) another person remains wholly and merely an object of consciousness, and not another consciousness.
>
> (Bakhtin, 1984a, pp. 292–293)

Matusov does not believe that equal rights are possible in a classroom based on a knowledge curriculum with pre-set goals for students. The power roles in the classroom mean that, despite the respect and duty of care teachers have towards students, students are expected to act in a particular way and towards specific goals as dictated by the teacher. As such, achieving the equality Matusov sees as necessary for a dialogic space to be opened seems unlikely. If the underlying point about dialogic relationships is true then Matusov's argument, against Wegerif's claim to be attempting a dialogic corruption of a dialectic system from within, has more weight. Indeed, anything short of a complete revolution in education is not going to be enough to achieve equality.

Matusov's work is the first time, in the examples of authors' work I have chosen, that value judgements are ascribed to the positions of dialectic and dialogic. Thus far, there has been critique of how the authors involved have combined approaches, but this is the first time that one approach has been selected because it is seen as morally superior. Matusov summarises his position on education as:

> The goal of education is not to make students have the same understanding as the teacher, but rather to engage them in historically valuable discourses, to become familiar with historically, culturally, and socially important voices, to learn how to address these voices, and to develop responsible replies to them without an expectation of an agreement or an emerging consensus.
>
> (Matusov, 2011, p. 115)

Matusov provides some clear arguments for the lack of compatibility between Vygotsky's and Bakhtin's theories based around their different treatments of human consciousness. Matusov goes further than Wegerif (section 4.1) and advocates a rejection of Vygotsky. He frames Vygotsky's work as representing a linear curriculum and Bakhtin's work as the more subtle interpretation of meaning making, which values individuals. His opposition to linking the two theories suggests that there is no way of reconciling the variety of meaning making with the curriculum, but instead that the linear curriculum structure is to be

rejected. As such, Matusov's work might suggest that I focus on one theory rather than a combining approach to analysis. However, this is not realistic in my context. I am working with teachers and students within a system that I am unable to reject. As an alternative, I can be aware of ethical considerations in my own connecting attempts and take care before attempting some of the closer degrees of networking. His way of networking Vygotsky's and Bakhtin's ideas in the articles I have included here represents "understanding others/ "making own theories understandable" approaches. Despite discourse being clearly demonstrated through his articles based on discussion with others in the field to improve his understanding of others' positions, I argue that Matusov's exploration of the underlying principles of the two theories does not represent a "contrasting" approach. Despite being "focused on stressing differences" (Prediger et al., 2008, p. 9) like "contrasting", Matusov does not do this in an attempt to connect the theories. Instead of finding "strong differences [that] can make the individual strengths of the theories visible" (Prediger et al., 2008, p. 9), Matusov is not seeking to find strengths in Vygotsky's approach. As such, his networking approach can only be focused on describing his position and trying to understand those of others. Matusov's writing raises the question of what happens at the extremes of these theories and his ethics-driven stance implies a real difficulty in relating dialogic teaching with the current state of affairs in education.

### 4.3 Elizabeth White

Elizabeth White is another author taking issue with the use of Vygotsky's and Bakhtin's work as extensions of one another. I have chosen to include her as one of my focus authors as she had produced commentary on others working in the field (White, 2014b). She also goes further than others in her attempts to identify the philosophical background for Bakhtin. White worked with Matusov as part of a symposium in 2011 (see Matusov & Miyazaki, 2014, p. 1), crediting him with "providing a means of entry into the dialogue" (2014a, p. 232) around dialogic and dialectic theory. Her writing argues that, for Vygotsky and Bakhtin, "both men's pedagogical provocations – both ontological and epistemological – have more to contribute to local and international schooling when interpreted against their philosophical origins" (2014a, p. 231). Once again, White uses the "Take a dialogue and remove the voices […]" (Bakhtin, 1986, p. 147) key quote about dialectics, used by others in their arguments (see Wegerif in section 4.1), reinforcing the significance of this quote in disputing connections between Vygotsky and Bakhtin.

Her 2014 article, *Bakhtinian Dialogic and Vygotskian Dialectic: Compatibilities and Contradictions in the Classroom*, is based around the idea that Bakhtin's and Vygotsky's "positions are incommensurably located alongside one another" (White, 2014a, p. 220). Issues are raised based on the incompatible "philosophical orientations" of Bakhtin and Vygotsky (White, 2014a, p. 222). White highlights what she sees as the current position within education, dialogics being subsumed as a subsection of dialectics, resulting in misunderstandings and missed opportunities (White, 2014a, p. 220). Like Matusov, White recognises key similarities between the dialectics of Vygotsky and the dialogism of Bakhtin; their similar backgrounds or, more specifically "the social, philosophical and political contexts in which they lived" (White, 2014a, p. 225), and their joint interest in "dialogue as central to human consciousness" (White, 2014a, p. 225). She also finds similarities in the way they "both recognize the significance of time and space on the present, as well as the role of language on thought, and were undoubtedly influenced by Marxist philosophy" (White, 2014a, p. 225). However, she believes these similarities are no more than a shared theme in their writing. Looking at the detail, White identifies important distinctions between the theories in how they address the ideas of identity, freedom of the individual, and the function of language.

In a similar manner to Wegerif and Matusov, White links Vygotsky with previous writers in dialectics such as Hegel, saying, "Vygotsky's position that knowledge leads to freedom is reversed by Bakhtin for whom there is no such thing as individual freedom or personal emancipation" (White, 2014a, p. 226). Freedom for Hegel does not have the same connotations as the word does when used colloquially. For Hegel, "The State is […] the embodiment of rational freedom" (Hegel, 2001, p. 62), where "Freedom is nothing but the recognition and adoption of such universal substantial objects as Right and Law, and the production of a reality that is accordant with them – the State" (Hegel, 2001, p. 75). A person who has achieved self-consciousness will choose to contribute to a rational society, this is their freedom. As White goes on to explain, "[f]or Vygotsky all higher mental functions are interiorized relations of social order, while for Bakhtin there is no such thing as an inner life. He believed that the word only has meaning when it is given form in dialogue" (White, 2014a, p. 227). White sees the differences in Vygotsky's and Bakhtin's approaches to the individual exemplified in their approaches to language:

> Bakhtin makes it possible to view private speech as social since, in his view, all language is interanimated by others even when there is no one else immediately

> present. Vygotsky, on the other hand, saw private speech from a developmental perspective, as a precursor to social language, and did not consider the notion of addressivity, or discourse, in his work.
>
> (White, 2014a, p. 227)

For White, then, the Hegelian idea that knowledge leads an individual to the freedom to contribute to society is contrary to Bakhtin's ever-shifting sense of identity where meaning is only found in interaction with others.

White's writing has, so far, reiterated that of Matusov. However, alongside her discussion of freedom, identity and the use of language, White makes a distinctive point about power dynamics in discourse, specifically with the "status of each partner in the learning process" (White, 2014a, p. 227). White references Vygotsky's positioning of the learner as "novice" and the teacher as "complete authority" (White, 2014a, p. 228), but highlights the fact that Bakhtin "paid little attention to relationships that are characterized by power and control" (White, 2014a, pp. 227–228) as a weakness in his approach. In section 4.2, I discussed Matusov's view that a dialogic framing avoided framing one person in an interchange as more powerful than another, but White sees a gap where Matusov sees a benefit. However, I have previously written about Bakhtin's use of the authoritative word (section 2.1.5), which White does not mention, so I would suggest that, instead of a gap in Bakhtin's theory where he does not pay attention to power relations, perhaps a better argument is that neither Vygotsky nor Bakhtin offer suggestions to mitigate the effects of the power dynamics inherent in their theories. The absence of a way of addressing power roles is a possible limitation when connecting these two theories. I am going to discuss the role of power in the work of Bakhtin and Vygotsky further in my discussion at the end of this chapter (section 4.5).

The main focus of White's writing is the philosophical influences of others on the writing of Vygotsky and Bakhtin, particularly when discussing the key points of dialectics and dialogics. For dialectics, White argues that:

> Bringing Marx, Spinoza and Hegelian dialectics to bear, Vygotsky's quest then was to emancipate the learner to higher psychological thought processes through the dialectic of everyday and scientific levels of thought, as a pathway to freedom and by paying attention to social context and history.
>
> (White, 2014a, p. 223)

On the other hand, White describes the dialogism of Bakhtin, as being:

> still heavily influenced by Marxism, an inevitable position as a member of a society that had no toleration for alternative views, there are strong indications that Bakhtin rejected Marxist ideals in favour of the unusual combination of Kantian ethics, Russian formalism, Dostoevskian polyphony and Rabelaisian carnivalesque. (White, 2014a, p. 224)

The number of philosophical influences White has traced here goes beyond that of Hegel and Marx highlighted by previous authors. White posits that, once you are listing this many influences, rather than saying Bakhtin ascribes to all of these then Bakhtin's work could instead be seen as "a philosophical contribution in its own right" (White, 2014a, p. 231). I am going to return to discuss the positioning of Vygotsky and Bakhtin according to White in more detail (see section 4.5.1).

White's links to Matusov, along with her ideas around power and the philosophical differences between Bakhtin and Vygotsky, make her work notable in the networking of Vygotsky and Bakhtin. Her work focuses on the differences between the two theories and as such represents a "contrasting" approach. Her approach to the exploration is more moderate than Matusov's and, as such, I suggest there is enough evidence of highlighting strengths as well as weaknesses to justify describing it as contrasting rather than one of the "understanding […]" approaches. In fact, White draws attention to the issue of power as a common thread for investigation between the two theories. White's position, claiming Bakhtin's philosophical influences being based on so many other philosophical positions that it constitutes a position of its own, is a unique approach to the philosophical discussion and one that I will discuss further (section 4.5.1).

### 4.4 Richard Barwell

I explained in section 1.1.2 that Barwell was my introduction to the work of Bakhtin. I have included his work here because of the specific examples of a dialogic and dialectic analysis he offers and his suggestions for a parallel use of the theories. Barwell, in his 2015 paper *Formal and Informal Mathematical Discourses: Bakhtin and Vygotsky, Dialogue and Dialectic,* takes up the discussion started by Wegerif and Matusov about the fundamental differences in the work of Bakhtin and Vygotsky. However, his networking approach is different from theirs as his work suggests that valuable insights can be gained from each of the theories applied separately to analyse the same data. His work has a close focus on the mathematics

classroom and language use within it, particularly from the perspective of a classroom containing multiple national languages, with the emphasis being on developing one of these above the others. By using the context of national languages, alongside the more subtle differences within them, his work served as an excellent entry point for me when I began my studies teaching in a multilingual classroom (see section 1.1.2).

Barwell begins by explaining that there is a general interest amongst mathematics educators in the role language plays in the learning process and "[t]he development of learners' mathematical language is generally conceptualised as a transition from students' informal, 'everyday', expressions of mathematical thinking, towards communication using more formal or conventional mathematical language" (Barwell, 2015, p. 2). Barwell explains that, despite previous researchers' extensive use of Vygotsky in their work, "Vygotskian theory […] has limitations, including that it over-emphasises an instrumental view of the role of language in learning and development, in which mathematical discourse is primarily understood as a tool for mathematical thinking" (Barwell, 2015, p. 2). In the place of Vygotskian theory, he offers, for consideration, Bakhtin's work and, more specifically, a discussion of the differences of Vygotsky's dialectic and Bakhtin's dialogic. In contrast to Matusov however, Barwell takes pains to point out that:

> [m]y purpose in examining these two perspectives is not necessarily to oppose them, synthesise them or argue that one is better than the other (in dialectic fashion, perhaps) but rather, dialogically, to set the different sets of assumptions and distinctions side by side so that their differences and possibilities may each be seen in the light of the other.
>
> (Barwell, 2015, p. 3)

Barwell's statement implies a different networking approach to the two theories that I will return to at the end of this section.

Barwell describes two key features of dialectic thinking that I have discussed previously: "First, dialectical thinking is based on binary oppositions and an examination of the movement between the opposing positions" (Barwell, 2015, p. 3); and "second […] dialectic thinking includes a notion of progress, of improvement, through which oppositions can be overcome" (Barwell, 2015, p. 3). He draws attention to Wegerif's argument that Vygotsky's work uses Hegel's and Marx's ideas to frame development as progress towards a unity and explains that, from a dialectic perspective:

> Learning mathematics is mediated by formal mathematical language, through a dialectic process in which learners move along the continuum or up the hierarchy. This movement does not mean that leaders cease to use informal mathematical language; these informal expressions are gradually subsumed into formulations that correspond increasingly closely to formal mathematical language.
>
> (Barwell, 2015, p. 4)

Formal and informal mathematics language is a new way of describing the dialectic move towards unity we have seen discussed by Wegerif and Matusov previously. Barwell then extends this line of thinking and explains that, if taken to its endpoint, Vygotsky's work leads to a point where "language […] and cognition are fused. Interaction about mathematics is first experienced collectively and is then 'individualised' to become individual cognition" (Barwell, 2015, p. 5). Linking to Wegerif's own links between identity and appropriation (section 4.1), Barwell says that "through learning to use mathematical language, students become particular kinds of […] people" (Barwell, 2015, p. 5). Contrary to Matusov's rather dim view of dialectics and its march towards a "preset math truth" (Matusov & Wegerif, 2014, p. E7), historically, dialectics has "highlighted the importance of student's informal language in the process of learning mathematics and formal mathematical language" (Barwell, 2015, p. 5, see section 2.2.2).

Barwell contrasts this dialectic framing of formal and informal language with a dialogic approach, explaining:

> A dialogic perspective on formal and informal discourse in mathematics classrooms highlights a relationship that is not binary. Rather than steady progress from informal mathematical language to formal mathematical language, or at least the goal of attaining one through the other, or even of steady advancement in the parallel development of both, the students in the example work at and through multiple mathematical discourses, voices and, in some cases languages. Of course, students need to learn formal mathematical discourse as part of learning mathematics, but this does not mean that it is somehow ready-made, with the teaching guiding the students towards it; nor is informal mathematical discourse simply a scaffold to reach more formal language. Rather, the students work at expanding their discursive repertoires, giving them a wider range of ways to make meaning in different mathematical situations.
>
> (Barwell, 2015, p. 13)

Here, Barwell reiterates the now familiar ideas about utterances, otherness (or difference) and the presence of multiple voices. However, what is particularly useful for my study is how he specifically links these concepts to formal and informal mathematical language. Barwell explains that there is an ongoing tension between the heteroglossia of informal languages and the unitary language of formal mathematics in every utterance (2015, p. 7), citing this as an example of "how all the participants are influenced by the tension between the centripetal and centrifugal forces of language" (Barwell, 2015, p. 12). Barwell's research looks at Canadian classrooms where English is an additional language for the mathematics learners. As a result, Barwell explains that, in addition to tensions between formal and informal mathematical language based on the fact that using formal mathematical language is a requirement of the curriculum, there is another tension at work here in the "institutional requirement to use English, as well as the broader politics of language in Canada, in which aboriginal languages have a marginal status" (Barwell, 2014, p. 920). Barwell is clear that these tensions are never fully resolved. He agrees with Matusov in interpreting "Bakhtin's sense that perfect control would mean that nothing is left to say" (Barwell, 2015, p. 7) and "differences open up possibilities for meaning" (Barwell, 2015, p. 7). Barwell's study focuses on mathematical language as opposed to mathematical methods and focuses on discourse in the classroom, rather than written methods.

Barwell's discussion of dialectic and dialogic positions is similar to that which has gone before in Wegerif, Matusov and White, but also goes a step further, giving examples of how a Vygotskian or Bakhtinian analysis of the same situation (in this case, a discussion about classifying polygons) would be different:

> A Vygotskian dialectic perspective highlights a process of socialisation as the students learn to talk and think about polygons in predetermined mathematical ways using a stable mathematical language. This perspective emphasises a series of oppositions, including the one between formal and informal mathematical language. A Bakhtinian dialogic perspective highlights the constant, local, situated and emergent nature of the mathematical discourses in the lesson. A relation is constructed between more and less formal ways of expressing mathematics […] More formal and less formal are not in opposition, but work together and in relation with other discourses. Each utterance reflects: multiple voices, including the

> teacher's and the students'; multiple discourses, including several versions of mathematical discourse; and multiple languages.
>
> (Barwell, 2015, pp. 11–12)

Barwell's examples of analyses are going to play an important role in the early stages of my methodological development (see section 6.3). As part of this valuable example analysis, Barwell highlights that, "From a Bakhtinian perspective […] it is not possible to cleanly define formal and informal mathematical discourse, since they depend on who is speaking, to whom, and in relation to what else has been said" (Barwell, 2015, p. 6), which is different from the formation of formal mathematics language in a dialectic approach where the formal expression is predetermined.

Barwell's writing is significant in that he approaches the debate from the point of view of someone working in classrooms with another layer of analytic challenge – that of students working in different national languages. As such, identifying tensions between heteroglossia and unitary language involved, in some cases, identifying different national languages as centripetal and centrifugal forces. In addition, Barwell's writing provides an example of two concurrent analyses of the same situation – one using Vygotsky's dialectics and one using Bakhtin's dialogics. Barwell summarises the distinction he sees between dialectic and dialogic approaches in the mathematics classroom as:

> In the former, the relationship between formal and informal mathematical language is understood in terms of the development of students' control of formal mathematical language to do mathematics. In the latter, the relationship between formal and informal mathematical discourse is understood relationally as the expansion of repertoires of ways of talking about mathematics.
>
> (Barwell, 2015, p. 3)

Barwell suggests that, whilst there are important distinctions between dialectics and dialogics, they should not be positioned in opposition to one another and there could be meaningful ways for them both to be used for analysis. As such, his approach to networking the two theories could be described as "comparing"/ "contrasting". He keeps the two theories separate, but makes a significant amount of meaning about the two theories by identifying similarities and differences. Of the pair, a "contrasting" approach focuses more on difference but, as Barwell takes a more neutral approach to networking these two theories, I suggest that his approach is closer to a "comparing" connecting strategy. One limitation to connecting theories after Barwell is that he focuses very carefully on the

spoken language being used which is typical of a traditional Bakhtinian model, however he then also restricts the Vygotskian analysis to the spoken word. As such, this networking strategy may require limiting the data being analysed in order for the analysis to be consistent with the underlying theories. However, Barwell's style of linking theories does suggest that the theories do not require a common philosophical basis and serves as an example of a study that can benefit from both analytical styles.

### 4.5 Discussion

The aim of this chapter (along with Chapter Three) has been to identify the "implicit views and explicit statements" (Radford, 2008, p. 320) that form the principles of Vygotsky's and Bakhtin's work. I have established connections between these theories and identified the arena in which I am working, so that I can begin to move towards developing a methodology and research questions for this project. This chapter has shown that the focus of identifying these parameters has shifted towards the philosophical background of Vygotsky and Bakhtin. As such, I am going to revisit the philosophical discussion I began in the previous chapter (section 3.5).

#### 4.5.1 Returning to The Philosophical Arguments

In Chapter Three, I discussed the Hegelian roots of Vygotsky (section 3.5.1) but described the lack of clarity around the philosophical basis of Bakhtin (section 3.5.2). In the current chapter, the discussion about the extent to which Bakhtin's and Vygotsky's theories are considered to be compatible has returned to the question of underlying philosophical positions. I return to the philosophical arguments here, in an attempt to learn more about the philosophical underpinnings of Bakhtin.

Gardiner (2000) and Dafermos (2018) are two authors who look specifically at the philosophical background of Bakhtin. Gardiner argues that Bakhtin's work is linked to dialectics, but a specific form of dialectics: "there are two major strains in modern dialectical thought - the logical and objectivisitic on the one hand, and the corporeal, intersubjective and 'dialogical' on the other" (Gardiner, 2000, p. 121). Gardiner argues that Bakhtin's "theoretical trajectory" can be closely paralleled by the latter. To illustrate the parallels, Gardiner makes links between, firstly, Merleau-Ponty's work returning "dialectics […] to its Socratic, dialogical roots, by placing the phenomenon of language at the centre of philosophical inquiry" (Gardiner, 2000, p. 128) and, secondly, Bakhtin's focus on the

dialogic meaning making inherent in language. Gardiner uses Merleau-Ponty's work to argue that there is a framing of dialectic that does not have an "inherent telos or ultimate resolution" (Gardiner, 2000, p. 132), instead, representing "the simultaneous conceptualization of identity and difference, unity and diversity - to grasp, in short, the fundamental ambiguity of human existence" (Gardiner, 2000, p. 136), whilst still retaining the element of change and development central to dialectics.

Gardiner argues that the "official Soviet Marxism" (Gardiner, 2000, p. 121) present in the Soviet Union during Bakhtin's life is a "crude and rather caricatured version" (Gardiner, 2000, p. 121) of dialectic theory and therefore the "depiction of Bakhtinian thought as wholly incompatible with dialectics is part of a pronounced desire to demonstrate his hostility towards Marxism per se" (Gardiner, 2000, p. 121) by authors sympathetic to Bakhtin but who want to distance themselves from Marxism. Gardiner admits that it is not possible to truly know how Bakhtin viewed Hegel and Marx and the issue is complicated further as "it is not always clear whether Bakhtin is referring specifically to Hegelian or Marxist dialectics, or both simultaneously" (Gardiner, 2000, p. 126). Gardiner suggests that Bakhtin and Merleau-Ponty both:

> [S]tress the principle of 'identity within difference', wherein individual subjects retain their distinctiveness and capacity for autonomous moral judgement and action, but at the same time must be understood as entities that are constituted in and through their communicative exchanges, or dialogues, with others, in the context of everyday sociality.
>
> (Gardiner, 2000, p. 139)

Maintaining the difference between individual and society, whilst recognising their interconnectedness, allows for "an 'incomplete synthesis', a partial and pragmatic consensus out of which can emerge forms of collective agency oriented towards progressive social change" (Gardiner, 2000, pp. 140). The suggestion of a middle ground in a dialectic framing, where difference is not always there to be overcome in a synthesis, is a move away from the extremes of interpretations such as Matusov's. Gardiner uses this moderate interpretation of dialectics to suggest "perhaps a sustained and productive interchange can yet occur between what remains of value in the critical and reflexive Marxist tradition and the sort of dialogical approach inspired by the work of Mikhail Bakhtin" (Gardiner, 2000, p. 140), saying that he sees a strong link between Marx's early work to focus on the individual rather than an ideal and Bakhtin's work.

In a similar manner, Dafermos (2018) argues that post-modernism has misinterpreted Hegel's and Marx's work and claims that, when Bakhtin criticises dialectics, he is in fact critiquing the Marxist State specifically rather than dialectics in general. Dafermos suggests that Bakhtin's comments were there to critique the official dogma of dialectical materialism being used in the Soviet Union at the time of his writing as opposed to a more general critique of Hegel. Dafermos cites Côté (2000) to claim it is possible to demonstrate Bakhtin was, in fact, in dialogue with dialectics, "beyond the superficial direct critique of Hegel, Bakhtin in fact responded actively and more often implicitly to Hegelian philosophy in his own works" (Côté, 2000, p. 26). In contrast, Dafermos links dialogics to roots in Ancient Greece in "the context of the polis as a community of actively participating citizens" (Dafermos, 2018, p. A2), claiming that the ideas were then "reborn" in Bakhtin's work (Dafermos, 2018, p. A2). In effect, there is the implication that there were no specific precursors to Bakhtin's work in a markedly different manner to Vygotsky's roots in dialectics. Dafermos is clear that "Bakhtin offered a classic formulation of the dialogic nature of consciousness […] which goes beyond purely linguistic or literary phenomena" (Dafermos, 2018, p. A3).

Dafermos recognises the strong arguments for incompatibility between dialectics and dialogism. He references to the key quote that has been used by Wegerif (section 4.1) and White (section 4.3), "Take a dialogue and remove the voices […]" (Bakhtin, 1986, p. 147) but argues "there is nothing more alien to dialectics than the idea of isolated, individual and abstract consciousness" (Dafermos, 2018, p. A7). Dafermos explains that "Marx's account of alienation and commodity fetishism offers a deep understanding of the distortion (degradation) of human relationships in the capitalist society. The relations between people appear to be relations between 'things' (commodities)" (Dafermos, 2018, p. A9). Compare this comment with the point Wegerif made about Bakhtin's approach (see section 4.1), "relationships between things are very different from relationships between voices" (Wegerif, 2008, p. 353). However, Dafermos argues that Marx offered a form of dialectics which "emphasizes internal, essential connections between people rather than a separated individual, an abstract consciousness" (Dafermos, 2018, p. A7). So, different versions of dialectics exist, some of which may be more compatible with Bakhtin's position.

Dafermos acknowledges that "dialectics has been conceived as a mode of thinking connected with a concrete form of knowledge production" (Dafermos, 2018, p. A10), whereas dialogue has focused on "communication between consciousnesses rather than with knowledge production" (Dafermos, 2018, p. A10). However, he suggests that parts of Bakhtin's writing, whilst focusing on the meaning making possible between consciousnesses, illustrate dialectic elements:

> Bakhtin's idea of an open, developing organic unity is a truly dialectical insight in the theorizing of human sciences [...] I don't claim that Bakhtin was a dialectical theorist, but only that it is possible to find influences of dialectics in his writings. In other words, there is no absolute gap or a rupture between dialogic and dialectic traditions but paradoxically, a dramatic relation between them might be detected.
>
> (Dafermos, 2018, p. A12)

If Dafermos sees links between the dialectic focus on knowledge and the dialogic relation between consciousnesses then an exploration of mathematics as language might be possible through connecting the two theories.

The prevailing philosophical view that has come through in this chapter is that, ontologically, there are sufficient differences between the theories of Bakhtin and Vygotsky to restrict the networking strategies that can be applied to connect them. However, as Barwell has illustrated, there is an argument that concurrent use of the two theories may have an analytical benefit. As a result, I am going to finish this chapter by summarising the discussion on connecting the theories based on their underlying principles so that I can decide what approach to take exploring an analytical methodology.

### 4.5.2 Points of Connection for Principles

Chapters Three and Four have continued the start I made in Chapter Two, exploring the underlying principles of Bakhtin and Vygotsky. I have focused on the networking strategies of other authors, almost adopting a meta-level "comparing"/ "contrasting" approach to my literature review, which has allowed me to improve my own understanding of the theories and their philosophical backgrounds. I recognise that there are a significant number of other authors writing in the field and that, by selecting a subset of them, I am possibly biasing my discussion. I have tried to highlight key authors who raised key points or offered new interpretations in a manner then picked up and utilised by others however, I discuss the implications of my choices further in section 10.2.2. In this section, I am going

to summarise some of the key points of Bakhtin's and Vygotsky's work with implications for connecting the theories at the level of principles, before moving on to think about the significance for developing a methodology, which will be the focus of the next subsection of this thesis (Chapters Five, Six, Seven and Eight).

In addition to the ability to explore each theory in more depth, in Chapter Three I discussed how a connecting theories approach not only takes two theories and offers a manner of networking to fit particular scenarios or cover particular issues not explored fully by one theory, but also offers the opportunity for extension beyond the scope of the original theories. These developments and extensions were made to the theories of Bakhtin and Vygotsky using "synthesizing" and "integrating locally" approaches. The authors I considered justified this networking approach based on the common focus of Bakhtin and Vygotsky on the sociocultural background and context in which texts are formed/signs are used/tools are selected. Both Vygotsky and Bakhtin base their theories on the significance of interaction with an other. They posit that social interaction is, through internalization or assimilation/appropriation respectively, what leads to knowledge (Vygotsky) or meaning making (Bakhtin). Essentially, this interaction leads, in turn, to change. The importance of context, social interaction and valuing of text/dialogue indicate where I can start considering methodology in connecting theories.

In this chapter, authors have offered arguments for limiting the level of connection between the theories of Bakhtin and Vygotsky. The authors in this chapter have argued, if Vygotsky's dialectic is a strict process of internalization of information from more knowledgeable individuals leading to the overcoming of difference and thus progress towards a pre-set goal, then his position is fundamentally different from Bakhtin's approach of meaning making in difference aiming for the "expansion of repertoires of ways of talking about mathematics" (Barwell, 2015, p. 3), where the idea of overcoming difference is antithetical. How the two theories treat difference has implications for power dynamics between those involved in discourse and for the concept of identity. These key points represent fundamentally different philosophical positions and raise issues of inconsistency if attempting a joint analysis. For example, Wegerif argues that Wertsch's conceptualising of signs and tools leads to an inconsistent interpretation of Bakhtin's work. Matusov goes further and rejects almost all forms of connection between the two theories. His argument is based on the individual freedoms offered by the different theories, and

claims that any benefits would be outweighed by the power relations he sees in Vygotsky's theories, which not only limits analysis, but potentially the freedoms of those involved in a dialectic system. The authors in this chapter have argued that Vygotsky's dialectics requires a difference between a more knowledgeable individual who leads less knowledgeable ones towards scientific, pre-determined outcomes in a quest for progress. Those with more knowledge are in a position of power over those with less. In a dialectic framework, knowledge is linked to self-consciousness and freedom. In contrast, despite Bakhtin's clear idea of the authoritative word (section 4.2), Matusov argues that a dialogic approach requires equality among participants for meaning making to take place. This is difficult to reconcile with a classroom environment where the students are working in the context of the institution and the authoritative word carried by the government via the school and teacher.

Due to the Soviet system in place at the time in which Vygotsky and Bakhtin were writing, the issue of power becomes difficult to analyse. For Vygotsky's dialectic approach, a power imbalance is seen as necessary and, based on Hegel's interpretation, contribution to a rational state is seen as a positive outcome. However, judging if a positive view of power and State matches Vygotsky's own view is difficult. Vygotsky's work does not aim to mitigate power relations and could be interpreted as echoing the structure of the Soviet Union at the time (hence Matusov's strong reaction). However, it could also be a necessary position to stay in the good graces of the government. In contrast, Bakhtin's authoritative word could be read as a critique of the power of institutions such as the Soviet state. However, as Gardiner argued (section 4.5.1) "it would be manifestly impossible to disentangle, in anything resembling a satisfactory fashion, Bakhtin's relationship to Hegel, never mind Marx" (Gardiner, 2000, p. 126) and, as such, to separate Bakhtin's writing from any influence by the State is impossible. To say that Bakhtin's writing was not influenced by Marx or was anti-Marxist is overly simplistic. Bakhtin's writing was undoubtedly influenced by writing when he did and through his own experiences being exiled. The influence of the political climate at the time of writing for both Vygotsky and Bakhtin is, in itself, an example of how important the context is for analysis and interpretation.

How the process of internalization or assimilation/appropriation is interpreted has significance for how identity is seen for Bakhtin and Vygotsky. For Vygotsky's theory, each individual is developing scientific concepts that are presented to them by the teacher via

the ZPD and are aiming to internalize knowledge via the mediational means of social and cultural tools in a process of overcoming. Wegerif argues that, as dialectic understanding is developing, the participants are retreating towards a single consciousness as students take social tools and internalize them to form their own identity. On the other hand, he argues that, for Bakhtin, identity is ever shifting (section 4.1). Due to the necessity of responsivity in every utterance, it is difficult to draw conclusions from individual utterances as to the identity of the person creating the utterance and, in fact, the implication that identity is never stable and fixed, but instead constructed through individual interactions with others.

The authors in this chapter argue that the disparities in approach to difference, power dynamics and identity mean that Bakhtin's and Vygotsky's theories are fundamentally incompatible. However, much of this argument is based on a reading of Vygotsky that is based on a traditional dialectic approach with Hegel as a strong influence. The overcoming stage of internalization in pursuit of a scientific ideal required in this interpretation of dialectics has led to the argument against the close networking of Bakhtin and Vygotsky. Extending Vygotsky's theory of internalization to the extreme of internalizing scientific concepts in pursuit of an ideal resulting in a retreat to "one abstract consciousness" (Bakhtin, 1986, p. 147) is something that Wells argues is ultimately unattainable (section 3.3.2). Even Matusov agrees, saying "[s]omeone might counter-argue that my critique of Vygotsky is a bit unfair: Vygotsky studied development, i.e., becoming and not being – he studied incomplete, developing life rather than full life" (Matusov, 2011, p. 103). Focusing on the extremes of a theory could equally be applied to Bakhtin with similarly unhelpful results. If Bakhtin's idea that meaning shifts in different contexts and with different participants, then there is a danger of extending to the point that every utterance is so unique not enough common meaning can be made to make sense of anything. In section 4.5.1, I have used the work of Gardiner and Dafermos to suggest that, instead, there may be different forms of dialectics so the underlying disparities between Vygotsky's and Bakhtin's theories are less clearly defined than some of the authors in this chapter have argued. Moving the focus away from the extremes of both ideas allows me to keep the two theories in the same interactional space.

Following Gardiner and Dafermos, I suggest there are a number of networking strategies that could be applied to Bakhtin's and Vygotsky's theories in order to allow a connecting theories approach at the level of principles, which will allow an analysis that reflects both

the structure of the curriculum and allows for context-specific meaning making. The authors in this chapter make it clear that there are issues with pursuing a combined analysis and the networking strategies I use with the two theories may have to be limited in order to avoid inconsistencies in the underlying philosophical positions. These points must be carefully considered. However, an approach focusing on a single analytical style risks discounting one aspect of the classroom experience and removes my study from the context in which I am working. Through this chapter, I have given examples of bringing theories together that do not require the theories to have a consistent philosophical basis. Barwell (section 4.4) argues that applying both theories could have something significant to contribute to an analysis and Wertsch has a similar approach, arguing that "attempts to account for complex human phenomena by invoking a perspective grounded in a single discipline are […] unlikely to be productive" (Wertsch, 1998, p. 7). Networking styles such as "understanding others/ "making own theories understandable" and "comparing"/ "contrasting" allow me to respect the ideas of the authors in this chapter that reject the close linking of Bakhtin and Vygotsky, whilst also testing the ideas of the authors in Chapter Three to see if the frameworks can be more closely held.

Additionally, I am hoping to benefit from the opportunity to explore each theory in more depth whilst retaining the opportunity to extend beyond existing theories. The use of a connecting theories approach will allow me to explore my framing of mathematics as a language in a Bakhtinian sense. An analytical approach that uses Vygotsky's idea which have a clear framing of mathematical methods within his mediational means, may allow me to see how a development of Bakhtinian principles in the same direction may be useful. If my framing of mathematical methods as a language in a Bakhtinian sense proves useful, this sets methods up as being subject to centripetal and centrifugal forces and, thus, links Bakhtin's literary theories directly with the teaching and learning of mathematics.

My intention with this project is to explore the benefits and drawbacks of a connecting theories approach to Vygotsky and Bakhtin in the context of analysing student and teacher perceptions of student methods. In the previous two chapters, I have used a range of networking strategies to describe the attempts of other authors writing in the field to bring the theories of Bakhtin and Vygotsky together. In the next subsection, I consider how, from these common principles, the theories can be connected at the level of methodologies.

## Chapter Five – Methodology: Planning for Perspectives

In Chapters Two, Three and Four, I discussed the key theoretical and philosophical underpinnings of Vygotsky and Bakhtin in an attempt to establish the principles, Radford's (2008) first possible point of connection, of the two theories. I concluded that, despite evidence the two approaches contained significant differences, the theories could both bring something significant to an investigation. The next step is to consider the methodological implications of the two theories to see if there are points of connection that can established between both.

Methodology is Radford's second key area to consider when looking at connections between theories. Based on the first area of connection, principles (P), Radford describes "[a] methodology, M, which includes techniques of data collection and data-interpretation as supported by P" (Radford, 2008, p. 320). I could look at a methodology for each theory separately and see what applying each of these to data could mean. Alternatively, one could look for a single methodological approach that can be applied to both theories, as well as options in between. Either way, the methodology I would need to employ has to meet the criteria of "operability and coherence" (Radford, 2008, p. 320) with the principles.

The aim for this subsection of my study (Chapters Five to Eight) is to use the networking strategies of Prediger et al. (2008) to continue to explore the similarities and differences between the theories to develop a methodological approach that represents the principles of Vygotsky and Bakhtin. I am going to begin by looking at methodological approaches of each of the theories independently before experimenting with connections that can be made between the methodologies, a process of testing and refining using my own data to get an idea of the networking strategies (Figure 1.2) that I could apply. As in the first part of my thesis when I considered the principles, the process of working on the methodological approaches of the theories will teach me more about the theories themselves, as well as highlighting possible connections.

In this chapter, I begin the process of translating the theories, philosophies and issues that I have discussed in the previous four chapters into implications for a methodology. In addition to considering the methodological implications of Bakhtin's and Vygotsky's theories, my exploration will include the influence of a dialogic or dialectic methodology on

the design, data collection, data presentation and data analysis of the study I carried out, based on the practical research problem I highlighted in Chapter One (section 1.2). I am going to go over the key features of the theory again, then plan the practicalities that will allow me to collect data that I can use to develop my deeper methodological understanding. As I have already mentioned, this approach means I am not going to detail a methodological approach first and then put this into action, but rather use the investigation to develop my approach.

### 5.1 Introduction

From the discussion of the previous chapter (section 4.6.2), I can summarise the following similarities and differences between the principles of the two theories.

Similarities:

- Socio-cultural background and context
- Impact of historical experience
- Presence of the other
- Valuing texts/discourse

Differences:

- How internalization or assimilation/appropriation treat overcoming difference between self and other
- Identity
- Power relations

In order to explore these similarities and differences through developing a methodology and analytical approach, I need to make sure that I look at the units of analysis, Bakhtin's utterance and Vygotsky's word meaning (or mediated action according to Wertsch (see section 3.1.2)). In addition, to gather information about the perspectives of teachers and students towards mathematical methods, the mathematical language they employ and to explore the idea of mathematical methods as language, I will need to create the opportunity for these methods to be the focus of discourse.

### 5.2 Methods

The principles of Vygotsky's and Bakhtin's theories, summarised above, help me focus on the type of data I wish to collect so that I can test different networking approaches. I will

focus on the collection of data based on texts and discourse that represent multiple perspectives and voices in context. The perspectives that I would like to focus on are those of the people most closely involved with the mathematical methods of students i.e., the students who generate them and the teachers who interpret them on a regular basis. I want to hear the voices of those directly concerned with students' choice of methods, which are also directly impacted by curriculum changes or examination-board instructions as to which methods students should be encouraged to use.

One of the aims of this project (section 1.1) is to give the opportunity for low prior attaining (LPA) students, whose methods are often undervalued by an examination system and curriculum that requires they approach their mathematics a certain way, to have their voices heard. In addition, my previous research (John, 2013) has focused on using the professional knowledge and experience of teachers as a valuable resource. Rather than treating teachers as agents of an "evil monological system" (Matusov & Wegerif, 2014, p. E3), I would, instead, like to incorporate their voices as part of my study. This opportunity for teachers to work together to share ideas about student methods has echoes of Wells's "collaborative communities of practice" (1999, p. 330, discussed in section 3.3.2), where teachers creating the ZPD amongst themselves are "constructing their understanding of the art of teaching through reflective practice" (Wells, 1999, p. 329). I have planned this project to give LPA students and teachers a chance to be heard.

Wertsch (1998) says, "[t]he essence of examining agent and cultural tools in mediated action is to examine them as they intersect" (p. 25). In order to examine these intersections from multiple perspectives, I planned to carry out a three stage study. I planned to run two series of discussion groups, one with teachers and one with students to discuss the mathematical methods used by students when approaching a mathematical task. The discussion groups would be followed by a series of classroom observations to look at mathematical methods as they are generated.

### 5.2.1 Artefact Generation

To give a platform to the voices of students and teachers, I planned to run two series of group discussions to discuss mathematical methods (see sections 5.2.2 and 5.2.3 for discussion of the design of these sessions). In order to focus these discussions, I decided that it would be useful to have specific written examples (what I have termed artefacts) of

students answering examination-style questions. The use of written solutions as a basis for discussion allows me to maintain my original focus on the methods of LPA students on examination-style questions. In addition, the use of examination-style questions allows themes about mathematical methods to come through in discussion so I can analyse these themes using the signs of Vygotsky and explore their framing as examples of Bakhtin's utterance. In order to create these artefacts, I wrote three examination-style questions to put to students from across age groups. The questions are based on my experience of similar questions from the work I am doing as part of my teaching job in preparing students to sit national external examinations at 16-years-old. The questions aim to be accessible for all, so that students of all ages and levels of prior attainment could write something, but they also increase in difficulty over the three questions so there is something there to challenge the more confident students. The presence of more challenging questions is consistent with my focus on LPA students to see how they approach these problems as they would be expected to in an examination.

I planned to issue the questions to all of the students I teach and have permission from the school to do this as working on problems constitutes part of their normal mathematics lessons (ethical issues discussed in more detail in section 5.3). The plan was for the questions to be issued to all students taught by me (approximately two hundred students across all year groups) in part of a normal lesson. When designing the questions, it was important that the students had an easy and subtle opt-out since their work forms part of my on-going investigation and will be shared with other participants. The importance of including a way for students to opt-out had an impact on the design of the question sheet (see Figure 5.1). I designed a question sheet divided into two halves. The two halves each contain three questions. The only differences between the two are a slight change in the diagram in question two and different numerical values in calculations.

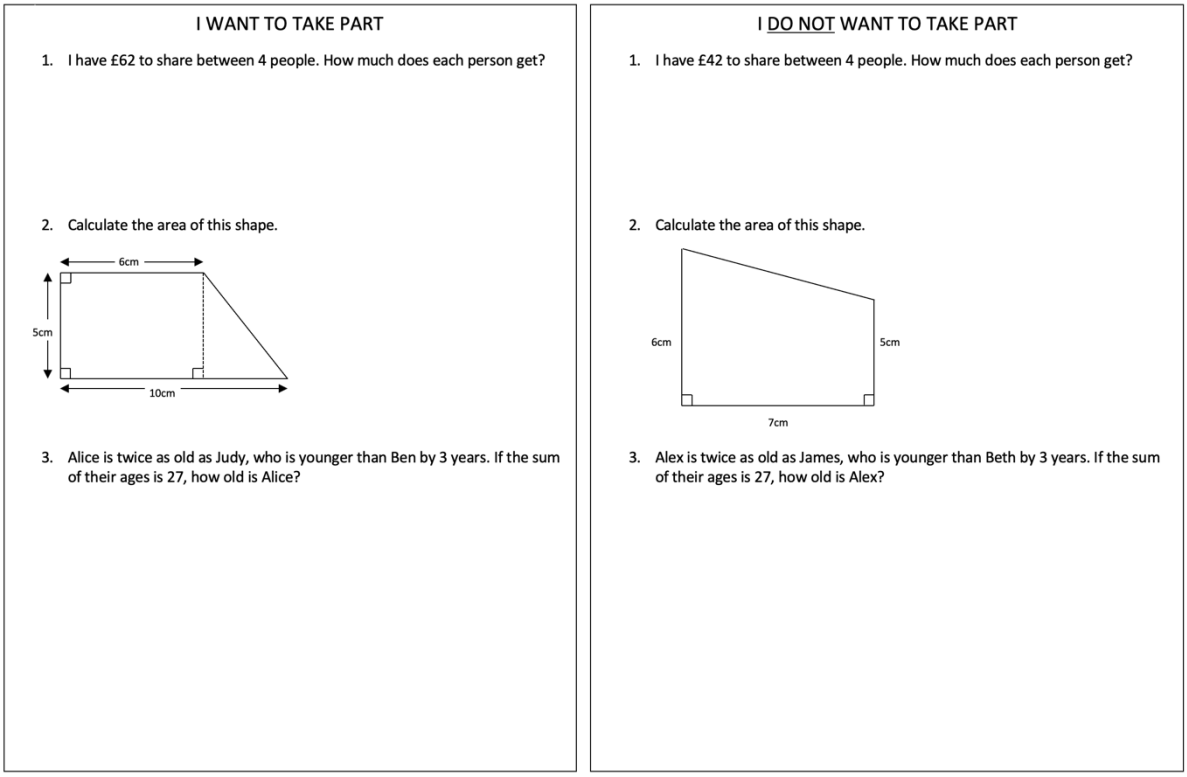

Figure 5.1 – Complete artefact (see also Appendix One)

The plan was for all students to be given the sheet and time to complete as many questions as they would like. They can choose to opt out completely by not engaging in the task at all, complete the questions on the left-hand side of the sheet (the "I want to take part" half), which will be used in the study or complete the questions on the right-hand side of the sheet (the "I do not want to take part" half) to more subtly opt-out of the study. By opting out, they do not stand out in any way (either to me as their teacher-researcher or to their peers) so they feel no pressure to participate based on what others are doing or out of a desire to please me. I also planned not to circulate to help students during the time they are working on these problems as I would usually. Circumstances would then be a little closer to how students would select methods in an examination but also attempt to make sure that no one who has chosen to opt-out feels scrutinised. Once responses were collected, I cut the pages in half, immediately discarding the half labelled "I do not want to take part" so I was left with the work of the students who have chosen to take part.

The two versions of Question One (Figure 5.2) are division questions that vary only in the amount being divided.

I WANT TO TAKE PART

1. I have £62 to share between 4 people. How much does each person get?

I DO NOT WANT TO TAKE PART

1. I have £42 to share between 4 people. How much does each person get?

Figure 5.2 – Artefact Question One

Question One was designed specifically to have the following features:

- A non-integer solution, introducing a little more challenge for students.
- Dividing by four, which lends itself to a variety of methods e.g., short division (sometimes referred to as the bus stop method, Figure 1.1), chunking (Figure 1.1), using fractions or halving and halving again. I wanted the students to have options when approaching the task.
- The question is based around money, a context that sometimes aids learners in checking answers or gives them something more tangible to think about, which can give them a way into a question they may not otherwise attempt.

Question Two (Figure 5.3) is an area problem based around a trapezium.

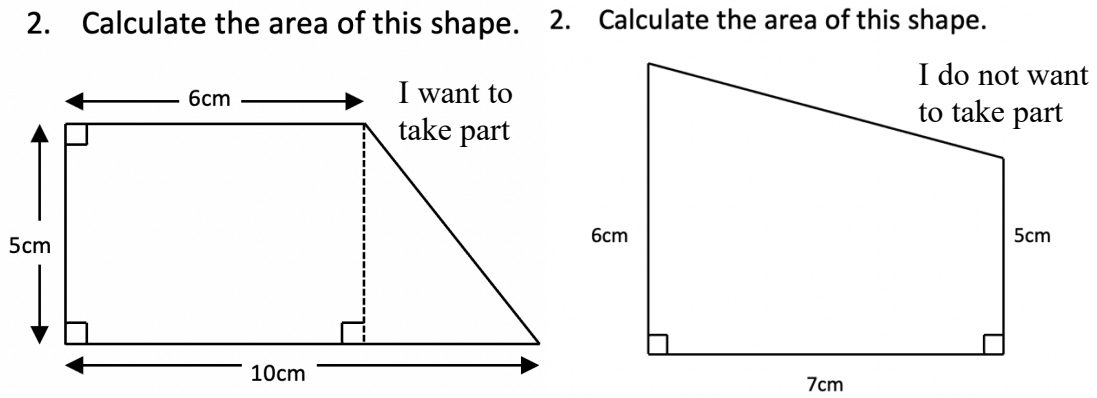


Figure 5.3 – Artefact Question Two

Question Two was designed to have the following features:

- Right-angled trapezium, encouraging students to think about alternatives to using the formula for the area of a trapezium. There are a variety of ways of solving this problem.
- Missing side length, requiring students to calculate one of the side lengths in order to use the technique of separating the shape into a rectangle and a triangle.

- Finding the area, a topic students easily confuse with finding the perimeter of a shape. This question will give me the opportunity to look at common misconceptions.

Question Three (Figure 5.4) is a question based on linked ages.

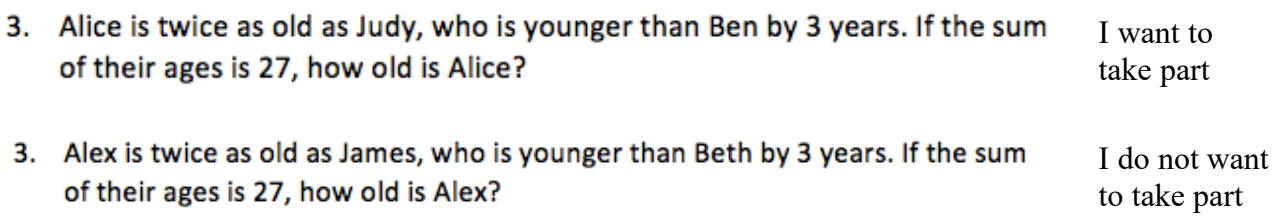


Figure 5.4 – Artefact Question Three

Question 3 was designed to have the following features:

- A variety of methods: a typical example of an examination-style question (for 16-year-olds) that can be solved using a variety of approaches such as an algebraic approach or a trial and improvement method.
- Higher level of challenge, a question challenging for all students.

The three questions form the basis for the discussion groups that I planned. I planned to analyse the discussion of mathematics rather than the artefacts directly, which is consistent with the focus on mathematical language I discussed in the previous two chapters, rather than the use of mathematics as language, which is something I am also going to use my data to investigate. Basing my analysis on group discussion also avoids any analysis of student methods based solely on my views and perceptions, which is contrary to the concept of learning or meaning making with others that Vygotsky and Bakhtin espouse. I am arguing that analysis of student methods by one person in isolation, as in the case of an examiner marking an examination paper, means that there are other interpretations that may be missed. By using these artefacts as a basis for sharing the perspectives of teachers and students, I planned to gather multiple ideas about the methods students are using – the more perspectives offered, the richer the resulting set of data. Bakhtin's meaning making currently focuses on discourse through utterances (section 2.2.1) and words as language. The different discussions based around mathematical methods will allow me to analyse the meaning making in the discourse using utterance or, in the case of Vygotsky, using his concepts of signs and tools, rather than my analysis of individual methods where I am still testing the framing that a mathematical method can be interpreted as an example of utterance.

### 5.2.2 Teacher Groups

The first of the discussion groups I planned to form, to gather a range of voices and share perspectives on mathematical methods, involved a group of colleagues from within my own school mathematics department. I want to utilise the experience and knowledge of my colleagues, something that we had few opportunities to share with each other at the time but is so valuable. Using Bakhtin's definition of voice (section 2.1.1) "the speaking personality, the speaking consciousness. A voice always has a will or desire behind it" (Holquist, 1981b, p. 434), I wanted to give teachers the opportunity to have their voices heard on the topic of student methods.

I planned a series of sessions (see Figure 5.5), each of which would be audio recorded and then transcribed in order to create texts for analysis. According to my provisional plan, the first session would focus on asking teachers to complete the same questions I asked the students to answer when they generated the artefacts (section 5.2.1). In the first instance, I planned to ask the teachers to complete questions as if they were simply doing the problems for themselves (with no other audience in mind), then I planned to ask them to complete the same three problems as though they were teaching a class of students. The problems would be the starting point for a discussion on the similarities and differences between the methods they have chosen.

In our second meeting, I planned to introduce artefacts, based on the same three problems, generated from the students that I teach (as discussed in section 5.2.1). The artefacts would have been through the process of cutting in half and having the opt out halves removed so some of the now A5 sheets will be blank, where students have chosen not to take part in the study. I plan to use these artefacts to broaden the discussion started in the first teacher group session about the similarities and differences between the mathematical methods.

I planned sessions three and four to be flexible so they could vary depending on the discussion that has taken place in the first two sessions. I wanted the teachers to have some choice in what to discuss. In Figure 5.5 I listed some of the suggestions I had for topics for these group sessions. I thought one possible focus for session three could have been on the artefacts generated by classes with a high number of LPA students as part of the artefact generation sessions (see section 5.2.1), in order to gather particular ideas and perspectives

from teachers on this specific subset of student methods. Another suggestion for the final teacher group was that teachers could bring along examples of student work taken from their own lessons, which would allow teachers to focus on aspects of student methods they were particularly interested in, especially as they would have been generated in that teacher's own classroom. The discussion from the preceding sessions would help shape the final two and dictate precisely how I set this up. For example, the teachers may decide to focus on LPA students for this session or the discussion by this point may have taken a different turn and the later discussion will reflect this.

Teacher Group Outline

Session 1
Blank copies of artefacts. Teachers complete:
- as if no one would see
- as if you were demonstrating to a class

Session 2
Student methods from artefacts.
- What's the same?
- What's different?

Session 3
Suggestion – Focus on low prior attaining from artefacts created in student groups.

Session 4
Suggestion – Bring own examples.

Figure 5.5 – Teacher group session outline (provisional)

I planned to transcribe these sessions between one session and the next with the idea of bringing ideas and themes that have come out of previous meetings to the next, making it a process that continually develops based on the path laid down by the individuals involved. Throughout these teacher group sessions, I planned my role to be that of teacher-researcher. Whilst running the sessions as a researcher, I was simultaneously going to participate in discussion as a teacher. The sessions (along with the student sessions in section 5.2.3) had multiple layers of interpretation, which made analysis more challenging. My analysis would be an interpretation of the teachers' own interpretation of students' methods.

### 5.2.3 Student Groups

Having set up and started running teacher discussion groups, I planned the second stage of my field work to involve setting up student discussion groups. These student discussion groups would run in a similar way to the teacher discussion groups, bringing together the voices of students to discuss mathematical methods and allowing their perspectives to be shared. I planned to work with a small group of LPA students, who do not often get the chance to contribute to the conversation about mathematical methods and the curriculum. As I have previously mentioned, in addition to generating discourse for my analysis, these group discussion sessions were a chance for these students to have their voices heard.

For the student sessions, I wrote a provisional plan (see Figure 5.6) with the idea that later sessions would depend on the outcomes of and ideas raised in the earlier sessions. The first of the four sessions would involve asking the students to complete the problem-set from the artefact. Their solutions would help facilitate a discussion based around their personal approaches. In the second session, I planned to introduce the artefacts generated by students earlier (see section 5.2.1). In this case, the artefacts would allow us to discuss the methods of other students. In the third session, I would use the solutions to the problems that the teachers completed in their first session, giving the students the opportunity to discuss the teachers' own methods in a way they do not usually get the opportunity to, allowing them to discuss the methods of the teachers in a safe environment. I also planned a flexible fourth session to work in the same way as the optional teacher sessions. The session would be shaped according to the outcomes of the previous sessions and give the students the opportunity to decide on their own focus. In Figure 5.6, I have made a provisional suggestion, that the students could bring examples of their work from lessons for discussion.

Student Group Outline

Session 1
Blank copies of questions.
- Which method would you use if no one would see it?
- If it was being seen by a teacher?
- If it was a test?

Session 2
Student methods from artefacts.
- What's the same?
- What's different?

Session 3
Teacher methods from artefacts created in teacher sessions.
- What's the same?
- What's different?

Session 4
Suggestion – Bring own examples.

Figure 5.6 – Student group session outline (provisional)

### 5.2.4 Classroom Observations

Having completed the teacher and student group sessions, the final stage of my fieldwork was going to be classroom based. I planned to carry out a series of classroom observations to capture discourse on mathematical methods in the classroom, allowing me to extend my exploration of methodology and analysis. This classroom observation would be planned in more detail once the discussion groups were complete, allowing me to focus specifically on elements that I have explored in those earlier stages. Classroom observation would also allow me to continue to highlight the voices of LPA students by directing the observations towards classes with a high proportion of these students.

My initial plans for classroom visits included tape recording segments of lessons and possibly sampling work produced by students during those segments. Combining recordings, notes and samples of mathematics from students was designed to maximise the number of overlapping perspectives I had. I was not planning to conduct video recordings at any stage of my data collection. There is an argument for using video recordings in order to include gesture in any analysis. Some writers in the field have taken steps to extend the

definition of a sign from the one I have explored in previous chapters. Their "semiotic bundle" (Arzarello et al., 2009, p. 99) includes gesture with the different interpretations of sign that I have discussed so far, such as written and spoken text (Arzarello et al., 2009, p. 100). Arzarello et al. argue that "within this wide conception of sign, we regard gestures as important semiotic resources in strict relationship with the more traditional signs (such as spoken or written language, mathematics symbols, and so on)" (2009, p. 99). Analysis of the semiotic bundle is designed to recognise that:

> gestures can play an important double role. First […] they can support thinking processes of students and promote the transition personal-institutional with suitable conversions from one sign to another […] Second, gestures have also a communicative function […] they allow alternative ways of embodying and organising information that the student is not able to express in purely verbal or formal ways.
>
> (Arzarello et al., 2009, p. 107)

Despite this argument for the inclusion of gesture and the value it may bring to an investigation in allowing one to "fully grasp the evolution of learning processes" (Arzarello et al, 2009, p. 101), it is a contested and significant step away from the text-focused work of Bakhtin and Vygotsky, and would have necessitated a significant increase in the amount of data I gathered and change in my approach to developing a methodology and analysis. In the group discussion stages, I aimed to gather perspectives on the written methods of students motivated by an interest in their approaches to examination-style questions. Physical gestures are not recorded as part of the examination. Video recording the teachers or students tackling the questions for the first session of each of the discussion groups may have shifted the focus away from the written text, a motivating factor behind the study, to the detriment of the analysis. There may be an argument for including video recordings at the classroom observation stage, focusing on the moment at which students apply mathematical methods to a task. However, the focus of this stage was to be directed by the outcomes of the discussion groups and, as such, I would look again at whether or not adding gesture to the data I collect at that stage is necessary.

### 5.2.5 Summary

The following table summarises the different stages I have outlined above. I have emphasised, throughout this chapter, how the details of each section will develop as the

project unfolds, so I will explain in later chapters how the fieldwork I completed varies from this plan:

| Stage | Pre-requisites | Expected Data |
|---|---|---|
| Preliminary – Artefact generation | Blank artefacts ready for completion | Approximately 150 written examples of student work |
| 1. Teacher groups | Artefact generation<br><br>Themes from the student groups (for the later sessions) | Examples of teachers' methods<br><br>Audio recording of discussions<br><br>Transcriptions of audio recordings |
| 2. Student groups | Artefact generation<br><br>Themes and artefacts from the teacher groups (for the final session) | Examples of methods from the students in the group<br><br>Audio recording of discussions<br><br>Transcriptions of audio recordings |
| 3. Classroom observations | A summary of any outcomes and exploration from the teacher and student groups sessions | Audio recordings of sections of lessons<br><br>Examples of student work from the lessons |

Figure 5.7 – Summary of the stages of data collection

### 5.3 Ethics

The ethical considerations of my study were identified as part of an unpublished assignment I wrote as a formative stage of my PhD research (John, 2016). I paraphrase my findings here.

Throughout the process of designing the data collection, I have to ensure I consider the ethical implications of my study. I am carried out my research in the school where I worked, which meant I knew the school, the staff and the students that took part. This made it easier to talk to people about permission or to have ongoing conversations about plans, withdrawal and how those involved felt about taking part. However, I also had to be careful to take my place in the school into account when considering the ethics or my project. I continued to work at the school after my fieldwork was complete and had to remember throughout that I not only represented the university as I went about this study, but also the school.

At the school level, I have obtained permission to carry out the research. The permission is not included as an appendix to maintain anonymity but is available on request. I have completed the ethical conversation with a fellow researcher from the University of Bristol process and, as an additional measure, spoke to a senior member of staff at school whose roles included safeguarding responsibilities. I have submitted my ethics form (Appendix Two) to the university ethics board and it has been approved (Appendix Three).

In addition to consent at the school level, I sought individual permission from those involved in the group sessions. Prior to the first meeting of the groups, I explained the plan and gave information about participants' right to withdraw, followed by recording the verbal consent of participants at the start of the session. As I discussed in sections 5.2.2 and 5.2.3, despite planning each group session with a particular theme, they are designed to be flexible and respond to what has come up in previous sessions. As a result of this potential for change, I sought verbal consent for recording from participants at the start of every session, which was captured at the start of the audio recordings. This is an attempt to acknowledge that "researchers can rarely […] know the full extent of what participation may entail" (Heath et al., 2007, p. 404), particularly in an evolving investigation such as this one.

I obtained written consent from the parent or legal guardian for any student taking part in student discussion groups. I also sought and obtained verbal permission from the student at the beginning of each group session. A blank copy of this consent letter is attached as Appendix Four. Completed versions are on file but not included here to ensure anonymity. In addition to seeking permission from legal guardians, the letters open communication with the families of the students as a matter of courtesy but also so they know how to report any concerns as the study goes on.

Power relationships within groups are a key issue for me to be aware of from an ethical standpoint. In section 4.5.2, I discussed how power relations are featured in the work of Bakhtin and Vygotsky, respectively, along with the points raised by those writing in the field (e.g., Matusov, section 4.2; White, section 4.3). As they feature in the wider discussion, I am hoping that the data produced through this investigation will allow me to look at power relations in more detail and build a response into my methodology as it develops (see section 8.2.6 for example). However, that does not mean I should not take care to

acknowledge the power relations in evidence as I set up the study. I must ensure that, as far as possible, I mitigate any potential effects of unequal power relations at each stage, particularly with regard to my own role. Of those making up the teacher groups for the discussion sessions, individuals hold different roles within the faculty and wider school. Some of the participants line manage one another or hold positions of responsibility within the faculty. Even though I am part of the department, my colleagues are aware that I am conducting research for a study and this can be intimidating. I had to make sure to emphasise that the point of the sessions was not to pass judgements on students, teachers or classes, but rather to gather a range of views and voices. As part of my initial explanation, I clarified what the data is being collected for and how it will be used. I also ensured the anonymity of all participants as part of my write up so they can be sure that comments, written methods and classroom observation data will not be linked to individuals.

For the student groups, the power imbalance is even more pronounced. Not only am I a figure of authority as their teacher, I am also bringing the aforementioned sense of external scrutiny with me as researcher. To help mitigate this, I, as with the teachers, explained the point of the research and how the data I collected would be used, before asking if any students wanted to volunteer to take part in the student group sessions. I also tried, as far as possible, to maintain a relaxed atmosphere for the group sessions and repeated the option for students to opt out if they wished at any point. This option of withdrawal was included in the permission letter to parents (see Appendix Four).

To carry out visits to classrooms in the final planned stage, I would have needed teacher authorisation to visit their classes. I would have offered any students who were uncomfortable with the arrangements the chance to study somewhere else for the duration of the session or, alternatively, the chance to sit away from where I was carrying out recordings. I would also have asked permission from each student from whom I wanted to take a copy of work. Teachers are wary of classroom observations as traditionally, they are carried out for the purpose of evaluation of the teacher, such as in an inspection of teaching or as part of an annual performance review. However, as these observations were planned as a continuation of the discussions that had been going on in the teacher groups, I hoped that those who took part would understand that they were not linked to any such judgement.

The key part of my ethical approach to this project was that I continued to reflect on the ethics of the unfolding situation and how my decisions impacted on the participants. I needed to be conscious of the power imbalances present in each stage and, through exploring the framing of power relations in Bakhtin and Vygotsky's work, I hoped to do just that.

This chapter has aimed to explain my approach to the next section of the study. I have described how I intend to develop my methodology and analytical framework by collecting data. In the next chapter, I begin to apply the principles I have discussed in the first four chapters to analysis of the collected data and explore how this application, using the networking strategies of Prediger (2008), might allow connections between Bakhtin's and Vygotsky's theories at the methodological level. The next chapter will mark a shift as I begin to report on the early stages of the data collection and analysis.

## Chapter Six – Methodology: Early Stages

Chapter Five looked at the practical aspects of gathering data about perspectives on mathematical methods among students and teachers as a start to examining the methodology strand of Radford's (2008) connecting theories framework. In exploring possibilities for my fieldwork and analysis from Vygotskian and Bakhtinian perspectives, I will be developing insight into networking the two theories. In this chapter, I am going to go over the early stages of my fieldwork and analysis, explaining how the study developed and some key methodological and practical points that became evident in these early stages. I will then go on to explain how these changes will impact the next stages of analysis.

I am going to begin by reviewing the collection of artefacts that I outlined in the previous chapter (section 5.2.1). I will then describe using the artefacts as the basis for a working group at the British Society for Research and Learning of Mathematics (BSRLM) conference in 2017, which acted as a pilot study to identify key points for development before I moved onto analysing the group discussion sessions. Finally, I will look at my analysis of the first teacher group discussion, which was carried out at an early stage in the process. Although lacking the detail of some of the later analysis, my first attempt at analysing a group session led to several significant methodological and practical decisions and served as an important building block to create some of the analytical tools that I go on to use.

### 6.1 Gathering Artefacts

I carried out a trial of the generation of artefacts during November 2016. I visited a class of 17-year-old students taught by one of my colleagues. These students did not take part in the final artefact collection and their work was not included in any final analysis. The main aims of this trial were to check the format of the question sheet, practice my explanation of the task and check that the questions would offer opportunities for a variety of methods as well as an appropriate range of levels.

The students were asked to complete a draft version of my questions. I used the opportunity to practise explaining their right to withdraw and how the two-part sheet worked. The student verbal feedback was that this was clear and they understood what they were required to do. Other feedback was positive and their only recommendation was

that the sheet did not offer enough space for their workings and they were required to continue onto the side of the sheet that was going to be cut off, which was obviously not ideal as part of their workings would be lost. The students also mentioned that they thought the youngest students (11-year-olds) would find the final question challenging, so I needed to be sure to mention that this is intentional and it is fine if students cannot complete all questions. Looking at the solutions from the trial group, not all of them got all of the questions correct. There were a variety of different methods used in order to answer the problems. The requirements that the questions got more difficult towards the end were satisfied, allowing for challenge for the older students, and that students were not bound to a particular method in answering.

In light of this feedback, I edited the size and layout of the sheet slightly so that the students had more space for their work. Copies were made in a different colour for each year group. The colour coding was for my own benefit so that I could easily see which classes had completed the sheets and, due to specific learning requirements, since certain students required coloured paper to make reading easier for them, I did not want their sheets to stand out from everyone else's. The colour coding was not shared with any teachers or students who took part in the discussion groups but was used to identify the artefacts from my lower prior attaining classes if I decided these were necessary to identify for a specific purpose in the group discussions, for example, in the final stages of the teacher discussion groups (see section 5.2.2). I then spent ten to fifteen minutes with each of my classes for them to complete the final artefacts that went on to be used throughout this study.

The students, overall, coped well with the demanding questions. During a normal lesson, I would circulate around the class offering help and support. I made the decision not to here so that I would not be able to see whether students were opting in or out as this may have added extra pressure. However, I could not help as I usually do in lessons if they got stuck. I also asked students to complete the questions without discussion so that it was their methods that they used rather than those of their classmates, but this may have had the unintentional effect of making the questions seem like a test. There was a fine line to tread between gathering methods based on students' own ideas whilst not adding pressure by replicating test conditions. Classes did take the process seriously and several students used the opt-out I had built in as I found blank halves in the final collection. So, they felt comfortable and had understood how the process worked.

In addition to any pressure added by replicating test conditions, the artefact generation stage did highlight an ethical issue that I previously identified (see section 5.3), that of the power imbalance of me working with students. One class, containing several low prior attaining (LPA) students, found completing the mathematical tasks in the artefact generation stage challenging. After my explanation of the project, the artefacts, the right to withdraw, and how artefacts would be used, one student asked, "So if we do the 'good' side […]". I had obviously unintentionally imbued one side of the activity sheet with more value than the other. I reexplained and they were happy to continue, but one student was visibly uncomfortable when completing the sheet. They handed me the sheet and said "I don't want to do it" and "I don't do well with pressure". Obviously, despite my attempts to make it clear the data I collected would be anonymous and give the students options, they were still feeling an innate pressure to take part. One student summed up the issue by saying, "You help us learn maths Miss so we'll help you with this". They sensed it was important to me. Is this fair? Should I discount the data from this class? Or is that an injustice for those who really wanted to take part and worked hard on challenging problems? The student who was obviously upset felt comfortable enough to hand me back their work to throw away (which I did). Several students used the opt-out side of the sheet, so they were not all scared into taking part. Is this a sign the system is working if they feel happy to use the opt-out that I have built in? I felt that the reaction of this class illustrated how difficult they found assessment situations, such as examinations, where there is no in-built opt-out.

Once I had gathered the artefacts, I wanted the opportunity to test their use as a foundation for discussion on the topic of mathematical methods. In order to see if they would work effectively as prompts to a discussion and also to give me an insight into what other decisions I would need to make before I moved to the group discussion stage. I ran a small pilot study by hosting a working group at a conference.

### 6.2 Pilot Study

The working group I ran took place at the March 2017 BSRLM (British Society for Research into Learning Mathematics) conference. The session involved participants forming small groups with each group given four randomly selected artefacts to discuss. I asked the groups to focus on the similarities and differences between the methods used by students, circulating amongst the groups as they talked. I then used the points I had heard as I

circulated to lead a final discussion involving all the participants. I wrote the session up as a conference paper (John, 2017), including here a short excerpt from the analysis section, which demonstrates my first attempts at analysing discussion based on the artefacts.

In the session, the groups considered examples of Question One (Figure 5.2). They were given copies of the original artefacts, which allowed participants to annotate the examples. I include below four of the annotated responses taken from that session (Figure 6.1, Figure 6.2, Figure 6.3 and Figure 6.4), which were published in the conference paper (John, 2017). Part of the discussion focused on Question One. Participants discussed how students made use of the context of the question, money, recognising that some students had referred back to the context in their solution (Figures 6.1 and 6.3) whereas some had not (Figure 6.2). The discussion raised the questions "Does the lack of a link back to the original problem show less understanding? Or could it be considered that the students have not answered the question?" (John, 2017, p. 4)

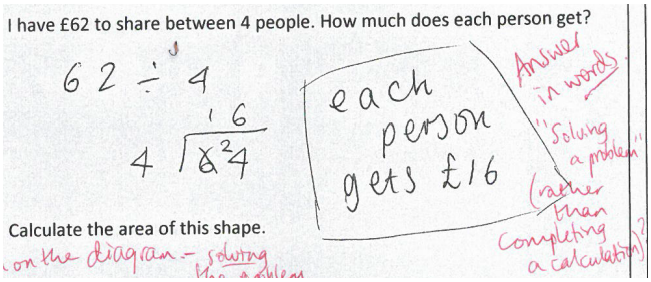


Figure 6.1 – Example of Question One as used in BSRLM conference paper (John, 2017)

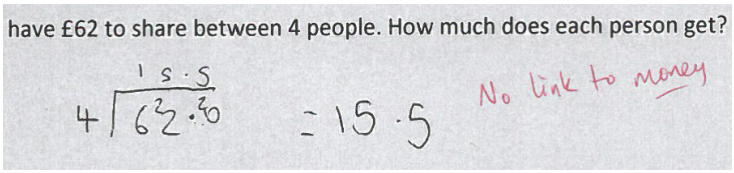


Figure 6.2 – Example of Question One as used in BSRLM conference paper (John, 2017)

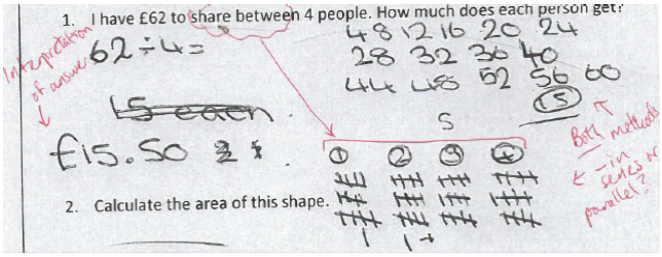


Figure 6.3 – Example of Question One as used in BSRLM conference paper (John, 2017)

Having discussed the use of units in solutions, the group considered a further example (Figure 6.4):

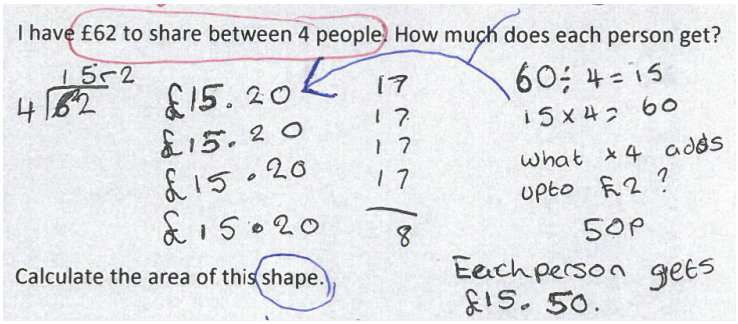


Figure 6.4 – Example of Question One as used in BSRLM conference paper (John, 2017)

The following is an extract of my analysis, taken from the paper:

> This student began by using the 'bus stop' algorithm for division, but misinterpreted their remainder as representing 20p – not the only student to do so. However, the student has corrected their mistake and reinforced that the remainder 2 is the same as £2 divided by 4. Their solution illustrates the difficulty some students had interpreting their answer to the 'bus stop' algorithm. However, this student has overcome this by linking their solution back to the problem.
>
> (John, 2017, p. 4)

The focus of the analysis in the extract above is on description, implying that we can find indications such as using units and "linking their solution back to the problem" to show that a student has or has not understood the question. The analysis is embedded in a linear idea of progress from not understanding to understanding based on whether students are using methods that represent formal or standardised mathematical language, which resembles a dialectic approach. The analysis does not contain identifiable elements of a dialogic approach and, despite reflecting aspects of a dialectic approach, does not provide the detail I need to identify connections between the two theories. Without audio recordings of the discussions, I did not have access to the data I needed and so the analysis is lacking the perspectives of others to add depth and variety to the interpretation that I am looking to the group stages to provide. However, the pilot study did illustrate that the artefacts prompted discussion about methods, which was promising for the next stages.

The pilot study also highlighted some practical issues with gathering data from group discussions. I was unable to audio record the conference session due to not being able to

obtain the permissions from participants, who had a free choice of sessions and therefore I had no idea who would be attending in advance. This made it difficult to include the voices of the participants, partly leading to some of my difficulties in analysis as the analysis was based on my ideas guided by what I had taken from the discussion and the notes participants added to the artefacts, rather than the spoken discussion of those present. There is the necessity for recording and transcription of the teacher and student group sessions as I had planned (see sections 5.2.2 and 5.2.3).

### 6.3 Outcomes

My first step in developing my analysis from this rudimentary starting point was to return to examples of similar analyses in the literature. One of the clearest summaries of the work of Bakhtin and Vygotsky being applied to analyse the same set of data comes from Barwell (2015). Barwell is one of the authors who has written about the clear differences between the approaches of Vygotsky and Bakhtin (see section 4.4). He does, however, allow parallel analyses from the two perspectives to lend new information to an analysis using a "comparing" networking strategy. Barwell looked at language use in classrooms with a mixture of national languages, focusing specifically on a discussion where the technical term polygon was introduced in a discussion of classification of shapes. Barwell analyses an excerpt of this lesson using first a Vygotskian perspective, then a Bakhtinian one:

> From a Vygotskian perspective, the students are being socialised into a discourse of formal categorisation, instantiated not just in the vocabulary, but in the associated syntax through which suitably mathematical statements about the categorisation can be made. Hence, 'polygon' is introduced as a key term of vocabulary, while the use of 'non-' to form complementary categories is an example of syntax. The differences in the students' suggested categories and those of the teacher are in opposition, and the teacher guides the students towards the formal mathematical account: 'there are shapes we call polygons'. She replaces the students' terms with standard mathematical language.
>
> From a Bakhtinian perspective, the differences apparent in the teacher's introduction are not so much in opposition as in dialogue with one another; the process of making sense of the word and the concept 'polygon' arises through the different ways of classifying shapes that preceded this moment, and so on. The teacher's remarks include multiple discourses (mathematical, textbook, teacher,

> school) and multiple voices, including the voices of the students whose ideas can be heard in the teacher's words. The teacher acknowledges the students' ways of classifying shapes, saying they were 'very good', but sets them to one side in order to focus on the more formal terms of polygon and non-polygon. These formal terms are marked as such by the expression 'we call', as in 'there are shapes we call polygons'. What is taken as more formal mathematical discourse arises through the interaction (rather than a priori), with the teacher's emphasis on what 'we call' some of the shapes arising in response to the students' own categorisation. When the teacher does this, she simultaneously constructs the students' classifications and the formulation they use to express them as less formal. This account captures a little of the tension between the unitary language (in this case, formal mathematical discourse) and heteroglossia (students' informal expressions) to which Bakhtin refers.
>
> (Barwell, 2015, p. 9)

I am not analysing classroom discourse until the final stage of data collection (see section 5.2.4) so the direct analysis of student-teacher interactions Barwell offers here is not the aim for the teacher or student groups I am analysing in the first two stages. Instead, from these analyses, I am looking for techniques I can take into the analysis of discussion between teachers or between students and teachers around student artefacts and the methods they represent. From Barwell's writing I can see that, for a Vygotskian analysis, I would look for differences and a sense of progress towards an existing concept of formal language. For a Bakhtinian analysis, I would look for ideas in dialogue with one another, evidence of different voices, the shifting and context-specific framing of technical terms as formal or informal and, in a wider sense, the tensions in evidence between heteroglossia and unitary language. These two approaches echo the principles of the theories I summarised in Chapter Four (section 4.6.2). For my discussion groups, participants may shape their discussion around methods in a particular way. Echoing a Bakhtinian or Vygotskian idea of formal and informal, there may be evidence of tensions present in the teachers' discussion or they may identify certain features of students' methods that have a particular interpretation after a Bakhtinian or Vygotskian fashion.

One element I am exploring, as I develop my understanding of Bakhtin's dialogics, is if dialogics can be used to discuss mathematical methods as examples of utterances and hence as examples of mathematical language. For example, if I applied these Bakhtinian

principles to an analysis of the mathematical methods highlighted by the notes and perspectives of those who participated in the BSRLM session in section 6.2, I can focus on the tensions at play in the solutions provided. For example, there is a tension highlighted in Figure 6.3. The student has recognised the task requires division and has reframed the question using standard mathematical notation by writing 62 ÷ 4. However, in order to solve the problem, the student has used two informal methods to achieve a solution, as identified by the note, "Both methods", of the participant from the BSRLM session. I could interpret this note as the participant having identified the tension between a unitary mathematical language and informal methods. An example of another tension, identified by the participants in their notes, could be that provided by the question itself. The students who produced the work in Figures 6.1 and 6.4 (and Figure 6.3 although it is crossed out) have been influenced by the language of the question to respond directly with a short, worded response. The added detail offered by applying Bakhtin's work on the utterance to the elements of methods identified by the participants is immediately evident here and is something I am going to continue to test in my next analytical attempt.

### 6.4 The First Teacher Group Session

Having reviewed the detail of Barwell's examples of dialogic and dialectic analysis (section 6.3), I moved to begin my analysis of the teacher group sessions in an initial attempt to use these techniques. I discussed in section 4.5.2 that I am going to begin by adopting an approach that allows for the two theories to be used in parallel and allows a focus on developing understanding. As such, I am going to adopt a "comparing"/ "contrasting" networking strategy to this analysis. Please note, I am going to address my transcription style in section 6.6 so there may be typographical errors and non-standard notation in the transcriptions in section 6.4.

The first teacher group session took place in May 2017, and involved five teachers and myself as teacher-researcher. The session focused on the teachers creating their own version of the artefacts as I explained in section 5.2.2 and summarised in Figure 5.5. From the transcription of this group session, I have chosen specific sections that lend themselves to an analysis along the lines of that which I have begun to explore. Further discussion of the selection of these sections takes place in section 6.5. As I initially approached the analysis, I considered the question of whether to discount my words as researcher, but I am also a teacher so have included them, aware that they will be affecting the language used by

the others. A fuller discussion of my dual role as teacher-researcher and the significance of this for the study can be found in section 8.2.6. There were seven sections initially selected for analysis, illustrating areas of theory I was interested in or a particular topic relevant to my context. I have retained three here as they prove the most useful for future exploration. As the sections selected were presented in chronological order, I have retained the original numbering below hence starting at 6.4.4. Other sections analysed are in Appendix Twelve.

### 6.4.4 Notation

In the external mathematics examination papers sat by students at 16 or 18 years of age, there are marks allocated for the correct presentation of solutions by students. Sometimes this takes the form of marks for including the units for a geometry question such as those detailed in the mark scheme for the Summer 2018 GCSE Mathematics Higher Paper 1H, Question 5 (Pearson Education Ltd., 2018a, p. 9). Sometimes this takes the form of presenting money with the correct units and rounded to two decimal places such as in the mark scheme for the Summer 2018 AS Level Mathematics Paper 1, Question 6 (Pearson Education Ltd., 2018b, p. 13) or presenting time in the correct format such as in the mark scheme for the Summer 2018 A Level Mathematics Pure Paper 1, Question 8b (Pearson Education Ltd., 2018c, p. 15).

Questions One and Two in my artefact are both reminiscent of mark scheme expectations in that they both include units of money or area. I had not intentionally included two such examples of these types of questions when designing the artefacts, but units had been mentioned during the BSRLM session discussions. I noticed when looking at the methods the teachers had written down that many had not used units and so decided to ask about it:

R: Who put units on their answer?
K: Me.
R: For both?
K: Yup.
M: Oh, I didn't now that you mention it?
K: Oh no, only for one.
I: Not for the last question.
K: Yes you did, there and there.
V: Not that one.
(All talking over each other)

M: I put units the first time but not the second time.

K: So did I, don't know why.

R: I did mine the other way round, when I was showing the students I made sure to put the units on, but when I was doing it myself I didn't bother.

K: You also didn't put it to the correct number of decimal places.

R: I also didn't bother with the correct number of decimal places, no I didn't which is really interesting, so hang on…

M: What's the correct number of decimal places?

R: If it's money then two theoretically.

M: Isn't this a question about area?

[clarified we're looking at Q1 still]

N: I didn't either.

R: You've got a pounds sign there.

N: I have got a pounds sign there.

R: Is that for you or is that for the students?

N: That's my one, I didn't bother with the students'!

V: I went back and tacked it on my one at some point. When I did it myself I basically stopped at 15.5 and then carried on and at some point there thought "I need a zero on that and a pounds sign on that."

The idea of the correct use of units is tied, not to a particular method or process but, instead, to the idea of a standard or correct notation. The teachers discuss the use not only of units but also the "correct" number of decimal places. No matter which method someone uses to work on Question One or Two, the teachers show that there is an expectation from the context of the question that the answer is presented in a particular way. A Vygotskian perspective would be that the teachers are familiar with this mathematical convention; they are socialised into the vocabulary and syntax of formal mathematics. Does the use of units or other mathematical conventions signify the mastery of a particular concept in that the user recognises the significance of these conventions for the context? The teachers discuss having the "correct" number of decimal places implying that there is an incorrect number that might show a lack of full mastery of their chosen mediated approach to the question. Without the correct presentation, the tool has not been applied successfully to the specifics of the question.

From a Bakhtinian perspective, the teachers could be said to have identified a tension working to standardise the presentation of a solution in a particular format. I have mentioned already (section 6.4.2) the idea that phrases such as bus stop could be seen as markers of speech genres or as indicating a particular tradition of terminology representative of a specific context. Could the units and rounding be considered as part of formal mathematical language? Were a student to give a verbal solution to question one such as "fifteen point five", this is not the same as them saying "fifteen pounds and fifty pence". The second phrasing clearly demonstrates the student linking a solution to the original context, which is what is expected in an examination question and in wider society when discussing money in business. The only difference here is that the teachers have identified the units via mathematical symbols on a page, rather than words. Students have to engage with the question dialogically to make meaning of the context so, as such, is there a Bakhtinian framing of the written solution where the style of presentation is seen as a marker of a speech genre in that it represents "relatively stable types of these utterances" (Bakhtin, 1986, p. 60)?

If these units and rounding are signs of a speech genre, then is this an example of students learning something for a wider sociocultural setting? Or an example that learning a convention from their wider experience applies within this speech genre as well? Are there many overlapping speech genres at work? Teaching students to use the correct units or the correct number of decimal places is significant in that, by doing so, we are introducing them to and helping them to use the markers of the speech genre known as formal mathematical language, which then has a knock-on effect into other contexts that students experience and vice versa.

### 6.4.5 Question Three

The third question from the artefact was the most complicated, designed to offer challenge to all students. In the group discussion, it was discovered that the teachers all used algebra to approach the problem:

> N: We've resorted to algebra!
> R: Yeah. Everybody! Everybody's has done it as an algebraic problem.

K: I was trying to think of a way of leading them through without leaping into algebra and I couldn't think…other than trial and improvement I couldn't think of an effective, time economic way of doing it.

Sections 6.2 and 6.4.1 demonstrated a range of possible approaches for Question One. In contrast, here, the teachers have recognised they have all utilised the same standard method for completing Question Three. They notice that they have all applied an algebraic method, however, saying "the teachers all used algebra" does not in any way imply that everyone's algebra is the same. Simply saying that someone has used an algebraic method is not sufficient to describe what the different teachers have done:

V: Is it worth talking about the thought process of who we chose as our variable in question three?
R: Oh yeah, actually, that's a good point.
V: I chose Judy, but looking around the table, did most people use Judy?
M: I chose Judy the first time when I got it wrong but the second time I chose Alice because she was actually the person we wanted to know about and it seemed easier to choose her straightaway because then she'd already be in the answer.
I: I chose Judy because you can place her without really changing other things twice - that's two times Judy and that's Judy plus three.
[M and V agreeing]
K: I chose Ben.
V: Yeah, which is the one I would definitely not choose.
K: Well I'd already noticed the difference between the 'if you want to take part' and the 'if you don't want to take part' being gender specific so I just chose Ben just to be difficult. No I just went for Ben because everything is based on Ben. I agree afterwards having looked at everyone else's things it would have been far easier to do Judy but…
V: Why is everything based on Ben?
K: Well Judy is based on Ben and Alice is based on Judy and therefore implicitly is based on Ben and therefore everybody is based on Ben, but unnecessarily complicated so anyway, as you can see (gesturing at workings).
V: Fair enough.

Even within the scope of algebraic methods, and if I narrow that down for the sake of argument to algebraic methods that lead to the correct answer, there are still different ways of putting together the solution. So, referring to an algebraic method is a simplification of what is going on, as there are variations and those variations lead to more or less complicated methods.

If I frame this discussion as looking at who has mastered a tool, then is this variation a finely tuned example of how some have mastered a faster or more efficient way of using this particular tool? Or is each variation a different tool altogether? Or, if I return to the mathematics as language position, with a Bakhtinian framing, is the small variation here down to how meaning is being made through the teacher and the question interacting and the subtlety of what is going on in that interactional space? Does the difference arise from the unique way each teacher uses their prior experience of examinations, the classroom, and student methods?

As well as considering how the teachers framed their algebraic method, discussing Question Three raised the issue of what value does an algebraic method have compared to a trial and improvement method?

> R: …you can do trial and improvement for it within certain limits can't you so if you know that she's going to be twice as old as this person who is younger than…that gives you some limits on how old they can each be to start with and the fact the sum has to be 27…
> V: So the mean is going to be about 9…
> K: So 56 is probably not a good guess!
> M: That's a good point.
> R: [to V] Say that again?
> V: So the mean's got to be 9 so there's a reasonable first guess for whoever the middle age is which is Judy isn't it? No, you don't even know what the middle age is do you…You would guess it's probably Ben because he's a bit older than…
> M: Yeah, Judy's the middle age, I mean Alice is the middle age.
> R: Or Ben's the middle age?
> N: Alice is the oldest isn't she?
> V: Yeah Alice is the oldest.

M: [rereading the question] Oh, okay.

N: Ben's got to be in between Judy and Alice.

V: Again, and actually if you do that and think well Alice is twice as old as Judy and Ben's only three years older than Judy, assuming Judy's more than three years old, then Ben's the middle so lets give him, if you're doing a trial and improvement, nine, which is the mean average of the ages, see where that takes you. It would happen to land you on the right set of numbers.

R: As if by magic!

K: However as a process for trying to do that it's not a terribly useful strategy for answering a lot of questions that are turning words into algebra.

V: No

R: No

V: However a process of saying roughly what do I expect it to be and then doing a sort of mean thing and then going is that about right if they're all about [inaudible] years old.

K: Exactly, as we were talking about that earlier, is that a reasonable answer.

…

K: Yeah, having an idea, as you say, having an idea a) what you're looking for and b) whether or not what you've got is a reasonable answer is certainly something I would want to encourage with them "Does that make sense, you've got an answer of -12. Does that make sense as an age?" You know?

Here, the teachers' discussion implies a far deeper understanding and link to the context of the question when using a trial and improvement method than the algebraic method, despite the algebraic method being seen as more sophisticated and the one used by all of the teachers. There is a sense that the teachers all turned to the algebraic method that they could rattle off to solve the problem, but the logical trial and improvement method actually required more discussion and seemed to be something that would be more difficult to communicate to students. If, from a Vygotskian position, concepts are built by an overcoming of difference via mastering tools then does the use of more sophisticated methods, as defined by an examination board or curriculum, automatically imply an understanding of the question and context? Or does this discussion, involved with trial and improvement, imply a higher level of engagement with the context, justifying the curriculum specifying such approaches?

### 6.4.7 How Many Methods?

In the previous sections, I have looked at discussions between the teachers as to how they completed the questions from the artefact, first as if they were doing it themselves and then as if they were teaching students. When the discussion turned to the variety of different methods available, the teachers start to relate this directly to their classroom practice:

> N: It's actually really interesting um process of looking at how differently people do things and how differently people teach things. It would be very useful as training to do this sort of thing more I think. To pick up tips and think about "oh yeah and do I do that as part of my practice, yes I normally write the unit."
> K: Less so about whether or not you remember to put something in that you know you should but more do you do it this way, do you do it that way or do you do it both cause I'm often unsure as to whether when I show them that there's more than one way of doing whether I've actually clarified things or if I've muddied it completely 'cause they start looking at you sort of a bit cross-eyed and glazed and you think OK maybe that was one method too many there.
> V: But then sometimes you can go "You know you can also do it this way" and they go "why didn't you tell us that months ago sir?"
> K: Well exactly.
> N: But also it allows them to be a bit more creative about the way that they approach doing questions if they've got choices about how they're gonna do it…
> K: As long as the choice doesn't paralyse them, yeah.
> M: It should also be like a better conceptual understanding cause you can link it to more different kinds of thinking and more different kinds of processes.
> K: At our level, yes.
> M: That's why you find it easy because you can see all the different, you can see what all the connections are whereas if you're just seeing a process that you can't really put it in its place unless you…
> K: But when we're answering question of that form we have a bank of different strategies for doing it which is what we're hopefully trying to give them is a batch of

different ways of being able to find these things, rather than just you do this, you do this, you do this and you write down whatever the answer is.

The discussion by teachers about knowing how many methods to teach is an excellent opportunity to think about what this variety of methods represents. If we consider a Vygotskian perspective, does the wide variety of methods represent multiple tools for approaching problems with these tools forming a spectrum from less to more formal? Working through multiple methods is a process of internalization on the journey to mastery of a concept so the aim of having multiple methods is to aid the path to this mastery. Introducing multiple methods as a teacher is to try and offer scaffolding towards more formal mathematics to aid in this progression. On the other hand, the discussion of the teachers sounds more like a Bakhtinian perspective, introducing multiple methods as an attempt to offer multiple representations so that they broaden the ways in which students can engage and make meaning with the task. Could these multiple methods be considered heteroglossia? Or the use of multiple methods suggested by multiple people as multiple voices? Or do the different methods offer different tensions for students depending on how students engage with them?

One question raised by the teachers here is, "I'm often unsure as to whether when I show them that there's more than one way of doing whether I've actually clarified things or if I've muddied it completely". Would the concern here go against the framing of supporting students with multiple ways of making meaning and different ways of tackling a question? Is it more about when to show them these methods? In which case, how do you decide which to do first? Does a framing of one method as more suitable than another lean more towards a toolkit analogy? Or can I interpret the situation using a Bakhtinian framing, where the more methods students know, the more mathematical language they are using so the more they are using the speech genre? Are those centripetal forces pulling them towards not one way of doing something, but a set of ways of doing something, a series of techniques to make meaning from these problems.

In addition to the discussion about the methods, this excerpt includes a line from one teacher about how much they value the process of the discussion with others: "It's actually really interesting um process of looking at how differently people do things and how differently people teach things. It would be very useful as training to do this sort of thing

more I think". Echoing Wells's ideas about the ZPD for teachers, the idea of learning from other teachers is in evidence. It is nice to see that the teachers are getting something back from their contribution.

### 6.4.8 Discussion

The second attempt at an analysis based on Bakhtin's and Vygotsky's ideas that I carried out in this section shows a development from my earlier analysis (section 6.2). By refocusing on the key characteristics of a Bakhtinian or Vygotskian analysis using Barwell's examples, I have started to demonstrate the deeper discussion made possible using these ideas. This is particularly evident in sections 6.4.4, 6.4.5, and 6.4.7, where the analysis considers a Bakhtinian perspective and Vygotskian perspective, allowing the highlighting of connecting points and differences. I would particularly like to continue connecting points and differences into the next analysis. However, there are elements that are still not working. In this section, I am going to discuss the strengths and weaknesses of my early analysis, highlight the points I wish to carry forward as I develop my understanding of the analytical approach and where I can make connections as part of the methodological exploration of connecting theories.

One element that has lent added depth to my trial analysis is the treating of the mathematical methods of students as examples of language and, as such, interpreting the discussion of these methods by teachers in the group discussions as identifying tensions that have had an effect on the methods. I mentioned, in section 2.3, that Bakhtin saw mathematics as something that could not be engaged with dialogically. Wegerif points out that Bakhtin "argues that voices and texts are constructed within and through dialogue" (2008, p. 353), so, this is true of the discussion of mathematics in the classroom and the interpretations of the mathematics in my discussion groups. However, through this preliminary analysis, I have attempted to demonstrate how the mathematical methods of students do represent different voices creating utterances with the intention of responsivity, under tensions introduced in the dialogic space opened up by the artefact questions. Framing mathematical methods as language explains some of the points brought up in the teacher dialogue. Wegerif (2008) has a framing of dialogic that involves a self-other-sign relationship where he offers "[a]ny sign taken to be a mediation between self and other […] must presuppose the prior opening of a space of dialogue within which such a sign can be taken to mean something" (p. 353). In this interpretation, the written

mathematics of students is the signs offered in a dialogic space opened up between themselves and the voices inherent in the questions and the voices of teachers and other influences on their mathematics prior to the moment when they engage with questions. I will continue to explore this idea in future analyses.

There are elements that still need developing in order to allow me to focus on applying a Bakhtinian and Vygotskian analysis. Firstly, I need to discuss further why I have chosen the excerpts that I have. For the provisional analysis I have just completed, I looked for sections that resonate with the perspectives I am discussing, but this means missing sections out. I would like to more clearly define what I am missing out and what I am selecting for analysis. There are lots of excerpts I can choose to illustrate the theories I am investigating. The key issue is how I choose so that I can go a step further and, using my planned "comparing"/ "contrasting" networking strategy, look at the differences and similarities in analysis to continue to develop my understanding of the two perspectives and establish possible points of connection.

Secondly, I need to standardise my transcription style. Whilst the style of transcription I have used here preserves the flow of conversation between participants, it would be useful to have a clearer sense of when in the conversation excerpts have been taken. This is partly due to the significance of context and time in both a Bakhtinian and Vygotskian approach, but also so that it is easier for me to refer to particular exchanges when I am analysing several at once.

### 6.5 The Unit of Analysis

I first discuss the unit of analysis in sections 2.1.1, 2.2.1 and 3.1.2. The notion of unit of analysis I am using is similar to that of Ernest (2016). He explains that "as well as serving as a methodological focus it is intended that a unit of analysis should be a prototype or microcosm that represents the key relationships as well as the entities of a study. I term this the ontological use or meaning of the term" (Ernest, 2016, p. 40–41), which is similar to the significance Radford places on the links between the principles and methodology of theories. Due to the dual theoretical stance of the project, the unit of analysis for Vygotsky and for Bakhtin need to be clearly defined and methods planned that take the unit of analysis into account. Commonly quoted units of analysis are word meaning for Vygotsky and utterance for Bakhtin (see my discussion of Wertsch's view, section 3.1), but Matusov

(2007) suggests that defining the unit of analysis is more complex and needs careful consideration. In accordance with Matusov's dialogic standpoint, his ideas around the unit of analysis aim to avoid holism and suggest that "due to its incomplete nature, the unit of analysis cannot be fully known before the research is started" (Matusov, 2007, p. 328). Matusov suggests that the unit varies depending on the participants, researcher and audience of the research. I need to carefully define my unit of analysis in a way that is specific to my own study.

Vygotsky defines the unit of analysis as that which is "further unanalyzable and yet retains the properties of the whole" (Vygotsky, 1986, p. 5). He suggests that "such a unit can be found in the internal aspect of the word, in word meaning" (Vygotsky, 1986, p. 5). Ernest (2016) discusses Vygotsky's early sense of word meaning as his unit of analysis but explains "tool-mediated action serves as his later and more developed unit of analysis" (p. 41), echoing Wertsch's interpretation of the unit of analysis for Vygotsky (see section 3.1.2) as the "individual(s)-operating-with-mediational-means" (Wertsch, 1991, p. 96).

In Chapter Two (section 2.1.3), I discussed the utterance as Bakhtin's unit of analysis where a word or combination of them acts where "an intense interaction and struggle between one's own and another's word is being waged, a process in which they oppose or dialogically interanimate each other" (Bakhtin, 1981, p. 354). Bakhtin is clear in his later writing that "the boundaries of each concrete utterance as a unit of speech communication are determined by a change of speaking subjects, that is, a change of speakers" (Bakhtin, 1986, p. 71). This is something captured by my transcription. However, I do not then simply focus in to attempt an analysis on individual speakers. Instead, I must remember that no utterance stands alone as "its beginning is preceded by the utterances of others, and its end is followed by the responsive utterances of others" (Bakhtin, 1986, p. 71). What I am trying to capture is the responses of others to an utterance, the sequence of utterances making up a dialogue and what that reveals about the tensions at play for the participants.

In order to make sure that my analysis holds true to these theoretical underpinnings, I must ensure my methods and analysis allow me to get a picture of these irreducible units of analysis. To put it another way, my level of zoom must be such that I am close enough to see the detail of the interactions between participants e.g., focusing on "individual(s)-operating-with-mediational-means" (Wertsch, 1991, p. 96), and far enough away that I do

not focus in on something smaller than the irreducible unit of analysis and therefore miss the reflection of the bigger picture provided by the unit of analysis. In the context of my study, when selecting sections of transcription for analysis, it is important that I choose sections which allow me to view examples of the units of analysis. Defining the boundaries of these extracts is important: too much discourse and the extract becomes unwieldy; too little and I will not get a clear picture of the tensions and interplay between participants. Using Bakhtin's notion of boundaries to an utterance could be extended to help here. Bakhtin's notion of utterance includes the property of finality; the idea that this change of speaking subjects happens when a speaker "has said (or written) everything he wishes to say at a particular moment or under particular circumstances." (Bakhtin, 1986, p. 76). Therefore, choosing an excerpt that covers one topic would fit with this criterion. From a Vygotskian perspective, could selecting based on a topic be seen as a discussion around one mediational tool used by a student? Or one situation in which a variety of mediational tools could be employed with a discussion about which has been selected? Focusing on one topic of discussion would reflect the particular challenges of my situation, where I am analysing teacher and student group discussions that involve their own tool/sign use, although the discussions themselves are based on students' use of tools/sign use in the artefacts. I discuss how I made my selections in more detail in section 7.1 with specific details in each section of 7.2.

### 6.6 Transcription Conventions

Having discussed the way in which I select sections of transcription for analysis, I want to look in more detail how I transcribe my audio recordings. It is from these transcriptions that I will be making my selections and there are some key decisions to be made about style and notation so that the transcription itself facilitates my analysis and reflects the theoretical positions I am drawing on. I have carried out an extended discussion of transcription conventions based on Ochs (1979) and my choices using an extract from the start of Teacher Group Session 2b and included the discussion as Appendix Five.

I tried four different transcription styles before arriving at the conventions detailed below. The decisions made were: to use contribution numbers rather than line numbers; to use full stops to denote a one second pause, with pauses under one second marked by (.) and over three seconds by, for example, (4) for a four second pause; to remove capitalisation at the start of sentences as this implies punctuation; to use a – to denote part of a contribution

has been missed out; actions will be noted in my transcriptions if they are audible on the tapes (italicised and right justified), if other actions are necessary to illuminate the discourse further, then these will be included in subsequent discussion. Putting this all together, I am going to attempt a final transcription of the section I used in my exploration in Appendix Five to summarise the discussion:

1 R: alright we're recording . is everybody happy to be tape recorded

2 N: yes

3 R: I need a verbal one sorry . I kn- stop it *laughing*

4 K: yes

5 R: thank you

6 K: I'm absolutely happy

7 R: ok we've got another teacher possibly joining us in a second (.) um so quick recap 'cause it's been a while

**Transcription Attempt Four**

My plan is to make sure that, in my initial transcriptions (presented in the appendices), all participants have been made anonymous and contribution numbers are in place. I will then use the transcription style I have used here in Transcription Attempt Four when I present extracts of transcripts for analysis.

### 6.7 Moving Forward

This chapter has summarised the early stages of my methodological exploration into analysing data using Bakhtinian and Vygotskian perspectives. I have shown two early attempts at analysis and highlighted aspects that I would like to take forward into later analysis as well as those that still need work. I have also carried out some technical tests based on areas of these early analyses that I did not find satisfactory in order to improve later attempts.

These technical tests have allowed me to explore the implications of the choices I am making with selecting excerpts and how I present my transcription. However, they also

raise theoretical questions. For example, as part of my exploration of transcription techniques (Appendix Five), I have considered Ochs's discussion about presenting transcription in such a way so as not to bias gesture over words (1979). The discussion led me to consider a bigger question of how I present the two authors I am working with. Does my choice of who to write about first in a discussion or analysis (for example I wrote about Bakhtin before Vygotsky in Chapter Two) introduce bias for me or for the reader? Many authors writing in the field present Vygotsky before Bakhtin. For those presenting the theories as linked and Bakhtin's work as an extension of Vygotsky's this makes sense but for those arguing that the two are separate theories, what is motivating that choice and is this a convention I wish to follow?

My next steps will be to apply the strengths of the analysis I have explored so far to the remaining teacher groups, whilst continuing to develop my analytical style in line with the theories I am using. At the start of the next chapter, I explain a more formal framework I am going to introduce in order to support this.

## Chapter Seven – Methodology: Teacher Groups

In Chapter Six, I explored the early stages of a methodology for analysing perspectives based on Vygotsky's and Bakhtin's theories. I explained my basis in Radford's connecting theories and my intention to initially adopt a "comparing"/ "contrasting" approach after Prediger et al. (2008) as a possible networking strategy at the level of connecting methodologies. As I explained in section 6.4.8, the discussion of connecting points and differences of key characteristics was particularly useful in my trial analysis. This chapter starts to develop the initial ideas into a formal analysis. To explore this, I begin by explaining the differences between my plans for the teacher groups and what took place. I will then summarise my findings from the last chapter to show how they guided my analysis. I will then look at the two teacher group sessions I have not already analysed (the first was used in Chapter Six, see section 6.4). Finally, I will discuss what I have found and what I wish to take forward to the analysis of my student sessions.

Originally (section 5.2.2), I planned four sessions with the last two having optional focuses based on the outcomes of earlier sessions (see Figure 5.5). Figure 7.1 below has been updated to reflect the sessions that took place:

Teacher Group Outline

Session 1
Blank copies of artefacts. Teachers complete:
- as if no one would see
- as if you were demonstrating to a class

Session 2a
Discussion with teacher group with some of those who participated in session 1.
Student methods – from artefacts.
- What's the same?
- What's different?

Session 2b
Discussion with teachers that could not make session 2a.
Student methods – from artefacts.
- What's the same?
- What's different?

Figure 7.1 – Updated teacher group session outline

Figure 7.1 shows the third and fourth sessions were replaced with two different versions of Session 2. Teacher Group Sessions 2a and 2b featured different staff members and ran at different times given the availability of staff. Staff involved in the study were under a lot of pressure in school at the time and I wanted to accommodate everyone who wanted to take part in the study, so modifications were necessary. I labelled the sessions 2a and 2b to reflect the fact that both sessions focused on the student artefacts.

To analyse Teacher Group Sessions 2a and 2b, I am going to draw together the analytical points made in Chapter Six to form a working framework. The framework will act as a reference point throughout my analysis, allowing me to evaluate my efforts more effectively.

### 7.1 A Working Framework

In Chapters Three and Four, I explained the interpretations of Bakhtin's and Vygotsky's theories by others in the field. Then, in Chapter Six, I used Barwell's (2015) interpretation of Bakhtin's and Vygotsky's positions to trial a preliminary analysis. However, to explore the methodological aspect of networking theories, and hence the methodology of my study, I need to summarise my own interpretation of Vygotsky's and Bakhtin's theories so I can be consistent in my application of the theories through my analysis.

In Figures 7.2 and 7.3 below, I have summarised what I see as the key aspects of both a Bakhtinian dialogic and Vygotskian dialectic perspective to act as an analytical framework. The summary serves as a reminder of the underlying principles of each and will help me relate my analysis or any questions that arise back to the key theories. The aim of the summary is to be practical by focusing on key principles, whilst still attempting to avoid reducing complex theories to a few bullet points. As a result, rather than treating the lists as a complete overview, they act as a guide, a starting point that still allows free exploration as I use "comparing"/ "contrasting" networking strategies as part of my analysis.

| Vygotskian Dialectic Analysis | | | |
|---|---|---|---|
| **Feature** | **Detail** | | **Links** |
| V1: Difference | Theories or ideas are in opposition and must be overcome.<br>Learning awakens developmental processes that occur with others and when internalized gives rise to independent development brought about by environment. | | Wegerif, 2008, p. 359<br><br>Vygotsky, 1978, p. 90 |
| V2: Internalization | An idea is experienced collectively and then individualised as it is internalized. Formal mathematical language is part of a cultural tool that students need to internalize so they can master the concepts. This process of internalization is the overcoming of difference and affects your identity.<br>However, internalization is not always fully realised and Wertsch, along with his toolkit analogy, suggests mastery as an alternative framing. | | Section 2.2.2<br><br>Section 3.1.2 |
| V3: Development | The overcoming of opposition and mastering of higher concepts through internalization marks a progression or development towards uniformity or the idea of "Universal Reason".<br>These higher levels affect earlier, less developed ideas.<br>In a school setting this is often seen as the teacher guiding students from informal to formal or standard mathematical language/methods. | | Section 3.5.1<br>Vygotsky, 1986, p. 203<br><br>Barwell, 2015, p. 4 |
| V4: Hierarchy | The idea of development towards an ideal means those more developed have power over those less as they influence them via the ZPD.<br>In a school setting, teachers use this power to lead students a particular way. | | Section 2.2.3<br><br>Section 4.2 |
| **Benefits** | | **Issues** | |
| It echoes the set-up of the National Curriculum with its ideas of progress, development, and mastery. | | Its notion of progression sets formal and informal mathematical language in a hierarchy and thus devalues the work of those who have not mastered formal methods | Matusov, Section 4.2 |
| It includes the idea of power relations between students and teachers and between the government and schools. | Matusov, Section 4.2 | Analysing the process of internalization requires returning to the starting point of development and, in practical terms, this not possible. | Vygotsky, 1986, pp. 235–236 |
| | | Argument against perfect scientific being. | Wells, Section 3.3.2 |

Figure 7.2 – Key features of a Vygotskian dialectic analysis

| Bakhtinian Dialogic Analysis | | | | |
|---|---|---|---|---|
| **Feature** | **Detail** | | | **Links** |
| B1:<br>Other | Interaction with the other or alien word means that differences are in dialogue with each other, rather than in opposition. One is not trying to make meaning by overcoming the other. Instead, the difference itself opens the possibility of meaning making.<br>The other not only allows for meaning making, but the difference between authoritative voice and internally persuasive word is key to "an individual's becoming". | | | Section 4.1<br><br>Bakhtin, 1981, p. 342 |
| B2:<br>Meaning making | Meaning created in the difference between self and other does not contribute to an ongoing progression or development in a linear sense. Instead, it is an "active understanding, one that assimilates the word under consideration into a new conceptual system, that of the one striving to understand".<br>In a dialogic approach, the aim of a lesson is to expand repertoires of mathematical language and build a wider range of ways to make meaning. | | | Bakhtin, 1981, p. 282<br><br>Barwell, 2015, p. 13 |
| B3:<br>Centripetal/Centrifugal forces | Representing the ongoing, non-binary tensions present in each utterance between a unitary language and the diversity of multi-voiced heteroglossia present in all speech that "makes it possible for students to express mathematical ideas meaningfully".<br>In a school context, unitary language may be framed as formal mathematical language but for Bakhtin, the definition of unitary language depends on the context. | | | Barwell, 2015, p. 8 |
| B4:<br>Speech genres | Act as unifying forces that develop within a group, for example, a particular profession or family, allowing a shared basis for meaning making.<br>Wertsch sees speech genres as mediational means. | | | Section 2.1.3<br><br>Section 3.1.2 |
| **Benefits** | | | **Issues** | |
| Tensions and speech genres are a practical way of working with the historical meaning and connotations brought to a situation by the participants. | | Section 3.1.2 | Some writers argue that Bakhtin's approach does not address power relations, but I would like to explore this as I have discussed Bakhtin's ideas about the authoritative voice, which clearly references power relations in exchanges. | White, Section 4.3<br><br>Section 2.1.5 |
| | | | No linear concept of progress/development. This goes against how what is happening in a school is conceptualised, particularly if I am focusing on methods chosen by students for examinations, which focus on demonstrating progress/development. | Barwell, 2015, p. 3 |

Figure 7.3 – Key features of a Bakhtinian dialogic analysis

In previous chapters, I began to develop a framing of mathematical methods as an example of written language. In section 6.4.8, I discussed Wegerif's idea that, if a sign is acting as "mediation between self and other" (2008, p. 353) then a dialogic space must be present for meaning making (B2). I am exploring the possibility that mathematical methods could be considered to be examples of such signs. So, a Bakhtinian analysis can be applied to the methods under discussion in the groups as well as to the discussion itself. Through this chapter, I am going to continue to test analysing not only the discussion of the teachers but also, where appropriate, analysing the methods under discussion as well.

In sections 7.2 and 7.3, I am going to use the framework above to analyse the transcripts of Teacher Group Session 2a and Teacher Group Session 2b. Full transcripts are available as Appendices Seven and Eight respectively. Following my discussion in section 6.5, I read through the Teacher Group Session transcripts from both a Bakhtinian and Vygotskian perspective and then selected extracts which allowed me to access the corresponding units of analysis. From a Bakhtinian perspective, I have selected sections of transcript that include an exchange of utterances trying to give enough text to trace where the interplay between those speaking makes meaning through the theme of a discussion. As such, when a theme changes or ends, the extract ends. From a Vygotskian perspective, extracts have been selected that allow discussion of one or more mediational means being employed, or a discussion of how mediational means may have led to mediated action. As mention of a particular mediations means may form part of a longer thematic conversation (which formed the start and end points of the Bakhtinian criteria for selection), there may be several examples in one extract shown here such as in section 7.2.1 where this is discussed in more depth. From the extracts I analysed, I have selected for inclusion those that allowed me to test the framework above or that highlight ideas I may have overlooked.

To structure my analysis, I have taken each section of transcript and looked at it first from a dialectic (Vygotskian) then dialogic (Bakhtinian) perspective using the key terms and ideas from the framework above as a guide. Where appropriate, I have also tested my dialectic analysis of mathematical methods alongside a dialogic one. To make it easier to link my framework with my analysis, I have coded each key feature, e.g., V1 for the first feature of a dialectic analysis.

### 7.2 Teacher Group Session 2a

In my analysis of Teacher Group Session 2a, I have identified extracts that focus on the influence of the curriculum, interpretation of different methods, incorrect methods, and the role of laughter.

#### 7.2.1 The Curriculum

During my analysis of Teacher Group Session 1 (section 6.4), I wrote about the tension in the discussion created by the demands of the curriculum on the teacher as well as the student. I am going to revisit the idea using the more detailed framework outlined above. The following extract is a discussion about Question One from the artefacts (Figure 5.2). From a dialogic perspective, this extract shows the meaning making taking place between two teachers, one of whom is reacting to a student method with knowledge of the examination mark scheme, whereas the other is not and how they use their varied positions to make meaning with one another but also with the student method. From a dialectic perspective, the teachers are using language as a sign system. One teacher is more knowledgeable on a particular aspect of the examination expectations and this teacher is linking their discussion to a variety of other tools and sign systems such as the examination mark scheme and the student method to explain how this particular student method does or does not fit the curriculum expectations:

133. R: see . I think this is really interesting look . what would you say to that as an answer for the first question it's fifteen point five as opposed to fifteen point fifteen point five zero
134. M: it is fifteen point five
135. R: you lose a mark in the GCSE now
136. M: do you
137. R: yeah so there was a question on one of the papers we looked at which was that bill question where you had to like you were debited this amount so you had to do it and then credited this amount or something . it was the foundation one and um . and a lot of mine 'cause what they got on their calculator at the end was like was like point two or something so they wrote point two not point two zero and they lost a mark for it

138. M: yeah, yeah I mean I always tell mine if it's money put two decimal places so that's interesting that they lose a mark now because actually that is that is fine it is fifteen and a half pounds so that is the same so they shouldn't really lose a mark
139. R: no it's the same at A level they have to have it to two decimal places .
140. M: what when they're doing money
141. R: yeah . always
142. M: fair enough . that is how money works I suppose .
143. R: yeah . it just seems harsh
144. M: It seems it seems . it should be explicit first of all in the question that that's what they're expecting . because it doesn't actually mean they don't understand money if they've written fifteen point five pounds
145. T: yeah
146. M: like they can understand perfectly well that fifteen point five is fifteen pounds fifty
147. R: yeah it's going the other way isn't it that when they write something like fifteen point five and then some of them in- interpret it as fifteen pounds and five don't they
148. M: yeah . that's true . yeah
149. R: when you see them go back the other way
150. S: somebody had written fifteen oh five as their answer
151. R: yeah
152. S: I think they'd just done the bus stop wrong . -

From a dialectic perspective, one teacher is holding the expectations of the examination mark scheme as the standard for mastery (contributions 135 and 137). The teachers discuss passing on to students the importance of using the standard presentation of the results (contribution 138) but recognise that students are still working towards use of the formal language, in this case, the requirements of the mark scheme under discussion that is being used to assess student progress and development (V3). However, teachers recognise anything informal is considered, by this mark scheme, as less good, labelling the requirements "harsh" (contribution 143).

From a dialogic perspective, the teacher discussion highlights a tension for teachers between the examination board's expectations regarding the presentation of solutions to a

context-based question and the fairness of this for students. The initial question is phrased to ask about rounding (contribution 133) but is quickly linked to an examination context (contribution 135) by the same teacher who continues (contributions 137 and 139) showing the focus on examination expectations when considering this question. Instead of focusing the discussion around rounding, the emphasis and language have rapidly shifted to discussing marks and examination papers. However, there is simultaneously a pull towards the welfare of the students in the discussion, with a second teacher (contribution 138) explaining that, despite their classroom practice being to remind students about rounding, their comment is more focused on rounding due to the context of money, rather than the examination expectations. The second teacher then suggests, "they shouldn't really lose a mark" (still contribution 138), implying that it is unfair. The first teacher then responds with "it just seems harsh" (contribution 143), seeming to react to the second teacher moving the focus away from the examination expectations. The tension between examination board expectations and what is fair for students is embodied here through the two teachers and their shifting language demonstrating the responsivity of the participants to the context and previous contributions.

If I proceed with analysing mathematical methods dialogically, then I could say that the teachers have identified the necessity for students to recognise the specific speech genre (B4) represented by the examination question. Gerofsky (1996) suggests "a description of the mathematical word problem as a linguistic genre" (p. 36), and, in a later article, links the worded problems specifically with Bakhtin's concept of speech genre (B4) by suggesting they have a recognisable "form and addressivity" (Gerofsky, 1999, p. 37). In the case of Question One, the speech genre (B4) around worded mathematical problems shapes contributions, including an expectation that solutions are presented in a certain way. Here, the teachers have recognised that the expectation is not stated by the question (contribution 144) and, as such, students would need to have prior experience of similar questions to understand this requirement. Using a dialogic analysis implies that examination questions carry the voices of others (B1). Those who decide education policy, those who have interpreted the policy for individual examinations and those who write examination questions are all represented in the wording of examination questions. In this specific circumstance, those voices include me as researcher in my design of the artefacts.

The curriculum, as other analysis in this chapter and the next will show, is a significant shaping factor in the discussions between teachers and students. From a dialectic perspective, the curriculum has influence over what methods are taught when and how they are to be presented. From a dialogic perspective, the curriculum is the source of a tension that helps define formal and informal mathematics based on individual classroom contexts.

### 7.2.2 Different Methods

Continuing my interpretation of mathematical methods using a dialogic approach led me to the next example. From a dialectic perspective, this extract illustrates the variety of mathematical tools and signs employed by students and the teachers using language to make judgements on the methods they have employed. This extract is particularly interesting as it deals not just with the variety of methods but also with the teachers' emotional response to those methods. From a dialogic perspective, this extract shows teachers' making meaning with a variety of student methods. If I interpret mathematical methods as a type of language, then the following extract is a good example of how multiple methods for Question One (Figure 5.2) could represent heteroglossia:

43. R: - I really like question one for like . loads of random methods . so this person has written out like their four times table
44. T: OK?
45. R: to try and get to sixty two . this person's done . like bus stop method . and then tried to check it afterwards
46. M: sounds cool
47. R: and not got the right total
48. M: got it wrong
49. S: some have gone remainder two and then gone it's twenty pence
50. R: so haven't understood
51. T: tallies
52. R: I've got this look
53. T: oh wow
54. M: that makes me so sad that someone's had to do that to work out sixty divided by four
55. R: did they get the right answer

56. S: no . one person will get sixteen and the rest will get fifteen .
57. R: OK so they
58. S: which is kind of
59. R: they haven't shared equally but also miscounted
60. M: they miscounted but it's such a stupid way of doing it.
61. R: yeah but there's a lot of them . if you look through then quite a lot of them have done tallies
62. T: tally charts yeah have done it right
63. S: yeah
64. R: for question one which really surprised me and you'll notice
65. S: this one has halved and halved again which is quite nice
66. R: yeah that's quite nice
67. S: to do four
68. M: that's surprise um what level is that that yellow paper
69. R: it varies so these are two different year groups completely different year groups and if you find one on a different colour *tapping on sheets*
70. M: yeah because the good thing about it is they've clearly understood what sharing sixty out is about if you see what I mean so they've conceptually understood it but it's really sad that they've got no skills whatsoever for actually making that happen
71. R: what would you like expect to see if you kind of gave them that sort of problem . like
72. M: Bus stop method . or just the answer . it's such a simple problem

In the first session (section 6.4.1), I analysed teachers discussing the methods they used for Question One in their own completion of the artefacts. The teachers mentioned "half and half again" and "bus stop… short division" among their approaches. In this extract, teachers are listing different methods students have used in their approach to answering the same question. What is striking from this extract are the methods identified by teachers that were not in evidence in the methods used by the teachers themselves, such as, writing out the four times table (contribution 43) and tallies (contributions 41 and 51).

One teacher has a specific method they might expect to see from a student (contributions 71–72), but the group have identified that many students have not selected that preferred method for their attempt at answering the question. Some of the alternatives students have

used are methods perceived as less formal, or have not led to the correct answer, provoking emotional reactions such as shock (contribution 53), sadness (contribution 54) and even disdain (contribution 60). A dialectic perspective might frame these teacher expectations as fitting with the sense of progress and development (V3) goals for students set by the curriculum. Teachers are expecting students to use methods that correspond to later development stages (V3) as dictated by the curriculum, for example, using the bus stop method over tallying. The fact that these students are using techniques from earlier stages (some successfully) is considered inferior to the use of more advanced methods. The emotional response, such as "it's really sad" (contribution 70), to students struggling with these techniques may reflect the teachers' understanding that these students are struggling to conform to the expectations of the curriculum and that, ultimately, they will struggle, in lessons, examinations and the wider world, as a result. The dialectic method of analysis has an advantage over the dialogic here as it does provide a reason for the emotional aspects of the teacher's reaction.

If I extend my analysis of mathematical methods from a dialogic perspective, the discussion again highlights the influence the centripetal force (B3) of the curriculum has had on the teachers involved. The teachers have picked up on the tension from the curriculum (as in section 7.2.1) that draws those working with it towards particular methods (contribution 72). Through their discussions, specific examples have illustrated that, working in counterpoint to this tension, is the heteroglot variation in mathematical language students have employed as they attempt to create a solution from the tensions they are working within i.e., the methods they have used. The voices of different students are evident in the different methods they have employed, including writing out a times table (contribution 43), tallies (contribution 51), and halving and halving again (contribution 65). The way the teachers engage with the heteroglossia of student methods, as they attempt to interpret the students' process of making meaning from the question, has raised surprisingly emotional reactions.

A teacher preference for a particular method or approach is a theme also in evidence when discussing Question Three (Figure 5.4). The extract below shows teachers contrasting two specific approaches to Question Three. From a dialogic perspective, the extract shows teacher discussion of two different meaning making methods employed by students. From a dialectic perspective, it shows the teachers using language and examples of student

methods to discuss two specific uses of mediational means by the students – algebra and trial and error:

106. M: quite a few people seem to have just divided the twenty seven by three . and then gone
107. R: 'cause they worked out there were three of them
108. M: that one must have been roughly that and then fiddled about a bit and worked out the answer
109. R: which is not horrendous
110. M: it's just . yeah it's such an interesting approach because that seems . like extra work to me to just start off guessing and then we'll go from there I suppose
111. R: do you think
112. S: she's done trial and error
113. T: that is right actually that does work
114. R: so whoever wrote Alice is twelve for the ones we were looking at earlier
115. S: Alice is twelve seems to be the right answer yeah
116. T: so how've they expressed that right . they've looked for who the youngest person is and given them an *x* value which I think's really nice .. and then you don't end up with lots of different letters
117. R: so so we're saying that the trial and error one just seems like it might be a lot of extra work compared to writing it as algebra
118. M: yeah because we've got good algebra skills so to us algebra is a quick way of solving problems

From a dialectic perspective "fiddled about a bit" (contribution 108) is considered "extra work" (contribution 110), suggesting this teacher may consider efficiency of method a sign of a more developed mathematician. The teacher could be interpreting efficiency as a higher level method, which echoes the idea of development (V3) from my framework. The efficacy of a method could be an indicator of someone who has internalized (V2) more of the concept by mastering better (in this case more efficient) tools and suggests a hierarchy (V4) of methods in the eyes of the teachers.

From a dialogic perspective, the use of "trial and error" (contribution 112) or "fiddled about a bit" (contribution 108) are the teachers' interpretations of the ways that students

are creating meaning with the question. With their "not horrendous" (contribution 109), the teachers recognise that students have brought their prior experience to bear on this particular problem, attempting to put together a solution that shows responsivity to the other (B1) who will be interpreting their solution. However, the teachers also acknowledge a difference between their prior experience and therefore approach to meaning making (B2) and that of the students, acknowledging that the teachers bring their prior experiences with algebra to the situation (contribution 118). As I mentioned in section 6.3, Barwell talks about the definition of formal mathematics changing based on the context. As teachers talking with other teachers, algebra is framed here as the formal method. For the students engaging with the problem, trial and error (known as trial and improvement in the curriculum) may have been the most formal method they know if they have not yet experienced the algebra required to solve this type of problem. As we saw in section 7.2.1, the teachers themselves may choose to hold trial and improvement as the formal method for students. This tension between algebraic and numerical approaches is clear for teachers (e.g., contributions 117–118). The numerical method may appeal to students who struggle with algebra, but the algebraic method is more efficient. Is this a distinct tension? Or merely a version of the formal/informal tension I have already discussed?

As part of the discussion about different methods, the teachers discuss Question Two (Figure 5.3) and how they select from the range of methods they are familiar with. The extract below is a specific example of how their prior experience might guide their meaning making or, alternatively, how prior experience might influence the mediational means selected:

157. R: I liked this one here which has got um .. they've turned it into a rectangle
158. M: yeah I've seen one like that yeah
159. R: so they've rewritten it as ten times five and then they've worked out the triangle and taken it away
160. M: I often when I'm teaching have people say why don't you just work out the whole thing and take that bit away if the rectangles this bit you've got a missing thing as well that's a
161. R: yeah it's really nice isn't it
162. M: really common intuition that people have

163. T: see I would probably trust doing this way more than I would the trapezium rule because I would look at it and go it kind of is a trapezium but is it . it's got a flat edge .. I'd sort of start questioning myself I don't know

164. M: yeah there's another one like that there .. it is it is a trapezium but the trapezium rule doesn't have the same sort of intuitive value in a way does it as adding a rectangle and a triangle and I mean you try and explain that you're finding the mean of the two sides they just glaze over ..

From a dialectic perspective, the teachers are discussing a range of methods or mediational means that could be considered as a hierarchy (V4). The two methods of finding the areas of separate easier shapes and then combining them to find the area of a trapezium would have been more accessible. Of these, finding the area of the square and the area of the triangle and adding them together could be considered more straightforward, whereas finding the area of a large rectangle and then subtracting the area of a triangle is harder for students to visualise and, as such, is seen less commonly. The third method the teachers discuss is using the trapezium rule, the formula for the area of the trapezium based on finding the average of the two parallel sides to reconfigure the shape as a rectangle and then multiplying this by the perpendicular distance between them. The formula is more challenging, firstly to remember and secondly to implement. What is particularly interesting about this extract is how the teachers frame themselves as not always being confident with selecting the trapezium rule as their method (contribution 163), which is a contradiction to a dialectic framing of teacher as expert. The teachers know the area of a trapezium rule and how to use it so would be considered to have mastered the tool. However, they are second-guessing themselves in their application of the formula to this type of question.

From a dialogic perspective, there is a tension between the intuitive methods of the teachers and their understanding that there is a standard method for approaching this topic, i.e., the use of the area of a trapezium rule. It echoes a tension acting on students but for different reasons. For students, the tension is acting to draw them towards the more formal method as they extend their range of ways of making meaning. They are working to appropriate the area of a trapezium rule. Teachers have theoretically already assimilated/appropriated this method but make it clear that this is not always their choice of method for approaching the question (contribution 163). So, why the tension? Perhaps

understanding is not enough to influence choice of method, but confidence is needed too. The discussion also implies this tension impacts on how teachers discuss this rule with students. Contribution 164 explains the difficulties of teaching the area of a trapezium rule, "I mean you try and explain that you're finding the mean of the two sides and they just glaze over", rather than making use of more "intuitive" methods.

The tension between intuitive methods and a standard method for teachers can be linked back to the tension between informal and formal mathematical methods. Informal mathematical language is represented by the heteroglossia of multiple methods and formal by the more unitary expectations of the examination boards and curriculum. So far, I have identified a few tensions but most of these could be seen as represented by the overarching tensions between the curriculum and informal methods. I suggest that part of the reason for this link is that the context I am investigating is that of discussion around examination questions. This does not mean other tensions are not present, just that they are harder to identify.

### 7.2.3 Incorrect Methods

In the previous section of this chapter (section 7.2.2, contributions 43–72), I have used teachers' discussions of the variety of methods used by students to frame conversations about the benefits of one method over another. In contrast, in the following extract, teachers are working hard to make sense of the methods that students have used but are struggling to see why students have written what they have. The extract is one of the only examples of teachers being unable to make sense of the methods of the students:

225. S: it's interesting isn't it . just looking at the area one there's just these random . there's some here that you just can't even . can't see what they've done
226. R: yeah this is it in some of them you have no idea where the numbers have come from.
227. S: this one's managed added together . fifteen five six and four .. to get thirty . I'm not even sure where they've got those numbers from .
228. R: no I I have nothing for you for that one
229. M?: so that's five
230. R: and that's six and there's ten at the bottom but I mean fifteen have they added those two

231. S: fifteen and a four
232. R: have they added that and then thought that was four
233. T: and the width is four there
234. R: yeah that's where the four's come from
235. T: so have they just taken all of the bits and added them up
236. S: I don't know this one's kind of randomly said that's seven and then . twenty eight from somewhere
237. R: seven times four
238. S: maybe that's what they add up to they've added done a perimeter maybe but they've randomly said seven . maybe they went five six oh I'll put seven
239. T: yeah, noticing a pattern
240. R: but it's interesting that even with the ones that are wrong you can kind of see where some of them have gone wrong and why they're wrong and some of them it's like that's just a number I have nothing I have no idea where your brain was when you did this

The students whose methods are under discussion here know that the question requires something of them, that it has something to do with the numbers on the picture and that it requires some sort of mathematical process to get there. However, I am analysing "a teacher's constructions of views of students' mathematics" (Morgan & Watson, 2002, p. 88). The teachers reading these methods are struggling to interpret the steps the students have made to write down a solution.

From a dialectic point of view, a student working on the problem may already have several tools at their disposal such as being able to multiply, add, and make sense of numbers or units. All of these tools would have been required to even begin to engage with the problem. The fact that the students have written something but have not fully solved the problem would indicate that they have not mastered the required concepts and are still working on the process of internalizing (V2) an effective method. Here the student is individual(s)-operating-with-mediational-means (Wertsch & Tulviste, 1992, p. 555), but, despite the mediational means at their disposal, their actions are not sufficient to complete the mathematical task. Guided by their understanding of standard methods and their overview of the progress expected of students, teachers are trying to find examples of recognisable tools or methods (contribution 238) that would indicate developing

understanding. The teachers here are trying to sort through the mediational means in order to work out the actions the students were attempting however, they do not have the information required to make sense of the student attempts. It would be necessary to talk or interact further with the students who selected these mediational means in order to further understand what their intentions were. In this case, the students would know best about their own methods. I discuss this idea in more detail in section 7.3.2.

From a dialogic point of view, the student is working to make meaning from the difference between their previous experience with similar problems and this question, with its echoes of different voices (their teacher, the expectations of an examination-style question and my voice as researcher). The written mathematics shows their attempts but could indicate that the student does not have enough experience of similar problems or that there was not enough diversity in their previous experience and thus the methods they have available to them. Teachers in this discussion are working to make their own meaning of the students' understanding by tracing the students' own attempts at meaning making (B2) from the written methods. In discussing what they see of student methods, the teachers are revoicing the students' attempts to make meaning. Layering multiple voices (those represented by the examination-style question, the other teachers in the discussion and that of the student represented by the method) could be a way for the teachers make their own meaning from the student methods. The teachers are working to make meaning in the difference between their experience, suggesting ways to approach the problem and the student's. The extract above is an example of what happens when the interpretative process breaks down. There is not enough shared prior experience between student and question or teacher and student's method, which could be explained as not enough familiarity with the speech genre (B4) for the student and teacher to have created a shared common ground for meaning making (B2). The student has written down what they have for a reason but there is not enough commonality for the teacher to make meaning from their contribution. The sense of other (B1) is strong in this extract but so is the amount of work going on to make meaning in these gaps.

It is worth noting that the work the teachers are putting in to interpret student methods as evidence of their mathematical thinking is based on the idea that the students have intention behind what they are writing down. The aim of the students is to take steps towards a solution. What is to say that the students did not panic and write down the first

thing they saw? The teachers have taken the work as genuine attempts at solving the problems and analysed accordingly. Mortimer and Scott said "there is always the possibility that the student finds what the teacher is saying unfamiliar, and struggles to relate it to their existing understandings. In such cases, teaching and learning become more demanding" (2003, p. 10), which is in evidence above. In the next section, I am going to analyse a discussion prompted by a slightly less straightforward student intention.

### 7.2.4 Laughter

The following extract is an example of a discussion based on one or two specific artefacts that were singled out in each group when they were used. From a dialogic perspective, this is an example of a series of exchanges where teachers make meaning with one specific student method. From a dialectic perspective, the extract includes teachers using language as their mediational means in trying to make sense of a particular set of signs and tools used by one student operating in a very particular way with mediational means:

165. T: someone's split this one into
166. M: crikey
167. S: oh my goodness
168. M: why did they stop at four
169. T: …needlessly into five parts
170. R: do you know why do you know why that one 'cause I can tell you that was one of my
171. T: "sound as a pound"
172. R: that was one of my [year group] because I told them I was interested in . different methods so they have gone out their way to do weird and wonderful methods for that question…
173. T: so five segments.
174. R: …and I saw it and I was like you so yes, so they've split the rectangle into four smaller rectangles and then decided to .. I was . not thrilled . I mean it was very funny .. which is what they were after

Without the additional context provided in contribution 172, the teachers are confused as to why anyone would approach Question Two in this manner. In section 7.2.2, I talked about how teachers view efficiency from a dialectic or dialogic perspective. Here, the

teachers have found a less efficient method from, theoretically, a more experienced student. To be clear, this student has not made a mistake, or got the incorrect answer but instead made a conscious decision to mock the process (albeit good-naturedly) by intentionally misinterpreting a comment I made as I talked to students before they started to complete the artefacts.

From a dialectic perspective, this student is using their understanding of the concept of finding the area to make fun of my explanation that my study was about mathematical methods. The student has demonstrated significant mastery of the concept in their choice of joke. Not only does the student have to have a good understanding of how to find the area of the shape, they also have to understand the requirements of the examination-style question they are attempting. The student has not just made something up to get a laugh, they have consciously identified one correct approach to answering this question and then heightened that method to the level of parody by making their chosen method inefficient to the point of being ridiculous. That is why it is funny and why the artefact has been picked out in the extract above.

By making this joke, the student is not just parodying the situation they are in as part of the research study, but also the requirements of the curriculum. It takes understanding to be able to step outside the framework to mock it effectively. The parodying here could represent resistance from inside the system, but the teachers have picked up on no malicious intent. Instead, the sense is that of an inside joke.

From a dialogic perspective, there is a clear influence of the centripetal force (B3) of the requirements of the curriculum in that the student has recognised the requirements of the question and selected a method to calculate a correct answer. However, in this example, the student has recognised the tension towards unitary language and has instead decided to toy with the idea of meaning making (B2) by selecting a method from their repertoire that is so unlikely to be used genuinely that this has become notable. The student is not only recognising the voice of the curriculum or examination board, but also my voice as researcher in the question. The student's awareness of the context has affected their approach to their choice of method. I would argue that this is one of the strongest examples so far of student voice being expressed through the mathematical methods they have chosen.

Bakhtin saw comedy as a feature that distinguished the novel from older, traditional forms of writing:

> In popular laughter, the authentic folkloric roots of the novel are to be sought. The present, contemporary life […] concepts were originally the objects of ambivalent laughter, at the same time cheerful and annihilating. It is precisely here that a fundamentally new attitude toward language and toward the word is generated. Alongside direct representation – laughing at living reality – there flourish parody and travesty of all high genres and of all lofty models embodied in national myth.
>
> (Bakhtin, 1981, p. 21)

In section 4.3, I discussed White's view that Bakhtin "paid little attention to relationships that are characterized by power and control" (2014a, pp. 227–228), but I suggested that Bakhtin's authoritative word gave a clear framing of power relations. Instead, I proposed a reframing of this argument that both Bakhtin and Vygotsky both lack practical suggestions to address the power imbalance present in the classroom. Vygotsky does not necessarily frame these power relations as something that needs addressing, but Bakhtin's authoritative word and the resulting closing down of heteroglossia (section 2.1.5) is seen as a negative. The presence of laughter is a specific way of bringing power relations to the fore and levelling voices as it:

> demolishes fear and piety before an object, before a world, making of it an object of familiar contact and thus clearing the ground for an absolutely free investigation of it. Laughter is a vital factor in laying down that prerequisite for fearlessness without which it would be impossible to approach the world realistically.
>
> (Bakhtin, 1981, p. 23)

Suggesting that only by "fearless" interaction can proper investigation take place raises the question of what happens to the intrinsic power relations present in any interaction between people? Specifically, in my study, what happens between students and teacher or between teachers? Matusov says that interaction "can be dialogical when the other consciousness is treated as having equal rights with one's own" (2011, p. 103). Does a lack of levelling suggest that without the breaking down of these power roles any investigation is not going to be credible?

White mentions carnivalesque in her writing as a specific form of laughter that Bakhtin is interested in through his study of Rabelais's work (2014c). Carnivalesque is a "temporary

suspension, both ideal and real, of hierarchical rank [that] created during carnival time a special type of communication impossible in everyday life" (Bakhtin, 1984b, p. 10). It can be described as authorised mockery as, although it ridiculed those in positions of power, it was permitted only on occasions such as feast days, as prescribed by those in control. According to Bishop (1990, pp. 49–50), paraphrased below, carnivalesque is:

- "Ambivalent" – uniting opposites, e.g., birth and death. It contains the idea of bringing the powerful down to earth, but a "fertile earth" that in some way grows our understanding, including the idea of crowning and uncrowning (p. 49).
- "Grotesque" – using low humour, for example jokes about bodily functions. The idea of the hideous made funny (p. 49).
- "Universal" – the comedy and laughter are directed at everyone, including the participants. They are laughing at themselves (p. 50).

These categories seem unlikely to be fulfilled in a classroom environment. The example, in the extract above, could be considered carnivalesque as it laughs at others as well as at itself. It also pokes fun at the power of the system (curriculum) in an officially sanctioned way – the student still answers the question. However, none of the grotesque elements are present here. Instead, Bakhtin's concept of parody seems a more appropriate term to use to describe what the teachers have picked up on.

In "the prehistory of novelistic discourse" (Bakhtin, 1981, p. 51), writing in a particular genre severely constrained authors and, as such, parody was an opportunity to hear the voice of the author. Hoy (1994) talks about the folkloric roots of football chants and how sharing these "ritual mantras" (p. 295) in a group of fans "establishes a kind of united self-identity" (p. 295) within subcultures. In the extract above, the student has intentionally misinterpreted my instructions and, through their parody, is making a statement about their identity within the context. Does the sense of identity through humour give an example of the element "of agency to dialogic learning that is missing in traditional behavioural and cognitive accounts of learning. This aspect of agency adds a normative or moralistic dimension in the theory" (Koschmann, 1999, p. 309)? Or does the way I can frame the laughter in a dialectic manner indicate that this is not unique to the dialogic approach?

Bakhtin says, "the process of parodying forces us to experience those sides of the object that are not otherwise included in a given genre or a given style" (Bakhtin, 1981, p. 55). So,

highlighting this example in discussion also adds to my understanding of speech genres (B4). Contrary to my earlier example (section 6.4.2), describing a speech genre is not a case of identifying the use of specific terms, i.e., teachers using the term bus stop for short division. Using an approach focused on individual terms as markers, speech genres quickly become based on specific contexts, such as, the speech genre of that school or even that specific group of teachers. Emerging here and from the discussion in section 7.2.3 is a way of thinking of a speech genre as a source of tension that is creating common meaning for participants by drawing communication towards forms/types/styles. What can be seen from the extract above is that an understanding of speech genre also allows for laughter. The shared understanding between teachers and student of the requirements of the speech genre are what allows the students' choice of method to be a talking point and a shared joke.

It is clear from discussing this extract that, despite my intention to follow a "comparing"/ "contrasting" approach with parallel dialectic and dialogic analyses, the dialogic interpretation has offered a greater depth of analysis. Due to the parallel approach, this is not offering an extension of existing theory as in more closely networked approach, but rather an extension of the depth of my own use of the existing theory, highlighting a possible gap or area of investigation. In this case, if laughter is a way of mitigating power relations for a dialogic investigation, what would be the equivalent in a dialectic one? The flexibility to move between analytical styles to provide a depth of analysis and the opening up of discussion around a point dealt with differently between the two theories is a key benefit of a connecting theories approach. I return to this discussion in section 8.2.3 where, in a similar manner, a dialectic analysis offers a greater depth of analysis.

### 7.3 Teacher Group Session 2b

In my analysis of Teacher Group Session 2a, I identified the significant impact of the curriculum on both students and teachers, looked at the variety of methods used for a question and how teachers make meaning from these methods, particularly how hard they work when the methods are incorrect. I have also begun to discuss the significance of laughter in unequal power relationships.

Teacher Group Session 2b echoed some of these themes. However, in the following sections, I have selected two extended discussions from the transcripts for analysis. These

are the discussions of student approaches to Question Two and Question One and raised opportunities to discuss classroom practice and the shifting sense of formal mathematics. The order in which the extracts are presented is the order in which the questions initially appeared in the discussion, linking to my ongoing focus on chronological presentation.

### 7.3.1 Classroom Practice

In section 7.2.3, I looked at teachers struggling to interpret student methods for Question Two. The participants in Teacher Group Session 2b also discussed the methods chosen by students to approach this question but, in the two extracts to follow, they go a step further than the discussion seen in previous extracts and suggest different ways in which their work in the classroom could address misconceptions:

48. K: not even sure what this one did we've got ten plus six plus five
49. R: they've just added all the numbers they can see
50. K: which is all the numbers they can find which is twenty one and then they've timesed it by four
51. N: because they know it's area.
52. K: so you times it by four it's bound to make area at least they've timesed it by something, but um
53. N: maybe it's the difference between the ten and the six they've multiplied by
54. K: maybe you're thinking slightly harder than they did!
55. N & R: *laughing*
56. R: yeah I've got another one there who's just added up the numbers and . another one there.
57. K: yes we have um well I think there's about . four to one who have done something attempting area as opposed to perimeter so that's a difficult problem
58. N: that's a valid point actually the the confusion between perimeter and area um how is there a best way to teach that so that people understand the difference between them

From a dialectic perspective, the teachers here are aware that students have not mastered the concept and are familiar with several ways in which student methods show this. For example, students adding up all numbers present in the picture (contribution 49) or calculating the perimeter instead of the area (contribution 57). Teachers are trying to

identify any indications that the students are in the process of internalizing (V2) the concept of calculating area based on the external tools they have used. In contributions 50–52, the teachers give credit to the student for multiplying, having interpreted this as an indication that the student had remembered this aspect of finding area. The higher level of understanding held by the teachers gives them a wider view of the stages of student development (V3), which, in turn, allows them to identify mistakes, be aware of possible misconceptions and be able to draw conclusions about partially correct methods as student work to overcome difference (V1).

From a dialogic perspective, the teachers are trying to make meaning (B2) with the other (B1) represented by the students' methods, interpreting specific aspects of the methods as being part of a method they are familiar with, and bringing their own prior experience of common mistakes made by students in the classroom to their interpretation of student methods. For example, contributions 50–52 show teachers linking one student's multiplication with their having made meaning (B2) from the question around area. Similarly, contributions 56–57 show teachers interpreting students who have added all the measurements together as having done this due to a confusion between perimeter and area.

Previous analysis has shown how mistakes are an excellent prompt for discussion (sections 7.2.2 and 7.2.3) but towards the end of this extract, teachers are not discussing an arithmetic error or algebraic slip, but a specific and repeated confusion between two technical mathematical terms. In contrast to the teachers in Group Session 2a, the teachers here return the conversation to classroom practice, discussing how to address the common student error of calculating the perimeter rather than the area. The following extract offers a specific example of the teachers bringing their own experiences of classroom practice to an issue raised during the discussion, thus illustrating some of the sociocultural elements that go into the choices teachers make in the classroom. The dialogic tracing of centripetal and centrifugal forces on meaning making is clearly reflected here. However, the extract also shows teachers discussing a range of mediational means that they can share and use with students in the classroom. As such, it illustrates not only the teachers here operating with mediational means, but also how they might introduce and use a variety of mediational means with their students. Teachers working together here to continue to

develop their own practice echoes Wells's discussion of the ZPD for teachers (section 3.3.2):

69. N: so does anyone have a magic way of teaching the difference between perimeter and area
70. K: well there's one thing but it relies on being our age I think
71. R: go on
72. K: well perimeter has the word 'rim' in it
73. R: yeah
74. K: and rim is the outside
75. R: I sometimes
76. K: but there's nothing that corresponds to area it's got the word 'r' in it but that doesn't help
77. N: I've seen the word perimeter spelt perim 'add' ter but that doesn't actually help
78. K: a) that doesn't help their spelling and b) that doesn't really help with anything else
79. R: no 'cause perpetuating that idea that to get the area you multiply and to get the perimeter you add is not overly helpful as soon as you're past you know
80. K: squares
81. R: yeah a rectangle or a square exactly
82. K: it it is to an extent because all areas are .
83. N: based on the area of rectangles
84. K: based on the multiplication of . the product of two orthogonal measurements . what you do afterwards
85. R: I'm going to use the word orthogonal with [class name] later this afternoon and see what happens
86. K: yeah see how it goes yeah
87. R: yes no I appreciate that but
88. K: well I thought you might so I mean there is there is a basis there I mean I always say to them . if it's perimeter you're adding if it's area you're multiplying but I don't sort of say well its only multiplying these two together . 'cause I've seen them with triangles multiplying all three numbers together and thinking that's going to be an area which is . how do you counter that idea . we used to teach area and perimeter together which was always a mistake because it's like teaching rounding to two

significant figures after you've just done rounding to two decimal places . it just means they don't understand how to do either of them 'cause they don't remember which is which so. No I haven't a foolproof method

89. R: the only thing that I've managed with some of mine is talking about perimeter fences like around the outside of stuff but that's just 'cause I grew up on an army base so that's how I think of it and if they play lots of like computer games and stuff it quite often comes up on that but
90. N: I mean I quite often look at you know the amount of fencing that you need to go around something as that's the perimeter and the squares metre squares of grass that you need to put inside that's the area ..
91. R: but other than that
92. N: I've got little centimetre squares that I also produce
93. R: oh
94. N: and lay it on the shape and go this is a centimetre square we can't measure it in lines we need to measure it in these for area
95. R: that's really nice like a visual thing 'cause especially for some of the ones you work with that must be really really useful
96. N: yeah and then I have my little centimetre cubes which I can
97. R: building little shapes with them
98. N: yes in fact that is what I was doing period two today
99. R: oh
100. K: we used to have a set of . cuboids
101. N: shapes
102. K: no cuboids er so that you . had a big square so yea big which was then sliced up into uh I supposed it was a thousand cubes and then the slices were in a hundred cubes and then there were ten slices of a hundred cubes and then you had columns that had ten in them and then you had little cubes that had one in them so you could see that the one
103. N: where have they gone
104. K: ummmm . who knows they were wooden so
105. N: were they in the SEN department I know
106. K: they were wooden so they probably got
107. V: I remember I remember things like
108. N: we used to have those

109. V: that from when I was in primary school
110. R: yeah for things like place value when you first start doing place value

In this extract, the teachers discuss practical solutions for helping students remember the difference between perimeter and area, such as breaking the word perimeter down (contributions 72–74) and linking the words to real-life contexts (contributions 89–90), before discussing the use of physical props to help (contributions 92–110). From a dialectic perspective, the memory aids discussed here are a particular type of tool that students can use in their mastery of the concept of area. Interestingly, the memory aids seem to be significant at an earlier stage of the development (V3) of students as the teachers are focusing on establishing definitions of "which is which" (contribution 88) for students rather than on routes to calculation. Teachers also demonstrate how the context of the classroom impacts their choice of language. Contributions 84–86 show teachers acknowledging that the use of formal mathematical language such as "orthogonal" may not be appropriate in the context of a particular classroom, perhaps because they think the students have not progressed enough and use of the term is a step too far along the path in the hierarchy (V4) of formal mathematical language. The use of a particular term, based on student development (V3), requires teachers to have a clear idea of the developmental path for students and raises the question of how, as a teacher, does one decide when it is appropriate to introduce a particular term? Is this dictated by the curriculum and, if so, why is it considered appropriate at one age but not another?

Having discussed memory aids, in contributions 92 – 110, the teachers begin to discuss the use of physical items to help develop the understanding of area and extend the discussion of using these for volume as well. The dialectic idea of mediational means frames the physical items as tools. Students would start by using the physical tool to help develop their understanding and would then go on to internalize (V2) the process so the physical tool was no longer required. The tools could be compared to the use of the pole in the common dialectic example of pole vaulting (see section 3.1.2). From the comments around the cubes being in "the SEN department" (contribution 105), where SEN refers to Special Educational Needs, or "when I was in primary school" (contribution 109) there is a sense that physical tools are more common for students who have not made as much progress as others and an expectation that, as students progress, they will move away from physical

tools. Physical tools like cubes are not allowed into national examinations sat by students of 16- or 18-years-old.

From a dialogic perspective, the discussion shows an appreciation on the part of the teachers that these memory aids rely on a particular set of experiences with teachers acknowledging "it relies on being our age" (contribution 70) and "only because I grew up on an army base" (contribution 89). By linking memory aids to real-life examples, teachers are exploring a shared grounding on which to make shared mathematical meaning with students in a classroom context. Another way in which the teachers are aware of the necessity for shared prior experience is in their framing of formal mathematical language. The teachers recognise that "orthogonal" is too formal for use in that classroom (contributions 84–86), echoing Barwell's suggestion that formal and informal terms vary based on context (2015, p. 9, discussed in section 6.3). For the purposes of teacher discussion, however, "orthogonal" is a formal mathematical term that those present are familiar with and is therefore appropriate for use. When the discussion moves to physical items, these could be an alternative source of meaning making (B2) for students, although their use has a less clear role from a dialogic perspective. I suggest the items do not represent methods or language in and of themselves, but how they are used by the teachers and students allows for a dialogic space to be formed between those in the classroom. So, the items could be considered to be facilitating discussion and dialogue. There is a tension evident here in the discussion between the use of physical items to improve conceptual understanding and the expectation of eventually removing these physical props. The tension is present for both teachers and students between how useful these items can be in facilitating discussion and, again, the expectations of the curriculum and examination boards.

From a dialectic perspective, the two extracts in this section show the pathway teachers see for students in developing their understanding of perimeter and area, using various tools to offer support, from memory aids to physical items. From a dialogic perspective, the extracts demonstrate teachers' awareness of how their language changes based on the context of their work and the wide range of ways in which this allows them to support students in making meaning. Once again, the presence of the curriculum is obvious, but there is a clear emphasis on classroom practice in a way that has not been seen in previous extracts.

### 7.3.2 Formal Mathematics in Context

The extracts in this section contain one of the clearest illustrations yet of how the idea of progression is present in the teachers' discussion of student methods:

164. N: so sharing sixty two pounds between four people . this person has . split the four into sorry the two into four pieces and the six into four pieces and then multiplied that answer by ten to make it sixty divided by four
165. R: mmmhmm
166. N: and then added the two parts together
167. K: the two parts together and got the right answer
168. R: that's really nice I've not seen that one before
169. N: an interesting way I mean I did think for God's sake learn how to do short division *laughter*
because they are really jumping through the hoops there to be able to work that out
170. R: they really are but like but does that do you think that implies they understand what they're I suppose the process
171. N: that shows their understanding of what they're working out
172. V: yes
173. K: yeah it's decomposition isn't it
174. N: six and that that six is actually in the tens column
175. R: yeah
176. N: so you have to multiply the answer by ten . that's quite that's quite interesting
177. R: yeah I quite like it as like as showing that I know what I'm doing
178. N: showing your understanding of what's happening
179. K: yeah what seems to have been happening
180. R: hang on let me get that [K about to write on one]
181. K: this is a spare one it's not
182. R: that's alright then
183. K: that's why I chose one that was not written on
184. R: that's fine go for it *laughing*
185. N: there's quite a lot more work there than

186. K: what seems to have been taught in primary schools is when you're saying share . twelve between .. oops four people is that they've been taught to draw four boxes or circles or squiggles
187. N: and go one two three
188. K: or whatever and then go one two three four five six seven eight nine ten eleven twelve which is all very functional and well and great for something like this but if you've got two thousand four hundred to share between four people well then it's not going to help you at all and it also
189. V: I mean I remember being taught like that in primary one
190. K: exactly
191. R: yeah
192. K: but it then becomes all consuming because you can't then do this sort of question because it won't work
193. R: yeah a lot of people have used that method and what they've done is I'm just trying to find a nice example I know there are some in here um is what they've done is they've ended up not sharing equally between the four people so there are two pots that have got fifteen in and two pots that have sixteen in
194. N: because you can't have a half
195. K: yeah because you haven't anything left but it also then just as you said why can't you flipping learn to do short division because it's so much quicker
196. N: it's far quicker
197. K: and so much easier
198. N: yeah people find that really hard . that division process
199. R: that short division process
200. N: and they and particularly when they're trying to problem solve they tend to resort to something that that they can follow their understanding of it rather than
201. K: yeah
202. N: because they don't understand what they're doing here they're just doing it by rote it's another one of those things that they're following

Here, the teachers have found an unusual sharing approach to Question One (contributions 164–168), expressing a preference for short division (contribution 169) before agreeing that the sharing method does show conceptual understanding (contributions 171–173) due to the specific addressing of place value. They then discuss similar methods and how

they are taught in primary schools (contribution 186) before repeating a preference for short division (contribution 195) despite recognising that short division is sometimes completed by rote rather than with any understanding (contribution 202).

From a dialectic point of view, the teachers have a clear idea of the progression students are expected to make. The idea of things being taught "in primary schools" (contribution 186) has a clear sense of time with development (V3) based on the curriculum expectations for different age groups. One teacher frames sharing between boxes or circles as something from earlier in a series of progressive steps and the group are clear on the limitations students reach using this sharing approach (contributions 188–195). Two separate teachers express a preference for short division (contributions 169 and 195) describing it as "quicker" (contribution 196) and "easier" (contribution 197). Their preference comes across strongly, with the additional "for God's sake" (contribution 169), despite their acknowledging that for some students "they don't understand what they're doing here they're just doing it by rote" (contribution 202). The discussion between teachers has raised an interesting point for me. Despite recognising conceptual analysis can be demonstrated by other methods, teachers here still prefer to see a short division method even though they have identified that students could learned the method by rote. Their preference could be due to a method fitting the expectations of the curriculum more closely or simply because they do not like to see wasted effort. The interesting question is how this would be manifested in a classroom, e.g., would teachers lead students specifically towards a short division method despite suspecting it was being learned by rote rather than with underlying conceptual understanding? In a classroom, teachers are facilitating the students experiencing the more efficient methods in a ZPD with the students working to overcome the difference. If this process of overcoming for a student means that taking on a new method makes it part of themselves, encouragement to overcome earlier methods in a prescribed manner could be considered an unacceptable use of power by of the teacher. Each new step is the student overcoming something about their person, about their self, which echoes Matusov's perspectives on dialectics (section 4.2).

From a dialogic perspective, the teacher discussion starts by showing a tension between less efficient methods, which still demonstrate conceptual understanding and more efficient methods that could have been learned by rote, like I have already discussed (section 7.2.2). However, the discussion extends further, exploring the influence of primary

school teaching (contribution 186) and acknowledging their own strong preferences (contribution 169), in one case with a teacher revoicing another by repeated "as you said why can't you flipping learn to do short division" (contribution 195) to reemphasise the point. Another way in which revoicing is used by teachers is with the comment "because you can't have a half" (contribution 194), which is a reference to the fact that, at a certain stage in primary school, it is acceptable for a short division problem to conclude with a remainder. However, once a student reaches secondary school, they would be expected to continue to divide this remainder leaving a decimal solution. The teacher is joking with their comment that "you can't have a half" (fractions are part of the primary curriculum) but is making a point about the curriculum expectation at that stage. The tension here is more subtle than just that between formal and informal mathematics. Barwell's idea that formal mathematics is context-based (section 6.3) could reframe this tension for the teachers as being between different formal mathematics; that of primary school (contribution 194), and that expected at secondary level by the teachers in this discussion (e.g., contribution 195). The influences and voices of their previous teachers or even the previous stages of the curriculum are in tension with the preferred method of the teachers involved in the discussion in a way that is potentially more complex than just a tension between informal and formal methods. The tension is not just present for the students attempting to make meaning in the context of the problem, but also for the teachers who, in interpreting student methods, try and work with the traces of previous teachers' voices. There are always multiple voices present in utterances and, in this chapter, I have suggested this is true of mathematical methods, but are some of the voices present stronger than others? Considering the mathematical methods from a dialogic point of view, as I have been attempting throughout, I would argue that this excerpt illustrates the heteroglossia present in each mathematical method, as the voice of the student along with prior influences such as primary teachers are in evidence, but also the teachers' appreciation of that heteroglossia.

Despite the strong preference given for one approach, implying a strong pull towards a unitary method for the teachers and, through them, for the students, teachers in this extract show appreciation for a wide range of methods. The unusual sharing approach (contribution 164) is valued as a way of creating meaning from the question. Value is a complicated term to use with its monetary and hierarchical connotations. Instead, I tried to think of value in terms of time. The analysis of Question One from the artefact required a

significant amount of interpretation by the teacher reading the method, especially the subtlety around the place value understanding indicated by multiplying by ten (contribution 164). The teacher has taken time to dig into what the student has written and attempted to make their own meaning (B2). In this sense, the teacher is valuing part of a repertoire of methods available to solve this problem. I wonder if this would be possible in an examination-marking situation. As Morgan argues, "[s]tudents who are creative or produce unusual work […] are thus at risk because the value to be placed on their work depends crucially on the idiosyncratic resources of the teacher assessing it" (Morgan & Watson, 2002, p. 99), so in an examination scenario, there is the danger that the examiner would not invest the same value. There is discussion between examiners about variation in methods as part of the process of writing a mark scheme, but once a mark scheme is agreed, options beyond those agreed are not suitable as examiners have to be consistent in how they apply it. As such, the interpretation of student methods by teachers is restricted.

Section 7.3.2 has illustrated how well dialectic theory is reflected in the framework of the curriculum and the habit of teachers to base discussions of methods around development (V3) and progress. However, it is evident from the second extract, how hard teachers are working to make meaning of student methods or pinpoint them in their development (V3) framework. A dialogic framework has demonstrated the value teachers lend to other methods, demonstrating meaning making (B2) efforts and the efforts teachers put in to make meaning of the students' own methods, suggesting that the teachers themselves are learning as part of the process. Is there a similar dialectic framing of teachers learning from students? Teachers developing their understanding of mathematics by working to overcome the difference in their understanding to that of the students seems contrary to the dialectic narrative of needing more experienced others to develop. Wells (section 3.3.2) argued that teachers are experiencing their own ZPD when in the company of other teachers, and the ZPD does allow for collaborative learning where the role of more knowledgeable others swaps between peers (Goos et al., 1999, pp. 39-40) but what about, in the classroom context, with no more developed teachers around and the issue of power roles introduced that prevents all participants being considered peers?

Alternative interpretations of the ZPD may offer a route for exploration. Goos et al. (1999) offer four conceptions of the ZPD based on their reading of Vygotsky. They suggest treating a classroom as a "community of learners" (p. 42) where the teacher's role is that of

facilitator and "the participating teachers worked from the premise that mathematics is learned through engagement in social and communicative activity, and they organized an environment in which students were actively engaged in mathematical sense-making" (Goos et al., 1999, p. 43). Their position marks a move away from a transmission model of mathematics teaching, but still has teachers in a position where they are in charge of setting up the learning situation and does not account for the learning of teachers themselves. Roth and Radford (2011) have explored the possibility of a symmetrical ZPD (sometimes called a bi-directional ZPD). They note that "the zone of proximal development is frequently thought of and applied in a one-sided manner that juxtaposes a more knowledgeable teacher or peer and a less knowledgeable learner" (Roth & Radford, 2011, p. 93). Acknowledging that "cultural knowledge… endows the teacher with a particular asymmetrical role" (Roth & Radford, 2011, p. 103), they argue that this is insufficient to explain the learning taking place. Instead, Roth and Radford suggest that, in a student-teacher exchange, "both participants need to learn: one learns mathematics and the other pedagogy (and often mathematics, too, for the teacher learns to see things from a different mathematical perspective)" (2011, p. 71). In which case, teachers could be overcoming the difference (V1) between their understanding of that student's understanding and the students' own, thus advancing their craft, but this requires the students to be experts on their own learning. There have been indications that this might be worth exploring. In section 7.2.3 I raised the suggestion that student input on the methods they selected would be valuable for teachers in their attempts to make sense of student methods. In this context the students could be considered the experts on their methods.

In their argument for a symmetrical or bi-directional ZPD with the framing of "the role of more knowledgeable other as alternating between the participants" (Abtahi et al., 2017), Roth and Radford (2011) suggest that this version of the ZPD "rests on a non-transmissive and non-authoritarian form of knowing" (p. 103) which has implications for some of the positions on power inherent in the dialectic approach suggested by authors such as Matusov (section 4.2). Others (e.g., Abtahi, et al., 2017) have gone further and explored the notion of a multi-directional ZPD, where they raise the possibility that the more knowledgeable other switches between parent and child or, indeed, that tools themselves can act as the "more knowledgeable other" (p. 285). Whilst my study does not go that far, Roth and Radford suggest that "[m]ore efforts have to be deployed to understand…zones of proximal development not only as zones of agreements but also of tensions, disagreements,

misunderstandings, conflict and subversion" (2011, p. 105). It is important to note, as evidenced by the use of terms such as "tensions" along with the theory they quote in developing this interpretation of the ZPD, that Roth and Radford are forming this interpretation by utilizing Bakhtin's theories as part of their close networking of the theories of Vygotsky and Bakhtin (as discussed in section 3.2). This has consequences for my application of this in analyses where I am not networking as closely. Is Roth and Radford's bi-directional ZPD potentially problematic as it is based on two different philosophical groundings?

The alternative formations of the ZPD offer alternative positions to the traditional dialectic framing of the relationship between teacher and student as one where a teacher passes knowledge to the student in a unidirectional manner, which would allow me to investigate the learning of teachers alongside that of students. A bi-directional ZPD also begins to suggest a way of mitigating traditional power roles in the classroom. These two points will be particularly relevant in Chapter Eight as I analyse the student discussion groups. I can also look for evidence of bi-directional ZPD formation and examine whether this framing is consistent with the networking strategies I am using. There remain questions such as how does a framing that has students as experts on their own learning fit with the influence of curriculum and examination rubrics to guide learning? If a bi-directional ZPD is not consistent with my networking strategy, an alternative might be in evidence, for example, a dialogic framing of both parties making meaning in a classroom context could give more options for exploring the learning of teachers in the classroom.

### 7.4 Revisiting the Working Framework

In section 7.1, I proposed an analytical framework to help guide my analysis of discussion and, throughout this chapter, I have used "comparing"/ "contrasting" approaches to carry out a parallel analysis of teacher discussions. In this section, I am going to revisit my analytical framework, highlighting successes and areas for further exploration to aid in my analysis of the student group sessions in the next chapter. I am also going to discuss the benefits of parallel analyses as opposed to an approach that focused solely on one theory.

The dialectic section of the framework proved useful, with all four key aspects I identified: difference (V1), internalization (V2), development (V3), and hierarchy (V4), in evidence throughout my analysis. The dialectic approach highlighted the impact of the

developmental framework of the curriculum on teacher discussion, such as in sections 7.2.2 and 7.3.2. From a teacher perspective, Vygotsky provides a way of analysing the structures the teachers are working in, particularly the curriculum and the examination board, which are filtered through the school to reflect the key themes of progress and development (V3). The teachers have shown a focus on effective methods that demonstrate conceptual understanding of a topic and, in Session 2b, teachers showed an interest in exploring the practicalities of teaching different methods (section 7.3.1) and the relevance to their classroom practice in a way that suggests the teachers learning from one another in their own version of the ZPD.

Areas of a dialectic analysis that I would like to explore further include a more detailed look at the process of internalization (V2). I have used "tools" throughout this chapter, but the discussion of physical items in section 7.3.1 has reminded me of the distinction between the tools and signs of Vygotsky (section 2.2.1) and I would like to look in more detail about how these are relevant in mathematical methods in the next chapter. Returning to a discussion of the different types of mediational means might help me talk more specifically about students overcoming difference (V1), which has not been discussed in detail in this chapter but may be more in evidence in the student group sessions.

The dialogic section of the framework was also useful. I was able to identify several tensions at work for the teachers in their discussions with one another (e.g., sections 7.2.1 and 7.2.2) and how the other (B1), represented by the other teachers and, a degree more remote, the other of the curriculum, shapes discussion. However, where the dialogic analysis has really come into its own is in my interpretation of the student mathematical methods as representing utterances from students. If I treat methods as examples of student voices shaped by tensions and capable of opening a dialogic space with others (B1), then my analysis has allowed discussion of the meaning making (B2) of teachers with the other (B1) of student methods (sections 7.2.2, 7.2.4, and 7.3.1) and how speech genres (B4) can explain teacher analysis of student methods (sections 7.2.3 and 7.2.4). The teachers have a clear idea of the centripetal force (B3) of the curriculum working to draw students towards a more unitary form of expression, but there are also many examples of the heteroglossia inherent in the students' work (sections 7.2.2 and 7.3.2). The dialogic approach has provided detail about teachers' hard work within the framework of the curriculum to make sense of the heteroglossia of students and has proved particularly

useful when discussing students who have misunderstood, half completed a question, used an unusual method or not had any idea of where to start. Teachers discussed different methods to help address these mistakes and showed understanding of the need to adapt approaches based on specific contexts with the sense of formal language or a formal method shifting depending on the situation.

In section 7.1, I identified power relations within a Bakhtinian framework as something I wanted to explore and, in section 7.2.4, showed how laughter helped me explore the ideas of speech genres (B4) and power relations. I have given an example of how, through laughter, "[t]he direct and serious word was revealed, in all its limitations and insufficiency, only after it had become the laughing image of that word – but it was by no means discredited in the process" (Bakhtin, 1981, p. 56). It will be interesting to see if I can find further examples of laughter in the student group sessions. Another aspect of a dialogic approach, not included in my analytical framework, is that of revoicing. I have used the idea of revoicing in this chapter (section 7.2.3), and Barwell (2015, p. 11) utilised it in his analysis as an example of teachers acknowledging heteroglossia. I have suggested that teachers revoicing student methods leads to a greater layering of voices that may help meaning making (B2). In my project, I have collected the voices of multiple participants and have layered them through my analyses to make meaning. Revoicing in the student groups could be interesting, particularly with regards to the students discussing teacher methods in session 3.

There are two areas of a dialogic analysis I would like to explore further in my student groups session analysis. Firstly, the idea of the difference between self and other (B1) is so important in Bakhtin's work and whilst it has been behind much of my analysis here, a more specific discussion of it in the student section may help develop the distinction between it and the sense of difference (V1) in Vygotsky's work. Secondly, I noticed that, in my analysis, instead of referring to centripetal and centrifugal forces, I instead often refer to tensions. The tension between the curriculum and those working in a school context was so evident that I found it difficult to identify other tensions at work and those I did find all linked back to variations on the theme of formal and informal mathematical methods. This is the scenario Wertsch warns against saying, "one might be tempted to say […] "the voice of the curriculum" can be heard […] Yet an account proceeding only to this point would not be getting to the heart of the matter. To understand why teachers' manuals are written as

they are, one must examine the forces at work on a more general cultural, historical, and institutional level" (1991, pp. 144–145). Whilst acknowledging that there will be many hidden tensions that I will be unable to account for without the shared background with all the individuals involved, are the tensions caused by the curriculum the only ones I can analyse here? In Chapter Eight, I am going to rename analytical framework B3 as tensions, rather than centripetal/centrifugal forces to see if that helps focus my analysis on tensions.

Chapter Seven has been an example of an attempt at connecting theories for Bakhtin and Vygotsky at the methodological level using a parallel analysis carried out using a "comparing"/ "contrasting" networking style. The analysis of extracts carried out here has benefitted from this in a number of ways. Firstly, there are examples where having both a dialectic and dialogic framework has allowed a deeper understanding of the two separate theories such as in section 7.2.1 where I discuss a framing of the significance of the curriculum from both a dialogic and dialectic perspective – a development from the dialectic focus on curriculum impact prior to this point. This example reflects Prediger, who cites the consideration of variations between two networked theories as an opportunity to increase understanding of both (Prediger et al. 2008, p. 9). Secondly, there are examples of where having parallel analysis has allowed analysis of more data than having one theory at play such as in section 7.2.4 where a dialogic analysis of laughter is discussed as a means of addressing power relations. Thirdly, the parallel analysis and direct "comparing"/ "contrasting" approach has extended the possibilities of what a dialogic analysis can offer such as in section 7.2.3 where I attempt to apply a Bakhtinian analysis of utterance directly to mathematical methods. In this example, I could directly analyse the mathematical methods using a dialectic approach, but there was no equivalent in a dialogic approach, which allowed me to realise there was potential for development. Without the comparison, I question whether my development of the dialogic method in this direction would have been prompted. Without the addition of the Bakhtinian-style analysis of mathematical methods, I would have been limited to analysing the discussion between group participants without the additional level of detail offered by analysis of the artefacts are working with. This would have had an impact when I come to consider the implications of my study for teaching and learning. If students' mathematical methods are considered to be utterances, and as such, subject to centrifugal and centripetal forces then the forces in play can be studied and teaching adapted accordingly, which serves as a direct link for Bakhtin's

literary theory background to the teaching and learning of mathematics. I discuss this in more detail in section 10.2.1.

As I move forward into the analysis of student groups, there are a number of elements of analysis I would like to continue to explore. Firstly, are there more examples of where a parallel analysis offers more depth or detail than the other? Secondly, there are elements beginning to come through in the analysis in this chapter that lean more towards the benefits of a "coordinating"/ "combining" approach where an analysis considers "the same phenomenon from different theoretical perspectives as a method for deepening insights on the phenomenon" (Prediger et al. 2008, p. 10). As such in my analysis of the student group sessions, I am going to experiment with testing networking strategies which hold the two theories more closely together in order to see if I can make use of the benefits of some of these strategies such as gaining more information about the phenomenon under investigation (from a "coordinating"/ "combining" approach). To do this, I am going to use the student group sessions analysis to explore how the overarching structure of my analysis impacts on my connecting theories within Radford's methodological component. I found it challenging deciding on a structure for the analysis of these teacher group sessions with many extracts that would have been interesting to analyse and repeating themes across different sessions. Thinking about the structure brought to mind Och's (1979) ideas on privileging within a transcription (Appendix Five), raising the idea that when I choose to begin my analysis by writing about a Vygotskian or Bakhtinian perspective first, this may privilege one perspective over the other. For the most part, I have used the order following the framing of the theories in the wider literature (Chapters Three and Four), but the literature tends to consider Bakhtin as an extension of Vygotsky so is this organisation that I have used representative of a parallel analysis after a "comparing"/ "contrasting" approach? In section 7.1 I have discussed how I have selected sections of transcript to capture units of analysis of both a dialogic and dialectic nature, but do I always need to consider both perspectives if I am working to connect theories or, as I have started to discuss, is one approach more appropriate than the other for analysing some extracts? For the student groups, I am going to look at privileging through analytical structure in more detail.

## Chapter Eight – Methodology: Student Groups

In Chapter Seven, I tested my analytical framework as part of connecting the theories of Vygotsky and Bakhtin at a methodological level, by using it to analyse the discussion of mathematical methods by teachers. My analysis followed "comparing"/ "contrasting" (Figure 1.2) approaches, which focused on developing "a better understanding of the […] theories" (Prediger et al., 2008, p. 9). In this chapter, I will continue to use my analytical framework to analyse student group sessions to develop my investigation of connecting theories (Radford, 2008). In addition to continuing the exploration of my analytical framework, I am interested to see how students' perspectives on mathematical methods compare to those of the teachers as well as how my role as teacher-researcher varies when in discussion with students rather than teachers.

The plan for the student group sessions was outlined in Figure 5.6 (section 5.2.3) but was adapted slightly in practice. I have summarised these changes in Figure 8.1.

Student Group Outline

Session 1
Blank copies of questions.
- Which method would you use if no one would see it?
- If it was being seen by a teacher?
- If it was a test?

Session 2
Student methods – from artefacts.
- What's the same?
- What's different?

Session 3
Teacher methods – from artefacts created in teacher sessions.
- What's the same?
- What's different?

Figure 8.1 – Updated student group session outline

Figure 8.1 shows that a final group session to be guided by student ideas did not take place. The sessions took place once a week and were shorter than the teacher sessions, but attendance of students dwindled from four students in the first session to two in the third. I discuss the impact of low attendance in the third session in section 8.4.4 but, due to the low attendance, I decided that continuing to a fourth session would not be productive.

### 8.1 Updated Analytical Framework

In section 7.4, I discussed aspects of the analytical framework that I would like to explore further: the internalization and mediational means of a dialectic approach; the other; and tensions of a dialogic approach. I also identified two aspects of a Bakhtinian dialogic analysis that had not appeared in my original framework: laughter, and revoicing. I have thus updated my framework (below) to reflect these changes with alterations in italics.

| **Vygotskian Dialectic Analysis** | | |
|---|---|---|
| **Feature** | **Detail** | **Links** |
| V1: Difference | Theories or ideas are in opposition and must be overcome.<br>Learning awakens developmental processes that occur with others and when internalized gives rise to independent development brought about by environment. | Wegerif, 2008, p. 359<br><br>Vygotsky, 1978, p. 90 |
| V2: Internalization | An idea is experienced collectively and then individualised as it is internalized. Formal mathematical language is part of a cultural tool that students need to internalize so they can master the concepts. This process of internalization is the overcoming of difference and affects your identity.<br>However, internalization is not always fully realised and Wertsch, along with his toolkit analogy, suggests mastery as an alternative framing. | Section 2.2.2<br><br>Section 3.1.2 |
| V3: Development | The overcoming of opposition and mastering of higher concepts through internalization marks a progression or development towards uniformity or the idea of "Universal Reason".<br>These higher levels affect earlier, less developed ideas.<br>In a school setting this is often seen as the teacher guiding students from informal to formal or standard mathematical language/methods. | Section 3.5.1<br>Vygotsky, 1986, p. 203<br><br>Barwell, 2015, p. 4 |
| V4: Hierarchy | The idea of development towards an ideal means those more developed have power over those less as they influence them via the ZPD.<br>In a school setting, teachers use this power to lead students a particular way. | Section 2.2.3<br><br>Section 4.2 |

| **Benefits** | | **Issues** | |
|---|---|---|---|
| It echoes the set-up of the National Curriculum with its ideas of progress, development, and mastery. | | Its notion of progression sets formal and informal mathematical language in a hierarchy and thus devalues the work of those who have not mastered formal methods | Matusov, Section 4.2 |
| It includes the idea of power relations between students and teachers and between the government and schools. | Matusov, Section 4.2 | Analysing the process of internalization requires returning to the starting point of development and, in practical terms, this not possible. | Vygotsky, 1986, pp. 235–236 |
| | | Argument against perfect scientific being. | Wells, Section 3.3.2 |

Figure 8.2 – Key features of a Vygotskian dialectic analysis

| Bakhtinian Dialogic Analysis | | | | |
|---|---|---|---|---|
| **Feature** | **Detail** | | | **Links** |
| B1:<br>Other | Interaction with the other or alien word means that differences are in dialogue with each other, rather than in opposition. One is not trying to make meaning by overcoming the other. Instead, the difference itself opens the possibility of meaning making.<br>The other not only allows for meaning making, but the difference between authoritative voice and internally persuasive word is key to "an individual's becoming". | | | Section 4.1<br><br>Bakhtin, 1981, p. 342 |
| B2:<br>Meaning making | Meaning created in the difference between self and other does not contribute to an ongoing progression or development in a linear sense. Instead, it is an "active understanding, one that assimilates the word under consideration into a new conceptual system, that of the one striving to understand".<br>In a dialogic approach, the aim of a lesson is to expand repertoires of mathematical language and build a wider range of ways to make meaning. | | | Bakhtin, 1981, p. 282<br><br>Barwell, 2015, p. 13 |
| B3:<br>*Tensions* | Representing the ongoing, non-binary centripetal/ centrifugal forces present in each utterance between a unitary language and the diversity of multi-voiced heteroglossia present in all speech that "makes it possible for students to express mathematical ideas meaningfully".<br>In a school context, unitary language may be framed as formal mathematical language but, for Bakhtin, the definition of unitary language depends on the context.<br>*Revoicing by others is one way in which heteroglossia can be recognised.* | | | Barwell, 2015, p. 8 |
| B4:<br>Speech genres | Act as unifying forces that develop within a group, for example, a particular profession or family, allowing a shared basis for meaning making.<br>Wertsch sees speech genres as mediational means. | | | Section 2.1.3<br><br>Section 3.1.2 |
| **Benefits** | | | **Issues** | |
| Tensions and speech genres are a practical way of working with the historical meaning and connotations brought to a situation by the participants. | | Section 3.1.2 | No linear concept of progress/development. This goes against how what is happening in a school is conceptualised, particularly if I am focusing on methods chosen by students for examinations, which focus on demonstrating progress/development. | Barwell, 2015, p. 3 |
| *Contrary to White's argument, Bakhtin's concept of laughter as a levelling tactic helps address unequal power relations introduced by the authoritative voice.* | | *Section 4.3*<br>*Section 7.2.4*<br>*Section 2.1.5* | | |

Figure 8.3 – Key features of a Bakhtinian dialogic analysis

I have applied the updated framework to the analysis in this chapter. In addition to the parallel Vygotskian and Bakhtinian analysis of previous chapters, I wanted to look at the

effect of the structure of the analytical process on the analysis itself. In order to do this, I varied the way in which I applied the analytical framework to the transcripts of the final two student group sessions. For the first session, I have continued to follow similar "comparing"/ "contrasting" approaches from the previous chapter in order to test this for the student groups. For the second and third student group sessions, I am going to attempt a "combining" networking approach in order to test if connecting Bakhtin and Vygotsky's theories more closely is possible and productive. At the start of sections 8.3 and 8.4, I will go into more detail about my choices and design for each analysis.

## 8.2 Student Group Session One

To analyse Student Group Session One, I am going to continue with the same approach I took with the teacher groups. I will analyse my transcripts using the updated analytical framework above and the "comparing"/ "contrasting" approaches I trialled in Chapter Seven to lead to a "better understanding of typical characteristics of the theoretical approaches in view" (Prediger et al., 2008, p. 10). I present for discussion here excerpts that highlight particular aspects of the framework, or extracts that suggest additions or alterations to my framework.

### 8.2.1 Bus Stop as Tool

Of the three questions in the artefact, the group were most confident answering and discussing Question One in this session. The following extract is an early discussion between the students and teacher (researcher) about a specific approach taken by several students and how to use this approach to solve Question One (Figure 5.2). From a dialectic perspective the extract is an example of a particular mediational means selected by the students and the teacher working with them through this method to reach a solution. From a dialogic perspective the extract shows the teacher making meaning with both the written work (such as in line 33) and the utterances of the students, and the students making meaning of the method through interaction with the utterances of the teacher as they work through the task:

9. R: - who would like to talk to me about question number one what sort of method did you decide you might need for question number one .
10. A: divide sixty-two by four

11. R: divide sixty two by four OK so that tells me what the question wants me to do which is a good first step isn't it OK um . what sort of methods do we have for doing sixty two by four 'cause some people have written down
12. P & E: bus stop
13. R: ooh bus stop ooh that was very quick . so some people have written it down in bus stop and some people haven't written it down in bus stop do we know how bus stop works
14. D: yes
15. R: yeah
16. A: you put sixty-two there and four there
17. R: and then do that nice so do you want to try it do you not want to try it you don't have to some people are like yes I want to try it
18. E: I know how to set it out it just doesn't work
19. R: you know how to set it out it just doesn't work show me what you mean (8)
20. E: do you divide just by six
21. R: right so the bit on the outside is the bit we divide by so six divided by four
22. E: which is .. one
23. R: good so that goes on the top
24. E: and you put the two there I think
25. R: yep nice
26. E: and then divide it by twenty-two
27. R: and then again four divided by twenty-two fours into twenty-two nice and be careful because it's a horrible one ok take your time it's horrible the fours into twenty two is the trick isn't it yeah so what's the problem with fours into twenty two
28. A: it doesn't go into the twenty-two
29. R: it doesn't go into the twenty-two so what might we need to do
30. A: decimals
31. R: decimals some sort of decimals
32. D: then you put like a zero then two above it and then yeah
33. R: oh I see so you've put like a decimal point and then zero and then ooh so you've got remainder two haven't you for yours yeah which is exactly right
34. P: you need to put the zero
35. R: you've got two remainder two but it's changing it into a decimal then isn't it yeah
36. P: oh yeah

From a dialectic perspective, the speed with which students identified their method of choice, rapidly answering "bus stop" (contribution 12) to the teacher's inquiry about methods, indicates the students' familiarity with a particular tool. However, one student has tried to use the bus stop method for division but is trying to divide the wrong way around by dividing four by sixty-two (contribution 20). The teacher corrects the order for the first step of the division (contribution 21), but the mistake is repeated by the student (contribution 26) and corrected by the teacher again (contribution 27). The student's understanding is at the preconceptual level and, despite having made the connection between bus stop and a division problem, the student is still mastering the tool. Here, the use of "bus stop" (contribution 12), in the discussion but also in their solution, could be the mathematical equivalent of Vygotsky's idea that "at the moment a child assimilates the meaning of a word, or masters an operation" (1978, p. 90) is the first step in development (V3). Here, the fact that the student cannot use the tool to come up with the correct answer has resulted in them framing it as a tool that "doesn't work" (contribution 18). For my analysis, I have used tool to describe a particular method, following Wertsch's toolkit analogy (section 3.1) as mentioned in my analytical framework (Internalization, V2). However, classifying the bus stop method as a tool is overly simplistic. Wertsch relabels Vygotsky's two branches, sign and tool, of mediational means as cultural tool and technical tool, making an abbreviation of both to tool simple. However, by using tool in both of these classifications, it masks the key difference between signs and tools, namely that they act on different things. Vygotsky's signs (Wertsch's cultural tools) act on the individual whereas Vygotsky's tools (Wertsch's technical tools) work on "mastering […] nature" (John-Steiner & Souberman, 1978, p. 127). Using tool to describe the bus stop method or describing it as part of a toolkit, erroneously implies an association with Vygotsky's tool/Wertsch's technical tool with the aim of affecting the external world, rather than the internal one. Instead, the bus stop method here is a sign (Vygotsky) or cultural tool (Wertsch). Mastering the bus stop method is orientated to an internal change.

From a dialogic perspective, this extract illustrates a range of different meaning making (B2) interactions taking place. What is under discussion is one student's attempts to make meaning between their previous experience or assimilated/appropriated meaning of similar questions and the specific demands of the question they were presented with. The dialogic space that occurred as a result of that difference has resulted in their attempt at

the bus stop method (that I am treating here as an example of mathematical utterance). Meaning making (B2) is made evident through the teacher and student exchange (e.g., contributions 18–21), so, in effect, what I am analysing is an utterance about an utterance. If I restrict myself to analysing the discourse between teacher and student, the students' rapid agreement as to the method they chose (contribution 12) indicates a centripetal force at work drawing students towards a common method to approach the question. However, the subsequent comment that "I know how to set it out it just doesn't work" (contribution 18) suggests that the student is still working to make meaning (B2) in the difference between themselves and the expectations of the question. The centripetal force is so strong, it is drawing them towards a method that does not lead to a correct answer above another method that may be more successful for them. The strength of the centripetal force at work here echoes the clear preference of teachers for the bus stop method as a solution to this question (section 7.2.2). This preference could be an example of the authoritative word, which "demands our unconditional allegiance" (Bakhtin, 1981, p. 343) and which may have resulted in this uniformity of approach. However, in section 8.2.2 (contribution 59), the same student discusses an alternative method, which suggests they have other methods in their repertoire. Whilst strong, the centripetal force does not reach the level of the authoritative word.

I mentioned in section 7.3.2 I was interested in seeking examples of where a bi-directional ZPD might be in evidence and contributions 17-20 could be one example of the role of learner swapping between the teacher and student, if only briefly. In contribution 19, the teacher asks the student to clarify what the student has done in their method, thus putting the student in the role of expert in their own learning. However, the manner in which the student replies (contribution 20), suggests the student is not confident in their own understanding of their method. I will discuss this in more detail in section 8.5.

Section 8.2.2 is an additional analysis included as Appendix Thirteen. As in Chapter Six, I have retained the section numbering to indicate chronology.

### 8.2.3 Question Two

The following extract features a discussion about Question Two (Figure 5.3) and finding the area of a trapezium. I chose this extract because it contains several themes. Firstly, the

discussion links to a previous teacher group discussion, secondly, the group use multiple methods in their discussion, and thirdly, a new tension emerges:

88. R: - OK then let's talk about question number two then right calculate the area of this shape so first things first does everybody understand what the question is asking us to do 'cause I think 'cause these are really difficult questions does anyone need to does anyone want to ask about what the question's asking us to do .. to find the area of the shape if I asked you to colour in the area of the shape could you do that . just like shade the area of the shape what would you colour in
89. P: all of it
90. R: yeah go on then show us . show us the area of the shape can you do that
91. P: like there *pen on paper noises*
92. R: yeah like that like a quick scribble so it's the space inside the shape isn't it guys yeah
93. A: does that count as it as well
94. D: yeah it does
95. R: yeah that's right
96. D: it's the whole thing
97. R: it is the whole thing lovely why do you think the line's there 'cause you were pointing at the line weren't you A
98. P: 'cause it's not the whole shape
99. D: oh my Dad told me this
100. R: it might not be a full shape we might have cut something off somewhere
101. A: I've forgotten
102. R: OK it might be a hint to help us do the question mightn't it does anyone remember how we work out the area of a shape then . it's something to do with those numbers around the outside isn't it
103. A: it's the same as that side
104. R: which side ooh the dotty line oh you're right the dotty line is the same as the five centimetres you're quite right well spotted good .. so if I was trying to work out the area inside I tell you what let's make it a little bit easier let's just cover that bit up let's cover up the triangle and let's just look at the rectangle how would I find the amount of space inside the rectangle
105. P: add up the sides

106. R: I could add up the sides but adding up the sides would tell me this wouldn't it does anyone know what hang on does anyone know what this is called if I'm going around the outside of something
107. D: perimeter
108. R: perimeter is that what you were going to say B I think you were so the perimeter is adding up all the numbers around the outside so what do we do if we're trying to find the space inside the shape we're not adding up .. what could we be doing (6) OK I tell you what let's have a look can I borrow somebody's pen and we can have a look on this one so if I'm doing the area of the shape you know the easiest way to find area is to count the squares so I could it like this couldn't (12) so the quickest way of doing it er yeah is by counting these up yeah so shall we do it together you ready one two three four five six seven eight nine ten eleven twelve it's just me counting now guys
109. A: yup
110. All: thirteen fourteen fifteen sixteen seventeen eighteen nineteen twenty twenty-one twenty-two twenty-three twenty-four twenty-five twenty-six twenty-seven twenty-eight twenty-nine thirty
111. R: so the space inside that shape is thirty squares so we say thirty centimetres squared so with the little two next to it .. can anyone spot a quicker way of doing that rather than just counting every single individual square like this oh *bell rings*
112. P: has it got something to do with the right angles
113. R: something to do with the right angles well that tells us the type of shape go on E what were you going to say
114. E: is it like count up them and them and then times it
115. R: ooh you count up them and them and you times it that sounds like a very sensible idea doesn't it
116. D: what
117. R: you multiply them together
118. A: yes
119. R: so this is telling you it's five squares that way
120. P: no you add them
121. R: no we've just done adding for perimeter haven't we and six squares that way so if you do five times six what number does that give us
122. A: thirty

From a dialectic perspective, there are a range of mediational means being employed here. Each leads to students being a step closer to internalization of the area of a shape. In this situation, the teacher is guiding students through stages for working out the area of the trapezium: counting squares (contribution 108); introducing multiplying (contribution 111); and guiding students away from confusing perimeter and area (contribution 106). I discussed the teachers' conversation about how to address the confusion between area and perimeter in section 7.3.2. The teacher discussion focused on memory aids to help students remember which is which. However, in the extract above, a student who can tell me to shade in the shape (contribution 91) tries to add up all the distances around the outside in order to calculate the space inside the shape (contribution 105), indicating a lack of understanding of the underlying concept. The student is not reading the word area and thinking it means distance around the outside then employing a method/tool they have to calculate that. The student is reading the word area, understanding that it is a measure of the space inside a shape, but thinks that adding the numbers around the outside of the shape is how you calculate the space inside. Reading a written method, such as the teachers in Section 7.3.2, there is no distinction between these two, different issues. For both, the teachers would see a question that says area followed by a method that attempts to calculate perimeter. The interpretation depends on "a teacher's constructions of views of students' mathematics" (Morgan & Watson, 2002, p. 88), an idea I first raised in section 7.2.3, which has significance for examination context.

The teacher and students employ different strategies for Question Two that are in evidence in the transcript: shading (contribution 91-92); writing (contribution 108); gesturing (spoken about in contribution 97); and talking (throughout). From a dialogic perspective, not all of these can be analysed as, for example, shading and, arguably, gesture, do not represent language in a Bakhtinian framework. There is a clear restriction to using a dialogic analysis in evidence here. The elements of gesture do fall under a dialectic signs and tools framework so could be analysed (albeit with significant adaptations to the data collection methods if this was to be a focus). As my study does not look at the use of gesture but focuses instead on spoken word and written methods, this is not key, but does point to a more significant restriction if I were using a primarily Bakhtinian analysis in a scenario such as the classroom where I had initially planned to carry out a final stage of my research. A dialogic analysis or, as I have chosen, a connecting theories approach would still allow this possibility. The writing is echoed by the discussion, so, focusing on the

spoken aspects here, there are two tensions in evidence that I have not discussed previously. The first is the tension (B3) between home and school (contribution 99), which can be difficult to see. The overt comment here gives some evidence of the tension at work in the discussion. Students who work on their mathematics at home with parents, siblings or tutors have the influence of those voices adding to their perspectives when they complete a mathematical problem, giving an insight to some of the sources of heteroglossia present in student methods. Another tension (B3) in evidence is that, for me, between my role as teacher and my role as researcher. I do not keep the discussion focused on what the students have written down, but instead work through the problem with them (e.g., contributions 108–111). I will discuss the tension of teacher-researcher more thoroughly in sections 8.2.5 and 8.3. These themes may not have been highlighted by a dialectic approach. It seems that there has been a discussion of mathematical methods at home which may have led to a ZPD environment for the student with a parent, but the dialectic framing does not have a clear explanation for how previous interaction influences the interaction in this student discussion group. The evidence in this extract suggests the two learning environments are interacting and the student has brought some remnant of the previous discussion to the context here but the question remains, how does the learning at home and the learning at school interact? The dialogic approach has a clear interpretation and framing of this with the interactions at home and the current discussion placed in tension with the student using all of these experiences to make meaning in their interaction with others in the group.

### 8.2.4 Power

One of the areas I discussed wanting to explore further in section 7.4 is how Vygotsky and Bakhtin respectively frame power relations and what evidence can be seen of this in the discussions. The extract below, continues discussion of Question Two and was chosen as the clearest example from the transcripts of evidence of the existence of power relations between students as well as the power relations between teachers and students:

127. R: OK so that tells us that one there OK so we could multiply the sides together couldn't we what about the triangle bit on the end then what might we need to do for the triangle bit what's the same and what's different about the triangle bit
128. A: it's different lengths

129. R: different lengths OK good so we don't actually know this little side length here but you've already spotted that that's five as well haven't we what about this bit at the bottom
130. A: is it half of ten so five
131. R: half of ten why do you think it's half of ten
132. A: because it's ten all the way
133. R: ten all the way well if it's ten all the way there and that bit there is six what do we think this bottom bit might be P do you want to have a look woah woah P is going to have a go it's ten all the way across and up to there is six what's this bit here got to be
134. P: five
135. A: four
136. R: ..six plus five would be eleven wouldn't it she's not helpful *students giggling*
137. D: I know
138. R: so six plus what gives me the ten there … hush you that's six squares and I want ten squares all together how many more squares would I need there (6)
139. P: would it be four
140. R: perfect really nicely done so four squares there so I could do you think I could times those numbers together like five for that bit and four for that bit would that find me the area of the triangle …
141. P: yeah
142. R: sorry I'm stealing your pen *all giggling*
five centimetres there and four centimetres there so we could do five times four and that gives us twenty centimetres squared . does that look right
143. D: sure
144. R: if I just times them together
145. A: don't you have to square them
146. R: that's what my square is there
147. A: oh
148. D: don't you have to have a square around it
149. R: I think we might need to do area again in lessons before the end of term *laughing*
can you see look that finds me that area of that rectangle do you remember
150. A: oh yeah
151. R: so what would I need to do as my final step

152. A: find out the other half
153. R: there was a really important word in what you've just said
154. A: half
155. R: half it so we don't want all of that do we we just want half of that
156. A: so ten
157. R: so that would be ten like that does that make sense
158. D: sure

From a dialectic perspective, the power dynamics in this extract follows the framing of those more developed (V3) leading others in a particular way. Firstly, the teacher is leading the students through the steps to find the area of the trapezium with comments such as "what about the triangle bit on the end then what might we need to do for the triangle" (contribution 127), and "what about this bit at the bottom" (contribution 129). The leading even extends to expecting specific formal mathematical terms from students such as "there was a really important word in what you've just said" (contribution 153). The teacher seems to be expecting a specific type of mediated action or, to require the use of a specific mediational means to lead to the required mediated action. This limiting to a specific mediational means is a restriction of the options for the individual operating with mediational means. The teacher role here is similar to what it might be in the classroom so is not unusual, although does have a sense of the hierarchy (V4) of those with more knowledge. However, there is evidence of a second more knowledgeable other in the discussion here in student A. The teacher, R, is discussing a method with student P and student A jumps in with the correct answer (contribution 135). The teacher is working hard to manage their contributions to try and give the other students space to contribute and to help them with their methods. Student A could be described as being in a position of power from a dialectic sense as they have offered correct solutions to questions (contribution 135) but, also, there may be evidence that their solutions are leading the contributions of other students with Student P echoing Student A's solution (contribution 139). It is not possible to know if Student P has learned anything from Student A's contribution, if they are just repeating the word, or if they have come to the correct conclusion another way. The student could be at the stage of internalizing (V2) where mastering the word is the start of a developmental process (section 2.2.2). The dialectic principle of the ZPD requires more developed (V3) others to be involved in discussion for students to make progress between what they can do themselves and what they can do

with the help of others (section 2.2.3), but there is an interesting dynamic here where the teacher is leading the students a particular way and then managing other participants who are interrupting that path, albeit with humour (e.g., contributions 136 and 138).

From a dialogic perspective, power is framed predominantly through the presence of the authoritative word (section 2.1.5). Authoritative discourse does not seek "a free appropriation and assimilation of the word" (Bakhtin, 1981, p. 343), but instead comes with the authority of an institution or particular person attached. The authority of an institution makes it straightforward to show why the teacher is in a position of power over the students involved, but it is more difficult to explain how one student's contributions might be examples of the authoritative word. The authoritative word "demands our unconditional allegiance" (Bakhtin, 1981, p. 343), not allowing the free assimilation of the word and as such restricts heteroglossia. One way, I suggest, more complex power dynamics might be analysed from a dialogic perspective is through looking for limiting of the voices of others (B1). By jumping in with their answer (contribution 135), Student A limits the contributions of Student P, but, equally, the teacher hushing Student A (contribution 138), even humorously, is limiting the voice of Student A and, in a wider sense, by managing the contributions of all students (e.g., contribution 133), the teacher is choosing which voices are acceptable in an interaction. The extract does show a second use of humour in a dialogic framework to mitigate the impact of the authority lent by the institution of the school, filtered through the teacher (first discussed in section 7.2.4). The teacher uses humour to make the point that the student jumped in by pulling a face (not in transcription as it is not audible for the tape but is the cause of the audible laughter in contribution 136) and a joking "hush" in contribution 138 directed at Student A to prevent it happening again. In this case, the humour is used by the teacher rather than by the students so does not have the same subversive impact.

### 8.2.5 Discussion

Through section 8.2, the dialectic framing strongly reflects the curriculum in conversations around, for example, particular methods the students have tried to use (section 8.2.1). The dialogic elements reflect a wider variety of sources of meaning making for example, the acknowledgment of interactions at home mixing with the interactions in school (section 8.2.3). The student group analysis has taken some themes in evidence through the teacher group sessions and echoed them in the context of student discussion. Themes of power, a

range of sources of meaning, and discussions around particular methods are all in evidence here although the context of student discussion has shifted the emphasis. The theme of power, particularly, has come through in this chapter. In section 8.2.4 the use of both theories to analyse the specifics has led to a substantially more developed argument by being able to see how each frames the situation. This analysis is supplementary to that of section 7.2.4 where I was specifically analysing the mitigation of power relations

The parallel analysis has also prompted me to think more about my role in the study. With the dialogic tensions between my role as teacher-researcher in section 8.2.3 and the dialectic and dialogic discussion of power in section 8.2.4, another theme that comes through strongly in the analysis of Student Group One is the shifting of my role in the research. I have written previously about my role as teacher-researcher (section 5.2.2), but analysis of the transcripts shows the way I embodied the dual role is different in the student discussion groups than it was in the teacher discussion groups. Throughout section 8.2, the extracts have shown me as teacher guiding students through methods for the different questions. I had, completely unconsciously it is worth noting, shifted into more of the role I have in the classroom. As a result, my language in section 8.2 has shifted. In analysis of the teacher groups, I referred to myself as "the researcher", whereas throughout the analysis above I have used "the teacher". The change happened organically as I was doing the analysis and reflects which aspect of my role felt more in evidence in the transcripts. However, at no point am I one or the other, so, referring to myself as either teacher or researcher is a misnomer. To reflect more accurately my inhabiting of the dual role, I am going to refer to myself as "I" or "me" through the other analyses in this chapter. Using first person pronouns will make it clear that I am not drawing a boundary between these two aspects of myself in these situations.

In Chapter Seven (section 7.4) I summarised examples of where analysis of the extracts was benefitted by a parallel analysis or, alternately, given more depth by one theory or the other and mentioned that I would continue to search for examples of situations where this was the case in the student groups sessions. Section 8.2.3 contains an example of once such situation as the transcription features discussion of a variety of mathematical methods, some of which cannot be analysed in a purely dialogic framework (although I am trying to extend this somewhat by analysing mathematical methods as utterances of the people that constructed them). In my discussion, I mention the restrictions of a dialogic analysis and

how useful it is to have had the option of assessing the situation from a dialectic perspective as well. The option to pick and choose one analytical approach over another in a "comparing"/ "contrasting" connecting theories approach is a benefit of having two to choose from. My next step is to investigate if connecting theories using a different networking approach might allow me to address some of the questions I have raised such as those around privileging one approach over another (discussion from section 7.4), or allow me to specifically address issues that have arisen from my analysis so far such as how power relations impact discussion. In the next section, I am going to start to experiment further with the structure of how I apply my analytical framework (see section 8.3 for more detail) and exploring my shifting role is going to play a key part.

### 8.3 Student Group Session Two

In my analysis so far, I have adopted a "comparing"/ "contrasting" approach to analysis, where the theories of Vygotsky and Bakhtin were kept separate and applied one after the other, which has allowed me to build an analytical tool. However, for the remainder of my analyses, I am going to try using the "coordinating"/ "combining" networking strategies (Figure 1.2). The idea is to tie together elements of the theories of Vygotsky and Bakhtin in order to build a "networked understanding of and empirical phenomenon or a piece of data" (Prediger et al., 2008, p. 10), which shifts my focus from using the data to build an analytical framework, to using the analytical framework to "look at the same phenomenon from different theoretical perspectives as a method for deepening insights" (Prediger et al., 2008, p. 10). Of the "coordinating"/ "combining" approaches, a "coordinating" approach "can only be done by theories with compatible cores" (Prediger et al., 2008, p. 11), whereas a "combining" approach "does not necessitate the complementarity or even the complete coherence of the theoretical approaches in view. Even theories with conflicting basic assumptions can be combined in order to get a multi-faceted insight into the empirical phenomenon in view" (Prediger et al., 2008, p. 11). Due to my discussion of underlying philosophical issues in Chapter Four, it is a "combining" approach I am going to take here.

For the analysis of Student Discussion Group Two, I applied a "combining" approach that takes a dialectic approach to reading the transcripts and then a parallel approach to analysis of each section of transcript the dialectic reading has highlighted. Student Discussion Group Two focused on the artefacts generated as discussed in section 6.1. A dialectic approach to reading the transcripts focused on the development (V3) of the

discussion over time and highlighted the role of the teacher in discussion. Specifically, I have chosen extracts that include the instructions I, as teacher, used during the discussion and the immediate events preceding and proceeding the instructions themselves. I have also, where appropriate, included follow up extracts that I have interpreted as including discussion that reflects directly back on the instructions used. At this level of extract selection, I looked at how my instructions have guided the students towards a particular action or even towards a particular mediational means to achieve the desired action or, as some of the authors considered in Chapter Four might interpret it, the teacher leading students towards a "preset math truth by making all other possible alternative truths cognitively impossible and socially/politically dangerous for the participants" (Matusov & Wegerif, 2014, p. E7). This approach to reading the transcript will result in a series of extracts providing a dialectic themed structure. I then used both a dialectic and dialogic analytical framework to analyse utterances and mediated action as in previous analyses, thus allowing an investigation focused on my area of interest, that of power relations, whilst allowing the option of analysing other aspects of student perspectives on mathematical methods.

### 8.3.1 Initial Instruction

The extract below is my initial instruction to students in Student Group Session Two and shows me asking students to look for specific artefacts:

7. R: - okay so what I thought we'd do is we'd just have a little look through and to start with all I'd like you to do is find something that you find something that you think is interesting so it can be because it's weird it can be because it's exactly what you did it can be because it's completely different to what you did so just have a little sort through I'll just move this microphone out of the way *rustling* while I'm eating my lunch um and you guys can have a little think and it might be that you pull out two or three it might be that you just find one and then you're going to try and tell me what it is that you found interesting about it

The instruction is intended to be gentle, using phrases such as "just have a little look through" and "all I'd like you to do". The instruction to "find something that you think is interesting" is similar to the directions I gave when using these artefacts with teachers (sections 7.2 and 7.3), intended to give students the room to interpret the instruction a

variety of ways. The students sort through the artefacts and begin to identify some for further discussion. The extract below shows one student discussing their selection:

9. R: go on E say that again
10. E: they didn't use any . like .. symbols they just wrote it down
11. R: there's lots of words in that one
12. E: and it looks really confusing
13. R: it does look really confusing lots and lots and lots and lots of words

A student has identified an example based on finding it confusing, a pattern which continues with students often focusing on the use of algebra in Question Three. The extract below illustrates how the students react to the algebra in solutions to Question Three:

25. E: I don't like that one it's using *x* and it doesn't say *x* in the question
26. R: ooh ok you don't like that question three I'm putting some of them over here because we're going to talk about them *paper rustling* in a little bit more detail in a minute does anyone else want to add any to there oh you've got a nice pile going on there . fabulous do you want to give me those or do you want to hold on to them you're going to give me those OK anybody else found any that they'd like to talk about so I've got one two three four five six at the moment
27. A: I just like looking through them
28. R: it's good isn't it I find it really interesting seeing how other people did this
29. E: they've used *x*s as well
30. R: they've used *x*s as well OK (9) are there any that you find interesting P either because they're weird or because you understand them or because you don't understand them or
31. P: um I think this one
32. R: you think this one
33. P: yeah
34. R: why is that one
35. P: it just looks it just looks quite interesting
36. R: it looks quite interesting shall I add it to my pile
37. P: yep
38. R: OK it's on my pile

39. E: that one's used *x*s as well
40. R: that one's used *x*s as well lots and lots of these have used *x*s shall I pull those ones out and put them on the pile so that you can get angry about them in a minute (5) I have to say
41. E: they've used even more *x*s
42. R: they've used every more *x*s .. OK this is my pile that E would like to set fire to because they've got lots of *x*s thank you kindly

From a dialectic perspective, the initial instruction has been successful in guiding students towards a specific action i.e., identifying artefacts, using the mediational means available i.e., the words of the discussion and the artefacts on the table. However, the lack of specificity has led to students predominantly identifying methods they do not understand to discuss later. I later attempted to push for a little more information (contribution 34) but am unsuccessful (contribution 35) with the student echoing my initial instruction back to me.

From a dialogic perspective, the students are starting to make meaning (B2) with the methods of others (B1), even if that meaning is that they do not understand the methods. This is reflected in the students' use of informal language here such as "that one's used *x*s" (contribution 39) rather than using the word algebra. The students' lack of confidence is in evidence through comments such as "I don't like that one" (contribution 25). The extract shows me revoicing the student contributions of informal mathematical language (e.g., contributions 40 and 42). Revoicing the students' utterances shows me valuing the different voices present and could have a reassuring effect on the students as well as me reassuring myself that I have heard their contributions accurately.

### 8.3.2 Second Instruction

My second instruction followed the initial sort of the artefacts and shows the move from an instruction to sort to an instruction to explain why they chose the examples they did:

44. R: - so what we're going to have a little look and do is we're going to have a little discussion about why we find them interesting and if we can work out what the people were trying to do with what they've written down does that sound helpful

does that sound clear everyone OK with what we're doing OK um so let me just put these out of the way . so shall we go through them *sheets placed forcibly on table* one by one would that be helpful and I've got a mixture here I've got some that people thought were interesting because they were clear or interesting because this or because they were confusing or or that sort of thing OK so . these are I think this is your pile of things you wanted to set fire to so I'll put all of those together as a pile so what do we think does anyone want to grab the one they suggested or didn't suggest and tell me why they put that one out there . so who chose (.) this one for example

From a dialectic perspective, this additional instruction was designed to encourage students to move beyond identifying methods they found interesting to attempting to explain what they found interesting about their choices and explaining, "then we can see if we can work out what the people were trying to do with what they've written down". In the teacher group sessions, teachers take the initial instruction and run with it, perhaps because teachers are used to analysing as part of their usual way of approaching student methods so no further prompting was necessary. Students, however, are less used to analysing methods of others (B1) and so more support was needed. From a dialectic perspective, my second instruction could be me trying to provide extra support in the development (V3) of the students' analysis of the methods of others, developing their use of the mediational means. I reinforce the instruction in the discussion that follows. The following extract shows students working to make sense of Question Three and they focus specifically on two methods:

67. R: - somebody else who chose another one that they wanted to talk about or if you see any where you'd like to go ooh that's a bit weird … what do we think
68. A: that one they didn't actually write anything they just
69. E: drew pictures
70. A: drew pictures
71. R: so we chose this one because for question number three we've already sort of started to have a conversation about this eww they've used *x*s thing which E brought up and we'll talk about a bit more in a second but for this one they've got like some pictures drawn so they've put like Alice is twice as old as Judy and then they've drawn some pictures and why do you think they've drawn those pictures . what do the pictures show us

72. E: the people
73. R: the people so what is important about the pictures do you think
74. P: they're getting smaller from the ages maybe
75. R: ooh that's nice so maybe they were trying to work out the order like who was the oldest and who was the youngest do you think it's helped them with the question
76. P&A: yes
77. R: maybe yeah because at least it got them in the right order hasn't it because that was quite confusing for us wasn't it like we really didn't like question three at all did we so that at least has helped them work out what age order the students were in yeah which is quite nice let's talk about this one because this was this was on your little pile here of things which you gave to me and went ewww so tell me why these ones here .. actually let's do this the other way round put these ones towards me and these can go in front of you so what was it about these question threes that you didn't like because we didn't like question three last week when we did it because it was quite difficult so what was it that made you go ewww for these ones
78. E: they've got *x*s on them
79. R: they've got *x*s on them what is it about the *x*s that we don't like
80. E: they aren't in the question
81. R: they aren't in the question OK do we agree with that
82. P: yeah
83. R: they're confusing I think I don't know why do you think they've put *x*s there then if they're not in the question .. anyone got any ideas 'cause I don't know so I thought I'd ask what do you think
84. A: it might just help them understand bit easier by writing out different ways
85. R: yeah it's kind of like it's almost like this way was one way of them trying to write it out yeah with symbols with like pictures so do we think these symbols are a different way of writing it out to try and help them understand
86. A: yeah
87. R: can you link any of the symbols that they've written down to anything that is in the question … so can you like spot where the numbers are the same like in the bits with the algebra and then in the question
88. E: twenty-seven and three
89. R: twenty-seven and three so what did they put equals twenty-seven and then what does this say

90. E: the sum of their ages is twenty-seven
91. R: so they're saying they have to get twenty-seven at the end which is quite clever isn't it
92. A: they have said that *x* is fifteen and then two *x* is thirty and then two *x* take away three is twenty-seven
93. R: Ahh that's interesting it's OK if you don't get it it was a really horrible question do you remember me saying last week that the questions were designed for like the year thirteens as well as like for the year sevens so went all the way across the board um I quite like this one here I know that the *x*s are really confusing why do you think this one why do you think I like this one why do you think they've got J A and B go on E
94. E: because they've used the beginning of the names
95. R: they've used the beginning of the names so this is kind of almost like when they're writing down the symbols they've kind of tried to do it for each person haven't they can you see that so that's quite nice I quite like that and then what have they done underneath
96. A: they've writ it out again
97. R: so they've used it then they've used it with these to try and work out their age which is quite nice really isn't it even though it's horribly confusing yeah OK nice so we're not a fan of the algebra is what you're telling me but can we see how it might be useful maybe in a few years once we've got a bit more practice with it
98. A: yeah
99. R: OK you're still not OK with it P neeayah . a bit what sorry
100. P: in the middle
101. R: in the middle OK that's OK to be in the middle -

The students have selected examples of two different methods for solving Question Three. The first with pictures to represent the ages of the people in the question (contributions 71-76) and the second algebraic methods, both with *x*s (contribution 78) and with the initials of the names (contribution 94). From a dialectic perspective, the pictorial method could be considered less formal than the algebraic methods with both examples of cultural tools (Wertsch, section 3.1.2), or signs (Vygotsky, section 2.2.1). From the transcript, the students are able to suggest an explanation for the pictorial method relatively quickly (contribution 74), whereas their confusion over algebraic approaches gave rise to their unimpressed comments from section 8.3.1 (e.g., contribution 25) that I allude to in this

extract (contributions 71 and 77). Contributions 73 to 76 are another example of a hint of an interaction that could be interpreted as representing a bi-directional ZPD. In contribution 74 the student offers an interpretation of the method that I as teacher-researcher had not considered before (as evidenced by the start of contribution 75). In this case, the student could be considered the expert and me the leaner. I discuss this further in section 8.5. Contributions 79 and 80 directly address the difficulty that students have with the algebraic representation of the question. Making the link from a worded problem to the use of algebra in order to solve the problem is a leap that requires significant prior experience to realise that algebra could be a useful tool. In making the decision to go through the problem with the students, I have tried to set up a learning opportunity for the students to engage with the methods of others and my own support in order to help them make progress in their ZPD. However, despite my help, producing a fully correct solution is beyond the students' current ZPD and the focus of our discussion is on translating the question into a form that might lead to a successful method at some point in the future (e.g., contributions 87–91), rather than producing a fully correct solution.

From a dialogic perspective, students are trying to make meaning (B2) from the different methods shown by students in the artefacts (e.g., contributions 84 and 94). However, they have very little common ground to use as the basis for meaning making (B2), possibly due to a lack of experience with the methods of algebra (contribution 78), or, as I previously discussed in section 7.2.1, with the genre of worded questions (contribution 80). I am trying to help by introducing other information that might help create common ground for meaning making (B2) so the students in the group move from trying to make meaning directly with the methods of the students in the artefacts to making meaning with me through discussion (e.g., contributions 87–90). Barwell suggests that, from a dialogic perspective, "students work at expanding their discursive repertoires, giving them a wider range of ways to make meaning in different mathematical situations" (2015, p. 13). I am helping them expand their "discursive repertoires" by using the link between pictures and the question that the students have already identified (contribution 74) as an example of a different way of representing the question, and then extending this to the algebra (contribution 85).

### 8.3.3 Third Instruction

The third instruction emerges as the result of a specific difficulty students have with my style of questioning. I present one extract first to put the instruction in context and illustrate how the students are struggling:

102. R: OK so any more that we think are weird or that we think are nice or that we understand
103. A: that one
104. R: this one shall we talk about this one for a second *laughing* OK so somebody tell me why do we think this one's weird compared to all the others
105. E: lots of writing
106. R: lots and lots of writing on it so let's have a look at the writing OK so for question number one OK so remember the question said I've got sixty-two pounds to share between four people how much does each person get do you remember we answered that one last week so it is says sixty two divided by two is thirty one thirty one divided by two is fifteen point five . so does it sort of do the right thing
107. A: yeah they're doing the right thing but they don't write it out in letters
108. R: they've just written it out as words and letters haven't they rather than what did we do instead
109. A: numbers
110. R: numbers and bus stop didn't we why why do you think they've written it out in words (4) there's not a right answer don't panic
111. P: they could make it might feel a bit easier with writing it in
112. R: they might feel more comfortable writing it in words good why didn't we write it in words when we did it like
113. P: it's a bit easier perhaps
114. R: what were you saying B
115. B: don't know
116. R: that's fine so you find it easier not writing it in words ..
117. P: it's alright I guess

In the extract above, the student has selected an artefact where the solution to question one has been written entirely in words, as a narrative, rather than a more standard mathematical way using symbols. The method was actually written by the student I

discussed in section 7.2.4, who had adopted a humorous approach to completing the artefact questions. The students are (understandably) finding it difficult to interpret the student's method for Question One (contribution 110). My dialectic reading of the transcript indicates that, specifically, the students are struggling to respond to a "why" question "why do you think they've written it out in words" (contribution 110). The question is an attempt from me for them to develop (V3) their analysis of the mathematical methods from the artefacts, but this is a step too far. I use "why" questions at other points in this session (for example section 8.3.4, contribution 93), despite the fact that there are no examples of me asking a direct why question like these during the teacher group sessions. Part of the reason for the change in style of questioning may be to do with my shift in identifying more closely as teacher for the student group sessions rather than the researcher emphasis in my role in the teacher group discussions.

I have used why questions in several different ways. I ask students why they think another person has done what they have, "why do you think they've written it out in words?" (contribution 110), which seems almost entirely pointless as, based on the theories I am discussing, it is not possible to know why someone has done what they have. However, there are also examples of why questions that the students are more comfortable answering such as "why do we think this one's weird compared to all the others" (contribution 104), which prompts a quick response from one student (contribution 105). Here, the why question relates directly to their experience and, as such, students find a reason more easily than questions that ask them to explain why another has made a specific choice. Recognising that the students find why questions more difficult, my third instruction followed and made use of the idea that the students are more comfortable answering questions about their own experience:

120. R: - I bet if we look through that massive pile again we could spot lots and lots of different methods shall we do that for a minute just spotting lots and lots of different methods shall we say the words thing works like what they've written down is correct but we wouldn't use it yeah that's fine let's have another look through this one then 'cause what could be quite fun to do is yes I know there are like two hundred of these but what could be quite fun to do is if we have another look at question one is see if we can find ones that we would do and ones that we wouldn't do does that

make sense so everyone got through and see if you can find like three or four which ones would you do and which ones wouldn't you do

In response to the difficulties students had answering questions about the motivations held by others, my third instruction focuses on asking students to think of their own responses, "see if we can find ones that we would do and ones that we wouldn't do". The following extract shows an example of the type of discussion that followed the new instruction and the refocusing on the students' own experience:

207. R: - go on then P which one did you choose
208. P: I thought maybe these two
209. R: OK were these ones you would do or wouldn't do
210. P: um this one I wouldn't do
211. R: OK why not
212. P: because it's a bit confusing
213. R: which bit particularly was confusing can you tell me
214. P: I think maybe a bit of the bus stop because . it's quite hard
215. R: yeah OK the bus stop is a bit tricky so they've used the bus stop so that's something you might not do and is it the same for this one or is this one different
216. P: I would do the adding up and add them instead of the bus stop because I like that one better
217. R: so you might write it out and then add it up that way
218. P: yeah
219. R: ah nice so that adding up method is nicer than the bus stop method
220. E: I'd kind of use this one
221. R: you'd kind of use this one OK explain to me what this one is doing
222. E: they've divided it by two and then divided it by two again
223. R: nice OK so divided it by two and divide by two
224. E: like halving
225. R: so it's kind of like that one except that one's in words and that one's in numbers
226. E: that one doesn't really explain what it is I guess
227. R: OK so we don't think the wordy one explains what it does whereas that one does -

In the extract above, students have selected artefacts based on methods they would and would not do (contribution 209). The focus on their own experience has allowed a more open discussion and they seem more confident discussing their own choices (contributions 212 and 214), going a step further to suggest choices they might make in the future (contribution 216). In the first group session, the students predominantly attempted the bus stop method for Question One (section 8.2.1), however, here, at least two of the students can identify alternative methods and explain why they would use an adding method (contribution 216) or repeated halving (contributions 220–224).

From a dialectic perspective, students experience difference (V1) and then work to internalize (V2) concepts in a process traditionally based around progressing towards more formal methods. The efficiency of one mediational means, halving and halving again (contributions 220–222), over another, the bus stop method, could mean that expressing a preference for repeated halving over the bus stop method is a move forward in Student E's progress caused by seeing the method used by other students and discussing it in sessions. The student is working towards employing that particular mediational means in a similar situation in the future. In contrast, choosing the adding method as a preferred method (contribution 216) could be considered to be a step backwards from the bus stop method to a less formal, less efficient method. Student P has expressed a preference for what could be considered a less sophisticated method, which might be a sign that they have also made progress. However, their progress is towards a method that might be more reliable for finding the correct answer the next time they come across a similar problem as opposed to attempting the bus stop method but being unsuccessful. The reasons for preferring one mediational means over another could vary from person to person. From a dialogic perspective, both of these students are making new meaning around Question One. By looking at the work of others (B1), students are "expanding their discursive repertoires" (Barwell, 2015, p. 13) by making meaning (B2) in the difference between themselves and others. In a dialogic framing, both more and less sophisticated methods are important to learning.

Being more aware of how I question and the impact on students is important for any future work, but it is not as simple as removing questions that ask students to think about the methods of others. It is important for students to engage with difference (V1) or others (B1) as an important source for their own mathematics. The extract above does not

exclusively focus why questions on the students' own experience. Examples of hidden why question, such as "explain to me what this one is doing" (contribution 221) also ask students about the unknowable ideas of others. However, they do not cause stress as previously, which may be because these questions are asked about instances where they have a greater mastery or more familiarity with the methods. Instead, the student explains their own interpretation (contribution 222). The next extract illustrates how more experience with a particular method can lead to a deeper discussion and more confident student contributions:

233. R: that's fine don't worry OK any more for any more right this one
234. E: fractions
235. R: fractions not keen on fractions
236. P: I've got a fraction one
237. R: you've got a fraction one yes we were talking about your fraction one
238. A: that one's four and that one's two
239. R: that one's four and that one's two OK that's an interesting one why have they used two for their fraction and they've used four
240. A: 'cause they've half-ed it and half-ed it again
241. R: mmmhmm
242. A: and they've gone into four as in fours in sixty two
243. R: in one go
244. A: yeah
245. R: so which would do you think you'd do one of these more likely than you would the other
246. A: that one
247. R: you'd do the halving and halving again and you're happy to write that as a fraction almost yeah OK nice alright

The extract above shows students recognising that, even within the use of fractions there is still a wide variation. Students have successfully identified two instances of artefacts that have fractions written for Question One, with one artefact representing the division by four as a fraction, "that one's four", and the second using the repeated halving method using fractions "that one's two" (both contribution 238). Student A has successfully identified the different approaches these methods represent (contributions 240 and 242), which is a

sophisticated analysis of small variations in the methods. From a dialectic perspective, the student could be showing progress, from the bus stop method many students defaulted to in section 8.2.1 when they completed the questions for themselves, to the more efficient method of repeated halving (contributions 245–247).

From a dialogic perspective, the presence of so many fraction examples in the artefacts has created a tension between their bus stop representations of the question from the last session and the use of fractions by others. The interaction with the other (B1) has opened up a significant space for meaning making (B2), illustrated by how students have been able to identify and discuss the nuances present in the different uses of fractions. Students have then been able to make links between their own ideas (e.g., halving and halving again from section 8.2.2) and the use of fractions to represent such an approach (contributions 239–240). As such, the discussion from the two group sessions so far has demonstrated changes in student perspectives on methods, or, at least, their perspectives on which methods they might choose to use.

### 8.3.4 Discussion of the Dialectic Structure

Section 8.3 was an attempt to structure an analysis using a "combining" approach to networking the two theories. The analysis was based on a dialectic reading of the transcripts to identify extracts showing how the session itself progressed and the influence of me as teacher, which led to ideas about how careful consideration of questions might be an area for further study. An in depth look at exactly how questioning impacts on unfolding discussion would have significance for the teaching and learning of mathematics as well as for research. The initial reading was followed by parallel dialogic and dialectic analyses, which allowed an exploration of student perspectives on mathematical methods.

Using a "combining" approach to networking theories for the analysis of Student Group Session Two has allowed me to narrow my focus to consider a specific issue, that of the impact of my questioning on discussion. As a result, I have utilised a key benefit of this approach, namely, that theories can be "combined in order to get a multi-faceted insight into the empirical phenomenon in view" (Prediger et al. 2008, p. 11), which has allowed me to draw specific conclusions about further study that may lead to changes in practice for me as researcher and teacher in a way that the more open parallel analysis did not. Simultaneously, the approach has allowed me to analyse other themes that have arisen in

the selected extracts such as students struggling to make sense of certain methods (section 8.3.2) and the different reasons a student might select a specific method (section 8.3.3).

There are elements of the structure that presented a challenge to my analysis. As I discussed elements of dialogic and dialectic theory within the overall dialectic structure, there were sections I wanted to move and group but could not as that have would have disrupted the chronological structure. Also, if a "combining" approach is based on combining different elements of theories, should my use of an overall dialectic framework mean that within it the analysis is strictly dialogic to reflect selecting one theory for one part (i.e., a dialectic approach for the initial reading of transcripts) and a second for the closer, more detailed analysis (i.e., the dialogic approach for analysis of individual exchanges)? In the final analysis, I am going to keep to a "combining" approach, this time with an overall dialogic framing followed once again by parallel analyses.

### 8.4 Student Group Session Three

For my analysis of Student Group Session Three, I am going to adopt a dialogic approach to the reading of transcripts as an alternative take on the "combining" approach I used in section 8.3, this time using the dialogic theory as the framing theory. In section 8.3, the dialectic selection of extracts allowed me to focus on a specific issue I was interested in. I hope to use a dialogic structure in a similar way, this time to investigate the point I made in my conclusion to Chapter Seven (section 7.4) about how challenging I was finding it to identify tensions that could not be considered as a subset of the wider tensions caused by the curriculum. Tensions are not always easy to identify in the utterances they influence. For this analysis, the "combining" approach allows me to focus on finding extracts where I can identify tensions (B3) and then select these for further analysis using parallel dialectic and dialogic approaches.

For the third student group session, I took the artefacts generated during the first teacher group session and shared them with the students so they could look at and discuss examples of teachers' work. The session was attended by only two students and myself and the dwindling attendance is why a fourth session (as planned in section 5.2.3) did not take place. The students present for the session three discussion had been present at all the previous student sessions and were perhaps the quietest of the group. It was lovely to get to hear their voices more clearly in session three.

### 8.4.1 Tension: Formal-Informal Mathematical Language

In the first group session, the students all attempted to use the bus stop method to answer Question One. In session two, the students began to identify alternative methods, such as repeated halving, that they thought could be useful and fractions were discussed as a way of representing division. The two extracts in this section have been selected as they show the students choosing examples of methods from two teachers from Teacher Group Session One which are distinct from the methods they have discussed previously. From a dialectic perspective, the extracts allow me to analyse the students' discussion of specific mediational means selected by the teachers in order to address the task and, in turn, the students use of these methods as mediational means to develop understanding of the problem. From a dialogic perspective the students are making meaning with the teacher methods (what I am taking as utterances), and through conversation with each other and me as teacher-researcher:

5. R: go on then show me put it flat on the table so we can both see if you put it there 'cause then I can read it upside down why did you choose this one as one that you wanted to talk about
6. B: 'cause it's different to um .. bus stop method
7. R: ooh OK so for question one they've used a different method do you want to explain what they've done
8. B: I think they're halving it
9. R: they're halving it OK yeah good so this here says sixty-two divided by two which is the same as halving lovely and then what did they do
10. B: they halved it again
11. R: lovely really nice and they got fifteen pounds and fifty pence at the end very nice that's a very interesting one 'cause that was very different to what we did wasn't it when we did it last time 'cause we all did bus stop lovely what a very good example P your go which one would you like to talk about
12. P: that one
13. R: this one OK so any particular part of it that you'd like to talk about
14. P: like this bit
15. R: this bit at the top go on

16. P: ‘cause looks a bit easier than bus stop and that it’s going down like B’s but um . it goes in fours then twos
17. R: lovely so how’s this one written what have they used for this one
18. P: column is it column
19. R: sort of what does it look like B
20. B: fractions
21. R: they kind of look like fractions don’t they yeah so sixty-two divided by four is a fraction and then thirty-one over two is a fraction and then fifteen point five at the end so you quite liked that because it looked easier right so both of these methods actually look a little bit easier than the bus stop method one that we chose don’t they is what we’re saying -

One student has identified an alternative method to the bus stop one that they used to solve the problem (contribution 6) and has made a good attempt to interpret the method the teacher used in the artefact (contributions 8 and 10). The other student has chosen a different example that they think looks more straightforward (contribution 16) and has spotted a subtle difference between the two methods, both of which have been written as fractions, hence the student saying, “it’s going down like B’s” (contribution 16). The “going down” refers to a reduction in the numerators as the teacher simplifies/divides and the “in fours then twos” (contribution 16) refers to the denominator changing as it is simplified. Strikingly, despite the detailed description of the differences in methods, the second student uses “column is it column” (contribution 18) to describe what they see and need the support of the other student (contribution 20) to identify that these are fractions. The student seems to be describing the literal arrangement of the numbers in a column one on top of the other in a fraction.

From a dialectic perspective, the student is able to discuss key features of the methods such as the fact that the denominator is decreasing, however, there is no indication that the student has connected the pattern of decreasing in twos to an understanding of why. A lack of use of technical mathematical terms suggests that the student is in the early stages of developing their understanding (V3) of using fractions to represent division. I have chosen to ask questions (contributions 15 and 17) or model fuller explanations (contributions 9 and 21) to help the student progress in their understanding and level of explanation. From a dialogic perspective, difference allows space for meaning to be made, but in parts of the

discussion the students and the creators of the artefacts have few points of common understanding. I am, therefore, trying to provide more points of commonality or offer more links so that the students in the group have more points for meaning making (B2). My close questioning may have narrowed down the options the students felt they had, acting as a centripetal force in suggesting a particular form and style of answer and guiding them in a particular direction. There is a sense of a "guess what is in my head" approach to questioning here (e.g., contributions 17–20), which would indicate to students that I am looking for specific responses. As such, there is tension (B3) between a specific technical term and the student's informal descriptions. The responsivity inherent in the utterance means that the students' contributions are always shaped towards my reaction so the closeness of my questioning is a powerful force shaping every utterance of theirs.

The student who has used "column" to describe fractions in the previous extract (contribution 18) uses the same terminology again later on in the session. The extract below is a specific, short exchange about one example of Question One:

84. R: OK so this is question one and the bus stop method go on
85. P: I think I wouldn't have tried that one 'cause I'd probably have tried something a bit easier
86. R: OK what would you use
87. P: I'd probably do like maybe add it up and do some columns
88. R: add it up using columns OK that sounds like a good idea and what would you add up
89. P: um like maybe them bits and see how much money they'd get
90. R: OK lovely

There is a degree of ambiguity over the use of column here. The student is discussing their ideas about how they would approach Question One, having been over a variety of ways to approach it both from teacher and other student examples over the three sessions. The student explains, "I'd probably do like maybe add it up and do some columns" (contribution 87), which can be interpreted in a number of ways. The contribution could be taken as a whole with the student indicating that they have remembered a repeated adding method we discussed in the previous student group session (section 8.3.3 contribution 216) and that they would use the column method for addition. However, in light of their earlier use

of “column” to refer to fractions (contribution 18) and the discussion over the sessions about how fractions could be an alternative method for Question One, a degree of ambiguity is introduced. The column method for addition makes more sense in the context of this specific extract and is how I interpreted the comment (contribution 88), but the previous extract shows that the student has another previous meaning for “column” and it is not definite that the previous correction of “fractions” by the other student present (contribution 20) or by my subsequent use of it (contribution 21) have been sufficient for the student changing their use of the term. The extract above may illustrate how the conclusions teachers draw about student methods from their comments are only one interpretation.

### 8.4.2 Tension: Individual Voices-Authoritative Word

The pattern of my comments shaping the discourse continues in the following extract where I speak to each student in turn about the artefacts they have selected and demonstrates the questions I used. The extract has been selected as, not only does it contain further examples of students identifying alternative methods used by the teachers, but also examples of a particular style of interaction between the utterances of teacher and student that I have interpreted as highlighting a tension:

21. R: - what about your other one P
22. P: um they added um they timesed the shape up so they get the
23. R: oh cool so this is the second question you want to talk to us about on this one OK so say that again
24. P: they like um timesed the six or the five to get
25. R: yep
26. P: and they timesed .. two and then four why have they timesed that one then
27. R: oh I see so this is because so is it because of what they the method they’ve used or is it because of where they’ve written the method that’s helpful
28. P: mmmmm where they’ve done it
29. R: OK ‘cause they’ve kind of done it inside the shape haven't they
30. P: yeah
31. R: yeah so the six times five is inside the rectangle bit and then the half times four times five is inside the triangle bit do you like that one as well B
32. B: mmm

33. R: for the same reason or a different reason
34. B: same
35. R: the same reason OK alright what about your second one is it under here is it that one there is it that one you chose right tell me why you chose that one
36. B: um 'cause on this one they use *x*s
37. R: ooh OK so for question three on this one they've used *x*s tell me how that helps why you chose that
38. B: umm (8) don't know why
39. R: OK so did you choose it because it was interesting rather than because it explained it
40. B: yeah
41. R: OK so you liked that they used *x*s for that because you thought it was what different to what we've done before or the same or
42. B: different
43. R: different to how they've done it before
44. R: OK um it's interesting that you've chosen question number three because we haven't really talked about question number three have we other than like lesson uh last session where we went um we didn't like the algebra which was quite funny OK so these are the ones that we like and are interesting particularly for question number one and maybe a little bit for question number two you thought that you'd found some things that might make it look a little bit easier is that about a good summary of what we've had a look at alright I've written down some questions which is why I keep looking at this piece of paper to make sure that I'm asking all the questions OK have a little look through again so if we out them back out so we can see them all um .. are there any that you don't understand and you'd like to ask questions about what they've done or why they've done it (4) so again take some time and look through them and have a think are there any that you think you don't understand and you'd like to ask some questions about you don't understand what they've done or why they've done it -

I have already discussed (section 8.3.3) the impact of the type of questions I ask. In the extract above, there are further examples of why questions (such as contribution 35 and 37). Despite my previous successful adaptation of questioning to focus why questions on students' own experiences, here the questions related to their own experience

(contribution 37) still result in the student struggling to answer (contribution 38). The extract above shows my reaction as teacher to the student's difficulty. From a dialectic perspective, my next comment (contributions 39–43) is an attempt to scaffold the learning of the students by giving suggestions for explanations and modelling phrasing. From a dialogic perspective, my intention was to try and support discussion but there is a tension (B3) for me as teacher-researcher between prompting and scaffolding to support students make their contributions and giving the students space to have their voices heard. The whole extract is a back and forth between me and the student with me asking a question and the student trying to answer (e.g., contributions 31–34). There is not a sense of free-flowing discussion but instead close questioning on my part. At the time, I thought I was helping, but as the analysis goes on, the tension (B3) between prompting for contributions and actually hearing individual voices becomes obvious.

In the above extract, in addition to the questions and prompting for the teacher role, there is also evidence of my researcher role, clarifying things not audible for the tape, for example, which question the student is gesturing at (contribution 23) or the position of writing on the page (contribution 29). Contribution 44 shows me consulting a paper copy of my plan for the student group session (Figure 8.1). The teacher-researcher tension (B3) is one that I am operating within, but is not something the students are necessarily aware of. They are used to me inhabiting the teacher role in our usual classroom interactions and, as I have previously discussed (section 8.2.5), the teacher role has been most in evidence for the majority of the student group sessions. Contribution 44 is significant as I am drawing the attention of the students to my role as researcher. The students are not acting in the teacher-researcher tension as I am, instead, the students are working in the tension (B3) between authoritative word and internally persuasive voice. They know that the work they are analysing is that of teachers, which they are used to being correct. The authoritative word of the teachers' methods are in tension with their own internally persuasive voice. Students are taking the teacher methods to be correct because they were written by teachers but the students are not convinced (contributions 39–40). The methods and ideas they are discussing are not yet part of their own internally persuasive voices.

### 8.4.3 Speech Genres: The Outcome of Tensions

In the previous two sections, I have tried to identify some of the tensions (B3) present in the group sessions. While the tensions themselves can be difficult to identify, one way that I could seek more information is to consider the speech genres (B4) that are intertwined with the tensions. The tensions that pull towards unitary language are context-specific and create their shared basis for meaning making (B2) by forming speech genres that guide and shape each utterance (section 2.1.3). By looking at features of the speech genre, I may be able to trace the tensions at work or at least the impacts of these tensions on the students.

Speech genres (B4) are difficult to outline. Wertsch explains that "[s]tudies of classroom discourse have yielded some general evidence that suggests the existence of fairly standard and rigid speech genres. One indication of this is that teachers […] are consistently reported as doing the majority of the talking" (1998, p. 120). My voice is certainly the most in evidence through these sessions. As I discussed in section 7.2.4, individual utterances, whilst being shaped by the presence of a speech genre, are not enough in themselves to describe a speech genre. Instead, I have been thinking about speech genres as restricting what is meant in a given situation as "the intentions permeating these languages become things, limited in their meaning and expression" (Bakhtin, 1981, p. 289). If a speech genre implies a certain common meaning for a situation, then it also precludes certain meanings. Following this approach, I selected the following extract where I argue that, from a dialogic perspective, the series of utterances in this extract suggest the presence of a speech genre. From a dialectic perspective the group are trying to disassemble the mediated action taken by the teacher who wrote this solution to Question Three whilst I as teacher-researcher try to help them link the teacher's choice of mediational means with the question:

44. R: - this one . yeah . OK let's have a look then so P why did you choose the one that you chose
45. P: 'cause it's got Js in
46. R: 'cause it's got Js in have you got Js in yours as well oh so we've chosen ones that have got sort of similar things ooh that's interesting shall we have a look then so let's put them next to each other and see right so can we spot things that are the same about both of those methods
47. P: they've got Js in
48. B: yeah

49. R: they've got Js in what else
50. P: and the two and the
51. R: two J and J plus three that's what you're pointing at isn't it B and then what have they done
52. P: they've got the same total
53. R: so what does total mean
54. P: like when they add it up and they get them
55. R: and they've got the same so yours have got the same here but what's different about yours to P's B
56. B: it's not labelled
57. R: it's not labelled so this one's got some names on it to explain what's going on so this person has said OK we're going to write Alice's age as two J now why do we think they've written Alice's age as two J can you see where that matches up with the question
58. B: twice as old as Judy
59. R: good yeah so it's kind of like a code isn't it they've written Alice is twice as old as Judy which is what the two J is for now Judy is is as old as Judy which is why they've written Judy equals J and then what about Ben how does that match up with the bit of the question
60. P: three years older than Judy
61. R: yeah or Ben is hang on or Judy who is younger than Ben by three years yeah so Judy's age plus three gives you Ben's age yeah why have they added them up .
62. P: 'cause they're going high I don't know ..
63. R: which bit of the question tells us we're going to we need to add them up
64. P: is it there like where
65. R: yeah so that's what they've done isn't it they've added them up and that's what they've got as their answer but there's a bit in the instructions that tells us we need to add them up can you see it …
66. P: is it like .
67. R: which bit up here in this writing tells us we need to add it up
68. P: is it Ben's three years
69. R: yep so that's that bit there isn't it . yeah
70. B: oh aged twenty-seven

71. R: mmmm good so the sum of their ages is twenty-seven can you see that and do you remember the sum is the same as add 'em up yeah so we've summed them up and we said the sum of their ages equals twenty-seven

In previous discussions, the students have, with prompting, begun to make links between the written question and the symbols representing it in the algebraic methods that appear in the artefacts. Here, the extract has several features that could begin to indicate the presence of a speech genre (B4). Contributions 45–49 show mirrored language patterns that I, by parroting the students, am encouraging. There are also language patterns typical of classroom discourse such as "[h]eavy reliance on the initiation-reply-evaluation (I-R-E) sequence" (Wertsch, 1998, p. 121), such as in contributions 57–59. Students are working with technical terms such as "total" (contributions 52–54) and "twice" (contribution 58), where the speech genre limits these terms to representing mathematical processes, i.e., add and double. By defining and making links using these technical terms, the students are showing familiarity with the speech genre. The way the students are able to make more connections between the worded question and the algebraic solutions present in the artefacts also shows how they have made meaning (B2) in a specific manner based on the familiarisation process they have been through over the three sessions. This process has begun to build ground for common meaning making (B2). I suggest that the use of specific patterns of speech that have been adopted by the group, specific technical terms, and some evidence that students are beginning to make links between the worded question and use of algebra show students are building familiarity with a speech genre.

### 8.4.4 Discussion of the Dialogic Structure

The "combining" approach I have attempted in section 8.4 used a dialogic reading of the transcripts to identify tensions (B3) before applying a parallel dialectic and dialogic analysis of each extract. Having concluded in Chapter Seven that the tensions I identified there could all be traced back to the presence of the curriculum, the dialogic structure here was an attempt to try and trace other tensions present in the group sessions. Based on my analysis, I have identified tensions that I had not in previous analyses, although these are still linked to the authoritative presence of me as the teacher in representing the institution of the school in interaction with students. To further expand my study of tensions, I either need to set up a study that looks at students working on mathematics in different contexts in order to try and trace other sources of tension or, alternatively, I could look at the forces

that have shaped the curriculum in order to better understand what its influence over the students carries with it.

Using a dialogic reading of my transcripts to select extracts for analysis seems to have offered fewer opportunities to provide a dialectic commentary at the parallel analysis stage than in section 8.3 where I used a dialectic structure with a parallel dialogic and dialectic exploration which highlighted new themes or ideas. The difficulty in running a parallel analysis after a dialogic selection of extract may be a sign of a problem in the "combining" approach to connecting the theories, however, there are difficulties in concluding the benefits and issues with a dialogic structure based solely on the analysis offered in section 8.4. I have mentioned already that there were fewer students present for the third student group session. As a result, there were far fewer contributions than the other two sessions (92 contributions compared to 204 and 351), which gave less material to work with. I was also limited in my search for tensions (B3) by the dominating presence of my own voice in the sessions. The students do not interact directly with one another in the same way that the teachers did either through lack of confidence or due to my overwhelming scaffolding. My voice is the clearest one. As a result, the dialogic structure I have attempted to use here has not been as effective in providing a vehicle for me to investigate the student perspectives. In addition, there has been less opportunity for me to consider a dialectic perspective as part of the parallel extract analysis in the same way I have been able to in previous analyses. I suggest that the latter issue is due to the specific dialogic concept of tensions (B3) forming the basis for selection, which means my analysis is focused on a dialogic approach rather than a balance between the two.

In addition to my analytical investigation, one of my other intentions for Student Group Session Three was to allow students the opportunity to comment on and critique the methods of teachers (section 5.2.3). However, as evidenced in the extracts I have presented in section 8.4, the students can relate the methods to their own likes and dislikes but the discussion has not reached the stage of a critique. My additional intention was not successful in providing the students the subversive opportunity to critique the work of teachers in a reversal of the traditional assessment roles.

### 8.5 Points of Connection for Methodology

Analysing the student discussions has facilitated further scrutiny of elements of my analytical framework (section 8.1), such as, the effect of the authoritative word and the role of speech genres (B4) in narrowing down options for speech; the role of the teacher in shaping the ZPD; and a closer look at students engaged with the process of internalization (V2). The analysis has also highlighted areas in which a Vygotskian dialectic or Bakhtinian dialogic analysis are less effective. For a Bakhtinian dialogic analysis, attempts to search for tensions (B3) have proved only moderately successful. There is a strong tension between the informal and formal mathematical language (including, I have suggested, mathematical methods), but other tensions have proved difficult to trace unless students have made a specific comment, for example, in section 8.2.3 where the student raised the tension between home and school. One side effect of the lack of other visible tensions is that there is then focus on me as teacher-researcher. For a Vygotskian dialectic approach, the framing of methods in a hierarchy (V4) focused on progress and development (V3) undervalues the variety of methods students have discussed and adopted.

In section 7.3.2 I raised the suggestion of a bi-directional interpretation of the ZPD. Despite looking for examples of a bi-directional framing of the ZPD in my analysis, that interpretation did not come through as clearly as the linear framing did. There are a number of outstanding issues around a bi-directional interpretation of the ZPD. The first is that the examples I have found of student and teacher interactions that could be indicators of a bi-directional ZPD (sections 8.2.1 and 8.3.2) may not be true representations of a bi-directional ZPD. I suggest that in sections 8.2.1, the teacher is not truly swapping places to become the learner with the question for the students to show the method they have used but that this is, instead, a different way of leading the student. The example in section 8.3.2 of a new way of interpreting a method and my suggestion in section 7.2.3 that student input could have illuminated the methods the teachers were discussing, suggests there is an interpretation of student as expert. If this is the case, then there remains the benefit that this is a dialectic ZPD interpretation which could be used to help mitigate some of the implications of power relations in teacher-student interactions. As such, there needs to be consideration of how a teacher might facilitate (after Goos et al., 1999) more opportunities for a bi-directional ZPD, particularly (in my context) if there a particular difficulty for this with low prior attaining (LPA) students. Finally, and I think most significantly, there is the issue that a bi-directional ZPD after Radford (section 3.2) is an example of networking the

theories of Bakhtin and Vygotsky at the "synthesizing"/ "integrating locally" and thus, the underlying philosophical differences I discuss in section 4.5.1 suggests that this level of networking may not be possible. As such, my connecting theories approach with the networking strategies I am attempting offer an alternative way of dealing with the more traditional interpretation of the ZPD. I return to this discussion and ideas for further investigation in Chapter Ten.

In Chapter Seven I focused on a "comparing"/ "contrasting" networking strategy, testing if parallel analysis of extracts from my transcripts would build an analysis that offered more than an analysis based on only one theory. I concluded there that the "comparing"/ "contrasting" of dialogic and dialectic analyses allowed me to analyse more data by allowing analysis of discussions that could be better framed by one rather than the other, allowing a greater depth of analysis by extending understanding of the two theories in the process of comparing or contrasting and also allowing me to extend Bakhtin's ideas around language towards mathematical methods. In Chapter Eight I have shown that connecting theories at the level of methodology is possible in a number of different ways with my attempt to extend my networking strategies from "comparing"/ "contrasting" approaches, seen in Chapter Seven and section 8.2, to a "combining" approach in sections 8.3 and 8.4, which allowed for "multi-faceted insight into the empirical phenomenon in view" (Prediger et al., 2008, p. 11). The "comparing"/ "contrasting" approach in section 8.2 contains examples of where one approach is more appropriate for analysing a specific aspect of interaction than another (section 8.2.3), as well as where bringing both to bear proves illuminating (the discussion of power in section 8.2.4). From my two attempts at a "combining" analysis in sections 8.3 and 8.4, the dialectic structure of section 8.3 gave a framework that allowed a view of the development (V3) of a discussion over the session and the significance of more developed others, whilst leaving space for parallel analyses of student perspectives on mathematical methods. The dialogic structure of analysis in section 8.4 allowed an in-depth analysis of tensions (B3) that led into a discussion of speech genres (B4), but left less room for a dialectic analysis at the parallel analysis level, although part of this may be due to a lack of data.

The use of a "combining" approach with a focus on a dialectic structure to highlight the impact of teacher instruction on the ensuing discussion (section 8.3) has proved useful in allowing me to narrow the focus of my analysis, which could be both a positive and

negative strategy for a researcher, depending on the aim of the study. A narrow focus on a specific issue might lead a researcher to a detailed, in-depth exploration of this issue. As my project was more exploratory, I found it useful to have a more open approach, in the earlier stages of the study (Chapter Seven), allowing for the investigation of a variety of themes. I have discussed (section 8.3.4) that a narrower focus offered by choosing one theory over another can restrict the view of other themes emerging for example, if this is used to select extracts, as I have done, then selection based on one theory may mean other interesting themes are missed as they would have been in sections of text not selected.

Based on the analysis in this chapter, were I to follow a "combining" approach at the methodological level in future research, I recommend using a dialectic structure as it proved useful here, and then either attempt parallel dialectic, dialogic analyses within or, perhaps, select a dialogic analysis for this secondary, closer, level of analysis. Prediger et al. use the term "grain" (2008, p. 11) to describe these varying levels of analysis. However, one thought that echoed with this design of a combining approach was the problem of Matusov and Wegerif's (2014) discussion about changing a system from within that I discussed in section 4.2. Could placing a dialogic approach within a dialectic frame be described in a similar manner i.e., could it be said that the framing theory has too much significance over the one being used on the closer analytical level? There is also an ongoing consideration after Ochs (1979) as to the bias that is introduced by experiencing or using one theory before another. I am also aware that, due to some of the arguments raised by the authors in Chapter Four, I have not pushed the connecting of theories at a methodological level beyond the levels of connection that allow for theories with incompatible philosophical groundings. I discuss this decision further in 10.1.4.

In Chapter Five, I mentioned a possible classroom observation stage of my fieldwork (section 5.2.4). I planned to analyse my teacher and student groups and then decide whether to proceed with observations. However, the window for these to take place was a particularly challenging one for the department, which had a significant impact on the time they had available to participate (as I already mentioned in my introduction to Chapter Seven) and would have increased the stress levels of staff whose classrooms I would have been using for any observations. In addition, if I had decided to audio record sessions, the differences between a group session and a classroom session would have required significant discussion and had I video recorded sessions, aspects such as gesture would

have required extensive expansion of my methodological discussion. I waited until all analysis of previous stages was complete to see if I had grounds to justify classroom observations. However, as stated in my COVID-19 impact statement (p. 4) the closure of schools in March 2020 took the decision out of my hands. The circumstances mean I have missed the opportunity to further test my analytical tool and the opportunity to further emphasise the voices of LPA students by centring them in further data collection. However, the identification of my shifting role towards a stronger emphasis on the teacher aspects of my role means that the teacher-student relationship is not missing entirely from my study.

Chapter Eight concludes my exploration of methodology, the second of Radford's areas for connecting theories (2008). I have tested a number of networking strategies to help establish to what extent the methodologies of the two theories can be connected. By connecting theories, I have been able to more clearly discern the benefits and possible uses of the theories I have chosen to connect. In the next chapter, I am going to address research questions, the last of Radford's three points of connection. I am going to look at examples of research questions that have been used previously in a Bakhtinian or Vygotskian study before suggesting research questions for further research based on suggestions that have come from the analysis here.

## Chapter Nine – Research Questions

Radford's writing on connecting theories focuses on three elements; principles, methodology and research questions (section 1.3.1). In Chapters Two, Three and Four, I focused on exploring connections between the principles of the two theories and in Chapters Five, Six, Seven and Eight, I focused on building a methodology that allowed me to explore connections between Bakhtin's and Vygotsky's approaches. Here, I am going to discuss the final area of connection: research questions.

I have chosen to write my thesis without placing research questions in the traditional place. Radford is clear that "[a] research question […] already presupposes a "theoretical perspective" from which to state it" (2008, p. 322). As my thesis has focused on establishing a theoretical and methodological basis on which to connect the theories of Vygotsky and Bakhtin, writing research questions before the principles and methodological considerations had been explored could have led to research questions that contradicted my later exploration or could have limited my study by providing an incorrect framework for analysis. In the place of research questions, I have used Radford's writing on connecting theories (2008) and the networking theories of Prediger et al. (2008) to focus my research.

If, as Radford posits, looking at connecting theories "leads to a deeper acquaintance and understanding of the theories" (2008, p. 319), completing the discussion and analysis of the last eight chapters means I am now in a position to investigate research questions based on the understanding I have developed of the underlying principles and methodology of Vygotsky and Bakhtin. Suggesting "[a] set, Q, of paradigmatic research questions (templates or schemas that generate specific questions as new interpretations arise or as the principles are deepened, expanded or modified)" (Radford, 2008, p. 320) based on these two theoretical positions could serve as an illustration and effective summary of what I have discussed throughout the thesis and serve as a guide to future research.

The chapter has two aims: to consider the third element of Radford's connecting theories; and to set up future research. In order to explore research questions as an area of connecting theories, I am going to begin by critiquing early research questions I drafted for my study before its current structure was established. I am then going to review examples of stated aims and research questions that have been used by other authors in the field.

Based on these wider examples, I will then discuss research questions to focus on elements I would like to study in future research.

### 9.1 Initial Questions

In the early stages of my study, when I still planned on following a more traditional project structure, I drafted several sets of research questions. My previous work had been structured based on research questions but the ones I drafted for the project did not seem to fit the evolving understanding I had of the theories involved. At the halfway point of the research, my writing focused on the perspectives of others towards the mathematical methods selected by students and I drafted the following three questions:

> What are teachers' perspectives on students' approaches to mathematical tasks?
> What are students' perspectives on students' approaches to mathematical tasks?
> What tensions are there between these perspectives and how do they manifest in a classroom setting?

The set of questions did not reflect the complexity of the study and question three, particularly, showed an emphasis on a Bakhtinian approach that was not representative of my intention to use a connecting theories approach.

I continued to revisit the questions as my research progressed and this reflection resulted in supplemental questions. Whilst helping me link the drafted research questions more closely to the theories of Bakhtin and Vygotsky, they focused on me and my unfolding understanding of the theories I was using rather than questions that, through investigating, would make a contribution to knowledge:

1. Questions about teacher perspectives
    a. What are teacher perspectives on students' mathematical methods?
    b. Do I need a dialectic and dialogic version of these first before trying to combine?
2. Questions about student perspectives
    a. What are student perspectives on students' mathematical methods?
    b. Do I need a dialectic and dialogic version of these first before trying to combine?
3. Questions about the dialogic/dialectic theory

    a. Can a dialogic and dialectic perspective be linked in order to bring more to an analysis than they each would if they were used independently?
    b. To what extent does the layering of dialogic and dialectic perspectives make for a useful analysis?
4. Questions about my own position.
    a. Am I writing my project from a dialogic standpoint? A dialectic standpoint? Both? Neither?
    b. To what extent does the use of both dialogic and dialectic perspectives in an analysis add to our understanding of the perspectives themselves?

The reflective questions, developed based on my original attempt at writing research questions, led me to realise that until I had explored how I was going to use both Bakhtin's and Vygotsky's theories, forming research questions that reflected the results of my exploration of networking approaches was not going to be productive. As an alternative way of focusing my research, I used Radford's connecting theories framework to guide me. Until I had explored the connections between Vygotsky and Bakhtin in more detail and, as a result, improved my understanding of the two, shaping research questions risked not reflecting the development of my ideas through the study.

Having spent the previous eight chapters exploring principles and methodologies along with networking strategies for connecting theories at each of these stages, I am in a stronger position to consider research questions that reflect the theories. Radford explains:

> It is precisely because research questions are dependent on their theories that abstracts for articles are very often difficult to write: the author is required to state her research questions without having had the chance to lay down the theory from which the research questions borrow their meaning.
>
> (Radford, 2008, p. 322)

Before I write my own research questions that will lay out my plans for future study, I am going to look at how others have formed research questions or shaped their studies according to the theories of Bakhtin and Vygotsky.

### 9.2 Research Questions in Wider Literature

Before attempting to form my own research questions, I looked to the wider literature to see how others had interpreted Vygotsky's and Bakhtin's work to shape their own studies.

In section 6.4, I used Barwell's (2015) work to help me begin to shape my analytical framework. The key themes of Bakhtin and Vygotsky that I used in my analytical framework can help by similarly guiding research questions, reflecting the fact that "in order to tackle a particular question, the question […] has to be framed in a form that the theory can deal with" (Radford, 2008, p. 321). Barwell's ideas helped form the basis of my analytical framework and key terms identified in the framework and through my exploration in the group discussion analysis can be used in research questions to make them specific to one theory or, if avoided, provide a more general research question. Accordingly, terms such as internalization (V2), development (V3), progress and mastery would indicate a research question formed from a Vygotskian position. On the other hand, research questions based on Bakhtin's theories may include terms such as meaning making (B2), tensions (B3) and other (B1). In addition to key terms, questions should reflect the principles of the theory, for example, a Bakhtinian study would not look for any sense of progress towards universal reason and a Vygotskian one would not focus on the development of an individual working in isolation.

Radford (2008) highlighted a variety of examples of how theories shaped research questions, including a Vygotskian example: "Vygotskian theorists would perhaps have re-framed the question in terms of designs of zones of proximal development, interiorization of knowledge and meaningful participation in social praxis" (p. 321). Other studies that used the work of Bakhtin, Vygotsky or some combination of both are a good source of information as to how theories can shape research questions or project aims. Radford's 2006 article based on a Vygotskian inspired perspective focused on the question, "how do the objects of mathematics, sciences, aesthetics and other disciplines proceed from their primary intrapersonal origin to their ideal objectivity?" (p. 40). Here, the question contains the progress over time inherent in a Vygotskian analysis and the development of ideas to a scientific, abstract ideal. Barwell's 2014 paper was part of a wider project on the learning of mathematics and languages in multilingual classroom contexts. Barwell found it productive to "identify situations in which manifestations of unitary language came into struggle with manifestations of heteroglossia in ways that appeared to have a bearing on the students' learning of mathematics" (pp. 915–916), which clearly reflects the theoretical standpoint of the author and his roots in Bakhtin's ideas with the use of unitary language and heteroglossia.

For those referencing both Bakhtin and Vygotsky in their work, their research questions reflect a more complex theoretical position. Radford's (2000) work draws on Vygotsky and Bakhtin along with Wertsch's interpretation of the two in a study that "seeks to provide explanations about how students come to use signs and appropriate their meanings in the course of their initiation into the social practice of algebra" (p. 245). The aim has drawn on Vygotsky's work on signs (section 2.2.1) and assimilation/appropriation used by Bakhtin (section 2.1.5), combining both into one focus. Matusov (2011), on the other hand, looks to "analyze these irreconcilable differences in details by considering the case of a museum discourse to see how Vygotsky and Bakhtin's approaches would respond to it" (p. 104). Matusov's focus here is necessarily more general in asking about responses that allow him to consider both perspectives, without combining technical terms from both so he can maintain his position of treating the two theories as distinct.

These examples, taken from authors I have previously referenced, show how the research questions and aims demonstrate the theoretical perspectives of the authors. Díez-Palomar and Olivé (2015), studying interactive groups (IGs), use a combination of Vygotsky's and Bakhtin's work, stating the following research questions:

1. What types of interactions takes place when children and/or adults come to a deep, consistent and meaningful understanding of mathematics ideas?
2. Can dialogic talk clarify how children learn [mathematics] through interaction within IGs?
3. What might we learn about the practice of dialogic talk that will assist us in reinterpreting the role of the tutor?

(Díez-Palomar & Olivé, 2015 p. 1301)

Questions two and three are specific to their idea of dialogic talk (Bakhtin) and reflect the social aspects of both Vygotsky's and Bakhtin's perspectives. However, question one does not. Suggesting that it is possible to trace a specific interaction to "deep, consistent and meaningful understanding" (Díez-Palomar & Olivé, 2015 p. 1301) is contrary to Vygotsky's and Bakhtin's ideas. Specifically, the idea that meaning and understanding build through many interactions over time and, for Bakhtin at least, is ever changing. I need to make sure that my research questions do not similarly contradict the theories they embody. Exploring these examples from other studies along with the key ideas I developed for my analytical

framework (sections 7.1 and 8.1) and throughout my analysis in chapters seven and eight, I can now suggest my own research questions.

### 9.3 Suggestions

Using the technical terms and thoughts on what can be analysed from the examples of other authors combined with the ideas from my analytical framework, I am now going to suggest my own research questions. Writing research questions for a study at the end defeats the point of research questions as a guide and focus. Instead, I have taken the areas I would like to develop for future study raised in previous chapters (e.g., section 8.3.3) and am suggesting research questions that could help structure future work.

When writing research questions for a study based on two theories, the networking approach of the study affects the research questions used. As I have explained (section 1.3.2), different networking strategies have different aims, for example the "comparing"/ "contrasting" approach focuses on developing understanding of the two underlying theories, whereas a "combining" approach aims to give fuller insight into a specific phenomenon (see discussion in section 8.3). The full list of aims offered by Prediger et al. is:

- understanding each other (and ourselves),
- better understanding of a given empirical phenomenon,
- developing a given theory, or
- overall (long-term) aim: improving teaching practice by offering orientational knowledge or design results.

(Prediger et al., 2008, p. 16)

The different aims have implications for the research question design. Strategies that require improved understanding of individual theories or developing a particular theory, require questions which are specific to each of the theories involved. On the other hand, approaches that aim to develop understanding of a particular phenomenon or the implications for teaching practice might, instead, require a question based on the phenomenon or teaching issue under examination. In this instance, the question must be one that is possible given the principles and methodologies of both theories.

The following table shows some of the areas of future study that I identified in previous chapters and suggests research questions. I have separated the questions into those that

would be relevant to a study seeking to develop understanding of a Bakhtinian or Vygotskian position from questions that are relevant to a study aiming to investigate a particular phenomenon or teaching issue:

| Area for Further Research | Questions Based on Vygotsky | Questions Based on Bakhtin | Questions Based on a Phenomenon/Teaching Issue |
|---|---|---|---|
| Continued investigation into the use of bi-directional or other ZPD interpretations which allow for the interchangeable role of learner in an exchange. | Are there alternative, dialectic interpretations of the ZPD that allow for the possibility of teacher learning in interactions with students? | | How can the learning of teachers be analysed in the classroom when a teacher is working with students? |
| An approach to interviewing low prior attaining students without limiting the contributions of students. | How could the choice of interview techniques allow discussion of a progress-based curriculum with students who are not successful according to the requirements of said curriculum? | How can I better structure discussion with low prior attaining students so as not to mask their voices? | How can I balance my role as teacher/researcher to maintain the familiar relationship established as their teacher whilst also achieving my research aims? |
| Tracing the roots of tensions. | | What socio-cultural elements have contributed to the tensions created by the curriculum? | |
| An understanding of how to mitigate or address power relations in research. | Does a Vygotskian approach always frame imbalanced power relations as necessary for development?<br>Does addressing power imbalances affect opportunities for progress? | How could laughter be better utilised in discussions or in a classroom order to allow more opportunities for frank exchanges of ideas between those in unequal positions of power? | To what extent do power relations impact on student contributions to the discussion of mathematical methods? |
| Giving teachers more information about the ways in which they talk about student methods and how that discussion is framed by different theories. | By involving teachers in reflective practice based on Vygotsky's work, could they improve how they construct the ZPD for student learning? | By involving teachers in reflective practice based on Bakhtin's work, could they be more conscious of how they introduce sources for student meaning making? | By involving teachers in reflective practice based on theoretical study, could they make specific classroom choices related to student methods?<br>How could reflective practice be better facilitated in the context of a busy, stressful school environment? |

Figure 9.1 – Suggested research questions

The table shows that the areas for future study I have identified require a variety of different approaches. Some do not involve networking theories, for example, an exploration of tensions, which is related to one specific theory. Whereas, for others, such as the further exploration of power relations, a choice can be made between parallel research

questions based on building understanding of specific theories or a more general question focusing on the teaching or research issue.

### 9.4 Points of Connection for Research Questions

I have focused my discussion on the impact of theory on the design of research questions, but it is important to recognise that the links between theory and research questions work in both directions. An understanding of the underlying theory is necessary to shape appropriate research questions but the research questions themselves shape developments and explorations in the theories. The two elements influence one another (Radford, 2008, p. 321), making well written research questions even more important.

The benefits of a connected theory approach are clear when it comes to designing research questions. As I have discussed (section 9.3), the aims of a project affect the degree to which the researcher chooses to network the two theories and hence the styles of research question chosen, but connecting theories offers more flexibility. For example, when taking the research focus of how best to interview low prior attaining students, a "comparing"/ "contrasting" approach would mean that research questions from both Bakhtin and Vygotsky could be applied, allowing for the flexibility of a parallel analysis I have already discussed (section 8.5). Alternatively, if a "combining" approach was being taken, the question based on a phenomenon or issue might be more appropriate to allow for a particular structure to be adopted. These questions may also be more appropriate than the individual theory ones if the closer connecting strategies were being used.

Analysing how effective research questions are without testing them in practice is not possible. However, this chapter has demonstrated that writing research questions, which connect theories using a variety of networking strategies, is possible. Research questions are so specific to individual studies that, as I argued in my introduction to this chapter, without a rigorous grounding in principles, methodology and a clear idea of any networking approach, the research questions risk being disconnected from the theoretical underpinning of any study. My hope is that, having carried out this project, I have already explored much of the underlying theory and, as such, it will be easier to ground any future research in the theory.

I have now considered each of Radford's three possible areas for connecting theories. In the next, and final, chapter, I am going to discuss the effectiveness of using Radford's connecting theories framework along with the strengths and weaknesses of my study.

## Chapter Ten – Conclusion and Evaluation

In my final chapter, I am going to summarise my findings using the connecting theories framework I have used throughout. My thesis has been divided into three subsections based on Radford's three areas of connection: Principles (Chapters Two, Three and Four); Methodology (Chapters Five, Six, Seven and Eight); and Research Questions (Chapter Nine). I am going to continue to follow the same structure in my summary discussion here. I will begin by reviewing the outcomes of my work on each area of connection in turn before summarising my overall findings. I will then move to evaluate the strengths and weaknesses of my study before finally reiterating the possible areas of further research I mentioned in the previous chapter (section 9.3).

### 10.1 Connecting Theories

Radford's (2008) three areas of connecting theories: Principles, Methodology, and Research Questions have provided the structure for this project. I have used Prediger's networking strategies (see Figure 10.1, a repeat of Figure 1.2) to explore different approaches to connecting the theories of Bakhtin and Vygotsky for each of the areas.

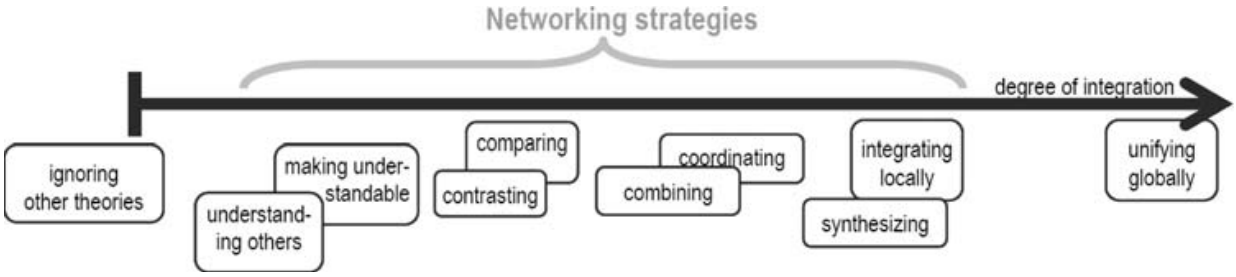


Figure 10.1 – Networking strategies (Prediger et al., 2008, p. 8)

To summarise the connections identified at each stage, I am going to take each area in turn and review my findings.

#### 10.1.1 Principles

Chapters Two, Three and Four contained my writing about connecting the principles of Bakhtin's and Vygotsky's theories, the "implicit views and explicit statements that delineate the frontier of what will be the universe of discourse and the adopted research perspective" (Radford, 2008, p. 320). In Chapter Two, I began by adopting "understanding others"/ "making understandable" approaches. I identified key aspects of each of Bakhtin's

and Vygotsky's theories in preparation for the more detailed discussion to come. In Chapters Three and Four, I explored the networking strategies of authors who have discussed the connections between Bakhtin and Vygotsky or who have used them both in their work. Chapter Three focused on authors who believe Vygotsky's and Bakhtin's work could be considered similar or as extensions of one another. Chapter Three demonstrated some of the benefits of applying networking strategies as each of the three authors I considered link the two theories closely in order to develop theory, from Wertsch's "integrating locally" approach to create his toolkit analogy (section 3.1.2) and develop a theory of mediated action to Wells's "synthesizing" approach to extending the concept of the Zone of Proximal Development (ZPD) (section 3.3.2). In contrast, in Chapter Four, the authors I examined tended towards the other end of the networking strategies spectrum, from the "making understandable"/ "understanding others" approaches of Matusov to the "comparing" strategy of Barwell, making a clear argument for the limitations of linking theories, particularly those based on different philosophical grounds.

Throughout the principles subsection of my study, I identified areas of possible connection between the two theories. I identified the sociocultural background, presence of other and valuing of texts present in both Bakhtin's and Vygotsky's theories as grounds for connecting the two theories. On the other hand, I identified the approach of each theory to difference, identity and power as significant disparities that meant any networking would have to be carefully considered. As a result of exploring connecting theories at the level of principles, I propose a "comparing"/ "contrasting" networking strategy for use with Bakhtin's and Vygotsky's theories, which permits the two theories to retain their own distinct features and accepts possible fundamental differences, particularly the philosophical incompatibilities argued by the authors featured in Chapter Four. In addition, "comparing"/ "contrasting" approaches allow for an investigation of "implicit assumptions and priorities in the core of the theories" (Prediger et al., 2008, p. 9), so, one could continue to develop their understanding of Bakhtin's and Vygotsky's theories in the context of a particular research problem.

### 10.1.2 Methodology

Chapters Five, Six, Seven and Eight focused on the second of Radford's areas for connecting theories. The methodology is made up of "techniques of data collection and data-interpretation" (Radford, 2008, p. 320), that built on the exploration of principles in the

first chapters. For this study, Chapters Five and Six explored the data collection and techniques I would need in order to be able to adopt a connecting theories approach to my analysis by developing a transcription style and testing early analytical techniques in order to apply networking strategies in an analysis.

In Chapter Seven, I took the early analytical techniques of Chapter Six and turned them into an analytical framework. I then put the framework into practice, analysing the teacher group sessions. The analysis followed "comparing"/ "contrasting" approaches, applying first one theory, then the other, which allowed me to discuss key areas of debate raised in the principles subsection of the thesis such as power relations for Vygotsky and Bakhtin, the role of teachers in the ZPD, and the distinctions between the developmental focus of Vygotsky and the widening of "discursive repertoires" (Barwell, 2015, p. 13) of Bakhtin, thus developing my understanding of principles beyond the areas I discussed in Chapters Two, Three and Four. The analysis here allowed me to consider the benefits of a "comparing"/ "contrasting" approach such as developing a deeper understanding of the theories involved, allowing flexibility to analyse sections of the transcript which may only have an interpretation with one theory and prompting the development of my application of Bakhtin's theory to mathematical methods.

In Chapter Eight, I updated my analytical framework to account for changes that arose in Chapter Seven and then continued with the "comparing"/ "contrasting" approach to networking for the first of the student groups to look at how my analytical framework worked in a student analysis rather than between teachers. Again, this gave me the opportunity to benefit from the connecting theories approach when one approach proved more useful for analysing a situation than the other. For the remainder of Chapter Eight, I wanted to shift the intention of my analysis away from developing my understanding of the theories as they pertained to the methodology, to improving my understanding of the phenomenon under consideration. As such, I trialled a more closely networked approach for my analysis of the final two student group sessions. I adopted a "combining" approach "in order to get a multi-faceted insight into the empirical phenomenon in view" (Prediger et al., 2008, p. 11). To do this, I used one of either Bakhtin's or Vygotsky's theories to shape my selection of extracts from the transcriptions and then applied the parallel analysis of the extract selected in this manner. Using either a dialectic or dialogic structure alongside a parallel analysis allowed me to focus on specific issues that had come up in earlier

discussion such as the impact of the pressures of the curriculum on interactions and tracing tensions beyond the curriculum. Of the two, a Vygotskian overall structure proved more productive for analysis using the "combining" approach. Moving forward, I propose that for those looking to connect the theories of Bakhtin and Vygotsky at a methodological level, it is possible to use a number of networking strategies depending on the intention of the connection. Exploring the theory in the methodology subsection of this project has allowed me to explore the boundaries and nuances of the two theories in a way that would not have been possible had an exploration been limited to a literature review chapter. I also consider alternative interpretations of the ZPD in an attempt to find a dialectic way of mitigating power relations by suggesting a way of students being experts on their own learning but only found a little evidence of a bi-directional ZPD. However, I have suggested instead that my experiments connecting the theories of Bakhtin and Vygotsky in a "combining" manner still allows for a framing of students as joint contributors to learning and the learning of teachers.

### 10.1.3 Research Questions

In Chapter Nine, I considered different approaches to writing research questions that would work for a study adopting a connecting theories approach based on the theories of Bakhtin and Vygotsky. Moving forward, I propose that questions that retained the distinct language of the two different theories and would be appropriate for a study that focused on improving understanding of theories, following networking strategies such as "comparing"/ "contrasting". On the other hand, questions that focused on a specific phenomenon or practical issue, and could be investigated using a networking strategy such as "combining".

### 10.1.4 Final Discussion of Connecting Theories

Through my project, I have taken Radford's ideas around connecting theories and applied them to a discussion of mathematical methods in order to explore where I am positioned in the debate around using both Vygotsky and Bakhtin in an analysis. By exploring networking strategies at the three key points of connecting theories: principles, methodology and research questions, I have established that, through either "comparing"/ "contrasting" or "combining" approaches, the two theories can indeed be applied to an analysis. The result is a level of insight into either theories or phenomena that goes beyond that of applying one theory in isolation. Radford suggests that connections happen at

different levels and the two theories do not have to be consistent throughout to be connected (Radford, 2008, p. 322), which ties into both the "comparing"/ "contrasting" and "combining" approaches, keeping the principles of the theories separate but offering connections at the level of methodology and research questions. This networking position allows me to benefit from the connecting theories approach in that it helps me deepen understanding both of the theories themselves and the phenomenon under examination, whilst steering clear of the closer connections that could be rendered invalid by discrepancies in the underlying philosophical positions of the authors. The specific networking approach depends on the motivation of the study in question, reflecting Radford's idea that "the possibilities of connectivity rest, in the end, on the goal of the connectivity and the possibilities afforded by the principles of the theories under consideration" (2008, p. 324).

Investigating possible areas of connection by exploring networking strategies has allowed me to investigate the approaches taken by others in the field and develop an analytical framework. I have been able to test this framework to analyse teacher and student discussion groups. In Chapter Eight, I tested analysing a specific phenomenon using a "combining" networking strategy. Having tried with both a Bakhtinian and Vygotskian structure, I suggested that an overall structure based on a Vygotskian perspective, followed by parallel analysis of specific extracts using my analytical framework could prove useful. One of the reasons a "combining" approach with this structure could be useful is because it reflects the strengths and weaknesses in both approaches. The dialectic perspective, with its focus on development and internalization or mastery worked well as an overall structure, as it echoes the specific expectations and progress over time represented by the curriculum and then passed on, via the teachers, to the students. However, a purely dialectic approach to analysing phenomena risks losing the subtlety of what is happening as social interaction begins to create meaning. A dialogic perspective gives a detailed sense of what is happening in the moments where meaning is being made from difference and the forces at work in the dialogic space opened by the difference. However, the dialogic approach, with its wide variety of sources of meaning available to those involved and the difficulty in tracing them makes an analysis challenging. Connecting dialogic and dialectic positions allows boundaries to be identified within which meaning is being created, without aiming for a complete abstraction of concepts, while simultaneously allowing for discussion and analysis of interaction and utterance in a way that is not so individualised

that it cannot be analysed. Such an approach avoids the extremes of the two theories, as discussed in section 4.5.2.

There is an argument that connecting theories may have led to a less focused study than if I had chosen one theory to base my study on at an earlier stage. However, writing about Bakhtin, I was always going to have to deal with the connections to Vygotsky as much of the literature discusses their links and without the connecting theories approach, I would not have been able to deal with this in as much detail. I hope that by examining and experimenting with the connection between Bakhtin and Vygotsky I have offered a suggestion for how the two theories can be productively used together in the future (see proposals in sections 10.1.1, 10.1.2 and 10.1.3). Focusing on one theory would also have meant that I missed out on the benefits of a connecting theories approach, namely, the flexibility to analyse data that can be better framed by one theory over the other, the greater depth offered by analysing data by theories and the opportunity to extend the applications of Bakhtin's theories around language. This extension is particularly important as in my exploration of mathematical methods as a language in a Bakhtinian sense, I see methods as subject to centripetal and centrifugal forces thus clearly linking Bakhtin's theories to teaching and learning of mathematics. I suggest further research in this area in section 10.3. With Vygotsky's theory representing the curriculum and Bakhtin's the detailed analysis of how socio-cultural settings impact meaning making, connecting the theories allowed me to investigate the original dilemma I had around the demands of the curriculum and associated examinations contrasted with the variety of approaches employed by students to answer questions and the teachers seeking to validate those methods. Having both theories allowed me to carefully select and test aspects of both theories to build an analysis that specifically fit my context.

To develop a study based on connected theories requires transparency and reflection at each stage. Basing a study on the more closely connected networking strategies is particularly challenging. Prediger et al. suggest that "the last two strategies […] must usually build upon the less integrative strategies and hence need more time to be evolved" (2008, p. 12). The approach has to be based on what is being investigated and the underlying principles of the theories to be networked must be investigated thoroughly.

## 10.2 Evaluation

My thesis is a multi-layered analysis, firstly, of connecting two theories and, secondly, of analysing the perspectives of teachers and students on mathematical methods. In this section, I will evaluate the strengths and weaknesses of my study.

### 10.2.1 The Strengths of My Study

My study set out to explore the possibility of connections between two theoretical positions using Radford's (2008) areas of connectivity. My exploration has taken the approaches of other authors in the field who have used the work of Bakhtin and Vygotsky, along with the critiques they raised, and built a functioning analytical framework that neither dismisses the differences between the theories, nor treats them as insurmountable. The resulting framework allowed detailed and specific analysis of data collected from my own context and has suggested a number of areas of further research based on both practical and theoretical questions. As I have mentioned in section 10.1.4, my hope is that this exploration of the possibilities of connecting the two theories contributes to the debate in the field around if and how these two theories can be used together. I have also aimed to show how care must be taken when connecting and any networking strategy carefully selected based on the researcher's intention. I feel my work has demonstrated the significance of considering each of Radford's possible areas of connection (2008).

In addition to exploring the theories of Bakhtin and Vygotsky, I have also explored the possibility of interpreting the mathematical methods of students as a form of language, with individual methods representing utterances with a clear sense of both a student's voice and the heteroglossia of the other voices that influenced the production of the utterance. This is not an extension as a benefit from a closely connected networking strategy, but instead a benefit of the "comparing"/ "contrasting" approach which allowed me to identify a gap and speculate that Bakhtinian framing could prove fruitful for an analysis. I have demonstrated in my analysis that teachers and students interact with the methods of others in a dialogic exchange and link these mathematical methods back to the influences on the people who have created them and the socio-cultural context in which they were working. Mathematical methods not only carry the student voice but are also shaped by responsivity in specific contexts and tensions. Bakhtin's argument that "[t]he entire methodological apparatus of the mathematical and natural sciences is directed towards mastery over mute objects, brute things, that do not reveal themselves in words"

(1981, p. 351) framed mathematics as a remote, unassailable discipline. However, this framing of mathematical methods means that the student voices in evidence are lost. The result is a flat "right or wrong" view of students' mathematical utterances, particularly in examinations, which discounts the unique contributions of students. I argue that mathematical methods are representations of the voices of students and we, as educators, researchers and assessors, do them a disservice not to think of their methods as such. I discuss further research on this point in section 10.3. but if mathematical methods are considered as utterances, then mathematics becomes situated, context-based and moulded by those using it rather than a remote, untouchable, "pure" discipline. In addition, if mathematical methods begin to be considered as a type of language then, like language, they should be taught with a sense of culture and history.

The group sessions, focused on mathematical methods, allowed opportunities for those that did not normally get the chance to have mathematical discussions. The teacher group discussions were particularly successful, with in depth discussion of mathematical methods that let me explore a number of key theoretical issues such as the role of laughter and examples of tensions. For teachers, taking time out of their schedules to focus on discussing mathematics was a rarity at the time. Also, students seldom had the opportunity to look at examples of student work form those outside their class. As a result, the transcripts were fascinating for me to analyse and both teachers and students mentioned an enjoyment of the process of sorting through examples of others' work. For instance, the following extract, taken from Teacher Group Session One (Appendix Six) pre standardisation of transcripts:

> N: It's actually really interesting um process of looking at how differently people do things and how differently people teach things. It would be very useful as training to do this sort of thing more I think. To pick up tips and think about "oh yeah and do I do that as part of my practice..."

Also, consider the following extract, taken from Student Group Session Two (Appendix Ten) and not used in the study so, again, transcript not standardised:

26. R: Anybody else found any that they'd like to talk about? So I've got one, two, three, four, five, six at the moment.
27. A: I like looking at them.
28. R: It's good isn't it. I find it really interesting seeing how other people did this.

One of the motivating factors behind my project was to give voice to teachers and students and to make sure they got something out of participating. They gave so generously of their time, it was important to me that those involved took something from the process. In this respect I believe I was successful.

A final contribution of my project was to highlight my own voice as teacher-researcher based in the specific context of my own school, working with my colleagues and students. The reflective element of the study has led to significant changes in my own understanding and led to a variety of areas of interest for further research.

### 10.2.2 The Weaknesses of My Study

Despite the strengths of the connecting theories structure, there are areas of my study that, in retrospect, may be considered weaknesses. The first is my focus on the work of particular authors (Chapters Three and Four). To condense the large body of prior work to a thesis necessitated choices about whose work to include and whose to leave out. These choices were considered carefully and at no stage have I tried to represent my study as a comprehensive write up of the entire field, but no doubt others working in the same way would have made other choices and, as such, potentially drawn different conclusions. The impact of exploring power relations through the prism of their writing is in evidence in my analysis. Matusov's particularly strong stance has had an obvious impression on my study, with its growing focus on the impact of power. It is important to say that the interpretations of power I focus on in this paper, such as the impact of the curriculum and the leading of students via the ZPD or Bakhtin's Authoritative Word are only two among many. Despite acknowledging the perhaps out of proportion influence of a limited group of authors, their influence allowed me to develop a significant line of thought with regard to power roles and their impact such as in sections 7.2.4 and 8.3. Nevertheless, including the work of others, might have led my study in other directions and with other interpretations of power, my analysis would be different.

As a result of the context of my study and the authors I chose for my literature review, my initial interpretation of the ZPD was focused on an overcoming and linear progress model. This interpretation was in line with the shape and influence of the National Curriculum and examination expectations that student use particular mathematical methods (section

7.2.1). Despite considering alternate forms of the ZPD (section 7.3.2, 8.5 and 10.1.2), I found little evidence of a bi-directional ZPD in my analysis of the student groups and suggest that the idea of a bi-directional ZPD formulation might represent a connecting theories position beyond that which I was taking based on philosophical inconsistencies. However, if a purely dialectic bi-directional formulation of the ZPD would help introduce a way of mitigating the power dynamics at work from a dialectic perspective. A lack of evidence of this in my analysis of the transcript, particularly of the student groups, indicates that more research is needed and there may have been other factors at play that affected the formation of a bi-directional ZPD. I wish to discuss this further in section 10.3.

A second potential weakness is that of platforming the voices of certain students. I have mentioned already that the teacher groups were something that I feel has worked well, giving a platform for teacher voices. However, the student groups were not as effective at maintaining the voices of low prior attaining (LPA) students. In section 8.3.3, I discussed how the style of questioning that I used during the student group sessions may have masked the voices of the students involved and may have contributed to a lack of opportunities for a bi-directional ZPD. These group sessions were one of my main ways of giving a voice to LPA students. I feel that, although the sessions were a start, they were not as effective as I wanted them to be. A more careful approach is needed to help structure group sessions when the participants find discussing mathematics difficult or, alternatively, different arrangements could be made to access these voices in a context in which they feel more comfortable.

### 10.3 Future Steps

By focusing on connecting the theories of Vygotsky and Bakhtin, my investigation has laid the foundation for a variety of future paths for study, either by suggesting the continued exploration of the use of the two theories as discussed in section 10.1.4, or by raising a number of practical areas for study. In Chapter Nine, I discussed a variety of research questions that might be appropriate for future research and, along with the discussion of the strengths and weaknesses I identified in section 10.2, I would like to focus on two areas for any future study. Firstly, extending my understanding of power roles, specifically by developing ways of accessing the voices of LPA students and exploring alternative formulations of the ZPD to consider the learning of teachers. Secondly, continuing my exploration of the analysis of mathematical methods through Bakhtin's view of language.

In order to continue to explore the power dynamics of learning, I would like to look at how laughter (dialogic) and a bi-directional ZPD (dialectic) could be used to give students, particularly LPA students, a voice (dialogic) or the opportunity to be experts on their own methods and mistakes (dialectic) within the examination and curriculum framework of the school. Part of this is in allowing the voices of LPA students to come to the fore. To do this, I would like to explore different techniques for discussion group settings such as the one I used or look at alternative ways of giving LPA students space to contribute. These techniques may involve choosing different questioning styles, setting the groups up differently, for example, making them more mixed groups so LPA students do not feel focused upon. Alternatively, I could look at their voices in an alternative setting in which they may be more comfortable. Extending my understanding of other ZPD interpretations would also allow me to consider the learning of teachers as I would like to investigate further instances where they are learning themselves, possibly in groups such as the teacher discussion group that formed part of my project, or in their classrooms, in their roles as teachers.

Secondly, I would like to continue to develop a Bakhtinian analysis of mathematical methods. Looking in more detail at the context surrounding students as they form written mathematical methods and the ways in which this is in evidence in the methods could provide more evidence of the centripetal and centrifugal forces at work or of the speech genres that are in play in school mathematics. The way that prior experience and utterance shape the mathematical methods of students is significant to teaching and learning as if we understand the impact of prior experience on the mathematical methods themselves, we can make changes to the elements of that experience we have influence on. We can also think about how the prior experiences that we do not have control over can be leveraged in the learning of mathematics.

For both of these areas of further investigation, one way to address them is to carry out classroom investigations. In a classroom context, the focus could be more effectively placed on LPA students in a familiar and, therefore, possibly more comfortable environment. It might be possible to capture more of the influences on mathematical methods as they are influenced and informed by the prior utterances and the discussion going on around them Classroom observations would also be useful to explore alternative formulations of the

ZPD, including the bi-directional ZPD and teachers' learning in the classroom environment. My COVID impact statement (p. 4) explains why classroom observation was not possible for my study and, in section 8.5, I explain why, even without these particular circumstances, classroom observations would have been beyond the scope of the study. However, testing my analytical framework in a classroom environment is an important next step.

### 10.4 Final Word

My study of connecting theories through this project has allowed me to make significant strides in exploring my role as teacher-researcher, both through interactions with teachers and students, and through theoretical exploration. My hope is that, through this study, I have contributed in some small way to Prediger et al.'s view that "to develop connectivity of theories means to reduce isolated approaches and gain more connected knowledge" (Prediger et al., 2008, p. 17) as part of the "mathematics education research community" (Prediger et al., 2008, p. 6), whilst simultaneously contributing space and time for the discussion of mathematics to the students and teachers of my own school.

I do not believe that I, as a teacher of mathematics am marching students towards a predetermined destination with no opportunities for deviation nor individuality in my teaching the mathematics curriculum. Equally, I believe that labelling students with a number or letter grade in a way that carries an inherent sense of their worth is morally questionable and is so in a way that is inherently unfair to students who find mathematics difficult. My PhD research has been an opportunity for me personally to engage with the questions of where I stand as a teacher and as a researcher.

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## Appendices

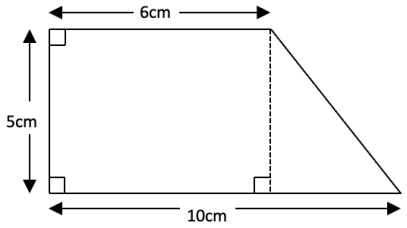
1. Blank Artefact
2. Ethics Form
3. Ethics Board Confirmation
4. Parental Permission Letter
5. Transcription Exercise
6. Transcript Teacher Group Session One
7. Transcript Teacher Group Session 2a
8. Transcript Teacher Group Session 2b
9. Transcript Student Group Session One
10. Transcript Student Group Session Two
11. Transcript Student Group Session Three
12. Additional Analysis of Teacher Group One
13. Additional Analysis of Student Group One

## Appendix One – Blank Artefact

### I WANT TO TAKE PART

1. I have £62 to share between 4 people. How much does each person get?

2. Calculate the area of this shape.

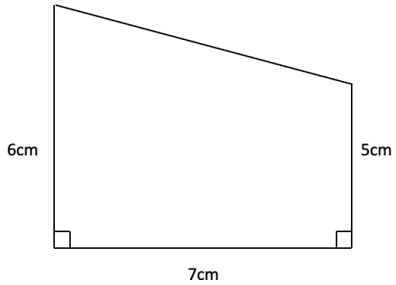



3. Alice is twice as old as Judy, who is younger than Ben by 3 years. If the sum of their ages is 27, how old is Alice?

### I <u>DO NOT</u> WANT TO TAKE PART

1. I have £42 to share between 4 people. How much does each person get?

2. Calculate the area of this shape.




3. Alex is twice as old as James, who is younger than Beth by 3 years. If the sum of their ages is 27, how old is Alex?

## Appendix Two – Ethics Form

GSoE RESEARCH ETHICS FORM

It is important for members of the Graduate School of Education, as a community of researchers, to consider the ethical issues that arise, or may arise, in any research they propose to conduct. Increasingly, we are also accountable to external bodies to demonstrate that research proposals have had a degree of scrutiny. *This form must therefore be completed for each piece of research carried out by members of the School, both staff and students*

The GSoE's process is designed to be supportive and educative. If you are preparing to submit a research proposal, you need to do the following:

1. **Arrange a meeting with a fellow researcher**
   The purpose of the meeting is to discuss ethical aspects of your proposed research, so you need to meet with someone with relevant research experience. A list of prompts for your discussion is given below. Not all these headings will be relevant for any particular proposal.
2. **Complete the form on the back of this sheet**
   The form is designed to act as a record of your discussion and any decisions you make.
3. **Upload a copy of this form and any other documents (e.g. information sheets, consent forms) to the online ethics tool at :** https://dbms.ilrt.bris.ac.uk/red/ethics-online-tool/applications**.**
   **Please note: Following the upload you will need to answer ALL the questions on the ethics online survey and submit for approval by your supervisor (see the flowchart and user guides on the GSoE Ethics Homepage).**

If you have any questions or queries, please contact the ethics co-ordinators at: gsoe-ethics@bristol.ac.uk

**Please ensure that you allow time before any submission deadlines to complete this process.**

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Prompts for discussion

You are invited to consider the issues highlighted below and note any decisions made. You may wish to refer to relevant published ethical guidelines to prepare for your meeting. See http://www.bristol.ac.uk/education/research/networks/ethicnet for links to several such sets of guidelines.

1. Researcher access/ exit
2. Information given to participants
3. Participants right of withdrawal
4. Informed consent
5. Complaints procedure
6. Safety and well-being of participants/ researchers
7. Anonymity/ confidentiality
8. Data collection
9. Data analysis
10. Data storage
11. Data Protection Act
12. Feedback
13. Responsibilities to colleagues/ academic community
14. Reporting of research

Be aware that ethical responsibility continues throughout the research process. If further issues arise as your research progresses, it may be appropriate to cycle again through the above process.

**Name(s):** Harriet John
**Proposed research project:** Using Teacher and Student Perspectives to Explore the Dialogic Pedagogy of Mathematical Progress
**Proposed funder(s):** Self-funded
**Discussant for the ethics meeting:** Danielle Sinclair.
**Name of supervisors:** Alf Coles, Laurinda Brown
**Has your supervisor seen this submitted draft of your ethics application?** Yes

The Project

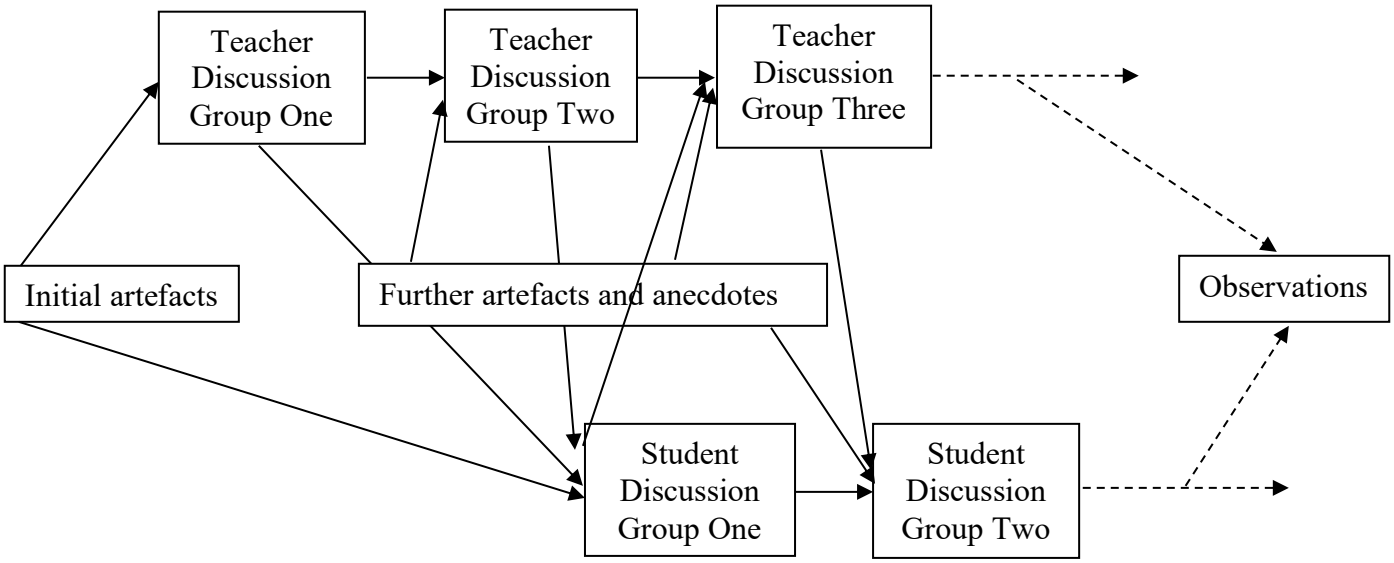
I am investigating text formation and meaning making in the mathematics classroom in order to explore what it is that helps students to make mathematical progress. In order to do this, I plan to carry out a three stage project. To begin with, I will form teacher groups with other teachers from my faculty. I am a teacher of mathematics in a secondary comprehensive school and want to bring the expertise of the other teachers in my department together in order to collect perspectives from different classrooms. My role within this group will be as both teacher and researcher. Before the first group session, I plan to create a set of artefacts in order to prompt discussion amongst the teachers as to the use of mathematical methods and progress by students. The artefacts will consist of the same mathematical problem, solved by students across different year groups. I will ask all of the students that I teach to complete the problem and then bring the solutions along to the first group session. This session's discussion will form the basis of the discussion for the next session, along with any artefacts or anecdotes the teachers would like to bring. This discussion will form the basis for the third discussion and so on. I would like the discussion group to be a cyclical process where the project is being continually moulded and developed as we work as a group. These sessions will be audio recorded so that I can review them and also so that I can feedback and agree a transcript of proceedings with the other members of the group.

For the second stage of the project, I would like to form a similar group with students. This will allow me to develop the understanding from the teacher groups by adding another perspective to the discussion. The student groups will be run in a similar way to the teacher groups. The first session will be based around a discussion of the same artefacts as the first teacher group was and then subsequent sessions will build on and develop from the preceding ones. The students taking part in the group will all be volunteers and, depending on uptake, I plan to take volunteers from a variety of year groups and from a variety of teachers' classes. Again, these sessions will be audio recorded, allowing me to agree a transcript of events with the participants and use this transcript to prompt future discussion, alongside any artefacts or transcripts that the students would like to bring themselves.

Finally, I plan to carry out a series of classroom observations. The form these classroom observations take will be partially based on the results of the discussions from the teacher and student groups. I will be looking for examples of text formation and meaning-making in context. The classrooms I would like to go into will be those belonging to the teachers involved in the group discussion stages and I plan to open

mine to them in return. From these lessons I intended to collect audio recordings, observation notes and samples of student work in order to be able to analyse the progress that is taking place.

Below, I have attempted to illustrate how each stage of the process is linked to the other stages. As you can see, with this number of links between stages, the project is going to be an emergent cyclical process where I, as researcher, will have to change and adapt at each stage based on what has gone before. This has obvious ramifications for my ethical considerations.




I will now take each part of the process and look at the key ethical issues for that stage.

Ethical issues

**Before starting**

Access has been arranged via the Senior Leadership Team at school. I have spoken to the Head for formal permission, then I spoke with members of the senior leadership team and my Head of Faculty for practical access. I will also speak with the safeguarding officer to ensure that I follow school permissions before each stage of the project.

I need to be aware that I am operating within two ethical frameworks – those of the university, and those of the school. As a member of staff at the school and a researcher at the University of Bristol, I have a responsibility to both institutions. I do not foresee a situation in which these responsibilities will clash, but it is a possibility that I need to be aware of. By maintaining a professional approach to the research and by leaving value judgements of teachers and students out of my data collection and analysis, alongside obtaining all possible permissions and consent at each stage, I can portray all participants in a non-judgemental manner. If I find that this is not

possible, then I will need to consult with my Head of Faculty or with my supervisor in order to resolve any clash between my two roles.

**The data collection process**

I will only collect the data that I have planned for my study. To collect unnecessary data would cause me to have to do extra work and would be unethical for the participants. I will make sure to follow all guidelines that I discuss here to ensure anonymity and wellbeing. Being a teacher, I do have an insight into the concerns of my colleagues and this will hopefully allow me to be sensitive to their concerns. I will also take care not to impose what I want the outcomes to be upon the data at the collection stage. This is a delicate balance as I am taking an active role in the research. It is also worth mentioning a withdrawal option for the researcher too. Time management will have to be done carefully as I am teaching full time as well as researching.

Stage One – The teacher groups

Before the teacher groups begin, I will be collecting artefacts from students. For this, I will give two tasks. If students wish to participate, they do the first one. If not, they complete the second. This way they can opt out if they are not comfortable without having to declare it outright. Individual permissions of students will be by their choice to complete the question and covered by the school's existing permissions. The students will not be asked to put their name on their work. I will use colour coded paper to identify the different year groups, but will not share the system with the teachers as we begin discussions to allow them to make judgements without any preconceptions as to age/year group.

I will ask for volunteers from staff to take part in the group. This is the only way to ensure that I am not imposing on teachers' valuable time. In order to make it clear that the other teachers have the right to withdraw at any time, I will make sure there is ongoing conversation about permission and withdrawal. Verbal permission will be obtained from participants at the group discussion stage (which will then appear in transcripts to be approved by the participants). I must be sure to include statements about use for future publication. The participants will be kept anonymous, but I plan to give them the choice of how they want to be referred to – an initial? A name they have chosen? Etc. It is important that the participants understand what my aims are at each stage of the project and what the data I am collecting is going to be used for. As the project develops, these could vary from stage to stage and I will have to make sure that I am continually reviewing the information given to the participants.

I do have to be aware that I am in a position of authority over some of the participants. I line manage one of the teachers who has expressed an interest in participating and I will have to manage this relationship carefully throughout the project, making it clear at all stages that my research does not involve value judgements over the participants' teaching. As I am not a member of the senior leadership team, it is not as worrisome to participants and I will make sure to make clear that my focus is not behaviour management/marking etc., but a very specific phenomenon within the classroom.

The teachers taking part will be invited to give feedback and check the accuracy of the record. This will in turn shape my analysis as the project I am planning is very much a collaborative process. They will also be given the opportunity to view the project once it has been written up.

Stage Two – The student groups
I will ask for volunteers from students for the voice groups. This is the only way to ensure that I am not putting pressure on students to take part, or potentially including vulnerable students. Permissions of parents will be sought as a courtesy at the student voice stage – It is important to do this as I am a representative of the school and have a responsibility to portray them in a positive way. I will also ask for verbal permission at the group discussion stage (which will then appear in transcripts to be approved by the participants). I must be sure to include statements about use for future publication. In order to make it clear that the students have rights of withdrawal I will make sure there is ongoing conversation about permissions and, as with the teacher groups, I will share up-to-date information about the data I'm collecting and the uses of it at each stage.

When it comes to possibly including students that I teach in the groups, care must be taken as the power relationships are different than those between a teacher and a student that I do not teach. This could potentially have an impact on my future teaching of the student. Group sessions are also preferable from a safeguarding point of view for me as well as for the students.

Above my ethical choices for the project, there is a school policy that covers the students' wellbeing, and safeguarding officers to ensure that this is being followed. I have chosen group sessions for both teachers and students in order to minimise the intensity and pressure to hold discussion. I will also use my knowledge of the school and staff, alongside my professional judgement (as a teacher) to help. There is a complaints procedure built into the existing school systems, so those would be the first recourse for any student, parent or teacher with a concern, followed by contacting my supervisor at the university.

Stage Three – Observations
The final observation stage will hopefully unfold from the group sessions and this may make it easier for the teachers involved to feel comfortable with my entering their classrooms to observe. If they find it a natural progression from the research they have already completed then it will make this normally intimidating observation process more comfortable. By opening up my classroom to them as well, I hope to make it clear that this is a cooperative learning process rather than any type of evaluation.

The permission of the teachers involved in observations will be covered by existing school permissions that allow observation, but the permission of teachers will be sought at this stage as a courtesy and the students will be given the right to leave the class if they do not want to take part.

**After the data collection**

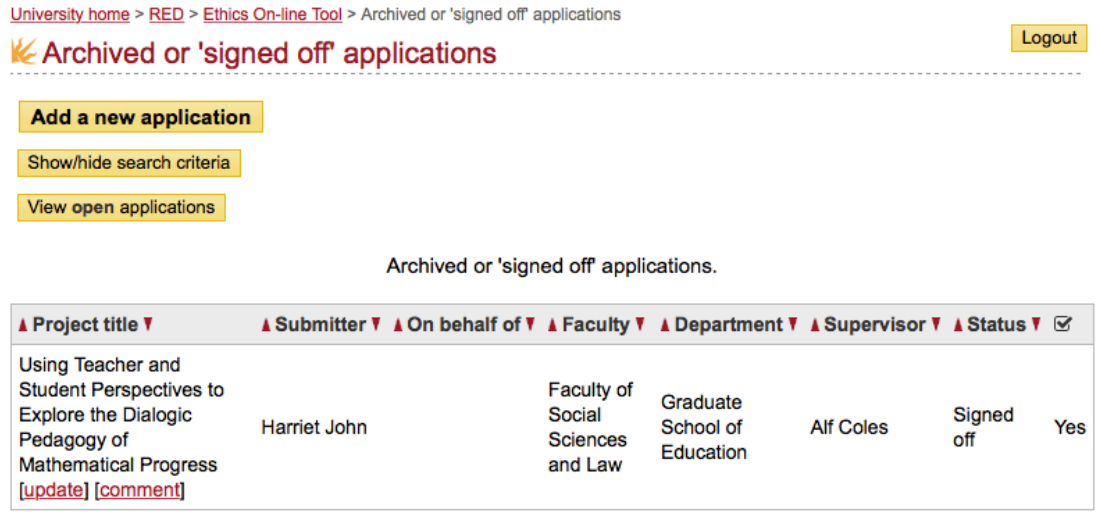
The security of the data I generate is key. Any electronic data capture (audio recordings etc.) will be stored on the secure, encrypted university server provided for this purpose. Other physical documents (photocopies of student work etc.) will be kept anonymously in a secure, locked location. My storage of data and the anonymity of my participants will adhere to the Data Protection Act (1998) in as much as it will only be used for the purposes I have agreed with the participants, it will be agreed as an accurate representation with the participants and destroyed once it is no longer useful. I am not collecting any specifically sensitive data, such as the participants' ethnic background or political opinions, so extra steps do not have to be taken to protect this.

I must be careful about any values/judgements that I bring to the data analysis. What I write about artefacts/other teachers/the school/my observations etc. reflects on the school and the university. It may also affect my professional relationships with my colleagues and students going forwards.

Permission for reporting my research will be sought at each stage of the research. Reporting will not involve any value judgements as I have already explained. Again, the reporting of my data will be in consultation with my supervisor and any ethical issues that arise in reporting sensitive information can be discussed within the support systems of school or university as they arise.

I will still be a teacher at the school so I will be around for the follow up and, despite exiting my role as researcher, I will still be seeing students/staff members and using the work in practice. Ideally, all participants taking part in the study gain something from the process.

## Appendix Three – Ethics Approval



University home > RED > Ethics On-line Tool > Archived or 'signed off' applications

Logout

Archived or 'signed off' applications

Add a new application

Show/hide search criteria

View open applications

Archived or 'signed off' applications.

| Project title | Submitter | On behalf of | Faculty | Department | Supervisor | Status | |
|---|---|---|---|---|---|---|---|
| Using Teacher and Student Perspectives to Explore the Dialogic Pedagogy of Mathematical Progress [update] [comment] | Harriet John | | Faculty of Social Sciences and Law | Graduate School of Education | Alf Coles | Signed off | Yes |

| ID | Name | Faculty | Department | Supervisor |
|---|---|---|---|---|
| 38442 | Miss Harriet John | Faculty of Social Sciences and Law | Graduate School of Education | Professor Alf Coles |

**Status**
Signed off

**Date added**
June 5, 2016

**Signed off date**
Aug. 4, 2016

**Is this a student project?**
Postgraduate Phd

**Project title**
Using Teacher and Student Perspectives to Explore the Dialogic Pedagogy of Mathematical Progress

**Estimated start date**
June 15, 2016

**Duration (months)**
24

**Project outline**

I am investigating text formation and meaning making in the mathematics classroom in order to explore what it is that helps students to make mathematical progress. In order to do this, I plan to carry out a three stage project. To begin with, I will form teacher groups with other teachers from my faculty. I am a teacher of mathematics in a secondary comprehensive school and want to bring the expertise of the other teachers in my department together in order to collect perspectives from different classrooms. My role within this group will be as both teacher and researcher. Before the first group session, I plan to create a set of artefacts in order to prompt discussion amongst the teachers as to the use of mathematical methods and progress by students. The artefacts will consist of the same mathematical problem, solved by students across different year groups. I will ask all of the students that I teach to complete the problem and then bring the solutions along to the first group session. This session's discussion will form the basis of the discussion for the next session, along with any artefacts or anecdotes the teachers would like to bring. This discussion will form the basis for the third discussion and so on. I would like the discussion group to be a cyclical process where the project is being continually moulded and developed as we work as a group. These sessions will be audio recorded so that I can review them and also so that I can feedback and agree a transcript of proceedings with the other members of the group. For the second stage of the project, I will form a similar group with students. This will allow me to develop the understanding from the teacher groups by adding another perspective to the

discussion. The student groups will be run in a similar way to the teacher groups. The first session will be based around a discussion of the same artefacts as the first teacher group was and then subsequent sessions will build on and develop from the preceding ones. The students taking part in the group will all be volunteers and, depending on uptake, I plan to take volunteers from a variety of year groups and from a variety of teachers' classes. Again, these sessions will be audio recorded, allowing me to agree a transcript of events with the participants and use this transcript to prompt future discussion, alongside any artefacts or transcripts that the students would like to bring themselves. Finally, I plan to carry out a series of classroom observations. The form these classroom observations take will be partially based on the results of the discussions from the teacher and student groups. I will be looking for examples of text formation and meaning-making in context. The classrooms I would like to go into will be those belonging to the teachers involved in the group discussion stages and I plan to open mine to them in return. From these lessons I intended to collect audio recordings, observation notes and samples of student work.

**Supporting information**

See ethics form uploaded.

**Appendix Four – Parental Permission Letter**

Developed from a school proforma as part of the ethical conversation. The unheaded, redacted version is included here to preserve anonymity. Completed responses from parents on file.

Dear Parents/Carers,

I am currently completing a PhD in Mathematics Education at the university of Bristol. Part of my research concerns the methods used by students when faced with mathematical tasks. One of the things I am doing in order to investigate this is a short series of group discussions with students and I am writing to ask permission for your child to take part.

The discussion groups will be made up of approximately five students who will be asked to complete some mathematical tasks and then discuss the methods they used. The work will not be marked or assessed, they are just an opportunity for students to discuss their ideas.

The group sessions will take place in lunchtimes (to fit around your child's other commitments) for around 20 minutes and there will be three sessions arranged over this term. Sessions will be voice recorded and any written solutions produced by the students may also be used in writing up the research. The student will remain anonymous throughout the investigation and all data collected will be kept in accordance with new GDPR regulations. Students have the right to withdraw at any stage.

Should you wish to ask any questions about the research, please contact me by email [redacted] or by phoning the school.

If you are happy for your child to participate in these sessions, I would be grateful of you would complete and return the reply slip below.

Yours sincerely,

Student Name:_____ Form:_____

I give permission for my son/daughter to participate in an educational research discussion and for their contribution to be anonymously included in any reporting of the research.

Signed:_____Parent/Carer Date:_____

## Appendix Five – Transcription Exercise

My aim in this appendix is to first show how I initially processed the audio data and then discuss different styles of transcription in order to decide which will be the most useful for my analysis. I will then summarise the choices I have made for my final transcription excerpts that will be used through all future analyses. To do this, I am using a short extract from the start of one of my transcriptions (Teacher Group Session 2b, Appendix Eight). I do not use this extract as part of my analysis so it serves a purpose here instead.

### Provisional Transcription

My provisional transcripts are available as Appendices Six to Eleven. The aim of the provisional transcription is to get a rough outline of the session, allowing me to note the entire session, and obtain an overview of all the contributions made by participants. The resulting documents are long and, although they have been proofread, there are still some sections that contain typographical errors and the occasional word that was inaudible in the first couple of attempts at transcription. Certain sections of the transcription may have been summarised, for example, general chat at the end as I was reaching to turn off recording devices or if a participant turned up late and I recapped the task for the session (shown in the transcription with a short description). I also made basic alterations to preserve the anonymity of the participants: in practice, changing how I refer to them; redacting the names of any students mentioned; and redacting one section of conversation between two teachers that referred to a student and their particular learning needs. A sample from this initial run through is shown below:

> R: Alright, we're recording. Is everyone happy to be recorded?
> V: yes.
> (K is nodding)
> R: I need a verbal one I'm afraid… [laughing] stop it!
> K: Yes, I'm absolutely happy.
> R: Ok, we've got another teacher possibly joining us in a second. Right, quick recap 'cause it's been a while…

**Transcription Attempt One**

The provisional transcription gives a good sense of what is happening in the early stages of the session as I gain verbal permission from participants and set the scene based on what had happened in the previous session. The transcription allows for clarity as to who is speaking when. I have chosen to adopt some conventions at this stage such as standardised spelling or common abbreviations for example '"cause". I have also used punctuation to denote the start and end of sentences, where participants have asked questions and made exclamations. Pauses are marked by ellipses and actions are denoted in square brackets.

The transcription shows that the first part of the session was relaxed and there was a good relationship between the participants with the joke at the start. It gives us enough detail to get a sense of what is going on in the background, e.g., rummaging through papers and participants nodding rather than giving verbal

permission, and shows how the scene was set for the rest of the session. However, there are inaccuracies in the transcription, for example, spelling mistakes or even a word missing or substituted for another. It is a first run through only, and the aim of it is to get a sense of the overall discussion rather than allow close analysis of details. As a result, whilst allowing me to identify key sections and place those sections within the wider dialogue, this stage of transcription is not completely reliable and may mask some important aspects such as the use of key words, and some of the more subtle interplay between participants, e.g., if several are speaking at once.

Clearly, refinements need to be made to this process. The obvious place to start is to revisit the transcription repeatedly and work to make it more accurate, but I wanted to experiment with standard transcription practices to see if they lent further detail or clarity to my study. As a result, I looked to Ochs (1979) and her writing on transcription practices. What follows is my attempt to use some of her suggestions and a discussion of how they are or are not useful to my study.

**Standard Transcription Practices**

The decisions I make around my transcription are not just a case of giving more clarity to who is saying what. Ochs (1979) writes that "transcription is a selective process reflecting theoretical goals and definitions" (1979, p. 44). What I choose to transcribe, how I lay out this transcription and, sometimes most importantly, what I choose to leave out, is important. My choices should reflect my theoretical stance and will impact on the conclusions I will be able to draw from my data. As a result, it is important for me to be able to justify these choices and explain why they are best for my study.

To begin with, I wish to address some of the key issues with my provisional transcription (the first transcription attempt). Specifically, the inaccuracies and the fact that, though this extract is easy to identify as the opening section of the transcript, other extracts would benefit from some sort of reference to make it clear where they come in the discussion, which will be particularly useful when I have gathered several quotes together in an analysis. My attempts resulted in Transcription Attempt Two:

1 R: Alright, we're recording. Is everybody happy to be tape-recorded?

2 N: Yes.

3 [K is nodding]

4 R: I need a verbal one sorry...I know… [laughing] Stop it!

5 K: Yes…

6 R: Thank you.

7 K: …I'm absolutely happy.

8 R: OK we've got another teacher possibly joining us in a second. Um so quick recap 'cause it's

9 been a while…

**Transcription Attempt Two**

Transcription Attempt Two is slightly different from the first in terms of content. Transcription errors have been corrected and one or two words that were missed on the original transcription have been added such as on line 4. I have adopted some further conventions such as writing numbers as words rather than digits and numbering my lines. These decisions have been made as they lend useful information to the reader. Using words rather than digits for numbers illustrates details such as how different people speak about numbers such as 102 which could be "one hundred and two", "one zero two" or "one oh two", which may reveal information about interpretation of place value for example. Understanding of place value is something that could be important to people interpreting mathematical methods, especially in examination scenarios.

The introduction of line numbers allows readers to see at what point in a dialogue extracts are taken from. Line numbers can also make it clearer in what order the exchanges take place, although this is not foolproof as there are times when participants are speaking simultaneously. If this is the case, the line numbers continue sequentially, but I have noted it in square brackets if multiple persons are speaking at the same time or I have used ellipses if one person is interrupting another such as in lines 5, 6 and 7. Line numbers do give a sense of time and context within the discussion, which is central to both a dialogic and dialectic investigation.

The alterations I have made here still result in a transcription that is easy to read and allows identification of sections of the session that may prove interesting to analyse further. However, these changes do not give the detail of pauses or tone of voice and there is a discrepancy in that I am giving details of actions that are not evident on an audio recording, for example the nodding on line 3. My transcriptions do occasionally describe actions that can be heard for example people arriving, doors opening, papers shuffling or people laughing. I have included descriptions of these actions as they often describe breaks in discourse or mark a change in topic. However, the nodding cannot be heard in the audio recording. It is taken from my recollection of the session rather than what I can hear and was added as it helped give some context to the exchange and to explain the joke. However, it may not have a place in the formal transcription as I have no record of the actions that took place in the sessions and cannot be consistent in my inclusion or exclusion of them. To include actions in my analysis would require video recording or note taking at the time and further literature review, which goes beyond the scope of this stage of my study (see section 5.2.4 for further analysis of this point).

To introduce more consistency and derive more detail from the recordings, I am going to repeat a section of this transcription using Ochs's suggestions for standardised transcription notation. A complete summary of Ochs's standardised notation can be found in her 1979 work (pp. 63–65), but I am going to use the following conventions here:

/ to mark an utterance boundary
// at the start and ] at the end to mark an overlap in speaking

, to denote a low rise in intonation
? for a high rise
. for a low fall (also can mark the boundary of an utterance)
! for an exclamatory utterance
((LF)) to denote laughter
- h for an audible in breath, h for an out breath
_____ marks emphasis
(.) for a brief pause

It is important to note that Ochs does not use utterance here in the same way that Bakhtin does. For Ochs, an utterance is a unit linked to a physical action bounded by "a single intonation contour and single breath group" (1979, p. 63), unlike Bakhtin for whom "[t]he boundaries of each concrete utterance as a unit of speech communication are determined by a change of speaking subjects, that is, a change of speakers" (Bakhtin, 1986, p. 71).

1 R: Alright/ h we're recording/ is everybody happy to be tape-recorded.

2 N: Yes/

3 R: I need a verbal one sorry/ I kn- ((LF)) Stop it!

4 K: Ye//s]!

5 R: //Th]ank you/

6 K: I'm absolutely happy/

7 R: OK we've got another teacher possibly joining us in a second/ Um so (.) quick recap 'cause

8 it's been a while/

**Transcription Attempt Three**

Here, the extra notation has captured a couple of items of interest. At the end of line one, the researcher is asking a question, but the intonation does not rise as it may on a traditional question. I have previously used a "?" to end this line of transcription, but according to the transcription style here a "." would be more appropriate to mark a low fall in tone rather than a high rise as would traditionally accompany a question.

I have removed the description from line 3 which explains a little of the context for the following two lines. The nodding is not audible on the recording, so, as I have previously discussed, it may not have a place in a formal transcription of the tape. All of these changes of notation have caused a slight problem in that they sometimes affect the line numbers. This will mean that the line numbers are inconsistent from the provisional transcripts, making them less useful to place separate sections within the overall session transcript.

Despite extending the use of standard notation to give a more detailed picture of what is happening on the recording, the extra notation interrupts words, losing the natural flow of what is happening and making the overall transcript less

readable. My analysis is interested in the dialogue between people and these extra details are interrupting the flow of that dialogue. From my discussion around the unit of analysis in the previous section, one of my key methodological aims is to keep the interplay between participants clear, particularly as this helps delineate the start and end of exchanges and, as such, where I begin and end my selected extracts.

In addition to standard notation, Ochs discusses the use of a table as a model transcription format (1979, p. 62). The following is a suggested style:

| | | Participant A | | Participant B | | Comments |
|---|---|---|---|---|---|---|
| Time | Line | Non Verbal | Verbal | Non Verbal | Verbal | |
| | | | | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |

This layout is designed to break down some of the prejudices that come with a Western left-to-right, vertically downward reading style, since "[m]aterial presented to the left tends to capture the reader's attention before material to the right does" (Ochs, 1979, p. 54). This table format places non-verbal actions on the left in an attempt to overcome the tendency by readers to background nonverbal interaction. However, as I have previously discussed, I am not in a position to detail the nonverbal information present in sessions, as this was not something I recorded at the time. In addition, due to the multiple participants, this table would be impractical in a formatting sense and would further disrupt the flow of the dialogue. As such, this format does not serve my purposes.

Ochs's point about the layout drawing attention to my voice as researcher as the first and last voice on each recording highlights the fact that, despite my wish to be a participant in the fair and equal exchange of ideas, I hold a particular position of power in these exchanges. I am the one controlling the recording (even if I am not the one guiding the discourse at various points), I am the one who has brought these people together for sessions and the one who will bring sessions to a close. Numbering the lines highlights this clearly through the simple fact that I am always line 1. The comment section does allow me to add notes such as the nodding that had previously been removed. However, this raises the same issues about adding context not in evidence on the audio recording and the temptation with this column is to include interpretation or analysis at a stage that is simply supposed to focus on recording what is heard. Obviously, there is interpretation going on anyway as "the problems of selective observation" (Ochs,

1979, p. 44) are still a danger, but steps can be taken to minimise this where possible.

**Discussion**

By considering different versions of the formatting of my transcription, I have highlighted some key areas that need addressing in any final version. I have discussed how or if I incorporate actions into any transcript, how I go about addressing punctuation such as question marks, and some key formatting issues such as line numbers becoming increasingly challenging to keep consistent between extracts embedded in my analysis and the initial transcriptions included as appendices. This final discussion aims to discuss each of these in turn and present a final transcription.

Of the four transcription attempts I have made, Transcription Attempt Two, with its incorporation of some of the standard transcription tools to make it easier to read and line numbers to make it easier to place within the wider transcript, has proved the clearest and most useful. I will, therefore, take Transcription Attempt Two as the starting point for my final attempt. I have made a specific choice not to use some of the formal frameworks and tools available to me due to the fact that they mask some of the interplay involved in the transcriptions, which is important to my analysis. To address the formatting issue of line numbers and make the numbering more practical to use, I have decided to use contribution numbers rather than line numbers. This will still give a sense of where in a transcript an extract has been taken from, but will mean that I am not forever tweaking numbering as words run over to the following lines. If I require more detail during an analysis, I can then introduce a more detailed breakdown of a contribution by line at that stage. I have also included a – to denote a break of contribution, for example if I only include the second half of a contribution due to a topic change halfway through, something often in evidence in my speech summarising one topic and moving on to another.

It could be argued that introducing standard punctuation and spelling, whilst making it more straightforward for a reader, is an act of interpretation on the part of the transcriber. Ochs discusses using phonetic transcriptions for young children but explains that "[a]s the child's pronunciation approaches adult norms, use of phonetic representation should be less critical" (Ochs, 1979, p. 61). As such, the speech of teenagers and adults in my transcriptions of student and teacher groups follows standard spelling with the inclusion of some modified orthography, e.g., 'cause, in line 8 (now contribution 7), adding to the readability or sense of natural exchange I have previously discussed. However, introducing standard punctuation is more difficult. In my analysis of Transcription Attempt Three, I discuss the question mark that had previously been used at the end of line (now contribution) 1. It is clear from this discussion that including punctuation such as question marks is an interpretative act on my part and is something that the standard transcription notation used in this attempt did address, albeit whilst making other parts of the transcription less clear. Standardising punctuation can also imply pauses, which may not have been there in the original discussion and, as such, could cloud my use of pauses to mark the end of exchanges. To address this, my final version will use full stops to denote a

one second pause, with pauses under one second marked by (.) and over three seconds by, for example, (4) for a four second pause. I will also remove capitalisation at the start of sentences as this implies punctuation.

Finally, with regards to the incorporation of actions into a transcript, there are some key actions that serve to highlight the context of the discussion, for example, the door opening or a new person joining. These actions will continue to be noted in my transcriptions as they are audible on the tapes. I will note them in italics and take advantage of the left-right bias I discussed above by placing them to the right of the corresponding transcription of discourse, clearly showing the emphasis is on the discourse. However, only audible actions will be included. If other actions are necessary to illuminate the discourse further, then these can be included in subsequent discussion.

**Appendix Six – Teacher Group Session One**

R: Excellent, it's working. Everybody happy that I'm recording this and for the purpose that I can use this later on.
All (V, K, N, I): yes
R: Fabulous ok so what we're going to do today is I'm going to give you some questions and I'm going to ask you to do them in slightly different ways and then we're going to talk about how you've done them and if there are similarities in the methods that we all use and if there are differences and see where that leads and that will then hopefully shape the next conversation that myself and whoever turns up to the next one will talk about. Is that OK?
All: Mmmm (nodding).
R: Fabulous, alright, N, you had a question?
N: I was going to ask if you wanted us to approach these questions in the way that our students would approach them or as teachers?
R: Perfect, OK, so what I'm going to give you (to N) it [your question] feeds into what I'm about to explain.
N: Oh I see, that's what I thought.
R: What I'm going to give you is a copy of some questions that I asked all of my students to do from year seven through to year thirteen they all did the same three questions and you'll notice this is split into two parts. They were asked if they wanted to take part or if they didn't and they filled whichever half of the sheet was appropriate for that. I'm assuming…please tell me if I'm wrong… that by agreeing to be here that you are happy to take part in this (laughing) it's going to form part of my PhD study so therefore will be written up and potentially put into my thesis but also into articles and mentioned at conferences and things like that so if you are at any stage not happy, not just necessarily now, if you want to disappear then please don't hesitate just walk out or let me know or whatever… the first time we're going to do this…is I would like you to do this and if we stick to the I want to take part halves as opposed to the other half um just if you were doing it for yourself if you were just scribbling down some ideas for yourself doing the question at home just to work out an answer that sort of thing.
Does that make sense to everybody? Does everybody want to take a couple of minutes to do that now? ... So whatever method you find springs to mind as it were...
[M arrives with biscuits – recap of recording, instructions and right to withdraw]
[working in silence]
M: (not serious) Is it a test?
R: It's not a test, no.
R: I forgot to say you don't need to write your names on the sheets. It's the first question all my classes asked "do we write our names on them" I was like "no you don't write your names on there"
K: (to K) are you just checking?
N: (to K who's leaning over her work) Oh have I made a mistake?
K: (agreeing) Mmmm
[laughter]
N: fours into 6 goes… Yes, so I have, I beg your pardon.
R: It's alright, it really doesn't matter…
[general chat]

M: I have got the last question wrong.
R: That's alright…it makes it…it's…either way it's interesting to talk about so that's fine. Ok we have a choice now. Do we want to talk about how we've all done this? Or do you want to do it my alternative way and then talk about it afterwards? What do you think? Keep going with the maths? OK, second one then. If you could maybe, I don't know, write a big 2 at the top of this so we know this is your second version. I forgot to bring a stapler. I was going to staple them all together.
**5 mins**
This time what I'd really like you to do is I'd like you to do it in the way that you were teaching it to a class of students, OK, and for arguments' sake let's say KS3 maybe a set 2, somewhere in the middle does that make sense? So if you were teaching it to a class of students, how would you present this problem?
[working in silence]
N: Question number two would kind of depend on whether I was teaching area of a trapezium or more simple shapes.
R: That's fine. You can do both methods or you can pick one. It's entirely up to you.
[working in silence]
M: I got it right this time!
[working in silence]
[clarifying only need to do one half of the sheet for M]
[working in silence]
R: (peeking at V's work) Just being nosy! OK excellent, shall we pool them all in the middle. Is that alright? Everyone labelled one and two on their sheets just so that when I'm collecting in I know which order we did them in? So, shall we start with… Did anyone do exactly the same thing on both?
K: Yes
V: Pretty much
I: I did two different methods for question one but the others are pretty much the same.
R: Shall we talk about question number one then? What sort of methods have we got?
N: Did both – half and half again and divide (correctly this time).
R: When you say divide correctly, what method did you use for that one?
N: I use what they call the bus stop… short division method
R:I can see lots of bus stops looking round
M: I wrote the bus stop out but I didn't actually use it in my own head. I just knew that it was going to be 15.50 but then I wrote it out again over here 'cause if I was doing it with students then I would probably do the bus stop method.
R: That's nice. What's that? Is that related to the bus stop method or is that just like an alternative?
M: That's just what you'd need to do so I wrote that down.
R: So that's 62 over 4 turned into a fraction.
M: Yeah 62 over 4. I'd probably write that like that if I was teaching.
R: Nice, as opposed to using divide symbols and stuff?
M: Yeah, exactly and then I'd probably do a bus stop method.
**10 mins**
R: What about you?

I: So I did bus stop when I did it myself and I thought about how I would teach it – I'd split it into 60 and into 2 and divided each of those by four and then put them back together.
R: People who used the same thing for both, is that because that's you do it that way and that's how you teach because you think that's the easiest method for doing it?
V: Um, I tend to encourage them to use the bus stop method whenever they possibly can because they forget and then when I prompt them "What method do you have for dividing?" "Don't know, don't know, oh bus stop!" "Can you do it like that?" "Yeah, probably."
R: Who put units on their answer?
K: Me.
R: For both?
K: Yup.
M: Oh, I didn't now that you mention it?
K: Oh no, only for one.
I: Not for the last question.
K: Yes you did, there and there.
V: Not that one.
(All talking over each other)
M: I put units the first time but not the second time.
K: So did I, don't know why.
R: I did mine the other way round, when I was showing the students I made sure to put the units on, but when I was doing it myself I didn't bother.
K: You also didn't put it to the correct number of decimal places.
R: I also didn't bother with the correct number of decimal places, no I didn't which is really interesting, so hang on…
M: What's the correct number of decimal places?
R: If it's money then two theoretically.
M: Isn't this a question about area?
[clarified we're looking at Q1 still]
N: I didn't either.
R: You've got a pounds sign there.
N: I have got a pounds sign there.
R: Is that for you or is that for the students?
N: That's my one, I didn't bother with the students'!
V: I went back and tacked it on my one at some point. When I did it myself I basically stopped at 15.5 and then carried on and at some point there thought "I need a zero on that and a pounds sign on that."
R: That's fair enough. Do we want to talk about the second one then? Alright so… V, you've written just some random numbers on your first one. Do you want to explain that?
[chat]
V: The rectangle was clearly 5 by 6, there weren't any numbers down the sides of the triangle, so I scribbled them in as 5 by 4 and then wrote the areas in the middle of the spaces and for the kids I wrote in the middle I still put random looking 5, 6 and 4 on the lines but in the middle of the spaces rather than just writing 30 I've done 5 x 6 = 30 and 5 x 4 over 2 equals… that's meant to be 10

even if it looks like a 6 or a b and I did the adding up in columns and showing it rather than doing 30 + 10 in my head.
R: K you've used two completely different methods pretty much.
K: Yeah…
R: [long pause] Would you like to expand upon that?
K: Errr, well, I, well, when you said it was key stage 3 a sort of set 2, well I would probably do it by compound shapes to start with because that way you can deal with any number of compound shapes rather than just this specific one which happens to have a different specific rule so I was working on the principle that I would rather show them a process they can apply to other things rather than just how to do it for that question.
R: Would you have done it differently if I'd said for a key stage 4 class or…?
K: That depends on what point we were in the topic. I think if we were just starting to introduce more complex shapes then no I would have done it the same as I did the second time as a compound shape. If we were getting on to the bit where we have a rule for doing trapezia then I would have used that so as N said it depends on what you've just been teaching them really.
R: Which did you do N for your two?
N: Umm I showed both when I was doing the teaching bit umm so but I didn't actually write down the answers I just showed what calculations you needed to do to work out the area of the trapezium or the area of a rectangle and a triangle. Actually if I was really teaching to a class I would do what V had done and write the calculations inside the shapes…
[interrupted by a student]
**15 mins**
…because that seems to really work with compound shapes when they are learning cause then they can see that they need to add the two bits together.
R: Which did you do yourself?
N: I worked out the area of the rectangle and the area of the triangle and then added them together.
K: Can I just say that what I wanted to do was to also do it by working out the area of the greater rectangle and the missing triangle and show them that you can do it that way as well
R: Ok, so multiple methods for that really. M?
M: In my head I just did 8 x 5 because it was clearly the average of 6 and 10 and then here I did write it out with the numbers for the… area of the trapezium rule and also I've written it as two separate shapes on there I haven't written the answers but obviously I would go through all the actual answers if I was really teaching it. They do need the area of a trapezium formula so it's not crazy to introduce but as K says it depends what you've been doing really.
R: And I, your beautifully meticulous method
I: Yeah, so I've basically did the same thing both times I was just more explicit about what I was doing for thinking about teaching the students. So I did it by compound shapes just because of the way that one's set out really lends itself to seeing it as a rectangle and a triangle whereas if it had been more of a trapezium with like triangles on both sides if you know what I mean I might have been tempted to use the trapezium formula instead but that just felt the most simple way of doing that one.

R: And beautifully laid out! I did mine using compound shapes both times, just differed how I wrote them so for the one I did for the students I wrote it directly on to the shape my methods and I wrote it out using the divide symbol here rather than timesing by a half because I know that some of mine find that a little bit tricky or the way that I did it which is writing as a fraction. I also remembered to put units on my answer when I showed it to the children! [joking with another teacher writing on their sheet] You don't need to add it now! So I put that in there cause I know that's another thing they quite often don't pay attention to and there is a mark allocated for it in various places in various tests which is a bit rubbish anyway, but important for them. Lets talk about question 3 because we've all pretty much used variations on exactly the same method. Anyone like to start?
N: We've resulted to algebra.
R: Yeah. Everybody! Everybody has done it as an algebraic problem.
K: I was trying to think of a way of leading them through without leaping into algebra and I couldn't think… other than trial and improvement I couldn't think of an effective, time economic way of doing it.
R: There's sort of, you can do trial and improvement for it within certain limits can't you so if you know that she's going to be twice as old as this person who is younger than… that gives you some limits on how old they can each be to start with and the fact the sum has to be 27…
V: So the mean is going to be about 9…
K: So 56 is probably not a good guess!
M: That's a good point.
R: [to V] Say that again?
V: So the mean's got to be 9 so there's a reasonable first guess for whoever the middle age is which is Judy is it? No, you don't even know what the middle age is do you… You would guess it's probably Ben because he's a bit older than...
M: Yeah, Judy's the middle age, I mean Alice is the middle age.
R: Or Ben's the middle age?
N: Alice is the oldest isn't she?
V: Yeah, Alice is the oldest.
M: [rereading the question] Oh, okay.
N: Ben's got to be in between Judy and Alice.
V: Again, and actually if you do that and think well Alice is twice as old as Judy and Ben's only three years older than Judy, assuming Judy's more than 3 years old, then Ben's the middle so lets give him, if you're doing a trial and improvement, 9, which is the mean average of the ages, see where that takes you. It would happen to land you on the right set of numbers.
R: As if by magic!
K: However as a process for trying to do that it's not a terribly useful strategy for answering a lot of questions that are turning words into algebra.
V: No
R: No
V: However a process of saying roughly what do I expect it to be and then doing a sort of mean thing and then going is that about right if they're all about [inaudible] years old.
K: Exactly, as we were talking about that earlier, is that a reasonable answer.
R: [to a teacher adding to method, laughing] Stop changing your answer!

N: It doesn't affect my answer! I just notated it wrongly here because Judy wasn't x – 3.
R: It's fine! I said to the students, it drove the students up the wall because I told them I wouldn't tell them what the right answer was. That didn't work, they all wanted to know what the right answer was.
**20 mins**
K: Yeah, having an idea, as you say, having an idea a) what you're looking for and b) whether or not what you've got is a reasonable answer is certainly something I would want to encourage with them "Does that make sense, you've got an answer of -12 does that make sense as an age?" you know?
R: It's interesting that we all went with the exact same method for the last one because the students definitely didn't.
M: What did they do?
R: Well bear in mind that some of the students were [year group], we had a really, this is the conversation for next time so there was a really interesting mixture of completely random trial and error, and then some really well thought out trial and error, and then some sort of half algebraic, half logical methods sort of leading on to fully algebraic methods at the end but not all of the algebraic methods then led back to…
[Another units discussion]
R: Ummm… can we just get rid of all the second ones for a second, is that alright? [paper sorting]
V: Is it worth talking about the thought process of who we chose as our variable in question 3?
R: Oh yeah, actually, that's a good point.
V: I chose Judy, but looking round the table, did most people use Judy?
M: I chose Judy the first time when I got it wrong but then the second time I chose Alice because actually she was the person we wanted to know about and it seemed easier to choose her straightaway because then she'd already be in the answer.
N: I chose Judy because you can place her without really changing the other things twice – that's two times Judy and that's Judy plus three.
[M and V agreeing]
K: I chose Ben.
V: Yeah, which is the one I would definitely not choose.
K: Well I'd already noticed the difference between the 'if you want to take part' and 'if you don't want to take part' being gender specific so I just chose Ben just to be difficult. No I just went for Ben because everything is based on Ben. I agree afterwards having looked at everyone else's things it would have been far easier to do Judy but…
V: Why is everything based on Ben?
K: Well Judy is based on Ben and Alice is based on Judy and therefore implicitly is based on Ben and therefore everybody is based on Ben, but unnecessarily complicated so anyway, as you can see (gesturing at workings).
V: fair enough.
R: OK, anything else? I just wanted to look at the ones [first version of solutions] and see if there's anything we noticed, but I don't think so. Anyone else got anything that we've done wildly differently when we did it for ourselves?
[considering questions]

I think that's everything. Excellent, thank you very much. So, next time if you fancy coming along again which will be at some point next term, I'm going to bring along some examples of… are people just changing their answers afterwards? [teachers were annotating on their solutions!]
M: I just couldn't work out where I'd actually gone wrong and that was what upset me!
[chat]
R: So next time I've got like 200 versions of these that the students did and what would be really nice to do is to bring some of those along and you can have a look and see what is the same to how we've been doing it and how we teach it and how they do things differently, cause they really do things differently to us.
**25 mins**
N: It's actually really interesting um process of looking at how differently people do things and how differently people teach things. It would be very useful as training to do this sort of thing more I think. To pick up tips and think about "oh yeah and do I do that as part of my practice, yes I normally write the unit."
K: Less so about whether or not you remember to put something in that you know you should but more do you do it this way, do you do it that way or do you do it both cause I'm often unsure as to whether when I show them that there's more than one way of doing whether I've actually clarified things or if I've muddied it completely 'cause they start looking at you sort of a bit cross-eyed and glazed and you think OK maybe that was one method too many there.
V: But then sometimes you can go "You know you can also do it this way" and they go "why didn't you tell us that months ago sir?"
K: Well exactly.
N: But also it allows them to be a bit more creative about the way that they approach doing questions if they've got choices about how they're gonna do it….
K: As long as the choice doesn't paralyse them, yeah.
M: It should also be like a better conceptual understanding cause you can link it to more different kinds of thinking and more different kinds of processes.
K: At our level, yes.
M: That's why you find it easy because you can see all the different, you can see what all the connections are whereas if you're just seeing a process that you can't really put it in its place unless you…
K: But when we're answering question of that form we have a bank of different strategies for doing it which is what we're hopefully trying to give them is a batch of different ways of being able to find these things, rather than just you do this, you do this, you do this and you write down whatever the answer is.
M: With units on.
K: With units on.
R: The really interesting stage is that, I'm going to do this process again, but with students and talk to them about how would you do it if they were just doing it for themselves, like in the back of your book or somewhere like that, and then how would you do it if your teacher was going to see it and how would you do it in an exam and how does that vary and do you feel like it needs to vary which I think is another sort of conversation. Right, almost tutor time. Thank you so much everybody!

**Appendix Seven – Teacher Group Session 2a**

1. R: Ok, we're recording, Hello everybody. Just so everybody's aware I am recording this. Is that OK with everybody? (others nod). I kind of need verbal permission.
2. M: yes
3. S: yes
4. T: yes
5. R: OK, thank you, that's excellent, OK, so, um basically this is the second one of these that we've done isn't it. The first one what I did was I gave everybody a copy of some questions that looked like this um, and this had gone out to students in my classes last year so 200 students from all of my classes last year had a look at these um and we had a go at doing them ourselves and then we had a little chat about the methods that we'd chosen to use in order to answer the questions. Does that sound vaguely familiar?
6. M: Mmmhmm.
7. R: Excellent, um, so what I thought I would do this time round for quite a quick session hopefully is I thought I'd bring along some examples of the students' work that they did on the same questions um, and it doesn't matter if you haven't seen the questions before and what I thought we'd do is just go with something really simple to start with which is have a little look, there's 200 examples here across all year groups 7 through 13 um, and see if you find something that interests you. Maybe it's a bit different, maybe it's really similar to what you thought, maybe they've done it, gone about it a completely different way because one of the things that we noticed from what we did was that it was broadly similar, weren't they, across all of the teachers and it's just interesting to have a look at the students and see if that holds true. So, have a little rummage, see what you can find.
8. T: Are these colour-coded for any particular reason?
9. R: Yeah, it's by year group but…
10. T: Ok, but don't worry about it?
11. R: Yes, but I wasn't going to tell you which one's which
12. S: I'm going to try and guess which is which now.
13. R: yeah, it won't be obvious because they're different like it'll be bottom set one year and top set another year so it's not obvious in the slightest. So feel free to have a little rummage through and find something that looks a little bit interesting.
    (paper shuffling noises)
14. R: Although I have to say it's quite easy to work out the [year group] because they did ??? methods.

**2 mins**

15. R: Some of them will be blank by the way. If they're blank it means they've opted out and filled in the other side of the sheet so they didn't have to do both.
16. M: I find this one interesting because I just don't understand what they've done at all.
17. R: Which question? Or all of them?

18. M: So question three.
19. R: Yep.
20. M: So I don't understand where they've got that equation from or why that would be correct.
21. R: OK so they've put 2x – 3 = 27
22. M: Yeah, So Alice is twice as old as Judy
23. R: (to others) Can you see that?
24. M: that's a really odd thing to have just come up with from just reading the question without having to do any processing I think. It might be that that is the equation but I don't honestly know without writing some things down first. So it's really amazing that they know that from just reading the question.
25. R: It sounds so like they've tried to formulate an equation haven't they and they've sort of translated
26. M: Yeah, but I would need to sort of do Alice and Jen, whoever it is, and Ben and then put them in terms of each other and substitute it all in and get an equation whereas they've just cracked straight on there. They've just got the thing.
27. R: I don't know if they've got the right equation…
28. M: I don't think they have
29. R: …I think they've just written down the bit of the question for one person like using algebra and equated it to the total as opposed to working it out
30. M: Yeah, they've just done it completely wrong basically
31. R: but it's nice going straight to the algebra from the… have you got the same thing?
32. T: No, I was going to say with that one what I think they've done is mentally order the people so Alice is twice as old as Judy who is younger than Ben so she's given somebody an x value and she may have, or he may have mixed those up in the wrong way but the 2x would be twice as old as.
33. R: So this is interesting because this doesn't use any algebra but that seems to have broken down the question in a far sort of clearer way than that person there.
34. M: Yeah.
35. R: Haven't they?
36. T: Yeah 'cause they've got their 2x – 3 = 27 whereas 27 – 3 = 24
37. R: and then they've halved it.
38. M: Does that work? That doesn't work does it?
39. S: It doesn't work they have to divide by three

**4 mins**

40. T: You can do this with the block method (?) as well, but I don't know if we've got any that have.
41. S: There's lots of random algebra, using letters.
42. M: This person's worked out the perimeter instead of, and they've not even done that properly, instead of the area.
43. R: [amused] With no symbols, they've just listed them. There's no plus there's no nothing we know nothing about that method. I really like

question 1 for like loads of random methods. So this person has written out like their four times table

44. T: OK?
45. R: To try and get to 62. This person's done like bus stop method and then tried to check it afterwards…
46. M: Sounds cool.
47. R: …and not got the right total.
48. M: Got it wrong
49. S: Some have gone remainder two and then gone twenty pence.
50. R: So haven't understood…
51. T: Tallies.
52. R: I've got this look.
53. T: Oh wow.
54. M: That makes me so sad that someone's had to do that to work out 60 divided by four.
55. R: Did they get the right answer?
56. S: No. One person will get 16 and the rest will get 15.
57. R: OK, so they…
58. S: Which is kind of…
59. R: Which is kind of, they haven't shared equally but also miscounted by 1.
60. M: They ??? such a stupid way of doing it.
61. R: Yeah…but there's a lot of them, if you look through then quite a lot of them have done tallies for question one which really surprised me…
62. T: (over the top) Tally charts, yeah, have done it right.
63. S: Yeah
64. R: …and you'll notice…
65. S: This one has multiplied to get which is quite nice.
66. R: Yeah. That's quite nice.
67. S: To do four.
68. M: What level is that, that yellow paper?
69. R: It varies so these are two different year groups [gesturing at the two examples being held up] completely different year groups and if you find one...
70. M: Yeah, because the good thing about it is they've clearly understood what sharing 60 out is about if you see what I mean so conceptually understood it but it's really sad that they've got no skills whatsoever for actually making that happen.
71. R: What would you like expect to see if you kind of gave them that sort of problem?
72. M: Bus stop method or just the answer. It's such a simple problem.
73. R: Yeah.
74. M: Do you mean what would I expect to see if it was an exam?
75. R: You know what… (gets distracted).
76. S: OK. Oh bless.
77. T: Trial and error for the sum of the ages?
78. R: Yeah, there are some of those that are quite successful, like that looks like trial and error…the first time they got that right.
79. T: Yeah they've just gone it totals 27 so that's probably it. Nice.
80. R: But, like, so many GCSE kids do that.

81. M: Yeah.
82. T: Yeah.

**6 mins**

83. R: So many GCSE kids would do that sort of question by doing trial and error and as long as they get the right answer it's valid. It's when they get the wrong answer that they don't get any method marks and, what's that, a four mark question?
84. M: Yeah.
85. S: Yeah.
86. M: Harsh, yeah.
87. R: Something like that. Can we find and other weird and wonderful ones?
88. T: I've got A plus J plus B for question three, and ooh we've got another algebra one so we've got the xs and ys, x, y, z.
89. R: OK, so the three of them add up to that and then
90. S: Oh god.
91. R: So that must be Judy, that's Alice
92. T: Yeah, that's better.
93. R: and I don't think they've done that one…
94. T: yeah, so that one's better so they've actually done it in terms of xs so they've got one age for sort of base age.
95. R: But they've started out like that, and they tried to conceptualise it here, but does that work?
96. T: So they've said j = x
97. R: Is that not supposed to be 2x
98. T: Judy is younger than Ben so she'd be youngest so you start with x and then Alice is twice as old as Judy…
99. R: Oh, no, no, I'm being an idiot!
100. T: …which would be two x…
101. R: I've only looked at 300 examples of these!
102. T: and then Ben is three years younger than Judy who is younger than Ben by three years. Judy is younger…Ooh…
103. R: It's not helpful is it?
104. T: No.
105. R: The wording of that question is really difficult.
106. M: Quite a few people seem to have just divided the 27 by three and then got…
107. R: For three of them
108. M: …that one must been roughly that and then fiddled about a bit and worked out…
109. R: Which is not horrendous…
110. M: It's just, yeah, it's such an interesting approach because that seems like extra work to me, to just start off guessing and then we'll go from there. I suppose.
111. R: Do you think…
112. S: She's done trial and error.
113. T: That is right actually, that does work.
114. R: So whoever wrote Alice is 12 for the ones we were looking at earlier…
115. S: Alice is 12 seems to be the right answer, yeah.

116. T: Alice is 12. So how've they expressed that right, they've looked for who the youngest person is and given them an x value which I think's really nice and then you don't end up with lots of different letters.
117. R: So we're saying that the trial and error one just seems like it might be a lot of extra work compared to writing it as algebra?

**8 mins**

118. M: Yeah, because we've got good algebra skills so to us algebra is a quick way of solving problems.
119. R: Yeah, this is the thing… 'cause all teachers did it algebraically didn't they when we had a look at it.
120. S: This person has got 42.
121. M: Aww.
122. R: Oh bless.
123. S: I don't know how they've done that because there's no workings, they've just come up with 42.
124. T: Some have written it in words "62 divided by 2 is 31. 31 divided by 2 is 50.5" in words. 15.5 sorry.
125. S: Why've they…?
126. R: I don't know. I did ask them to show their methods clearly! They might have thought I meant like, write it out, it's a surprising number of them when I say show your workings really clearly try and write it in words or explain what you've done write it in words as opposed to…
127. M: It's true.
128. T: They've done it for all of it.
129. R: Yeah.
130. M: I think that in itself is a really interesting thing just because it suggests that people don't think that maths has logical and explanatory power.
131. R: In itself.
132. M: it's a sort of tool but they don't think it's a tool that can really tell you something in itself.
133. R: See, I think this is really interesting look. What would you say to that as an answer for the first question. 15.5 as opposed to 15.50.
134. M: It is 15.5
135. R: Yeah, but you lose a mark in the GCSE now.
136. M: Do you?
137. R: Yeah, so there was a question on one of the papers we looked at which was that bill question where you had to like, are debited this amount so you had to do it and then credited this amount or something, it was the foundation one, and um, and a lot of mine, 'cause what they got on their calculator at the end was like .2 or something so they wrote .2 not .20 and they lose a mark for it now.
138. M: Yeah I mean I always tell mine if its money put two decimal places, so that's interesting that they lose a mark for that that now because that is actually fine, it is 15 and a half pounds so that is the same so they shouldn't really lose a mark.
139. R: No, it's the same at A level, they have to have it to two decimal places.
140. M: What, when they're doing money?
141. R: Yeah, always.
142. M: Fair enough, that is how money works I suppose.

143. R: Yeah. It just seems harsh.
144. M: It seems, it seems. It should be explicit first of all in the question that that's' what they're expecting because it doesn't actually mean that they don't understand money if they've written 15.5 pounds.
145. T: Yeah.
146. M: They can understand perfectly well 15.5 is 50p.
147. R: It's going the other way isn't it that when they write something like 15.5 and then some of them interpret it as 15 pounds and 5 don't they.

**10 mins**

148. M: Yeah, that's true, yeah.
149. R: When they go back the other way.
150. S: Somebody had written 15.05 as their answer.
151. R: Yeah, exactly.
152. S: I think they'd just done the bus stop wrong, but…
153. M: They've used trial and error, they've used algebra.
154. R: So, yeah, I was about to say about question 2, when we did question 2 and we talked about it in the group that we did before basically everybody did sort of area of a rectangle area of a triangle and added together and part of the discussion that we had was that because the way that question is laid out lends itself to that but there are some interesting methods that the students have used for question 2. Has anyone seen anything…
155. M: Yeah, working out the perimeter for example?
156. R: (laughing) Yeah "by doing it wrong"!
157. R: I like this one here which has got, um, they've turned it into a rectangle,
158. M: Yeah, I saw one like that.
159. R: So they've rewritten it as 10 x 5 and then they've worked out the triangle and taken it away.
160. M: I often when I'm teaching have people say why don't you just work out the whole thing and take that bit away if the rectangles this bit, you've got a missing thing as well that's a…
161. R: Yeah, it's really nice.
162. M: …really common intuition that people have.
163. T: See I would probably trust doing this way more than I would the trapezium rule because I would look at it and say "it kind of is a trapezium, but is it? It's got a flat edge…" I'd sort of start questioning myself.
164. M: Yeah, there's another one like that there. It is a trapezium, but the trapezium rule doesn't have the same sort of intuitive value in a way does it as adding a rectangle and a triangle and I mean you try and explain that you're finding the mean of the two sides and they just glaze over.
165. T: someone's split this one into…
166. M: Crikey!
167. S: Oh my goodness!
168. T: …needlessly into five parts!
169. M: Why did they stop at four?
170. R: Do you know why, do you know why that was? Cause that was…
171. T: [reading from sheet] "sound as a pound"

172. R: That was one of my [year group] because I told them I was interested in different methods so they have gone out their way to do weird and wonderful methods for that question…
173. T: so five segments.
174. R: …and I saw it and I was like "YOU….!" So yes, so they've split the rectangle into four smaller rectangles and then decided to… I was… not thrilled. I mean, it was very funny, which is what they were after.

**12 mins**

175. T: 10 x 6…
176. S: They're kind of halfway there. They've made the rectangle and then forgotten to take the triangle off afterwards so they've kind of done 5 x 10 and then they've failed there.
177. T: Yeah, 5 x 6, yeah some people have just done the rectangle and not done the triangle.
178. R: But it is interesting isn't it because we were talking about the fact that the question definitely set you up to do rectangle triangle and yet some people are still doing…
179. M: Yeah it does. It's that line isn't it. If that line wasn't there lots of people would struggle with it a lot more cause not everyone would think of putting it in.
180. R: Yeah, I think so as well. Any others? Go for it.
181. T: Someone's got 22 centimetres squared, I'm not really sure how they've got that.
182. R: I've got 78 just squared – there's no unit on that one.
183. T: Oh wow.
184. R: So how've we've done that? Oh, they've found perimeter for each section.
185. T: Yes, that's perimeter, it'd have to be because that's 6…
186. R: Except that's not!
187. T: …6 and 10. This one definitely is perimeter.
188. S: 65. What's that? I don't know how you'd get 65. They've done 6 x 10 plus 5 by the looks of it
189. R: They've just done something random with the numbers?
190. S: They've just randomly combined the numbers.
191. R: they've done 6 x 10 there to get the 60 and they've worked out that this is 4 which is quite sophisticated really considering they've just multiplied random numbers there.
192. M: I always think one of the most interesting things that you see in maths in a way is people just doing something completely random because that seems to really be a symptom of not expecting it to make sense at all so they've got no sense that maybe they're trying to actually work a thing out they've gone oh there's probably a thing do something with numbers oh what can I do? Oh I'll just check them all together. That must be a thing.
193. R: Well I find myself sometimes talking to like [year group] at the moment and I say if it's an area/volume problem you do not just multiply together all the numbers that you see and then inevitably the next question is volume of a cuboid, right you just multiply together all the numbers you can see. I hear myself saying it and I'm like NO!

194. T: There's some nice circling on question 3 to join the statements together on the question there. So they're trying to work out which bits of information fit with what.

**14 mins**

195. R: So it's like grouping it and not just identifying.
196. M: A bit of set theory.
197. R: Yeah 'cause there's quite, there's a couple I've seen where they've like circled key bits. This one is nice look. So they've, Alice is twice as old as, so they've written the algebra above there…
198. M: Yeah, that's a good idea
199. T: That's good.
200. R: …above the names which is quite nice on that one.
201. M: [discussion of paper colours!]
202. T: There's another trial and error for question 3 as well where they've had to go through three different versions.
203. R: So this is an interesting one so for this one which way would you read this? So would you read this as she's written a loads of ages out for Alice and a load of ages out for, and then worked out the others and then gone back and said this, or do you think she's done that one and then gone back.
204. T: I think she's done them by rows
205. S: I think so, otherwise you'd just stop wouldn't you?
206. T: Yeah.
207. R: This is another example of someone attempting to explain it using words though isn't it. So Ben is at least three, Alice is an even number less than half of 27.
208. T: OK.
209. M: That's good thinking that Ben is at least three
210. T: And that Alice is an even number. That's really nice.
211. R: To try and, yeah, help narrow it down.
212. S: Yeah so then they have started by doing Alice's ages.
213. R: Mmm, yes exactly so Alice is an even number less than half of 27 so yeah, that's, which is why she's started there and not started with the Ben is at least three as she could have started there and done a lot more iterations than just…
214. T: Lovely logic for trial and error or trial and improvement.
215. R: And it's interesting isn't it because we really try and guide them away from this
216. T: Yeah.
217. M: Mmm.
218. R: Though, having said that, I haven't for [year group] this year because the odds of them getting an algebra method right are so slim I've said that if you get a question like this you're going to have to do trial and error because we can't do the algebra for it is the thing which worries me slightly.

**16 mins**

219. M: It's tricky yeah, you need proper skills really and a sort of proper understanding.

220. R: Yeah I mean the number of people who tried to do this by algebra and messed it up, cause as I said, this goes 7 through 13 and even like [year groups] were struggling with it I think last year.
221. S: I think it's because of the wording isn't it as well you sort of struggle to tap into even to read it and make sense of it. It's hard.
222. R: It's a foul one it really is horrible
223. M: Yeah. I agree.
[M excuses themself to prep for lessons]
224. R: I'm happy to keep going if you guys are?
225. S: It's really interesting. Just looking at the area one there's just these random, there's some here that you just can't see what they've done.
226. R: Yeah, this is it. In some of them you have no idea where the numbers have come from.
227. S: This one's managed, added together 15, 5, 6 and 4 to get 30 I'm not even sure where they've got those numbers from!
228. R: No, I have nothing for you for that one.
229. M?: I mean, that's 5.
230. R: And that's 6 and there's 10 at the bottom but I mean 15? Have they added those two?
231. S: 15 and a 4?
232. R: Have they added that and then thought that was 4?
233. T: And the width is 4 there.
234. R: Yeah that's where the four's come from.
235. T: So have they just taken all of the bits and added them up?
236. S: I don't know. This one's kind of randomly said that's 7 and then 28 from somewhere?
237. R: 7 x 4?
238. S: Maybe? That's what they add up to. They've added, done a perimeter maybe. But they've randomly said 7. Maybe they went 5, 6, oh I'll put 7.
239. T: Yeah, noticing a pattern.
240. R: But it's interesting that even with the ones that are wrong you can kind of see where some of them have gone wrong and why they're wrong and some of them it's like that's just a number. I have nothing. I have no idea where your brain was when you did this.
241. S: 6 x 5 = 30 x 10 and then 60 x 5 = 300 and they've gone with 300.
242. T: It's a good number!
243. R: Just pick your favourite! Pick your favourite.
244. S: I like the person who said for question 3 "this is a riddle not a maths question".
245. R: Accurate.

**18 mins**

246. T: Well yeah.
247. R: I quite like this one. I've not noticed this one before. So they've kind of identified who's the oldest and he's the youngest and kind of tried to go through and say which one's oldest and which one's youngest. Are they right? Did they get the right order?
248. S: Looks like it. That's right. That's the right number.
249. T: She's twice as old as Judy.

250. R: who's younger than Ben by three. So they've kind of gone through there but they haven't got any other options, they've just gone 12, 9, 6 straight away so don't know, do you reckon there was some trial that happened there that they didn't write down? Or?
251. S: Yeah, probably they were just thinking about it weren't they.
252. R: It's like the way they've ordered that is really nice.
253. T: Yeah, so they've got the youngest that side
254. R: And then, they haven't double-checked it. So it's really… Just the range though. Just the range of ways they've approached the problems.
255. T: So if that were 14, then Judy would have to be 7.
256. R: Yeah, and then Ben would have to be ten.
257. T: And then 21 plus ten…
258. R: Would be too big. Yeah. [more paper sorting] Just why it's written like that. Looking at it with all the words! I did say show your method. I quite like this with the fractions. So they've split this look 60 and 2 and they've done 60 over 4 and 2 over and worked that out.
259. S: That's different.
260. T: That's nice.
261. R: And we still don't have a five zero. It's still five on there.
262. S: This one they've managed to do the bus stop wrong and then use it to get the right answer, [laughter] 'cause they've said reminder 5 and then turned it into .5 which gives them the right answer.

**20 mins**

263. R: Ooh, which does give them the right answer!
264. R: It's amazing isn't it, what we think is a really straight, like you were saying before M, that like a really straightforward answer, a really straightforward question and they struggle with it so much just because it's a decimal answer. If I'd given them 60…
265. S: Yeah, they'd have been OK. Or more of them would have.
[Bell rings. Rush to pack up!]

**Appendix Eight – Teacher Group Session 2b**

1. R: Alright, we're recording. Is everyone happy to be recorded.
2. N: yes.
3. (K is nodding)
4. R: I need a verbal one I'm afraid... [laughing] stop it!
5. K: Yes, I'm absolutely happy.
6. R: Ok, we've got another teacher possibly joining us in a second. Right, quick recap 'cause it's been a while. Last time when we had this group we had a few of us and we sat down and tried out some problems if you remember. Two sides of the sheet and we looked at the methods we would use to tackle the questions and then we looked at similarities and diffffereces between the methods that we chose, and what we did it two, one like doing it for ourselves ad then we did like teaching it to class and how would we vary our methods - if at all. OK, so what I've got for you today is approximately 200 samples of students doing the same questions as we did last time and what I'd essentially like us to do for 15-20minutes is have a little look through and see if you can, see what you find interesting essentially, is there anything in there that I'd wildly different that sticks out or that you particularly like so we're just going to have a sort through and you can tell me what you like is that alright?
7. N: Yes
8. R: Fabulous so go for it, there's hundreds of them so have a rummage and see what you can find. [rummaging noises] I don't feel like I've quite got through all of them!
9. K: There's quite a lot of blank ones.
10. R: Yes so that's kind of the opt out version if you see what I mean so there will be some blank ones where they decided to do the other half of the sheet which I then had to throw away for ethical reasons if you see what I mean.
11. N: That one's quite interesting.
12. R: Ooh, OK
13. N: Because it's a trapezium question and so many students just cannot remember the formula for a trapezium and so they tend to get into the pattern of right-angled trapeziums...trapezii?
14. R & K: Trapezia!
15. N: They work out the rectangle and they work out the triangle and I'm never quite sure if that's... if that's... I mean that's the correct way to work that out but I always point out the trapezium rule because otherwise, when they don't have right-angled trapeziums they're going to get stuck.

**2 mins**

16. R: That's a really nice example.
17. N: I know why they do that...
18. R: Yeah, and there's another one there that does it really clearly and
19. K: I mean there's dozens of them in here that do it the same way. I haven't found a single one yet that does it the ??? Way
20. N: It kind of implies that that's the way you should do it by putting that dotted line there.??? Angled.
21. R: Yes, so part of that comes from the question which is nice.

22. R: what else, this one's interesting. This one looks like they've tried to turn it into a rectangle
23. N: Mmmm
24. R: And then maybe they would do it as a whole rectangle and then subtract that
25. N: ten times fives
26. K: Yep
27. R: The calculations don't support that. The calculations, they've done five times six to find the smaller one and then four times five which is this bit and then halved it.
28. N: So what they've done is...
29. K: They've done the same...
30. N: They've worked out two rectangles.
31. K: Mmm
32. R: Oh I see what you mean
33. N: Oh and then they've taken away rather than
34. K: Yes
35. N: They haven't seen that you could make it a big rectangle, they've just kept it as two...
36. R: I take it back then!
37. K: I was interested in and I'm sorting through to see
38. R: Oh you are sorting into piles!
39. K: The proportion of people who actually worked out the area as opposed to the perimeter!
40. R: *laughs* Yep, that was a really common one.
41. K: Because quite a few.. well actually no [looking at the piles], quite a few of my class would quite happily work out the perimeter
42. R: OH there are definitely some that have done perimeter
43. K: Yeah, I've got two of them, but of those that did actually work out the area, there's a wildly variant set of answers to what the area actually is even those most of them have done that method of splitting it into two shapes.
44. N: Is that because they can't multiply..
45. K: Well most likely yes.
46. N: Or add, what's going on, why are they... that's forty centimetres, that's forty centimetres..
47. R: So that's an interesting way of categorising it - those that have attempted to find area...
48. K: Not even sure what this one did we've got ten plus six plus five...

**4 mins**

49. R: They've just added all the numbers they can see
50. K: Which is all the numbers they can find which is twenty one and then they've timesed it by four.
51. N: because they know it's area.
52. K: So you times it by four it's bound to make area? At least they've timesed it by something, but...
53. N: Maybe it's the difference between the ten and the six they've multiplied by?
54. K: Maybe you're thinking slightly harder than they did!

55. N & R: *laughing*
56. R: Yeah, I've got another one there who's just added up the numbers and another one there.
57. K: Yes, we have, well I think there's about four to one who have done something attempting area as opposed to perimeter which is??
58. N: That's a valid point actually the confusion between perimeter ad area. Is there a best way to teach that so that people understand the difference between them...

[Teacher V entering classroom]

59. V: Hello, sorry.
60. R: Hi, we are recording if that's OK, we need a verbal permission.
61. V: Oh yes, of course.

[repeat opt out etc. conversation]

62. R: That's better than this one who just decided he'd lost his voice when I asked him.
63. K: I was going to sign for you, yeah!
64. R: These are two hundred examples of student's work on the same three questions that we did in our. Last group sessions
65. K: "this one!" Delightful!
66. R: Ummm would you like to have a little sort through. We're just finding things that we find interesting...
67. V: OK.
68. R: Or that you think are different or that you want to talk about.
69. N: So does that one have a magic way of teaching the difference between perimeter and area?
70. K: Well, there's one thing but it relies on being our age I think.
71. R: Go on.
72. K: Well perimeter has the word 'rim' in it.
73. R: Yeah.
74. K: And rim is the outside
75. R: I sometimes..
76. K: But there's nothing that corresponds to area. It's got the word 'r' in it but that doesn't help!
77. N: I've seen the word perimeter spelt perim 'add' ter but that doesn't actually help.
78. K: A) that doesn't help their spelling and B) that doesn't really help with anything else.
79. R: No, 'cause perpetuating that idea that to get the area you multiply and to get the perimeter you add is not overly helpful as soon as you're past...
80. K: Squares.
81. R: Yeah, a rectangle or a square, exactly.
82. K: It is to an extent because all areas are...
83. N: Based on the area of a rectangle
84. K: based on the multiplication of... the product of two orthogonal measurements.

**6 mins**

85. R: I'm going to use the work orthogonal with [class name] later this afternoon and see what happens.
86. K: Yeah, see how it goes!

87. R: Yes, no I appreciate that but...
88. K: Well, I thought you might! So I mean there is a basis there I mean I always say to them if it's perimeter you're adding if it's area you're multiplying but I don'tsort of say well its only multiplying these two together cause I've seen them with triangle multiplying all three number together and thinking that's going to be an area well how do you counter that idea.. WE used to teach area and perimeter together which was always a mistake because it's like teaching rounding to two significant figures after you've just done rounding to two decimal places. It just means they don't understand how to do either of them cause they dont' remember which is which. ??? Foolproof method.
89. R: The only thing I've managed with some of mine is talking about perimeter fences like around the outside of stuff, but that's only because I grew up on an army base so that's how I think of it, and if they play lots of like computer games and stuff it quite often comes up on that but...
90. N: I mean, I quite often look at you kn ow the amount of fencing you'd need to go around something as perimeter and the squares, you know, metre squares of grass that you need to put inside, that's the area.
91. R: But other than that...
92. N: I've got little centimtre square that I also produce...
93. R: Awww
94. N: And go this is a centimetre square, we can't measure it in lines we need to measure it in these for area.
95. R: That's really nice, like the visual thing for area. Especially for some of the ones you work with that must be really useful.
96. N: Yeah, and then I have my little centimtre cubes which I can.
97. R: building little shapes with them?
98. N: Yes. In fact, that is what I was doing period two today.
99. R: Awww.

**8 mins**

100. K: We used to have a set of...
101. N: Shapes?
102. K: No, cuboids. So that you had a big square so yea big which was then sliced up into, uh I supposed it was a thousand cubes ad then the slices were in a hundred cubes and then there were ten slices of one hundred cubes and then you had columns that had ten in them and then you had little cubes that had one in them so you could see???
103. N: Where have they gone?
104. K: Ummmm...Who knows! They were wooden so
105. N: The SEN department? I know
106. K: They were wooden so they probably got
[All talking over one another]
107. V: I remember, I remember things like
108. N: We used to have
109. V: That from when I was in primary school
110. R: Yeah, for things like place value, for when you first start doing place value.
111. K: Yeah, exactly
112. V: Yeah

113. K: but also they're really good for volume as you could see the how many of those make on of those. It's not just going to be ten it's not going to be a hundred it's going to be ten that way and ten that way to make one face and then ten that way to make a solid.
114. R: Yeah. We have got some of the centimetre cube very small multi link things in the cupboard but not really enough to make anything and you'd be there FOREVER
115. K: That's 'cause everybody's got some
116. R: stashed somewhere, yes
117. K: rather than the set being available for everybody to use, everybody's got some in their classrooms.
118. N: I also have at the moment the 3D shapes
119. R: Oh that's where the tray's gone, that's alright, and you've got the dice?
120. V: Only for this lesson, yes
121. R: *laughing*
122. N: No, I kept mine over the weekend, sorry. So many of them are looking at volume at the moment I thought I'm going to have those.
123. R: [at the same time] No, it's fine, it's alright.
124. N: Because they've also got slices so that you can show the number of layers.
125. K: Yeah, the prisms do.
126. N: Yeah, they have slices
127. K: Which is good so that you can see what the cross-sectional area is.
128. V: Um, I was just going to say that for years, when I've, when we've been recapping perimeter and area have drawn a rectangle, filled it with the word area written in bubble writing written in perimeter round the outside. Which, area is too short a word to write it all around the outside and perimeter is too long a word to fit in bubble writing in the middle of my

**10 mins**

129. R: Oh that's nice!
130. V: And I actually I found that quite a lot of kids can quite accurately reproduce that image probably more can accurately than can get area and perimeter right before they think about it, but as a sort of image to remind them very quickly as they go into an exam thing 'cause I almost never see them get them the wrong way around when they try and draw that.
131. R: That's nice
132. N: OK, I shall try that.
133. K: That's interesting because when I introduce this topic as, obviously as a recap, I say what's the perimeter of the shape or at least what's the area of the shape they can nearly alway tell me it's the space inside the shape and yet when they come to doing it they don't actually associate what they've just said with the method.
134. V: Yeah, Yeah. They definitely...
135. K: They can repeat parrot fashion what they've been told.
136. V: Yeah, and that's the same thing, they can produce that and then if you force them to actually think about that where have you written the word perimeter, ummm, yeah, but actually I have been telling my year elevens

when finding area and perimeter, being able to repeat it parrot fashion is important 'cause then when they get to perimeter with algebra and they just have to write the bits of algebra with the plus sign in between them they don't get to that point even though they can say well you add up all the lengths around the outside. Well, what are all the lengths? X, 3x, x and 3x + 2 so what do you need to do? Add them up? So what are you going to do? I dont't know.

**12 mins**

137. K: but at least by that stage they've got two p marks for the process
138. V: If they'd written down "I need to add up all the lengths around the outside and I'm going to do something with that" and then try and do something with that then the stuff's inside their head somewhere.
139. N: It's that difference between the conceptual understanding and the doing by rote you know because you go over and over and over well what you need to do is add up the sides, but they don't actually, they're not linking that with the length around the perimeter of the shape that their working out or, be accuse I mean that's easier than the area because are they don't think about counting in squares they're thinking about either base times height because that's what they've been taught, or ...there's two numbers and you multiply them by each other and they do that because they've been taught it by rote rather than thinking about what it is they're finding out.
140. K: In year seven when I'm introducing it I have Arnold the ant who starts at the top left hand corner of a shape and has to walk all the way around until he gets back to where he started from.
141. N: That's nice, filling in all of the missing numbers.
142. K: Yeah, and how far he's walked and also for area how many of his family, and there are thousands of ants, can fit inside.
143. N: Yep.
144. K: So, but that doesn't work on year eights upwards and they seem to have... no, actually some of those who I taught in year seven have said no I remember Arnold when I mentioned it.
145. N: Yeah, I like Arnold.
146. V: Could he not be Perry the ant?
147. R: Yeah, that was what was annoying me was that it starts with A and...
148. K: No.
149. R: and it's area so
150. N: but the ant also has to live in the area so
151. R: I supposed yeah.
152. K: Yeah, the ants live in the area and he's... they're alliterative ants as well.
153. R: *giggles*
154. N: Maybe he's Perry A[redacted] or something.
155. R: And the A[redacted] family live in the area.
156. N: And the A[redacted] family live in the area.
157. R: Nice, nice.
158. K: And he's just outside now.
159. R: Ok, do we want to have a look at something else?

**14 mins**

160. N: This one...

161. R: OK. Is this question one?
162. N: Is very interesting
163. R: Yeah
164. N: Sharing sixty two pounds between four people, this person has split the four into, sorry the two into four pieces and the six into four pieces and then multiplied that answer by ten to make it sixty divided by four...
165. R: MmmHmm
166. N: and then added the two parts together..
167. K: And got the right answer
168. R: That's really nice, I've not seen that one before
169. N: An interesting way. I did think 'for God's sake learn how to do short division' [laughter] because they are really jumping through the hoops there to work that out.
170. R: But like, but do you think that implies they understand what they're, the process?
171. N: That shows their understadning, their working out
172. V & K: Yes. Yeah
173. K: It's ??? Isn't it
174. N: And that that six is actually in the tens column...
175. R: Yeah
176. N: so you have to multiply the answer by ten
177. R: Yeah, I quite like it, like, as showing what you're doing
178. N: Showing your understanding.
179. K: What's seems to. have been happening
180. R: Hang on, let me get that [K about to write on one]
181. K: This is a spare one
182. R: That's alright then
183. K: That's why I chose one that was not written on!
184. R: [laughing] That's fine, go for it
185. N: There's quite a lot more work there than..
186. K: What seems to have been taught in primary schools is when you're saying share 12 between, oops, four people is that they've been taught to draw four boxes Circe or squiggles
187. N: one, two, three
188. K: or whatever and the go one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve which is all very functional and great for something like this, but if you've got two thousand four hundred to share between four people well then it's not going to help you at all. And also...
189. V: I mean, I remember being taught like that in primary one.
190. K: Exactly.
191. R: Yeah
192. K: But then it becomes all consuming because you can't then do this sort of question becasue it won't work.

**16 mins**

193. R: Yeah. A lot of people have used that method and what they've done is, I'm just trying to find a nice example, I know there are some in here, and what they've done is they've ended up not sharing equally between the four people. There are two pots with fifteen in and two pots that have sixteen in.

194. N: Because you can't have a half
195. K: Yeah because you haven't anything left. But it also then, as you said, why can't you flipping learn to do short division? Because it's so much quicker
196. N: It's far quicker
197. K:...and so much easier.
198. N: Yeah. People find that really hard, that division process.
199. R: That short division process
200. N: And particularly when they're trying to problem solve, they tend to resort to something where they can follow their understanding of it rather than it
201. K: Yeah
202. N: Because they don't understand what they're doing here, they're just doing it by rote it's another one of those things where they're following a thing and...
203. K: [reading from artefact] "One person will get sixteen and the others wil get fifteen." Well that's not even right!
204. R: No.
205. N: Then if you're counting sixty two, that's quite a lot.
206. R: Yeah, exactly.
207. K: so, even having done that the method then falls down
208. N: Yeah
209. K: because they didn't a) they didn't do it correctly or this miscounted how many that they were
210. N: Yeah, they missed one
211. K: ... were there in the first place
212. R: I think person started and then gave up because they realised their circles were too small and it was going to take them forever, and there's a couple of people who have done it as kind of a check so there's one somewhere... Oh I don't have a hope of finding it in the net couple of minutes where, um, they've done it one way and then they've done it a different way as well to check and see if it's right.
213. K: Well that's good. There's an awful lot who've got it perfectly which is rather nice.
214. R: Mmm. I saw quite a few of these where they've done...
215. N: Remainder two
216. R: remainder two
217. K: The wrong question... Oh I see
218. R: ...and then turned it into pounds and pence
219. N: Lots of students can cope with dividing whole numbers but don't know what to do with remainder 'cause I think that's another thing that happens at primary school level is that they're taught that ??? 15 whole times with two left over and they don't have to sort of transfer that into continuing division.
220. K: This one has just simply counted up in fours which is the other technique...
221. V: Yup
222. K: ...you just keep on counting up in fours until you get what you want.

**18 mins**

223. K: Which yes, functional again...
224. R: I'm going to steal that one
225. K: but for a question more complicated than the one you've given them is no use whatsoever.
226. V: And similarly forty is ten fours twenty is five fours, twenty four is six fours they want forty add twenty two so halfway between twenty and twenty four is twenty two.
227. R: They've got the right answer
228. V: So it's ten and five and a half
229. K: At least there's an understanding of number there
230. R: Painful
231. V: there's a.. painful
232. K: Painfully long winded
233. V: there's a method to it that's very lucky it comes to a half
234. R: Yeah, I like that one [pointing to another example]
235. K: I refer the honourable lady to the answer my colleague gave a few moments ago, Oh my word, yeah well that's just ???? Convoluted
236. R: Yeah, it's
237. N: Mmm
238. M: In fractions
239. V: I was surprised, this person did fractions writing it as sixty two over four and not then simplifying that down to thirty one over...
240. K: two
241. V: ... two but saying sixty two over four equals forty over four, twenty over four, two over four
242. R: Mm
243. V: and adding them together.
244. K: Numerous methods...
245. N: How did they get there?
246. V: ten, five and a half. That's, I thought, initially when I saw it that's what they implied but they're not it's just shrunken it ??? Writing all their divisions fraction-wise.
247. R: what were you going to say [to K]?
248. K: [looking blank]
249. R: You started a sentence, I can't remember what it was now... multiple methods, that wat it
250. K: Multiple methods uh, surprising number that have actually got the right answer or near enough the right answer. The remainder, as you say, being, thrown them.
251. R: Yeah.
252. K: I think decomposition works it's just that this is not a good example because it's a lot quicker to do it other ways than by decomposition.

**20 mins**

253. R: Hmm. Yeah. This is it isn't it. Whether it works and whether it's efficient. [checks time] Is anyone dashing off? Just double checking
254. N: No I
255. V: I don need to get some lunch at some point
256. R: Yeah, I think lunch would be
257. K: I think that would be a good idea

258. R: That's fine.
259. K: Bar charts? Tally charts and..
260. R: yeah, but again it
261. K: IT's that same idea of four pots and...
262. V: yeah
263. R: Yup. Let's talk about, are we all done with
264. V: Very, very few halving and halving agiain.
265. K: Yes.
266. R: Yeah, I know, frustrating isn't it
267. V: I'm surprised by how
268. K: The problem with teaching the trick about dividing by four is that they'll want to know why they can't then do it for five or six or eight
269. V: Yes, for six, half it half it and half it again!
270. N: *laughs*
271. K: [joking] Huh?
272. R: There's one in here somewhere where they've halved it 'cause that means over two, halved it again becasue that means presumably over three and halved it again to get it as...
273. N: OH.
274. R: to get it something as or they've tried to do something so they've obviously remembered the halve it and halve it again thing, but not the umm the rest of it so. OK.
275. N: And finally..
276. R: Do we want to talk about three? WE don't have to...
277. K: Very interestingly some of the comments I mean just the two I've looked at here some that are clearly lept straight in with algebra becasue ti was obvious, it had to be algebra others who aren't that happy with algebra judging by the way they did the division who just going for numbers
278. N: Trial and error?
279. K: Sort of trial and error basically, more errors than trial, umm
280. V: I've got algebra all written out in words. Like, the whole page was written out in words
281. K: Good grief, no numbers at all!
282. V: No! Anywhere!
283. K: Interesting.
284. V: but you know, reasonable amounts of you know a plus b plus j is twenty seven. Two j plus j plus three plus j is twenty seven, three j is twenty four
285. K: Perhaps they're ??? At maths?

**22 mins**

286. N: Or they've had it really drummed into them that you're never allowed to condense anything and write it in bullet form, everything has to be written out in full sentences
287. K: That is in bullet form
288. R: OK, yup,
289. V: but, doing it algebraically, but just surprised to be able to manipulate the algebra but not to write it down

290. K: That must mean a ll the comprehension's going... did they get the right answer?
291. V: No, they thought that two j plus j plus another j came to three j because they hadn't written things down clearly they just lost track.
292. K: I was thinking that all the processing must have been going on in their head and they must have been writing it down as they were doing it.
293. V: Yeah, absolutely right
294. R: Yeah, literally what was going on in their head.
295. V: But it was... I can see they were on exactly the right track, it was just three j rather than four j.
296. K: There are however, a large number who just went...
297. N: Can't do that
298. K: What? No idea, question mark.
299. R: Which considering it's a horribly wordy... horrible worded question, but it is a fairly standard GCSE exam style question and for at least three of the different colours in front of you they should have been able to at least write something down. Like if you write the first line of the algebra you get the mark in the exam don't you so they should at least have been able to write down the a plus b plus j equals twenty seven for example or try and reformat that to you know, two x plus, whatever
300. K: Oh God no! [In response to the one A is holding up]
301. N: This one's really complicated. So this one is Alice is two brackets n minus three, Judy is n minus three and Ben is n.
302. K: Well they've started with Ben and worked backwards.
303. N: Yeah.
304. K: So Ben is three years older than Judy so he's the n
305. V: Yep
306. K: So he's the n, she;s the n minus three
307. N: Exept, no Julie is three years younger
308. K: And then Alice is twice as old as Judy so that's correct.

**24 mins**

309. R: Yeah, one of the things we talked about didn't we when we were doing our methods was that depending on who you choose as your starting algebra point, who you make n or who you make x or whatever it simplifies or otherwise the rest of the..
310. N: Whereas this one here started with A who was not Alice
311. R: Oh gosh.
312. V: yes, it's A for Judy
313. N: A for Judy
314. R: Amazing
315. N: But then they've tried to formulate that so that
316. V: Oh, they're right
317. N: yes
318. R: It's fine, it's just confusing
319. N: Yeah.
320. R: And might throw and examiner marking it.
321. K: So I would have always said to them, if the question is find out who is Alice, we discussed this before. If the question is who is Alice, how old is Alice, to do everything in terms of Alice. But actually that would lead to

some fairly complicated fractions here for Ben so I think that having started with...

322. R: Is that what this person has tried to do?
323. K: Yes, start with what Ben is and work backwards is actually a better choice at this point.
324. R: So.
325. V: Yeah, although you get half an Alice and half an Alice, which is...
326. K: An Alice and a half and two Alices...
327. V: ...but the fraction...
328. K: ...How many beans make five?
329. V: Indeed.
330. R: Brilliant. OK. Everybody happy.
331. V: Happy.
332. R: Right, do you want to know why these were so weird now? I said I want to see a variety of methods that's what I'm interested in looking at, so they went out of their way to be weird and wonderful so
333. V: *laughing*
334. R: So this colour is [year group]
335. K: So all the ones I'm looking at
336. R: being able to, so writing for this one is really unusual
337. N: Right, OK, yeah
338. R: When I asked them to find the area of the trapezium has split it into four smaller rectangles before finding the area, so any of that colour, prepare for weird and wonderful

**26 mins**

339. N: OK
340. K: This one's too boring simple then, this one's just as per normal.
341. R: Yes, some of them did.
342. V: Ah, so that was one of the ones who did sixty two over four equals forty over four equal ten, equals twenty over four equals five eqauls a half.
343. R: Mmm, yeah. Which is all completely correct, but may explain slightly why you would not do that in
344. K: Mmm
345. V: Apart from having equals everywhere
346. K: Yes, there are too many equalses that are not equivalent.
347. R: Anyway yes, I should let you go so that everyone can have some lunch. Thank you so much.
348. N: Thank you.
349. R: That was really helpful.

**Appendix Nine – Student Group Session One**

1. R: We're recording as you can see the little thing's ticking up so what I need you to do for me first as we said is to say that you're happy to be recorded so if we just go around the circle is that OK?
2. All: yes
3. R: now then, lets repeat the way this is going to work is that that when I type it up and everything I'm going to make sure that nobody's names are used it's going to be what we call completely anonymous so you don't need to worry about that so it doesn't matter at all. What I thought we'd do today is…look, i wrote a little plan. This is what I'm doing with the teachers and this is what I'm doing with you guys so i kind of thought we'd try and a few sessions if we can manage that by the end of term That's like one a week until the end of term and then you're done. And today what I thought we'd do is this. What I've brought along is I've brought along some questions so you have one each so you can have a little look and I did this in class with some of my students last year and what we did with this is you'll notice there are two kind of halves to these questions OK..do you want to borrow a pen lovely? Here you go. And what they could do is they could choose which side they wanted to do and if they wanted to take part and whether they didn't want to take part. I'm not going to do it the same way with you guys as you've all said that you were interested in taking part so what I thought we'd do is have a little look at the questions on the left hand side, so this side of the paper together if that's OK. Now then, the questions, there are three of them and they're not very nice questions OK. I chose nasty questions. Don't look like that. I promise they're not that horrible! So what I thought we'd do is just take 30 seconds, I don't necessarily want you to do the questions because as I've said they're really horrible, is have a little look at them and see what it is you might do as your first step. So what might be the first thing that you try and do for that question. Does that make sense? so what I'm not expecting is for anyone to go through and go [silly voice] "Haha! I've done it correctly" OK [laughter] (I know this is going to sound really weird when I'm typing it up later) but all I want you to do is to have a little look and have a think about what methods you know that could help you with the questions. So take thirty seconds while I open the biscuits loudly and have a think about that. You can scribble on it, write all over it if you want to any ideas you've got. Not literally just scribble it out A - hilarious, hilarious you are.
4. A: I know!
5. R: And help yourself to a biscuit if you want one.
6. A: The whole packet?
7. R: You won't have any left for next week if you eat them all now! Anything at all - I don't mind about spellings, don't worry about mistakes. Literally nothing. Whatever you think might be useful. And you don't have to start with the first question, you can start with the second question. The last one's really horrid.
8. P: Oops.
9. R: It's fine, keep going. Whatever you want to write is fine. [silence while they work] As soon as you think you've written everything that's useful,

feel free to grab a biscuit and just relax. [more silence] just finish off the bit you're thinking about then we'll have a chat. [more silence] OK, some people will have written loads of things down, some people wont have written anything down because it's all up in their heads and that's absolutely fine. Do you want 30 more seconds? [couple more second] I'm just going to scribble down this number [to a student]! OK, what I thought we'd do, rather than look at anybody's answers, ok, what I thought we'd do is just talk about what sort of things we might want to try and do for each question. Is that OK? And you can talk as much as you like or as little as you like and it find it too stressful you can just disappear I don't mind OK. Would anyone like a biscuit before we start? No one been sitting there eyeing the biscuits but too polite to take one?

**5 mins**

[no one wants my biscuits. I move them off the table so we have space to work] OK, I'll put them over here for now. If you want them then just let me know. OK, so let's start with question number one for now shall we. Is everyone looking at question number 1? And feel free again to scribble or not to scribble what ever you want to do. Who would like to talk to me about question number one? What sort of method did you decide you might need for question number 1?

10. A: divide sixty two by four
11. R: divide sixty two by four OK so that tells me what the question wants me to do which is a good first step. UM, what sort of methods do we have for sixty two by four because some people have written…
12. P & E: bus stop
13. R: Oo! That was very quick OK so some people have written it down in bus stop and some people haven't written it in bus stop. Do we know how bus stop works?
14. D: Yes.
15. R: Yes?
16. A: You put sixty there and four there [student gesturing to writing]
17. R: and then do that. Nice. So do you want to try it? Do you not wat to try it? some people are like yes, I want to do it!
18. E: I know how to set it out it just doesn't work.
19. R: You know how to set it out it just doesn't work? Show me what you mean.
20. E: Do you divide just by six?
21. R: So the bit on the outside is the bit we divide by so six divided by four
22. E: Is one?
23. R: good, so that goes on the top.
24. E: and you put the two there?
25. R: Yep. Nice.
26. E: And then divide it by 22
27. R: And then, again, four divided into twenty two, fours into twenty two. And be careful, take your time, its a horrible one. the fours into twenty two is the trick isn't it. So what's the Robles with fours into twenty two?
28. A: It doesn't go into twenty two
29. R: It doesn't go into twenty two so what might we need to do?
30. A: Decimals.

31. R: Decimals.
32. D: Then you like put a zero then two above it and then yeah.
33. R: Oh I see so you've put like a decimal point and then zero and then ooh so you've got remainder two haven't you for yours yeah? Which is exactly right…
34. P: You need to put the zero.
35. R: You've put two remainder two but it's changing it into a decimal then isn't it yeah.
36. P: Oh yeah
37. R: Why have you put a zero there at the end of yours?
38. D: I don't know.
39. R: You're not wrong, I'm just interested that's all.
40. D: I don't know.
41. A: To make it a whole number so it's easier to calculate.
42. R: Yeah. That's fine. What are we trying to work out? What sort of question are we working on?
43. D: Um, how much money people get.
44. R: It's money so why do we think we have two numbers after the decimal point for money?
45. D: Because it would be stupid if it wasn't?
    [laughter]
46. R: Why do we write money with two numbers after the decimal point?
47. A: 'Cause then it's easier to work it out because it's a whole. I dunno.
48. E: What?
49. R: What you…
50. A: It's easier to take away and add.
51. R: it is, yes, you're right, if you fill in all the gaps. What units do we use for money ladies? What do we use? Pounds and then what?
52. A: Pence
53. D: Pence. p?
54. R: [gesturing at those who haven't spoken much] Let them talk!
55. D: No! [jokingly]
56. R: Pounds and then what?
57. A: Pence
58. R: Pennies yeah and we normally have pennies with two decimal places after the pounds sign. OK, very nice. do we know any other methods other than bus stop for doing dividing. Does anyone know of any others? I'm just going to sit there and break??? So did we all use bus stop for…
59. E: You could half it and half it again maybe.
60. R: Ooh, that's quite a nice method, we could halve it and halve it again, [to the rest] did you hear that? So if we halve i sixty two…stop it [to student playground with pencil]… all I'm going to hear is clattering on this recording [giggling]. So what were you, no hang on you were starting to write it down, what were you writing down.
61. B: Half of it?
62. R: half of sixty two, what is is it, what's were you writing down.
63. B: 31

64. R: 31 perfect really nice. Go ahead, I just spotted you doing and thought "Ooh, fabulous that's really good. So that's sixty two divided by…if i've halved it what have I did I died it by?
65. P: two
66. R: Two, but i'm trying to divide by four so what do I need to do once I've halved it?
67. A: Halve it again.
68. R: Halve it again. So what could I do if I'm doing thirty one divided by two? Let's see. I think P wants to tell me [P looks a bit panicked] No, no, no no pressure if you don't want…
69. P: I'm not really sure.
70. R: could we do thirty divided by two? If we were sharing thirty pounds between us do you know how much we'd each get?
71. P: We'd get… is it…I don't remember.
72. R: Ooh, can we give her a hand?
73. E: Fifteen
74. R: Fifteen each? Does that work?
75. P: Yeah.
76. R: Fifteen each, that makes thirty doesn't it. What about if we've got one pound and we're sharing with each other what..
77. P: fifty p
78. R: Fifty p so that would be fifteen pounds and fifty p wouldn't it?
79. P: Mmmhmm.
80. R: Which is the answer so that's another really good method that E's suggested that gets us the right answer. Nice one E. Anyone got any other methods for number one. No? so we've got bus stop and then halving it and halving it again. Which do like better? Which do you prefer because you all prefer…
81. A: bus stop
82. R: …went like busstop straight away but do we actually find that easier?
83. A, E, D, P: yes.
84. R: Some of us, but I don't think you do do you [to B]? Can you tell us why?
85. B: I don't know, I think it's easier.
86. R: Yeah? It's a bit… Yeah because you were saying it's a bit confusing because you weren't sure which way round to do it. Is that right?

**10 mins**

87. B: Yeah.
88. R: OK. Alright. Shall we talk about question number two? Anyone got anything else to say about question number? OK then, let's talk about question umber two. Alright, calculate the area of this shape so first things first does everyone understand what the question is asking us to do 'cause I think these are really difficult questions. Does anyone need to… does anyone want to ask about what the question is asking us to do? It asks you to find the area of the shape. If I asked you to colour in the area of the shape, could you do that? Just like shade the area of the shape, what would you colour in?
89. P: All of it.
90. R: Go on then, show us, can you show us that?
91. P: Like there.

92. R: Yeah like that, like a quick scribble. So it's the space inside the shape isn't it guys.
93. A: Does that count as it as well
94. D: Yeah it does
95. R: Yeah that's right
96. D: It's the whole thing.
97. R: It is the whole thing. Lovely. Why do you think the lines there? Cause you were pointing at the line weren't you A.
98. P: 'Cause it's not the whole shape
99. D: Oh my Dad told me this.
100. R: It might not be a full shape, we might have cut something off somewhere.
101. A: I've forgotten.
102. R: OK, it might be a hint to help us do the question mightn't it. Does anyone remember how we do the area of the shape then? It's something to do with those numbers around the outside isn't it.
103. A: It's the same as this side [pointing to the left hand side of the shape].
104. R: Which side ooh, you're right. The dotty line is the same as the five centimetres you're quite right, well spotted. So if I was trying to find out the area inside…I tell you what, let's make it a little bit easier, let's cover that bit up, let's cover up the triangle let's just look at the rectangle. How would I find how much s space was inside the rectangle?
105. P: Add up the sides?
106. R: I could add up the sides, but adding up the sides would tell me this. Can anyone tell me what this is called? hang on A. Does anyone know what this is called if we're going around the outside?
107. D: Perimeter.
108. R: Perimeter. Is that what you were going to say B? I think you were. Perimeter is finding the distance around the outside. What about if we're trying to find the space inside the shape? We're not adding up….
[no ideas forthcoming]
OK, I tell you what, can I borrow someone's pen and we can have a look on this one [using a blank copy]. If I'm doing the area of the shape, the easiest way to area is to count the squares so i could it like this couldn't I [drawing in lines] so the quickest way of doing it is by counting these up so, shall we do it together [they join in to start, then tail off], one, two, thee, four, five, is, seven, eight, nine, ten, eleven, twelve…it's just me counting now guys
109. A: Yup
110. All counting together: thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty, twenty one, twenty two, twenty three, twenty four, twenty five, twenty six, twenty seven, twenty eight, twenty nine, thirty.
111. R: So the space inside that shape is thirty squares so we say thirty centimetres squared so with the little two next to it. Can anyone spot a quicker way of doing that rate her then just counting every single individual square [bell rings] Oh!
112. P: Has it got something to do with the right angles?
113. R: something to do with the right angles well that tells us the type of shape, go on E, what were you going to say?
114. E: Is it like count up them and them and then times it.

115. R: OOo you count up them and them and you times it.
116. D: What?
117. R: You multiply them together. Does that make sense?
118. A: Yes
119. R: So this is telling you it's five…
120. P: [talking to E] so you add them?
121. R; no, we've just done adding for perimeter haven't we…and six squares that way so if you do 5 times six, what does that give us?
122. A: thirty!
123. R: Thirty. Is everyone happy to stay for two or three more minutes to finish discussing this question and then go to lunch?
124. A: OK
125. R: does anyone need to dash off now because they desperately need to be somewhere? It's fine if the answer's yes.
126. ???: no [shaking heads]
127. R: OK. So that tells us that one there. So we could multiply the sides together couldn't we. What about the triangle bit on the end. What might we need to do for the triangle bit then. What's the same and what's different about the triangle bit.
128. A: It's different lengths.
129. R: It's different lengths. So we don't actually know this little side length here, but you've already spotted that that's five as well haven't we. What about this bit at the bottom?
130. A: Is it half of ten, so five.
131. R: Half of ten. Why do you think it's half of ten?
132. A: because it's ten all the way.
133. R: ten all the way. Well if it's ten all the way and that bit there is six, what do we think this bottom bit might be. P, do you want to have a look? Woah, woah, P is going to have a go. It's ten all the way across and this bit is six, what's this bit got to be [all of this is being pointed to on the picture at the same time].
134. P: Five?
135. A: Four.
136. R: [pulling face at A for jumping in, girls giggling]. Six plus five would be eleven wouldn't it…she's not helpful.
137. D: I know!
138. R: so six plus what gives me the ten there? [To A] Hush you. That's six squares and I need ten squares all together.
139. P: Would it be four?

**15 mins**

140. R: Perfect, really nicely done. It would be four squares there. So do you think I could times those numbers together, like five for that bit and four for that bit. Would that find me the area of the triangle?
141. P: Yeah

[A giggling in background, chat about R stealing a pen]

142. R: Five centimeters there and four centimetres there so we could do five times four and that gives us twenty centimetres squared. Does that look right.
143. D: Sure.

144. R: If I just times them?
145. A: Don't you have to square them?
146. R: That's what my square is there [pointing at the squared sign on the cm]
147. A: Oh
148. D: [joking] don't you have to it a square around it
149. R: I think we might need to do area again in lessons before the end of term. Can you see right that finds me the area of that rectangle [drawing in the complete 5x4 rectangle over the triangle of the picture].
150. A: Oh yeah.
151. R: so what would I need to do as my final step?
152. A: Find out the other half.
153. R: there was a really important word in what you've just said…
154. A: Half.
155. R: So we don't want all of it do we we just want half of it.
156. A: Ten!
157. R: So that would be ten. Like that.
158. D: Sure.
159. R: What did people write down that was different for that question? Did anyone do something that's as different or wasn't quite right?
160. A: No
161. R: No, OK, fine. Let's just talk about three super quickly, is everyone OK to do that for a couple of minutes because lots of people scribbled stuff down so I'd be really interested to find out what we were thinking for number three. OK, number three, talk to me then.
162. A: Work out Ben's age.
163. R: Work out Ben's age. Why are we working out Ben's age?
164. A: Because then it's three less than Ju…Alice
165. D: [same time] It says work out Alice
166. P: What?
167. R: So she's suggesting we find out Ben's age first
168. P: Isn't it thirty
169. A: Ooh.
170. R: OK, why did you say thirty P?
171. P: because the sum of them is 27 and he's three years…
172. R: He's three years younger
173. P: Oh yeah, I've gone wrong.
174. R: So it's something to do with those three years
175. D: That's what I wrote down
176. A: So is he 24?
177. P: yeah
178. R: OK, so it's 24 is we take off three from Ben's age.
179. E: I've divided it by two.
180. R: Why've you divided it by two?
181. E: because it says Alice is twice as old as Judy.
182. R: OK, so you could divide by two…
183. A: Do you times them?
184. R: Or we could… Oh I see it depends where we start doesn't it with this question.
185. P: Ugh.

186. R: No, it's OK, some of my A level students got confused by this so you're doing brilliantly to be even just discussing it.
187. A: Oh my God!
188. R: What about you D. What did you try doing?
189. D: I don't know any more!
190. R: Well you've written loads of stuff down. I was watching, it was very clever, go on.
191. D: Well, uh, Don't want to read it.
192. R: Don't want to read it? OK, Are you happy for me to read it and you can tell me what you meant?
193. D: Sure.
194. R: OK, so we've done 27 take away three. Why did we take away three?
195. D: Umm, showing the age of Ben and then Alice
196. R: OK, so take away three because younger than. What about the 24 divided by two?
197. D: To show that Judy is twice as young as Alice
198. R: OK, that's fine. So that's what we've got. And then we've got twelve at the end.
199. E: Could could it be like function machines?
200. R: Yeah, it could be function machines. So we need like a number that we put in in order to get this at the end. That's a good idea. I like that idea. Anyone got a different idea? What did you write down B?
201. B: Um, just the numbers.
202. R: [mishearing] the same thing? Can I see? Is that alright? I can look behind your pencil case. Very nice so we did 27 take away three to get Ben, so kind of bits of what we've already talked about which was really nice. Guys, that was… Anyone got anything else to tell me either about question one or two or three. No? A's like, can I have a biscuit now?
203. A: Food!
204. R: That was super, super helpful, thank you so much. What we're going to do next time if you're OK with it is I've got some examples of what other students did on these questions so I thought we'd get those out and we can have. Little chat about them and you can say oh I like that or I hadn't thought of that or oh that's hilarious, they got that wrong [giggles from the girls]. Is that OK for next time? Yeah? So can I keep these [the sheets]? Don't write your names on them, don't write your names on them, I'll tippex it for you lovely don't worry [student scribbling out name] so these are anonymous, but I'm going to hang onto these just in case they're helpful I genuinely can't emphasise enough how helpful you've been. Thank you.

**Appendix Ten – Student Group Session Two**

1. R: OK, let's go round the circle. We've got three people so far. We might have more once they've had their lunch so I need your agreement that you're happy for me to record it and that you're happy to be here and that you know you can disappear whenever you like. So happy with that?
2. A: Yeah
3. E: Yeah
4. P: Yeah
5. R: Fabulous. That's much better than when I do it with the teachers who just sit there and nod at me and I'm like "I need you to SAY yes" and they sit there like *nods* and that's very awkward. Last time we discussed, um you guys had a go at doing some of your own problems didn't you and then we had a lovely little discussion afterwards sort of about what you thought about the questions and the sorts of ideas you had. Do you remember that?
6. A&E: yeah.
7. R: Fabulous. So what I've got for you here is about 200, approximately, examples of the same questions but done by other people and as you already starting saying "ooh, that's not a [year group], ooh, that's not a [year group]" okay there is a big mixture here so I asked all my students last year from year seven through to year thirteen to have a go at these questions so there are lots and lots and lots of different types and styles of answers okay so what I thought we'd do is we'd just have a little look through and to start with all I'd like to to do is find something that you..find something that you think is interesting. So it can be be because it's weird, it can be because it's exactly what you did, it can be because it's completely different to what you did so just have a little sort through. I'm just going to move this microphone out of the way of my lunch, um and you guys can have a little think. It might be that you pull out two or three, it might be that you just pull out one and then you're going to try and tell me what it is that you found interesting about it.
8. E: why does that say..
9. R: Go on E, say that again.
10. E: They didn't use any symbols, they just wrote it down.
11. R: There are lots of words in that one
12. E: and it looks really confusing
13. R: It does look really confusing lots and lots and lots and lots of words.
14. E: I thought it said ??? But it said ???
15. R: Any others? So that's a really good example (talking about the one E's just found)
    [lots of paper shuffling]
16. E: That looks confusing.
17. R: Ok, why does that look confusing? Put it down so we can all see it.
18. E: I don't know, it just looks really complicated.
19. P: There's a lot of workings out.
20. R: There are a lot of workings out you're right. Can you find one that you don't think is confusing? Is there one in there that you think is really clear?
    [bell rings, door opens, another student enters]

21. R: Hello B, come and join us, we've only just started. Are you happy that we're tape recording? (B nods). I need you to say it out loud my lovely for the tape recording.
22. B: Yeah!
23. R: Fabulous, so what we're doing is, if you remember last time we did this, you guys did some work for me then we had a little talk about it yeah? So this time we're having a look at other people's examples and you're trying to find examples that you think are interesting so it's interesting because it's really different or it's interesting because it's the same or some or something like that. Why did you pull that one out A?
24. A: Ummmm, it kinda of looks,
25. E: I don't like that one. It's using x and it doesn't say x in the question.
26. R: Ooh, ok you don't like that for question three. I'm putting some of them over here because we're going to talk about them in a little bit more detail in a minute. *is making piles of the ones the students choose*. Does anyone else want to add any to there? Oh, you've got a nice pile going on there. Fabulous. Do you want to give me those or do you want to hold on to them? You're going to give me those. OK. Anybody else found any that they'd like to talk about? So I've got one, two, three, four, five, six at the moment.
27. A: I like looking at them.
28. R: It's good isn't it. I find it really interesting seeing how other people did this.
29. E: They've used 'x's as well.
30. R: They've used 'x's as well (sounding outraged)! Ok. Are there any that you find interesting P? Either because they're weird or because you understand them or because you don't understand them?
31. P: Um, I think this one.
32. R: You think this one?
33. P: yeah.
34. R: Why that one?
35. P: It just looks… it just looks quite interesting.
36. R: It looks quite interesting. Shall I add it to my pile?
37. P: Yes.
38. R: Ok, it's on my pile.
39. E: That one's used 'x's as well.
40. R: Lots and lots of these have used 'x's. Shall I pull those ones out and put them on the pile so you can get angry about them later.
41. E: They've used even more 'x's.
42. R: They've used every more 'x's. OK. This is my pile that E would like to set fire to because they've got lots of 'x's. Thank you kindly.

    **5 mins**

    Alright. We'll give B a couple of seconds to keep looking as she has only just joined us…(silence for a while while B sorts)… Ok, you found some? Do you want to add any to our pile or do you want to discuss these one with us?
43. B: I'll discuss.
44. R: OK. Are you sure? That's fine. So shall we scooch all these together so we get them out of the way…(tidying up noises)… So, what we're left with is… so what we're going to have a little look and do is we're going to have a little discussion about why we find them interesting and then we can see if

we can work out what the people were trying to do with what they've written down. Does that sound helpful? Does that sound clear? (Others nodding) Ok, so we're all clear with what we're doing, so…so shall we go through them one by one, would that be helpful? (Laying out sheets) and I've got a mixture here, some that people found interesting because they were clear or interesting because this or because they were confusing or that sort of thing. OK, so. I think this is your (nodding at E) pile of things you wanted to set fire to so I'll put all of those together as a pile. So, what do we think? Does anyone want to grab the one they suggested (or didn't suggest) and tell me why they put that one out there? So, who chose this one for example *taps on one example*.

45. A: Think that was me.
46. R: Think that was you. Ok, so, this one's got lots and lots of…sorry, go on, you tell me why you chose it go on, sorry.
47. A: because it's got lots of workings out.
48. R: Because it's got lots of workings out
49. A: and it's done different.
50. R: and it's done differently. Why is it done differently? What about it is different to what you've done before?
51. A: Ummm…
52. R: Let's narrow it down a bit shall we? Because that might make it easier. OK, question number one do you remember what we did last week for question number one?
53. A: Bus stop.
54. R: We did bus stop and then we talked about, you mentioned about another method where like you halved it and halved it again. Do you remember that? Can anyone explain what they've done for their question?
55. A: They've counted up in fours.
56. R: They've counted up in fours. Does that do the same thing?
57. A: Yeah.
58. R: But why don't we like it as much?
59. E: Cause it takes longer.
60. R: it takes a lot longer doesn't it, yeah? You (talking to A) went "Ugh" cause they've written lots and lots of things down.
61. A: yeah.
62. R: did they get the right answer?
63. P: I don't know.
64. E: Yeah.
65. R: I think they might have done, looking around at some of the others. It looks about right doesn't it, yeah. Alright, so that was interesting because they've done it very differently to like, some of the things we've seen before. Alright so that was a really good example, do you want to talk about some of the other questions on there or do you want to do a different one?
66. A: I don't mind.
67. R: Shall we try a different one? Let me put that to one side for a sec, alright who chose another one that they wanted to talk about? Or if you say any where you'd like to say "ooh, that's a bit weird". What do we think?
68. A: That one because they didn't actually write anything. They just…
69. E: Drew pictures.

70. A: Drew pictures.
71. R: So you chose this one because for question number three we've already sort of started to have a conversation about this whole oh they've used 'x's thing which E brought up and we'll talk about a bit more i a second, but for this one they've got like some picture drawn so they've put like Alice is twice as old as Judy and then they've drawn some pictures. And then they've drawn some pictures. Why do you think they've drawn those pictures. What do the pictures show us?
72. E: The people.
73. R: The people. So what is important about the pictures do you think?
74. P: They're getting smaller from the ages maybe?
75. R: Ooh, that's nice! So maybe they were trying to work out the order. Like who was the oldest and who was the youngest. Do you think it's helped them with the question?
76. P&A: Yes.
77. R: Maybe, yeah, because at least it got them in the right order hasn't it. Because we really didn't like questions three when we looked at it. We found it really confusing. So that at least has helped them work out what age order the students were in, yeah, which is quite nice. Let's talk about this one because this was on your little pile here which you gave to me and went "ewww". So tell me why these ones here actually let's do this the other way round, put these ones towards me and these ones go in front of you. So what was it about these question threes that you didn't like? Cause we didn't like question three last week when we did it because it was quite difficult. So what was it that made you go ewww for these ones?
78. E: they've got 'x's all over them.
79. R: They've got 'x's all over them. What is it about the 'x's that we don't like?
80. E: They aren't in the question.
81. R: They aren't in the question. OK. Do we agree with that?
82. P: Yeah
83. R: They're confusing. So why do you think they've put 'x's there then if they're not in the question? Has anyone got any ideas? 'Cause I don't know so I thought I'd ask. What do you think?

**10 mins**

84. A: It might just help them understand easier… by writing out different ways.
85. R: Yeah. It's almost like this way was one way of them trying to write it out yeah with symbols with like pictures so do we think this is a different way of writing it out to help them understand?
86. A: Yeah.
87. R: Can you link any of the symbols that they've written down to anything that is in the question? So like can you spot where the numbers are the same, like in the bits with the algebra and then in the question.
88. E: twenty seven, three
89. R: twenty seven three so what did they put? Equals twenty seven and then, what does this say?
90. E: All the ages add up to twenty seven
91. R: So they've saying they have to get twenty seven at the end which is quite clever isn't it.

92. A: Why have they put that x is fifteen and then two x is thirty? And then two x take away three is twenty seven.
93. R: Huh. That's interesting. It's OK if you don't get it. It was a really horrible question. Do you remember me saying last week that the questions were designed for like the year thirteens as well as like for the year sevens so went all the way across the board. Um, I quite like this one here. I know that the 'x's are really confusing. Why do you think this one, why do you think I like this one? Why do you think they've got J, A and B. Go on E.
94. E: Because they've used the beginning of the names.
95. R: They've used the beginning of the names. So this is kind of almost like when they're writing down the symbols they've kind of tried to do it for each person haven't they. Can you see that? So that's quite nice, I like that. And what have they done underneath?
96. A: They've writ it out again.
97. R: So they've used it then they've used it with these to try and work out the range which is quite nice really isn't it even though it's horrible and confusing. Yeah? Ok, nice. So you're not a fan of the algebra is what you're telling me but can we see how it might be useful, maybe in a few more years once we've got a bit more practice with it?
98. A: Yeah. (Others nodding, except…)
99. R: Ok? You're still not OK with it P? (She pulls a face) Neeayah… (she mutters something) A bit what sorry?
100. P: In the middle.
101. R: In the middle. Ok, that's OK to be in the middle. Right. Ok. So that's that pile. Let's pop those out the way Ok, and so we're back to this… we've got left.

**12 mins**

102. R: Ok, so any more that we think are weird or that we think are nice or we understand…
103. A: That one.
104. R: (laughing) This one? shall we talk about this one for a second? OK, so somebody tell me why do we think this one's weird compared to all the others
105. E: writing on it.
106. R: Lots and lots of writing on it. So lets have a look at the writing OK, so for question number one, OK, so remember the question said that I've got sixty two pounds to share between four people, how much does each person get? Remember we answered that one last week. So it is says "sixty two divided by two is thirty one. Thirty one divided by two is fifteen point five." So does it like do the right this?
107. A: yeah but they don't write it out in symbols (numbers maybe?) they've written it out in letters.
108. R: They've just written it out as words and letters haven't they, rather than, what did we do instead?
109. A: Numbers.
110. R: numbers and bus stop didn't we. Why do you think they've written it out in words. (Silence). There's not a right answer, don't panic.
111. P: they could make it…might feel a bit easier with writing it in... (trails off)

112. R: they might feel more comfortable writing it in words. Good. Why didn't we write it in words when we did it?
113. P: It's a bit easier perhaps
114. R: (thought B had whispered something) What were you saying B?
115. B: Don't know.
116. R: That's fine. So you find it easier not writing it in words?
117. P: It's alright I guess (very reluctantly)
118. R: OK. So why do we use the bus stop method then? Cause if…
119. A: It's quicker than writing it all out.
120. R: You don't get ???have to do it using bus stop (maybe)??? Like that because we talked about it last week didn't we so your method was the halving and halving it again method wasn't it. Which is kind of what they've done there with their different steps. I bet if we look through that massive pile again we could spot lots and lots of different methods. Shall we do that for a minute, just spotting lots and lots of different methods. Shall we say the words thing, what they've written down is correct but we wouldn't use it (lots of nods). Yeah? That's fine. Let's have another look through this one [pile] then cause what could be quite fun to do is, yes I know there are like two hundred of these, but what could be quite fun to do is if we have another look at question one is see if we can find ones that we would do and ones that we wouldn't do. Does that make sense? So everyone got through and try and find like three or four. Which ones would you do and which ones wouldn't you do?

    **14 mins**
121. A: I wouldn't do that one.
122. R: (Mis hears) you would do that one?
123. A: Wouldn't
124. R: You wouldn't do that one, alright hang on, hold on to it for now and we'll talk about it in a second.
125. E: I don't get this one.
126. R: Ok, so you wouldn't do this one?
127. E: No.
128. R: Ok, hold onto it then.
129. E: Well it's like confusing because…I don't know…
130. R: no, no, go on, tell me
131. E: because sometimes I would like, if I got the answer just to check I would like add it all up again
132. R: So this one's done the bus stop method
133. E: yeah and then add…
134. R: Added up his answer to check it? Ok, so that's sort of works, but you wouldn't have done it for that question. Ok, why not?
135. E: No, I dunno, I just…
136. R: Just wouldn't ??? Anyone else.
137. E: I wouldn't choose that one because it's using fractions.
138. R: Ooh, that ones got fractions so we…
139. A: And that one's got fractions too.
140. R: So did you pull that one out because you wouldn't do that either? OK. Why don't wee like fractions (genuine question)? Or why wouldn't we use fractions?

141. P: ‘Cause they’re confusing.
142. R: (Laughing) because they’re confusing. Why are they confusing?

**15 mins**

143. R: Can you tell me what it says? What does it say?
144. P: (incorrectly) forty two
145. R: What does that say? Slowly.
146. P: sixty two…
147. R: (at the same time, prompting) two
148. P: over four
149. R: Ok, so how does that relate to the question? Where have those numbers come from?
150. P: Oh, sixty two pounds and four people.
151. R: and four people. So do you reckon we could write it like that as a fraction? But it’s still not your first choice of method?
152. P: No.
153. R: Ok. (Students sorting sheets) Oh, that’s an interesting one. Why did you pick that one up B? ‘Cause this one’s got a kind of five bar gate tally system thing going on hasn’t it. Why did you pick that one up?
154. B: because it’s different to the others.
155. R: because it’s different to the others. OK. Tell me why it’s different. What have they done? Can you explain it?
156. B: They’ve used a tally?
157. R: (to another student who’s messing around hiding the sheet we’re talking about - jokingly) Stop it! Put it flat so I can see it. (To B) They’ve used a tally OK. So why do you think they’ve used a tally? What are they trying to do?
158. B: I don’t know.

**16 mins**

159. R: Where do you think the tally has come from? Can you help her P? Where do you think it’s come from?
160. P: It’s like the lines are all adding up to the same number. Maybe?
161. R: Oh! OK, so do you think they’re sat under one person and gone line line line line bar line line line. Do you think they did that as they went along? Or…
162. P: Yeah.
163. R: They could have done couldn’t they.
164. A: This person’s used it as well.
165. R: Or they could have gone along and shared it out one for this person, one for this person.
166. P&E: Yeah.
167. P: Yeah, that sounds right.
168. R: Does that make a bit more sense? (Finally processing A’s comment). Oh this one’s used a tally as well! Look at that one. What’s that one? There’s a lot of lines there and some circles as well.
169. A: Yeah, I reckon that’s going up to sixty…
170. R: So you reckon that’s sixty..
171. A: That’s sixty two yeah
172. R: That’s sixty two and then what do you think the circles are then?
173. A: and then… oh there’s four see one, two, three, four and there’s how many left over.

174. R: Nice. So they've tried to do it between four people.
175. A: Yeah
176. R: And it's kind of worked hasn't it.
177. A: Yeah, and there's two left
178. R: and they've kind of… Do you think think that this person is more successful than this person or that this person's more successful than this person?
179. A: Ummmmm, they've got the same answer.
180. R: They've got the same answer. Ok. So this person's said fifteen pounds each
181. A: And that one said sixteen
182. R: and that one said sixteen, sixteen and fifteen
183. A: Yeah, so they've both got it wrong.
184. R: So they've both got it a bit wrong, but they're both pretty close aren't they.
185. A: Yeah
186. E: Mmm.
187. R: So this person.. what's this person not done?
188. A: Hasn't…
189. R: 'Cause they've shared it out haven't they?
190. A: They haven't put the remainder
191. R: They haven't added the remainder, they haven't added it together. I think it adds up to sixty two, but if I went round this circle and said "there's fifteen pounds for you, and fifteen pounds for you and sixteen pounds for you and sixteen pounds for you" that would be what?
192. E & A: Equal.
193. R: Equal. So they haven't shared it equally, but they have made sixty two, whereas this person has got fifteen pounds each which only makes sixty so they've kept two quid for themselves…
194. A: that's probably why they put two there
195. R: …which was not part of the deal. That's probably why there's two in the middle. Nicely spotted I like it. OK, anybody else got something? Ooh, is this a "yes I would do" or "no I wouldn't do"?
196. A: No I wouldn't do.
197. R: Why wouldn't we do this one?
198. A: Because I don't know what method they've used.

**18 mins**

199. R: ….Can we work out what method they've used? Can we look at this and see what they've done? So they've got a sixty two and four and that's kind of set up like the bus stop method and then what's this times by ten doing? Any ideas?… It's not very straightforward.
200. A: No.
201. R: So what they've done is, it kind of looks like they've done the four times ten to get forty and then taken it away and found the difference. Can you see? But that's not one you'd do?
202. A: No.
203. R: OK, that's fair enough. Any more? P, did you pull any out you wanted to talk about?
204. P: Well, I did look at this…

205. R: The one they've crossed out!
206. P: No this one I looked at and went Ah.
207. R: Oh ,OK, well you can talk to me about that one then. Go on then P. Which one did you choose.
208. P: I thought maybe these two.
209. R: Are these ones you would do or wouldn't do?
210. P: Um, this one I wouldn't do.
211. R: OK, why not.
212. P: because it's a bit confusing.
213. R: Which bit particularly is confusing. Can you tell me?
214. P: I think maybe a bit of the bus stop because it's a bit hard.
215. R: Yeah, ok, the bus stop is a bit tricky so they've used the bus stop which is something you might not do and is it the same for this one? Or is this one different?
216. P: I would do the adding up and add them instead of the bus stop because I like that better.
217. R: So you might write it out and then add it up that way?
218. P: Yeah
219. R: Ah, nice, so that adding up method is nicer than the bus stop method.
220. E: I'd kind of use this one.
221. R: You'd kind of use this one? OK, explain to me what this one is doing.
222. E: They've divided it by two and then divided it by two again.
223. R: Nice ok, so divided it by two…
224. E: Like halving.
225. R:…so it's kind of like that one. Except that one's in words and that one's in numbers.
226. E: that one doesn't really explain what it is or does.
227. R: Ok, so we don't think the wordy one explains what it does. Ok. Fair enough. Explain in general so that we understand? Or explain in a mathematical way.
228. A: Wait, what one?
229. R: this one (pointing at the wordy one). Do we think it doesn't explain in general, like the process or it doesn't explain in like a mathematical way, or is it the same thing?
230. E: Doesn't explain in a mathematical way.

**20 mins**

231. R: (to a panicked-looking B) It's fine not to know. That's OK.
232. B: I don't know.
233. R: that's fine don't worry. Ok any more for any more? Right this one?
234. E: fractions
235. R: fractions, not keen on fractions?
236. P: I've got a fraction one
237. R: you've got a fraction one? Yes we were talking about your fraction one
238. A: that one's four and that one's two
239. R: (confused) that one's four and that one's two? (Gets it) oh that's an interesting one. Why have they used two for their fraction and they've used four?
240. A: cause they've half-ed it and half-ed it again
241. R: mmmhmm

242. A: and they've gone into four. As in fours in sixty two
243. R: in one go
244. A: yeah
245. R: so do you think you'd do one of these more likely than you would the other?
246. A: that one
247. R: you'd do the halving and halving again and you're happy to write that as a fraction almost? Yeah ok, nice, alright.
248. E: I'm confused with this one
249. R: alright, lets have a look then. Ugh, I'm confused with this one. (Shows the others) can everyone see that one? So what on earth do we think is going on here?
250. E: I have no idea
251. A: counted up in fours
252. R: but they haven't started at four have they? Where have they started
253. E&A: forty.
254. R: so why do you think they started at forty if they were counting in fours?
255. A: because they knew ten fours are forty
256. P: don't divide it as much maybe?
257. R: yeah, so maybe they know that not already so they don't have to do as much dividing and they're just looking at what comes after that. That's quite nice, I'd not spotted that one before. Good spot.
258. A: I found that one
259. R: you found that one, ooh, I think B's going to, wait, B's going to talk to me first. B? Did you find one you wanted to talk to me about?
260. B: umm, not really
261. R: not really? What's that one you're holding? What method have they used? (B shows her paper), ooh, do we know that method?
262. P: that's taking away
263. R: taking away. Let's have a look then. Oh yes, they've taken away forty there and they're left with 22. And then what have they done there? What's this bit doing down here?
264. B: um is that bit times?
265. R: they've timesed as well haven't they. What are they timesing by?
266. B: five, I think
267. R: so, what have they timesed by five though to get that? Do we think?
268. K: um, fifteen?
269. R: not sure? It's kind of done this hasn't it, look, they've spotted the four and they've done four times ten to get forty and four times five to get twenty and then they've done their subtracting instead so I guess for someone who's not very happy with dividing, that might be a nice alternative where they do timesing and taking away.

    **22 mins**
270. A: I would do this one.
271. R: ok, why? This is a divide by two, divide by two one
272. A: they've worked it all out and then they've added it up to make the same answer.
273. R: so both of you have chosen ones where you would add up at the end to check your answers haven't you.

274. A: yeah.
275. R: would you do that when you were like doing it in a test or in an exam?
276. A: yeah
277. R: you sure? You didn't do it the other day though when you did it with me
278. E: I don't know if I would
279. A: I did! Cause I added it up to make sure it's right
280. R: oh, I take it back then. Ok. Did you do it in your head or did you do it on the paper?
281. A: on the paper
282. R: you'd it on the paper so I can see that
283. A: to double check
284. R: you didn't though (to E), is there like a reason you wouldn't or you just…
285. E: I probably just forgot
286. R: probably just forgot? Ok, fine.
287. E: ??
288. R: that's alright, don't worry. That's fine ok, we've got five minutes left. Shall we leave these and go back to the ones we thought were weird before for a minute, yeah? I think I've still got a little pile of ones we pulled out because they were strange. (Student hands over another sheet of paper) is that to go on the pile of things we think are strange? (Student nods) is that because of question three?
289. P: that one, because they're using different factions than the other fractions.
290. R: ooh. This one is using lots of different fractions, let's talk about this one then, so (shuffling papers) … I've confused myself now I think these are all ones we pulled out before. Ok, so, up here I have got, let's just squidge these together a little bit so they're a little bit mor out of the way, um, I have got some mor examples that we pulled out because they were confusing. Who would like to (student trying to tidy up discarded papers) it's alright, don't worry about making them neat, just squidge them together, I'll do that when you guys are in tutor later. So why these ones are ones that we pulled out because they're weird?
291. A: fractions?
292. R: fractions again. Which ones were the factions ones?
293. A: that one over there.
294. R: that one over their. This one? Was a fractions one was it? Ok, why didn't we like this one. Cause actually there's kind of a few methods on here. Can anyone tell me some of the methods they've got on here? Cause yeah, I can see the fractions here.
295. A: oh, they've done it all different
296. E: they've done that
297. R: what's that?
298. P: they've done all fours
299. A: they've done all possible answers
300. E: added up
301. R: they've done their fours, right P, good and then they've added up and then you said that these fractions were weird weren't they, didn't you
302. A: because…

303. R: what on earth have they done there? Why is that only twenty two out of four?
**24 mins**
304. A: because it'll come to five and then two over four because there's two left over so it'll equal five and then there's fifty at the end
305. R: oh I see so they've kind of done it section by section almost so they've got the twenty two divided by four which gives you five and the two divided by four which gives you zero point five because that two would have been left over from that twenty two, yeah.
306. A: yeah
307. R: what's the bit they're missing here? What did they do here that left them with twenty two over four?
308. A: because they haven't worked out how you get fifteen
309. R: they haven't worked out that bit. They've worked out the five and they've worked out the fifty but they haven't worked out where this bit comes from have they, or they haven't written it down, they've obviously worked it out cause they've got the answer. Ok, so that's a weird one but we can kind of see where bits of that come from, yeah? Something we'd do or not do?
310. E&A: not do.
311. P: didn't get it
312. R: still didn't get it? Ok, would you guys do this (to E and B)
313. P: um, no (B shaking head)
314. R: why not P?
315. P: quite confusing
316. R: it is quite confusing, that's fine. Alright, one, two, three, four, five, six to go. Any more? What about these ones? What was it about these ones that made us go "ugh"?… did anyone pull one out because their question two was slightly strange? Or they didn't know what was going on for question two?
317. A: that one cause they've squared it.
318. R: ooh, so there's a squared bit on here. Ok, so this one's done six times five equals thirty centimeters squared and that confused us a little bit did it?
319. A: yeah, because I don't know why they squared.
320. R: you don't know why they squared was there. Ok, what have these two people done? So these two people have done something similar with their question twos. What have they done here with this extra finagle bit drawn one the end.
321. A: they, um, worked out the (bell goes, student pauses and looks at the door)
322. R: (reassuring) finish your sentence
323. A: ummm, like perimeter instead
324. R: why is that shape (the one on the other half of the paper for those who don't want to be included) easier to do than that shape? Without the extra bit on the end?
325. A: oh, because you get told where it's already there. You know that's already four centimetres up there, so…
326. R: OK
**26 mins**

327. A: it's easier to work out… and then you know that's five centimetres and then cause that's five centimetres.
328. R: so it's easier to have that straight up rather than that which is slanted I see ok.
329. E: there's also another one that like cut that into four pieces
330. R: there was
331. E: and then numbered them
332. P: oh yeah
333. R: do you think that was easier?
334. P: yeah
335. R: lets have a look, hang on, let me find it
336. E: I don't think it was in…
337. A: I found it and put it in
338. R: if you're late for registration you can tell them, oh yeah, I know I kept it one side. Do we think that's easier or do you think that's the same? Or do you think it's more difficult than just that bit there.
339. A: I think it's easier.
340. E: it's more difficult to see the work
341. R: OK (to A). You think it's easier, why do you think it's easier?
342. A: I think they've added, like, they've worked out how you can get, like, they've worked out every section.
343. R: OK, so you like it because it breaks it down.
344. A: yeah
345. R: (To E) You think it's more confusing cause there's more working?
346. E: yeah
347. R: is that what you said? What about you guys (to P and B)
348. P: Mmm… maybe.
349. R: maybe, but we're not really sure. Ok, anything else you want to add whilst we're here? Or is everybody happy that they've had their chance to explain which ones they thought were weird and which ones they thought weren't.
350. P: yeah
351. R: you've been absolute superstars and you need to get to registration so let me turn this off, hang on…

**Appendix Eleven – Student Group Session Three**

1. H: Ok, I think we're recording. Are you both happy for me to record this conversation?
2. B&P: Yep
3. R: Fabulous. We know by know that we have to say that out loud. OK, so this is our third session isn't it, our third lunchtime doing this is that right? The first time we talked about how you would do some of these problems, and then the second lunchtime last week we looked at how other students had tried these problems didn't we and you talked about what you liked about them a nd what you didn't like about them and which ones made you go uhhhh, like this umm, so what I thought we'd do for the third one is that I have got some examples of how teachers would do the problems so I thought you could have a little look at the teachers' methods and then you get to say what you like about those and what you don't like so that's, that would be a little bit interesting wouldn't it it's a little bit different. Let me move that a little bit out of your way (moves mic) and let me show you what I've got here, so, take my post it note off 'cause I kind of filed these and then forgot I was going to use them again, so let's have a little look. So what I've got here is the same thing again, these haven't been chopped in half so these are the same problems again, but this time, these are done by the teachers there we go, so, sort of a similar thing to last time when we looked at them. What I want you to have a look at is which of these do you think, like, really clearly shows what the teacher's done? And which of them do you like, sort of as methods, which of them make sense to you as methods? Does that make sense? Is that alright? So take a couple of minutes, have a look through, feel free to, like, steal them from each other and different sides of the table OK. Have a little read, a little think.
   [Silence, papers rustling]
   **2 mins**
   Maybe see if you can choose one or two and steal them out of the pile and say "right, I'd like to talk about this one". [Silence, papers rustling]. Are you both happy that you have something we can talk about? Yeah? Yeah? OK. Who would like to start? Would you like to start B?
4. B: Yes
5. R: Go on then. Show me, put it flat on the table so we can both see it - put it there 'cause then I can read it upside down. Why did you choose this one as one that you wanted to talk about?
6. B: 'cause it's different to the um bus stop method
7. R: Ooh, OK, so for question one they've used a different method. Do you want to explain what they've done?
8. B: I think they're halving it?
9. R: They're halving it, OK, good do this here says sixty two divided by two which is the same as halving, lovely, and then what did they do?
10. B: They halved it again?
11. R: Lovely, really nice, and they got fifteen pounds and fifty pence at the end. Very nice. that's a very interesting one, 'cause that was very different to what we did wasn't it when we did it last time 'cause we all did bus stop,

lovely, what a very good example. P, your go. Which one would you like to talk about?

12. P: That one.
13. R: This one? OK. So, any particular part of it that you'd like to talk about?
14. P: Like this bit…
15. R: [talking at the same time] this bit at the top? Sorry, go on.
16. P: cause looks a bit easer than bus stop and that its going down like B's, but it….goes in twos.
17. R: Lovely, so how's this one written? What have they used for this one? **4 mins**
18. P: Is it…sort of column?
19. R: What does it look like B?
20. B: Fractions
21. R: They kind of look like fractions don't they? Yeah? So sixty two divided by four is a fraction and then thirty one over two is a fraction and then 15.5 at the end. So you quite liked that because it looked easier? Right, so both of these methods actually look a little bit easier than the bus stop method one that we chose don't they is what we're saying???? Lovely, OK, and anything else that you'd like to, because you had two didn't you. Is that right? What about your other one P?
22. P: Um, they added, um they timesed the shape up. So they get the
23. R: Oh cool so this is the second question you want to talk to us about on this one. Ok, so say that again.
24. P: They, like, timesed the six or the five so ???
25. R: Yep
26. P: And they timesed two and then four why have they timesed that one there?
27. R: Oh I see, so this is because… so is it because of what they…. The method they've used, or is it because of where they've written the method that's helpful?
28. P: Mmmmmm, where they've done it.
29. R: Ok. 'Cause they've kind of done it inside the shape haven't they.
30. P: yeah
31. R: yeah so the six times five is inside the rectangle bit and then the half times four times five is inside the triangle bit. Do you like that one as well B?
32. B: Mm.
33. R: For the same reason or a different reason?
34. B: Same.
35. R: The same. Ok. Alright. What about your second one? Is it under here is it that one? Is it that one you chose? Right, tell me why you chose that one.
36. B: Um. Because for this one they use xs.
37. R: Ooh, OK so for question three on this one they've used xs. Tell me why that helps, tell me why you chose that.
38. B: Um. [long pause] Don't really know.
39. R: Ok, so did you choose it because it was interesting rather than because it explained it?
40. B: Yeah.

41. R: OK, so you liked that they used xs for that because you thought it was what, different to what we've seen before, or the same, or…
42. B: Different.
43. R: Different to how they've done it before.

**6 mins**

44. R: OK. It's interesting that you've chosen question number three because we have't really talked about question number three have we, other than like lesson, uh, last session where we didn't like the algebra which was quite funny. OK, so these are the ones that we like and are interesting, particularly for question number one and maybe a little for question number two you thought that you'd found some things that might make it look a little bit easier. Is that about a good summary of what we've had a look at? [nods] Alright. I've written down some questions which is why I keep looking at this piece of paper - to make sure that I'm asking all the questions. OK, have a little look through again so I'll it them back out so we can see them all, um, are there any that you don't understand and you'd like to ask questions about what they've done or why they've done it? So again, take some time and look through them. Are there any that you think you don't understand and you'd like to ask some questions about? You don't understand what they've done or why they've done it…[long pause while they sort through]… does everyone want to choose one or maybe two that they want to ask some questions about? You're choosing that one are you P? B, at that you're not sure about? It can be any question. I don't mind if it's question one or two or three.

**8 mins**

I'll move that one out so you can see cause that one's hiding under P's at the moment… [another long pause] Do they all make sense or… you could have a look at one of the questions threes couldn't you because that's been quite a confusing question for us hasn't it overall so do you want to pick one of the ones for question three that you think you might like to talk through? This one? Yeah? Ok, let's have a look then. So, P, why did you choose the one that you chose?

45. P: 'Cause it's got Js in.
46. R: 'Cause it's got Js in. Has yours got Js in as well [to B] Oh So we've chosen ones that have got sort of similar things! Oh, that's interesting, so let's put them next to each other and see. Right, so can we spot things that are the same about those methods?
47. P: They've got Js in.
48. R: They've got Js in. What else?
49. P: And the two and the…
50. R: two J and J plus three that's what you're pointing at isn't it B, and then what have they done?
51. P: They've both got the same total.
52. R: So what does total mean?
53. P: Like, when they add it up and they get them.
54. R: When they get the same so yours have got the same here but what's different about yours to P's B?
55. B: It's not labelled?

56. R: It's not labelled. So this one's got some names on it to explain what's going on. So this person has said 'OK we're going to write Alice's age as two J, now why do we think they've written Alice's age as two J? Can you see where that matches up with the question?
57. B: Twice as old as Judy.
58. R: Good, yeah. So it's kind of like a code isn't it. They've written Alice is twice as old as Judy which is what the two J is for. Now Judy is as old as Judy which is why they've written Judy equals J and then what about Ben? How does that match up with the bit of the question?
59. P: Three years older than Judy.
60. R: Yeah. Or Ben is… hang on… or [reading from the question] 'Judy who is younger than Ben by three years'. Yeah.

**10 mins**

61. R: So Judy's age plus three gives you Ben's age. Why have they added them up?
62. P: 'Cause they're going high? I don't know.
63. R: Which bit of the question tells us we need to add them up?
64. P: Is it there [points to total number in workings] where like…
65. R: Yeah, so that's what they've done. They've added them up and that's what they've got as their answer. But there's a bit in the instructions that tells us we need to add them up can you see it?
66. P: Is it like [still looking at the workings]…
67. R: Which bit up here [gestures at question text] tells us we need to add it up?
68. P: Is it 'Ben's three years'…
69. R: yes, so that's that bit there isn't it? [point to workings again] yeah?
70. B: Uh, 'aged twenty seven'
71. R: Mmmmm Good. So 'the sum of their ages is twenty seven' can you see that so the sum is the same as add 'em up yeah? So we've summed them up and said [points at the equals sign] the sum of their ages equals [points at the digits 27] twenty seven. Very clever. So it's kind of a code isn't it. We've kind of used letters to decode what the question says. Do you think looking at that thing there four J plus three equals twenty seven is easier than trying to read the question and make sense of it or not?
72. P: Yeah, I think it makes a bit of sense.
73. R: A bit of sense yeah? Do you think it's easier than reading the wordy question B, or do you think it's about the same or do you think it's more difficult?
74. B: it's about the same.
75. R: About the same. That's alright. That's fair enough. Then what they've done is they've looked for four times something plus three gives me twenty seven and they've worked out what that number must have been. Can you see that. Good. So good examples of something that looks a little bit confusing but well done on thinking through and seeing how it relates to the question. Very, very impressed. Last thing, because I think the bell's about to go and you two need to dash off and grab some lunch don't you.

**12 mins**

Um, my last question is can you see anything in here, and we'v kind of talked about this a little bit already, 'cause you've talked about how some of

the methods for question one look like they were easier than the bus stop method right, and that was the one you used, are there any other things in here that you spot as being very very different to the methods that we used when we tried to do it. So if I spread them out again, are there any of these that you think 'ooh, I definitely wouldn't have done that'…[long pause]… OK, P's got one. [to B] You want to go with that one? OK. You happy with that one P? OK. Right, B's turn to go first. Why have you chosen that one as 'oh no, definitely not, I wouldn't have done this'.

76. B: 'Cause I'm not sure what they've doing there with the arrows.
77. R: Ooh, nice, OK, so we're looking at question three and this one's got something which not a lot of the others has got which is they've shown their steps with arrows when they're solving the algebra. Which is, is this just something you wouldn't have thought to do, is that right?
78. B: Yeah.
79. R: OK, lovely, that's a really good example. Do you know what they're trying to do with this? Can you see what's happening to the equation in the middle?… So what's different between that line and that line? What's gone away?

**14 mins**

80. B: The three?
81. R: The three. So these arrows are showing us that they've taken the three away from that side and then done the same thing on that side can you see? So that's what they're trying to to do. They're trying to explain the method that they've used for each side, but if that's something that you would do that's absolutely fine. [Lunch bell rings for their lunch slot]
82. R: Last bit, go on then P.
83. P: Um, I think that bit 'cause…
84. R: OK, so this is question one and the bus stop method, go on.
85. P: I think wouldn't have tried that one 'cause I'd probably have tried something a bit easier.
86. R: OK, what would you use.
87. P: I'd probably do like add it up and do some columns.
88. R: Add it up using columns, OK. That sounds like a good idea, and what would you add up?
89. P: Um, maybe like them bits and see how much money they'd get.
90. R: OK. Lovely. Alright. Any questions about anything we've talked about over the last few weeks?
91. B&P: [at the same time] No.
92. R: Ok. Well, thank you so so much for all of your help. It's been an absolute pleasure I hope it hasn't been too awful giving up a little bit of your lunchtime every week and it's been absolutely brilliant and thank you coming. Is that alright? Fabulous. Enjoy the rest of your day.

## Appendix Twelve – Additional Analysis of Teacher Group One

### 6.4.1 Different Methods

The first theme I identified from the transcription of the meeting was that of the variety of ways of referring to methods. During the course of the session, the teachers used a lot of different ways of referring to their own mathematics. "Method" is used frequently, for example:

> K:… maybe that was one method too many

But also process and strategy:

> M: That's why you find it easy because you can see all the different, you can see what all the connections are whereas if you're just seeing a process that you can't really put in its place unless you…
> K: But when we're answering questions of that form, we have a bank of different strategies for doing it which is what we're hopefully trying to give them is a batch of different ways of being able to find these things, rather than just you do this, you do this, you do this and you write down whatever the answer is.

By far, the most commonly used way of discussing mathematics seen on the page is "do", "doing" or "done", evident in the previous quote, but also here:

> I: Yeah, so I've basically did the same thing both times I was just more explicit about what I was doing for thinking about teaching the students. So, I did it by compound shapes just because of the way that one's been set out really lends itself to seeing it as a rectangle and a triangle whereas if it had been more of a trapezium with like triangles on both sides if you know what I mean I might have been tempted to use the trapezium formula instead but that just felt the most simple way of doing that one.

All of these terms appear to describe the same thing so, for clarity, the term process/strategy/method refers to the practical steps the person has taken to

"do" the problem or to the tools they have selected to solve the problem or the particular mathematical approach they have decided to employ when coming into contact with the mathematical situation. The actual thing that underlies all these methods, or the object they are demonstrating conceptual awareness of, is division. Referring back to Vygotsky's mediated action triangle (see section 2.2.1, Figure 2.1), or Wertsch's mediated action (section 3.1.2), these tools are the link between the stimulus and response or the mediating action of the agent. They are all different ways of interacting with/building meaning with the concept of division. From a Bakhtinian perspective, I am suggesting that these methods are examples of utterances shaped by the surrounding tensions.

One discussion about methods was particularly interesting to me:

> R: …What sorts of methods have we used?
> N: Did both - half and half again and divide (correctly this time).

For me, this discussion is interesting because I interpret this comment as teacher N seeing "half and half again" and "divide" as (a) two different things, and (b) the only two ways of doing this question. In addition, based on the tools idea I have just discussed, "half and half again" would be considered as a tool or method, whereas "divide" would be the underlying concept. When pressed, the teacher clarifies:

> R: When you say divide correctly, what method did you use for that one?
> N: I used what they call the bus stop…short division.

So, in fact, would I consider "half and half again" and "bus stop… short division" to be the two different methods? Following Wertsch (section 3.1.2), different methods could then be seen as different tools for approaching division. Using this idea, students learning different methods for this problem would be seen as adding to their toolkit of methods for completing a division problem. I am not directly analysing the artefacts, but I can use the toolkit analogy to look at the discussion between teachers about methods.

Having discussed tools and concepts, one question raised is the difference between methods and concepts. Barwell talks about formal and informal mathematical language (sections 4.4 and 6.3). I am arguing that there is a similar framing available with mathematical methods, but is there also the same set up for concepts? Or is there a clear change in sophistication for different concepts, hence my choice of questions that get more challenging through the sheet I have designed? It seems logical that a question about forming and solving equations (Question Three) is more conceptually difficult than that of dividing by four (Question One) but is that supported by Bakhtin and Vygotsky or is it more intrinsically tied to signs and meaning making than that? A hierarchical framing of concepts implies there is some remote mathematics out there that is accessed via the methods that we use, abstracting from the idea that nothing exists independently. Every concept is formed in context so is this remote concept built by overlapping use of mediational means (Vygotsky) or utterances (Bakhtin)?

### 6.4.2 The Meaning of Bus Stop

One of the methods or tools the teachers discussed in the previous section is the bus stop method. It is clear that within this dialogue, the phrase bus stop has a particular meaning beyond any day-to-day definition regarding a stopping place for buses. The phrase bus stop in this context refers to a standard method used to solve a problem involving division and is sometimes called short division. It is clear from the teacher conversations that this method is widely used and that bus stop is a term they are comfortable using:

> R: I can see lots of bus stops looking around
> M: I wrote the bus stop
> …
> I: So, I did bus stop

The teachers are using the phrase bus stop in a way that they expect the other teachers to understand and, indeed, in this context, bus stop is an established term for short division used widely, carrying with it a set number of

connotations. In an analysis, can I therefore use particular terms such as bus stop as markers of speech genres (section 2.1.3) that have developed within a particular context? Would the use of bus stop be an indicator of formal mathematical language or informal mathematical language, referring, as it does, to a formal, short division method? I would argue that the use of bus stop here reflects Barwell's idea that, from a Bakhtinian perspective, the sense of what is formal and what is informal is relative to the context.

### 6.4.3 Using Student Speech

As part of the discussion about the bus stop method, I noticed a technique used by one teacher who talked about prompting students to use the bus stop method:

> V: Um, I tend to encourage them to use the bus stop method whenever they possibly can because they forget and then when I prompt them "What method do you have for dividing?" "Don't know, don't know, oh bus stop!" "Can you do it like that?" "Yeah, probably".

The student voices are not direct quotes, but more a construction of students' words based on prior experience. A similar example features in section 6.4.5. The teacher's use of students' words is not a literal revoicing of students as we see from the teacher in Barwell's example (2015). In his example, the teacher takes students' ideas around the definition of a polygon and revoices them using more formal mathematical language:

> In the teacher's utterance, the use of 'closed' responds to the student's 'there aren't any holes' and a relation is established between them. This relation is dialogic; it features the presence of more than one perspective, and is clearly shaped by the tension between centripetal and centrifugal forces of language. In the teacher's revoicing, in particular, this tension is apparent. By reformulating the student's perfectly understandable expression, the teacher both acknowledges and demonstrates heteroglossia; there are multiple ways to say the same thing in

mathematics.

(Barwell, 2015, p. 11)

However, the extract I have highlighted could still be an example of using the words of others. Does the inclusion of student words invoke similar formal/informal tensions as their use in the example Barwell gives? The teacher's use of mock student words could be to illustrate their encouragement of students towards an approach. The teacher might be using the technique to lend extra weight to the point they are trying to make by showing student agreement with their comment. Or is it a more overt example of how we use the words of others all the time?

### 6.4.6 Teachers Being Wrong

The transcript contains examples of the teachers in the group discussing instances where they have made mistakes or made corrections:

R: (to K) are you just checking?
N: (to K who's leaning over her work) Oh have I made a mistake?
K: (agreeing) Mmmm
[laughter]
N: fours into 6 goes… Yes, so I have, I beg your pardon.

M: I have got the last question wrong.
R: That's alright…it makes…it's…either way it's interesting to talk about so that fine.

R: [to a teacher adding to a method, laughing] Stop changing your answer!
N: It doesn't affect my answer! I just notated it wrongly here because Judy wasn't $x$ - 3.

R: … are people just changing their answers afterwards? [teachers were annotating on their solutions]

> M: I just couldn't work out where I'd actually gone wrong and that was what upset me!

To be clear, I am in no way implying that the teachers do not understand the mathematics, or that they are unfamiliar with any of the methods (although with the wide variety of possible methods available for different questions, suggesting all teachers are aware of all methods ever is also not what I mean). However, acknowledging that teachers make mistakes does go some way towards countering the narrative of the teacher as some sort of infallible expert representing the curriculum that is present in a dialectic approach. By reframing the teacher role from that of a transmitter of knowledge or an organiser of a zone of proximal development (ZPD) then, still from a dialectic perspective, is it sufficient for teachers to have a grasp of what is more formal so they can extend the repertoire of the students? Can framing teachers as having a better scientific understanding, better mastery of the tools, a wider variety of tools in their toolkit, move a dialectic framing of teachers to a closer, more relative position of expert?

The relative position of expert echoes the suggestions Barwell was making about the context-based definitions of formal and informal mathematics for a dialogic approach. For a dialogic approach, does the teacher represent one who has had more experience in meaning making in a mathematical context, being more fluent in the speech genre of mathematics? Or, do teachers simply have more to bring to a space of mathematical inquiry? Reframing teachers as those who are constantly making new meaning themselves allows room for teachers who are learning and making mistakes themselves, placing emphasis on the process of learning for students seeing that. This seems to be a long way from the master/slave relationship that Matusov talks about (section 4.2). Is there, instead, a tension for teachers, here, in terms of being the person with the more formal representations versus still learning themselves?

## Appendix Thirteen – Additional Analysis of Student Group One

### 8.2.2 Repeated Halving

The following extract is about an alternative method for Question One suggested by the same student who had struggled to use the bus stop method effectively in section 8.2.1 and shows the first alternative to the bus stop offered by students:

58. R: - do we know any other methods other than bus stop method for doing dividing does anyone know of any others ..
    *snapping noise*
    you're just going to sit there and break your pencil *students giggling*
    so did we all use bus stop kind of
59. E: you could half it and half it again maybe
60. R: ooh that's quite a nice method we could halve it and halve it again did you hear that so we could halve if we halve it sixty-two stop it *pen dropping on table all giggling*
    all I'm going to hear is clattering on this recording so what would happen if we halved sixty-two then does there you were no hang on hang on you were starting to write it down what were you writing down
61. B: half of it
62. R: half of sixty-two what is it what were you going to write down
63. B: thirty-one
64. R: thirty-one perfect really nice go ahead yeah I just spotted you doing and I thought ooh fabulous that's really good so that's um sixty-two divided by if I've halved it what have I have I divided it by
65. P: two
66. R: two so that's divided by two but I'm trying to divide by four so what do I need to do once I've halved it
67. A: halve it again
68. R: halve it again .. so what could I do if I'm doing thirty-one divided by two let's see if P wants to tell me no no no no pressure if you don't want to that's absolutely fine
69. P: I'm not really sure

70. R: not really sure could we do thirty divided by two does that work if we were sharing thirty pounds between us do you know how much we'd each get
71. P: we'd get . is it…I don't remember um .
72. R: ooh can we give her a hand
73. E: fifteen
74. R: fifteen each does that work yeah
75. P: yeah
76. R: fifteen each that makes thirty doesn't it what about if we've got one pound and we're sharing between each other
77. P: fifty p
78. R: fifty p so that would be fifteen pounds and fifty p wouldn't it yeah .
79. P: mmmhmm ..
80. R: which is the answer so that's another really good method that E's suggested that gets us the right answer nice one E

In the extract above, one student suggests repeated halving as an alternative to the bus stop method for Question One (contribution 59). The teacher then offers specific guidance, for example, in contributions 64–68 the teacher is supporting the student in making the link between halving and dividing by two. From a dialectic perspective, the teacher is trying to help students who have struggled to employ the bus stop method by discussing use of an alternative tool or mediational means which aims to help the students achieve the desired action – in this case, a correct solution to the mathematical task. This alternative method is based on halving by scaffolding through the Zone of Proximal Development (ZPD). By using scaffolding questions (e.g., contributions 66 and 70), and the support of other students (contribution 72) the teacher is helping one student to complete something they cannot do unaided (contribution 69) but can with support (contribution 77). One student started to write down the result of the first halving with no prompting (contribution 61), suggesting they needed less scaffolding. The students could be considered to be at different stages of internalizing (V2) the concept of halving. The use of the ZPD is in evidence in the

extract in section 8.2.1, as well as the teacher supporting students to use the bus stop method.

From a dialogic perspective, by using the student's phrasing of "half and half again" (contribution 59, echoed in contribution 60) alongside "divided by two" (contribution 70), and introducing the context of money from the problem to help with the decimal solution (contribution 76), the teacher is trying to help students draw on multiple voices or discourses (Barwell, 2015, p. 9). Drawing on a variety of voices and discourses achieves the effect of layering voices (as I am arguing that the question represents another utterance and therefore other voices (B1), e.g., that of the examination board), providing the students with multiple voices with which to make meaning (B2). By introducing the question to provide context for the students, the teacher is using the students' familiarity with using money to help, which is different to how the context for Question One has been discussed previously. In section 7.2.1, teachers discussed relating solutions back to the context for examination marks, which I suggested could be seen as an indicator of mastery in a dialectic approach. However, its use here, to support meaning making (B2) for students, suggests a different significance to the context. The tension (B3) here between abstract "thirty divided by two" and context "sharing thirty pounds between us" (both in contribution 70) is being utilised by the teacher as part of the process.