Dynamics of high-energy excitations in few-particle Hubbard models

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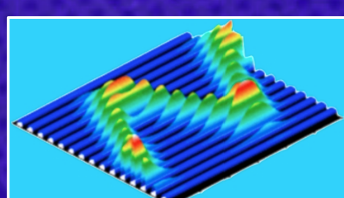


Nonlinear Schrödinger Equation:
Theory and Applications
Heraklion, May 20 – 24, 2013

Motivations (I)

1. The physics of interacting quantum systems is extremely broad, and stems at the heart of many hot topics in research.

   In the strong interaction regime, quantum transport becomes dominated by **correlated tunneling**, with *dramatic* deviations from single-particle tunneling...

   → **macroscopic effects**:

   - metal-insulator (Mott) transition,
   - superconductivity,
   - ferromagnetism and anti-ferromagnetism,
   - etc...

➡ **The study (and control) of the dynamics of few interacting particles in the presence of coherent driving with external fields and investigation of the role of particle statistics in correlated tunneling phenomena**

Motivations (II)

2. Quantum phenomena can be simulated in other physical contexts, by exploiting fundamental analogies between different fields. E.g.:

- cold atoms in optical lattices (quantum simulators) [see e.g. Nature Phys. 8, 267–276 (2012)]

- optical waveguide arrays (classical simulators) [see e.g. Laser & Photon. Rev., 1–19 (2008)]

→ **The proposal and realization of photonic structures mimicking low-dimensional Hubbard Models of few strongly-interacting particles to visualize in Fock space many-body quantum phenomena never observed in truly quantum systems**

Outline

1. Introduction: What connection with the NLSE?
2. Photonic simulators of correlated particles
3. Fractional Bloch Oscillations
4. Dynamic Localization of "*Doublons*" and Coherent Destruction of Correlation
5. Correlated-tunneling of Anyons and Correlated Bloch Oscillations of Anyons
6. Conclusions and Developments

Introduction:
(What connection with the NLSE?)

Introduction (I)

The Nonlinear Schrödinger Equation (NLSE) is a universal model equation, encountered in different contexts.

Within many-particle physics, the NLSE can be derived from the the Many-Body Schrödinger Equation (MBSE):

$$i\hbar \frac{\partial}{\partial t} \hat{\Psi} = [\hat{\Psi}, \hat{H}] \quad \text{with} \quad \hat{H} = \hat{H}_0 + \hat{H}_{int}$$

where

$$\hat{H}_0 = \int d\mathbf{r}\, \hat{\Psi}^\dagger(\mathbf{r}) \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) \right] \hat{\Psi}(\mathbf{r})$$

$$\hat{H}_{int} = \frac{1}{2} \int d\mathbf{r} \int d\mathbf{r}' \hat{\Psi}^\dagger(\mathbf{r}) \hat{\Psi}^\dagger(\mathbf{r}') V_{\rm int}(\mathbf{r} - \mathbf{r}') \hat{\Psi}(\mathbf{r}') \hat{\Psi}(\mathbf{r})$$

and $\hat{\Psi} = \hat{\Psi}(\mathbf{r}, t)$ is the particle field operator, obeying commutation (anti-commutation) relation for bosons (fermions).

Introduction (II)

For a cold and dilute gas of bosons, under a semiclassical approximation (Bogoliubov prescription)

$$\hat{\Psi}(\mathbf{r},t) = \Phi(\mathbf{r},t) + \hat{\zeta}(\mathbf{r},t)$$

$$\langle\hat{\zeta}\rangle = \langle\hat{\zeta}\hat{\zeta}\rangle = \langle\hat{\zeta}^{\dagger}\hat{\zeta}\rangle = \langle\hat{\zeta}^{\dagger}\hat{\zeta}\hat{\zeta}\rangle = 0$$

$$\Phi(\mathbf{r},t) \equiv \langle\hat{\Psi}(\mathbf{r},t)\rangle$$

condensate mean field

number of particles $N = \int d\mathbf{r}\, |\Phi(\mathbf{r},t)|^2$

the MBSE becomes the Gross-Pitaevskii equation (i.e. NLSE):

$$i\hbar\frac{\partial}{\partial t}\Phi(\mathbf{r},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) + g|\Phi(\mathbf{r},t)|^2\right]\Phi(\mathbf{r},t)$$

for contact interaction approximation: $V_{\text{int}}(\mathbf{r}-\mathbf{r}') = g\,\delta(\mathbf{r}-\mathbf{r}')$

Introduction (III)

In a lattice, a truncation of the MBSE without any semiclassical approximations, gives rise to another universal model: the Hubbar Model (HM).

The essence of the derivation:

1. Expansion of the particle field in Wannier States

$$\hat{\Psi}(\mathrm{r}) = \sum_{j,\sigma,b} \hat{a}_{j,\sigma,b}\, w_b(\mathrm{r}-\mathrm{r}_j)$$

j : lattice site index

σ : single-particle spin quantum number (m_z)

b : single-particle lattice band index

$w_b(\mathrm{r}-\mathrm{r}_j)$: j-th Wannier state in the b-band

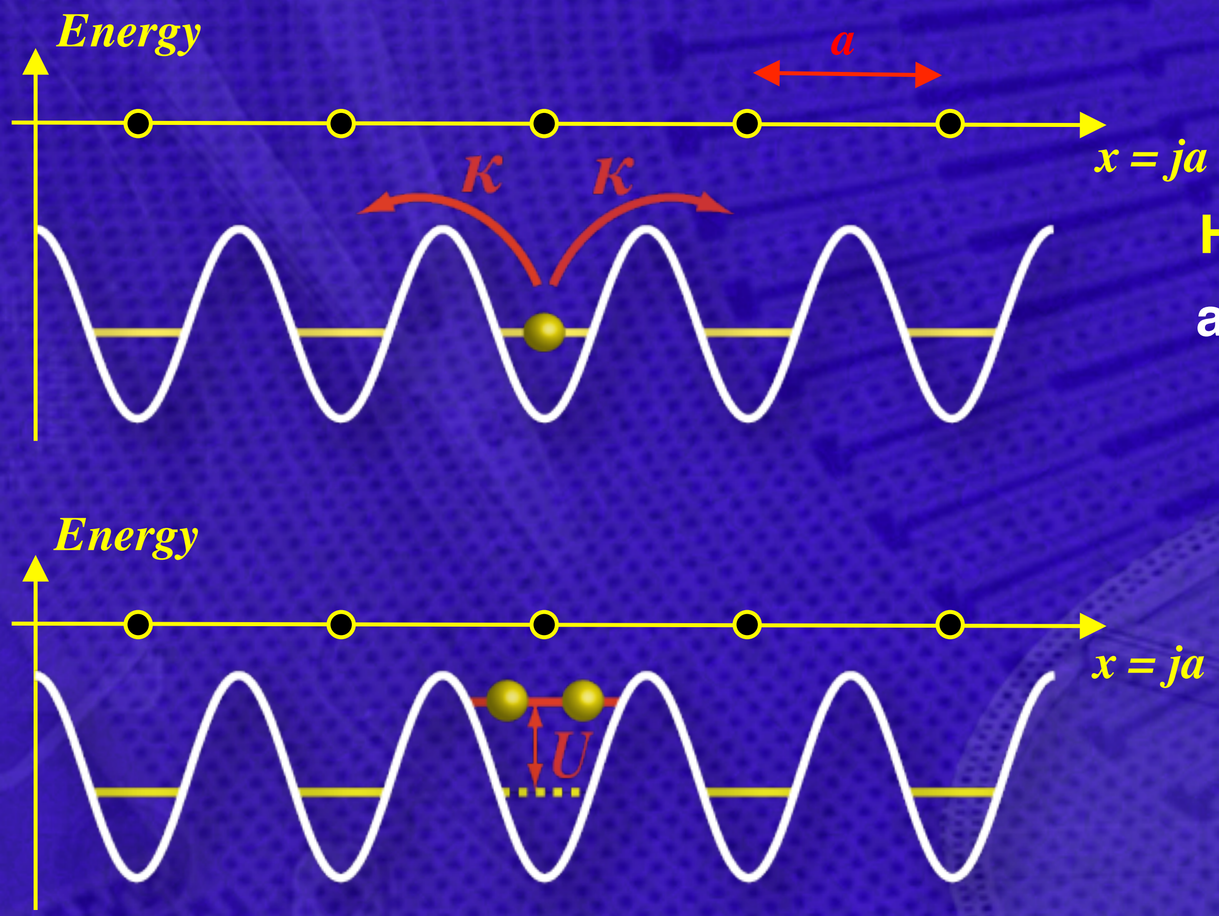
$\hat{a}_{j,\sigma,b}$ / $\hat{a}^{\dagger}_{j,\sigma,b}$ / $\hat{n}_{j,\sigma,b}$: annihilation / creation / number operators for 1 particle of spin σ at site j in band b

2. Particle interaction untill a given lattice neighbor and within a given sub-set of bands...(i.e. "*truncation*")

Introduction (IV)

The simplest (most popular) version of the HM:

1D single-band nearest-neighbor uniform tight-binding lattice



Single-particle **Hopping** (Tunneling) at nearest neighbors

On-site particle **Interaction** (e.g. Coulomb repulsion)

Introduction (V)

The ("*usual*") Hubbard Hamiltonian

$$\hat{H}_{HM} = \hat{H}_{HOP} + \hat{H}_{INT} + \hat{H}_{EXT}$$

$$\hat{H}_{HOP} = \sum_j \sum_\sigma -\kappa \left(\hat{a}^\dagger_{j,\sigma} \hat{a}_{j+1,\sigma} + \hat{a}^\dagger_{j+1,\sigma} \hat{a}_{j,\sigma} \right)$$ **Hopping**

$$\hat{H}_{INT} = U \sum_j \prod_\sigma \hat{n}_{j,\sigma}$$ **Internal Interaction**

$$\hat{H}_{EXT} = \sum_{j,\sigma} \varepsilon_j \hat{n}_{j,\sigma}$$ **External Interaction**

$$\kappa = \int_{-\infty}^{+\infty} dx\, w(x) \left[-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V_{LATTICE}(x) \right] w(x-a)$$

$$U = g \int_{-\infty}^{+\infty} dx \left| w(x) \right|^4$$

$$\varepsilon_j = \int_{-\infty}^{+\infty} dx\, V_{EXT}(x) \left| w(x - ja) \right|^2 \simeq V_{EXT}(ja)$$

Integrals involving single-particle Wannier States

Introduction (VI)

In a lattice...

$$\hat{H}_{NLSE} = \hat{H}_{LAT} + \hat{H}_{NL} + \hat{H}_{EXT}$$

Semiclassical approximation for a bosonic condensate of many interacting particles

$$\hat{H}_{LAT} = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_{LAT}(x)$$

$$\hat{H}_{NL} = g\left|\Phi(x,t)\right|^2$$

Nonlinearity due to interaction

$$\hat{H}_{EXT} = V_{EXT}(x)$$

$$\hat{H}_{HM} = \hat{H}_{HOP} + \hat{H}_{INT} + \hat{H}_{EXT}$$

Fully quantum description of few interacting particles (bosons, fermions, anyons)

$$\hat{H}_{HOP} = \sum_j \sum_\sigma -\kappa\left(\hat{a}^\dagger_{j,\sigma}\hat{a}_{j+1,\sigma} + \hat{a}^\dagger_{j+1,\sigma}\hat{a}_{j,\sigma}\right)$$

$$\hat{H}_{INT} = U\sum_j \prod_\sigma \hat{n}_{j,\sigma}$$

"*Nonlinearity*" due to interaction

$$\hat{H}_{EXT} = \sum_{j,\sigma} \varepsilon_j \hat{n}_{j,\sigma}$$

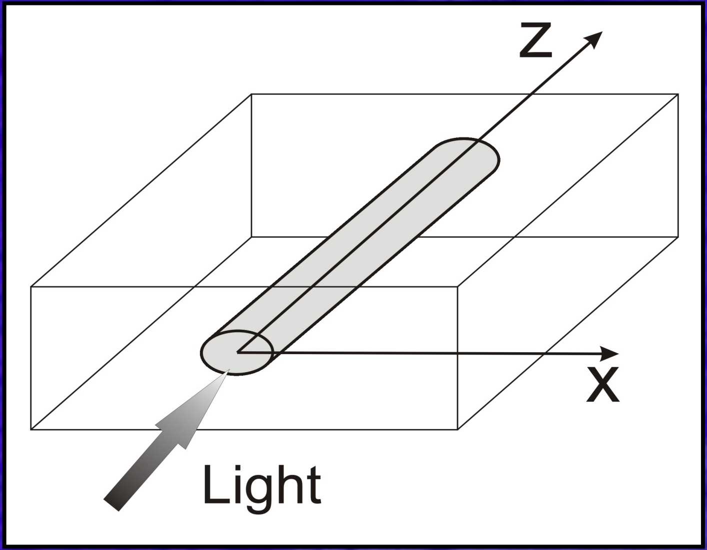
- possibly over-simplified, but very reach and interesting
- typically very difficult to solve! (very few 1D problems allow analytical solution, and in higher dimensions even numerical approach is challenging)

Photonic simulators of Hubbard Models

Photonic simulators of HMs (I)

The Quantum-Optical analogy (well known analogy)

2-D Electric field distribution in a guiding structure propagating in z direction



$$E(x,z,t) = \psi(x,z)e^{ik_0 n_s z}e^{-i\omega t}$$

…obeys Paraxial Wave Equation:

$$i\bar{\lambda}\frac{\partial\psi}{\partial z} = -\frac{\bar{\lambda}^2}{2n_s}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi$$

being… $\begin{cases} \bar{\lambda} = \lambda / 2\pi \\ V(x) = n_s - n(x) \end{cases}$

By changing… $\begin{cases} \bar{\lambda} \to \hbar \\ n_s \to m \\ z \to t \end{cases}$

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi$$

Schrödinger equation

See e.g. D. N. Christodoulides, F. Lederer, and Y. Silberberg, ***Nature* 424**, 817 (2003) and S. Longhi, **Laser Photon. Rev.** *3*, 243 (2009).

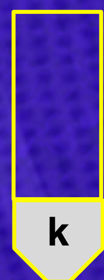
Photonic simulators of HMs (II)

The Fock space: the space of states in QFT

Start from the single particle space of states H_1.

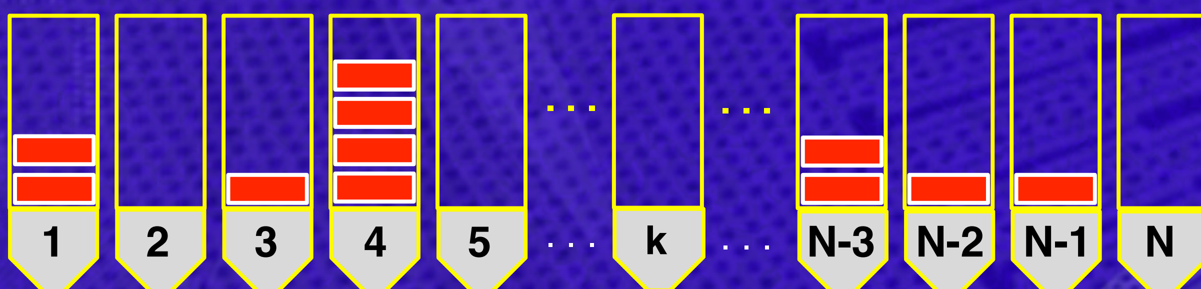
Suppose that the size of H_1 is N (i.e. N basis vector states).

Two ingredients:



The k-th basis state of H_1

Single quantum of excitation



e.g.

A basis element of the Fock Space with $Q = 11$ quanta

$$| \, 2 \, , 0 \, , 1 \, , 4 \, , 0 \, , \ldots \, 0 \, , \ldots \, 2 \, , 1 \, , 1 \, , 0 \, \rangle$$

$$| \, n_1 \, , n_2 \, , n_3 \, , n_4 \, , n_5 \, , \ldots \, n_k \, , \ldots \, n_{N-3} \, , n_{N-2} \, , n_{N-1} \, , n_N \, \rangle = \prod_k \frac{1}{\sqrt{n_k\,!}} \left(\hat{a}_k^\dagger \right)^{n_k} |0\rangle$$

With $\hat{a}_k^\dagger |n_k\rangle = \sqrt{n_k + 1}\; |n_k + 1\rangle$ the creation operator, and $|0\rangle$ the vacuum state.

Photonic simulators of HMs (III)

Within Fock Space representation, particle statistics determines the possible values of n_k for each k :

Bosons:	**Fermions:**
$n_k = 0,1,\ldots Q$	$n_k = 0,1$
$\hat{a}_k\hat{a}_l - \hat{a}_l\hat{a}_k = 0$	$\hat{a}_k\hat{a}_l + \hat{a}_l\hat{a}_k = 0$
$\hat{a}_k^\dagger\hat{a}_l - \hat{a}_l\hat{a}_k^\dagger = \delta_{k,l}$	$\hat{a}_k^\dagger\hat{a}_l + \hat{a}_l\hat{a}_k^\dagger = \delta_{k,l}$
(Bose-Hubbard Model)	**(Fermi-Hubbard Model)**

$\left\{\left|\Phi_q\right\rangle\right\}$ Fock space basis set (indexed by $\boldsymbol{q}$, i.e. a sequence of single-particle quantum numbers)

$\left|\Psi(t)\right\rangle = \sum_q c_q(t)\left|\Phi_q\right\rangle$ General state of the many-body quantum system

Our photonic simulator put each Fock basis in correspondence with an individual optical waveguide of the photonic structure:

→ $\left|c_q(t)\right|^2 \leftrightarrow P_q(t)$ (Optical power along waveguide $\boldsymbol{q}$)

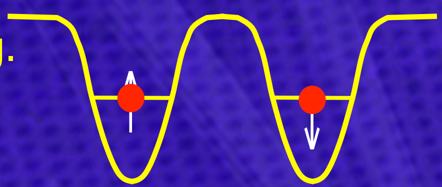
Photonic simulators of HMs (IV)

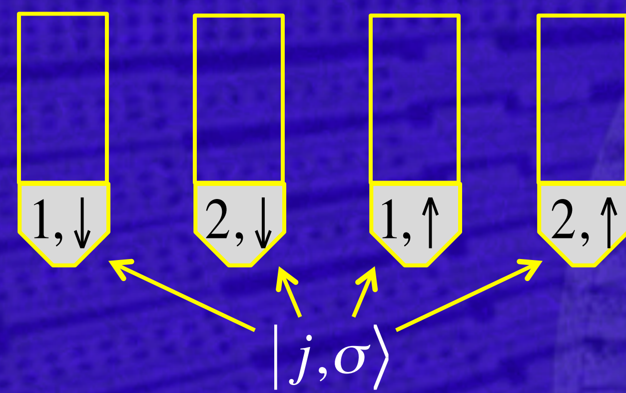
<u>The simplest Hubbard model</u>: electrons on 2-site lattice (double well)

Site index $j = 1,2$

1-particle Spin $\sigma = \downarrow, \uparrow$

E.g.

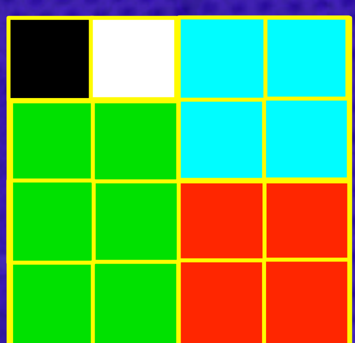




Single particle basis states (4 elements)

Fock Space : $|n_{1\downarrow}, n_{2\downarrow}, n_{1\uparrow}, n_{2\uparrow}\rangle$ with $n_{j\sigma} = 0,1$ → 16 element basis

Fock Space Sectors



- 0 electrons
- 1 electron
- **2 electrons**
- 3 electrons
- 4 electrons

$|1,1,0,0\rangle$, $|0,0,1,1\rangle$ — 2-electrons triplet basis states

$|1,0,1,0\rangle$, $|1,0,0,1\rangle$, $|0,1,1,0\rangle$, $|0,1,0,1\rangle$ — 2-electrons singlet basis states

→ <u>**mimicked by 2x2 square photonic lattice**</u>

Photonic simulators of HMs (V)

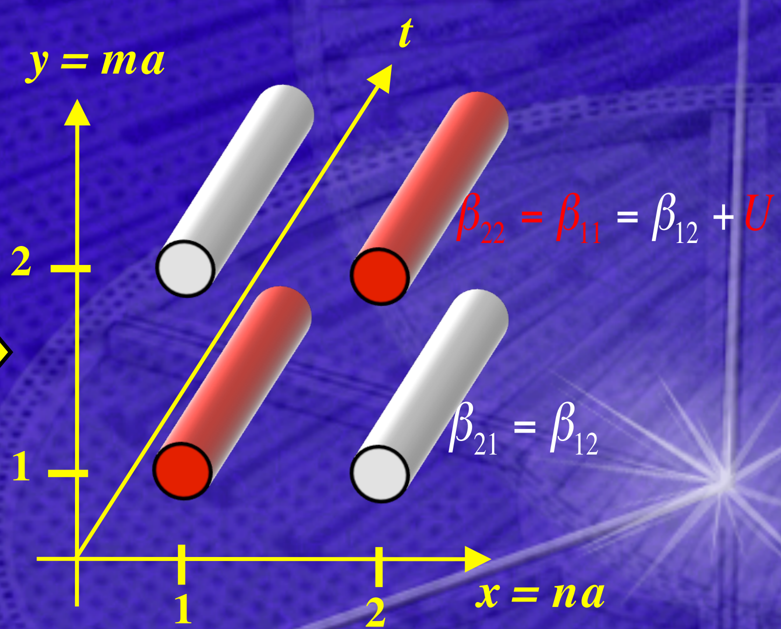
Photonic analog of the 2 interacting electrons in a double well

$$\hat{H}_{HM} = -\kappa\left[\hat{a}^{\dagger}_{1,\downarrow}\hat{a}_{2,\downarrow} + \hat{a}^{\dagger}_{2,\downarrow}\hat{a}_{1,\downarrow} + \hat{a}^{\dagger}_{1,\uparrow}\hat{a}_{2,\uparrow} + \hat{a}^{\dagger}_{2,\uparrow}\hat{a}_{1,\uparrow}\right] + U\left(\hat{n}_{1,\uparrow}\hat{n}_{1,\downarrow} + \hat{n}_{2,\uparrow}\hat{n}_{2,\downarrow}\right)$$

$$i\hbar\frac{d\left|\Psi(t)\right\rangle}{dt} = \hat{H}_{HM}\left|\Psi(t)\right\rangle \Leftarrow \left|\Psi(t)\right\rangle = \sum_{n,m=1}^{2} c_{n,m}(t)\,\left|\Phi_{n,m}\right\rangle, \text{ with: } \begin{cases} \left|\Phi_{1,1}\right\rangle = \left|1,0,1,0\right\rangle \\ \left|\Phi_{1,2}\right\rangle = \left|1,0,0,1\right\rangle \\ \left|\Phi_{2,1}\right\rangle = \left|0,1,1,0\right\rangle \\ \left|\Phi_{2,2}\right\rangle = \left|0,1,0,1\right\rangle \end{cases}$$

$\left\langle\Phi_{n,m}\right|$ (projection)

$$i\frac{d}{dt}\left\|\begin{matrix} c_{11} \\ c_{12} \\ c_{21} \\ c_{22} \end{matrix}\right\| = \left\|\begin{matrix} U & -\kappa & -\kappa & 0 \\ -\kappa & 0 & 0 & -\kappa \\ -\kappa & 0 & 0 & -\kappa \\ 0 & -\kappa & -\kappa & U \end{matrix}\right\| \left\|\begin{matrix} c_{11} \\ c_{12} \\ c_{21} \\ c_{22} \end{matrix}\right\|$$



Photonic simulators of HMs (VI)

Visualization of correlated tunneling (I)

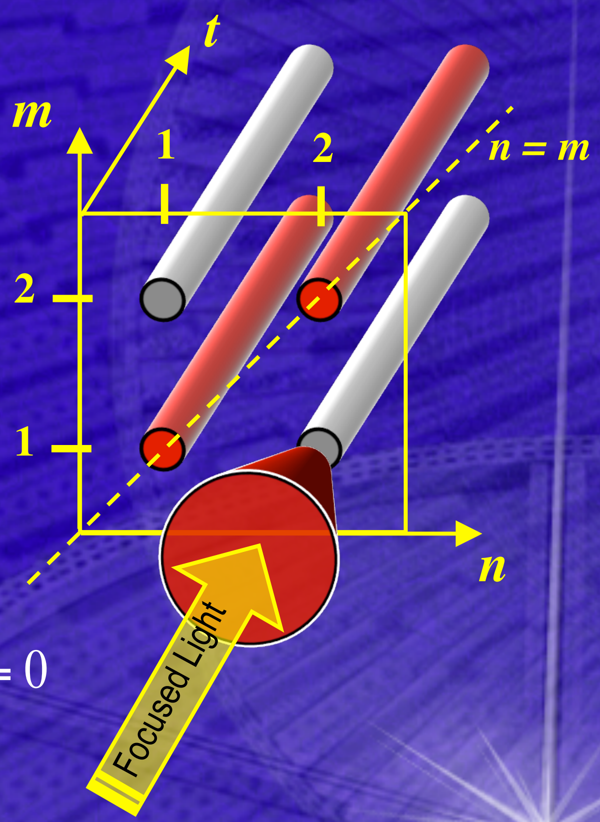
The effect of particle interaction can be visualized in Fock Space by looking at the tunneling dynamics in the photonic symulator.

The *Return (survival) Probability*:

$$P(t) = \left|\langle\psi(t)|\psi(0)\rangle\right|^2 = \sum_{n,m=1}^{2}\left|c^*_{n,m}(0)c_{n,m}(t)\right|^2$$

The *Spin Imbalance* between the sites:

$$N_{12}(t) = \frac{1}{2}\langle\psi(t)|(\hat{n}_{1,\uparrow} - \hat{n}_{1,\downarrow}) - (\hat{n}_{2,\uparrow} - \hat{n}_{2,\downarrow})|\psi(t)\rangle$$



Initial condition (input light excitation):

$$|\psi(0)\rangle = |0,1,1,0\rangle \quad \text{i.e.} \quad c_{21}(0) = 1, \ c_{11}(0) = c_{12}(0) = c_{22}(0) = 0$$

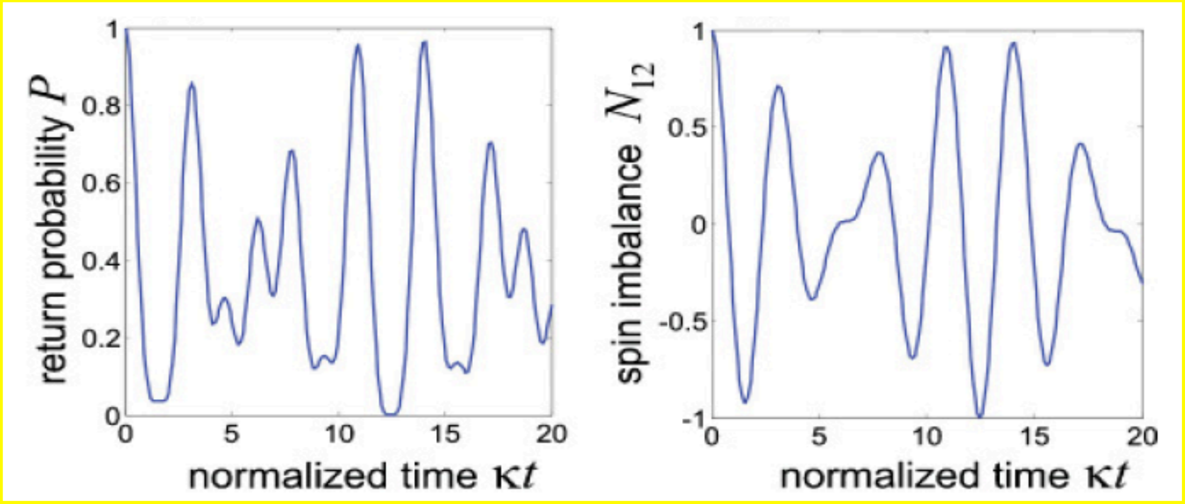
$P(t) = \left|c_{21}(t)\right|^2$ Fractional Power in the input waveguide

$N_{12}(t) = \left|c_{21}(t)\right|^2 - \left|c_{12}(t)\right|^2$ Fractional Power imbalance between off-diagonal waveguides

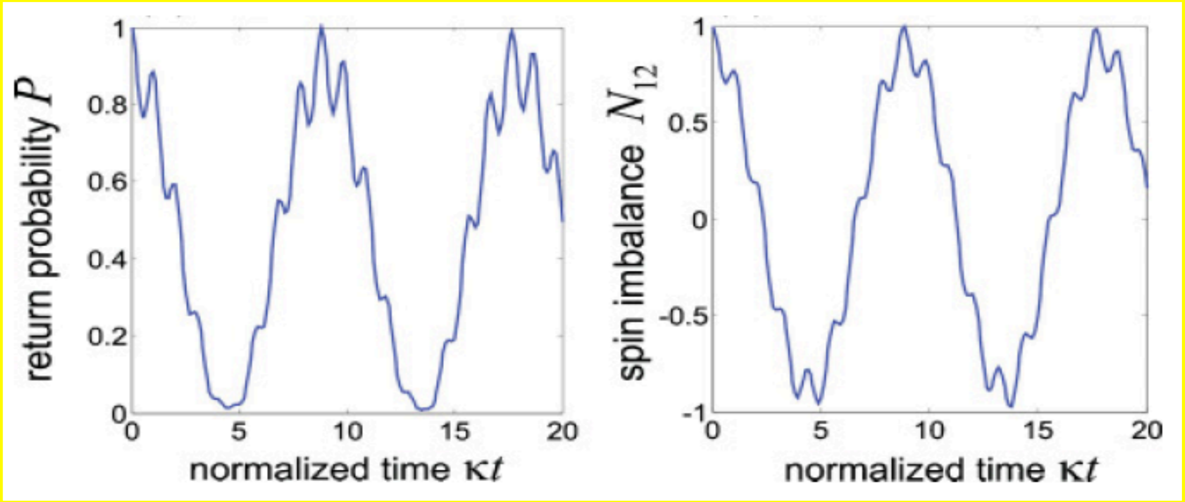
Photonic simulators of HMs (VII)

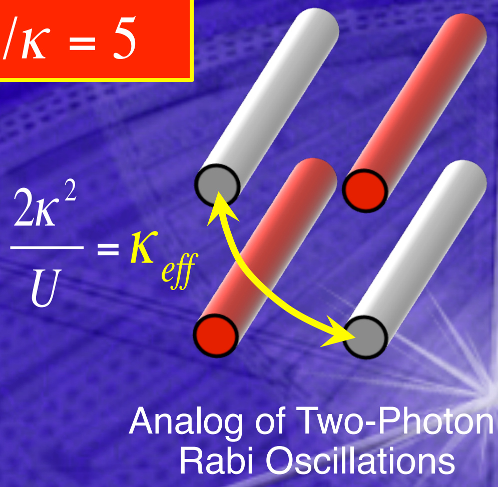
Visualization of correlated tunneling dynamics (II)

$U/\kappa = 0.5$



$U/\kappa = 5$





Phys. Rev. B. 84, 033102 (2011)

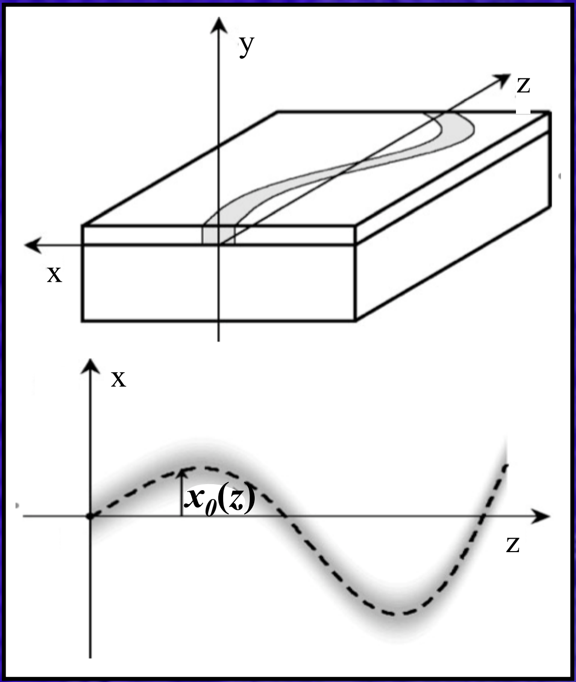
Photonic simulators of HMs (VIII)

Mimicking external fields in the photonic simulator

Paraxial and scalar optical wave equation

$$i\lambda \frac{\partial \psi}{\partial z} = -\frac{\lambda^2}{2n_s}\nabla^2_{x,y}\psi + V(x - x_0(z), y)\psi.$$

$x = x_0(z)$ equation of the optical axis



Kramers-Henneberger transformation (+ EIM)

$$x' = x - x_0(z), \quad z' = z,$$

$$\phi(x', z') = \psi(x', z') \exp\left[-i\frac{n_s}{\lambda}\dot{x}_0 x' - i\frac{n_s}{2\lambda}\int_0^{z'} d\xi\, \dot{x}_0^2(\xi)\right]$$

Schrödinger equation for a particle of mass n_s in an external driving force field

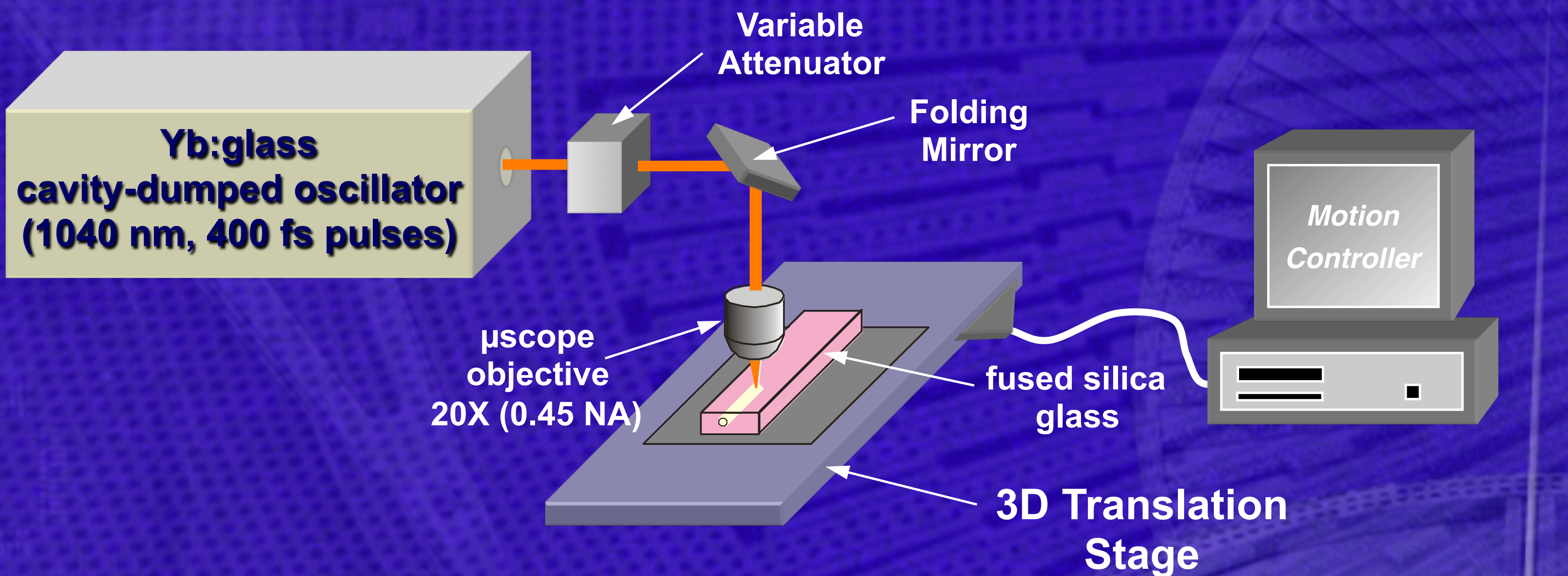
$$i\lambda \frac{\partial \phi}{\partial z'} = -\frac{\lambda^2}{2n_s}\frac{\partial^2 \phi}{\partial x'^2} + V_e(x')\phi - Fx'\phi \equiv \mathcal{H}_0\phi - Fx'\phi,$$

$$F(z') = -n_s \ddot{x}_0(z')$$

Phys. Rev. E **67**, 036601 (2003)

Photonic simulators of HMs (IX)

Fabrication of photonic simulators: the fs-laser writing



- Fuoresce at 650 nm under 633 nm illumination (living for few days)
- Writing parameters:
  - Pulse Energy: 300 nJ
  - Repetition Rate: 20 kHz
  - **Tuning writing speed: ~ 1-50 mm/s** ⇦ **Control of Refractive Index change**
  - Focusing range: 500 μm

J. Opt. A: Pure Appl. Opt. **11**, 013001 (2009)

provided by experimental group of Dr. Roberto Osellame (IFN-CNR)

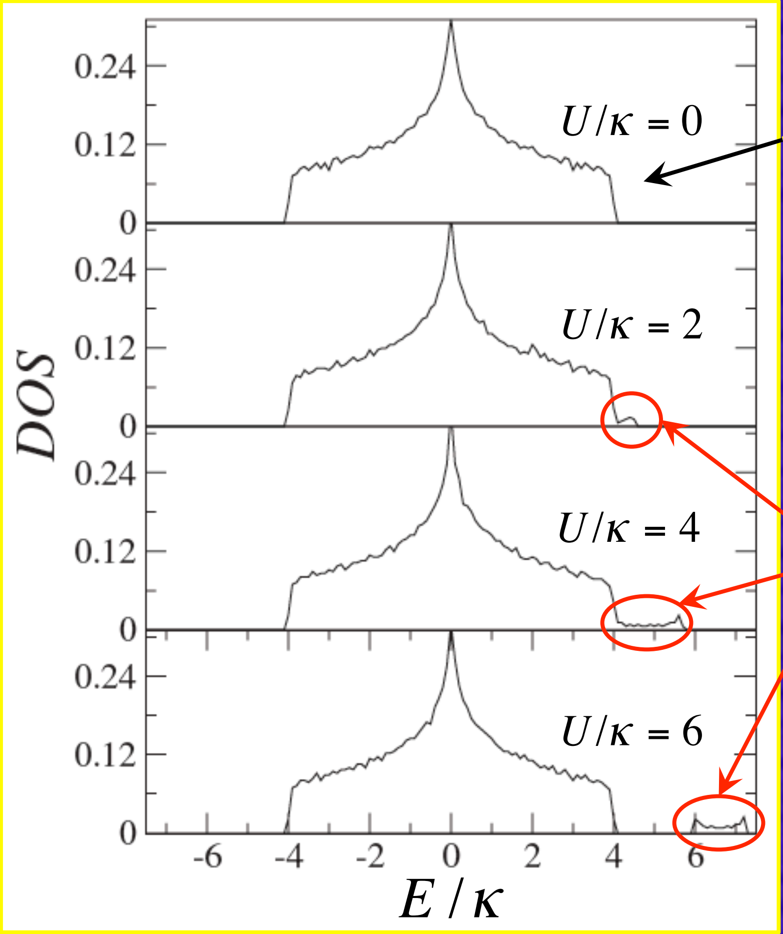
Fractional Bloch Oscillations

Fractional Bloch Oscillations (I)

The Hubbard model for a 1D lattice with 2 electrons

$$\hat{H}_{HM} = -\kappa \sum_{n=1}^{L-1} \left(\hat{a}^\dagger_{n,\downarrow} \hat{a}_{n+1,\downarrow} + \hat{a}^\dagger_{n+1,\downarrow} \hat{a}_{n,\downarrow} + \hat{a}^\dagger_{n,\uparrow} \hat{a}_{n+1,\uparrow} + \hat{a}^\dagger_{n+1,\uparrow} \hat{a}_{n,\uparrow} \right) + U \sum_{n=1}^{L} \hat{n}_{n,\uparrow} \hat{n}_{n,\downarrow}$$

(no external driving field)



...same as for 1 electron on a 2D tight-binding lattice (band of "single-particle" Bloch states)

The particles behave as two individual un-correlated particles

...a NEW BAND emerges from the "single-particle" Bloch band

$$d(E_j) = \sum_{n,m=1}^{L} |n-m| \left| c^{(j)}_{n.m} \right| << 1$$

(band of "bound-particles", molecular, states)

The particles behave as a composite particle: the DOUBLON

Numerical simulations for $L = 100$ sites
[W. S. Dias *et al.*, Phys. Rev. B **76**, 155124 (2007)]

Fractional Bloch Oscillations (II)

The photonic simulator of correlated Bloch Oscillations (I)

$$\hat{H}_{HM} = -\kappa \sum_{n=1}^{L-1} \left(\hat{a}^{\dagger}_{n,\downarrow} \hat{a}_{n+1,\downarrow} + \hat{a}^{\dagger}_{n+1,\downarrow} \hat{a}_{n,\downarrow} + \hat{a}^{\dagger}_{n,\uparrow} \hat{a}_{n+1,\uparrow} + \hat{a}^{\dagger}_{n+1,\uparrow} \hat{a}_{n,\uparrow} \right) + U \sum_{n=1}^{L} \hat{n}_{n,\uparrow} \hat{n}_{n,\downarrow}$$

$$+ \sum_{n=1}^{L} F \hat{x} \left(\hat{n}_{n,\uparrow} + \hat{n}_{n,\downarrow} \right)$$

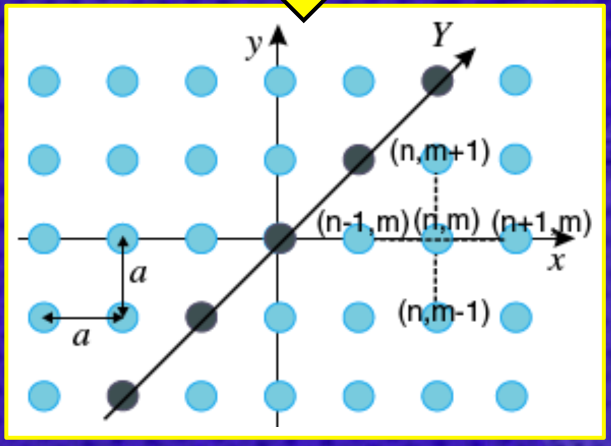
Uniform static driving force (e.g. $F = eE_{el}$)

$$i \frac{dc_{n,m}}{dt} = -\kappa \left(c_{n+1,m} + c_{n-1,m} + c_{n,m+1} + c_{n,m-1} \right) + U \delta_{n,m} c_{n,m} + F \frac{a}{\lambda} (n+m) c_{n,m}$$

$$F_Y = -n_s \frac{d^2 Y_0(t)}{dt^2}$$

2D square array of waveguides with a defect line (detuning along the main diagonal)

- $n_S + \Delta n$
- $n_S + \Delta n_1$
- n_S



Circular bending of the array in the plane of the main ($n = m$) diagonal

$$R = \frac{n_S}{\sqrt{2F\lambda}}$$

Fractional Bloch Oscillations (III)

Engineering of the photonic simulator

Structure parameters:

$$\left.\begin{aligned} \lambda &= 980 \text{ nm} \\ n_S &= 1.45 \\ \Delta n &= 1 \times 10^{-2} \\ \Delta n_1 &= 9.65 \times 10^{-3} \\ a &= 8.6\ \mu\text{m} \end{aligned}\right] \rightarrow \begin{aligned} &\kappa \simeq 4 \text{ cm}^{-1} \\ &U = (\Delta n - \Delta n_1)/ƛ \simeq 4\kappa \end{aligned}$$

$$T_B = \frac{2\pi}{Fa} = 2.25 \text{ cm}$$

$$F \simeq 3.2 \times 10^3 \text{ cm}^{-2}$$

$$R = \frac{n_S}{\sqrt{2}Fƛ} \simeq 21 \text{ cm}$$

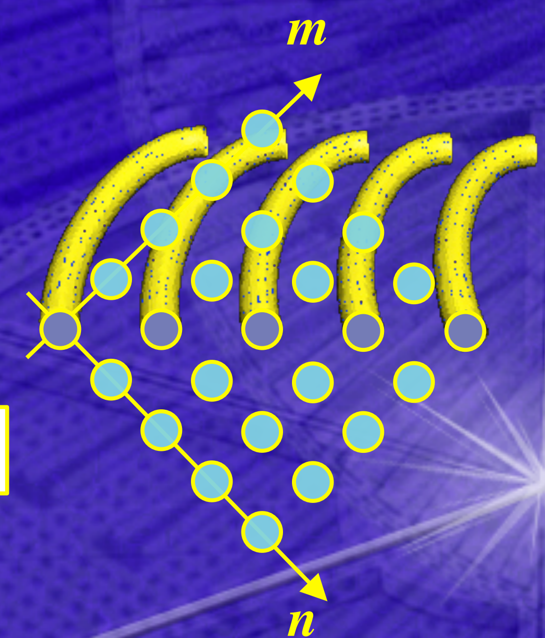
Excitation conditions (system preparation) :

For coupled-mode equations simulations

$$c_{n,m}(t=0) = Z\exp[-(n-n_0)^2/w^2 - (m-m_0)^2/w^2]$$

For paraxial wave equation photonic simulations

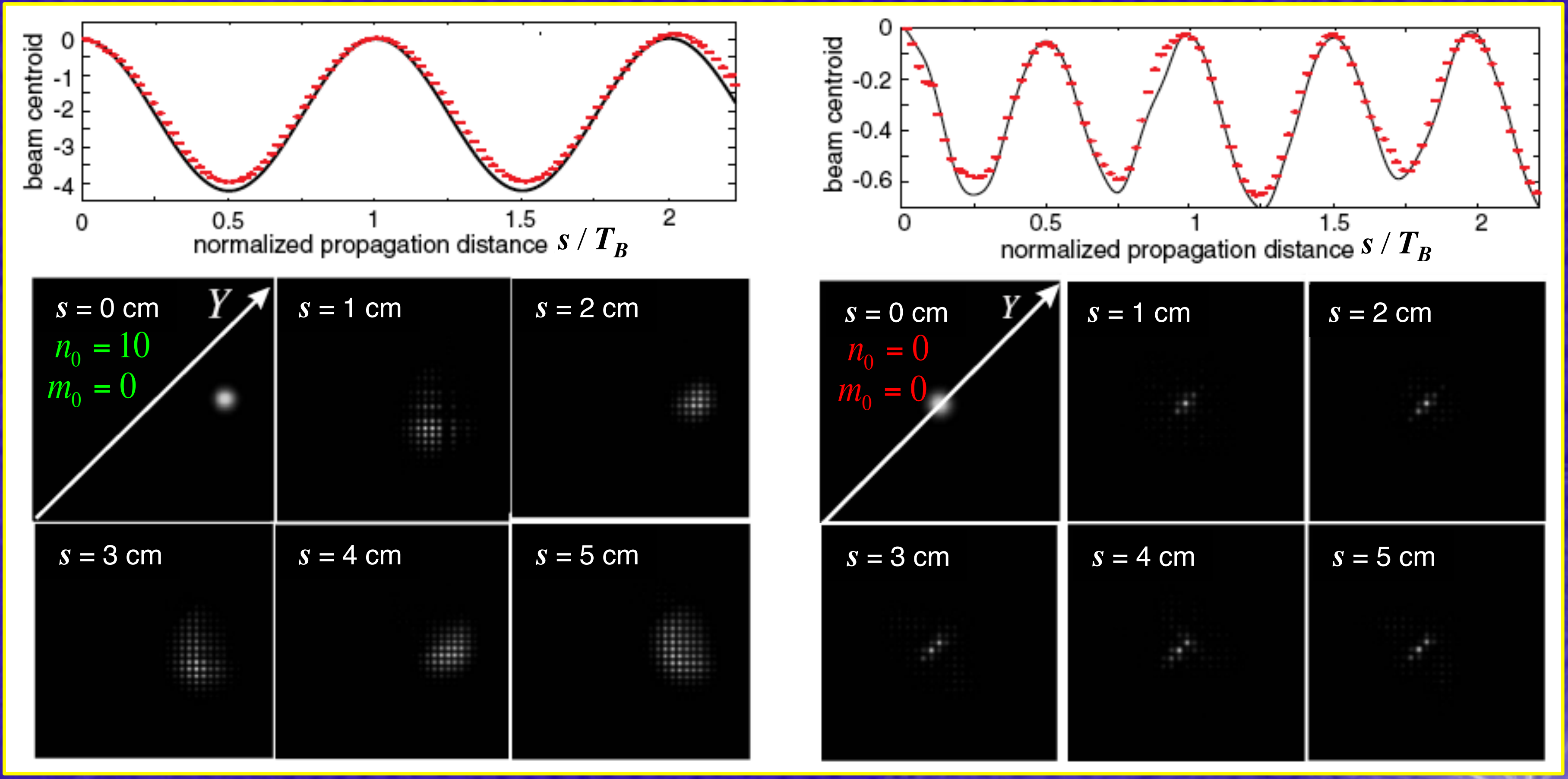
$$\phi(x,y,0) = \exp[-(x-n_0a)^2/(wa)^2 - (y-m_0a)^2/(wa)^2]$$



Opt. Lett 36, 3248 (2011)
Phys. Rev. B 86, 075143 (2012)

Fractional Bloch Oscillations (IV)

Visualization of correlated Bloch oscillations



Bloch oscillations of a single particle in a 2D square lattice

$$\omega_B = Fa = \boxed{e} E_{el} a$$

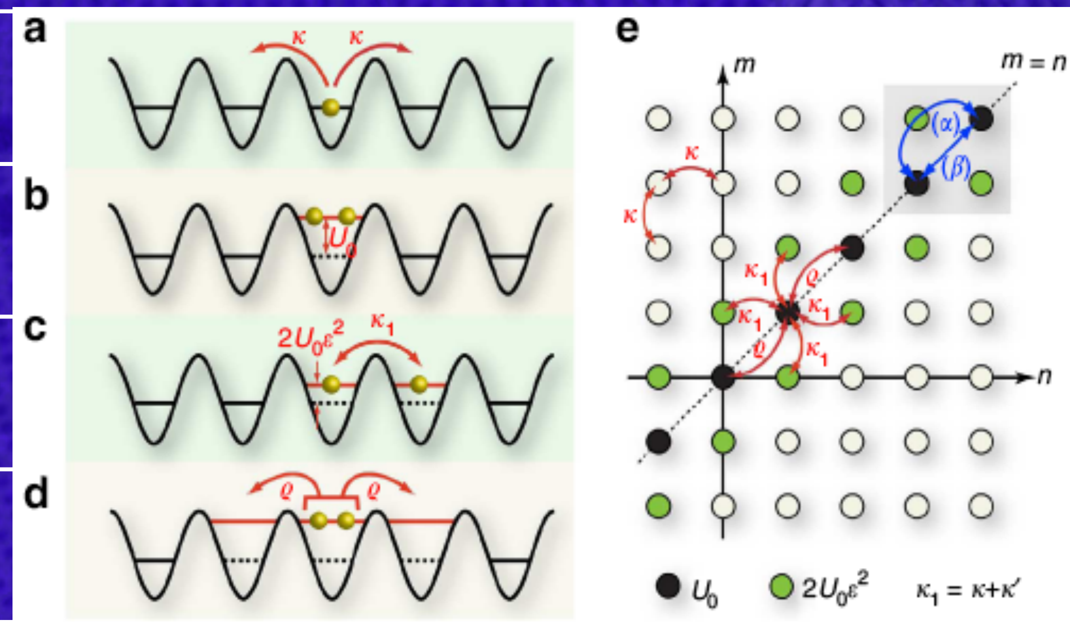
Bloch oscillations of the particle pairs → Doubling of the Bloch frequency!

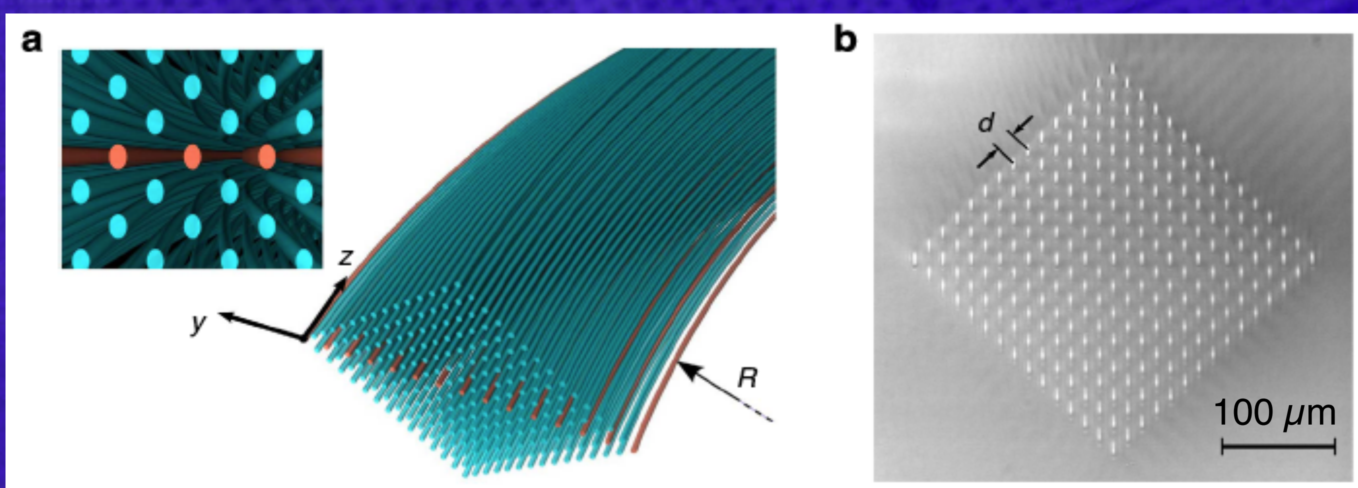
$$\omega_B = Fa = \boxed{2e} E_{el} a$$

Fractional Bloch Oscillations (V)

Experiments in photonic lattices: Design

	Single-particle tunneling
	On-site particle interaction
higher-order processes	NN particle interaction and Conditional single-particle tunneling
higher-order processes	Direct tunneling of doublons



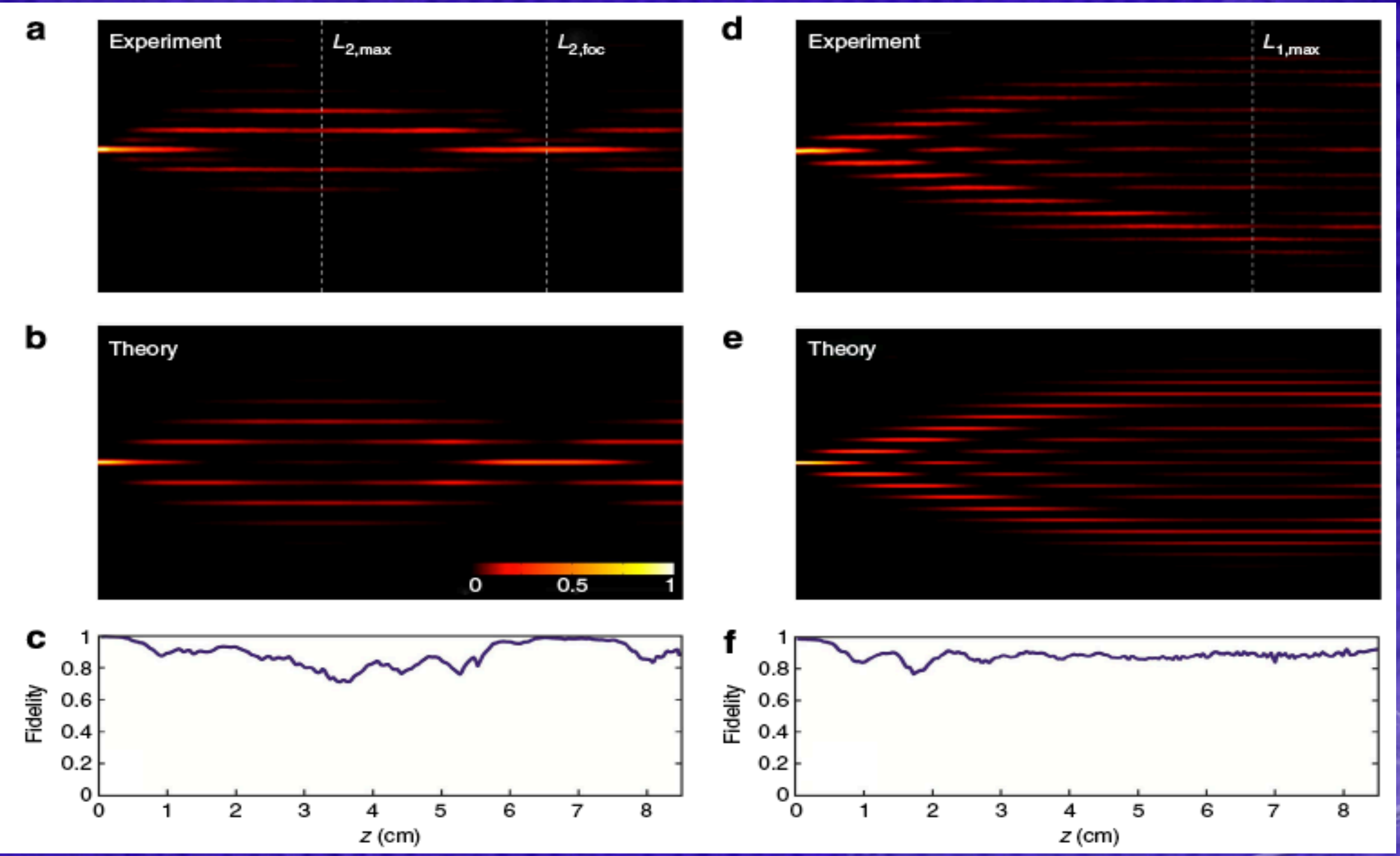


fs-laser written
15 x 15 2D
curved
waveguide array
(Osellame's group
@IFN-CNR Milano)

Fractional Bloch Oscillations (VI)

Experiments in photonic lattices: Results

Two interacting particles — A single particle



Nature Comm. 4, 1555 (2013)

First experimental observation of fractional BOs

Dynamic Localization of Doublons and Coherent Destruction of Correlation

Dynamic Localization of Doublons (I)

The AC-driven HM for a 1D lattice with 2 electrons

$$\hat{H}_{HM} = -\kappa \sum_{n=1}^{L-1}\left(\hat{a}^{\dagger}_{n,\downarrow}\hat{a}_{n+1,\downarrow} + \hat{a}^{\dagger}_{n+1,\downarrow}\hat{a}_{n,\downarrow} + \hat{a}^{\dagger}_{n,\uparrow}\hat{a}_{n+1,\uparrow} + \hat{a}^{\dagger}_{n+1,\uparrow}\hat{a}_{n,\uparrow}\right) + U\sum_{n=1}^{L}\hat{n}_{n,\uparrow}\hat{n}_{n,\downarrow}$$

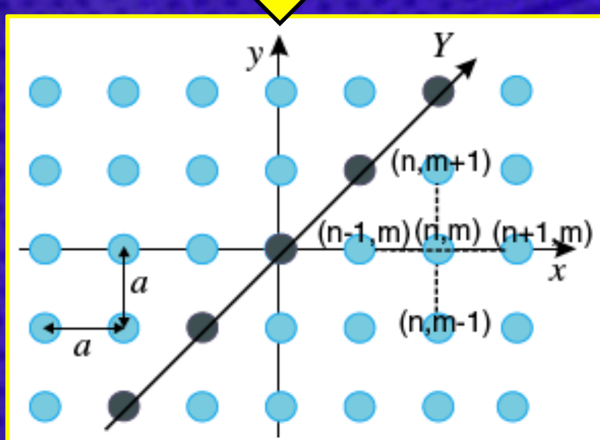
$$+\sum_{n=1}^{L} F(t)\hat{x}\left(\hat{n}_{n,\uparrow} + \hat{n}_{n,\downarrow}\right)$$

Dynamic driving force [e.g. $F(t) = eE_{el}(t)$
$E_{el}(t) = E_0 \cos(\omega t)$]

$$i\frac{dc_{n,m}}{dt} = -\kappa\left(c_{n+1,m} + c_{n-1,m} + c_{n,m+1} + c_{n,m-1}\right) + U\delta_{n,m}c_{n,m} + \frac{a}{\lambda}\left[nF_x(t) + mF_y(t)\right]c_{n,m}$$

2D square array of waveguides with a defect line (detuning along the main diagonal)

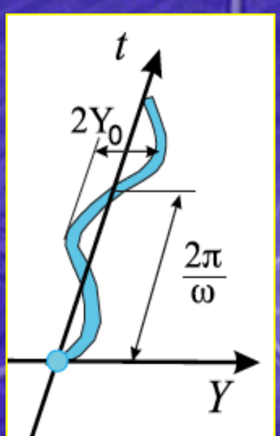
- $n_S + \Delta n$
- $n_S + \Delta n_1$
- n_S



$$F_Y(t) = -n_S\frac{d^2Y_0(t)}{dt^2}$$

$$Y_0(t) = Y_0\cos(\omega t)$$

$$Y_0 = \frac{\sqrt{2}eE_0}{n_S\omega^2}$$



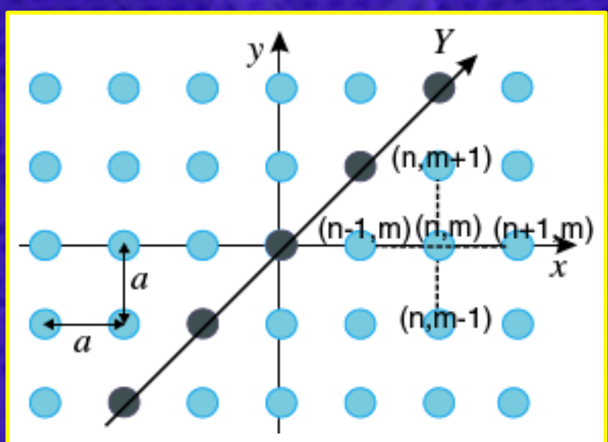
Dynamic Localization of Doublons (II)

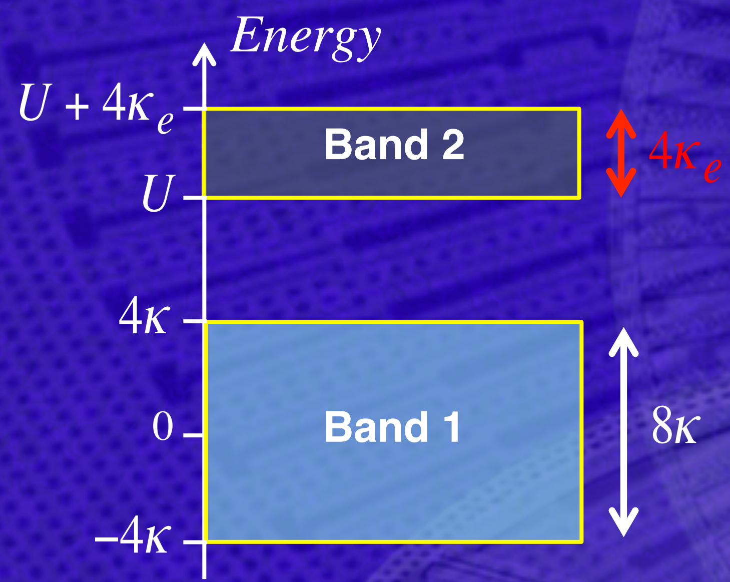
The strong interaction limit of the undriven HM

In the limit of $U \gg \kappa$ the photonic lattice reduces to:

- two semi-infinite 2D TB lattices for the single particle
- a 1D TB lattice for the doublons with tunneling rate $\kappa_e = 2\kappa^2/U$

Undriven lattice (straight optical axis)





$$\left.\begin{aligned} \kappa / U &= \varepsilon \\ \omega / U &\sim \varepsilon \\ eE_0 a / U &\sim \varepsilon \\ c_{n,m}(0) &= A_n(0)\delta_{n,m} \end{aligned}\right\}$$

Multiple Scale

Asymptotic Analysis

(low frequency weak field driving)

$$\left\{\begin{aligned} c_{n,m}(t) &= A_n(t)\delta_{n,m}\exp(i2\kappa_e t) + O(\varepsilon) \\ i\frac{dA_n}{dt} &= -\kappa_e\left[A_{n-1} + A_{n+1}\right] + 2eE_0 a/\lambda\!\!\!^{-}\cos(\omega t) n A_n \end{aligned}\right.$$

Dynamic Localization of Doublons (III)

Engineering of the photonic simulator

$$i\frac{dA_n}{dt} = -\kappa_e\left[A_{n-1} + A_{n+1}\right] + 2e\,E_0 a/\bar{\lambda}\cos(\omega t)\,nA_n$$

CMEs of a "single particle" of charge $2e$ in a 1D TB lattice with renormalized tunneling rate, driven by an AC electric field.

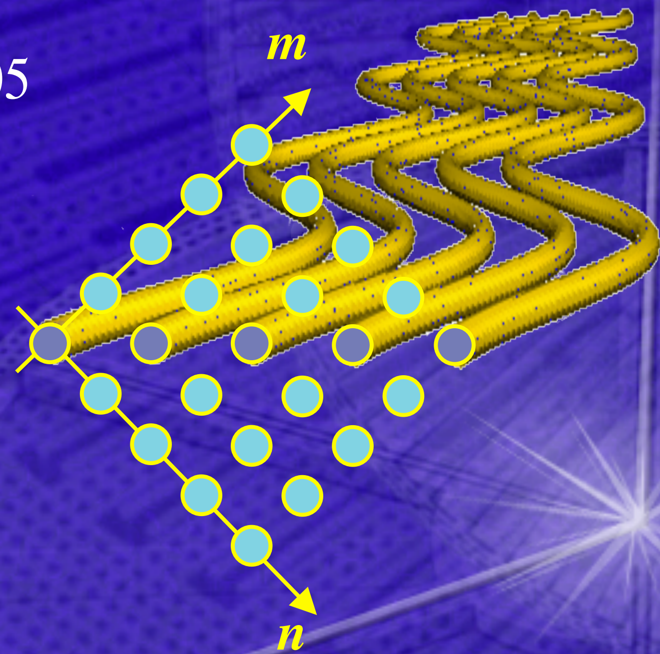
Floquet theory: collapse of quasi-energies (i.e. DL) provided that:

$$J_0\left(\frac{2eE_0 a}{\bar{\lambda}\omega}\right) = 0 \quad \text{i.e.} \quad \frac{2eE_0 a}{\bar{\lambda}\omega} = \sqrt{2}\,n_s a/\bar{\lambda}\,\omega Y_0 \simeq 2.405$$

$$\begin{aligned} \lambda &= 980 \text{ nm} \\ n_s &= 1.522 \\ \Delta n &= 1\times 10^{-2} \\ \Delta n_1 &= 9.05\times 10^{-3} \\ a &= 8.6\ \mu\text{m} \end{aligned}$$

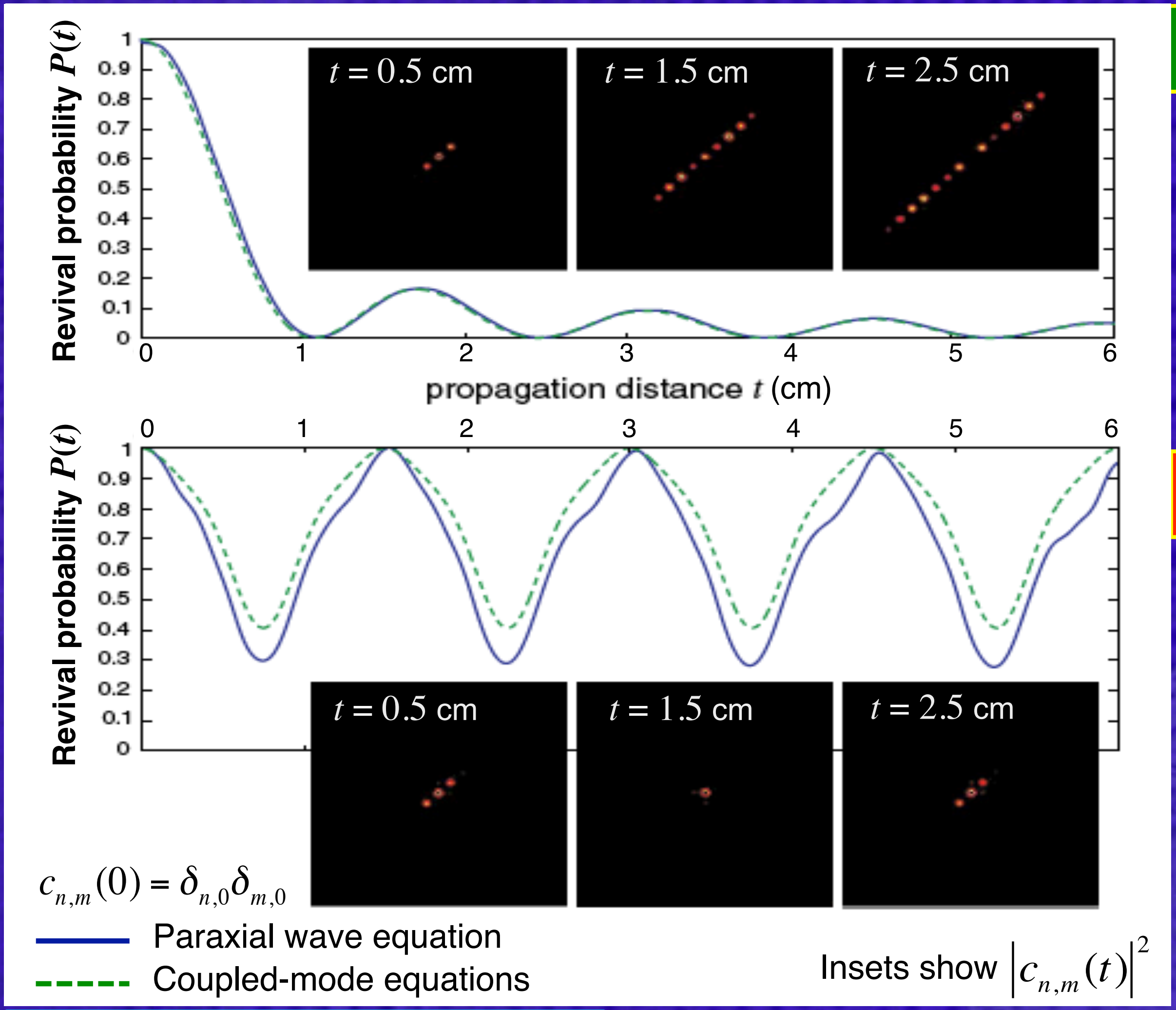
$$\begin{aligned} &\kappa \simeq 3.9 \text{ cm}^{-1} \\ &U = (\Delta n - \Delta n_1)/\bar{\lambda} \simeq 8\kappa \\ &\omega \simeq \kappa \simeq 4.187 \text{ cm}^{-1} \\ &\quad (\text{i.e. } T = 2\pi/\omega = 1.5 \text{ cm}) \end{aligned}$$

$$Y_0 = \frac{2.405\bar{\lambda}}{\sqrt{2}\,a n_s \omega} \simeq 48.4\ \mu m$$



Dynamic Localization of Doublons (IV)

Visualization of correlated dynamic localization



No driving

Discrete diffraction of doublons

$$P(t) \simeq |J_0(\kappa_e t)|^2$$

Driving...

Dynamic Localization of doublons

Opt. Lett. 36, 4743 (2011)

Coherent Destruction of Correlation (I)

Let's consider again the AC-driven HM:

$$i\frac{dc_{n,m}}{dt} = -\kappa\left(c_{n+1,m} + c_{n-1,m} + c_{n,m+1} + c_{n,m-1}\right) + U\delta_{n,m}c_{n,m} + \frac{a}{\lambda}e\sqrt{2}E_0\cos(\omega t)(n+m)c_{n,m}$$

We are interested in a different regime of strong interaction:

→ high-frequency and strong-field AC-driving

$$\left.\begin{aligned} \kappa / U &= \varepsilon \\ \omega / U &\sim 1/\varepsilon^2 \\ eE_0 a / U &\sim 1/\varepsilon^2 \end{aligned}\right]$$

Multiple Scale Asymptotic Analysis

$$\begin{cases} c_{n,m}(t) = A_{n,m}(t)\exp[-i(n+m)(eE_0 a/\omega)\sin(\omega t) - iU\delta_{n,m}t] \\ i\dfrac{dA_{n,m}}{dt} = -\boxed{\kappa J_0\left(eE_0 a/\hbar\omega\right)}\left[A_{n+1,m} + A_{n-1,m} + A_{n,m+1} + A_{n,m-1}\right] \end{cases}$$

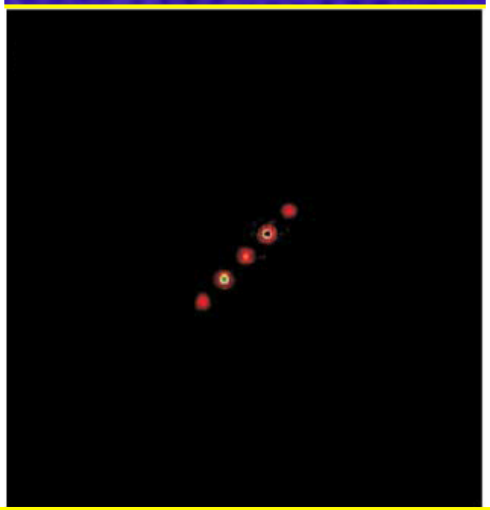
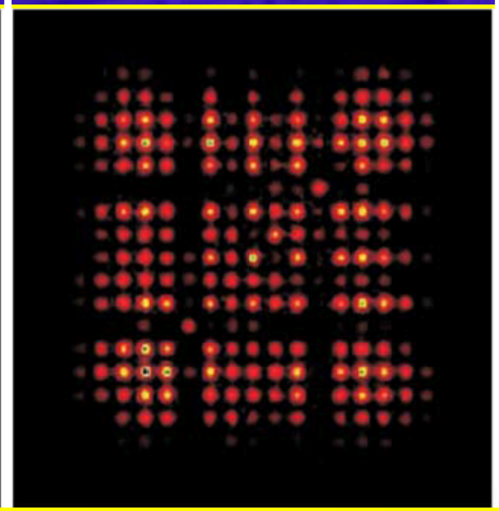
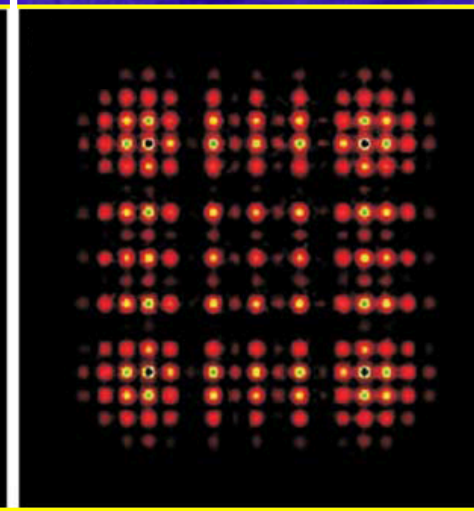
$$\kappa_{eff} = \kappa J_0(\bar{z})$$

⇒ CMEs of Homogeneous Straight Array

The defect diagonal is made invisible.

→ Particle interaction (correlation) is dinamically cancelled!

Coherent Destruction of Correlation (II)

1P tunneling	$\kappa \simeq 3.9\ \mathrm{cm}^{-1}$	$\kappa \simeq 3.9\ \mathrm{cm}^{-1}$	$\kappa J_0(\bar{z})$
2P Interaction	$U \simeq 25\kappa$	$U \simeq 25\kappa$	$U = 0$
AC driving	$\omega = 0$ $Y_0 = 0$	$\omega \simeq 10\,\kappa \simeq 40\ \mathrm{cm}^{-1}$ $Y_0 = \frac{\bar{z}\,\bar{\lambda}}{\sqrt{2}\,a n_s \omega} \simeq 7.6\ \mu m$ $(\bar{z} = 1.841)$	$\omega = 0$ $Y_0 = 0$
Occupation probability $\lvert c_{n,m}(t = 3\ cm)\rvert^2$ **for initial condition** $c_{n,m}(0) = \delta_{n,0}\delta_{m,0}$			
	Discrete diffraction on 1D diagonal sublattice Correlated tunneling	Discrete Diffraction on the whole 2D lattice Single-particle (uncorrelated) tunneling	

Opt. Lett. ***36***, 4743 (2011)

Correlated-tunneling of Anyons and Correlated BOs of Anyons

Correlated-tunneling of Anyons (I)

Anyons: what?

$\psi(B,A) = \psi(A,B)$ → Symmetric under exchange → Bosons

$\psi(B,A) = -\psi(A,B)$ → Antisymmetric under exchange → Fermions

$\psi(B,A) = \exp(i\theta)\psi(A,B)$ → More generally... → Anyons (Abelian)

Low-dimensional quasi-particles with non-trivial exchange statistic:

Generalized Commutation Relations on a 1D lattice (for Abelian Anyons)

$$\hat{a}_l \hat{a}_k^\dagger = \delta_{l,k} + \exp[-i\theta\epsilon(l-k)]\hat{a}_k^\dagger \hat{a}_l$$

$$\hat{a}_l \hat{a}_k = \exp[i\theta\epsilon(l-k)]\hat{a}_k \hat{a}_l$$

θ : **Statistical Exchange Phase**

l,k : lattice site index

$$\epsilon(l-k) = \begin{cases} 1, & \text{for } l > k \\ 0, & \text{for } l = k \\ -1, & \text{for } l < k \end{cases}$$

BOSONS $\theta = 0$

$\theta = \pi$ PSEUDOFERMIONS

The statistic is site-dependent.

Regardless θ, two anyons on the same site behave as bosons.

Correlated-tunneling of Anyons (II)

2-anyons dynamics on a 1D lattice

$$\hat{H} = -J\sum_l (\hat{a}_l^\dagger \hat{a}_{l+1} + \hat{a}_{l+1}^\dagger \hat{a}_l) + \frac{U}{2}\sum_l \hat{n}_l(\hat{n}_l - 1)$$

anyon-Hubbard Hamiltonian

$$|\psi(t)\rangle = (1/\sqrt{2})\sum_{n,m} c_{n,m}(t)\hat{a}_n^\dagger \hat{a}_m^\dagger |0\rangle$$

Fock space representation

$$i\frac{dc_{n,m}}{dt} = -J[c_{n+1,m} + c_{n-1,m} + c_{n,m-1}\exp(-i\varphi_{n,m-1}) + c_{n,m+1}\exp(i\varphi_{n,m})] + U\delta_{n,m}c_{n,n}$$

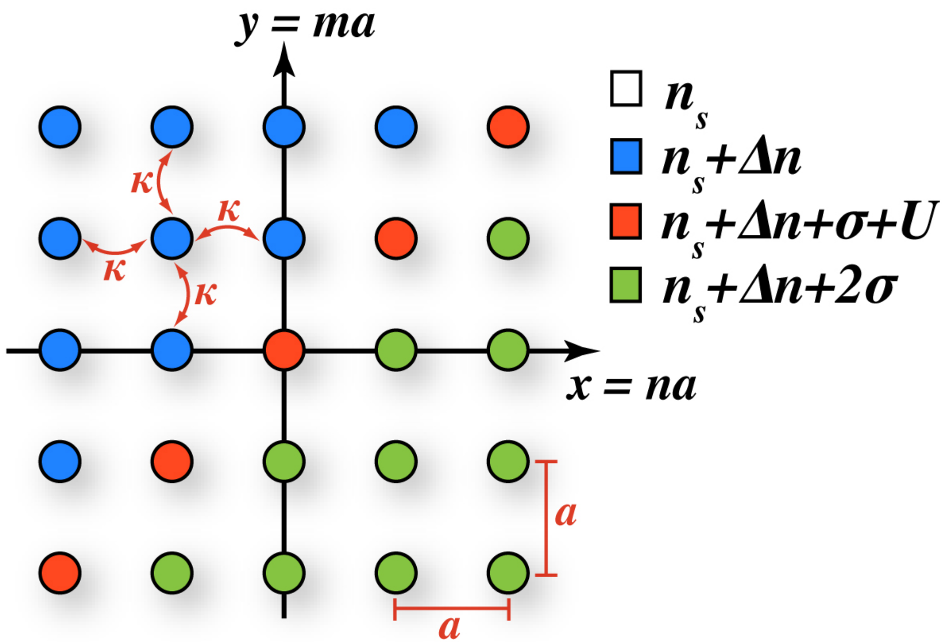
$$\varphi_{n,m} = -\theta \text{ for } n = m, m+1 \text{ and } \varphi_{n,m} = 0$$

(CMEs for Anyons on a 1D Lattice)

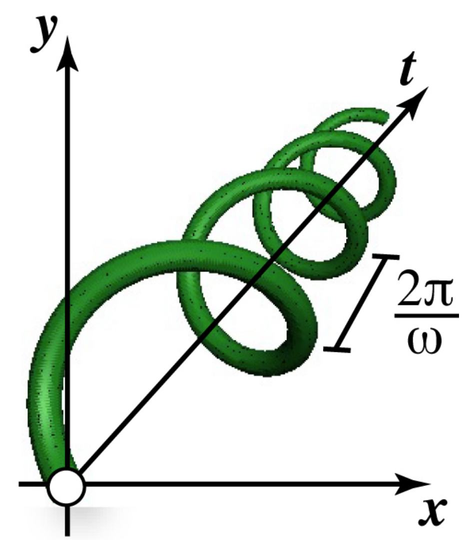
Note the presence of a site-dependent phase factor in the coupling rate due to the statistical exchange phase θ.

Correlated-tunneling of Anyons (III)

Photonic realization of anyonic tunneling on a lattice



$$\beta_{n,m} \simeq \begin{cases} \beta_0, \text{ for } m \geq n+1 \\ \beta_0 + \sigma + U, \text{ for } m = n \\ \beta_0 + 2\sigma, \text{ for } m \leq n+1 \end{cases}$$



Helical optical axis

$$\begin{cases} x_0(t) = A\cos(\omega t) \\ y_0(t) = A\cos(\omega t + \phi) \end{cases}$$

$$i\frac{dc_{n,m}}{dt} = -\kappa\left(c_{n+1,m} + c_{n-1,m} + c_{n,m+1} + c_{n,m-1}\right) + \beta_{n,m}c_{n,m} + \frac{n_s a}{\lambda}\left(n\frac{d^2x_0}{dt^2} + m\frac{d^2y_0}{dt^2}\right)c_{n,m}$$

Correlated-tunneling of Anyons (IV)

Design of the photonic simulator (I)

$$i\frac{dc_{n,m}}{dt} = -\kappa\left(c_{n+1,m} + c_{n-1,m} + c_{n,m+1} + c_{n,m-1}\right) + \beta_{n,m}c_{n,m} + \frac{n_s a}{\lambdabar}\left(n\frac{d^2x_0}{dt^2} + m\frac{d^2y_0}{dt^2}\right)c_{n,m}$$

$\omega, \sigma >> \kappa$ (High Frequency Modulation Limit)

$U = 0$ (No interaction)

$\sigma = M\omega$
$J_0(\Gamma) = J_M(\Gamma)$
$\Gamma = n_s aA\omega / \lambdabar$

} Resonance Conditions

$$i\frac{da_{n,m}}{dt} = -J\left\{a_{n+1,m} + a_{n-1,m} + a_{n,m-1}\exp\left(-i\varphi_{n,m-1}\right) + a_{n,m+1}\exp\left(i\varphi_{n,m}\right)\right\}$$

Precisely the CMEs of the anyon-Hubbard Hamiltonian, with:

$J = \kappa J_0(\Gamma)$ (Renormalized coupling)

$\varphi_{n,m} = -M\phi\left(\delta_{n,m} + \delta_{n,m+1}\right)$ → $\theta = M\phi$

The exchange statistic phase θ can be controlled by the driving parameter ϕ, i.e. by ellipticity of the helix!

Correlated-tunneling of Anyons (V)

Design of the photonic simulator (II)

$\lambda = 633$ nm
$n_S = 1.45$
$\Delta n = 0.001$
$a = 12\ \mu$m

$\kappa = 1.5\ \text{cm}^{-1}$

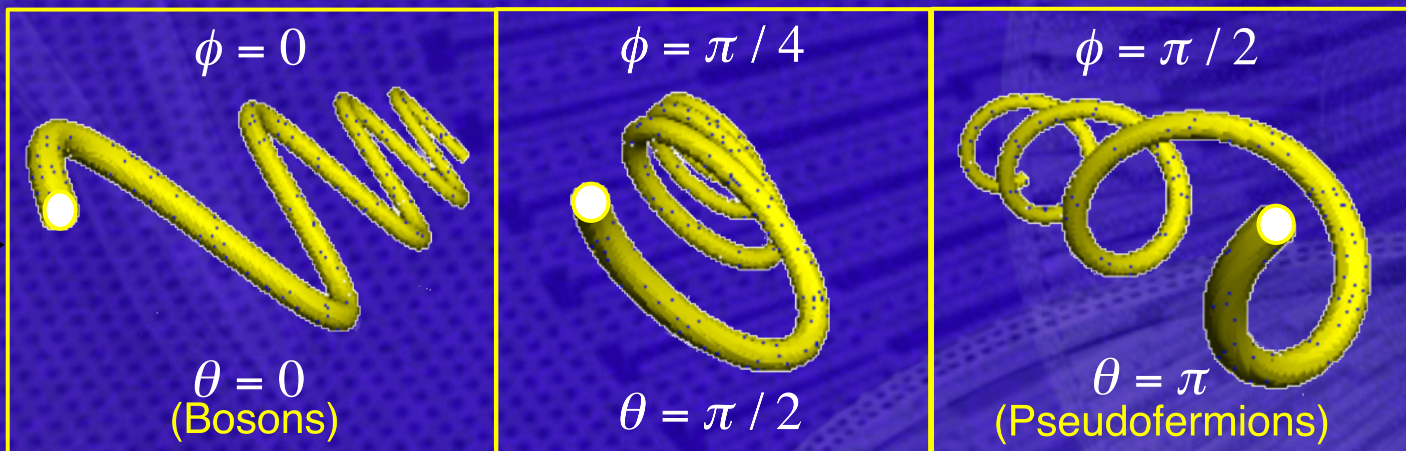
$J = \kappa J_0(\Gamma) \simeq 0.5\ \text{cm}^{-1}$
$(\Gamma \simeq 1.84)$

$\omega = 30J \simeq 15\ \text{cm}^{-1}$ $(T \simeq 0.42\ \text{cm})$

$\sigma = M\omega = 2\omega \simeq 30\ \text{cm}^{-1}$

$\Delta n_1 = \Delta n + \sigma / \lambda\!\!\!{}^{-} \simeq 0.00105$

$\phi = \theta / 2$



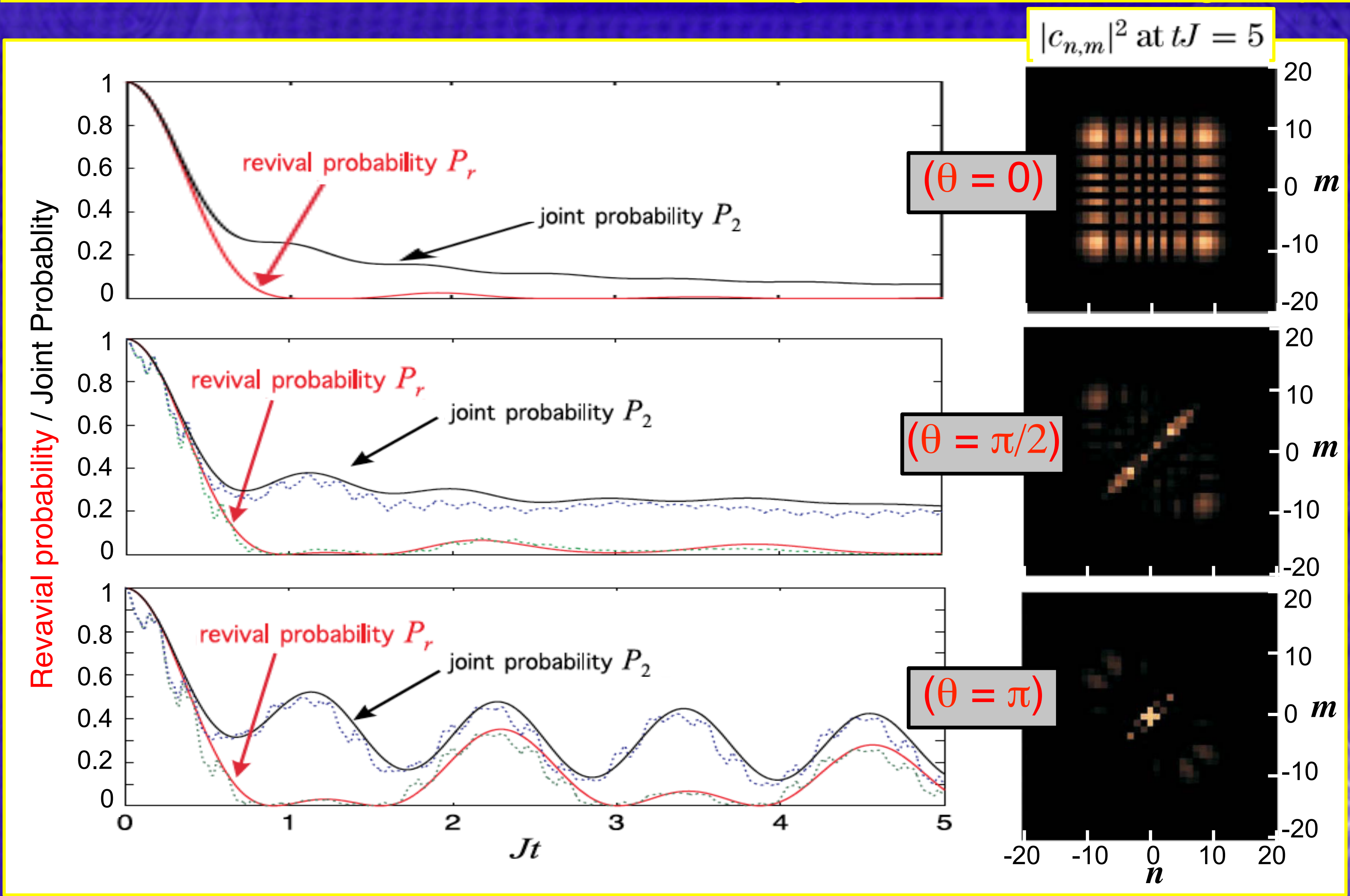
Initial condition (input light excitation): $c_{n,m}(0) = \delta_{n,0}\delta_{m,0}$

$P_r(t) = \left|c_{0,0}(t)\right|^2$ Revival Probability

$P_2(t) = \sum_n \left|c_{n,n}(t)\right|^2$ Joint Probability to find both particles at the same site

Correlated-tunneling of Anyons (VI)

Visualization of correlated-tunneling of non interacting Anyons

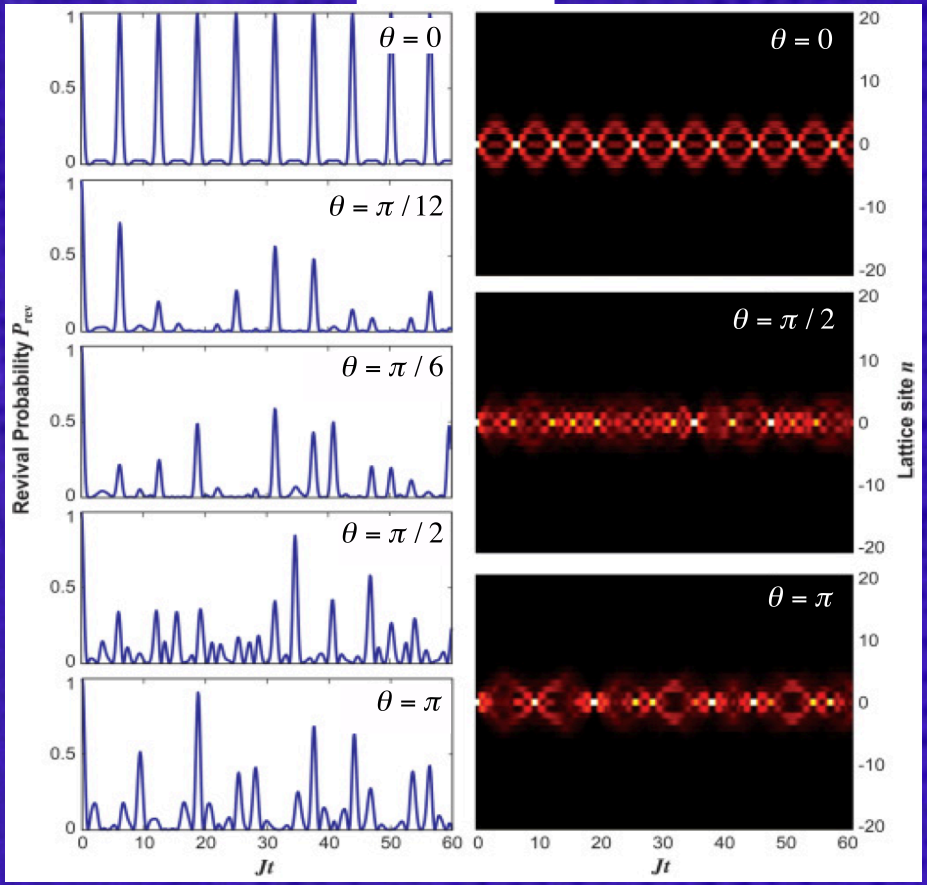


Opt. Lett. ***37***, 2160 (2012)

Correlated-BOs of Anyons (I)

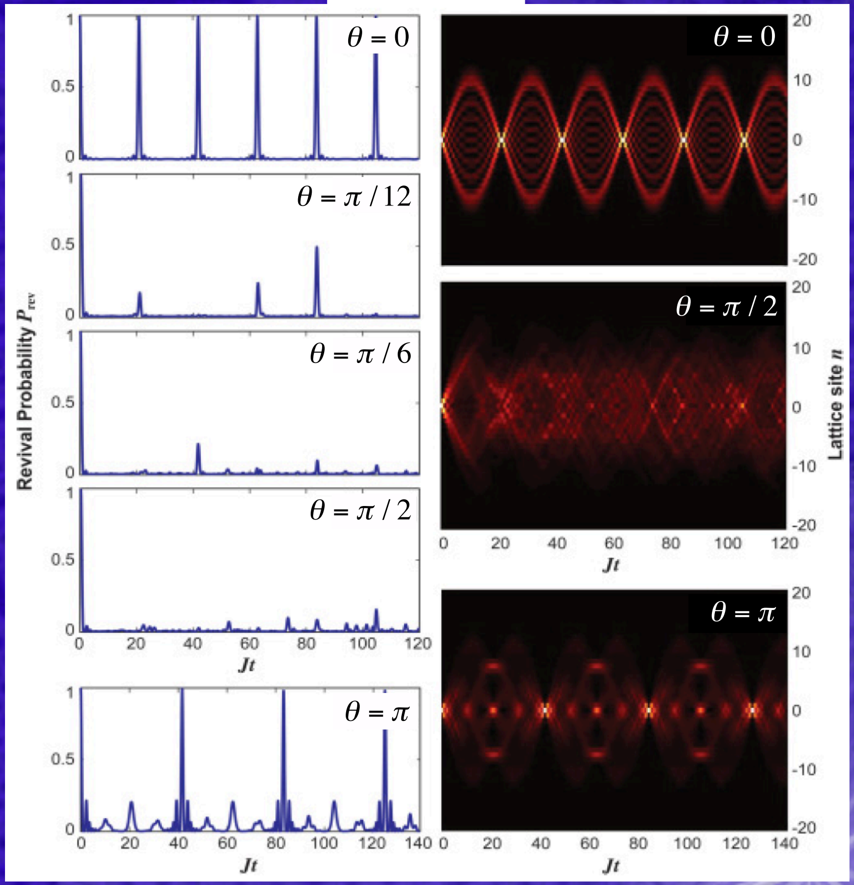
For non-interacting anyons, correlated BOs are *generally* degraded, but not always...

Strong driving $F/J = 1$



Only Bosons exhibit BOs (uncorrelated)

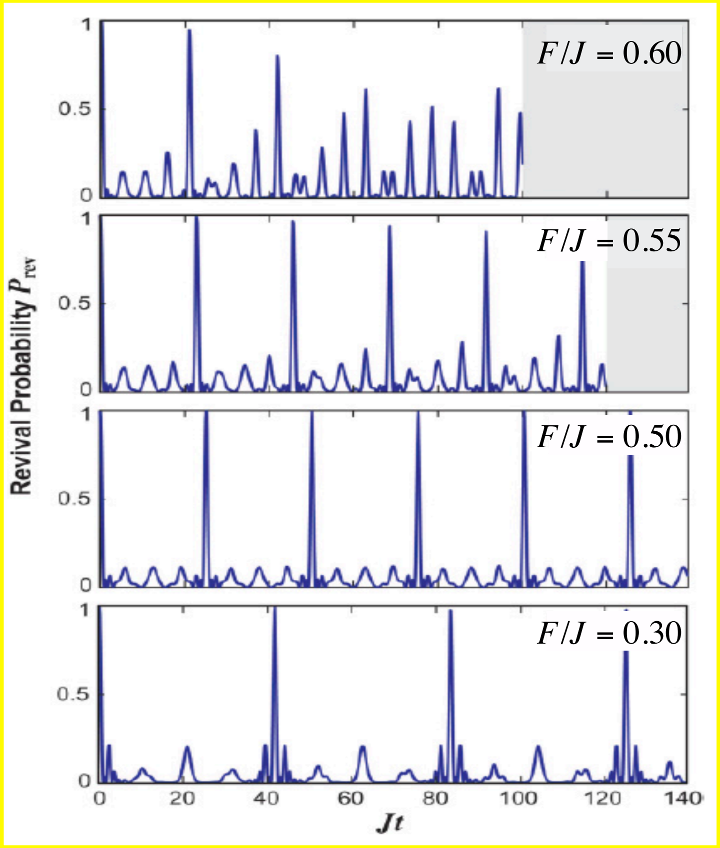
Weak driving $F/J = 0.3$

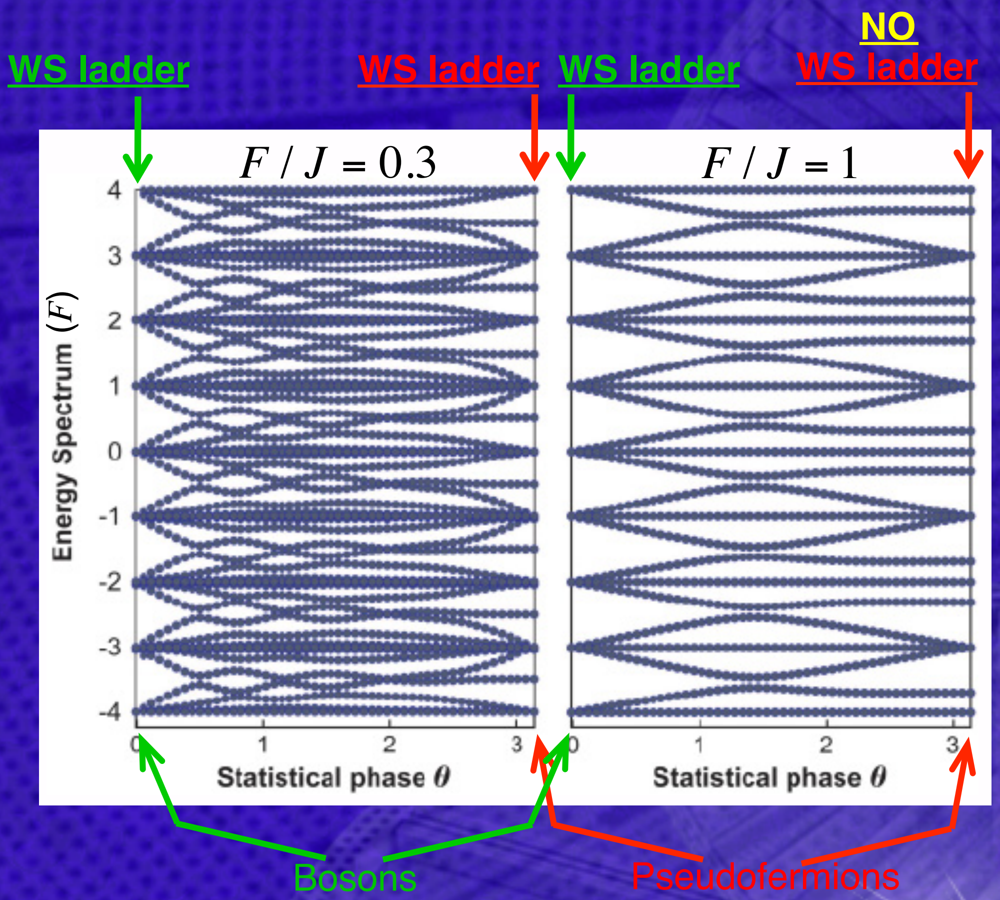


Pseudofermions exhibit correlated BOs at half the frequency of uncorrelated BOs

Correlated-BOs of Anyons (II)

For non-interacting Pseudofermions, the ratio F/J decides the existence of BOs!





In the strong interaction regime, BOs turn out to be insensitive to statistics
→ doubling of the BO frequency (as for 2 fermions or 2 bosons) regardless θ

Phys.Rev. B 85, 165144 (2012)

Conclusions and Developments

Conclusions

1. Theoretical investigation of the dynamics of few strongly interacting particles in one dimensional lattices under coherent driving with external (DC and or AC) fields: prediction of new physical features

2. Design and realization of classical simulators of few interacting particles based on optical waveguide arrays fabricated by fs-laser writing:

   ¶ a fundamental **drawback** on scalability...

   - 2 particles on a 1D lattice, or many particles in a DW, ok
   - More than 2 particles on a lattice? NO

   ¶ but also important **advantages**...

   - classical simulators of individual quantum systems in extremely low density (2 quanta) [challenging for quantum simulators]
   - direct access to Fock space allows ease of system preparation
   - optical phenomena can easily embed loss and gain, allowing simulation of correlated phenomena of non-Hermitian HMs

Developments

Theoretical study of novel quantum phenomena of correlated particles, in driven one-dimensional systems:

- Correlated super-Bloch oscillations (under DC+AC driving fields) (Phys. Rev. B 86, 075143, 2012)
- Many-particle quantum decay and trapping: The role of statistics and Fano resonances (Phys. Rev. A 86, 012112, 2012, and an experimental paper is in preparation, in collaboration with Osellame's group....)
- Quantum transport in bipartite lattices via Landau-Zener tunneling (Phys. Rev. A 86, 043633, 2012)
- Existence of low-energy doublons in ac-driven anisotropic HMs (Phys. Rev. A 87, 013634, 2013)
- Field-induced ferromagnetism (under next-nearest neighbor tunneling) (Europhys. Lett. 101, 67006, 2013)
- Tamm-Hubbard surface states embedded in the continuum (J. Phys.: Condens. Matter 25, 235601, 2013)
- "Klein tunneling" of correlated particles (beyond on-site interaction) (Eur. Phys. J. B 2013 in press)

Thank You
for
Your Attention

Field-induced ferromagnetism

$$\hat{H} = \hat{H}_{hop1} + \hat{H}_{hop2} + \hat{H}_{int}$$

Nearest-neighbor and next-nearest-neighbor single particle tunneling:

the (static) κ_1-κ_2 Hubbard Model

Ferromagnetic ordering iif $-\kappa_2 / \kappa_1 > r \sim 0.25$
(i.e. fundamental state with saturated Spin)
[see Pieri et al. Phys. Rev. B 54, 9250 (1996)]

$$\hat{H} = \hat{H}_{hop1} + \hat{H}_{hop2} + \hat{H}_{int} + \hat{H}_{drive}$$

the driven κ_1-κ_2 Hubbard Model

Ferromagnetic ordering for any value of κ_1 and κ_2
under <u>(relatively) high frequency driving</u> (i.e. $\omega / \kappa_1 > 1$)
and proper <u>value of the ratio</u> E_0 / ω

Analytical proof is provided for **<u>any particle density</u>** (below half filling) in a one-dimensional tight-binding lattice

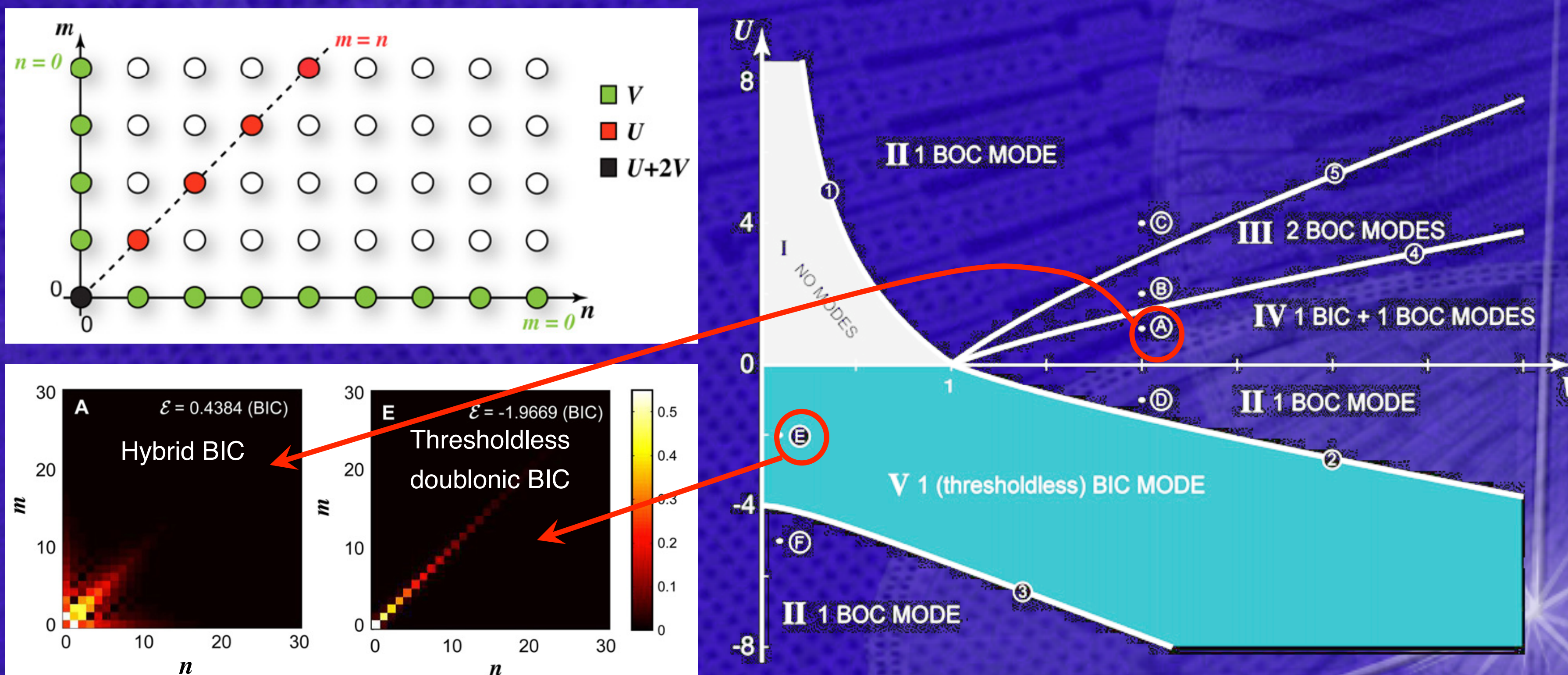
Europhys. Lett. **101**, 67006 (2013)

Tamm-Hubbard states in the continuum

$$\hat{H} = \sum_{k=0}^{\infty}\left[-\left(\hat{a}_k^{\dagger}\hat{a}_{k+1} + \hat{a}_{k+1}^{\dagger}\hat{a}_k\right) + \frac{U}{2}\hat{a}_k^{\dagger 2}\hat{a}_k^{2}\right] + V\hat{a}_0^{\dagger}\hat{a}_0$$

Static semi-infinite 1D TB lattice with an impurity at the edge

Fock space representation for the two-interacting bosons (a 2D lattice)



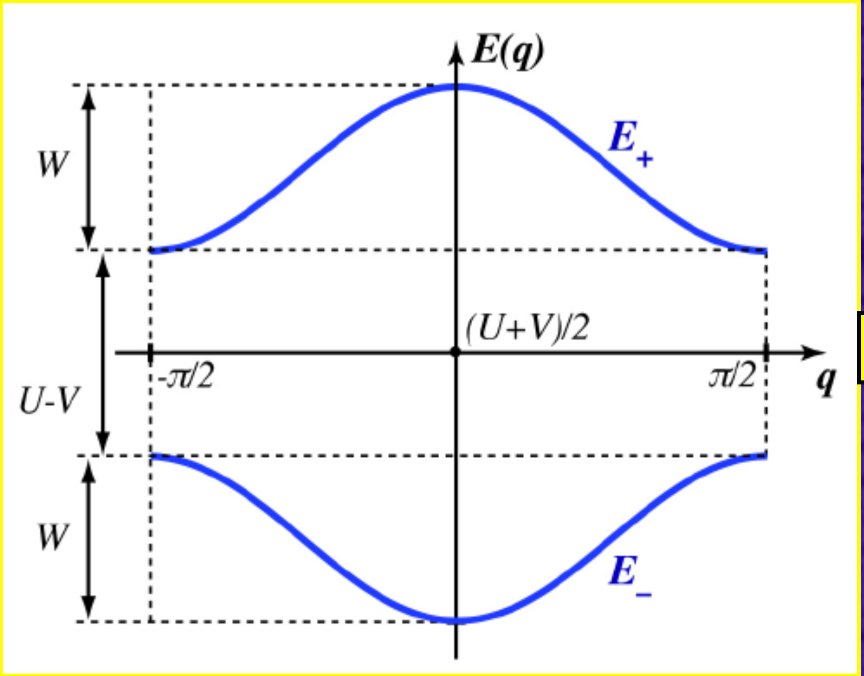
J. Phys.: Condens. Matter **25**, 235601 (2013)

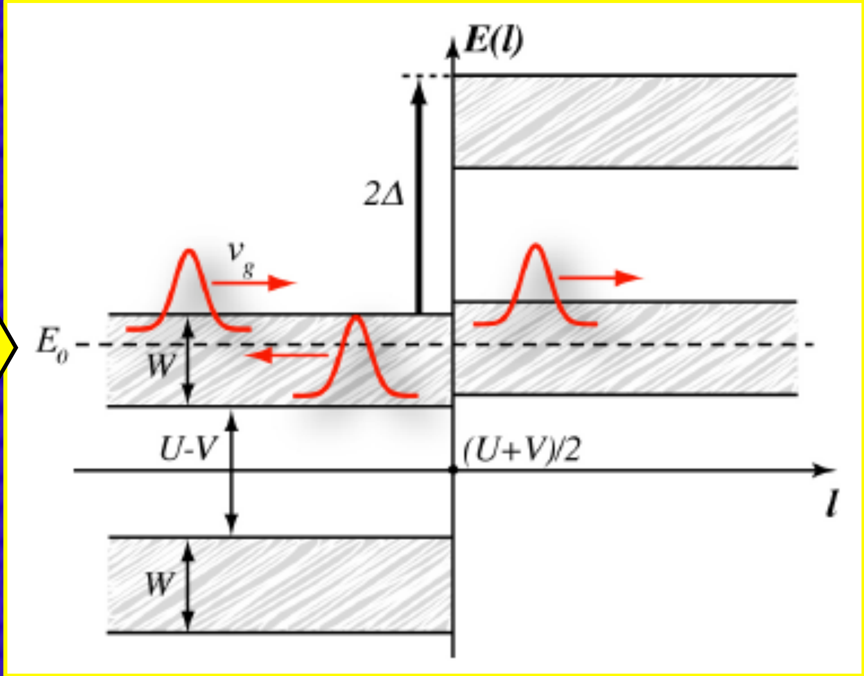
Klein tunneling of correlated particles

$$\hat{H} = -J\sum_l \hat{a}_l^\dagger\left(\hat{a}_{l-1} + \hat{a}_{l+1}\right) + \frac{U}{2}\sum_l \hat{n}_l(\hat{n}_l - 1) + V\sum_l \hat{n}_l\hat{n}_{l+1} + \sum_l \epsilon_l \hat{n}_l$$

On-site and nearest-neighbor particle interaction + Potential Step $\epsilon_l = \begin{cases} 0 & l < 0 \\ \Delta & l \geq 0 \end{cases}$

Two minibands for doublons





Eur. Phys. J. B 2013 (in press)

Thank You
for
Your Attention

The Hubbard Model

The space of states in many-body quantum physics

Quantum Field Theory (QFT) do not consider "**particles**" but "**quanta**":

- Particles can be **labeled** (and thus exchanged).
- Quanta are entities that can be "aggregated" but not labeled.

Ex. A, B are two labels for particles.

The states of particle A → 1-particle Hilbert space $H_1(A)$
The states of particle B → 1-particle Hilbert space $H_1(B)$
The states of particles A and B → $H(A,B) = H_1(A) \otimes H_1(B)$? NO!

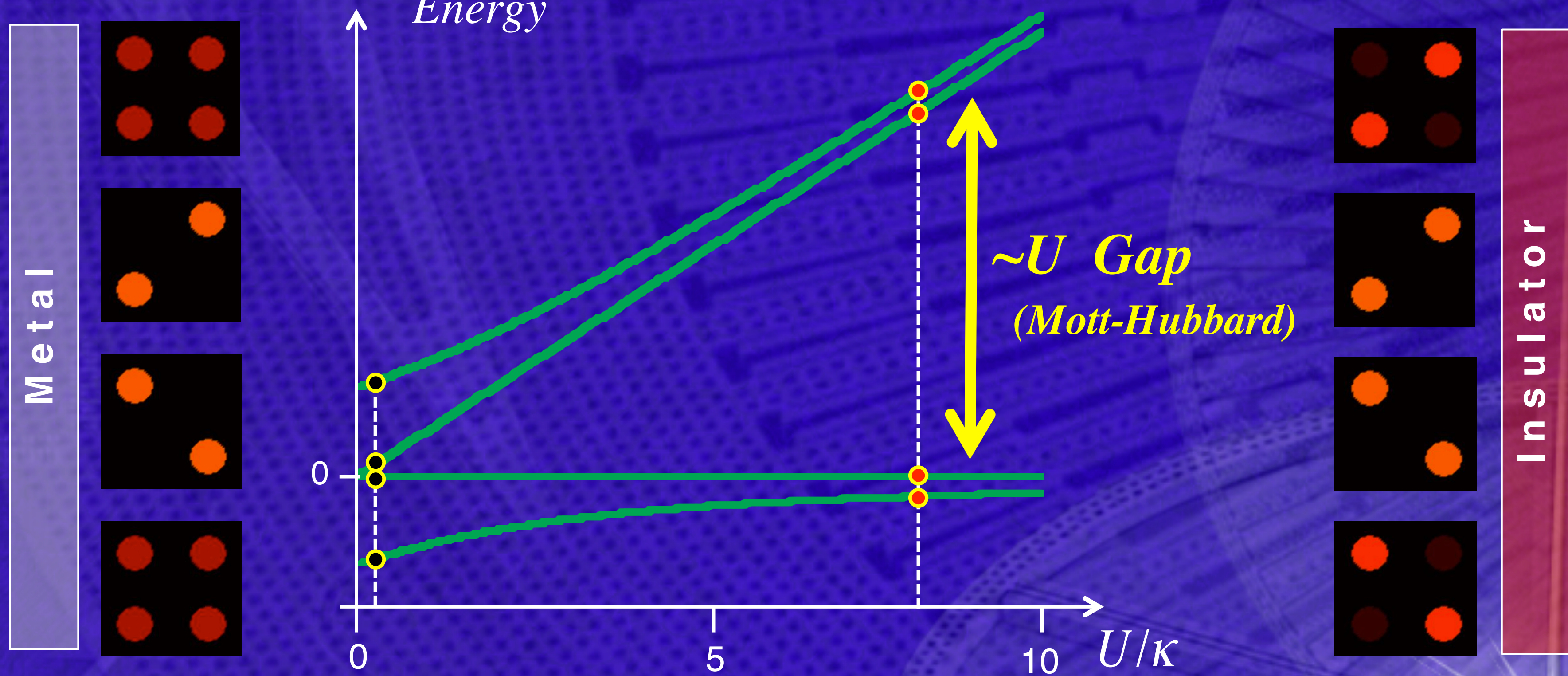
In $H_1(A) \otimes H_1(B)$ are symmetric, anti-symmetric, and non-symmetric states!
for Bosons : $\psi(B,A) = \psi(A,B)$ (symmetric under exchange)
for Fermions : $\psi(B,A) = -\psi(A,B)$ (anti-symmetric under exchange)

Solution 1: In the "particle view" we have to "*manually*" exclude the unphysical states lacking the proper exchange symmetry.
Solution 2: In the "quanta view" we refuse the physical meaning of particle labelling (so we miss the identity of particles) and build up the **Fock Space**

Correlated-tunneling in a Double Well

A photonic analog of the toy-model of Mott transition

Particle density $n = Q / L = 1$ → so-called half-filled system



A large energy gap is established between collective eigenstates with single site occupacy and collective eigenstates with double site occupacy

→ NO charge transport by nearest-neighbor hopping → INSULATOR

Correlated-tunneling of Anyons

Anyons: why?

- Anyons have been theoretically proposed more than 30 year ago.
  J. Leinaas and J. Myrheim, ***Nuovo Cimento B* 37**, 1 (1977).
  F. Wilczek, ***Phys. Rev. Lett.* 49**, 957 (1982).

- Anyons are invoked in the theoretical modelling of 2D systems.
  E.g.: Fractional quantum Hall effect (discovered in 1982)
  D. Arovas, J. R. Schrieffer, F. Wilczek, ***Phys. Rev. Lett.* 53**, 722 (1984).

- The experimental evidence of anyons is controversial.

- Very recent proposal to create anyons in 1D lattices by atom optics:
  C. Vitoriano et al., ***Phys. Rev. Lett.* 102**, 146404 (2009).
  T. Keilman et al., ***Nat. Commun.* 2**, 361 (2011).

- Anyon physics is mostly unexplored even on the theoretical side.

- Anyons are particularly interesting with respect to correlation:
  → correlated-tunneling even in the absence of interaction!

Correlated-tunneling of Anyons

More details on the design of the photonic simulator

$$i\frac{dc_{n,m}}{dt} = -\kappa\left(c_{n+1,m} + c_{n-1,m} + c_{n,m+1} + c_{n,m-1}\right) + \beta_{n,m}c_{n,m} + \frac{n_s a}{\bar{\lambda}}\left(n\frac{d^2x_0}{dt^2} + m\frac{d^2y_0}{dt^2}\right)c_{n,m}$$

Phase transformation: $a_{n,m} = c_{n,m}\exp\left[i\beta_{n,m}t + i\frac{n_s a}{\bar{\lambda}}\left(n\frac{dx_0}{dt} + m\frac{dy_0}{dt}\right)\right]$

$$i\frac{da_{n,m}}{dt} = -\kappa\left\{a_{n+1,m}\exp\left[-i\eta_{n,m}(t)\right] + a_{n-1,m}\exp\left[i\eta_{n-1,m}(t)\right] + a_{n,m+1}\exp\left[-i\rho_{n,m}(t)\right] + a_{n,m-1}\exp\left[i\rho_{n,m-1}(t)\right]\right\}$$

$$\eta_{n,m}(t) = \frac{n_s a}{\bar{\lambda}}\frac{dx_0}{dt} + \left(\beta_{n+1,m} - \beta_{n,m}\right)t = -\frac{n_s aA\omega}{\bar{\lambda}}\sin(\omega t) + \left(\beta_{n+1,m} - \beta_{n,m}\right)t$$

$$\frac{n_s aA\omega}{\bar{\lambda}} = \Gamma$$

$$\rho_{n,m}(t) = \frac{n_s a}{\bar{\lambda}}\frac{dy_0}{dt} + \left(\beta_{n,m+1} - \beta_{n,m}\right)t = -\frac{n_s aA\omega}{\bar{\lambda}}\sin(\omega t + \phi) + \left(\beta_{n,m+1} - \beta_{n,m}\right)t$$

Averaging of the exponential factors in the high frequency modulation limit $(\omega \gg \kappa, U)$ **under resonance condition** $\sigma = M\omega$ **(with** M **an even integer).**

e.g.: $\left\langle \exp\left[-i\eta_{n,m}(t)\right]\right\rangle = \lim_{T\to\infty}\frac{1}{T}\int_0^T \exp\left[-i\eta_{n,m}(t)\right]dt = \begin{cases} J_0(\Gamma), \text{ for } n \neq m,\ m-1 \\ J_M(\Gamma), \text{ otherwise} \end{cases}$

Developments

1. Theoretical study of novel quantum phenomena of correlated particles, in driven one-dimensional systems:
   - Correlated super-Bloch oscillations (under DC+AC driving fields) (Phys. Rev. B 86, 075143, 2012)
   - Many-particle quantum decay and trapping: The role of statistics and Fano resonances (Phys. Rev. A 86, 012112, 2012, and an experimental paper is in preparation....)
   - Coherent destruction of tunneling of two interacting bosons in a tight-binding lattice (Phys. Rev. A 86, 042104, 2012)
   - Quantum transport in bipartite lattices via Landau-Zener tunneling (Phys. Rev. A 86, 043633, 2012)
   - Existence of low-energy doublons in ac-driven anisotropic HMs (Phys. Rev. A 87, 013634, 2013)

2. Design and realization of photonic simulators for extended HMs to visualize at a classical level quantum effects never observed so far in a truly quantum system: e.g. super-Bloch oscillations, Block-Zehner oscillations, PT-symmetric HMs....