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Rejoinder: Analysis of AneuRisk65 data*

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We would like to thank the various groups who analyzed the AneuRisk65 data, providing a broad range of analyses with many interesting results. This dataset offers the possibility of performing registration and analyses at various data levels. Cheng et al. (2014) and Xie, Kurtek and Srivastava (2014) registered the three dimensional vessel centrelines, likewise in Sangalli et al. (2009, 2010). Staicu and Lu (2014) and Gervini (2014) opted instead for the corresponding one-dimensional curvature functions. The registration methods used are varied, some (see Cheng et al., 2014; Sangalli, Secchi and Vantini, 2014) only allowing for shift or linear warpings of the asbcissa parameters, others (see Staicu and Lu, 2014; Xie, Kurtek and Srivastava, 2014; Gervini, 2014) allowing for very flexible warpings. One difficulty with these data is that the portion of reconstructed vessel morphology has different length across subjects: for some subjects, only the terminal tract of the Inner Carotid Artery (ICA) is observed and reconstructed, for other patients a long portion of the arterial tract preceding the distal part is also available. The observed vessel lengths are significantly different for the considered groups of subjects – Upper and Lower-No groups – due to the different location of the scanned volume. Some registration methods (see Cheng et al., 2014; Sangalli, Secchi and Vantini, 2014; Gervini, 2014) explicitly tackle this issue, whilst others (see Staicu and Lu, 2014; Xie, Kurtek and Srivastava, 2014) map all curves to a common domain. On the other hand, we believe that the more flexible warping functions considered by these latter methods might have counterbalanced the problem, at least to some extent, by re-stretching toward the terminal part shorted arteries and by unwinding longer ones. After registration, most groups (see Cheng et al., 2014; Staicu and Lu, 2014; Gervini, 2014) try to discriminate Upper and Lower-No groups analyzing the vessel shape or some morphological feature such as vessel radius and curvature, similarly to Sangalli et al. (2009). As expected, when a rich family of warping functions is used, the discrimination improves if the phase variability captured by the warping functions is also explicitly considered in the discrimination method. Xie, Kurtek and Srivastava (2014) instead perform an unsupervised clustering of the vessels based on their three-dimensional shapes, analogously to Sangalli et al. (2010).

Here are some comments specific to each data analysis.

Cheng et al. (2014) analyze the data using techniques from shape analysis. The vessel centrelines are registered in the three-dimensional space by

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considering rigid transformations (translations and rotations) of these three-dimensional curves and simple shifts along their curvilinear abscissas. The different lengths of the vessel centerlines are appropriately considered by extrapolating shorter curves; the extrapolated tracts are then given nearly zero weights in the weighted Procrustes method used for registration and in the subsequent analysis. Differently from Sangalli, Secchi and Vantini (2014), since only shifts in the abscissa are considered, the registration method does not try to match carotids with different sizes. We were very happy to see that the many insights the authors give on the data, concerning for instance vessel morphological features and differences between the Upper and Lower-No group's centerlines, are in full agreement with our findings discovered during the AneuRisk project.

Staicu and Lu (2014) instead consider the one-dimensional vessel curvature profiles. These are registered by first mapping all curvature functions to a common abscissa interval and then by applying the Fisher-Rao registration method proposed in Srivastava et al. (2011). The optimal warping functions thus obtained are also used to register the vessel radius profiles. Hence, the amplitude variability of the aligned radius profiles and that of the curvature profiles are explored by means of functional principal component analysis, with a modified inner product that accounts for the phase variability captured by the warping functions. Since the latter are computed after having mapped all vessels to a common domain, these warpings implicitly include information about the vessel length, that is significantly different in the Upper and Lower-No subject groups and is directly correlated with the location of the aneurism, or its absence, as recalled above. We believe that this feature may imply an artificial discrimination between the two groups. We were in any case positively impressed by seeing that the results fully support the findings in Cheng et al. (2014) and in Sangalli et al. (2009). This is particularly interesting to notice given the different data considered in the registration (centerline curvature functions vs centrelines), and the different registration method, which allows for very flexible warpings instead of the shift and linear warpings considered in Cheng et al. (2014) and in Sangalli et al. (2009) respectively.

Xie, Kurtek and Srivastava (2014) use and extend the Fisher Rao approach to curves in three-dimensional space, warp the three-dimensional centerlines and analyze their shape. Differently from Cheng et al. (2014) and similarly to Staicu and Lu (2014), the curves are first mapped to a common domain and very flexible warpings are then allowed. The three-dimensional shapes of the vessel centrelines are hence studied by functional principal component analysis and clustering in the defined shape space.

Finally, Gervini (2014) proposes a rather different approach. The author analyzes the curvature profiles and proposes a likelihood-based warping method, combined with a logistic regression model that aims at discriminating subjects in the Upper and Lower-No groups. Short carotids are here treated as incomplete data. Considering a rich class of warping functions, the author finds that the discrimination results improve significantly when the warping functions are included explicitly in the logistic model.

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