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Event-triggered dynamic output quantized control for 2-D switched systems*

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Abstract. It is well known that designing mode-dependent event-triggered control (MDETC) brings challenging difficulties to theoretical analysis, especially for two-dimensional (2-D) switched systems. Therefore, for 2-D switched Fornasini–Marchesini local state-space (FMLSS) systems, this paper designs a MDETC to investigate global exponential stabilization almost surely (GES a.s.). A MDETC based on dynamic output quantization control scheme is designed, which not only has a wide range of practicability, but also greatly saves network bandwidth resources. By constructing mode-dependent Lyapunov functions that include two time directions, some novel sufficient conditions are provided such that the switched FMLSS system achieves GES a.s. Unlike most previous results, our results do not require each mode to be stable, not even after adding control. Finally, numerical experiments are provided to verify the validity of our main theoretical results.

Keywords: 2-D switched system, dwell time, event-triggered, quantization, stabilization.

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1 Introduction

2-D switched systems have received extensive attentions now because they have been applied in many fields, including flight control systems, multichannel power transmission systems, and thermal reaction process [4, 8]. 2-D switched systems can be represented by different models, including Roesser model, Attasi model, and Fornasini–Marchesini (FM) model. Especially, FMLSS model includes Roesser model and Attasi model as a special case [1, 5, 24]. Compared with switched Roesser model, switched Attasi model, and switched FM model, switched FMLSS model has wider applications. Therefore, a lot of research works on these systems have reported, e.g., Benzaouia et al. studied the stability for a class 2-D discrete-time switched systems by using a state feedback controller in [1]. The authors in [24] proposed an average dwell time (ADT) method to investigate the exponential stability control for a class 2-D switched systems, and output feedback stabilization controls for switched FMLSS system were studied in [5]. To guarantee stability for a switched system, it is generally required that all the switching modes to be stable and ADT to be bounded below. In the case that some modes are unstable, the upper bound of the dwell time (DT) of the unstable mode should not exceed a threshold [18]. This restrictive condition definitely leads to conservative results. Now, the stability control schemes for the 2-D switched systems, in which the part modes of systems are not stable, are rarely studied.

Event-triggered control (ETC) scheme is an efficient technology to save communication resources [20,21,31]. Although ETC has been extensively studied in one-dimensional (1-D) nonswitched systems [2, 3, 16, 21, 22] and 1-D switched systems [10–13, 27], few studies are found concerning ETC with respect to 2-D switched systems. In addition, most of the above results on ETC for switched systems are derived by directly adopting the control technology applicable to nonswitched systems, which often leads to unrealistic results such as the control schemes are often restricted by subsystem stability, system switching signal interval events and minimum sampling interval time, and so on. In order to overcome the problems mentioned above, recently the authors in [25] designed a mode-dependent event-triggered (ET) static output control scheme for 1-D discrete-time switched linear systems, which extended the control schemes for 1-D systems in [19, 32]. However, the design of the control scheme in [25] is very complex, and thus, its practicality is poor. Recently, the authors in [26] have extended the ETC to discuss the stability control with respect to switched FMLSS systems. Unfortunately, the given conditions are very strict such as requiring that each mode without control is stable, and the ADT must be greater than a given threshold. Therefore, by developing a new mode-dependent ETC, we will study GES a.s. for 2-D switched systems.

Apart from ETC, output feedback control is another effective method in saving network bandwidth sources. In addition, output feedback control is also an effective method to solve the measurability of the internal state of the system [9, 14]. In fact, the states with respect to most systems usually cannot be measured, and therefore, designing a controller based on the dynamic output is very valuable. Recently, designing controller based on output of system has been widely applied to 1-D switched systems [6, 33]. For instance, in previous representative studies, the authors in [33] and [6] designed dynamic output

H_∞ control for 1-D continuous-time switched systems and 1-D discrete-time switched systems, respectively. However, few authors focus on the output feedback control for 2-D systems, let alone 2-D switched systems. In recent years, based on the quantization technique, which is another effective method to save network bandwidth sources [15, 29, 34], the authors in [17] proposed an output quantized control for 2-D discrete switched complex networks.

Based on the above discussions, this paper studies output quantized ETC for a class of 2-D discrete-time switched system to ensure that the system achieves complete GES a.s. Our main contributions are as follows.

(i) An ETC based on dynamic output control scheme is designed for a 2-D switched system. It not only has a wide range of practicability, but also greatly saves network bandwidth resources, thereby ensuring the performance of the system.

(ii) The designed ETC scheme is mode-dependent (MDETC scheme) and general, and thus, new sufficiently mode-dependent conditions are obtained to make sure that the 2-D switched system finally achieve GES a.s. Our obtained sufficient conditions do not require each mode with or without control to be stable.

(iii) By using mode-dependent Lyapunov function analysis techniques, the relationship between the triggering interval and the ADT is well handled. Compared the studies involving ETC for 2-D switched systems in [26], our results do not require that the ADT of each mode must be greater than the predetermined threshold.

The rest structure is as follows. An ETC is designed in Section 2. The theoretical analyses are given in Section 3. An example of switched FMLSS system is presented in Section 4. The conclusions are given in Section 5.

Notations. $\mathbb{R}^{n\times m}$ denotes $n\times m$ real matrix. A^{T} is the transpose of the matrix A. The matrix $A>0$ ($A<0$) represents that A is positive (negative). $\mathrm{sym}\{A\}=A^{\mathrm{T}}+A$. I_n and $0_{n\times m}$ are the n-dimensional identity matrix and $n\times m$-dimensional zero matrix, respectively. $\|\cdot\|$ is the binary norm in Euclidean space. $\mathbb{Z}^+=\{0,1,2,\dots\}$.

2 Problem formulation and preliminaries

Based on FMLSS model in [7], the authors in [5] proposed the following switched FMLSS model:

$$\begin{aligned} x(i+1,\,j+1) &= A_{1\mathfrak{d}(i,j+1)}x(i,\,j+1)+A_{2\mathfrak{d}(i+1,j)}x(i+1,\,j),\\ y(i,j) &= C_{\mathfrak{d}(i,j)}x(i,j), \end{aligned} \tag{1}$$

where $x(i,j)=(x_1(i,j),x_2(i,j),\dots,x_{n_x}(i,j))^{\mathrm{T}}\in\mathbb{R}^{n_x}$ and $y(i,j)=(x_y(i,j), x_y(i,j),\dots,y_{n_y}(i,j))^{\mathrm{T}}\in\mathbb{R}^{n_y}$ represent the state vector and output vector, respectively. $\mathfrak{d}(i,j):\mathbb{Z}^+\times\mathbb{Z}^+\to\mathcal{R}=\{1,2,\dots,R\}$ denotes the switched signal for modes. $A_{1\mathfrak{d}(i,j)}=(a_{1\mathfrak{d}(i,j)})_{n_x\times n_x}$, $A_{2\mathfrak{d}(i,j)}=(a_{2\mathfrak{d}(i,j)})_{n_x\times n_x}$, $C_{\mathfrak{d}(i,j)}=(c_{\mathfrak{d}(i,j)})_{n_y\times n_x}$ are known constant matrices. The initial conditions of system (1) are

$$x(0,j) = \chi_1(0,j), \quad x(i,0) = \chi_2(i,0), \quad i,j \in \mathbb{Z}^+,$$

and

$$\chi_1(0,0) = \chi_2(0,0).$$

For the needs of further research, it is always assumed that the boundary conditions with respect to system (1) are bounded, denoted by

$$\limsup_{N_1\to\infty} \mathbf{E}\Biggl\{\sum_{j=0}^{N_1} \bigl\|\chi_1(0,j)\bigr\|\Biggr\} + \limsup_{N_2\to\infty} \mathbf{E}\Biggl\{\sum_{i=0}^{N_2} \bigl\|\chi_2(i,0)\bigr\|\Biggr\} < \infty.$$

Assumption 1. Assume that the switching time of system (1) occurs at i or j. Then $\mathfrak{d}(i,j)$ depends on $i+j$. Thus, there are

$$\mathfrak{d}(i,j) = \mathfrak{d}(t) \quad \text{for all } i+j=t,\ t \in \mathbb{Z}^+.$$

System (1) is switched randomly. Under Assumption 1, the following random switching rules are satisfied.

Definition 1. (See [30].) For $g,h \in \mathcal{R}$, $\mathfrak{d}(t)$ switches from the gth mode to the hth mode with the transition probability (TP)

$$\mathbf{P}\bigl\{\mathfrak{d}(t+1) = h \bigm| \mathfrak{d}(t) = g\bigr\} = p_{gh},$$

where $0 \leqslant p_{gh} < 1$ for $g \neq h$, and $p_{gh} = 0$ for $g = h$, $\sum_{h=1}^{R} p_{gh} = 1, g,h = 1,2,\dots,R$. $P = (p_{gh})_{R\times R}$ is an irreducible TP matrix and has a unique stationary distribution $\kappa = (\kappa_1, \kappa_2, \dots, \kappa_R)$.

Let $\mathfrak{d}_t := \mathfrak{d}(t)$ be a piecewise function of time t, where $t = i+j$. Then a time sequence of mode switching $\{t_\nu = i_\nu + j_\nu,\ \nu \in \mathbb{Z}^+\}$ satisfies $0 = t_0 < t_1 < t_2 < \cdots < t_\nu < \cdots$ and $\lim_{s\to+\infty} t_\nu = +\infty$ when $\mathfrak{d}(t) = r \in \mathcal{R}$ for $t \in [t_{\nu-1}, t_\nu)$. Let $\mathfrak{d}_t = \mathfrak{d}(t) = r$ and denote $d_r(\nu) = t_\nu - t_{\nu-1} < +\infty$. Referring to [17], the following conditions are provided.

Assumption 2. $\mathbf{E}[d_r(\nu)] = \alpha_r$, $r \in \mathcal{R}$, with $\alpha_r > 0$.

System (1) with control are represented as follows:

$$\begin{aligned} x(i+1,\, j+1) &= A_{1\mathfrak{d}(i,\, j+1)} x(i,\, j+1) + A_{2\mathfrak{d}(i+1,\, j)} x(i+1,\, j) \\ &\quad + B_{1\mathfrak{d}(i,j+1)} u(i,\, j+1) + B_{2\mathfrak{d}(i+1,\, j)} u(i+1,\, j), \\ y(i,j) &= C_{\mathfrak{d}(i,j)} x(i,j), \end{aligned} \tag{2}$$

where $u(i,j) \in \mathbb{R}^{n_u}$ represents the input of controller. $B_{1\mathfrak{d}(i,j)} = (b_{1\mathfrak{d}(i,j)})_{n_x\times n_u}$ and $B_{2\mathfrak{d}(i,j)} = (b_{2\mathfrak{d}(i,j)})_{n_x\times n_u}$ are known constant matrices.

Definition 2. (See [17].) System (2) is said to achieve GES a.s. if

$$\limsup_{t\to\infty} \frac{\ln \sum_{i+j=t} \|x(i,j)\|}{t} < 0$$

for any initial conditions $x(0,j) = \chi_1(0,j)$, $x(i,0) = \chi_2(i,0)$, $i,j \in \mathbb{Z}^+$, with the boundary condition

$$\limsup_{N_1\to\infty} \mathbf{E}\left\{ \sum_{j=0}^{N_1} \|\chi_1(0,j)\| \right\} + \limsup_{N_2\to\infty} \mathbf{E}\left\{ \sum_{i=0}^{N_2} \|\chi_2(i,0)\| \right\} < \infty.$$

Next, we will give the mode-dependent ET condition. When $\mathfrak{d}(t) = r$, $t \in [t_\nu, t_{\nu+1})$, denote $t_\nu^\phi = i_\nu^\phi + j_\nu^\phi$, $\phi = 0,1,2,\dots$, the ϕth sampling instant in the time interval $[t_\nu, t_{\nu+1})$, and the sampling instant t_ν^ϕ satisfies

$$t_\nu = t_\nu^0 < t_\nu^1 < t_\nu^2 < \cdots < t_{\nu+1},$$

where $t_\nu = i_\nu + j_\nu$ and $t_\nu^\phi = i_\nu^\phi + j_\nu^\phi$. Note that $t_\nu^0 = t_\nu$ is the first sampling time and the start time of the rth mode on $[t_\nu, t_{\nu+1})$, i.e., the sampling instant t_ν^ϕ is strictly dependent on the system mode $\mathfrak{d}(t)$. The mode-dependent ET condition is designed as follows:

$$t_\nu^{\phi+1} = \min\left\{ t > t_\nu^\phi \;\middle|\; \delta(i,j)^{\mathrm{T}} \Theta_r \delta(i,j) > \varepsilon_r y(i_\nu^\phi, j_\nu^\phi)^{\mathrm{T}} \Theta_r y(i_\nu^\phi, j_\nu^\phi) + \frac{1}{2}\alpha\beta^{\lambda t} \right\} \tag{3}$$

with $t = i + j$, $t_\nu^\phi = i_\nu^\phi + j_\nu^\phi$, $\delta(i,j) = y(i_\nu^\phi, j_\nu^\phi) - y(i,j)$, $\phi \in \mathbb{N}_+$. $0 < \Theta_r \in \mathbb{R}^{n_y \times n_y}$, $r \in \mathcal{R}$ are the matrices that needs to be designed. The scalars $0 < \varepsilon_r < 1$, $0 < \alpha$, $0 < \beta < 1$, and $\lambda > 0$.

Remark 1. Note that t_ν is the switching instant of the rth mode and $t_\nu^0 = t_\nu$, which implies that ET condition (3) is mode-dependent. In addition, compared with mode-independent ET condition for the 2-D switched system in [26], in this paper the DT of the mode is not restricted by its sampling intervals because the first sampling time of any mode is equal to its switching instant, i.e., $t_\nu^0 = t_\nu$. The sampling interval of ET condition (3) is not constrained by the sampling intervals. Therefore, our ET mechanism has a wider range of applications than that in [26]. Moreover, it should be noted that, although the ET conditions for 1-D switched linear system in [25] are also mode-dependent, they are cumbersome and thus not very practical. Furthermore, the corresponding delays are brought by using the segmentation method for achieving the ET scheme in [25].

Remark 2. Obviously, the ET condition (3) is affected by the parameters $\varepsilon_{\mathfrak{d}}$, α, λ. In the application, choosing a suitable combination of the three parameters $\varepsilon_{\mathfrak{d}}$, α, λ to adjust the sampling interval according to the network with different bandwidth networks is more flexible than only one parameter of these ET conditions being adjusted in [3, 12, 26].

Next, the quantized dynamic output ETC is designed:

$$\begin{aligned}\hat{x}(i+1,\,j+1) &= \hat{A}_{1\mathfrak{d}(i,\,j+1)}\hat{x}(i,\,j+1) + \hat{A}_{2\mathfrak{d}(i+1,\,j)}\hat{x}(i+1,\,j)\\ &\quad + \hat{B}_{1\mathfrak{d}(i,\,j+1)}y\big(i_\nu^\phi,\,j_\nu^\phi+1\big) + \hat{B}_{2\mathfrak{d}(i+1,\,j)}y\big(i_\nu^\phi+1,\,j_\nu^\phi\big),\\ w(i,j) &= \hat{C}_{\mathfrak{d}(i,j)}\hat{x}(i,j),\\ u(i,j) &= q\big(w(i,j)\big),\end{aligned} \tag{4}$$

where $\hat{x}(i,j) \in \mathbb{R}^{n_d}$ and $n_d \leqslant n_x$, $\hat{A}_{1\mathfrak{d}(i,j)} \in \mathbb{R}^{n_d\times n_d}$, $\hat{A}_{2\mathfrak{d}(i,j)} \in \mathbb{R}^{n_d\times n_d}$, $\hat{B}_{1\mathfrak{d}(i,j)} \in \mathbb{R}^{n_d\times n_y}$, $\hat{B}_{2\mathfrak{d}(i,j)} \in \mathbb{R}^{n_d\times n_y}$, $\hat{C}_{\mathfrak{d}(i,j)} \in \mathbb{R}^{n_u\times n_d}$ are the control gains, which need to be designed later, $t_\nu^\phi = i_\nu^\phi + j_\nu^\phi$ is the ET time, $w(i,j)$ represents the output vector. $o(i,j) = (o_1(i,j), o_2(i,j), \dots, o_{n_u}(i,j))$ is the output. $q(o(i,j)) = (q_1(o(i,j)), \dots, q_{n_u}(o_{n_u}(i,j)))^{\mathrm{T}}$ is a quantizer with $\tau = 1, 2, \dots, n_u$ satisfying $q_\tau(\cdot) : \mathbb{R} \to U_\tau$ with $U_{\mathfrak{d}(i,j)} = \{\pm o^\iota_{\mathfrak{d}(i,j)} \colon w^\iota_{\mathfrak{d}(i,j)} = \varsigma^\iota_{\mathfrak{d}(i,j)},\ 0 < \varsigma_{\mathfrak{d}(i,j)} < 1,\ \iota = 0, \pm1, \pm2, \dots\} \cup \{0\}\}$, $\bar{o}_0 > 0$. For all $v \in \mathbb{R}$, denote the quantizer $q_{\mathfrak{d}(i,j)}(v)$ as

$$q_{\mathfrak{d}(i,j)}(v) = \begin{cases} o^\iota_{\mathfrak{d}(i,j)} & \text{if } \frac{1}{1+\varrho_\tau}\bar{o}_i < v \leqslant \frac{1}{1-\varrho_\tau}\bar{o}_i,\\ 0 & \text{if } v = 0,\\ -q_{\mathfrak{d}(i,j)}(-v) & \text{if } v < 0,\end{cases}$$

where $\varrho_\tau = (1-\varsigma_\tau)/(1+\varsigma_\tau)$. Observing system (2) and controller (4), the following relationship can be found.

(i) The information of system (2) is transmitted to controller (4) through the output variable $y(i,j)$;

(ii) Controller (4) obtains the control state vector $u(i,j)$ according to the output variable $y(i,j)$ of system (2) and then applies $u(i,j)$ to system (2). That is, controller (4) performs feedback control according to the dynamic output of system (2).

Remark 3. Note that the controller (4) combines the ET scheme and quantization technique based on dynamic output, which greatly saves network bandwidth resources. Meanwhile, the control technique in [17] is further improved here.

Let the boundary conditions controller system (4) be ultimately bounded, which are similar to that of system (1). There exists a Filippov solution $\varDelta_{\mathfrak{d}(i,j)}(i,j) \in [-\delta_{\mathfrak{d}(i,j)}, \delta_{\mathfrak{d}(i,j)})$ of the quantizer $q_{\mathfrak{d}(i,j)}(v)$ such that $q_{\mathfrak{d}(i,j)}(v) = (1 + \varDelta_{\mathfrak{d}(i,j)}(i,j))v$. Hence, system (2) can be rewritten as

$$\begin{aligned}\hat{x}(i+1,\,j+1) &= \hat{A}_{1\mathfrak{d}(i,\,j+1)}\hat{x}(i,\,j+1) + \hat{A}_{2\mathfrak{d}(i+1,\,j)}x(i+1,\,j)\\ &\quad + \hat{B}_{1\mathfrak{d}(i,\,j+1)}y\big(i_\nu^\phi,\,j_\nu^\phi+1\big) + \hat{B}_{2\mathfrak{d}(i+1,\,j)}y\big(i_\nu^\phi+1,\,j_\nu^\phi\big),\\ u(i,j) &= [I_{n_u} + \varLambda_{\mathfrak{d}(i,j)}]\hat{C}_{\mathfrak{d}(i,j)}\hat{x}(i,j),\end{aligned} \tag{5}$$

where $\Lambda_{\mathfrak{d}(i,j)} = \operatorname{diag}(\Delta_{1\mathfrak{d}(i,j)}, \Delta_{2\mathfrak{d}(i,j)}, \dots, \Delta_{n_u\mathfrak{d}(i,j)})$. Combining (1) and (5) derives the following augmented system:

$$\begin{aligned} x(i+1,\, j+1) &= A_{1\mathfrak{d}(i,\, j+1)}x(i,\, j+1) + A_{2\mathfrak{d}(i+1,\, j)}x(i+1,\, j) \\ &\quad + B_{1\mathfrak{d}(i,\, j+1)}\big[I_{n_u} + \Lambda_{\mathfrak{d}(i,\, j+1)}\big]\hat{C}_{\mathfrak{d}(i,\, j+1)}\hat{x}(i,\, j+1) \\ &\quad + B_{2\mathfrak{d}(i+1,\, j)}\big[I_{n_u} + \Lambda_{\mathfrak{d}(i+1,\, j)}\big]\hat{C}_{\mathfrak{d}(i+1,\, j)}\hat{x}(i+1,\, j), \\ \hat{x}(i+1,\, j+1) &= \hat{A}_{1\mathfrak{d}(i,\, j+1)}\hat{x}(i,\, j+1) + \hat{A}_{2\mathfrak{d}(i+1,\, j)}\hat{x}(i+1,\, j) \\ &\quad + \hat{B}_{1\mathfrak{d}(i,j+1)}\big[C_{\mathfrak{d}(i,\, j+1)}x(i, j+1) + \mathfrak{d}(i,\, j+1)\big] \\ &\quad + B_{2\mathfrak{d}(i+1,\, j)}\big[C_{\mathfrak{d}(i+1,\, j)}x(i+1,\, j) + \mathfrak{d}(i+1,\, j)\big]. \end{aligned} \tag{6}$$

Letting $\eta(i,j) = (x(i,j)^{\mathrm{T}}, \hat{x}(i,j)^{\mathrm{T}})^{\mathrm{T}}$, system (6) becomes

$$\begin{aligned} \eta(i+1,\, j+1) &= (\mathcal{A}_{\mathfrak{d}(i,\, j+1)} + \mathcal{D}_{\mathfrak{d}(i,\, j+1)})\eta(i,\, j+1) \\ &\quad + (\mathcal{B}_{\mathfrak{d}(i,\, j+1)} + \mathcal{E}_{\mathfrak{d}(i,\, j+1)})\eta(i+1,\, j) \\ &\quad + \mathcal{F}_{\mathfrak{d}(i,\, j+1)}\delta(i,\, j+1) + \mathcal{G}_{\mathfrak{d}(i+1,\, j)}\delta(i+1,\, j), \end{aligned}$$

where

$$\begin{aligned} \mathcal{A}_{\mathfrak{d}(i,j+1)} &= \begin{pmatrix} A_{1\mathfrak{d}(i,j+1)} & B_{1\mathfrak{d}(i,j+1)}\hat{C}_{\mathfrak{d}(i,j+1)} \\ \hat{B}_{1\mathfrak{d}(i,j+1)}C_{\mathfrak{d}(i,j+1)} & \hat{A}_{1\mathfrak{d}(i,j+1)} \end{pmatrix}, \\ \mathcal{B}_{\mathfrak{d}(i+1,j)} &= \begin{pmatrix} A_{2\mathfrak{d}(i+1,j)} & B_{2\mathfrak{d}(i+1,j)}\hat{C}_{\mathfrak{d}(i+1,j)} \\ \hat{B}_{2\mathfrak{d}(i+1,j)}C_{\mathfrak{d}(i+1,j)} & \hat{A}_{2\mathfrak{d}(i+1,j)} \end{pmatrix}, \\ \mathcal{D}_{\mathfrak{d}(i,j+1)} &= \begin{pmatrix} 0_{n_x\times n_x} & B_{1\mathfrak{d}(i,j+1)}\Lambda_{\mathfrak{d}(i,j+1)}\hat{C}_{\mathfrak{d}(i,j+1)} \\ 0_{n_d\times n_x} & 0_{n_d\times n_d} \end{pmatrix}, \\ \mathcal{E}_{\mathfrak{d}(i+1,j)} &= \begin{pmatrix} 0_{n_x\times n_x} & B_{2\mathfrak{d}(i+1,j)}\Lambda_{\mathfrak{d}(i+1,j)}\hat{C}_{\mathfrak{d}(i+1,j)} \\ 0_{n_d\times n_x} & 0_{n_d\times n_d} \end{pmatrix}, \\ \mathcal{F}_{\mathfrak{d}(i,j+1)} &= \begin{pmatrix} 0_{n_x\times n_y} \\ \hat{B}_{1\mathfrak{d}(i,j+1)} \end{pmatrix}, \qquad \mathcal{G}_{\mathfrak{d}(i+1,j)} = \begin{pmatrix} 0_{n_x\times n_y} \\ \hat{B}_{2\mathfrak{d}(i+1,j)} \end{pmatrix}. \end{aligned}$$

Letting

$$E_{a1} = \begin{pmatrix} I_{n_x} \\ 0_{n_d\times n_x} \end{pmatrix}, \qquad E_{a2} = \left(0_{n_d\times n_x}, I_{n_d}\right),$$

one also has

$$\begin{aligned} \mathcal{D}_{\mathfrak{d}(i,j+1)} &= E_{a1}B_{1\mathfrak{d}(i,j+1)}\Lambda_{\mathfrak{d}(i,j+1)}\hat{C}_{\mathfrak{d}(i,j+1)}E_{a2}, \\ \mathcal{E}_{\mathfrak{d}(i+1,j)} &= E_{a1}B_{2\mathfrak{d}(i+1,j)}\Lambda_{\mathfrak{d}(i+1,j)}\hat{C}_{\mathfrak{d}(i+1,j)}E_{a2}. \end{aligned}$$

Denoting

$$\xi(i,j) = \left(\eta(i+1,\, j+1)^{\mathrm{T}},\, \eta(i,\, j+1)^{\mathrm{T}},\, \eta(i+1,\, j),\, \delta(i,\, j+1)^{\mathrm{T}},\, \delta(i+1,\, j)^{\mathrm{T}}\right)^{\mathrm{T}},$$

we have

$$
\begin{aligned}
\eta(i+1,\, j+1)^{\mathrm{T}} &= (I_b, 0_{b\times 2c})\xi(i,j) \triangleq E_1\xi(i,j),\\
\eta(i,\, j+1)^{\mathrm{T}} &= (0_{b\times b}, I_b, 0_{b\times d})\xi(i,j) \triangleq E_2\xi(i,j),\\
\eta(i+1,\, j)^{\mathrm{T}} &= (0_{b\times 2b}, I_b, 0_{b\times 2n_y})\xi(i,j) \triangleq E_3\xi(i,j),\\
\delta(i,\, j+1)^{\mathrm{T}} &= (0_{n_y\times 3b}, I_{n_y}, 0_{n_y\times n_y})\xi(i,j) \triangleq E_4\xi(i,j),\\
\delta(i+1,\, j)^{\mathrm{T}} &= (0_{n_y\times (2b+c)}, I_{n_y})\xi(i,j) \triangleq E_5\xi(i,j),\\
y(i,\, j+1) &= C_r(0_{n_x\times b}, I_{n_x}, 0_{n_x\times e})\xi(i,j) \triangleq C_rE_6\xi(i,j),\\
y(i+1,\, j) &= C_r(0_{n_x\times 2b}, I_{n_x}, 0_{n_x\times f})\xi(i,j) \triangleq C_rE_7\xi(i,j),
\end{aligned}
$$

where $b = n_x + n_d$, $c = b + n_y$, $d = c + n_y$, $e = d + n_d$, $f = n_d + 2n_y$. In addition, system (6) can be expressed as

$$
\eta(i+1,\, j+1) = \mathcal{H}_{\mathfrak{d}}\xi(i,j) \tag{7}
$$

or

$$
\begin{aligned}
\eta(i+1,\, j+1) = {}& \big[\mathcal{A}_{\mathfrak{d}(i,j+1)}E_2 + \mathcal{B}_{\mathfrak{d}(i,j+1)}E_3 + \mathcal{F}_{\mathfrak{d}(i,j+1)}E_4 + \mathcal{G}_{\mathfrak{d}(i+1,j)}E_5\big]\xi(i,j)\\
&+ \big[E_{a1}B_{1\mathfrak{d}(i,j+1)}\varLambda_{\mathfrak{d}(i,j+1)}\hat{C}_{\mathfrak{d}(i,j+1)}E_{a2}E_2\\
&+ E_{a1}B_{2\mathfrak{d}(i+1,j)}\varLambda_{\mathfrak{d}(i+1,j)}\hat{C}_{\mathfrak{d}(i+1,j)}E_{a2}E_3\big]\xi(i,j)
\end{aligned} \tag{8}
$$

with $\mathcal{H}_{\mathfrak{d}} = (0_{\tilde{b}\times\tilde{b}}, (\mathcal{A}_{\mathfrak{d}(i,j+1)} + \mathcal{D}_{\mathfrak{d}(i,j+1)})E_2, (\mathcal{B}_{\mathfrak{d}(i,j+1)} + \mathcal{E}_{\mathfrak{d}(i,j+1)})E_3, \mathcal{F}_{\mathfrak{d}(i,j+1)}E_4, \mathcal{G}_{\mathfrak{d}(i+1,j)}E_5)$, where $\tilde{b} = n_x + n_y$.

Our aim is to realize 2-D switched FMLSS system GES a.s. by using ET condition (3) and controller (4), i.e., for system (1), $\limsup_{t\to\infty} \ln\|x(i,j)\|/t < 0$ for any initial conditions with bounded boundary conditions. Therefore, it only need to prove that $\limsup_{t\to\infty} \ln\|\eta(i,j)\|/t < 0$ as any initial conditions of system (1) and controller system (4) with bounded boundary conditions.

Lemma 1. (*See* [23].) *For given matrices $\mathcal{X}$, $\mathcal{Y}$, $\mathcal{Z}$ with $\mathcal{Z}^{\mathrm{T}}\mathcal{Z} \leqslant I$ and scalar l, there holds*

$$
\mathcal{X}\mathcal{Z}\mathcal{Y} + \mathcal{Y}^{\mathrm{T}}\mathcal{Z}^{\mathrm{T}}\mathcal{X}^{\mathrm{T}} \leqslant l\mathcal{X}\mathcal{X}^{\mathrm{T}} + l^{-1}\mathcal{Y}\mathcal{Y}^{\mathrm{T}}.
$$

Lemma 2. (*See* [28].) *If $\mathfrak{d}(t)$, $t = i + j$, and κ are the mode-dependent ADT switching signal and the stationary distribution of P, respectively, then it derives that, for all $r \in \mathcal{R}$,*

$$
\lim_{t\to\infty}\frac{T_r(t)}{t} = \bar{\kappa}_r \quad a.s.
$$

with $\bar{\kappa}_r = \kappa_r\alpha_r/(\sum_{l=1}^{R}\kappa_l\alpha_l)$, $T_r(t)$ is the total time of $\mathfrak{d}(t) = r$ on the time interval $[0, t]$.

3 Main results

The GES a.s. for system (2) under controller (4) will be discussed. In the later analysis, by using new techniques, the Lyapunov function in Theorem 1 is designed to be mode-dependent. In addition, using the ergodic theory, the properties of the stationary distribution κ are extremely important to our theoretical analysis. Finally, the control gains of controller (4) are given by solving a set of LMIs in Theorem 2.

Theorem 1. *Assume that the control gains* $\hat{A}_{1r}$, $\hat{A}_{2r}$, $\hat{B}_{1r}$, $\hat{B}_{2r}$, $\hat{C}_r$ *are all known and Assumptions* 1–2 *hold. For pregiven constants* $\mu_r \geqslant 1$, $\beta_r > 0$, *if there exist matrices* $0 < Q_r^1 \in \mathbb{R}^{(n_x+n_d)\times(n_x+n_d)}$, $0 < Q_r^2 \in \mathbb{R}^{(n_x+n_d)\times(n_x+n_d)}$, $0 < \Theta_r \in \mathbb{R}^{n_y\times n_y}$ *and invertible matrices* $U_r \in \mathbb{R}^{(n_x+n_d)\times(n_x+n_d)}$, $V_r \in \mathbb{R}^{n_y\times n_y}$ *such that, for* $r \in \mathcal{R}$, *the following matrix inequalities are satisfied*:

$$\Upsilon_r < 0,$$

$$\left(Q_r^1 + Q_r^2\right) \leqslant \mu_r\left(Q_{\breve{r}}^1 + Q_{\breve{r}}^2\right), \quad \breve{r} \in \mathcal{R} \text{ and } \breve{r} \neq r, \tag{9}$$

$$\sum_{r=1}^{R} \bar{\kappa}_r\left[\frac{\ln \mu_r}{\alpha_r} + \ln \beta_r\right] < 0, \tag{10}$$

then, under controller (4), *system* (2) *completes GES a.s., where*

$$\Upsilon_r = \begin{pmatrix} \Upsilon_{11r} & \Upsilon_{12r} \\ \Upsilon_{12r}^{\mathrm{T}} & \Pi_r \end{pmatrix}$$

with

$$\begin{aligned}
\Upsilon_{11r} &= \mathrm{diag}\left(\mathcal{H}_{1r}, \mathcal{H}_{2r}, \mathcal{H}_{2r}, -l_r^{-1} I_{n_y}, -l_r^{-1} I_{n_y}, -l_r I_{n_y}, -l_r I_{n_y}\right),\\
\Upsilon_{12r} &= \Big((U_r E_1)^{\mathrm{T}}, \big(V_r(E_4 + C_r E_6)\big)^{\mathrm{T}}, \big(V_r(E_5 + C_r E_7)\big)^{\mathrm{T}},\\
&\qquad (\hat{C}_r E_{a2} E_2)^{\mathrm{T}}, (\hat{C}_r E_{a2} E_3)^{\mathrm{T}}, \big(B_{1r}^{\mathrm{T}} E_{a1}^{\mathrm{T}} E_1\big)^{\mathrm{T}}, \big(B_{2r}^{\mathrm{T}} E_{a1}^{\mathrm{T}} E_1\big)^{\mathrm{T}}\Big)^{\mathrm{T}},\\
\Pi_r &= \mathrm{sym}\big\{E_1^{\mathrm{T}} \mathcal{A}_r E_2 + E_1^{\mathrm{T}} \mathcal{B}_r E_3 + E_1^{\mathrm{T}} \mathcal{F}_r E_4 + E_1^{\mathrm{T}} \mathcal{G}_r E_5 - E_1^T E_1\big\}\\
&\qquad - \beta_r E_2^{\mathrm{T}} Q_r^1 E_2 - \beta_r E_3^{\mathrm{T}} Q_r^2 E_3 - \varepsilon_r E_4^{\mathrm{T}} \Theta_r E_4 - \varepsilon_r E_5^{\mathrm{T}} \Theta_r E_5,\\
\mathcal{H}_{1r} &= Q_r^1 + Q_r^2 - U_r - U_r^{\mathrm{T}}, \qquad \mathcal{H}_{2r} = \varepsilon_r \Theta_r - V_r - V_r^{\mathrm{T}}.
\end{aligned}$$

Proof. When $\mathfrak{d}(t) = r \in \mathcal{R}$, $t = i + j + 1 \in [t_{\nu-1}, t_\nu)$, $s \in \mathbb{N}_+$, consider the following Lyapunov function:

$$V_r\big(\eta(i,j), t\big) = V_r^1\big(\eta(i,j), t\big) + V_r^2\big(\eta(i,j), t\big),$$

where

$$\begin{aligned}
V_r^1\big(\eta(i,j), t\big) &= \eta^{\mathrm{T}}(i,j) Q_r^1 \eta(i,j),\\
V_r^2\big(\eta(i,j), t\big) &= \eta^{\mathrm{T}}(i,j) Q_r^2 \eta(i,j).
\end{aligned}$$

It is derived from (7) that

$$\begin{aligned}
&\varDelta V_r^1\bigl(\eta(i,\, j+1),\, t\bigr) - (\beta_r - 1)V_r^1\bigl(\eta(i,\, j+1),\, t\bigr)\\
&\quad = V^1_{\mathfrak{d}(t+1)}\bigl(\eta(i+1,\, j+1),\, t+1\bigr) - \beta_r V_r^1\bigl(\eta(i,\, j+1),\, t\bigr)\\
&\quad = \eta^{\mathrm{T}}(i+1,\, j+1)Q_r^1\eta(i+1,\, j+1) - \beta_r\eta^{\mathrm{T}}(i,\, j+1)Q_r^1\eta(i,\, j+1)\\
&\quad = \xi^{\mathrm{T}}(i,j)\bigl[E_1^{\mathrm{T}}Q_r^1E_1 - \beta_r E_2^{\mathrm{T}}Q_r^1E_2\bigr]\xi(i,j).
\end{aligned} \tag{11}$$

Similarly, for $V_r^2(\eta(i,j), t, r)$, one has

$$\begin{aligned}
&\varDelta V_r^2\bigl(\eta(i,\, j+1),\, t\bigr) - (\beta_r - 1)V_r^2\bigl(\eta(i,\, j+1),\, t\bigr)\\
&\quad = V^2_{\mathfrak{d}(t+1)}\bigl(\eta(i+1,\, j+1),\, t+1\bigr) - \beta_r V_r^2\bigl(\eta(i,\, j+1),\, t\bigr)\\
&\quad = \xi^{\mathrm{T}}(i,j)\bigl[E_1^{\mathrm{T}}Q_r^2E_1 - \beta_r E_3^{\mathrm{T}}Q_r^2E_3\bigr]\xi(i,j).
\end{aligned}$$

By (8) one has that

$$\begin{aligned}
&2\xi^{\mathrm{T}}(i,j)E_1^{\mathrm{T}}\bigl[\mathcal{A}_rE_2 + \mathcal{B}_rE_3 + \mathcal{F}_rE_4 + \mathcal{G}_rE_5 - E_1\\
&\quad + E_{a1}B_{1r}\varLambda_r\hat{C}_rE_{a2}E_2 + E_{a1}B_{2r}\varLambda_r\hat{C}_rE_{a2}E_3\bigr]\xi(i,j) = 0.
\end{aligned} \tag{12}$$

By Lemma 1 it can be derived that, for scalar $l_r > 0$,

$$\begin{aligned}
&\mathrm{sym}\bigl\{E_1^{\mathrm{T}}E_{a1}B_{1r}\varLambda_r\hat{C}_rE_{a2}E_2\bigr\}\\
&\quad \leqslant l_r^{-1}E_1^{\mathrm{T}}E_{a1}B_{1r}B_{1r}^{\mathrm{T}}E_{a1}^{\mathrm{T}}E_1 + l_rE_2^{\mathrm{T}}E_{a2}^{\mathrm{T}}\hat{C}_r^{\mathrm{T}}\hat{C}_rE_{a2}E_2,
\end{aligned} \tag{13}$$

$$\begin{aligned}
&\mathrm{sym}\bigl\{E_1^{\mathrm{T}}E_{a1}B_{2r}\varLambda_r\hat{C}_rE_{a2}E_3\bigr\}\\
&\quad \leqslant l_r^{-1}E_1^{\mathrm{T}}E_{a1}B_{2r}B_{2r}^{\mathrm{T}}E_{a1}^{\mathrm{T}}E_1 + l_rE_3^{\mathrm{T}}E_{a2}^{\mathrm{T}}\hat{C}_r^{\mathrm{T}}\hat{C}_rE_{a2}E_3.
\end{aligned} \tag{14}$$

Substituting inequalities (13) and (14) into (12) yields that

$$\begin{aligned}
&\xi^{\mathrm{T}}(i,j)\bigl\{\mathrm{sym}\bigl\{E_1^{\mathrm{T}}\mathcal{A}_rE_2 + E_1^{\mathrm{T}}\mathcal{B}_rE_3 + E_1^{\mathrm{T}}\mathcal{F}_rE_4 + E_1^{\mathrm{T}}\mathcal{G}_rE_5 - E_1^TE_1\bigr\}\\
&\quad + \bigl[l_r^{-1}E_1^{\mathrm{T}}E_{a1}B_{1r}B_{1r}^{\mathrm{T}}E_{a1}^{\mathrm{T}}E_1 + l_rE_2^{\mathrm{T}}E_{a2}^{\mathrm{T}}\hat{C}_r^{\mathrm{T}}\hat{C}_rE_{a2}E_2\\
&\quad + l_r^{-1}E_1^{\mathrm{T}}E_{a1}B_{2r}B_{2r}^{\mathrm{T}}E_{a1}^{\mathrm{T}}E_1 + l_rE_3^{\mathrm{T}}E_{a2}^{\mathrm{T}}\hat{C}_r^{\mathrm{T}}\hat{C}_rE_{a2}E_3\bigr]\bigr\}\xi(i,j) \geqslant 0.
\end{aligned} \tag{15}$$

Noting that

$$\begin{aligned}
y(i,j+1) &= C_r(0_{n_x\times b}, I_{n_x}, 0_{n_x\times e})\xi(i,j) \triangleq C_rE_6\xi(i,j),\\
y(i+1,j) &= C_r(0_{n_x\times 2b}, I_{n_x}, 0_{n_x\times f})\xi(i,j) \triangleq C_rE_7\xi(i,j),
\end{aligned}$$

it derives from ET condition (3) that

$$\varepsilon_r\xi^{\mathrm{T}}(i,j)\bigl\{[E_4 + C_rE_6]^{\mathrm{T}}\varTheta_r[E_4 + C_rE_6] - E_4^{\mathrm{T}}\varTheta_rE_4\bigr\}\xi(i,j) + \frac{1}{2}\alpha\beta^{\lambda t} \geqslant 0 \tag{16}$$

and

$$\varepsilon_r\xi^{\mathrm{T}}(i,j)\big\{[E_5+C_rE_7]^{\mathrm{T}}\Theta_r[E_5+C_rE_7]-E_5^{\mathrm{T}}\Theta_rE_5\big\}\xi(i,j)+\frac{1}{2}\alpha\beta^{\lambda t}\geqslant 0. \quad (17)$$

It follows from (11), (15), (16), and (17) that

$$\begin{aligned}&V_r\big(\eta(i+1,\,j+1),\,t+1\big)-\beta_r\big[\Delta V_r^1(\eta(i,\,j+1),\,t)+V_r^2\big(\eta(i,\,j+1),\,t\big)\big]\\&\quad\leqslant \xi^{\mathrm{T}}(i,j)\Sigma_r\xi(i,j)+\alpha\beta^{\lambda t}\end{aligned} \quad (18)$$

with

$$\begin{aligned}\Sigma_r={}&E_1^{\mathrm{T}}\big(Q_r^1+Q_r^2\big)E_1\\&+\varepsilon_r[E_4+C_rE_6]^{\mathrm{T}}\Theta_r[E_4+C_rE_6]+\varepsilon_r[E_5+C_rE_7]^{\mathrm{T}}\Theta_r[E_5+C_rE_7]\\&+l_rE_2^{\mathrm{T}}E_{a2}^{\mathrm{T}}\hat{C}_r^{\mathrm{T}}\hat{C}_rE_{a2}E_2+l_rE_3^{\mathrm{T}}E_{a2}^{\mathrm{T}}\hat{C}_r^{\mathrm{T}}\hat{C}_rE_{a2}E_3\\&+l_r^{-1}E_1^{\mathrm{T}}E_{a1}B_{1r}B_{1r}^{\mathrm{T}}E_{a1}^{\mathrm{T}}E_1+l_r^{-1}E_1^{\mathrm{T}}E_{a1}B_{2r}B_{2r}^{T}E_{a1}^{\mathrm{T}}E_1\\&+\mathrm{sym}\big\{E_1^{\mathrm{T}}\mathcal{A}_rE_2+E_1^{\mathrm{T}}\mathcal{B}_rE_3+E_1^{\mathrm{T}}\mathcal{F}_rE_4+E_1^{\mathrm{T}}\mathcal{G}_rE_5-E_1^{T}E_1\big\}\\&-\beta_rE_2^{\mathrm{T}}Q_r^1E_2-\beta_rE_3^{\mathrm{T}}Q_r^2E_3-\varepsilon_rE_4^{\mathrm{T}}\Theta_rE_4-\varepsilon_rE_5^{\mathrm{T}}\Theta_rE_5.\end{aligned}$$

Now, we will prove that $\Sigma_r<0$. It follows from Schur equivalence that $\Sigma_r<0$ is equivalent to

$$\hat{\Upsilon}_r=\begin{pmatrix}\hat{\Upsilon}_{11r} & \hat{\Upsilon}_{12r}\\ \hat{\Upsilon}_{12r}^{\mathrm{T}} & \Pi_r\end{pmatrix}<0, \quad (19)$$

where

$$\begin{aligned}\hat{\Upsilon}_{11r}&=\mathrm{diag}\big(\hat{\mathcal{H}}_{1r},\hat{\mathcal{H}}_{2r},\hat{\mathcal{H}}_{2r},-l_r^{-1}I_{n_y},-l_r^{-1}I_{n_y},-l_rI_{n_y},-l_rI_{n_y}\big),\\ \hat{\Upsilon}_{12r}&=\big(E_1^{\mathrm{T}},\,(E_4+C_rE_6)^{\mathrm{T}},\,(E_5+C_rE_7)^{\mathrm{T}},\,(\hat{C}_rE_{a2}E_2)^{\mathrm{T}},\\&\qquad(\hat{C}_rE_{a2}E_3)^{\mathrm{T}},\,\big(B_{1r}^{\mathrm{T}}E_{a1}^{\mathrm{T}}E_1\big)^{\mathrm{T}},\,\big(B_{2r}^{\mathrm{T}}E_{a1}^{\mathrm{T}}E_1\big)^{\mathrm{T}}\big)^{\mathrm{T}},\\ \hat{\mathcal{H}}_{1r}&=-\big(Q_r^1+Q_r^2\big)^{-1},\qquad \hat{\mathcal{H}}_{2r}=-\varepsilon_r^{-1}\Theta_r^{-1}.\end{aligned}$$

Premultiplying $\mathrm{diag}(U_r,V_r,V_r,I_{n_y},I_{n_y},I_{n_y},I_{n_y})$ and postmultiplying $\mathrm{diag}(U_r^{\mathrm{T}},V_r^{\mathrm{T}},V_r^{\mathrm{T}},I_{n_y},I_{n_y},I_{n_y},I_{n_y})$ to inequality (19), one has that

$$\check{\Upsilon}_r=\begin{pmatrix}\check{\Upsilon}_{11r} & \check{\Upsilon}_{12r}\\ \check{\Upsilon}_{12r}^{\mathrm{T}} & \Pi_r\end{pmatrix}<0, \quad (20)$$

where

$$\begin{aligned}\check{\Upsilon}_{11r}&=\mathrm{diag}(\check{\mathcal{H}}_{1r},\check{\mathcal{H}}_{2r},\check{\mathcal{H}}_{2r},-l_r^{-1}I_{n_y},-l_r^{-1}I_{n_y},-l_rI_{n_y},-l_rI_{n_y}),\\ \check{\Upsilon}_{12r}&=\big(\big(U_rE_1^{\mathrm{T}}\big),\,\big(V_r(E_4+C_rE_6)\big)^{\mathrm{T}},\,\big(V_r(E_5+C_rE_7)\big)^{\mathrm{T}},\,(\hat{C}_rE_{a2}E_2)^{\mathrm{T}},\\&\quad(\hat{C}_rE_{a2}E_3)^{\mathrm{T}},\,\big(B_{1r}^{\mathrm{T}}E_{a1}^{\mathrm{T}}E_1\big)^{\mathrm{T}},\,\big(B_{2r}^{\mathrm{T}}E_{a1}^{\mathrm{T}}E_1\big)^{\mathrm{T}}\big)^{\mathrm{T}},\\ \check{\mathcal{H}}_{1r}&=-U_r(Q_r^1+Q_r^2)^{-1}U_r^{\mathrm{T}},\qquad \check{\mathcal{H}}_{2r}=-V_r\varepsilon_r^{-1}\Theta_r^{-1}V_r^{\mathrm{T}}.\end{aligned}$$

By using the inequalities

$$-U_r(Q_r^1+Q_r^2)^{-1}U_r^{\mathrm{T}} \leqslant Q_r^1+Q_r^2-U_r-U_r^{\mathrm{T}},$$
$$-V_r\varepsilon_r^{-1}\Theta_r^{-1}V_r^{\mathrm{T}} \leqslant \varepsilon_r\Theta_r-V_r-V_r^{\mathrm{T}}$$

and $\Upsilon_r < 0$, one has that inequality (20) holds, i.e., it derives that $\mathfrak{d}_r < 0$. Therefore, by (18), one has, for $i+j+1=t\in[t_{\nu-1},t_\nu)$,

$$V_r\bigl(\eta(i,j),\, t+1\bigr) \leqslant \beta_r V_r\bigl(\eta(i,j),\, t\bigr) + \alpha\beta^{\lambda t}.$$

There exist small enough positive constants $\check{\beta}_r$, $m\in\mathcal{M}$, such that

$$\sum_{m=1}^{M}\bar{\pi}_r\biggl[\frac{\ln\mu_r}{\alpha_r}+\ln\hat{\beta}_r\biggr] < 0 \tag{21}$$

and

$$V_r\bigl(\eta(i,j),\, t+1\bigr) \leqslant \hat{\beta}_r V_r\bigl(\eta(i,j),t\bigr) - \check{\beta}_r V_r\bigl(\eta(i,j),t\bigr) + \alpha\beta^{\lambda t} \tag{22}$$

with $\hat{\beta}_r=\beta_r+\check{\beta}_r$. Denote

$$\psi(t)=\max\biggl\{\psi_r(t)=\sqrt{\frac{\alpha\beta^{\lambda t}}{\check{\beta}_r[\lambda_{\max}(Q_r^1)+\lambda_{\max}(Q_r^2)]}},\ r\in\mathcal{M}\biggr\}.$$

When $\|\eta(i,j)\|\geqslant\psi(t)$ with $i+j=t$, it follows from (22) that

$$V_r\bigl(\eta(i,j),\, t+1\bigr) \leqslant \hat{\beta}_r V_r\bigl(\eta(i,j),t\bigr). \tag{23}$$

Hence, it derives from (23) that, for $i+j+1=t\in[t_{\nu-1},t_\nu)$,

$$\begin{aligned}
V_r\bigl(\eta(1,t),\, t+1\bigr) &\leqslant \hat{\beta}_r\bigl[V_r^1\bigl(\eta(0,t),t\bigr)+V_r^2\bigl(\eta(1,\, t-1),\, t\bigr)\bigr],\\
V_r\bigl(\eta(2,\, t-1),\, t+1\bigr) &\leqslant \hat{\beta}_r\bigl[V_r^1\bigl(\eta(1,\, t-1),\, t\bigr)+V_r^2\bigl(\eta(2,\, t-2),\, t\bigr)\bigr],\\
&\cdots\\
V_r\bigl(\eta(t,1),\, t+1\bigr) &\leqslant \hat{\beta}_r\bigl[V_r^1\bigl(\eta(t-1,\, 1),\, t\bigr)+V_r^2\bigl(\eta(t,0),t\bigr)\bigr].
\end{aligned} \tag{24}$$

By (23) and (24) it is obtained that

$$\sum_{i+j+1=t+1} V_r\bigl(\eta(i,j),\, t+1\bigr) \leqslant \hat{\beta}_r \sum_{i+j=t} V_r\bigl(\eta(i,j),t\bigr). \tag{25}$$

Then it follows from (9) and (25) that, for $t \in [t_{\nu-1}, t_\nu)$,

$$\begin{aligned}
&\sum_{i+j=t} V_{\mathfrak{d}(t)}(\eta(i,j),t) \\
&\quad< \hat{\beta}_{\mathfrak{d}(t-1)} \sum_{i+j=t-1} V_{\mathfrak{d}(t-1)}\bigl(\eta(i,j),\, t-1\bigr) \\
&\quad< \hat{\beta}_{\mathfrak{d}(t_{\nu-1})}^{t-t_{\nu-1}} \sum_{i+j=t_{\nu-1}} V_{\mathfrak{d}(t_{\nu-1})}\bigl(\eta(i,j), t_{\nu-1}\bigr) \\
&\quad< \mu_{\mathfrak{d}(t_{\nu-1})} \hat{\beta}_{\mathfrak{d}(t_{\nu-1})}^{t-t_{\nu-1}} \sum_{i+j=t_{\nu-1}} V_{\mathfrak{d}(t_{\nu-1})}\bigl(\eta(i,j), t_{\nu-1}\bigr) \\
&\quad< \mu_{\mathfrak{d}(t_{\nu-1})} \hat{\beta}_{\mathfrak{d}(t_{\nu-1})}^{t-t_{\nu-1}} \hat{\beta}_{\mathfrak{d}(t_{s-2})}^{t_{\nu-1}-t_{s-2}} \sum_{i+j=t_{s-2}} V_{\mathfrak{d}(t_{s-2})}\bigl(\eta(i,j), t_{s-2}\bigr) \\
&\quad< \cdots \\
&\quad< \prod_{r=1}^{R} \mu_r^{N_r(t)} \hat{\beta}_r^{T_r(t)} \sum_{i+j=0} V_{\mathfrak{d}(0)}\bigl(\eta(i,j), 0\bigr),
\end{aligned}$$

and thus,

$$\underline{\lambda} \sum_{i+j=t} \bigl\|\eta(i,j)\bigr\| < \prod_{r=1}^{R} \mu_r^{N_r(t)} \hat{\beta}_r^{T_r(t)} \sum_{i+j=0} V_{\mathfrak{d}(0)}\bigl(\eta(i,j), 0\bigr)$$

with $\underline{\lambda} = \min\{\lambda_{\max}(Q_r^1) + \lambda_{\max}(Q_r^2),\ r = 1, 2, \dots, R\}$.

Therefore, one has that

$$\begin{aligned}
\frac{\ln \psi(t)}{t} - \frac{\ln \underline{\lambda}}{t} &\leqslant \frac{\ln \sum_{i+j=t} \|\eta(i,j)\|}{t} \\
&< \frac{\ln \sum_{i+j=0} V_{\mathfrak{d}(0)}(\eta(i,j),0)}{t} - \frac{\ln \underline{\lambda}}{t} \\
&\quad + \sum_{r=1}^{R} \left[\frac{N_r(t)}{t} \ln \mu_r + \frac{T_r(t)}{t} \ln \hat{\beta}_r\right].
\end{aligned} \tag{26}$$

According to (21) and the fact

$$\lim_{t\to\infty} \frac{N_r(t)}{t} = \frac{\lim_{t\to\infty}(T_r(t)/t)}{T_r(t)/N_r(t)} = \frac{\bar{\kappa}_r}{\alpha_r} \tag{27}$$

and combining (26) and (27), one has

$$\begin{aligned}
&\limsup_{t\to\infty} \left\{ \left(\frac{\ln \sum_{i+j=0} V_{\mathfrak{d}(0)}(\eta(i,j),0)}{t} - \frac{\ln \underline{\lambda}}{t} \right) + \sum_{r=1}^{R} \left[\frac{N_r(t)}{t} \ln \mu_r + \frac{T_r(t)}{t} \ln \hat{\beta}_r\right] \right\} \\
&\quad = \limsup_{t\to\infty} \left\{ \sum_{r=1}^{R} \left[\frac{N_r(t)}{t} \ln \mu_r + \frac{T_r(t)}{t} \ln \hat{\beta}_r\right] \right\} = \sum_{r=1}^{R} \bar{\kappa}_r \left[\frac{\ln \mu_r}{\alpha_r} + \ln \hat{\beta}_r\right] < 0.
\end{aligned} \tag{28}$$

Moreover,

$$\limsup_{t\to\infty}\left(\frac{\ln\psi(t)}{t}-\frac{\ln\lambda}{t}\right)<0. \tag{29}$$

Therefore, (26), (27), (28), and (29) yield that

$$\limsup_{t\to\infty}\frac{\ln\sum_{i+j=t}\|\eta(i,j)\|}{t}<0,$$

i.e., system (2) achieves GES a.s. □

It should be noted that one of the hardships in implementing the ET mechanism is the existence of Zeno-behavior. Controller (4) can effectively avoid the Zeno phenomenon. The system considered in this paper is discrete-time and the sampling time is dependent on the DT of mode, so there will be no such phenomenon that the control is triggered infinitely in a finite time-interval.

Remark 4. Under our controller design scheme, Theorem 1 does not require that all the subsystems (modes) of the switched system be stable. However, among the only existing research results on ETC for 2-D switched systems in [26], their control scheme requires that each subsystem or mode must be stable (see the condition related to modal stability $\mu > 1$ in Theorem 2 in [26]). In addition, Theorem 1 shows that in order to realize achieves GES a.s., even after the control is added, it is not necessary for all modes to be stable, but only for some modes to be stable. Thus, it can be seen that the sufficient conditions given in Theorem 1 for the 2-D switched system to complete GES a.s. are very general.

Remark 5. Theorem 1 is also valid in the case of arbitrarily ADT and minimum DT with respect to the modes of the switched system. However, the research results in [26] illustrate that the ADT must be greater than a given threshold, seeing the condition $\tau_a > \tau_a^*$ in Theorem 2 in [26], where τ_a is the ADT, and τ_a^* is a predetermined threshold. It can also be shown from this aspect that our method is less conservative and therefore has better application value.

Based on Theorem 1, the control gains of the quantized dynamic output ETC are now designed for system (2). Letting

$$\mathcal{A}_{1r}=\begin{pmatrix}A_{1r} & 0\\ 0 & 0_{n_d\times n_d}\end{pmatrix},\qquad \mathcal{A}_{2r}=\begin{pmatrix}0 & B_{1r}\\ I_{n_d} & 0\end{pmatrix},\qquad \mathcal{A}_{3r}=\begin{pmatrix}0 & I_{n_d}\\ C_r & 0\end{pmatrix},$$

$$\mathcal{B}_{1r}=\begin{pmatrix}A_{2r} & 0\\ 0 & 0_{n_d\times n_d}\end{pmatrix},\qquad \mathcal{B}_{2r}=\begin{pmatrix}0 & B_{2r}\\ I_{n_d} & 0\end{pmatrix},\qquad \mathcal{B}_{3r}=\begin{pmatrix}0 & I_{n_d}\\ C_r & 0\end{pmatrix}$$

and

$$\mathcal{F}_{1r}=\begin{pmatrix}0 & 0_{n_x\times n_u}\\ I_{n_d} & 0\end{pmatrix},\qquad \mathcal{F}_{2r}=\begin{pmatrix}0\\ I_{n_y}\end{pmatrix},$$

$$\mathcal{G}_{1r}=\begin{pmatrix}0 & 0_{n_x\times n_u}\\ I_{n_d} & 0\end{pmatrix},\qquad \mathcal{G}_{2r}=\begin{pmatrix}0\\ I_{n_y}\end{pmatrix},$$

$$\mathcal{C}_{1r} = \begin{pmatrix} 0 & I_{n_u} \end{pmatrix}, \qquad \mathcal{C}_{2r} = \begin{pmatrix} I_{n_d} \\ 0 \end{pmatrix},$$
$$\mathcal{K}_r^1 = \begin{pmatrix} \hat{A}_{1r} & \hat{B}_{1r} \\ \hat{C}_r & 0 \end{pmatrix}, \qquad \mathcal{K}_r^2 = \begin{pmatrix} \hat{A}_{2r} & \hat{B}_{2r} \\ \hat{C}_r & 0 \end{pmatrix},$$

it follows that

$$\mathcal{A}_r = \mathcal{A}_{1r} + \mathcal{A}_{2r}\mathcal{K}_r^1\mathcal{A}_{3r}, \qquad \mathcal{B}_r = \mathcal{B}_{1r} + \mathcal{B}_{2r}\mathcal{K}_r^2\mathcal{B}_{3r},$$
$$\mathcal{F}_r = \mathcal{F}_{1r}\mathcal{K}_r^1\mathcal{F}_{2r}, \qquad \mathcal{G}_r = \mathcal{G}_{1r}\mathcal{K}_r^2\mathcal{G}_{2r}, \qquad \hat{C}_r = \mathcal{C}_{1r}\mathcal{K}_r^1\mathcal{C}_{2r}.$$

Then $\varUpsilon_r > 0$ is equivalent to $\varOmega_r > 0$, denoting

$$\varOmega_r = \begin{pmatrix} \varOmega_{11r} & \varOmega_{12r} \\ \varOmega_{12r}^{\mathrm{T}} & \varOmega_{22r} \end{pmatrix},$$

where

$$\begin{aligned}
\varOmega_{11r} &= \operatorname{diag}\big(Q_r^1 + Q_r^2 - U_r - U_r^{\mathrm{T}},\, \varepsilon_r\varTheta_r - V_r - V_r^{\mathrm{T}},\, \varepsilon_r\varTheta_r - V_r - V_r^{\mathrm{T}}, \\
&\quad - l_r^{-1}I_{n_y},\, -l_r^{-1}I_{n_y},\, -l_rI_{n_y},\, -l_rI_{n_y}\big), \\
\varOmega_{12r} &= \big(E_1^{\mathrm{T}}U_r^{\mathrm{T}},\, (E_4 + C_rE_6)^{\mathrm{T}}V_r^{\mathrm{T}},\, (E_5 + C_rE_7)^{\mathrm{T}}V_r^{\mathrm{T}},\, \big(\mathcal{C}_{1r}\mathcal{K}_r^1\mathcal{C}_{2r}E_{a2}E_2\big)^{\mathrm{T}}, \\
&\quad \big(\mathcal{C}_{1r}\mathcal{K}_r^1\mathcal{C}_{2r}E_{a2}E_3\big)^{\mathrm{T}},\, E_1^{\mathrm{T}}E_{a1}B_{1r},\, E_1^{\mathrm{T}}B_{2r}E_{a1}\big)^{\mathrm{T}}, \\
\varOmega_{22r} &= \operatorname{sym}\big\{E_1^{\mathrm{T}}\mathcal{A}_{1r}E_2 + E_1^{\mathrm{T}}\mathcal{B}_{1r}E_3\big\} - \beta_rE_2^{\mathrm{T}}Q_r^1E_2 - \beta_rE_3^{\mathrm{T}}Q_r^2E_3 \\
&\quad - \varepsilon_rE_4^{\mathrm{T}}\varTheta_rE_4 - \varepsilon_rE_5^{\mathrm{T}}\varTheta_rE_5 + \operatorname{sym}\big\{E_1^{\mathrm{T}}\mathcal{A}_{2r}\mathcal{K}_r^1\mathcal{A}_{3r}E_2 + E_1^{\mathrm{T}}\mathcal{B}_{2r}\mathcal{K}_r^2\mathcal{B}_{3r}E_3 \\
&\quad + E_1^{\mathrm{T}}\mathcal{F}_{1r}\mathcal{K}_r^1E_4 + E_1^{\mathrm{T}}\mathcal{G}_{1r}\mathcal{K}_r^2\mathcal{G}_{2r}E_5\big\}.
\end{aligned}$$

The ET dynamic output quantization controller of system (2) is designed based on the following theorem.

Theorem 2. *Assume that Assumptions* 1–2 *are satisfied. For pregiven constants* $\mu_r \geqslant 1$, $\beta_r > 0$, *if there exist matrices* $0 < Q_r^1 \in \mathbb{R}^{(n_x+n_d)\times(n_x+n_d)}$, $0 < Q_r^2 \in \mathbb{R}^{(n_x+n_d)\times(n_x+n_d)}$, $0 < \varTheta_r \in \mathbb{R}^{n_y\times n_y}$ *and invertible matrices* $U_r \in \mathbb{R}^{(n_x+n_d)\times(n_x+n_d)}$, $V_r \in \mathbb{R}^{n_y\times n_y}$ *such that, for* $r \in \mathcal{R}$, (9) *and* (10) *are satisfied, and the following LMIs are satisfied*:

$$\varOmega_r < 0. \tag{30}$$

Then, under controller (4)*, system* (2) *completes GES a.s. In addition, the control gains* $\mathcal{K}_r^1$, $\mathcal{K}_r^2$ *in system* (4) *and the triggering parameters* $\varTheta_r$ *in* (3) *can be obtained directly by solving LMIs* (30).

Proof. Since $\varUpsilon_r > 0$ in Theorem 1 is equivalent to $\varOmega_r > 0$, the proof processe is the same as that of Theorem 1. □

4 Numerical examples

A numerical example with respect to discrete-time switched FMLSS system will be provided to analyze the feasibility and validity of our main theories later. Consider system (1) with three modes and take the following system parameter values:

Subsystem (1_1):

$$A_{11} = \begin{pmatrix} 0.43 & 0.2 \\ 0.2 & 0.2 \end{pmatrix}, \qquad A_{21} = \begin{pmatrix} 0.2 & 0.3 \\ 0.1 & 0.3 \end{pmatrix}, \qquad C_1 = \begin{pmatrix} 0.1 & 0.1 \end{pmatrix};$$

Subsystem (1_2):

$$A_{12} = \begin{pmatrix} 0.1 & 0.5 \\ 0.1 & 0.3 \end{pmatrix}, \qquad A_{22} = \begin{pmatrix} 0.25 & 0.1 \\ 0.01 & 0.1 \end{pmatrix}, \qquad C_2 = \begin{pmatrix} 0.1 & 0.1 \end{pmatrix};$$

Subsystem (1_3):

$$A_{13} = \begin{pmatrix} 1.6 & 0.1 \\ 0.1 & 1.2 \end{pmatrix}, \qquad A_{23} = \begin{pmatrix} 2.3 & 0.1 \\ 0.1 & 1.5 \end{pmatrix}, \qquad C_3 = \begin{pmatrix} 0.3 & 0.1 \end{pmatrix}.$$

Under Assumption 1, the switching signal $\mathfrak{d}(t) = \mathfrak{d}(i,j)$, $i,j \in \mathbb{Z}^+$, are mutually independent random variables, and choose the TP matrix as

$$P = \begin{pmatrix} 0 & 0.6 & 0.4 \\ 0.5 & 0 & 0.5 \\ 0.7 & 0.3 & 0 \end{pmatrix}.$$

Taking $\alpha_1 = 3.25$, $\alpha_2 = 3.5$, $\alpha_3 = 2$, the unique stationary distribution of P can be calculated as $\kappa = (0.37, 0.32, 0.31)$.

Taking initial conditions

$$x_1(0,j) = \begin{cases} 0.1 & \text{if } j \leqslant 20, \\ 0 & \text{if } j > 20, \end{cases} \qquad x_2(0,j) = \begin{cases} 0.2 + 0.2\sin(j) & \text{if } j \leqslant 20, \\ 0 & \text{if } j > 20, \end{cases}$$
$$x_1(i,0) = \begin{cases} 0.1 & \text{if } i \leqslant 20, \\ 0 & \text{if } i > 20, \end{cases} \qquad x_2(i,0) = \begin{cases} 0.2 + 0.2\sin(j) & \text{if } i \leqslant 20, \\ 0 & \text{if } i > 20, \end{cases}$$

we get the trajectory of system (1) shown in Fig. 1, from which one can see that system (1) without control is unstable. Corresponding switching signal is presented in Fig. 2. Figure 2 shows three modes 1, 2, and 3 of system (1), which correspond to subsystems (1_1), (1_2), and (1_3), respectively. Hence, according to Remark 4, the ETC for 2-D switched systems in [26] cannot be used for our numerical example.

Next, the control gains in dynamic output quantized ETC (4) are designed. To realize this goal, assume that, for system (2),

$$B_{11} = \begin{pmatrix} 0.4 \\ 0.2 \end{pmatrix}, \qquad B_{21} = \begin{pmatrix} 0.5 \\ 0.1 \end{pmatrix}, \qquad B_{12} = \begin{pmatrix} 2 \\ 0 \end{pmatrix},$$
$$B_{22} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \qquad B_{13} = \begin{pmatrix} 0.3 \\ 0.1 \end{pmatrix}, \qquad B_{23} = \begin{pmatrix} 0.2 \\ 0.2 \end{pmatrix},$$

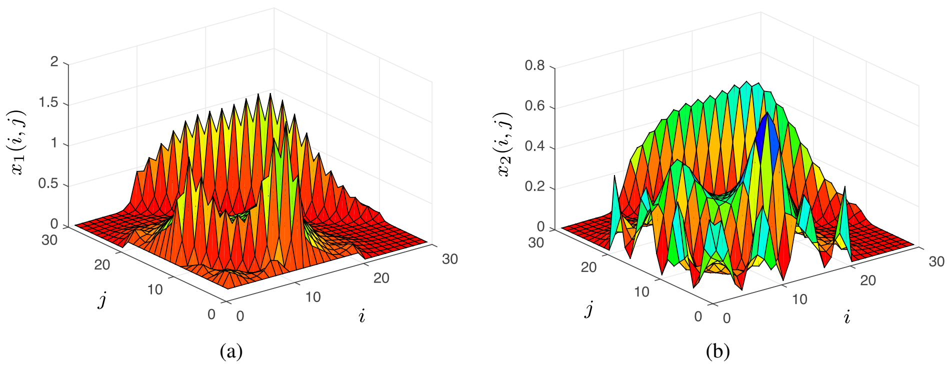


Figure 1. Trajectories for system (1) with the switched signal $\mathfrak{d}(t)$ that is indicated in Fig. 2, where $0 \leqslant i \leqslant 30$, $0 \leqslant j \leqslant 30$: (a) $x_1(i,j)$; (b) $x_2(i,j)$.

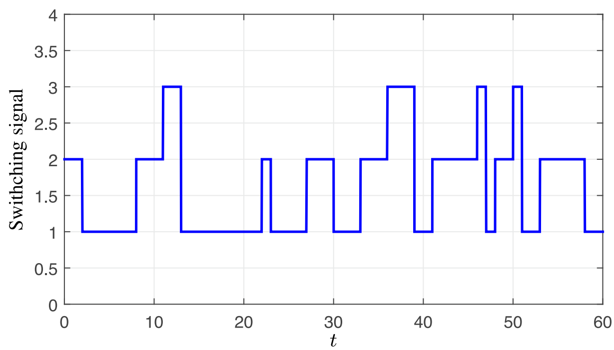


Figure 2. Switching signal $\mathfrak{d}(t)$ for system (1), where $t = i + j$.

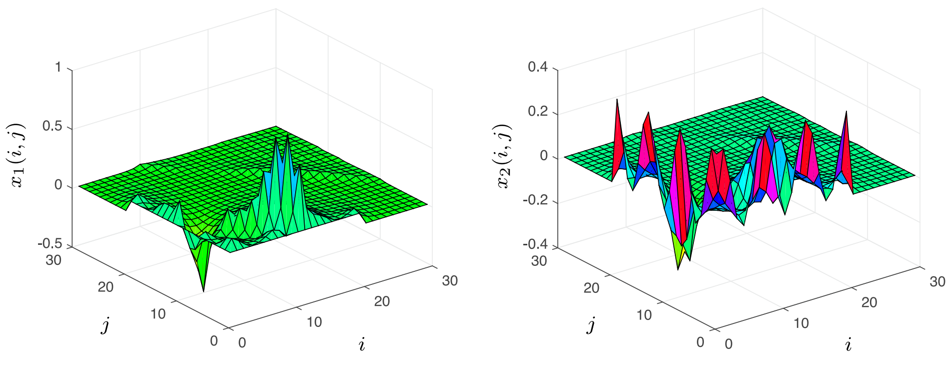
and take $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.8$, $\alpha = 1.5$, $\beta = 0.5$, $\lambda = 5$. By solving the feasible solution for LMIs (30) in Theorem 2, it follows that the control gains in system (4) are

$$\mathcal{K}_1^1 = \left(\begin{array}{c|c} \hat{A}_{11} & \hat{B}_{11} \\ \hline \hat{C}_1 & 0 \end{array}\right) = 10^{-5} \left(\begin{array}{cc|c} -4.5 & -0.047 & 0.41 \\ -5.65 & -47.59 & -3.62 \\ \hline -2.32 & -4.97 & 0 \end{array}\right),$$

$$\mathcal{K}_1^2 = \left(\begin{array}{c|c} \hat{A}_{21} & \hat{B}_{21} \\ \hline \hat{C}_1 & 0 \end{array}\right) = 10^{-5} \left(\begin{array}{cc|c} -0.66 & 1.59 & -0.46 \\ 4.97 & 2.80 & -3.60 \\ \hline -2.32 & -4.97 & 0 \end{array}\right),$$

$$\mathcal{K}_2^1 = \left(\begin{array}{c|c} \hat{A}_{12} & \hat{B}_{12} \\ \hline \hat{C}_2 & 0 \end{array}\right) = 10^{-4} \left(\begin{array}{cc|c} -5.30 & -2.99 & 2.10 \\ 15 & -1.75 & 2.11 \\ \hline 3.55 & -2.77 & 0 \end{array}\right),$$

$$\mathcal{K}_2^2 = \left(\begin{array}{c|c} \hat{A}_{22} & \hat{B}_{22} \\ \hline \hat{C}_2 & 0 \end{array}\right) = 10^{-4} \left(\begin{array}{cc|c} 6.14 & -0.08 & 0.31 \\ -8.25 & 4.18 & 0.34 \\ \hline 3.55 & -2.77 & 0 \end{array}\right),$$



(a) (b)

Figure 3. Trajectories for system (2) with dynamic output quantized ETC (4) and the switching signal $\mathfrak{d}(t)$ shown in Fig. 2: (a) $x_1(i,j)$; (b) $x_2(i,j)$.

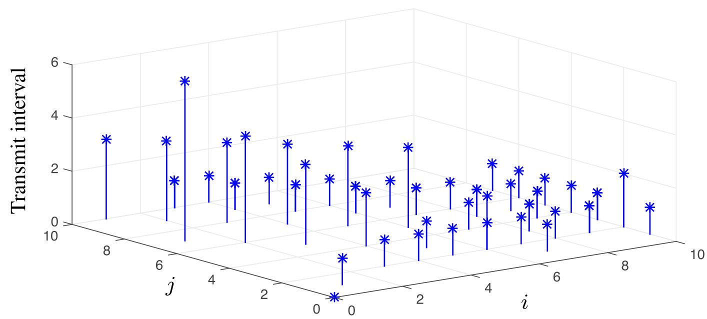


Figure 4. Evolution of the mode-dependent ET mechanism with respect to condition (3).

$$\mathcal{K}_3^1 = \left(\begin{array}{c|c} \hat{A}_{13} & \hat{B}_{13} \\ \hat{C}_3 & 0 \end{array}\right) = 10^{-4} \left(\begin{array}{cc|c} -26.2 & 41.1 & -0.83 \\ -10.6 & -13.7 & 0.93 \\ \hline -0.08 & 3.91 & 0 \end{array}\right),$$

$$\mathcal{K}_3^2 = \left(\begin{array}{c|c} \hat{A}_{23} & \hat{B}_{23} \\ \hat{C}_3 & 0 \end{array}\right) = 10^{-5} \left(\begin{array}{cc|c} 8.12 & -4.36 & 9.06 \\ 1.52 & 2.79 & -1.59 \\ \hline -0.83 & 39.14 & 0 \end{array}\right),$$

and the triggering parameters in (3) are

$$\varTheta_1 = 0.452, \qquad \varTheta_2 = 0.619, \qquad \varTheta_3 = 0.511.$$

In order to show the effectiveness of our ET dynamic output quantization controller, take the same switched signals as that in Fig. 2 and let the initial conditions with respect to the controller system (4) be $\hat{x}(0,j) = x(0,j)$, $\hat{x}(i,0) = x(i,0)$. Then Fig. 3 indicates that system (2) is stabilized under our designed controller. Moreover, Fig. 4 indicates the corresponding evolution of the ET mechanism.

5 Conclusions

The GES a.s. of 2-D switched systems are investigated by designing mode-dependent ET dynamic output quantized controller. Note that 2-D systems are special dynamic systems that depend on two independent variables, which makes their dynamic behavior analysis much more difficult than 1-D systems. Especially, for 2-D control systems that further involves switching, the related stability and stabilization research results are extremely rarely. Now, the studies on ETC schemes are fewer used for the studies of 2-D switched discrete-time systems. Therefore, the mode-dependent ETC schemes that incorporate other novel technologies, including quantization control, output control, and so on, are extremely difficult because they lacks sufficient early study results that can be used for reference. Nevertheless, this paper has given general sufficient conditions to ensure the GES a.s. of 2-D switched systems under the action of the designed controller. The sufficient conditions of LMIs are obtained, and the restriction that all modes must be stable is abandoned. Finally, the theoretical results are verified numerically. In terms of numerical value, it should be noted that the constraints associated with mode switching and mode stability in Theorem 2 are $\mu_r \geqslant 1$ and $\beta_r > 0$, $r \in \mathcal{R}$, respectively. However, in most previous researches, their main conclusions require $\mu > 1$, $\mu = \max\{\mu_r, r \in \mathcal{R}\}$ and $0 < \beta < 1$, $\beta = \max\{\beta_r, r \in \mathcal{R}\}$. Obviously, our condition is more general, which makes the feasible solution domain for numerical LMI solution larger, and thus the LMI in Theorem 2 is easier to solve. In addition, (30) in Theorem 2 can be directly obtained with LMI in MATLAB software, and then all control gains can be obtained. Although the mode-dependent ETC mechanism is a newly emerging method, it has been applied in some fields such as T-S fuzzy control system [2] due to its advantages in saving network bandwidth.

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