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Single-cavity dual-comb fiber lasers and their applications

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Single-cavity, dual-comb lasers are those specially designed mode-locked lasers that can emit more than one, asynchronous ultrashort pulse trains with stable repetition frequency difference between them. Unlike the longstudied, widely-used femtosecond lasers generating one stable pulse train, systematic investigation on them and their potential dual-comb applications only began, based on the fiber laser platform, around a decade ago, despite sporadic and limited reports of similar lasing phenomena since the beginning of the mode-locked laser studies. From a historic perspective, the birth of this novel technology is the lucky outcome of the timely collision of perpetual search for novel pulsing laser dynamics and concerted pursuit of open-minded solutions for out-of-lab dual-comb systems in the 2010s. In this review article, first, the current schemes to implement single-cavity dual optical frequency comb fiber lasers and their applications are summarized, based on the concept of multiplexed mode-locked lasers. The characteristics of reported singlecavity, dual-comb fiber lasers are discussed as well as their applications in spectroscopy, ranging, Terahertz (THz) spectroscopy, and asynchronous optical sampling (ASOPS). Finally, the more recent development of singlecavity, multi-comb lasers is presented.

KEYWORDS

single-cavity dual-comb, optical frequency comb, dual-comb, fiber laser, femtosecond laser

1 Introduction

Optical frequency comb (OFC) has discrete and equally spaced 'comb teeth' in the optical frequency domain, while their tooth spacing falls into the microwave frequency range. Realized by femtosecond pulsed lasers with octave spanning spectra and precisely controlled carrier envelope offset frequency (f_{ceo}), the frequency stabilized OFC technology [1, 2] has found applications in many areas of high precision measurement. Dual optical frequency comb scheme was proposed to further simplify the system configuration for broadband measurement while retaining the advantages of OFC [3]. It uses two optical frequency combs with slightly different mode spacing. The coherent beating frequencies from the two combs are well within the radio-frequency range and can be easily picked up and processed by the existing electronic system. Thus, fast, large bandwidth and high resolution measurement can be realized with low-bandwidth electronics. Therefore, it was applied to time/frequency transferring [4], time synchronization [5], optical spectroscopy [6–8] and absolute-distance ranging

[9]. It should be noted that, as the technology gained greater popularity, the concept of OFC had evolved into a broader one by covering other types of light sources, including microcavities and modulated continuous wave lasers, with sometimes limited bandwidth and less regulated optical frequency components. Also, along with the drastically growing interests in dualcomb techniques over the past decade, the focus of many studies had been shifted towards their applicability to the inthe-field applications and those that are less demanding in performance but more in system complexity. In spite of its great success in laboratory demonstrations, the complex and expensive phase-locked OFC laser systems, essential for the dualcomb techniques, has become a major obstacle for real-world applications. How to reduce the complexity of the dual-comb source while maintaining the mutual coherence of the dual combs had become one of the important issues for further development of the dual-comb technology [10].

Fiber mode-locked lasers are favored by many OFC systems due to its simple structure and low cost. In recent years, singlecavity dual-comb fiber laser technology that generates two OFC pulse trains at the same time has become a new type of lowcomplexity dual-comb light source. It could drastically reduce the complexity of a dual-comb source to that of a single fiber modelocked laser, and thus has become a hotly investigated research topic.

2 Single-cavity dual-comb technology

The development of the optical frequency comb has been closely correlated with the evolution of ultrafast mode-locked laser technologies. During its early days in 1990s, the OFC source had been mostly built based on the then-already-mature solidstate lasers, such as Ti: sapphire lasers [2]. However, in the ensuing two decades, ultrafast fiber lasers have been growing at a fast pace with a number of innovations with the aim of improving mode-locking stability, pulse energy and pulse width/spectral bandwidth. They'd been quickly adopted as the mainstream platform for OFC generation with their OFC noise performance rivaling that from other lasers, because of their additional advantages in the system compactness, ruggedness, and optical-fiber-compatible spectral window. In the first decade into the 21st century, most dual-comb demonstration had been carried out by fiber-laser-based, frequency-stabilized OFC systems.

Unlike their solid-state counterparts that was once plagued by the unwanted bi-directional oscillation issue in their early stages of development [11], it had been much easier to control the light path for pulses in the passively mode-locked fiber lasers, thanks to the availability of a plethora of single-mode, fiber-optic components like isolators. In the 90s, the research community in general had its eyes set on bringing the output power, bandwidth and other characteristics of these lasers on par with those of others. Except for scant reports of peculiar dual-wavelength lasing observation [12-15], little attention had been given to the multi-pulse lasing in fiber lasers. A few studies with revived interests on bi-directional mode-locking in solid-state lasers had been carried out, targeting the laser gyroscope applications [16-18]. Additional attention had been diverted to more complex pulsing phenomena in the 00s, as the fiber laser technology matured and its studies became more widely spread. There had been several reports of picosecond, dualwavelength pulse generation in the 1,560 nm spectral window from passively mode-locked Erbium fiber lasers [19-21]. It was later observed in 2009 that these pulses could have different periods due to dispersion from a normal dispersion laser [22]. In 2008, it was also demonstrated that bidirectional mode-locking can be realized in a ring fiber laser by removing the isolator and repetition rate difference on the order of tens of Hz was also observed [23].

Coincidentally, demonstrations of the dual-comb techniques, first proposed in 2002 [3], had been achieved for several important applications around the same period of time [6, 7, 9, 24]. Built on the momentum and following the same technical path of the hugely successful fully-frequency-stabilized OFC technology, they quickly garnered great attention as one of the most promising OFC techniques. Nevertheless, mutual coherence of two combs, instead of frequency stabilities of each comb, is now the most critical factor affecting the dualcomb performance, and this is readily ensured for the systems based on the traditional OFC sources. This minute yet important change could spur the dual-comb source technologies to a major shift in paradigm in realizing dual-comb light sources, in order to address growing concerns about the complexity challenges faced by dual-comb systems. Several alternative approaches, such as post compensation by digital processing [25] and electro-optic frequency combs [26], began to emerge.

Dual-wavelength generation of subpicosecond pulses with a stable repetition rate difference from an Erbium fiber laser was realized by tuning the spectral profile of the gain fiber and utilizing both the 1,530 and 1,560 nm windows in 2011 [27]. Realizing the potential to replace the sophisticated dual-comb sources at the time with such a simple free-running laser, the aforementioned laser was soon applied to demonstrations of asynchronous optical sampling (ASOPS) [28] and coherentdetection dual-comb ranging [29]. A universal approach to achieve stable asynchronous ultrashort pulse was proposed by introducing the concept of "multiplexing" into the field of ultrafast lasers [30]. In such a Multiplexed Mode-Locked Laser (M²L²), the cavity is designed to allow the simultaneous oscillation of pulses with different characteristics in one of the propagation dimensions. In the following years, numerous single-cavity dual-comb lasers and dual-comb systems based on them have been achieved, and so far they can still be categorized into the following four categories: wavelength-



multiplexed, polarization-multiplexed [31], directionmultiplexed, and pulseshape-multiplexed [32] lasers corresponding to the four dimensions of light propagation, just as envisioned by the concept of M^2L^2 [30].

On the other hand, as the emergence of single-cavity dual-comb laser opens up a new possibility for realizing lowcomplexity dual-comb systems, it is still necessary to verify the applicability of such simple light source to dual-comb applications, due to their significantly different characteristics to those of standard comb sources. Along with the continuous improvement of the dual-comb fiber lasers over recent years, successful demonstrations in many important dual-comb applications have been carried out, as shown in Figure 1.

2.1 Single-cavity dual-comb fiber lasers

Multiplexing principle of single-cavity dual-comb lasers as shown in Figure 2. Simultaneous lasing at multiple wavelengths in the laser gain spectrum is often realized by introducing spectral filtering [27–29, 33–62]. This scheme, firstly applied to many continuous-wave lasers, has a long



history, but little attention had been paid to the characteristics of the repetition frequencies of the output pulses [12, 13]. There had been conflicting reports of asynchronous or synchronous pulsing phenomena. On the other hand, when splitting gain spectrum into more than one sub-windows, the resulting mode-locked spectral bandwidth is often severely limited, and the long pulsewidth would render the output less useful for most dual-comb applications [19–21, 63]. The

ability of controlling the gain spectrum shape and bandwidth [27] enables the generation of femtosecond pulses in dualwavelength mode-locked fiber lasers. Due to the difference in cavity group velocity dispersion (GVD) at the wavelengths of the mode-locked pulses, the pulses would oscillate in the laser cavity asynchronously with slightly different repetition rate [22, 27]. The repetition frequency difference can then be designed by changing the intracavity dispersion. Various optical filtering schemes based on Lyot filtering [28, 29, 33-44, 46, 48, 56, 59, 60], Sagnac filtering [45, 47, 49, 50, 55], multimode interference [51, 52], spatial light filtering [53, 54, 62, 64] had been later applied to realize single-cavity dualwavelength mode-locking. Studies on the self-starting process of the wavelength-multiplexed laser source [41, 51], their environmental stability [41, 46, 49, 54], tunability in the repetition frequency difference [47, 48] and pulse dynamics [57, 58, 61] has further carried out.

As to the spatial multiplexing scheme, for the fiber lasers constructed of single-mode fibers, propagation direction in the cavity would be the only possible variable to differentiate different pulses in this physical dimension. In the laser cavity based on the free-space optical path, the phenomenon of bidirectional simultaneous oscillation of pulses was found in early laser research [11, 65]. In order to suppress phenomena such as spatial hole burning, optical isolators had been routinely used to achieve unidirectional operation of laser in the cavity. An all-fiber bidirectional mode-locked laser based on carbon nanotube saturable absorber was realized by removing the intracavity optical isolator [23]. Later on, simultaneous modelocking in the clockwise and counterclockwise directions in the ring cavity has become the common implementation of the bidirectional lasing [23, 30, 66-84]. Furthermore, in order to avoid the effects of competition between bidirectional pulses, there are also a variety of bidirectional fiber laser structures that share only part of the optical cavity [79, 83-86]. Unlike the wavelengthmultiplexed cases, it is thought that the difference in repetition rates between the two pulse trains likely originates from direction-dependent birefringence, non-linear phase shift, and the GVD at different lasing wavelengths [78]. In a study on a bidirectional, dual-wavelength mode-locked laser, it is revealed that the seemingly different types of laser implementations, like the bidirectional or multi-wavelength ones, could be regarded as examples in different transmission dimensions of a family of M²L². Furthermore, the idea of "multiplexing" was introduced into the design of a mode-locked laser cavity in order to realize asynchronous dual-comb generation [30]. The possibility of these multiplexed mode-locked laser as a dual-comb laser source was laid out as well [30]. Dual-comb spectroscopy measurements based on a bidirectional dual-comb laser were demonstrated [66], and further studies on directional multiplexed lasers in the 2 µm wavelength [67, 73], generation regimes of mode-locked [77, 82] and with high output power [72] had been carried out. Other designs adopting a partially sharedcavity structure to improve the self-starting and polarizationmaintaining properties of the laser had also been explored [69, 70, 74, 87, 88], sometimes at the expense of mutual stabilities of the dual combs.

Polarization-multiplexed lasers are to generate two pulse trains with orthogonal polarization states from one laser cavity [31, 89-97]. Conventional fiber mode-locked laser structures are either composed entirely of single-mode fibers or polarization-maintaining fibers. Concerned about the bandwidth limiting effect that may be introduced by birefringence, the former hopes to keep the birefringence in the optical cavity as low as possible, while the latter is designed to maintain a single polarization state operation. After the concept of M²L² was proposed, it is natural to look beyond the above two physical dimensions, and the scheme to realize polarization multiplexed fiber lasers was investigated. By introducing a short section of birefringent fiber into the cavity to break the degenerate transmission of polarization states in the cavity, it is demonstrated that two vector soliton pulse sequences with different repetition rates and orthogonal polarizations can be emitted stably [31]. The spectra of polarization-multiplexed mode-locked lasers overlap with each other fairly well, and can be directly applied to dual-comb measurements without non-linear spreading [89, 91]. Based on this concept, various studies, like dual-comb lasing utilizing residual birefringence in FP fiber cavities [98], polarization-multiplexed dissipative soliton generation [90], spectroscopy measurement applications using polarization-multiplexed fiber laser [92] had been carried out. Polarization-maintaining, polarization multiplexing schemes had been also developed based on partially shared-cavity structures [94, 99, 100].

Multiplexing based on temporal pulseshapes seems to be an even more daunting task. In order to enable pulses with distinct temporal profiles to mode-lock in the same cavity, unlike conventional lasers, multiple mechanisms that can contribute to the pulse shaping process need to be introduced. In a cavity with a physical saturable absorber, both polarization-dependent loss and birefringence can be introduced by using a polarizer with a polarization-maintaining fiber pigtail. In this way, the birefringence and polarization-dependent loss produce the Lyot filtering effect, which has a narrowing effect on the pulse bandwidth; on the other hand, the polarizer and the non-linear effect in the cavity can form the non-linear polarization rotation effect under sufficient pulse energy. It, combined with the physical mode-locking device, as hybrid mode-locking mechanism, can expand the pulse spectrum. Thereby, a pulseenergy-dependent, pulse shape/spectrum control scheme can be achieved. For the pulse with low pulse energy in the cavity, the linear filtering plays a more significant role, and the mode-locked spectrum is narrow. On the other hand, for the pulse with a sufficiently large pulse energy, the non-linear effect is more prominent, which significantly broadened its spectrum. The laser realized the simultaneous generation of ultrashort pulses



with different energies and time-domain characteristics. Two output pulses have overlapped spectra, albeit with very different bandwidths, and slightly different repetition frequencies due to a small center wavelength offset. The beat signal of the dual-comb also proves that the two pulses have good mutual coherence [32].

Repetition frequency difference (Δf) and repetition frequency (f) are the keys in dual-comb application. Δf affects the sampling rate, and f/ Δf is the scaling factor in the dual-comb application [10]. Figure 3 shows the relationship between f/ Δf and Δf for various types of sofar-reported single-cavity dual-comb lasers with a completely shared cavity. Δf is determined by the cavity dispersion, either chromatic or polarization-dependent, based on the types of dual-comb lasers, and varying the cavity parameters, such as GVD, birefringence can, thus, change Δf of the laser output. Due to the relatively large GVD or birefringence one can apply to a cavity, the wavelength-multiplexed and polarization-multiplexed lasers can realize relatively large Δf , which is beneficial for those measurement applications requiring a higher update rate. On the other hand, Δf of the other two types of lasers are generally much lower, which gives them certain advantages in realizing larger $f/\Delta f$ factors. A larger $f/\Delta f$ factor, i.e. a larger down-conversion ratio could be beneficial for those applications with a limited radio frequency (RF) detection bandwidth while targeting a broader frequency range, like THz spectroscopy [38, 39]. For partially-shared cavities, on the other hand, tuning Δf can be done by directly adjusting the lengths of corresponding light paths.

The pulse energy is the another key parameter in many dualcomb applications. Figure 4 summarizes the reported performance of dual-comb fiber lasers from that perspective. It can be seen that the average output power for most of the single-cavity dual-comb lasers falls around 1 mW, with one exception of a normal-dispersion, directional-multiplexed with an about 70 mW output. This is mainly due to the limitation posed by the intracavity pulse interactions, and could be ameliorated by extra-cavity optical amplification at moderate costs of system complexity. Since this power level falls short of the requirements of many applications, there had been quite a few demonstrations of amplified dual-comb sources for optical spectroscopy [34, 35, 37] and THz spectroscopy [38, 39, 42] with sufficient mutual coherence and stability, despite the expected impact from the non-linear and spontaneous emission noise from the optical amplification scheme.

It is also noted that, since the demonstrations of the proposed single-cavity, dual-comb fiber lasers and their dual-comb applications, dual-comb lasers based on other laser platforms had also emerged. Bi-directional Ti: sapphire lasers [101–103], spatially-multiplexed thin-disk laser [104], polarization-multiplexed thin-disk lasers [105–110], polarization-





multiplexed solid lasers [111–116] and even bi-directionalpumped microresonator [117] had been investigated. Compared to fiber lasers, some of them can achieve higher repetition frequency or higher output power, and they help to broaden the spectrum and scope of dual-comb laser technology.

2.2 Applications of single-cavity dualcomb lasers

The application schemes of single-cavity dual-comb lasers are shown in Figure 5, and can be roughly divided into two types: timedomain and frequency-domain measurements. The time-domain measurement includes ASOPS and ranging, and the frequencydomain measurement includes THz spectrum and optical spectrum measurements.

ASOPS pump-probe application was the first demonstration realized by a single-cavity dual-comb laser source for dual-comb applications. In 2012, the dual-wavelength dual-comb laser was used to demonstrate the measurement of the carrier dynamics of a semiconductor optical amplifier (SOA) with over 10 nanoseconds scanning range, sub-picosecond time domain sampling step, and hundreds of Hz update speed [28]. It was realized with a very simple experimental setup without the mechanical delay line. Pump-probe experiments were also performed using a bidirectionally mode-locked laser [118]. A polarization-multiplexed solid-state laser was used to measure the time constants in the SESAM dynamic model, which were consistent with the classical model [119].

Ranging, as another typical time-domain measurement, was soon preliminarily demonstrated sequentially using wavelength-multiplexed and polarization-multiplexed dual-comb lasers to achieve coherent distance measurement [29, 89]. Low-power dual-comb distance measurement based on incoherent reception was also realized [33, 120]. With a dual-wavelength dual-comb fiber laser with improved performance, ranging accuracy over a distance range of 70 m was determined to be on the order of μ m in a measurement time of 1 s [44]. Several related improved techniques [55, 110, 121] had been developed.

High-quality dual-comb spectroscopy measurement using a free-running laser was a more challenging task considering the various factors affecting the quality of the spectroscopic result. While the previous studies had obtained temporal interferogram patterns indicating likely promising mutual coherence between pulses from a dual-comb laser, goodquality spectroscopy results could not be obtained until a dual-comb fiber laser with sufficient passive mutual stability performance was developed. After several years of incessant improvement on the fiber laser setup, a dual-wavelength fiber laser achieved about 250 Hz mutual linewidth between the dual combs, lower than the repetition frequency difference of 1,250 Hz, and mHz fluctuations in the repetition rate under free-running [34]. difference Through the measurement of acetylene gas cell and high-Q micro-ring resonators, а picometer spectral resolution was demonstrated, with well-matched spectroscopic results with the known database. Spectroscopic measurement of hydrogen

cyanide gas was also carried out using a bi-directional fiber laser [66, 101]. Later on, doppler-limited hydrogen cyanide absorption spectra were measured at low pressures by polarization-multiplexed dual-comb laser [92]. These demonstrations verified the applicability of free-running single-cavity dual-comb lasers to spectroscopy applications with modest resolution requirements. Though fceo's of the two combs are not actively controlled, passive coherence illustrated by the good-quality spectroscopy results suggests the existence of correlated $f_{\rm ceo}$ variations, in contrast to the random drifts between them if two independent lasers are used. Since then, the spectral measurement technology based on single-cavity dual-comb technology in the 1 µm [107, 108], 1.6 μ m [35], and 2 μ m [45, 67] bands have also attracted the attention of many researchers. In addition, the advantages of the low complexity of the single-cavity dual-comb laser source system have also render it more useful to new applications stressing the low-cost or on-site requirements, such as multicomponent gas concentrations measurement in combustion [36] and fiber grating sensing system [37, 40, 68, 93].

Different from the spectroscopy measurement technology in the optical frequency band, the carrier-envelope offset frequency of the optical comb will not be converted to the terahertz or RF frequency band in the spectral measurement at those frequencies. Therefore, the requirement for the frequency stability of the carrier-envelope offset of the dual-comb light source is more relaxed. THz frequency measurement scheme based on the dual-wavelength laser source [38] and the THz time-domain spectroscopy measurement using the wavelength-multiplexing [42] and direction multiplexing [76] dual-comb lasers have achieved GHz spectral resolution. Further, an adaptive-sampling dual-THz comb spectral measurement method based on a dualwavelength laser greatly expands the averaging time window [39]. The absorption peaks of the methane cyanide gas in the vibrationally excited state in the low-pressure methane cyanide and nitrogen gas mixture can be distinguished, thanks to the MHz-level ultrahigh THz spectral resolution. Recently, the application of coherent anti-Stokes Raman scattering spectroscopy and microscopy with a single cavity dual optical frequency comb was reported [71, 75].

3 Single-cavity multi-comb fiber lasers and their applications

If additional combs can be introduced to a dual-comb system, such a tri-comb or multi-comb system could further expand the capability for certain applications [122]. However, a tri-comb system based on three frequency-stabilized optical frequency combs would be even more complex and expensive. Therefore, single-cavity multi-comb technology could be a more attractive alternative. However, from the perspective of

multiplexing schemes, it is challenging to realize the generation of triple-comb or multi-comb in other physical dimensions, except for wavelength. Wavelength -multiplexed laser would also be limited by the finite accessible spectrum width, which strongly limits the performance of multi-comb signals. The concept of Multi-dimensional Multiplexed Mode-Locked Laser (M³L²) was proposed [123]. Multiple combinations in the wavelength- and polarization-dimensions in the optical cavity were leveraged to realize a single-cavity, three-comb or even multi-comb light source [123]. On the application side, dead-zone-free RF frequency measurement was demonstrated with a single-cavity tri-comb laser with wavelength multiplexing [124]. A real-time ranging scheme based on a multi-wavelength tri-comb laser demonstrated an extended ambiguity range to the order of tens of kilometers [125]. A dual-asynchronous sampling scheme based on wavelength-multiplexed tri-comb source has realized the demodulation of fast-varying spectral modulations [126]. It should be noted that these demonstrations had been carried out with marginal increase in system complexity, due to the application of multi-comb fiber lasers.

4 Conclusion

We have reviewed the history and summarized the recent progress of single-cavity dual-comb fiber lasers and their applications. While hardly-known or recognized over a decade ago, single-cavity, dual-comb fiber lasers have become an attractive topic for researchers around the world. The essence of this technology is removing the "smart" feedback control system and replacing it with a properly designed, "dumb" yet self-stabilized physical system based on a good understanding of laser physics. This renders the unique advantages of the technology in system complexity, power consumption, cost, and compactness, which make it an alternative over the more universal, powerful yet more complicated frequency-stabilized OFC solutions. Such lasers could also play an important role in broadening the reach of the dual-comb techniques by unlocking those application areas that are highly sensitive to system complexity or size. On the other hand, it is not unexpected that, as such lasers are based on somewhat unconventional laser designs, there could be some performance limitations like the adjustability and stability of the repetition frequency difference, the pulse energy as well as intracavity phenomena unique to these lasers. While new mode-locked laser technologies based on more sophisticated ultrafast pulse dynamics or new functional devices keep on emerging, the dual-comb lasers, by far, mostly are still based on the traditional cavity and conventional soliton configurations. With further investigations into the dual-comb dynamics in the cavity and better understanding of ways to control and manipulate them, it is expected that more innovative solutions that further overcome the current performance

limitations would be found in the near future. It is foreseeable that more would delve into further investigating or solving these issues.

Author contributions

JY: wrote the initial manuscript, made graph, and revised the second drafts. XZ: obtained funding, designed and reviewed the manuscript. LZ: contributed sections of the initial drafts. ZZ: critically read, discussed, and edited the content prior to submission. All authors read and approved the manuscript.

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