

# Imagining Circles: A perceptual model for the Arc-Size Illusion

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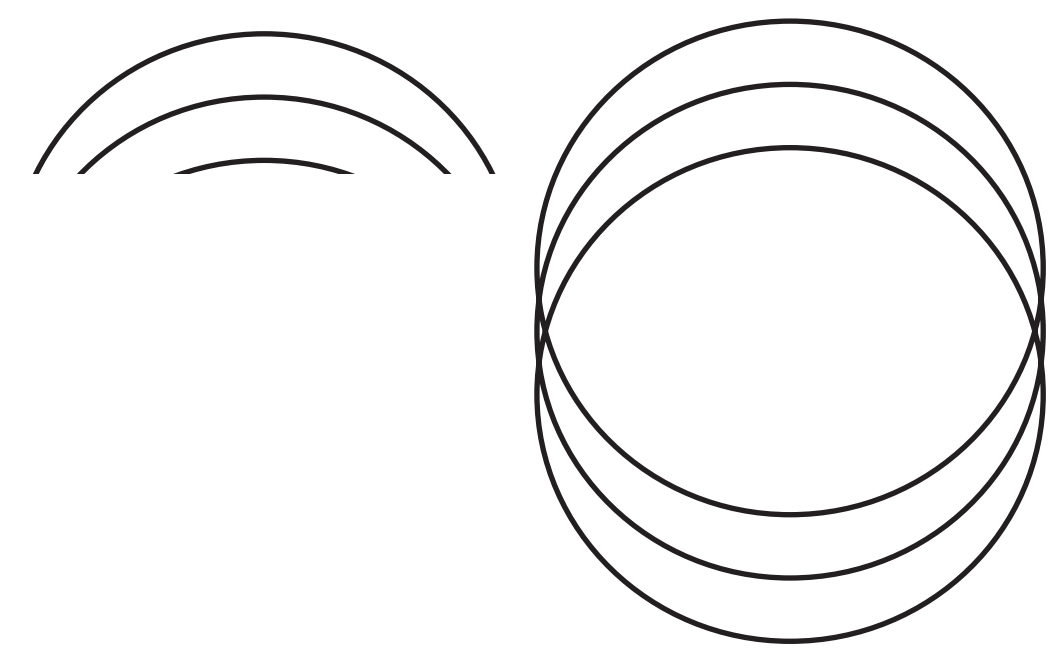
## BACKGROUND

Previous studies suggest a predominant role for co-circularity in:

- Contour detection (Achtman et al., 2003)
- Glass pattern detection (Wilson & Wilkinson, 1998; Wilson et al., 1997)
- Texture detection (Motoyoshi & Kingdom, 2010)

Co-circularity is an important feature in natural scenes (e.g. Elder & Goldberg, 2002) and points of maximum curvature are important for object recognition (e.g. Attneave, 1954; Biederman, 1987) and shape discrimination (Loffler et al., 2003).

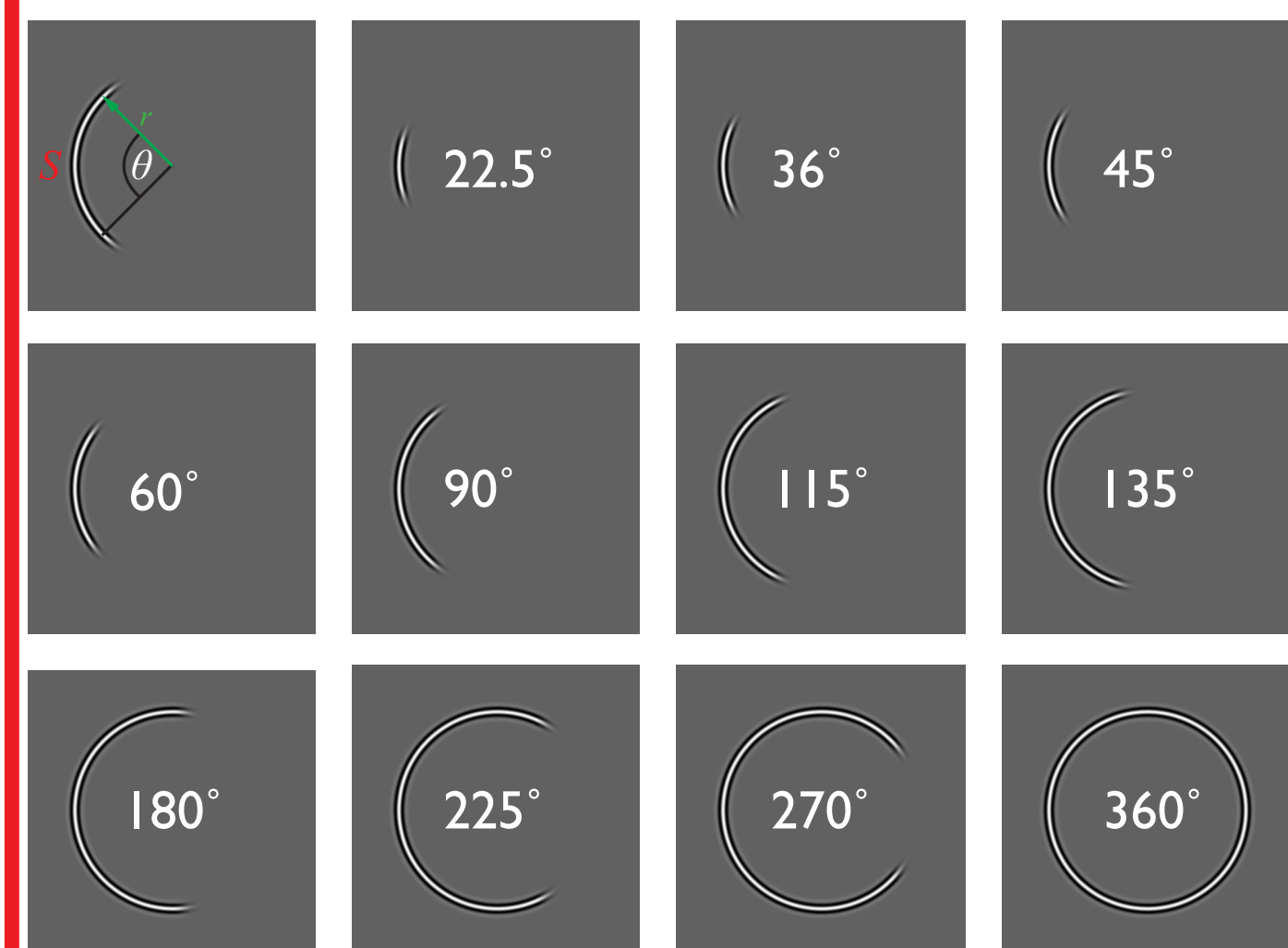
Short arcs are perceived as flatter than long arcs of the same radius, a phenomenon termed the "Arc-size Illusion" (ASI) (Virsu, 1971, Virsu & Weintraub, 1971).



## AIM

The aims were to measure the ASI, to develop a perceptual model for the ASI and to investigate its influence on other experiments that require curvature judgments.

## METHODS

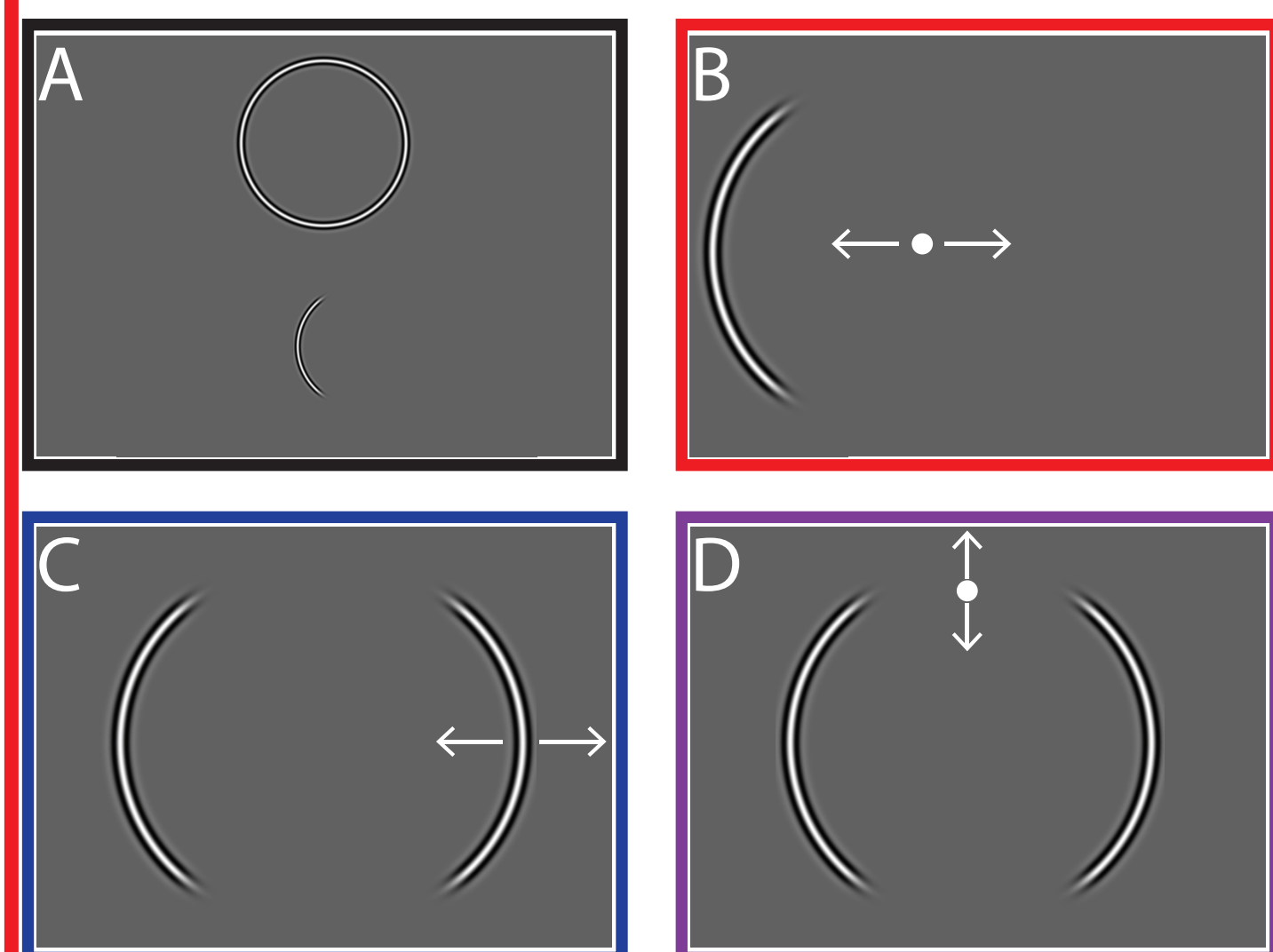


**STIMULI**

- Circular segments of varying arc lengths:
- Experiment 1 & 2:  $\pi/8$  and  $2\pi$  (for  $r=1^\circ$ )
- Experiments 3 & 4:  $\pi/8$  and  $2\pi$  ( $r=1^\circ$ )
- Three reference radii:  $R_{ref}=1^\circ, 2^\circ$  &  $3^\circ$
- D4 cross-sectional luminance profile;
- SF =  $8\text{ c/deg}$
- Contrast was ramped down at the boundaries.

### PROCEDURE

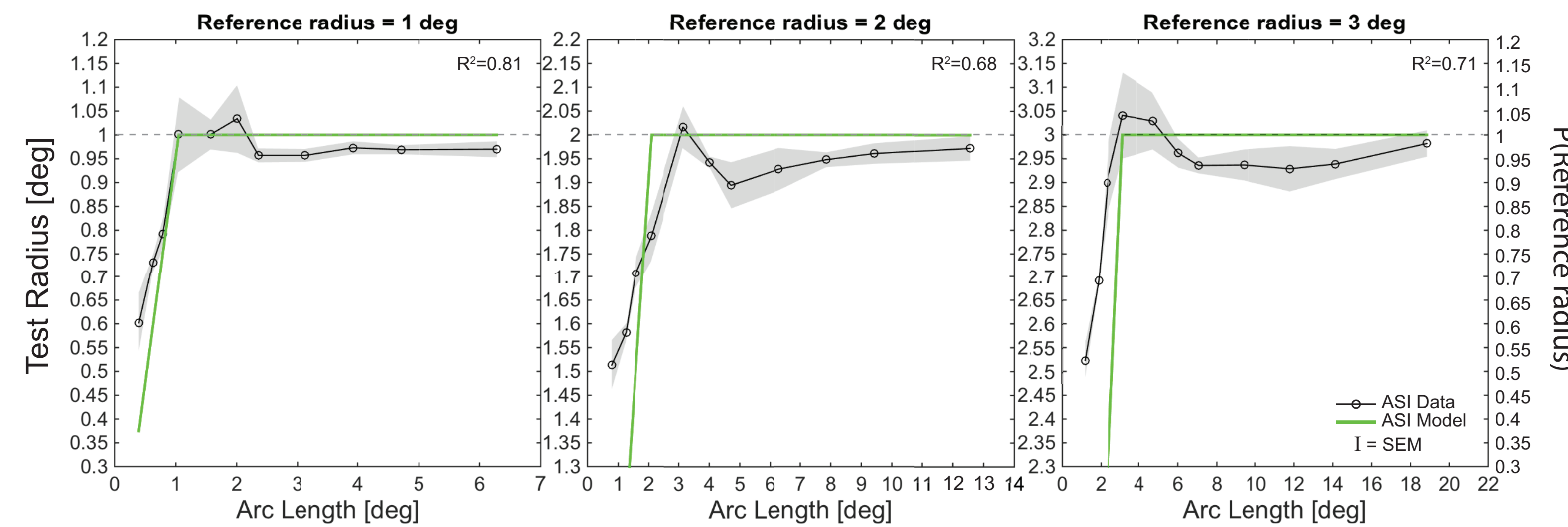
**Method of Adjustment**  
Subjects moved a white circular test dot (5 Pixels;  $0.093^\circ$ ) (Exps. 2 & 4) or one of the arcs (Exp. 3) by pressing keys on a numeric keypad. In all experiments, all reference radii were randomly interleaved. Each stimulus was tested 20 times; 3 repetitions. Viewing distance: 120 cm



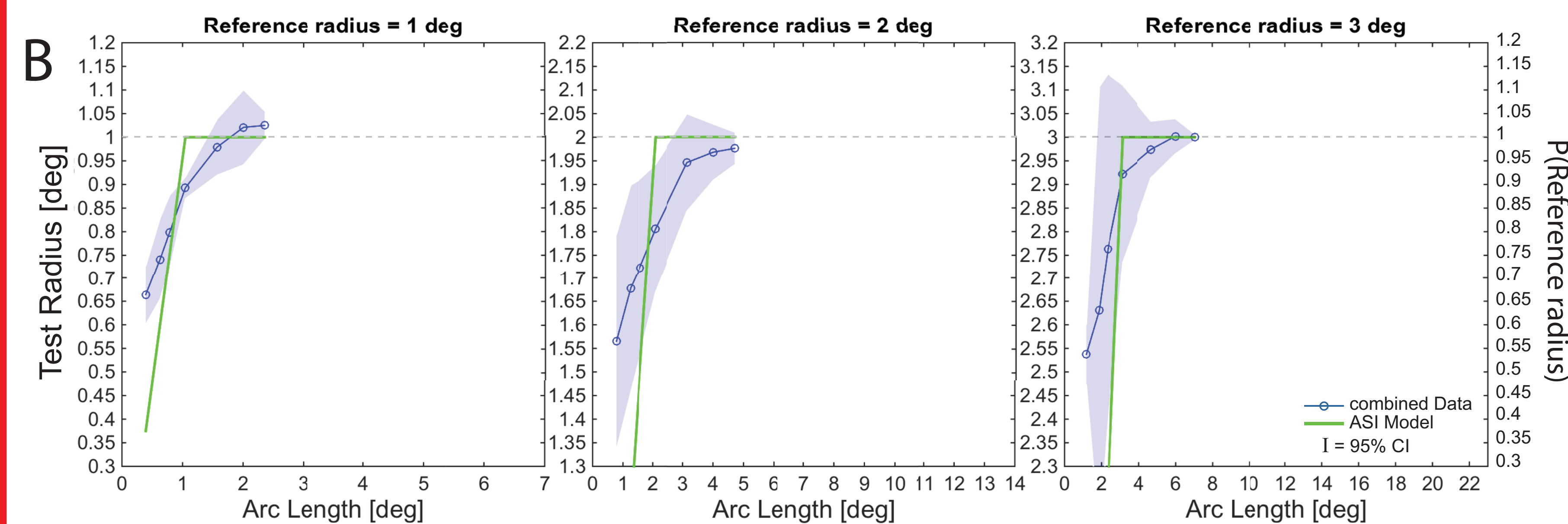
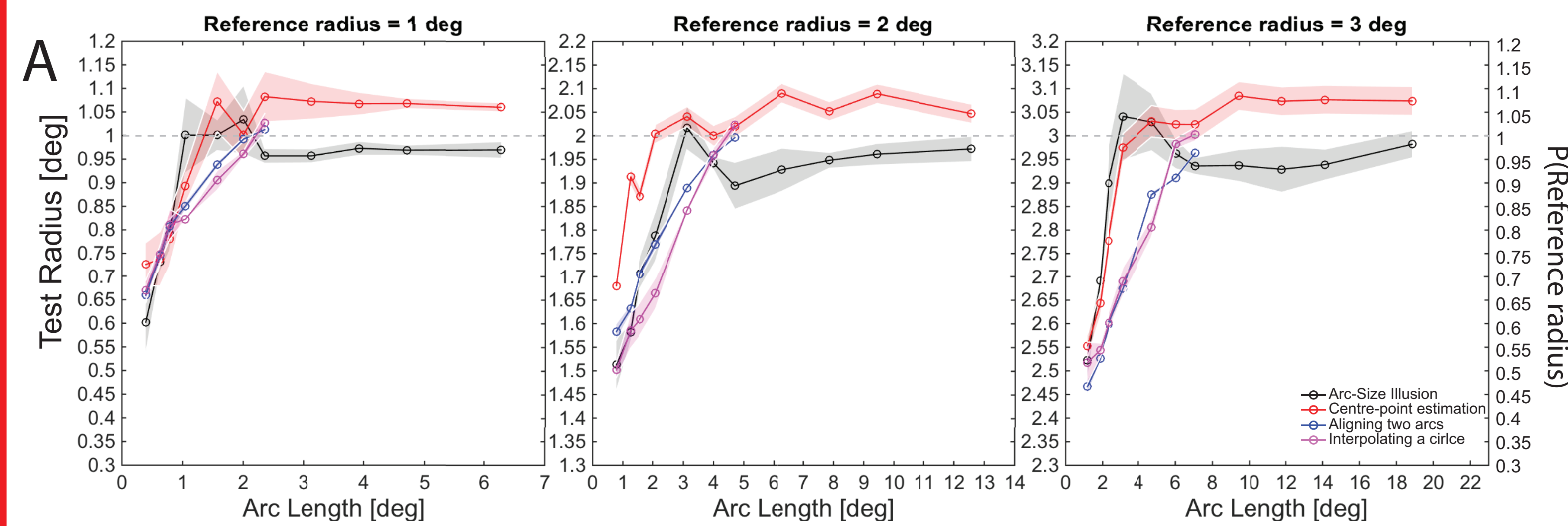
### EXPERIMENTS

Four psychophysical experiments were carried out:  
1. Measuring the ASI  
2. Estimation of the centre-point of a circle  
3. Adjusting two arcs to form a circle  
4. Extrapolation of curvature

## RESULTS

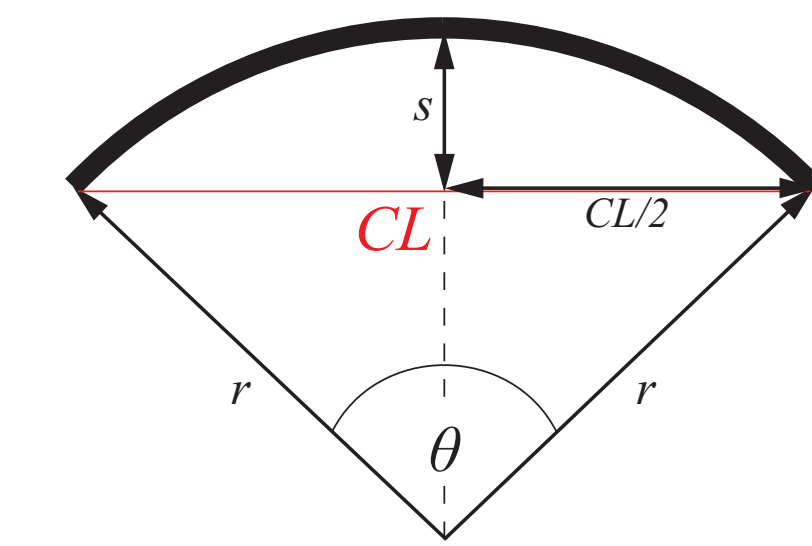


- curvature misjudgement is largest for very short arc lengths; the curvature of the test radius is consistently perceived flatter as the curvature of the reference circle
- as a consequence subjects increase the curvature (decrease the radius) of the test arc
- for shortest arc lengths (16th of circle) subjects set the test radius to on average 0.73 of the reference radius
- curvature misjudgement decreases with increasing arc length up to a 6th of a circle, after which it asymptotes and subjects judge curvature accurately
- A repeated measures ANOVA with test radius size ( $1^\circ, 2^\circ$  &  $3^\circ$ ) and arc lengths as factors revealed a significant main effect for arc length ( $F_{10,30}=26.774; p<.0001$ ), but no statistically significant between the different radii ( $F_{2,6}=14.91; p>.05$ ), which demonstrates that the ASI is independent of radius size and, therefore, scale invariant



- For all experiments, curvature misjudgment follows the same pattern as observed for the ASI
- Results from the first seven tested arc lengths have been compared statistically by a repeated measures ANOVA with test radius size ( $1^\circ, 2^\circ, 3^\circ$ ), experiment (1-4) and arc lengths as factors.
- Tests showed significant main effects for radii ( $F_{2,6}=10.195; p=.012$ ), experiment ( $F_{3,9}=4.073; p=.044$ ) and arc length ( $F_{6,18}=19.89; p<.0001$ )
- subsequent post-hoc T-Tests (corrected for multiple comparisons; Bonferroni) revealed no statistically significant differences ( $p>.05$ )
- Data from all four 4 experiments were averaged (bottom graph) and differences between the ASI model and data were statistically analyzed by T-Test
- Results show that the ASI model significantly overestimates the Test radius for the shortest arc length ( $p<.05$ ), but accounts well for the data

## MODEL



- Various geometrical features in the stimuli are potentially available for constructing a metric that encodes curvature.
- the chord ( $CL$ ); the sagitta or sag ( $S$ ); the arc length; the area enclosed by the chord and the arc; the central angle subtended by the test arc ( $\theta$ )
- The successful metric needs to predict a large underestimation of curvature for short arc lengths, a monotonic decrease in curvature misjudgment with increasing arc length and no misjudgment of arc lengths greater than about a sixth of a circle.

We tested numerous metrics, and found that the simple feature  $\theta$  gave the best account of the misjudgment portion of the data.

## DISCUSSION

- Short arcs are perceived flatter as long arcs of the same radius - ASI
- Perceived curvature is scale-invariant
- The central angle  $\theta$  gave the best account for curvature judgment
- $\theta$  is a scale invariant feature
- Curvature is computed for arcs up to a sixth of a circle
- The ASI can explain results in other tasks of curvature judgement

## CONCLUSION

Using new data and a model of the arc-size illusion we show that perceived curvature is scale-invariant, that is a curve appears similarly curved irrespective of viewing distance, even though its curvature in the retinal image changes with viewing distance. Second we show that curvature is computed only for arcs up to a sixth of a circle in length. These two properties of curvature perception are shown to predict the results of a series of experiments that involve curvature judgments.