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Telemobility Technical Report

Omnichannel retail fulfillment strategies

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16. Abstract

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Abstract

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CHAPTER 1

Introduction

1.1 Background

The COVID-19 global pandemic had a profound impact on a wide range of human activities. In the United States (U.S.), a significant portion of the population had worked and studied at home for an extended period of time due to lockdown orders. Even though shopping, considered an essential service, had never been restricted, many Americans voluntarily forwent their regular trips to physical stores out of safety concerns, in favor of online shopping. Not surprisingly, this behavioral change resulted in an incredible growth spurt for e-commerce. According to the U.S. Department of Commerce[1], the total e-commerce sales in the first quarter of 2022 were $250 billion, a 15.34% increase compared to the first quarter of 2021, and a 122.04% increase from the first quarter of 2020. They accounted for 14.3% of the total retail sales in the U.S., compared to 11.7% in the same period of 2020.

Amazon, the quintessential online retailer, is among the clear beneficiary of the pandemic. In the first quarter of 2021, Amazon recorded a 44% year-over-year growth in the revenue and 220% growth in the profit[2]. In 2019, the share of the U.S. households total spending with Amazon was 2.4% (that is, for every $100 U.S. households spent that year, $2.4 went to Amazon). This number jumped to 3.3% in 2020 and then to 3.6% in 2021[3]. In contrast, Walmart's share of the U.S. households' total spending rose only 0.2 percentage points during the same period, from 2.8% in 2019 to 3.0% in 2021. It is worth noting Walmart is by no means a "traditional" brick-and-mortar retailer, though the lion's shares of its revenues still come from the sales in physical stores. It has invested heavily and gained a growing footprint in e-commerce. In the first three quarters of 2021, Walmart took 6.6% of all online sales in the U.S., next only to Amazon's 41%[4].

Evidence began to emerge that suggests some pandemic-induced changes in human behaviors might stick. Of the thousands of consumers responding to a survey conducted by the EY Future Consumer Index in 2021[5], 40% state they will be "less

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[1]https://www.census.gov/retail/index.html

[2]https://www.businesswire.com/news/home/20210429006037/en/Amazon.com-Announces-First-Quarter-Results

[3]https://www.pymnts.com/opinion/2022/the-opportunity-in-offering-banks-a-bnpl-path/

[4]https://www.statista.com/statistics/274255/market-share-of-the-leading-retailers-in-us-e-commerce/.

[5]https://www.ey.com/en_us/consumer-products-retail/future-consumer-index-cycle-6-how-a-

inclined to be involved in experiences outside the home" in the future. 60% respondents were still visiting brick-and-mortar stores less at the time of the survey, despite reopening was in full swing through much of 2021. Moreover, 43% thought they will shop more often online for the products they previously bought in stores.

Any retailer who sees the writing on the wall knows their survival in the future depends on the creation of an *omnichannel* shopping experience that transcends the boundaries of conventional online/offline channels. There is no doubt that e-commerce has given birth to omnichannel retail and will continue to drive its growth in the years to come. That does not mean, however, the in-store shopping experience is bound to fade into oblivion. Physical stores still account for the vast majority of U.S. retail sales, especially in product categories like food and beverage (93%) and health and personal care (89%)[6]. Little wonder, then, as the brick-and-mortar retailers try to break through e-commerce barriers, Amazon has been aggressively pushing in the opposite direction. As of 2022, Amazon operates over 600 physical stores in North America, including more than 500 Whole Foods locations, Amazon Fresh (grocery), Amazon Go (cashier-less convenience stores), and recently launched Amazon Style (apparel).

Physical stores appeal to shoppers by allowing them to touch, feel and even try products, a unique experience that cannot be easily replaced or imitated by e-commerce. Moreover, large physical stores provide opportunities to enrich and diversify one's choice of channels through which orders can be fulfilled. This competitive edge came in handy when the pandemic dramatically tightens up labor markets and squeezes supply chains everywhere. The retailers such as Walmart and Target discovered their large number of physical stores allowing them to quickly scale up novel, pandemic-friendly fulfillment channels such as buy-online-pick-up-in-store (BOPS) and buy-online-drive-up-to-curbside (BODUC). Consumers who are wary of infection risks but intolerant of long delays find these options especially attractive. Physical stores do not just reduce the delivery demand. They can also speed up same-day or faster delivery, often hailed as the Holy Grail for a positive consumer experience in e-commerce. The high density of these stores and their proximity to consumers relative to Amazon-style fulfillment centers promise shorter delivery distance per order, less wait time for consumers, and ultimately lower operating costs. Hitherto Amazon's supply chain management and delivery capability appear to have kept up with its growing business reasonably well, without using physical stores as "front-end" warehouses. This was accomplished in part by working with thousands of delivery service providers who collectively hired hundreds of thousands of drivers dedicated to package delivery[7]. However, even a delivery workforce of this size can fall short sometimes. In fact, the share of Amazon orders arriving late jumped more than 4 percentage points (from 11.4% to 15.9%) after the pandemic demand surge set in[8]. A

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year-of-pandemic-changed-consumers

[6]https://www.emarketer.com/content/why-amazon-keeps-experimenting-with-physical-stores

[7]By the end of 2021, Amazon reportedly employs a quarter million couriers through 3000 providers, see https://www.cnn.com/2021/12/21/tech/amazon-delivery-night/index.html.

[8]https://www.statista.com/statistics/1220033/share-of-amazon-orders-arriving-late/

relevant question is *which fulfillment strategy – placing inventory in front-end ware-houses so that delivery can be initiated from there, or hiring an army of drivers to deliver directly from the fulfillment centers – is more efficient, and under what circumstances.* The answer to this question may play a crucial role in shaping the future landscape of omnichannel retail, as the key players are contemplating the next move to retain or grow their market shares.

The shift to omnichannel retail places the emphasis squarely on consumer experience. As Balis [Bal21] put it, "Old truth: You are competing with your competitors. New truth: You are competing with the last best experience your customer had". However, the laser focus on customer experience may be at odds with other pressing societal priorities. Notably, the "click-to-instant-deliver" experience made possible by the membership-based express delivery service such as Amazon Prime and Walmart Plus is extremely popular among customers, but it also forces the retailer to make multiple trips to a customer's home, often by large trucks, to deliver what can be bought in a single trip to a store by a passenger sedan. This shopping habit, once adopted by millions of Americans, could considerably increase the vehicle miles traveled (VMT), hence greenhouse gas emissions, contributed by the retail sector.

Interestingly, early studies [MHS01; EMC10] have concluded e-commerce tends to have a benign impact on the environment. For example, Edwards, Mckinnon, and Cullinane [EMC10] found e-commerce's home delivery operation is likely to generate less CO_2 emissions than an equivalent shopping trip. However, today's retail industry is very different from the early 2000s, when few had the expectation of receiving any online orders within a couple of days, let alone on the same day. A recent report by the Texas A&M Transportation Institute[9] anticipates the potential traffic impact of e-commerce growth in three areas: (i) regulating the use of personal vehicles by people acting as independent delivery contractors is difficult; (ii) the growth in urban logistics facility near or within urban centers may add additional traffic to road networks; and (iii) the increasing demand for express delivery may bring additional traffic to residential areas during off-peak periods. Clearly, delivery-induced traffic has become a concern for transportation planners, operators, and policymakers. However, whereas e-commerce generates more delivery trips, on the one hand, it also reduces the number of personal shopping trips on the other. The rise of omnichannel retail, especially the new fulfillment channels such as BOPS and BODUC, complicates the picture further because it seeks to balance customer volume between fulfillment channels that have different traffic implications. Taken together, it remains an open question *whether and how the continual evolution of omnichannel retail will disrupt the operation of surface transportation infrastructure.* A related question is *what policies can be implemented to mitigate this impact.*

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[9]https://policy.tti.tamu.edu/freight/how-will-e-commerce-growth-impact-our-transportation-network/

1.2   Overview

To recapitulate, this research is primarily motivated by the following questions that arise at the nexus of omnichannel retail and transportation.

1. What are the factors and circumstances that influence the retailer's optimal fulfillment strategy?

2. How should an omnichannel retailer allocate resources between different fulfillment channels to maximize its profit, while anticipating customers' channel choice preferences?

3. Will the fulfillment strategies adopted by the retailer have a significant traffic impact? If so, are there any ways to mitigate it from the policy point of view?

We will focus on the first two questions in this report, and leave the third to the next phase of this research.

   We propose a *theoretical* approach that centers on a stylized parsimonious model of omnichannel retail. The full-fledged model will be developed in three sequential phases, though only the first two phases are covered in this report.

   The first phase creates a base model that is complex enough to differentiate the most important fulfillment channels, but still simple enough to render useful insights. The model considers a retailer that owns a large distribution center and a front-end store with limited space. The store can either provide traditional in-store shopping, uses its space to fulfill online orders, or do both. The retailer can also fulfill online orders from the distribution center, either through a membership-based express service or a regular delivery service. As a starting point, customers are assumed to be *homogeneous*, and given three channels to choose from: in-store, online with membership (which promises express delivery), and online without membership. The customer choice depends on the utility of each channel, which is a function of the ratio between the number of users and the "capacity" of that channel. Thus, a fulfillment channel can be "congested" in the model the same way as a road can be congested by an excessive amount of traffic. The retailer's fulfillment strategy consists of three types of decisions: inventory in the store; price differentiation across channels; and express delivery capacity. In making these decisions, the retailer must anticipate the uncertainty in the total demand, which follows a random distribution, and the split of customers across different channels, which settles at a Nash equilibrium of a congestion game played by identical customers. Thus, the base model may be viewed as a *Stackleberg congestion game*.

   In the second phase, the base model will be extended to accommodate customers' heterogeneous channel preferences. Specifically, we choose to model channel-specific hassle costs as random variables that follow a joint distribution, from which the demand density can be derived for any given vector of hassle costs. This extension requires a rather different analysis, especially for the consumers' channel choice. Instead of deriving the channel choice according to Nash equilibrium, this model determines

it according to the specification of heterogeneity. Due to the complex interactions between fulfillment decisions and channel choice in this case, however, the model is not particularly amenable to analysis, except for highly simplified versions.

The third and future phase will attempt to connect channel heterogeneity to customers' home locations, thus adding a spatial structure into the model. With this feature, we propose to specify the retailer's delivery operations with greater details, which will enable us to track the total vehicle miles traveled in the system. The base model assumes the retailer only sells one genetic product. In this phase, we will also relax the above assumptions by allowing the retailer to carry more than one product and tailor fulfillment strategies to each product.

To test the proposed model and generate managerial insights, case studies will be constructed using publicly available empirical data wherever possible. By designing and experimenting with counterfactual scenarios, we will seek answers to the first two research questions posed at the beginning of this section.

1.3   Organization of the report

The remainder of this report is organized as follows. Chapter two reviews related studies, focusing on omnichannel retail, channel conflict management, and the impact of e-commerce on the transpiration system.

The main results are reported in Chapters three and four. Chapter three describes the base model formulated as a bi-level program. At the upper level, a retailer (the leader) chooses fulfillment strategies to maximize profit, whereas at the lower level the customers (the followers) pick a shopping channel to place their order for a genetic product. Using optimality conditions we transform the bi-level program into a single-level problem that can be solved analytically. We show that, in the absence of a hard constraint on the size of the front-end store, the retailer would always adjust its fulfillment strategies such that only one channel is used. This is precise because all customers are homogeneous in the base model. The analytical results are validated using numerical experiments. In Chapter four, customer heterogeneity is characterized using a joint distribution of two hassle costs, each for one channel. This extension leads to a new and (conceptually) simpler lower-level problem but a much more complicated objective function in the resulting single-level problem. To obtain insights we analyze a special version of the extended problem, in addition to performing numerical experiments.

Chapter 5 sketches a work plan for Phase 3 of this research. It consists of the following activities: (i) adding a spatial structure and analyzing traffic implications; (ii) adding product heterogeneity and analyzing cross-channel assortment congruity; and (iii) conducting case studies to answer the research questions.

Literature Review

The related studies are organized into four categories, according to the perspective from which they approach the omnichannel/e-commerce problem. Sections 2.1 and 2.2 take the perspective of the retailer and the manufacturer, respectively. In Section 2.3 we consider studies that focus on customers' preferences, experience, and migration across channels. Finally, Section 2.4 covers the analysis of e-commerce's traffic impact.

2.1 Retailer perspective

With the advent of e-commerce, there was growing attention on omnichannel retail and management strategies from the retailer's perspective. Brynjolfsson, Hu, and Rahman (2013) provided a comprehensively discussion on this topic. Gao and Su (2017) built a rational expectation equilibrium (REE) model to study retailer's stock decision and consumer's channel choice with and with BOPS (buy-online-pickup-in-store). The RE model was developed by Deneckere and Peck's (1995) model and Dana's (2001) model. Using the model, they found BOPS may not a suitable for best-sellers in stores but it can help expand market coverage by mitigating stock-out risk and improving customers' accessibility. Built on Gao et al.'s work, Kong et al. (2020) analyzed the pricing strategy with BOPS as one of the fulfillment channels. The results show differentiated pricing leads to a higher profit, especially if the BOPS channel has a low operating cost. Besides BOPS, other services designed to improve customers' shopping experiences are also considered in the literature. Zhang, Xu, and He [ZXH18] considered a reserve-online-pickup-and-pay-in-store channel (ROPS, in contrast with BOPS). ROPS may reduce return rate, help retailer expands the market, and increases cross-selling profits. Gallino and Moreno [GM14] empirically validated that sharing the inventory availability information can increase the number of customers visiting the stores. Bell, Gallino, and Moreno [BGM18] demonstrated that an online retailer can increase demand and operational efficiency by opening offline showrooms. Hübner, Holzapfel, and Kuhn [HHK16] explored the distribution system in omnichannel retail including the associated return processes. They suggest developing a high-level-of-service express delivery system is the key to winning the e-commerce battle. In sharp contrast, Jindal et al. [Jin+21] find the attributes that customers value the most are a large assortment, competitive prices, and purchase convenience. Their conclusion is that attempting to increase express delivery may not be a cost-effective strategy.

2.2   Manufacturer perspective

Since the early days of e-commerce, manufacturers have been wrestling with the dilemma of creating their own online retail channels to compete with the existing distribution channels. There is a large body of studies on channel conflict management between traditional retail channels and direct online channels. Chiang, Chhajed, and Hess [CCH03] suggested that the introduction of the direct channel may motivate retailers to perform more effectively. Although direct channels may take retailers' sales away (cannibalization), this reduction is often accompanied by a wholesale price drop. The combined effect actually benefits the retailer in equilibrium. This conclusion was verified by Tsay and Agrawal [TA04]. The authors examine several strategies, including adjusting wholesale pricing, incentivizing the retailer to divert customers toward the direct channel, and conceding the demand fulfillment function entirely to the retailer. Adding a direct channel is not necessarily detrimental to the retailer, given the arrangement may lead to price adjustment that could benefit both when the direct channel becomes more convenient and less costly than the traditional channel [Cat+06]. Hua, Wang, and Cheng [HWC10] and Wang, Wang, and Wang [WWW13] considered a game between retailer and manufactures. The former assumes the manufacturer plays the role of a Stackelberg leader in which it sets lead time and price for the direct channel, and wholesale price for the retailer, whereas the retailer would set the retail price. The latter regard the retailer as the Stackelberg leader who offers a markup purchasing contract to two competitive manufacturers. Other manufacture-retailer dual channel studies consider channel distribution strategy for different products [WWW16], sustainability [WZN17], and direct channel as a determent (e.g., to the launch of discount stores by retailers) [CPL18]. Unlike these studies, this research attempts to build a model for an omnichannel retailer that provides both online and offline channels, similar to real-world retail giants such as Walmart and Amazon.

2.3   Customer perspective

Customers' shopping preference for offline channels has been widely studied (e.g., [Mor79]; [Cre97]; [YDM98]). For online shopping, previous research has identified a range of factors affecting customers' preferences, such as prior purchase experience [BPV03] and website environment cues [CS08]. Hausman and Siekpe [HS09] examined the influence of website interface on consumers' purchase preferences. Chiu, Lin, and Tang [CLT05] studied the differences in the shopping preferences between males and females. Ha and Janda [HJ14] discussed the influence of customized information on online shopping preferences.

In the context of omnichannel retail, besides customers' shopping preferences, retailers are also interested in how customers migrate between channels. Ansari, Mela, and Neslin [AMN08] developed an approach to model and evaluate customers' channel migration. Neslin and Shankar [NS09] summarized the key issues in customer

management. After analyzing customers' responses to the channel migration strategy, they suggested retailers should adopt a channel migration strategy that let customers voluntarily migrate between channels or reward them to do so. Xu and Jackson [XJ19] examined the impact of channel attributes, including transparency, uniformity, and convenience, on customers' channel choices. They found channel transparency and uniformity have a positive impact on customers' channel choices by reducing their perceived risks.

2.4   Traffic perspective

One of the earliest studies of the impact of e-commerce on transportation systems is conducted by Matthews, Hendrickson, and Soh (2001). They compared the cost of online and traditional in-store distribution channels, using book sales as a case study. In the in-store channel, a book is shipped from the publisher through various distributors and warehouses and finally to a retail outlet. The customer purchases the book at the retail store and brings it home. In the online channel, a book is shipped from a publisher to a single warehouse by truck and then delivered through air freight to a regional airport or hub, from where it is transported by a delivery truck to the customer's home. They found online channel costs less and has few environmental effects when a return rate of 35% is assumed for the in-store channel. Their findings are also confirmed by Edwards, Mckinnon, and Cullinane [EMC10]. They focused on the last-mile leg and compare the level of carbon emissions from a conventional non-food shopping trip with those of delivering non-food items to the home. They found, on average, the home delivery operation is likely to generate less CO2 than the typical shopping trip.

More recently, Shao et al. [Sha+16] built a model to explore the relationship between e-commerce on traffic congestion. This paper indicates, to use the in-store channel, customers impose greater pressure on the road network, while e-commerce lessens this pressure by the scale economies in delivery (a vehicle is assumed to deliver multiple orders), but offers a lower value (due to delayed consumption, lower level of services, the potential for return, etc.) to consumers. Niu, Mu, and Li [NML19] examined the impact of two O2O strategies, uniform pricing policy, and differentiated pricing policy, on traffic congestion. They found uniform pricing reduces demand size in the online channel (hence reducing congestion cost) but increases the total profit when the logistic cost is low.

A Model of Omnichannel retail

Consider an omnichannel retailer that provides both online and in-store shopping services in a city where it operates a distribution center (DC) and a store. The store is located inside the city center (hence closer to customers) but has limited space A, whereas the DC has almost unlimited space but is located outside the city. For simplicity, we assume the retailer carries a composite product that is sold online at a constant price of p_0 per unit. The retailer may add a markup when the product is sold in-store, thus bringing the total price to p.

In this study, we focus on the retailer's store inventory, in-store pricing, and same-day delivery strategies. First, the retailer needs to decide how much of the store
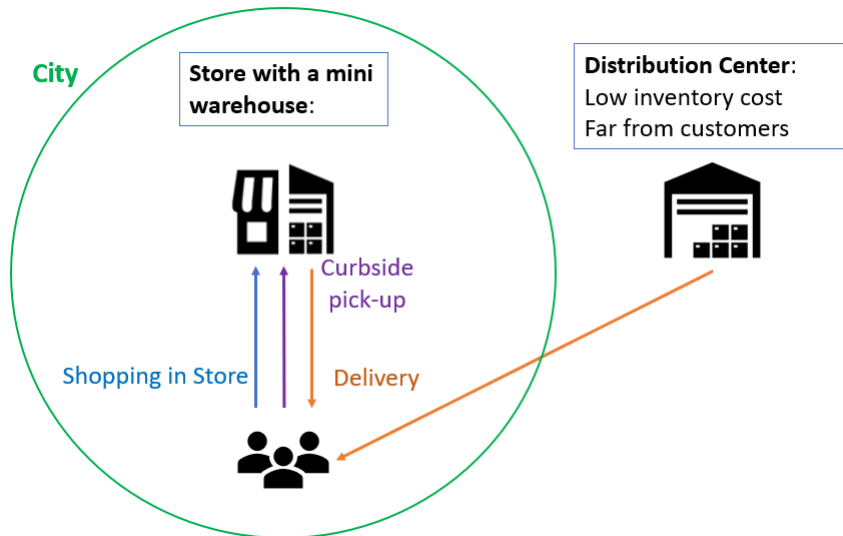

Figure 3.1: Omnichannel distribution

space should be allocated to hold q_s units of the products on the shelf for in-store customers. Note that the retailer does not have to allocate all the store space to in-store shopping. Instead, a portion of the product, denoted as q_w, can be stored in a warehouse dedicated to fulfilling online orders. Although the inventory cost is higher in the store than in the DC, fulfilling online orders from the store may save delivery costs. The retailer replenishes both store inventories (shelf and warehouse) daily. Second, the retailer needs to determine its same-day delivery capacity (in the form of vehicle-hour), denoted as t, and the premium the customers must pay for the service (in the form of a membership fee), denoted as r. Finally, the retailer has to determine the in-store price p.

On the demand side, we assume the number of potential customers targeted by the retailer is a random variable $\tilde{d}$, with a cumulative distribution function (CDF) F and a probability density function (PDF) f. Each customer purchases one unit of the composite product a week. To acquire the product, they shop with a frequency of e times per week. Unless they have the same-day delivery membership, we assume $e = 1$. The customers have three shopping channels to choose from: in-store, online with same-day delivery, and online without same-day delivery.

Assumption 1. *We introduce the following assumption to further simplify the analysis.*

1. *A customer always chooses the shopping channel that provides the highest utility, defined as the product's value less the sum of the customer's shopping cost and price.*

2. *Customers are homogeneous in their valuation of the product and the channel cost. The value of the product is v, and the hassle costs associated with an online and in-store channel are h_l and h_s, respectively.*

3. *Only customers shopping online may return the product after the purchase. The return probability is exogenous.*

4. *The premium customers pay for same-day delivery service is non-refundable.*

5. *Online orders can be delivered either from the warehouse or from the store (fulfilled at the mini-warehouse). Delivering from the store is cheaper because the store is closer to an average customer.*

6. *The retailer can capture all demands coming to the store or subscribe to the delivery membership, but only a portion (denoted by an exogenous parameter γ) of the other demands.*

7. *The unit cost of the product unrelated to the decisions considered herein is treated as exogenous denoted as c_p. Included in c_p is the cost to acquire the product and to move it through the supply chain to reach the city. c_p covers the delivery from the DC to either the store or a customer's home, and thus is independent of fulfillment channels*

3.1   Customer's choice

We next discuss a customer's shopping payoff in detail. Recall that the customer can choose one option from a set of options $\mathcal{I} = \{s, m, o\}$, where s, m and o stands for in-store shopping, online shopping with the same-day delivery (referred to as membership hereafter), and online shopping without the same-day delivery (referred to as online hereafter), respectively. Let $x_i, i \in \mathcal{I}$ be the proportion of the customers who choose channel i. We use $\mathbf{x}$ to represent a vector of this allocation.

The utility associated with shopping online without the same-day delivery membership fee

$$u_o = \theta(v - p_0) - h_l. \tag{3.1}$$

where h_l is the hassle cost of waiting incurred due to the lack of the express delivery service, and θ denotes the probability that a customer keeps the product after the purchase, treated as a constant per Assumption 1.2. Users of online channel do not pay for delivery because it is included in the price per Assumption 1.6. Since the retailer's decisions concerned herein have no effect on the online channel, u_o is a constant and treated as a fallback choice. Without loss of generality, we assume $u_o \geq 0$, which implies even the fallback choice will yield a non-negative utility.

For shopping in-store, the customer incurs a hassle cost of h_s, which depends on, among other things, the travel cost and the effort to locate the product from the store. Since the customer does not know how many others would choose to shop in-store, there is a possibility that the product might end up out of stock in-store. Let y_s be the probability that the customer assigns to the event that the product is available in the store. We assume, in the event, the customer encounters a stock-out, they will simply forgo the purchase. Accordingly, the utility for a customer shopping in-store is given by

$$u_s(x_s) = y_s(v - p) - h_s. \tag{3.2}$$

Note that y_s depends on $x_s\tilde{d}$, i.e., the random number of customers who actually show up in the store, and q_s, the inventory of the product. Let $\Omega = [\underline{D}, \overline{D}]$ be the support of $\tilde{d}$. Let $\mathcal{D}$ denote the set of all potential customers, and consider a given customer $i \in \mathcal{D}$. For a demand realization d, the probability that Customer i enters the market is given by $d/\overline{D}$. Accordingly, the joint probability that the realized demand is d and Customer i is among them is $df(d)/\overline{D}$. Conditional on the presence of the customer in the market, the probability that the realized demand happens to be d is given by (Deneckere and Peck, 1995)

$$P(\tilde{d} = d | i \in \mathcal{D}) = \frac{df(d)/\overline{D}}{\int_{\underline{D}}^{\overline{D}} zf(z)/\overline{D}\mathrm{d}z} = \frac{df(d)}{E(\tilde{d})} \equiv \frac{df(d)}{\bar{d}}. \tag{3.3}$$

Thus, the probability that Customer i can successfully shop in-store is given by

$$y_s = \int_{\underline{D}}^{\overline{D}} \frac{\min(x_s z, q_s)}{x_s z} \frac{zf(z)}{\bar{d}}\mathrm{d}z = \frac{\int_{\underline{D}}^{\overline{D}} \min(x_s z, q_s)f(z)\mathrm{d}z}{x_s\bar{d}} = \frac{E[\min(x_s\tilde{d}, q_s)]}{x_s\bar{d}}. \tag{3.4}$$

Thus, y_s equals the expected in-store sales divided by the expected number of customers choosing in-store shopping.

Proposition 1 (Utility of in-store shopping). *The utility of in-store shopping u_s is a monotonically decreasing function of the share of in-store shopping x_s and a monotonically increasing function of the shelf stock q_s when $q_s \leq \overline{D}x_s$.*

Proof. Since u_s increases with y_s, we only need to show y_s is monotonically decreasing with x_s and increasing with q_s when $q_s \leq \overline{D}x_s$. We first rewrite y_s as follows.

$$y_s = \begin{cases} \frac{q_s}{x_s \overline{d}} & q_s < \underline{D}x_s \\ 1 - \frac{1}{\overline{d}} \int_{q_s/x_s}^{\overline{D}} (z - \frac{q_s}{x_s}) f(z) dz & q_s \in [\underline{D}x_s, \overline{D}x_s] \\ 1 & q_s > \overline{D}x_s \end{cases} \tag{3.5}$$

It is easy to see that y_s is a decreasing function of x_s and an increasing function of q_s when $q_s \leq \underline{D}x_s$ (i.e., the stock is lower than the minimum possible in-store flow given by the share x_s). For $q_s \in [\underline{D}x_s, \overline{D}x_s]$, applying the Leibniz rule yields

$$\frac{\partial y_s}{\partial q_s} = \frac{1}{x_s \overline{d}}[1 - F(\frac{q_s}{x_s})]; \quad \frac{\partial y_s}{\partial x_s} = \frac{q_s}{x_s^2 \overline{d}}[F(\frac{q_s}{x_s}) - 1], \tag{3.6}$$

Since $F(\cdot) \leq 1$, we have $\partial y_s / \partial q_s \geq 0$ and $\partial y_s / \partial x_s \leq 0$. The proof is completed. $\square$

If the customer chooses the membership channel, they too could incur a hassle cost h_l in the event the product cannot be delivered on time. We use y_m to denote the probability that the customer assigns to the event that the product is delivered on the same day. Accordingly, the utility associated with the membership channel is

$$u_m(x_m) = \theta(v - p_0) - r - (1 - y_m)h_l = u_o - r + y_m h_l. \tag{3.7}$$

Similar to y_s, the probability y_m depends on the customer flow $x_m \tilde{d}$, and the maximum amount of product the retailer can fulfill through the membership channel, denoted as $q_w + q$, where q_w and q are units of products fulfilled through from warehouse and DC, respectively. We shall discuss how q and q_w are related to the same-day delivery capacity t later. We have

$$y_m = \frac{E(min(x_m \tilde{d}, q + q_w))}{x_m \overline{d}}. \tag{3.8}$$

Corollary 1 (Utility of online shopping with same-day delivery). *The utility of online shopping with same-day delivery u_m is a monotonically decreasing function of the share of online shopping with same-day delivery x_m and a monotonically increasing function of the same-day delivery capacity q_d ($q_d = q + q_w$), when $q_d \leq \overline{D}x_m$.*

Customers always choose a channel to maximize their utility. At equilibrium, the following conditions must be satisfied:

$$x_i^* > 0 \rightarrow u_i^* = \bar{u}; \forall i \in \mathcal{I} \tag{3.9}$$

$$u_i^* \leq \bar{u}, \forall i \in \mathcal{I}, \tag{3.10}$$

where

$$\sum_{i \in \mathcal{I}} x_i^* = 1; x_i^* \geq 0, \tag{3.11}$$

and $\bar{u}$ is the maximum utility any customer could achieve. In words, nobody could improve their own utility by unilaterally switching to a different channel at the equilibrium.

Figure 3.2 illustrates the equilibrium solution for given retailer decisions. The blue, red, and green curves represent the utility of channel s, m, and o, respectively. Of the three channels, the utility of s and m decreases with x_i whereas that of o is depicted as a horizontal line. For simplicity we shall assume customers tend to prefer s or m to o if the number of users of channel s and m is sufficiently small. This is consistent with the designation of the online channel as the fallback option in our model. Under this assumption, there are three possible equilibria. The first is achieved when the red and blue curves do not intersect within the feasible range, e.g.,
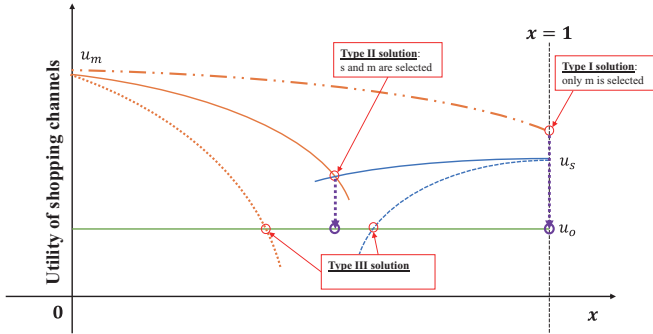
Figure 3.2: Illustration of the equilibrium solution.

$u_m(1) > u_s(0)$. (or $u_s(1) > u_m(0)$ In this case, everyone chooses the membership channel (Type I). The system arrives at the second equilibrium if the intersection between the red and blue curves lies above the green line, in which case only o is not used (Type II). The third case emerges when the intersection of the two curves rises above the line. They would not intersect, however, because the equilibrium utility will be kept at u_o and all three channels will be used (Type III).

3.2   Retailer's optimization problem

The retailer's goal is to maximize the expected profit by choosing the five design parameters, q_s (in-store shelf inventory), q_w (in-store warehouse inventory), t (same-

day delivery capacity), p (in-store price), and r (same-day delivery membership fee) while anticipating the customers' channel choice. To model the same-day delivery operation, let τ and τ_w be the vehicle time needed to deliver one unit of product from the DC and from the warehouse, respectively, and assume $\tau_w < \tau$. Recalling q is defined as the number of products delivered from the DC by channel m, we have

$$t = q\tau + q_w\tau_w. \tag{3.12}$$

Hereafter, we replace t with q as the decision variable that determines delivery capacity and represent $\{q_s, q_w, q\}$ with a vector variable $\mathbf{q}$. Both τ and τ_w may depend on e, the shopping frequency of the same-day delivery membership holders. Recall that we assume all customers purchase one unit of the product a week. Thus, the shopping frequency does not affect how much they buy, but merely how many trips the retailer must make to deliver them. The delivery cost, however, does not necessarily double when the shopping frequency doubles, because delivering the same amount of product in two separate trips means each time only half the cargo space is needed. The current model, however, is not amenable to specifying the actual relationship between τ/τ_w and e. We shall address this issue later using a model with spatial heterogeneity.

We are now ready to formulate the retailer's decision problem as a mathematical program with equilibrium constraints (MPEC).

$$\max g(\mathbf{q}, p, r, \mathbf{x}) = (p - c_p)E(\min(x_s\tilde{d}, q_s)) + \alpha E(x_s\tilde{d}) - (c_sq_s + c_wq_w)$$
$$+ \left(E(x_m\tilde{d}) + \gamma E(x_o\tilde{d})\right)\left(\theta(p_0 - c_p) - c_r(1 - \theta)\right)$$
$$+ rE(x_m\tilde{d}) - c_d(\tau_wq_w + \tau q) \tag{3.13a}$$

subject to:

$$a_wq_w + a_sq_s \leq A; \tag{3.13b}$$

$$\mathbf{q} \geq 0, p \geq 0, r \geq 0 \tag{3.13c}$$

$$x_i(u_i - \bar{u}) = 0, \forall i \in \mathcal{I}; u_i \leq \bar{u}, \forall i \in \mathcal{I}; \tag{3.13d}$$

$$\sum_{i \in \mathcal{I}} x_i = 1; \mathbf{x} \geq 0. \tag{3.13e}$$

In the objective function (3.13a), the first and second terms are the expected revenue from in-store sales and cross-sales, respectively. The cross-sale revenue is proportional to the expected number of in-store shoppers. The third term is the total in-store inventory cost, in which c_s and c_w are the unit inventory costs in-store and warehouse respectively. Note that the inventory cost at DC is normalized to zero. The fourth term is the revenue from online sales less the extra cost associated with the return. Here, $\gamma \in [0, 1]$ represents the share of online shoppers who end up shopping with the retailer. In other words, $(1-\gamma)E(x_o\tilde{d})$ represents the loss of customers to competitors. Finally, the fourth and fifth terms are the revenue generated from the sale of same-day delivery memberships and the delivery cost, respectively.

Constraint (3.13b) dictates that the amount of products placed in the store is restricted by the size of the store A, where a_w and a_s are the areas occupied per unit product in the warehouse and on the shelf, respectively. Constraints (3.13d) - (3.13e) rewrite the equilibrium conditions as a set of complementary conditions.

Assumption 2. *To simplify the analysis, we further assume*

1. *The unit inventory cost is proportional to the area occupied by the product, i.e.,*

$$\frac{c_s}{c_w} = \frac{a_s}{a_w}. \tag{3.14}$$

2. *Storing the product in and delivering it from the warehouse is cheaper than delivering it from DC, i.e.,*

$$0 < c_w + c_d \tau_w < c_d \tau \to \mu_0 \equiv \frac{c_d \tau}{c_w + c_d \tau_w} > 1; \mu_1 \equiv \frac{c_d(\tau - \tau_w)}{c_w} > 1. \tag{3.15}$$

Proposition 2 (Equilibrium corresponding to profit-maximization)**.** *To maximize the expected profit, the retailer's decision will always push the customers to settle at a Type III equilibrium, i.e., $u_m(x_m^*) = u_s(x_s^*) = u_o$ and $x_m^* + x_s^* + x_o^* = 1$.*

Proof. Since there are only three possible equilibria under the assumption, we only need to show the retailer's profit is not maximized under either Type I or Type II equilibrium. Without loss of generality, suppose $u_m(0) \geq u_s(0) \geq u_o \geq 0$. For Type I equilibrium, we have $u_m(1) > u_s(0) > u_o$. Note that

$$u_m(1) = \theta(v - p_0) - r - (1 - y_m(1))h_l.$$

Clearly, the retailer can increase its profit by charging a higher membership fee r until $u_m(1) = u_s(0)$, a Type II equilibrium. We next show Type II equilibrium cannot secure profit maximization either. At such an equilibrium, we have

$$u_m(x_m^*) = u_s(x_s^*) > u_o, x_m^* + x_s^* = 1,$$

and

$$u_s(x_s^*) = y_s(x_s^*)(v - p) - h_s, u_m(x_m^*) = \theta(v - p_0) - r - (1 - y_m(x_m^*))h_l.$$

Note that y_m and y_s are affected neither by in-store shopping price p nor by the membership fee r. Thus, the retailer can simultaneously raise both p and r (hence lowering u_s and u_m) while leaving the system at Type II equilibrium. It is easy to see the incentive to continue this price hike would only disappear when the system arrives at a Type III equilibrium, i.e., $u_s = u_m = u_o$. This completes the proof.   □

Since $u_m = u_o$ at a profit-maximization equilibrium, the membership fee at equilibrium is

$$r = y_m h_l. \tag{3.16}$$

Similarly, setting $u_s = u_o$ yields

$$p = v - \frac{h_s - h_l + \theta(v - p_0)}{y_s} \tag{3.17}$$

The numerator of the second term above, $h_s - h_l + \theta(v - p_0)$ may be viewed as the expected customer gain from the in-store channel (note that it equals $y_s(v - p)$). Hereafter we replace this term with the symbol ρ for simplicity and note that $\rho \geq h_s$ if in-store shopping is a viable channel (cf. Eq. (3.2)). We shall also replace $\theta(p_0 - c_p) - c_r(1 - \theta)$ in the fourth term of (3.13a) with ψ. Namely,

$$\rho \equiv h_s - h_l + \theta(v - p_0); \psi \equiv \theta(p_0 - c_p) - c_r(1 - \theta). \tag{3.18}$$

Note that ψ can be interpreted as the gross profit of selling one unit of product through an online channel.

3.3   Solution analysis

We proceed to analyze the solution to the retailer's optimization problem. Let us introduce $\mathbf{x}' = [x_m, x_s]$ and recognizing $x_o = 1 - x_m - x_s$ enables us to eliminate the flow conservation condition (3.13e). Invoking Eqs. (3.4), (3.8), (3.16) and (3.17), we rewrite Problem (3.13) as

$$\max g(\mathbf{q}, \mathbf{x}') = (v - c_p)y_s x_s \bar{d} + (\alpha - \rho)x_s \bar{d} - (c_s q_s + c_w q_w) + (x_m + \gamma(1 - x_m - x_s))\psi \bar{d}$$
$$+ h_l y_m x_m \bar{d} - c_d(q_w \tau_w + q\tau) \tag{3.19a}$$

subject to:
$$a_w q_w + a_s q_s \leq A; \tag{3.19b}$$
$$\mathbf{q} \geq 0; \mathbf{x}' \geq 0 \tag{3.19c}$$

To solve the above problem, we first fix $\mathbf{x}'$, and write the Lagrangian of (3.19) as

$$\mathcal{L}(\mathbf{q}|\mathbf{x}') = (v - c_p)y_s x_s \bar{d} + (\alpha - \rho)x_s \bar{d} - (c_s q_s + c_w q_w) + (x_m + \gamma(1 - x_m - x_s))\psi \bar{d}$$
$$+ h_l y_m x_m \bar{d} - c_d(q_w \tau_w + q\tau) - \phi_1(a_s q_s + a_w q_w - A) + \phi_2 q_s + \phi_3 q_w + \phi_4 q, \tag{3.20}$$

where $\phi_j \geq 0, j = 1...4$ are Lagrangian multipliers corresponding to the respective constraints.

3.3.1 Unlimited store space

We first consider the case when the store area A is so large that all demand can be fulfilled through the in-store channel. Since in this case Constraint (3.19b) is always inactive, the first-order conditions lead to

$$\frac{\partial \mathcal{L}}{\partial q_s} = (v - c_p)\left(1 - F\left(\frac{q_s}{x_s}\right)\right) - c_s + \phi_2 = 0; \tag{3.21a}$$

$$\frac{\partial \mathcal{L}}{\partial q_w} = -c_w - c_d \tau_w + h_l\left(1 - F\left(\frac{q + q_w}{x_m}\right)\right) + \phi_3 = 0; \tag{3.21b}$$

$$\frac{\partial \mathcal{L}}{\partial q} = -c_d \tau + h_l\left(1 - F\left(\frac{q + q_w}{x_m}\right)\right) + \phi_4 = 0 \tag{3.21c}$$

Per Assumption 2.2, we can show $\phi_4 > \phi_3 \geq 0$. Thus, $q = 0$. That is, if the retailer has access to an unlimited store area, it makes no sense to fulfill the membership channel from DC. If q_w or q_s is also zero, the problem is reduced to a trivial one, since only one channel (store or membership) will be used. We thus focus on the case where $q_w > 0$ and $q_s > 0$. This leads to the following solution

$$\frac{q_s^*}{x_s} = F^{-1}\left(1 - \eta\right); \quad \frac{q_w^*}{x_m} = F^{-1}\left(1 - \sigma\right); \tag{3.22}$$

$$\eta \equiv \frac{c_s}{v - c_p}; \quad \sigma \equiv \frac{c_d \tau_w + c_w}{h_l}. \tag{3.23}$$

Given any $\mathbf{x}'$, Condition (3.22) suggests both q_s^*/x_s and $(q_w^*)/x_m$ must be a constant in order to maximize the profit. Thus, it dictates the retailer's best response to a given user choice pattern, provided the distributional information about the demand (i.e., $F(\cdot)$) is known. Since y_s^* and y_m^* are functions of q_s^*/x_s and $(q_w^*)/x_m$ respectively (cf. Eq. (3.5)), they are determined accordingly. Let us introduce $\beta_s^* \equiv q_s^*/x_s$ and $\beta_m^* \equiv (q_w^*)/x_m$, and interpret β_s^* (β_m^*) as the total number of customers needed to exhaust the shelf (same-day delivery) capacity, corresponding to the choice probability x_s (x_m). Eq.((3.22)) suggests β_m^* only depends on the ratio between the unit storage and delivery cost from WH ($c_d \tau_w + c_w$) and the extra waiting cost h_l. The smaller the ratio, the greater the optimal capacity for the membership channel. When $c_d \tau_w + c_w > h_l$ (i.e., $\sigma > 1$), the membership channel should be abandoned because it is too expensive to operate relative to the value it creates. Similarly, β_s^* is determined by the ratio between c_s (in-store inventory cost) and $v - c_p$ (the profit potential). If $\eta > 1$ (i.e., the shelf inventory cost is higher than the gross profit potential), the in-store channel will not be used. The larger the inventory cost relative to the profit potential, the less attractive the store channel. If the total cost exceeds the profit potential, then the store channel should not be used at all.

Thus, for the given $\mathbf{x}'$, the retailer's decisions are summarized as

$$q_s = \beta_s^* x_s, q_w = \beta_m^* x_m, q^* = 0, r^* = y_m^* h_l, p^* = v - \rho/y_s^*. \tag{3.24}$$

Thus, the optimal membership premium r^* and in-store price p^* are independent of customer choice in this case. Also, since $y_i^*(i = m, s)$ increases with β_i^* per Eq. (3.5), so are r and p. q_i^*, on the other hand, is proportional to x_i. Replacing $\mathbf{q}$ in Eq. (3.19a) with $q_s^* = \beta_s^* x_s, q_m^* = \beta_m^* x_m, q^* = 0$, we arrive at

$$\max g(\mathbf{x}') = (v - c_p)y_s^* x_s \bar{d} + (\alpha - \rho)x_s \bar{d} - (c_s\beta_s^* x_s + c_w\beta_m^* x_m) + (x_m + \gamma(1 - x_m - x_s))\psi\bar{d}$$
$$+ h_l y_m^* x_m \bar{d} - c_d \tau_w \beta_m^* x_m \tag{3.25}$$

subject to:
$$x_m + x_s \leq 1; x_m \geq 0, x_s \geq 0. \tag{3.26}$$

The objective function is linear in both x_s and x_m, and its derivatives with respect to them are

$$\frac{\partial g}{\partial x_s} = (v - c_p)\bar{d}y_s^* + (\alpha - \rho)\bar{d} - c_s\beta_s^* - \gamma\bar{d}\psi; \tag{3.27a}$$

$$\frac{\partial g}{\partial x_m} = (1 - \gamma)\bar{d}\psi + h_l\bar{d}y_m^* - (c_d\tau_w + c_w)\beta_m^*. \tag{3.27b}$$

Furthermore, the difference between the two is

$$\Delta = \frac{\partial g}{\partial x_s} - \frac{\partial g}{\partial x_m} = \left[\bar{d}((v - c_p)y_s^* + \alpha) - (\bar{d}\rho + c_s\beta_s^*)\right] - \left[\bar{d}(\psi + r^*) - \sigma h_l\beta_m^*\right]. \tag{3.28}$$
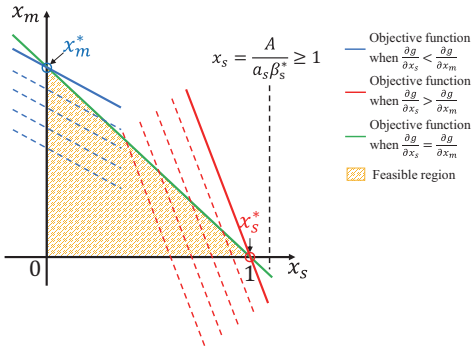
Figure 3.3: Optimal channel distribution with unlimited store space.

Here in the first and second brackets are the marginal net profits of in-store and membership channels, respectively. In each bracket, the first term represents the marginal gain, and the second the marginal cost. For the in-store channel, for example, the gain is the sum of cross-sale ($\bar{d}\alpha$) and the gross profit ($\bar{d}(v - c_p)y_s^*$), whereas the cost comes from inventory ($c_s\beta_s^*$) and the price pressure due to the competition from the online channel ($\bar{d}\rho$).

To summarize, the optimal channel choice must be all or nothing with unlimited store area, depending on the sign of Δ in Eq. (3.28). If $\Delta > 0$, everyone will be directed to the in-store channel; otherwise, the membership channel dominates; see Figure 3.3 for an illustration. When $\Delta = 0$, the channel choice becomes arbitrary.

3.3.2 Limited store space

If the demand is large enough to exhaust the store area, the resource constraint (3.19b) will always be activated, because the store is a preferred location for fulfilling both the in-store and the membership channel (Assumption 2.2). Ignoring the trivial case where $q = 0$, we have $\phi_1 > 0, \phi_2 = \phi_3 = \phi_4 = 0$ in Eq. (3.20). Applying the first-order conditions then leads to the following results:

$$\phi_1 = \frac{c_d(\tau - \tau_w) - c_w}{a_w} > 0; \frac{q_s^*}{x_s} = F^{-1}(1 - \mu_1\eta); q_w^* = \frac{A - a_s q_s^*}{a_w}; \frac{q_w^* + q^*}{x_m} = F^{-1}(1 - \mu_0\sigma);$$
(3.29)

where η and σ are defined in Eq. (3.23) and μ_0 and μ_1 are defined in Eq. (3.15). Similarly, the retailer's optimal response is to maintain $\beta_s^* = q_s^*/x_s$ and $\beta_m^* = (q_w^*+q^*)/x_m$ at a constant level. The only difference is that the activated resource constraint moves the position of β_s^* and β_m^* away from the upper bound of the demand distribution (note that both μ_1 and μ_0 are larger than 1 per definition).

We then rewrite the decision variables for the given $\mathbf{x}'$ as

$$q_s = \beta_s^* x_s; q_w = \frac{A - a_s \beta_s^* x_s}{a_w}; q = \beta_m^* x_m - q_w; r^* = y_m^* h_l; p^* = v - \rho/y_s^*, \quad (3.30)$$

and transform Problem (3.19) into the following:

$$g(\mathbf{x}') =(v - c_p)y_s^* x_s \bar{d} + (\alpha - \rho)x_s\bar{d} - \left(c_s\beta_s^* x_s + c_w\frac{A - a_s\beta_s^* x_s}{a_w}\right) + ((1 - \gamma)x_m + \gamma - \gamma x_s)\bar{d}\psi$$

$$+ h_l y_m^* x_m \bar{d} - c_d\left((\tau_w - \tau)\frac{A - a_s\beta_s^* x_s}{a_w} + \tau\beta_m^* x_m\right).$$
(3.31)

Again, this is a linear function of x_s and x_m, with the marginal profit for each being specified as

$$\frac{\partial g}{\partial x_s} =(v - c_p)\bar{d}y_s^* + (\alpha - \rho)\bar{d} - c_s\beta_s^* + \frac{c_w a_s\beta_s^*}{a_w} - \gamma\bar{d}\psi - \frac{c_d a_s\beta_s^*(\tau - \tau_w)}{a_w}$$

$$=(v - c_p)\bar{d}y_s^* + (\alpha - \rho)\bar{d} - \gamma\bar{d}\psi - \mu_1 c_s\beta_s^*; \quad (3.32a)$$

$$\frac{\partial g}{\partial x_m} =(1 - \gamma)\bar{d}\psi + h_l\bar{d}y_m^* - c_d\tau\beta_m^*. \quad (3.32b)$$

The difference between the two marginal profits reads

$$\Delta = \frac{\partial g}{\partial x_s} - \frac{\partial g}{\partial x_m} = \left[\bar{d}((v - c_p)y_s^* + \alpha) - (\bar{d}\rho + \mu_1 c_s\beta_s^*)\right] - \left[\bar{d}(\psi + r^*) - \mu_0\sigma h_l\beta_m^*\right].$$
(3.33)

Comparing it to Eq. (3.28), we can see the two expressions are nearly identical except both cost items in Eq. (3.28) ($c_s\beta_s^*$ and $\sigma h_l\beta_m^*$) are scaled by a constant larger than 1, which is related to the relative advantage of fulfilling membership orders from WH (see Eq. 3.15).
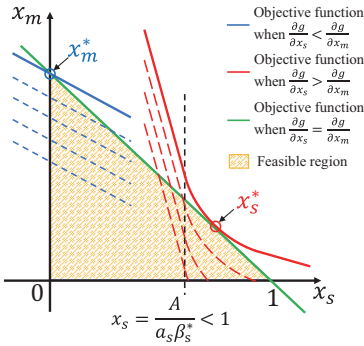
Figure 3.4: Optimal channel distribution with limited store space.

If $\Delta < 0$, the retailer prefers to direct a greater share of customers to the membership channel, because it has a larger marginal profit. Since there is no hard constraint on the capacity of that channel, the optimal solution is obtained when everyone uses it, i.e., $x_m^* = 1, x_o^* = x_s^* = 0$ (the top left corner of the feasible region in Figure 3.4).

The calculus is different when $\Delta > 0$. In this case, the retailer prefers the in-store channel, but x_s could reach such a level that it alone consumes the entire store space. This happens when $x_s = A/(a_s\beta_s^*)$. Beyond this point (i.e, on the right side of the vertical black dashed line in Figure 3.4), the retailer still has the incentive to direct more customers to the store channel as long as $\partial g/\partial x_s > \partial g/\partial x_m$. However, as more and more customers visit the store with a fixed shelf capacity, the best response strategy that holds β_s^* (hence y_s^*) as a constant can no longer be kept. Instead, we replace $\beta_s^* x_s$ in (3.31) with a constant $q_s = A/a_s$, and recognize y_s as a function of x_s. This leads to a new objective function

$$
\begin{aligned}
g(\mathbf{x}') =& (v - c_p)y_s x_s \bar{d} + (\alpha - \rho)x_s \bar{d} - c_s \frac{A}{a_s} + ((1 - \gamma)x_m + \gamma - \gamma x_s)\bar{d}\psi \\
&+ h_l y_m^* x_m \bar{d} - c_d \tau \beta_m^* x_m
\end{aligned}
\tag{3.34}
$$

This would alter the marginal profit of x_s to

$$
\frac{\partial g}{\partial x_s} = (v - c_p)\left(\bar{d}y_s + \frac{A}{a_s x_s}(F(\frac{A}{a_s x_s}) - 1)\right) + (\alpha - \rho)\bar{d} - \gamma \bar{d}\psi.
\tag{3.35}
$$

In addition, we have

$$
\frac{\partial^2 g}{\partial x_s^2} = -\frac{A^2(v - c_p)}{a_s^2 x_s^3}f(\frac{A}{a_s x_s}) < 0.
\tag{3.36}
$$

The marginal profit of x_m remains the same because the best response concerning the membership channel is not limited by the store space (hence β_m^* is still a constant).

Since $\partial^2 g/\partial x_s^2 < 0$, any additional increase in x_s beyond $A/(a_s\beta_s^*)$ is bound to reduce the marginal profit, as well as lower y_s. As one consequence, the retailer must lower the in-store price p as compensation, as shown in Eq. (3.17). There are two possible outcomes. The first occurs when $\partial g/\partial x_s$ is reduced to the level of $\partial g/\partial x_m$ before x_s reaches 1. In this case, both channels will be used at the optimum (illustrated by the red circle x_s^* in Figure 3.4). The other outcome emerges if $\partial g/\partial x_s > \partial g/\partial x_m$ even when $x_s = 1$, i.e., everyone will be directed to the in-store channel (even if the capacity is insufficient).

3.4   Special case: uniform demand distribution

In this section, we consider a special case where the demand follows a uniform distribution between $\overline{D}$ and $\underline{D}$ with a CDF $F(d)$ and a PDF $f(d)$ as follows:

$$
F(d) = \begin{cases} 0 & d < \underline{D} \\ \frac{d-\underline{D}}{\overline{D}-\underline{D}}; & \underline{D} \le d \le \overline{D} \\ 1 & d > \overline{D} \end{cases} \; ; \quad f(d) = \begin{cases} \frac{1}{\overline{D}-\underline{D}}; & \underline{D} \le d \le \overline{D} \\ 0 & else \end{cases} \tag{3.37}
$$

Accordingly, the successful shopping probabilities for in-store and membership channels, $y_i, i = s, m$, becomes

$$
y_i = \begin{cases} \frac{q_i}{x_i \bar{d}} & q_i < \underline{D}x_i; \\ 1 - \frac{(\overline{D}-q_i/x_i)^2}{\overline{D}^2 - \underline{D}^2} & q_i \in [\underline{D}x_i, \overline{D}x_i]; \quad , \forall i \in [m, s] \\ 1 & q_i > \overline{D}x_i, \end{cases} \tag{3.38}
$$

3.4.1   Unlimited store space

When $\overline{D}$ and $\underline{D}$ are sufficiently small relative to A, the store space can be viewed as unlimited. From Eqs. (3.22), and (3.37), we have

$$
\beta_s^* = (1-\eta)\overline{D} + \eta\underline{D}; \quad \beta_m^* = (1-\sigma)\overline{D} + \sigma\underline{D}. \tag{3.39}
$$

Clearly, the smaller η or σ, the more profitable and attractive their respective channel. The validity of the formula requires both η and σ are less than 1, for otherwise, their respective channel becomes nonprofitable. The marginal profits of the two channels (Eq. (3.27)) become

$$
\frac{\partial g}{\partial x_s} = (v - c_p)\mathcal{Q}(\eta) + (\alpha - \rho)\bar{d} - \gamma \bar{d}\psi; \tag{3.40a}
$$

$$
\frac{\partial g}{\partial x_m} = h_l \mathcal{Q}(\sigma) + (1-\gamma)\bar{d}\psi, \tag{3.40b}
$$

where $\mathcal{Q}(\cdot)$ is a quadratic function taking the following form

$$
\mathcal{Q}(w) = \frac{(\overline{D}-\underline{D})}{2}w^2 - w\overline{D} + \bar{d}. \tag{3.41}
$$

It is easy to verify that $\mathcal{Q}(w)$ is a decreasing function of w when $w \in [0,1]$. More specifically, when $w = 1$ (the direct cost of operating a channel equals the profit potential), $\mathcal{Q}(\cdot) = 0$; and when $w = 0$ (the direct cost of operating a channel is zero), $\mathcal{Q}(\cdot) = \bar{d}$. Assume the most favorable conditions for both channels (i.e., $\sigma = \eta = 0$) and replace ψ and ρ with their respective definitions, we have

$$
\Delta = \frac{\partial g}{\partial x_s} - \frac{\partial g}{\partial x_m} = \bar{d}((1-\theta)(v - c_p + c_r) + \alpha - h_s), \tag{3.42}
$$

where $1 - \theta$ is the return rate of shopping online and c_r is the unit return cost. As expected, excluding the considerations for costs (inventory and delivery), the advantage of the store channel derives primarily from cross-sale and lower return rates.

3.4.2   Limited store area

When $\overline{D}$ and $\underline{D}$ are large enough to exhaust all store space, the assumption of uniform distribution leads to

$$\beta_s^* = (1 - \mu_1\eta)\overline{D} + \mu_1\eta\underline{D}; \quad \beta_m^* = (1 - \mu_0\sigma)\overline{D} + \mu_0\sigma\underline{D}. \tag{3.43}$$

Accordingly, Eq. (3.32) becomes

$$\frac{\partial g}{\partial x_s} = (v - c_p)\mathcal{Q}(\mu_1\eta) + (\alpha - \rho)\bar{d} - \gamma\bar{d}\psi; \tag{3.44a}$$

$$\frac{\partial g}{\partial x_m} = h_l\mathcal{Q}(\mu_0\sigma) + (1 - \gamma)\bar{d}\psi. \tag{3.44b}$$

Note that

$$\mu_1 - \mu_0 = \frac{c_d\tau_w}{c_w}\left(\frac{c_d\tau}{c_d\tau_w + c_w} - 1\right) > 0. \tag{3.45}$$

Suppose the retailer can reduce the delivery time τ from DC while keeping everything else equal. This will reduce both μ_0 and μ_1, hence benefit both channels. This result is a bit counter-intuitive since one is inclined to think reducing the DC delivery time should impact the store channel negatively. However, μ_1 decreases more since $\mu_1 - \mu_0$ decreases with τ according to Eq. (3.45). Thus, paradoxically, improving the delivery time from DC would help the in-store channel, not the membership channel.

If $\Delta < 0$, the retailer will abandon the store channel completely, and use the entire store space as WH to fulfill online orders. Otherwise, substituting y_s in Eq. (3.35) with Eq. (3.38) leads to

$$\frac{\partial g}{\partial x_s} = \begin{cases} (\alpha - \rho)\bar{d} - \gamma\bar{d}\psi, & \beta_s < \underline{D}; \\ (v - c_p)\frac{\beta_s^2 - \underline{D}^2}{2(\overline{D} - \underline{D})} + (\alpha - \rho)\bar{d} - \gamma\bar{d}\psi, & \beta_s \in [\underline{D}, \overline{D}] \\ (v - c_p)\bar{d} + (\alpha - \rho)\bar{d} - \gamma\bar{d}\psi, & \beta_s > \overline{D} \end{cases} \tag{3.46}$$

where $\beta_s = q_s/x_s = A/(a_s x_s)$. Obviously, $\partial g/\partial x_s$ is a continuous monotonically increasing function of β_s when $\beta_s \in [\underline{D}x_s, \overline{D}x_s]$, and a constant when $\beta_s < \underline{D}$ and $\beta_s > \overline{D}$.

Note that $\beta_s > \overline{D}$ cannot be optimal because reserving a capacity that exceeds the absolute upper bound of the demand will produce redundancy. The retailer can always bring its inventory down from that level without losing any revenue. Suppose instead $\beta_s < \underline{D}$ at the optimum. In this case, the value of x_s has reached the maximum (i.e., 1, or everyone is directed to the in-store channel) and $\partial g/\partial x_s$ has

reached the lower bound but is still no less than $\partial g / \partial x_m$ (in-store channel is still more attractive than other channels). It follows that $(\alpha - \rho)\bar{d} - \gamma \bar{d}\psi \geq \partial g / \partial x_m \to \alpha \bar{d} \geq \partial g / \partial x_m + \gamma \bar{d}\psi + \rho \bar{d} \to \alpha \geq \psi + \rho$. Thus, for this corner case to materialize, in-store cross-sale profit (α) must be so large that it exceeds the sum of the gross profit of the online channel (ψ), and the customer gain of the in-store channel (ρ). This condition is unlikely to hold in practice.

We finally consider the case where β_s lies in $[\underline{D}, \overline{D}]$, i.e., the share of the in-store channel will reach a demarcation point such that $\partial g / \partial x_s = \partial g / \partial x_m$. This condition leads to

$$\hat{\beta}_s = \sqrt{\frac{2(\overline{D} - \underline{D})}{v - c_p} \left((\rho - \alpha + \psi)\bar{d} + h_l \mathcal{Q}(\mu_0 \sigma) \right) + \underline{D}^2}. \tag{3.47}$$

The interior solution can be summarized as

$$x_o = 0, x_s = \frac{A}{a_s \hat{\beta}_s}, x_m = 1 - \frac{A}{a_s \hat{\beta}_s}; \tag{3.48a}$$

$$q_s = \frac{A}{a_s}, q_w = 0, q = \left(1 - \frac{A}{a_s \hat{\beta}_s} \right) \left((1 - \mu_0 \sigma)\overline{D} + \mu_0 \sigma \underline{D} \right). \tag{3.48b}$$

3.5   Access to store inventory information

Many retailers have begun to offer various Buy-Online and Pickup-in-Store (BOPS) options in recent years. Such an option gives the customer access to in-store inventory information, which can in theory reduce the probability of encountering an unexpected stock-out event to zero. Thus, the information can make the in-store channel more attractive to customers. To the retailer, however, it is a mixed blessing. On the one hand, a better shopping experience in the store can boost sales there. On the other hand, as customers will shy away from the store once the stock-out occurs, the benefits from cross-sales are bound to drop (even if the retailer can sell more of the product through the store channel).

In this section, we assume the retailer makes the inventory information freely available to all customers through its e-commerce platform. This assumption changes the utility of the in-store channel to the following

$$u_s = (v - p) - h_s. \tag{3.49}$$

Since the customer only visits the store if they find the product is available, the probability of shopping success y_s is always 1. Although we do not analyze it here, we note the utility of a BOPS option would take a similar form but impose a different (potentially smaller as the search cost is avoided) hassle cost. By setting $u_s = u_o$, we find the in-store price now reads

$$p = v - \rho, \tag{3.50}$$

where ρ is defined in Eq.(3.18). The objective of the retailer's design problem becomes

$$\max g(\mathbf{q}, \mathbf{x}) = (v - \rho + \alpha - c_p)y_s x_s \bar{d} - (c_s q_s + c_w q_w) + (x_m + \gamma(1 - x_m - x_s))\psi\bar{d}$$
$$+ h_l y_m x_m \bar{d} - c_d(\tau_w q_w + \tau q) \tag{3.51}$$

Compared to the objective of the original Problem (3.19), the main difference concerns the effect of cross-sale: it is limited by the in-store stock volume $(y_s x_s \bar{d})$ rather than the in-store customer volume $(x_s \bar{d})$. Put in another way, these two quantities are identical, now the access to information is provided ($y_s = 1$).

3.5.1   Unlimited store area

When A is sufficiently large, the first-order conditions of the retailer's design problem with the objective Eq.3.51 lead to

$$\beta_s^* = \frac{q_s^*}{x_s} = F^{-1}(1 - \bar{\eta}); \quad \beta_m^* = \frac{q_w^*}{x_m} = F^{-1}(1 - \sigma); \tag{3.52}$$

$$\bar{\eta} \equiv \frac{c_s}{v - \rho + \alpha - c_p}; \quad \sigma \equiv \frac{c_d \tau_w + c_w}{h_l}. \tag{3.53}$$

Similarly, the result suggests the retailer's best response is to maintain constant β_s^* and β_m^*. While information availability does not change the form of β_m^*, it does change that of β_s^*. To highlight this difference, we use $\bar{\eta}$ to denote the constant defining β_s^*. Comparing $\bar{\eta}$ and η, we can see that, if the cross-sale effect is sufficiently strong (i.e., $\alpha > \rho$), $\bar{\eta} < \eta$. This means, everything else equal, β_s^* would increase (or the in-store channel becomes more attractive) after the inventory information is provided.

Replacing $\mathbf{q}$ in Eq. (3.51) with $q_s^* = \beta_s^* x_s, q_m^* = \beta_m^* x_m, q^* = 0$ yields

$$\max g(\mathbf{x}') = (v - c_p + \alpha - \rho)y_s^* x_s \bar{d} - (c_s \beta_s^* x_s + c_w \beta_m^* x_m) + (x_m + \gamma(1 - x_m - x_s))\psi\bar{d}$$
$$+ h_l y_m^* x_m \bar{d} - c_d \tau_w \beta_m^* x_m \tag{3.54a}$$

subject to:
$$x_m + x_s \le 1; x_m \ge 0, x_s \ge 0. \tag{3.54b}$$

The objective is a linear function with respect to $\mathbf{x}'$, and assuming the demand is uniformly distributed within $[\underline{D}, \overline{D}]$, the marginal profits are:

$$\frac{\partial g}{\partial x_s} = (v - c_p + \alpha - \rho)\mathcal{Q}(\bar{\eta}) - \gamma\bar{d}\psi; \tag{3.55a}$$

$$\frac{\partial g}{\partial x_m} = h_l \mathcal{Q}(\sigma) + (1 - \gamma)\bar{d}\psi, \tag{3.55b}$$

Compared to Eq. (3.40a), the marginal profit given by Eq. (3.55a) is augmented by $(\alpha - \rho)(\mathcal{Q}(\bar{\eta}) - \bar{d})$. Recall $\mathcal{Q}(\bar{\eta}) \le \bar{d}$, the effect of this change on the profitability of

the store channel depends on the sign of $\alpha - \rho$. If $\alpha - \rho > 0$, the effect is negative; otherwise, it is positive. Thus, the provision of inventory information hurts the in-store channel *if and only if* the retailer makes more money from cross-sale α than direct sale ($\rho = v - p$). Such a retailer will also react to this reduction in profitability by attempting to direct more customers to the in-store channel (since β_s^* increases after the information become available, as indicated in Eq. (3.52)).

3.5.2 Limited store area

When the store area A is not sufficiently large, the sub-game equilibrium solution given a certain $\mathbf{x}$ becomes

$$\beta_s^* = (1 - \mu_1 \bar{\eta})\overline{D} + \mu_1 \bar{\eta}\underline{D}; \quad \beta_m^* = (1 - \mu_0 \sigma)\overline{D} + \mu_0 \sigma\underline{D}. \tag{3.56}$$

Accordingly, Eq. (3.55) becomes

$$\frac{\partial g}{\partial x_s} = (v - c_p + \alpha - \rho)\mathcal{Q}(\mu_1 \bar{\eta}) - \gamma\bar{d}\psi; \tag{3.57a}$$

$$\frac{\partial g}{\partial x_m} = h_l \mathcal{Q}(\mu_0 \sigma) + (1 - \gamma)\bar{d}\psi. \tag{3.57b}$$

If $\Delta = \dfrac{\partial g}{\partial x_s} - \dfrac{\partial g}{\partial x_m} < 0$, the retailer will use the entire store space as WH to fulfill online orders. Otherwise, the marginal profit with respect to x_s becomes

$$\frac{\partial g}{\partial x_s} = (v - c_p + \alpha - \rho)\left(\bar{d}y_s + \frac{A}{a_s x_s}(F(\frac{A}{a_s x_s}) - 1)\right) - \gamma\bar{d}\psi, \tag{3.58}$$

and substituting y_s with Eq.(3.38) yields

$$\frac{\partial g}{\partial x_s} = \begin{cases} -\gamma\bar{d}\psi, & \beta_s < \underline{D}; \\ (v - c_p + \alpha - \rho)\frac{\beta_s^2 - \underline{D}^2}{2(\overline{D} - \underline{D})} - \gamma\bar{d}\psi, & \beta_s \in [\underline{D}, \overline{D}] \\ (v - c_p + \alpha - \rho)\bar{d} - \gamma\bar{d}\psi, & \beta_s > \overline{D} \end{cases} \tag{3.59}$$

In this case, neither $\beta_s > \overline{D}$ nor $\beta_s < \overline{D}$ can be optimal. That $\beta_s > \overline{D}$ cannot be optimal is, again, because reserving a capacity over the absolute upper bound of the demand does not make sense. If $\beta_s < \underline{D}$, the marginal profit turns negative ($-\gamma$), and thus cannot be optimal either. This is precisely because the retailer lost the profits generated from cross-selling to customers who would have come to the store even when the product is out of stock, had the inventory information not been available.

The solution corresponding to the case when β_s lies in $[\underline{D}, \overline{D}]$ is as follows:

$$\hat{\beta}_s = \sqrt{\frac{2(\overline{D} - \underline{D})}{v - c_p + \alpha - \rho}\left(\psi\bar{d} + h_l \mathcal{Q}(\mu_0 \sigma)\right) + \underline{D}^2}; \tag{3.60a}$$

$$x_o = 0, x_s = \frac{A}{a_s \hat{\beta}_s}, x_m = 1 - \frac{A}{a_s \hat{\beta}_s}; \tag{3.60b}$$

$$q_s = \frac{A}{a_s}, q_w = 0, q = \left(1 - \frac{A}{a_s \hat{\beta}_s}\right)\left((1 - \mu_0 \sigma)\overline{D} + \mu_0 \sigma \underline{D}\right). \tag{3.60c}$$

Heterogeneous Customers

For simplicity the base model presented in Chapter 3 assumes all customers be identical in their valuation of the product (v) and the hassle costs of different channels (h_s and h_l). It turns out, with homogeneous customers, the profit-maximizing retailer always prefers one channel to others, and in absence of hard constraints (e.g., store space), all customers will be guided to that channel. In reality, customers are anything but homogeneous. They may reside in different parts of the city, which means the time required (hence the hassle cost incurred) to travel to the store varies from one customer to another. Another source of heterogeneity is customer preferences, such as the value attached to the time spent on travel, waiting for online orders, and searching the product in the store. Allowing heterogeneous customers in the model means different channels may appeal to different demand segments. Thus, the retailer's decisions are not designed to maximize the output of one channel, but rather to achieve an optimal channel portfolio that takes advantage of customers' heterogeneous preferences.

In this chapter, we set out to add customer heterogeneity into the omnichannel retail model. To maintain tractability, we focus on the two hassle costs, h_s and h_l, allowing them to vary continuously across the population. Thus, the relative magnitude of the two costs determines the channel preference of a customer under a given omnichannel design. Throughout this chapter, we will also assume customers have access to the inventory information in the store and thus would only choose the in-store channel if the product is available.

Assumption 3. *To characterize customers' heterogeneous preferences, we assume*

1. *customers are distributed within a rectangle defined by the lower and upper bounds of the two hassle costs, i.e., $\{(h_l, h_s)|h_s \in [0, H_s], h_l \in [0, H_l]\}$ with a joint probability density function $f_h(l, s)$.*

2. *shopping with the online channel is always a viable option for any customer living in the city, i.e., $u_o \geq 0$, which dictates $H_l \leq \theta(v - p_0)$.*

4.1   Determination of market share

In the base model, the market share is determined according to a Nash equilibrium, at which identical customers will face the same utility regardless of which channel they choose. Thus, the focus there was to find a customer flow pattern that balances "congestible" channel resources to reach equilibrium. With heterogeneity, the mechanism is quite different: each user's own unique channel preference will determine, collectively, the market share of different channels.

We write the utilities of the three channels in the following and note that the in-store channel has no out-of-stock risk y_s thanks to the availability of information.

$$u_s = (v - p) - h_s; \tag{4.1a}$$

$$u_o = \theta(v - p_0) - h_l; \tag{4.1b}$$

$$u_m = \theta(v - p_0) - r - (1 - y_m)h_l. \tag{4.1c}$$

Under the assumption that a customer always selects the channel with the maximum utility for their own unique (h_s, h_l), the market share of the three channels can be determined using a 2-D diagram shown in Figure 4.1-a. In the diagram, the dashed red line marks the demarcation between the in-store channel and online channel: anyone whose (h_s, h_l) falls above (below) that line prefers the in-store (online) channel. In a similar vein, the dashed green line marks the demarcation between in-store and membership channels, and the dashed blue line between membership and online channels.

Since the three demarcation lines must intersect at the same point per definition, they divide the population into three segments, each for one channel, as highlighted by the color-shaded areas in Figure 4.1-a. That intersection corresponds to the customer indifferent to all three channels, whose hassle costs are

$$h_l(I) = \frac{r}{y_m}; h_s(I) = \frac{r}{y_m} + (v - p) - \theta(v - p_0). \tag{4.2}$$

For simplicity, this point will be referred to as *the indifferent point*, marked as I in Figure 4.1-a. The coordinates of the other points highlighted in the plot are

$$h_l(A) = 0; \qquad h_s(A) = (v - p) - \theta(v - p_0); \tag{4.3a}$$

$$h_l(B) = 0; \qquad h_s(B) = (v - p) - \theta(v - p_0) + r; \tag{4.3b}$$

$$h_l(C) = H_l; \qquad h_s(C) = (v - p) - \theta(v - p_0) + r + (1 - y_m)H_l; \tag{4.3c}$$

$$h_l(D) = H_l; \qquad h_s(D) = (v - p) - \theta(v - p_0) + H_l; \tag{4.3d}$$

$$h_l(E) = \theta(v - p_0) - (v - p); \qquad h_s(E) = 0; \tag{4.3e}$$

$$h_l(F) = \frac{\theta(v - p_0) - (v - p) - r}{1 - y_m}; \quad h_s(F) = 0. \tag{4.3f}$$

A few remarks about the diagram are in order here. First, not everyone who prefers the store channel can shop in the store. Instead, the actual flow of in-store channel

(a) Market share

(b) Impact of membership premium r

(c) Impact of store price p

(d) Impact of on-time delivery probability y_m
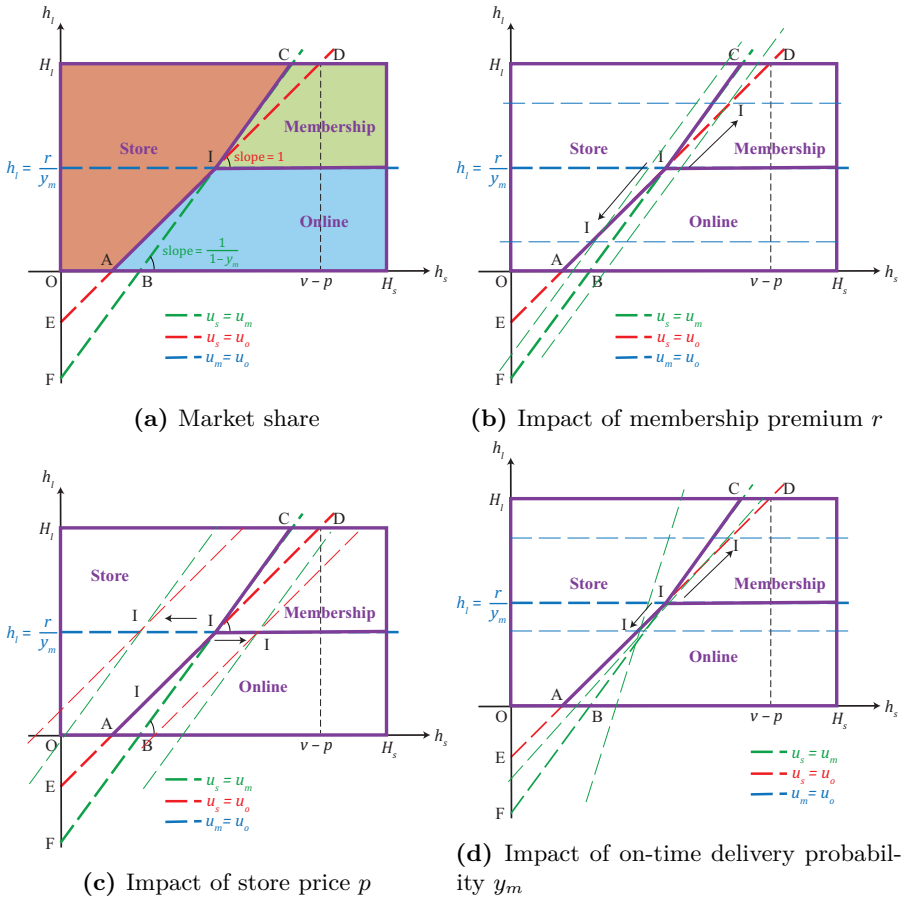
Figure 4.1: Illustration of channel choice with heterogeneous customers.

customers is capped by the store space. When it reaches the capacity, the retailer will lose those who prefer to shop in store but cannot find the product there. On the other hand, the customers who prefer the membership channel will always be served, even though not everyone will get the product delivered on time as promised (i.e., $y_m < 1$). Second, Eq. (4.2) can be used to tell how the coordinates of the indifferent point moves with the retailer's decisions. Specifically,

- A higher membership premium r will move point-I to the northeast corner along the dashed red line, leading to a smaller share for the membership channel and a larger share for both in-store and online channels, see Figure 4.1-b. When $r = y_m \theta(v - p_0)$ (i.e., when point-I, point-C, and point-D merge together), the membership share is reduced to zero. On the other hand, a lower r shifts point-I toward the southwest direction, benefiting the membership channel at

the expense of the other two. When point-I, point-A, and point-B overlap, no one will use the online channel.

- A higher (lower) store price p moves the indifferent point horizontally to the left (right), see Figure 4.1-c for an illustration. When the indifferent point moves to the left, the market share of the in-store channel shrinks, and those of the other two channels grow. Clearly, when the green line crosses the northwest corner of the feasible region, the in-store channel will be abandoned by all customers.

- The on-time delivery probability y_m for the membership channel is related to the delivery capacity t, which is related to the sum of q_w and q. A higher delivery capacity will push y_m toward 1. This will have two consequences. First, it will lower the dashed blue line. Second, it will force the dashed green line to turn counterclockwise. Together, the change will shift the indifferent point along the red line to the southwest direction.

Let S_s, S_m, and S_o be, respectively, the area corresponding to the in-store, membership and online segments highlighted in Figure 4.1-a, and $S = \sum_i S_i = H_l H_s$. Then, the market share of channel $i \in \{s, m, o\}$ can be expressed as

$$x_i = \frac{\iint_{S_i} f_h dS_i}{\iint_S f_h dS}. \tag{4.4}$$

Note that S_i can be written as a function of the retailer's decision variables (r, p, q, q_w, q_s). Thus, for a given vector of decisions, the market share $\mathbf{x}$ is uniquely determined.

4.2 Formulation of the general model

We are now ready to formulate the retailer's optimization problem as follows:

$$\max g(\mathbf{q}, p, r) = (p + \alpha - c_p)E(\min(x_s \tilde{d}, q_s)) - (c_s q_s + c_w q_w)$$
$$+ (x_m + \gamma x_o)\bar{d}\psi + r x_m \bar{d} - c_d(\tau_w q_w + \tau q) \tag{4.5a}$$

subject to:

$$a_w q_w + a_s q_s \le A; \tag{4.5b}$$
$$r \le y_m \theta(v - p_0); \tag{4.5c}$$
$$\mathbf{q} \ge 0, r \ge 0. \tag{4.5d}$$

Constraint (4.5c) imposes a natural bound on the membership premium r (no customer would use the membership channel when the upper bound is hit per the assumptions). The channel market share x_i in Eq. (4.5a) is given by Eq. (4.4). Noting y_m is an intermediate variable that has complex interactions with many other variables through Eq.(4.4), we propose to simplify the above optimization problem by

replacing q by y_m. This is achieved by invoking $q + q_w = \beta_m x_m$, and noting β_m is a function of y_m as per Eq.(3.8). It yields the following problem:

$$\max g(q_s, q_w, y_m, p, r) = (p + \alpha - c_p)y_s x_s \bar{d} - (c_s q_s + c_w q_w)$$
$$+ (x_m + \gamma x_o)\bar{d}\psi + r x_m \bar{d} - c_d(\tau \beta_m x_m - (\tau - \tau_w)q_w) \quad (4.6a)$$

subject to:

$$a_w q_w + a_s q_s \leq A; \quad (4.6b)$$
$$r \leq y_m \theta(v - p_0); \quad (4.6c)$$
$$q_s \geq 0, q_w \geq 0, r \geq 0; \quad (4.6d)$$
$$0 \leq y_m \leq 1, \quad (4.6e)$$

With the new formulation (4.6), the optimal solution can be obtained by applying the first-order optimality conditions. However, the complication is that moving decision variables shifts the demarcation lines in Figure 4.1, hence the shape, not just the area, of the three market segments. This means the retailer's profit given by (4.6a) would be a non-smooth, piece-wise function of the decision variables. Since optimizing such a function is not generally amenable to analysis, a numerical algorithm is needed to solve the most general version of Problem (4.6). We shall present such an algorithm in Section 4.4. Before we do that, let us first analyze a special version of (4.6) that has a smooth objective function.

4.3   Special case

In order to simplify the analysis of the heterogeneous model, we introduce a couple of additional assumptions.

Assumption 4. *In addition to Assumption 3, we further assume:*

1. *The demand is uniformly distributed within a range $[\underline{D}, \overline{D}]$.*

2. *Customers are uniformly distributed in the rectangle area formed by the lower and upper bounds of their hassle costs.*

3. *The retailer adopts a uniform pricing strategy, i.e., it always sets $p = p_0$.*

4. *All store space is devoted to shelf inventory. That is, $q_w = 0$.*

5. *The retailer always keeps a constant probability of on-time delivery for its membership channel. That is, y_m is fixed.*

6. *The store space A is unlimited, i.e., Constraint (4.5b) will never be activated.*

7. *$H_s = v - p_0$ and $H_l = \theta(v - p_0)$.*

Assumption 4.2 implies that the share of each channel i, defined in Eq. (4.4), can be simplified as

$$x_i = \frac{S_i}{S}. \tag{4.7}$$

Assumptions 4.3 and 4.4 each fix one of the five decision variables (p and q_w).

Assumption 4.5 fixes the level of service for the membership channel y_m. With Assumption 4.1, we can invoke Eq. (3.38) to establish a one-to-one correspondence between y_m and q/x_m, i.e.,

$$\frac{q}{x_m} = \overline{D} - \sqrt{(1 - y_m)(\overline{D}^2 - \underline{D}^2)} = \beta_m^h. \tag{4.8}$$

Since y_m is given, the decision variable q can be represented by x_m, hence is eliminated from the optimization problem. By promising an unlimited store space, Assumption 3.5 further avoids the analytical challenges posed by the complementarity condition.

We can write the utilities associated with different channels as

$$u_s = v - p_0 - h_s; \tag{4.9a}$$
$$u_o = \theta(v - p_0) - h_l; \tag{4.9b}$$
$$u_m = \theta(v - p_0) - r - (1 - y_m)h_l. \tag{4.9c}$$

Assumptions 4.7 restricts the feasible range of the hassle costs with its natural upper bounds. Note that if $h_s > v - p_0$, the store channel can never yield a positive utility. The same applies to the membership channel when $h_l > \theta(v - p_0)$. Moreover, given Assumption 4, point-E should always lie above point-F in Figure 4.1-a, and they both always lie below point-O. Also, the slopes of all three demarcation lines will remain constant. This means the shape of the three channel segments will not change with the remaining decision variables (r and q_s). Expressly, the three areas can be specified as:

$$S_o = -\frac{1}{2y_m^2}r^2 + \frac{H_l}{y_m}r; \tag{4.10a}$$

$$S_m = \frac{1 + y_m}{2y_m^2}r^2 - \left(H_l + \frac{H_l}{y_m}\right)r + \frac{(1 + y_m)H_l^2}{2}; \tag{4.10b}$$

$$S_s = H_l r + H_s H_l - \frac{(1 + y_m)H_l^2}{2} - \frac{1}{2y_m}r^2. \tag{4.10c}$$

Then, x_i can be obtained using Eq. (4.4). Clearly, in this special case, x_i only depends on r.

Now, the retailer's decision problem is to find the optimal store inventory q_s and membership premium r to maximize the profit, i.e.,

$$\max g(q_s, r) = (p_0 + \alpha - c_p)E(\min(x_s\tilde{d}, q_s)) - c_s q_s + (x_m + \gamma x_o)\bar{d}\psi + (r\bar{d} - c_d\tau\beta_m^h)x_m \tag{4.11a}$$

subject to:

$$r \leq y_m \theta(v - p_0); \tag{4.11b}$$

$$q_s \geq 0, r \geq 0. \tag{4.11c}$$

Assuming Constraint (4.11b) be inactive, the first-order condition with respect to q_s is

$$\frac{\partial g}{\partial q_s} = (p_0 + \alpha - c_p)\left(1 - F(\frac{q_s}{x_s})\right) - c_s = 0, \tag{4.12}$$

which leads to

$$\frac{q_s}{x_s} = F^{-1}\left(1 - \frac{c_s}{p_0 + \alpha - c_p}\right) = \beta_s^h. \tag{4.13}$$

Thus, regardless of the choice of r, the optimal q_s must always equal the product of a constant, β_s^h, and the market share of the in-store channel, x_s, which is a function of r. In other words, like the market share, q_s can also be expressed as a function of r. Replacing q_s in Eq. (4.11) with $\beta_s^h x_s$, we arrive at the following optimization problem with respect to r:

$$\max g(r) = \sum_{i \in \{o,s,m\}} \kappa_i x_i + \bar{d}r x_m \tag{4.14}$$

where

$$\kappa_s = (p_0 + \alpha - c_p)y_s\bar{d} - c_s\beta_s^h; \kappa_m = \psi\bar{d} - c_d\tau\beta_m^h; \kappa_o = \gamma\psi\bar{d}, \tag{4.15}$$

From Eq. (4.10) we have

$$\frac{\partial x_m}{\partial r} = \frac{(1 + y_m)r}{\theta y_m^2 H_s^2} - \frac{1}{H_s} - \frac{1}{y_m H_s}; \tag{4.16a}$$

$$\frac{\partial x_o}{\partial r} = -\frac{r}{\theta y_m^2 H_s^2} + \frac{1}{y_m H_s}; \tag{4.16b}$$

$$\frac{\partial x_s}{\partial r} = -\frac{r}{\theta y_m H_s^2} + \frac{1}{H_s}; \tag{4.16c}$$

Thus, the optimality condition requires

$$\frac{\partial g}{\partial r} = \sum_{i \in \{o,s,m\}} \kappa_i \frac{\partial x_i}{\partial r} + \bar{d}\left(x_m + r\frac{\partial x_m}{\partial r}\right) = \iota_0 r^2 + \iota_1 r + \iota_2. \tag{4.17}$$

where

$$\iota_0 = \frac{3\bar{d}}{2\theta y_m^2 H_s^2}; \tag{4.18a}$$

$$\iota_1 = \frac{\kappa_m - \kappa_o - 2\bar{d}\theta y_m H_s(1 + y_m)}{\theta y_m^2 H_s^2}; \tag{4.18b}$$

$$\iota_2 = \frac{\kappa_o + y_m \kappa_s - (1 + y_m)\kappa_m}{y_m H_s} + \frac{\bar{d}\theta(1 + y_m)}{2} \tag{4.18c}$$
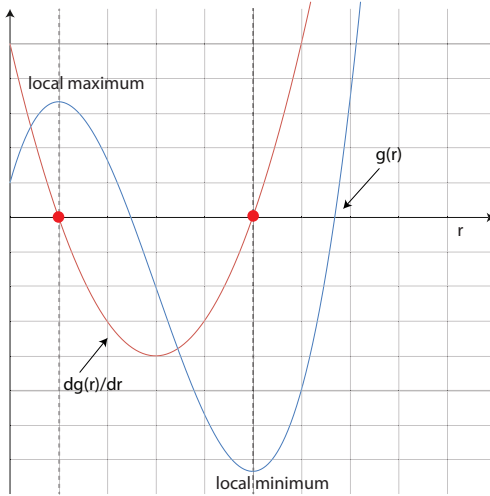
Figure 4.2: Illustration of the profit as a function of the membership premium and its derivative in the special case.

A "reasonable" system will pose several conditions on these parameters. First, ι_1 should always be negative. Otherwise, the derivative of g will continue to increase as r increases from zero, indicating the profit will indefinitely grow at a faster pace, an implausible scenario. Second, ι_2 should always be positive. Had this not been the case, the derivative of g will be negative when $r = 0$. It means, even at $r = 0$, the retailer would still lose money by slightly raising the membership premium. This could only occur if the delivery cost is so high that offering a membership channel is entirely unprofitable. Third, $\partial g/\partial r = 0$ should have two real roots, and the optimal premium is the smaller of the two. This requirement derives from the fact that the smaller root is located where $g(r)$ achieves a local maximum, see Figure 4.2 for an illustration. If $\partial g/\partial r = 0$ admits no real roots, the profit will always rise with r, never experiencing a downturn. That the smaller root must also be positive is guaranteed by the fact that $\iota_2 > 0$, as discussed above. Finally, the left root of $\partial g/\partial r = 0$ should be smaller than $y_m H_l$, or the profit would not peak even when r reaches a level that would render the membership channel unattractive to everyone.

To summarize the above discussion, the optimal solution to the special version of the retailer's problem can be obtained by

$$r^* = \frac{-\iota_1 - \sqrt{\iota_1^2 - 4\iota_0\iota_2}}{2\iota_0}. \tag{4.19}$$

In addition, the following conditions should be satisfied:

$$\iota_1 < 0, \iota_2 > 0, \iota_1^2 - 4\iota_0\iota_2 > 0, r^* < y_m H_l. \tag{4.20}$$

4.4   Solution algorithm for the general case

To solve Problem (4.6), we devise a specialized gradient ascent algorithm. The most computational challenging task is to evaluate the objective function (4.6a), which not only is non-smooth (as the function form changes with the decision variables) but also involves double integration (see Eq. (4.4)).

Figure 4.3 and Figure 4.4 enumerate all the possible functional forms for (4.6a). When $r/y_m \in (\underline{h_l}, \overline{h_l})$, there are ten different scenarios, see Figure 4.3. When $r/y_m \leq \underline{h_l}$ or $r/y_m \geq \overline{h_l}$, there are seven scenarios each, see Figure 4.4-(a) and 4.4-(b) respectively. All twenty-four cases are represented as a piecewise linear function that can be evaluated numerically. The gradient of the function is then evaluated via automatic differentiation (we use the implementation provided by PyTorch in this study).

Algorithm 1 gives the pseudo-code. Because the problem is non-convex and non-linear, the gradient ascent method can be easily trapped by local maxima. To address this issue, we always solve the problem multiple times, each starting from a randomly selected initial point. The best solution obtained in all runs is taken as an approximate global solution. In each iteration, we only move the current solution along one dimension (see line 10 in Algorithm 2), because the complex constraint structure makes it counter-productive to attempt ascending in more than one dimension.

Algorithm 2 describes a heuristic procedure aiming to choose a dimension along which the potential to improve the objective function value is the greatest. The algorithm first divides the five variables into two groups based on the constraint structure. Then, for each group, we pick from all variables that are not "stuck" (i.e., ascending along the gradient is still feasible) the one that has the largest directional derivative (line 5-12 in Algorithm 2). We then compare the two candidates from each group and make a choice based on the magnitude of their directional derivatives (lines
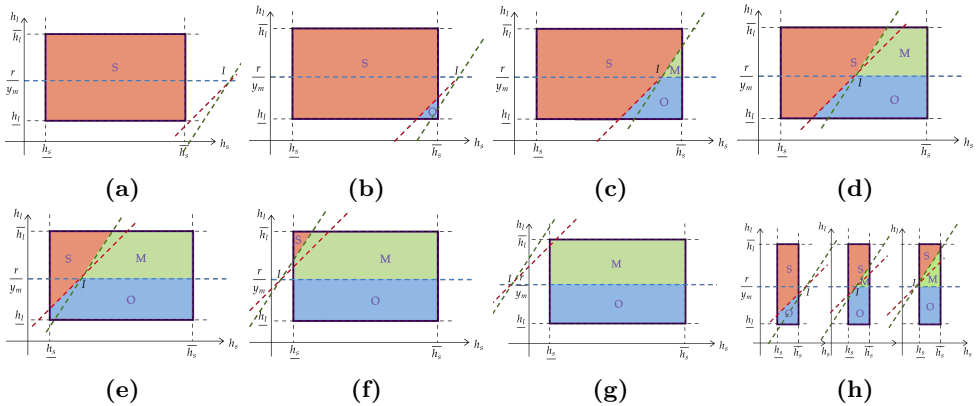
Figure 4.3: Market share when $r/y_m \in \left(\underline{h_l}, \overline{h_l}\right)$.

(a) Market share when $r/y_m \leq \underline{h_l}$.

(b) Market share when $r/y_m \geq \overline{h_l}$.
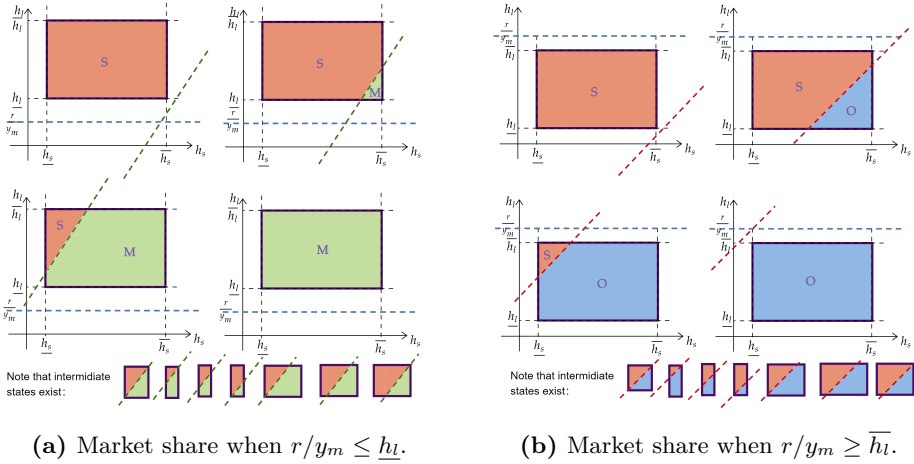
Figure 4.4: Market share when $r/y_m \notin \left(\underline{h_l}, \overline{h_l}\right)$.

13-22 in Algorithm 2).

Once the ascending dimension is determined, a line search is performed using a backtracking algorithm. If the line search fails to improve the objective function value, no change will be made, we simply set the current solution equal to the previous one. Each time the algorithm fails to make a meaningful improvement, we record it as an idle iteration. The algorithm is terminated when either the maximum number of iterations is reached or the number of consecutive idle iterations reaches a predetermined limit.

Algorithm 1 Gradient ascent algorithm for the general model.

1: **Input:** The number of random initial solutions IP, the maximum number of iterations allowed U for each initial solution, convergence criteria ϵ_1 and ϵ_2, and the maximum number of idle iterations E.
2: Set initial point indicator $ip = 0$.
3: **while** $ip < IP$ **do**
4:     **Initialization**
5:     Select a random initial solution $\mathbf{y}_0$ from the feasible region defined by (4.6b)-(4.6e).
6:     Evaluate $g_0(\mathbf{y}_0)$ using (4.6a).
7:     Set the iteration index $u = 0$, and the idle iteration index $e = 0$.
8:     **while** $u < U$ and $e < E$ **do**
9:         Set vector $\mathbf{b}_u$ such that $\mathbf{b}_u[i]$ = True if $\mathbf{y}_u[i]$ is involved in a binding constraint.
10:        Choose a decision variable identified as ith in the vector using Algorithm 2,
11:        Compute the derivative $g'_i = \frac{\partial g(\mathbf{y}_u)}{\partial \mathbf{y}_u[i]}$.
12:        Set $u = u + 1$ and the new solution $\mathbf{y}_{(u)} = \mathbf{y}_{u-1}$.
13:        **Line search**
14:        Initialize step size β.
15:        **while** $\beta > \epsilon_2$ and $g(\mathbf{y}_{(u)}) - g(\mathbf{y}_{(u-1)}) \leq 0$ **do**
16:           Update $\mathbf{y}_u[i] = \mathbf{y}_{u-1}[i] + \beta g'_i$.
17:           Update $g(\mathbf{y}_{(u)})$ and set $\beta = \beta/2$.
18:        **end while**
19:        **if** $g(\mathbf{y}_{(u)}) - g(\mathbf{y}_{(u-1)}) < 0$ **then**
20:           Set $\mathbf{y}_{(u)} = \mathbf{y}_{(u-1)}$ and update $g(\mathbf{y}_{(u)})$.
21:        **end if**
22:        **if** $|g(\mathbf{y}_{(u)}) - g(\mathbf{y}_{(u-1)})| < \epsilon_1$ **then**
23:           Set $e = e + 1$.
24:        **else**
25:           Set $e = 0$.
26:        **end if**
27:     **end while**
28:     Update the global optimal solution $\mathbf{y}^*$ with the best solution recorded for ip.
29:     Set $ip = ip + 1$.
30: **end while**
31: **Output:** $\mathbf{y}^*$.

Algorithm 2 Dimension selection method in gradient ascent

1: **Input:** The vector of constraint binding status $\mathbf{b}_u$, the gradient $\frac{\partial g_u(\mathbf{y}_u)}{\partial \mathbf{y}_u[\cdot]}$, gradient thresholds ϵ_1 and ϵ_2.

2: **Initialization:**

3: Separate the five decision variables into two groups $K_1 = ix(\{q_s, q_w\})$, $K_2 = ix(\{p, r, y_m\})$, where $ix(a)$ returns the index of variable a.

4: Initialize $i_l = -1, l = 1, 2$.

5: **for** $l = 1, 2$ **do**

6:      Set $j = 0, G = 0$.

7:      **for** each $j \in K_l$ **do**

8:          **if** $\mathbf{b}_u[j]$ is False or ascending along the gradient is feasible **then**

9:              If $\frac{\partial g(\mathbf{y}_u)}{\partial \mathbf{y}_u[j]} > G$, set $G = \frac{\partial g(\mathbf{y}_u)}{\partial \mathbf{y}_u[j]}, i_l = j$.

10:          **end if**

11:      **end for**

12: **end for**

13: **if** $i_1 = -1$ **then**

14:      Set $i^* = i_2$.

15: **else**

16:      **if** $\frac{\partial g(\mathbf{y}_u)}{\partial \mathbf{y}_u[i_2]} \geq \epsilon_2$ **then**

17:          Set $i^* = i_2$.

18:      **else if** $\frac{\partial g(\mathbf{y}_u)}{\partial \mathbf{y}_u[i_1]} \geq \epsilon_1$ **then**

19:          Set $i^* = i_1$.

20:      **else**

21:          Set $i^* = i_2$ when u is even and $i^* = i_1$ when u is odd.

22:      **end if**

23: **end if**

24: **Output:** i^*.

CHAPTER 5

Case Studies

In this chapter, we conduct case studies to better understand the trade-offs between various retail decisions and to test the sensitivity of the proposed omnichannel models to important inputs. We will start with cases where the analytical results are available for validation, and then deal with more realistic scenarios. In what follows, we first describe the settings of the experiments, including how the parameters are estimated from real-world data (Section 5.1). Then, Sections 5.2 and 5.3 report and analyze results for the base model and the general model, respectively.

5.1 Experiment settings

We assume (i) the daily demand be uniformly distributed between $\underline{D} = 4000$ to $\overline{D} = 8000$; (ii) the default price for one unit of the generic product be $p_0 = \$50$; (iii) the product be valued by all customers at $v = \$80$; (iv) the unit cost of the product (including any costs not covered by the fulfillment decisions considered in our model of omnichannel retail) be $c_p = \$40$ (this corresponds to a 20% gross profit margin when the product enters into the final fulfillment stage). We note that $p_0 = \$50$ is roughly the amount that an average household spend at Amazon and Walmart combined in a week in 2020. This number is estimated using per capital retail spending in 2020 (about $17000), the average number of persons per household (2.6), and the collective share of Amazon and Walmart in retail (about 6%). Since each household is assumed to purchase one unit per week, the retailer is serving on average $6000 \times 7 = 42,000$ households, even though on any given day, only 6000 (the average demand per assumption) actually show up in their store or online platform. Given about 123 million households in the US, we can see ours is roughly a 1/3000 model of the US retail market configured according to the combined sales of Walmart and Amazon.

We construct two representative omnichannel retailers, modeled after Walmart and Amazon, respectively, referred to hereafter as Retailer W and Retailer A. We next discuss how the model parameters are estimated for each retailer.

5.1.1 Retailer W

For the in-store channel, the hassle cost h_s is estimated based on the average travel time and fuel cost for a round trip, plus a search cost. Holmes [Hol11] estimates

the average distance between an American home and the nearest Walmart is about 6.7 miles in 2005. According to the annual report from Walmart's official website, there were 3800 stores in the U.S. in 2005. This number increased to 4743 in 2021[1]. Accordingly, we estimate the current average distance to the nearest Walmart store is about 6 miles. Moreover, each customer is assumed to set a 5-minute penalty to the search for the product. Then, h_s is estimated at \$9, using an average value of time (VOT) of \$19 (as estimated by National Bureau of Economic Research[2]), an average driving speed of 25 miles per hour (mph), and an average fuel cost of \$0.15 per mile.

For the two online channels (with or without membership), the hassle cost h_l is set according to the willingness to pay for same-day delivery, which is estimated at 10% of the product[3]. Thus, since the product is priced at \$50, the default value for h_l is \$5. We also estimate Walmart has an online-sale return rate of 20%, which is about the average across the entire e-commerce sector[4]. Thus, the parameter $\theta = 0.8$.

In 2020, Walmart had a total revenue of \$559B and spent about \$44B on inventory[5]. Thus, we estimate the inventory cost amounts to about \$2 for one unit of our generic product (assuming about half of the inventory costs are incurred in the processes covered by our model). Moreover, We conjecture the cost of storing the product on the shelf is three times as expensive as storing it in WH (due to the different requirements for space). This leads us to set $c_w = \$1$, and $c_s = \$3$. According to the inventory cost, we set the required space for on-shelf and WH storage as $a_s = 3.6m^2$ and $a_w = 1.2m^2$, respectively. This estimation of the unit storage space may seem arbitrary, but it need not concern us, since how many products the store can accommodate depends not only on the unit storage space but also on the total store area A. To "calibrate" a benchmark model, we shall determine the value of A that, when plugged into the base model, will yield the channel shares observed for Walmart (about 87% in-store sales). Since A will be determined after a_w and a_s are selected, the scale of the two parameters does not matter.

Estimating cross-sale profit is not easy. Previous studies [LSW05; VY08; SLL14; GM14] suggest retailers may generate 13% to 177% more profit from cross-selling. However, since the consumption of the generic product per household is fixed in our setting, it is difficult to explicitly model sales coming from "other" products. Instead, we shall assume cross-sales generate an extra 10% profit over the sales of the generic product (i.e., $\alpha = \$5$) – that is, the total sales remain the same, but the cross-sale effect helps boosts the profit margin. We will test the impact of α in the sensitivity analysis. To estimate the return cost c_r, we refer to Amazon's flat return charge per shipment, which ranges from \$2 to \$8[6]. Since we could not find Walmart's policy on

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[1]https://stock.walmart.com/investors/financial-information/annual-reports-and-proxies/default.aspx

[2]https://www.nber.org/papers/w28208.

[3]https://www.statista.com/statistics/1011198/willingness-to-pay-for-same-day-delivery-from-outdoor-specialty-retailers-us/

[4]https://www.invespcro.com/blog/commerce-product-return-rate-statistics/

[5]https://www.macrotrends.net/stocks/charts/WMT/walmart/revenue

[6]https://www.amazon.com/gp/help/customer/display.html?nodeId=GXM7UWCH63ZJHAVP

return charges, we set c_r to a conservative value of \$3 in the range given by Amazon.

The unit delivery cost c_d is estimated as the driver's wage plus the fuel cost per hour. The wage for a Walmart driver is set to \$20[7]. As for the fuel cost, we estimate an average delivery speed of 15 miles/hour at a fuel efficiency of 10 miles/gallon and a fuel price of \$4/gallon. Thus, $c_d = 18 + 1.5 \times 4 = \$24$/hour. To estimate the delivery time per order (τ_w and τ), we first note Amazon hires about 250,000 drivers, and according to information found on the internet, each driver delivers about 100-200 packages per day. Suppose each driver works 10 hours a day and recalls Amazon's 2021 revenue is about \$450B. We can estimate the value of each package at about \$33 and delivering a package costs 5 minute delivery time (assume 120 packages delivered for 10 hours). Thus, to deliver our unit product valued at \$50 requires $50/33 \times 5 = 7.5$ minutes. The vast majority of Amazon's packages should be delivered from DC. Since Walmart has much fewer DCs than Amazon, the value of its per capita delivery time from DC should be larger than 7.5 minutes. However, without a spatial model, it is difficult to estimate the difference with proper justification. For now, we will set $\tau = 10$ minutes and $\tau_w = 4.8$ minutes for Walmart.

The default parameter values for Retailer W are reported in Table 5.1.

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[7] https://www.indeed.com/cmp/Walmart/salaries/Delivery-Driver

Table 5.1: Default values of main parameters used in the experiments for Retailer W.

Variable	Default Value	Description
h_s	\$9	hassle cost for in-store shopping
h_l	\$5	waiting cost for normal online shopping.
p_0	\$50	original price of the product (for online shopping).
v	\$80	customer's valuation for the product.
c_p	\$40	cost to acquire the product.
$\overline{D}$	8000	upper bound of the total demand.
$\underline{D}$	4000	lower bound of the total demand.
α	\$10	cross-sale profit.
c_s	\$3	in-store inventory cost.
c_w	\$1	inventory cost for the mini-warehouse.
c_r	\$3	return cost per order.
c_d	\$26/hour	unit delivery cost (labor and fuel).
γ	0.5	retailer's online shopping market share.
θ	0.8	probability that an online order is not returned.
τ_w	4.8 minutes	delivery time per order from mini-warehouse.
τ	10 minutes	delivery time per order from DC.
a_s	3.6 m^2	area occupied by a unit of product in store.
a_w	1.2 m^2	area occupied by a unit of product in the mini-warehouse.

5.1.2   Retailer A

Retailer A differs from Retailer W *only* in the following aspects.

1. The number of store locations operated by Amazon is about one-eighth that by Walmart. Accordingly, the average distance between a customer and the nearest store for Retail A is assumed to be 50% more than that for Retailer W. Thus, h_s is set to $12 for Retailer A.

2. Retailer A requires less delivery time per order from DC because it has a greater number of DCs than Retailer W. As explained earlier, τ is set to 7.5 minutes for Retailer A.

3. Retailer A enjoys a lower-than-average online order return rate because it has more experience with e-commerce. The average return rate of Amazon in 2021[8] was around 5% to 15%. Accordingly, we set $\theta = 0.9$ for a return rate of 10%.

4. Retailer A boasts a greater supply chain efficiency. We estimate this advantage be translated to a 50% reduction in the per capita return cost. Accordingly, c_r is set to $2.

5. Retailer A's delivery driver makes $18 an hour based on Glassdoor [9], about $2 less than that for Walmart couriers. Thus, c_d is set to $24 per hour.

Table 5.2 summarizes the values of all five differentiating parameters for each retailer.

5.2   Base model

In Section 5.2.1, we first calibrate the model for Retailer W, or Model-W in short — by matching the model-produced channel shares with real-world observations — to estimate the floor space A compatible with the scale of our model (1/3000). Using

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[8]https://www.envisionhorizons.com/blog/using-amazon-return-data-to-your-advantage-and-reducing-return-rates

[9]https://www.glassdoor.com/Hourly-Pay/Amazon-Delivery-Driver-Hourly-Pay-E6036_D_KO7,22.htm

Table 5.2: Parameters that take different values for Retailers A and W.

Variables	Retailer A	Retailer W
In-store shopping hassle cost (h_s)	$12	$9
Return cost per order (c_r)	$2	$3
Delivery cost per hour (c_d)	$24	$26.
Return rate ($1 - \theta$)	0.1	0.2
Delivery time per order from DC (τ)	7.5 minutes	10 minutes

the calibrated parameter A we then build Model-A and compare it with Model-W in Section 5.2.2. Section 5.2.3 conducts sensitivity analyses on several critical parameters.

5.2.1 Benchmark

The current share of Walmart's online sales is 13% of the total sales. We found, for the given demand range and other parameters, when the store area $A = 22800$ square meters, the model yields a share of about 13% for the online channel.

The optimal decision variables, the channel shares, and the optimal profit of the calibrated Model-W (treated as the benchmark) are given below:

$$\text{Decisions:} \quad q_s^* = 6333, q_w^* = 0, q^* = 579, p^* = 51.69, r^* = 3.70;$$
$$\text{Channel share:} \quad x_s^* = 0.87, x_m^* = 0.13, x_o^* = 0;$$
$$\text{Profit:} \quad g^* = \$73574.14;$$

To maximize profit, Retailer W should use all store space for in-store shopping inventory ($q_s^* = 22800/3.6 = 6333$) and maintains a same-day delivery capacity of $q^* + q_w^* = 579$ per day. The in-store and membership channels will on average each attract, respectively, 5220 and 780 customers per day. The stock-out probability in the store is 1.0% ($1 - y_s^*$) and the on-time delivery rate for the membership channel is 74.0% (y_m^*). Retailer W has to levy a 3% markup for its in-store sales, in order to offset the relatively high inventory cost. The membership premium r is $3.7 per week or about $15/month. This is slightly higher than what Walmart Plus is charging for its membership ($12.95).

In total, Retailer W generates a profit of $73574.14 per day, translated to $1.75 per household or $3.23 per square meter of store floor space. Using this profit, we can estimate Retailer W's annual total profit amounts to $73574.14 \times 365 \times 3000 \simeq \$80\text{B}$. To put these numbers in perspective, Walmart turned a total profit of about $130B in 2020 and its U.S. market sales approximately account for 70% of the total sales. Thus, we estimate Walmart's total profit generated by the U.S. market is about $91B, fairly close to our estimate. Also, Walmart has about 4700 stores in the US, with an average floor space of 16740 square meters (180000 square feet) per store. Thus, its profit per square meter of store floor space per day is about $3.17, which is, again, close to our estimate.

5.2.2 Retailer W vs. Retailer A

For Retailer-A, its floor space is estimated to be about one-eighth of that for Retailer-W, which amounts to $A = 2850$ square meters for Retailer-A.

The results of the main decision variables for both retailers are reported in Table 5.3. We find Retailer A abandons the in-store channel altogether, preferring instead to use all store space for the fulfillment of orders from the membership channel. The

Table 5.3: Optimal solutions for Retailer A and Retailer W.

Retailer	x_s	x_m	x_o	q_s	q_w	q	g	p	r
A	0	1	0	0	2250	3350	$62580.0	$0	$4.4
W	0.87	0.13	0	6333	0	579	$73574.14	$51.69	$3.70

retailer would make about $11000, or 15%, less profit than Retailer W. At $4.4 a week (roughly $18/month[10]), its membership premium is almost 20% higher than Retailer W's. Evidently, Retailer A prefers the membership channel to the in-store channel because it enjoys a lower return rate (10% vs 20% for Retailer W) for online orders and a higher delivery efficiency (both c_d and τ are smaller).

Table 5.4 details and compares the revenue and cost components associated with each channel for both retailers. Both retailers would yield about $300k in revenue per day. This number scales up roughly to an annual revenue of $328.5B. In comparison, Amazon's total revenue in 2020 is about $386B, which presumably includes sales outside the U.S. For Retailer A, the income from membership sales amounts to about 10% of the total revenue, an order of magnitude higher than that for Retailer W.

Retailer W spends much more on inventory (almost an order of magnitude higher) whereas Retailer A spends much more on delivery and reverse logistics (about 5 times higher). Adding inventory, delivery, and return costs together amount to about 6% and 7.4% of total sales for Retailers A and W, respectively. Thus, Retailer A has a leaner operation, thanks to its much smaller physical footprint. However, Retailer

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[10]As a comparison, Amazon currently charges $15 for Amazon Prime membership.

Table 5.4: Revenue and cost table for Retailer A and Retailer W.

Revenue	Retailer A	Retailer W
Store sales revenue	$0	$266964.24
Online (membership) sales revenue	$270000	$31154.62
Online (membership) fee revenue	$26400	$2879.91
Total sales revenue	$296400	$300998.77
Cost		
Inventory cost	$2250	$19000
Delivery cost	$14370	$2561.28
Return cost	$1200	$467.32
Acquisition cost	$216000	$231501.70
Total Cost	$233820	$253530.30
Profit		
Cross-sale profit	$0	$26105.67
Total profit	$62580	$73574.14
Gross profit ratio	21.1%	24.4%
Inventory cost ratio	0.8%	6.3%
Delivery cost ratio	4.8%	0.9%
Return cost ratio	0.4%	0.2%

W has a higher profit margin: 24.4% compared to 21.1% for Retailer A. At first glance this result is puzzling. How could a seemingly more efficient business be less profitable? A closer look reveals the culprit is cross-sales. Abandoning the in-store channel means Retailer A forgoes all potential benefits from cross-sales. These benefits can be substantial. Indeed, at the default level set in our model ($5 earned profit per store visit), more than a third of all profits earned by Retailer W are attributed to cross-sales. Without this extra profit, the in-store channel would not have been chosen by Retailer W.

5.2.3 Sensitivity analysis

Given the importance of cross-sales benefits and the difficulty of directly measuring it in practice, this section performs a sensitivity analysis on the parameter α (the profit in dollar values per store visit generated from cross-sales). We also test the model's sensitivity to the online return rate $(1 - \theta)$ and the gasoline price. The latter is not a parameter of our model but affects the estimation of in-store hassle cost h_s and the hourly delivery cost c_d. In each case, the sensitivity results for both Retailers W and A are reported and compared.

Figure 5.1 shows how the main model outputs, including channel share (plots (a)-(b)), pricing (plots (c)-(d)), inventory (plots (e)-(g)) decisions and total profit (plot (h)) vary for both retailers as α increases from 0 to $20. The most important observation from Figure 5.1 is the existence of a "lower bound" for α, over which each retailer's preference would switch from the membership channel to the in-store channel. For Retailer W, the threshold is just shy of $5, whereas it is about $5.6 for Retailer A. In other words, to make the in-store channel attractive, the cross-sale effect needs to be stronger for Retailer A than Retailer W, but the discrepancy is relatively small. After α exceeds that threshold, the share of the instore channel continues to climb (accompanied by a slow decline in the price in-store, see Figure 5.1-(c)), though much more lowly for Retailer A than for Retailer W (see Figure 5.1(a-b). For Retailer W, there is actually an "upper bound" on α (about $16), at which the share of the in-store channel maxes out at 1.0. Thus, if the cross-sale effect is sufficiently large, Retailer W would prefer to have everyone in the store. This does not mean it has enough space to meet all demands all the time. Instead, the strategy simply generates enough cross-sale profits to offset the sales lost to stock-out.

When α lies below their respective lower bound, both retailers will use all store space to fulfill online orders (Figure 5.1(e-f)). In fact, this is the only fulfillment channel for Retailer W in this case, since its physical space is sufficiently large. On the other hand, fulfilling orders from DC is critical to Retailer A, but not so much to Retailer W (Figure 5.1(g)). Indeed, Retailer W would only fulfill orders from DC when α lies between the lower ($4.75) and upper ($15) bounds.

Finally, Figure 5.1(d) indicates the membership premium is not affected by α at all. As for the total profit, we note (see Figure 5.1(h)) (i) it gradually increases with α after it exceeds the lower bound; (ii) when α is below the threshold, Retailer
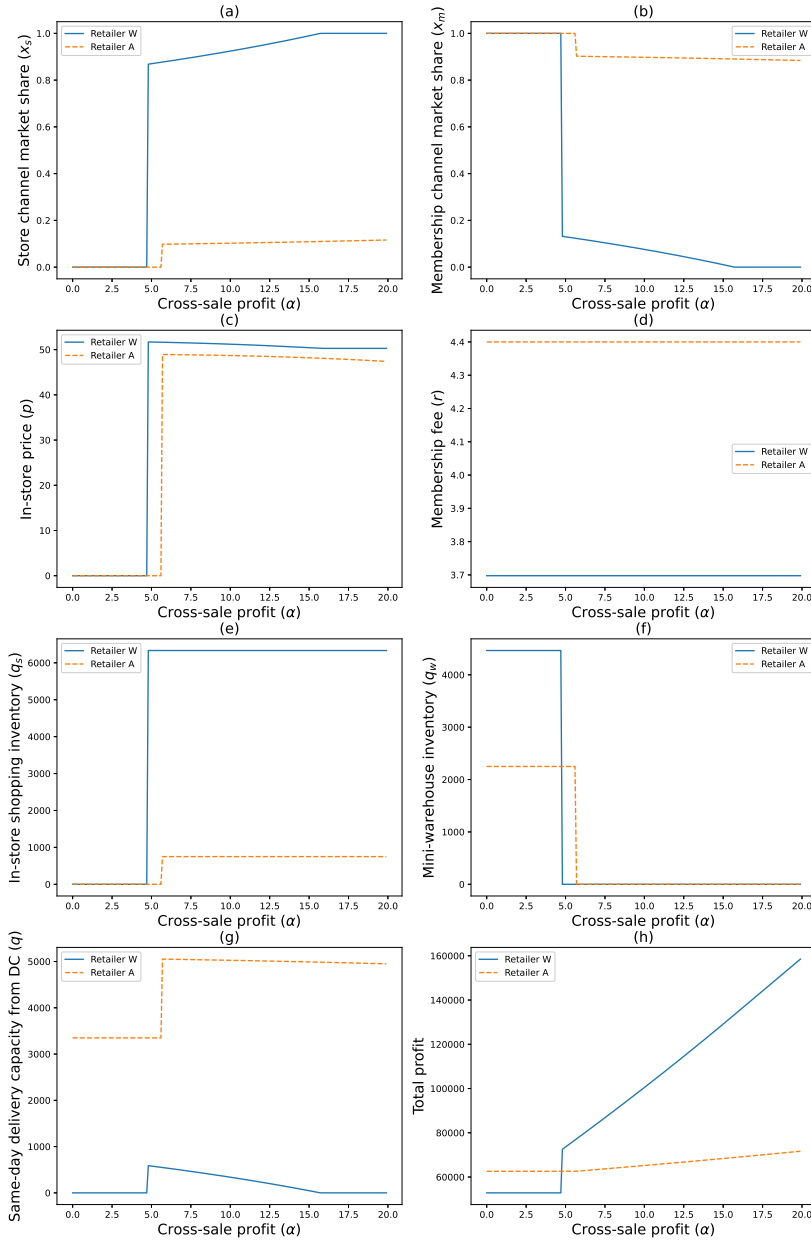
Figure 5.1: Sensitivity of the cross-sale profit (α) on (a) share of in-store channel x_s, (b) share of membership channel x_m, (c) in-store price p, (d) membership premium r, (e) in-store channel capacity q_s, (f) in-store warehouse capacity q_w, (g) delivery capacity q, and (h) total profit g.

A is more profitable than Retailer W; and (iii) due to its larger physical presence, Retailer W's profit is much more sensitive to α than Retailer A. Finally, the little jump in profit at the lower bound for Retailer W is a curious anomaly that requires explanation. Specifically, why is there a jump for Retailer W, but not for Retailer A? The direct reason is that the store was fully consumed for Retailer A, but not for Retailer W. This can be seen from the fact that only Retailer W delivers nothing from DC when α is below the threshold (i.e., $q = 0$, see Figure 5.1(g)). When α crosses the line, the retailer's channel preference suddenly shifts, and consequently, all consumers, indistinguishable as they are, are induced to switch channels simultaneously. This movement consumes all store space, abruptly activating the space constraint and causing a jump in the profit. In reality, such a discontinuity is unlikely to occur, because consumers are heterogeneous. We shall address this issue in Chapter 4.

In Figure 5.2, we let θ (1 minus the online return rate) vary from 0.5 to 1.0 for each retailer and compare the outputs of their corresponding models. Similar to α, there is a lower bound for the return rate – 0.12 for Retailer A and 0.18 for Retailer W – below which the membership channel dominates (see Figure 5.2(a-b)). Over that lower bound, there is a sudden drop in the share of the membership channel, followed by a graduate decline as the return rate continues to rise. Interestingly, a worsening return rate would strengthen the in-store channel's appeal, allowing the retailer to charge a higher price in-store (Figure 5.2(c)). For both retailers, the in-store price p rises from about \$50 at the activation of the in-store channel to about \$60 when the return rate reaches 50%.

When the return rate exceeds the threshold, the retailers first stop fulfilling orders from WH (see Figure 5.2(f)). The number of orders delivered from DC first has a sudden jump at the threshold, and then begins to decrease as the return rate is further worsened ((see Figure 5.2(g)). In the case of Retailer W, it is eventually reduced to zero after the return rate hits about 45%.

Again, we see the return rate has no impact whatsoever on the membership premium (Figure 5.2(f)), but its impact on the total profit is intriguing. For Retailer A, the profit decreases monotonically with the return rate (Figure 5.2(g)). This is easy to understand: a higher return rate reduces sales and drives up the return cost. We find the same is true for Retailer W, up to the point where the return rate reaches the threshold. When that threshold is exceeded, however, the trend is suddenly reversed. First, there is a jump in profit similar to what we have seen in Figure 5.1(g). More importantly, the profit for Retailer W continues to shoot up as the return rate increases. How could lowering the return rate for its membership channel be a bad thing for Retailer W's profit? The reason is the conflict between Retailer W's strong preference for the in-store channel and the effect of the return rate on the customer's preference for the membership channel. When the return rate is reduced, customers find the membership channel more attractive. However, the retailer still wants them to use the store channel. To avoid its own membership channel cannibalizing the market share of the more profitable in-store channel, the retailer must lower the price in-store to offset the improvement in the utility of the membership channel. This hurts its profit but is still the best action available. We do not observe this para-
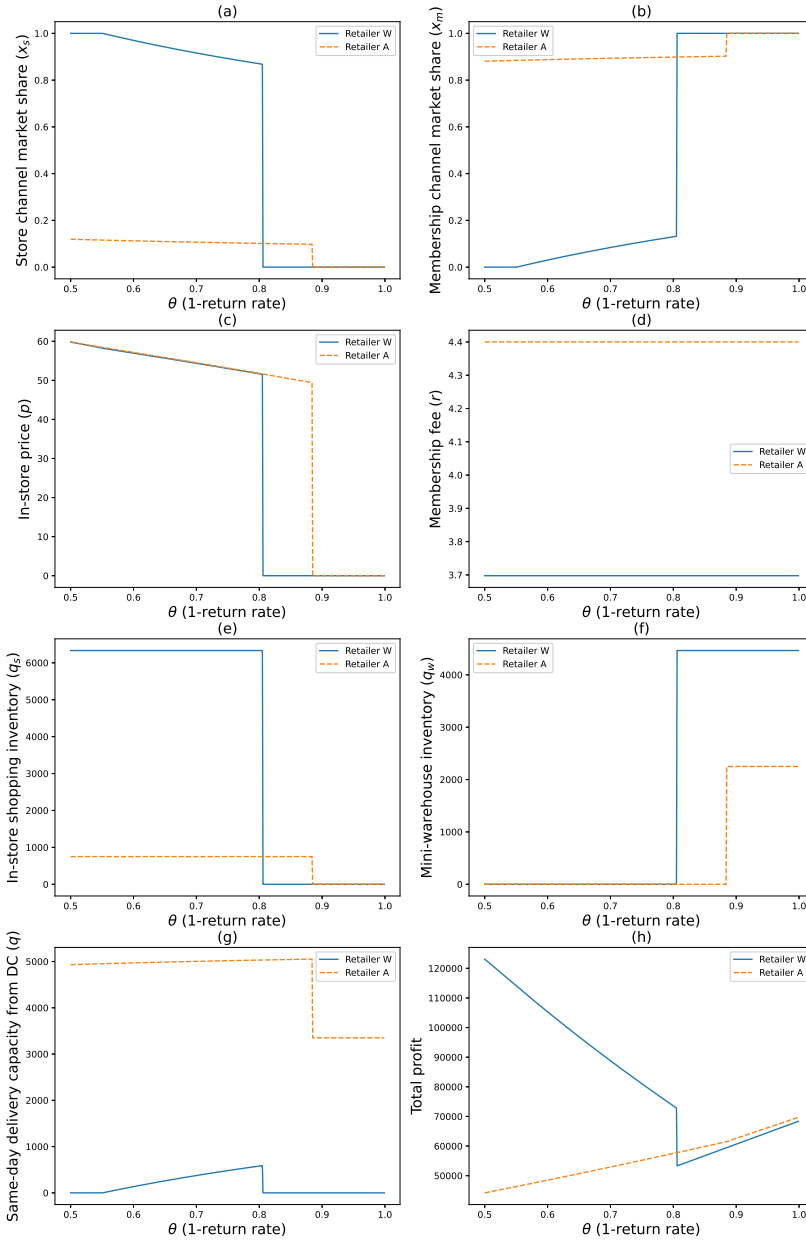
Figure 5.2: Sensitivity of θ (1-return rate) on (a) share of in-store channel x_s, (b) share of membership channel x_m, (c) in-store price p, (d) membership premium r, (e) in-store channel capacity q_s, (f) in-store warehouse capacity q_w, (g) delivery capacity q, and (h) total profit g.

doxical phenomenon for Retail A because its much smaller physical space limits the profitability of the in-store channel.

Figure 5.3 explores the impact of fuel price, which ranges from \$2/gallon to \$8/gallon, on the performance of omnichannel retail. Recall that, in our model, the fuel price is positively associated with the hassle cost for the in-store channel h_s and the hourly delivery cost c_d. The default fuel price is \$3 dollars/gallon, comparable to the U.S. national average in early 2021. It turns out the fuel price has a rather dramatic effect on Retailer W. Once the fuel price exceeds about \$3.5, the in-store channel is abandoned by the retailer(Figure 5.3(a-c) because the elevated hassle cost discourages customers from driving to the store.

To counteract the rising fuel price, both retailers would lower their membership premium r while reducing the delivery capacity q. A smaller q is bound to increase the likelihood of failing to fulfill the membership orders on time (see Figure 5.3(d, f, g)). That is, y_m will drop, dragging down the membership channel utility. Thus, a rising fuel price will force both retailers to lower the level of service at a discounted price. This is somewhat counter-intuitive because a customer would naturally expect the membership to go up when a higher fuel price drives up the delivery cost. Finally, while a higher fuel cost hurts both retailers, it hurts Retailer W much more. Indeed, when the fuel cost exceeds \$3.5 dollars, Retailer A outperforms Retailer W in terms of the total profit, thanks to its more efficient online operations.

5.3   General model

In Section 5.3.1, we validate the analytical results for the special case of the general model (Figure 4.2) of both Retailers A and W. We then solve the general model with different hassle distributions f_h in Section 5.3.2 and Section 5.3.3.

5.3.1   Special case

In the special model, y_m is an input rather than a variable. In our experiments, we set y_m to be 74% and 88% for Retailer W and Retailer A, respectively, which are consistent with the results in Section 5.2.2. Unless otherwise specified, the parameter values listed in Table 5.1 and Table 5.2 will be used for Retailer W and Retailer A, respectively. Assumption 4 also stipulates that A is unlimited, $p = p_0$ and $q_w = 0$. Indeed, the only fulfillment decision left to be optimized is the membership premium r.

Table 5.5: Optimal solutions of the special model with customer heterogeneity.

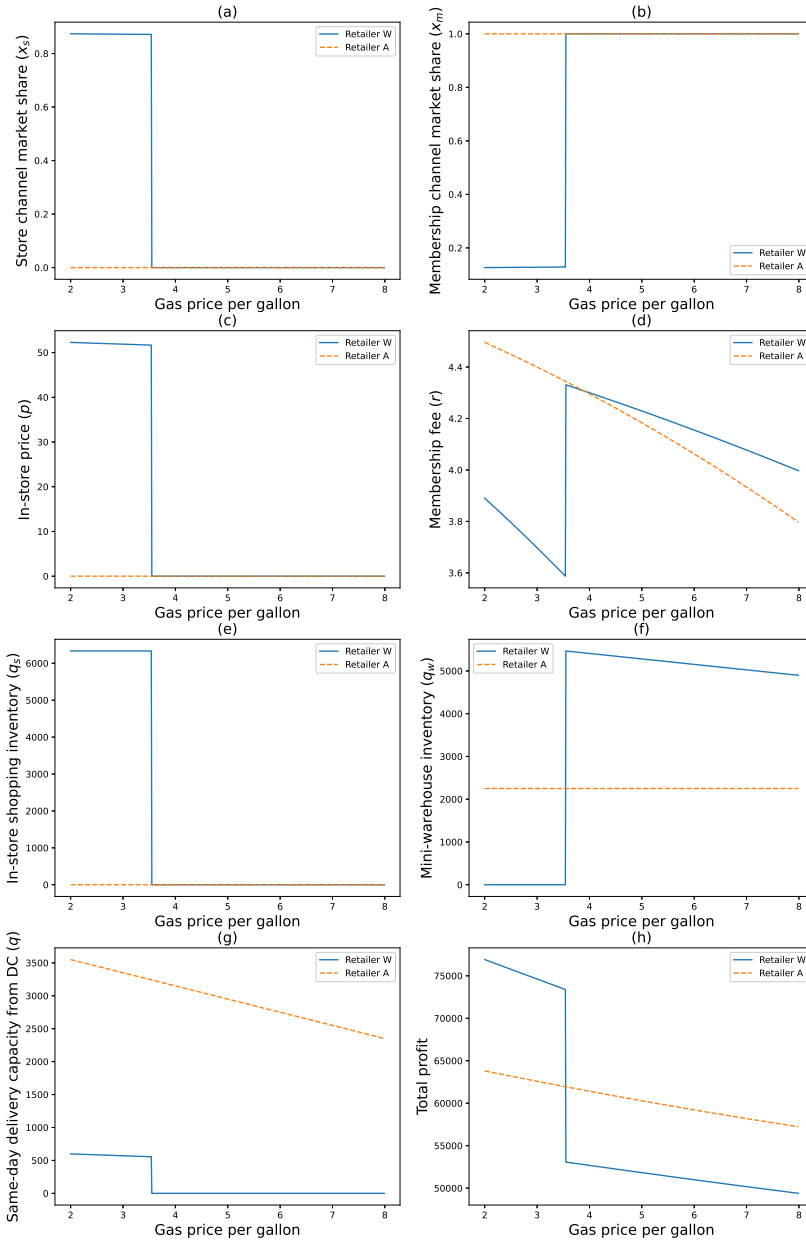| Retailer	x_s	x_m	x_o	q_s	q	r	g
A	0.51	0.24	0.24	3689	1365	\$7.93	\$61237.21
W	0.54	0.20	0.26	3893	908	\$7.10	\$55695.96

Figure 5.3: Sensitivity of the fuel price on (a) share of in-store channel x_s, (b) share of membership channel x_m, (c) in-store price p, (d) membership premium r, (e) in-store channel capacity q_s, (f) in-store warehouse capacity q_w, (g) delivery capacity q, and (h) total profit g.

The main results are reported in Table 5.5. As we can see, Retailer A's more efficient delivery operations allow it to charge a higher membership premium (about \$8/week), which yields a 10% higher profit than Retailer W. Whereas the two would build a similar in-store capacity, Retailer A can afford to maintain a delivery capacity almost 50% higher than that of Retailer W (1356 vs. 908). Both retailers leave about a quarter of all customers to the regular online channel. Retailer A fares slightly better in terms of the total share of customers served, with a 2 percentage point advantage.

Using the default inputs to the model, Figure 5.4 plots the function g (solid lines) and its derivative dg/dr (dashed lines) for Retailers W (blue) and A (red). We can see the shape of both curves closely resembles their counterparts in Figure 4.2, indicating all conditions listed in Eq. (4.20) are indeed satisfied. Within the reasonable range of r (0-\$10), the two profit curves are very similar, although the profit for Retailer A always stays above that for Retailer W.

5.3.2   General case with uniform hassle cost distribution

Assuming hassle costs be uniformly distributed, we solve and compare three general models, each corresponding to a different value of γ, the percentage of the customers choosing the regular online channel who decide to stay with the retailer. If $\gamma = 0$, for example, it means the retailer will lose all regular online channel customers. By default, $\gamma = 0.5$. We also test the cases where γ is set to 0 or 0.8. Like before, we first calibrate the model using the current share of Walmart's online sales. We fix the floor space A at the value obtained before and γ at 0.5. When $\overline{H_l} = 10$ and $\overline{H_s} = 15$,
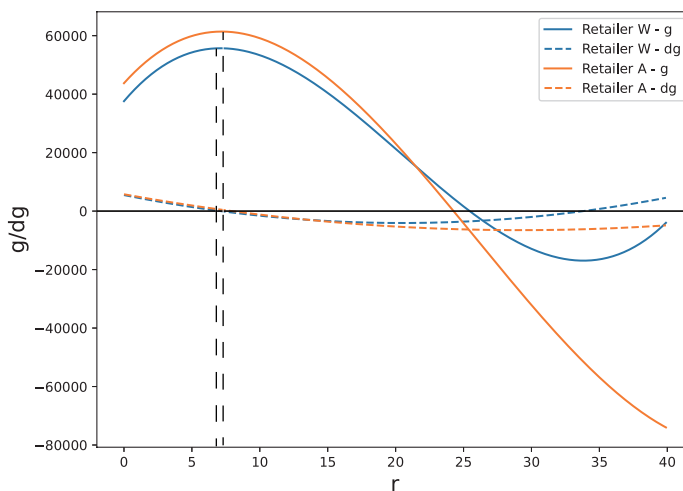

Figure 5.4: Illustration of the profit as a function of the membership premium and its derivative in the special case.

Model W approximately produces the observed market share. Thus, we use these values as calibrated upper bounds.

In the default setting, as shown in Table 5.6, we find considering customer heterogeneity causes a major shift in the fulfillment strategies for both retailers. A considerable amount of customers, 15% for Retailer W and 29% for Retailer A, are now left to the regular online channel. Compared to the optimal solution of the base model, the share of the membership channel decreases from 13% to zero for Retailer W, and the share of the in-store channel increases from zero to 14% for Retailer A. The main reason is, there will always be customers whose waiting costs (h_l) are relatively low when h_l is uniformly distributed from 0 to H_l and these customers would like to select the normal online channel which means the retailers will lose half of them $(\gamma = 0.5)$. So, to maximize the profit, Retailer W tends to expand the in-store channel, which is the most profitable channel for it, to reduce the market share of the normal online channel, while Retailer A has to decrease the membership fee and expand its in-store channel to limit the market share of its normal online channel. The in-store prices have risen dramatically. The markup for in-store sales, on the other hand, reaches 40% and 20% of the online price for Retailers A and W, respectively. Such high prices become viable because many customers have a cost structure that tolerates them. For example, a customer with a high h_l and h_s would stick to the membership channel and be willing to pay more for a high premium. On the other hand, a customer with a higher h_l and a low h_s would stick to the in-store channel even for fairly high in-store markups. Thus, the ability to "exploit" these high-value customers drives up the price. After customer heterogeneity is taken into consideration, Retailer W makes about 50% more money while Retailer A makes about 2% less. Table 5.7 further shows the total sales for Retailer A drop about 16% (from about $300K to about $250K) compared to the base model. The gross profit ratio increases substantially, especially for Retailer W, who gains almost 7 percentage points.

When γ is reduced from 0.5 to 0, the retailer loses all regular online sales. In response, they ramp up both the membership channel and in-store channel by respectively lowering the membership premium and in-store markup, see Table 5.6. In the end, Retailer A will lose about 5% of all sales, and about 5% of the profits. For Retailer W, its lost sales amount to about 12% and the lost profit about 2.5%. The gross profit ratio decreases a little for both retailers when γ decreases from 0.5 to 0. The reason is that the regular online channel has a fixed profit ratio of 20% per

Table 5.6: Optimal solutions of the general model with customer heterogeneity.

γ	Retailer	x_s	x_m	x_o	q_s	q_w	q	p	r	g
0	A	0.14	0.84	0.02	750	0	3088	$70	$0.1	$58593
	W	0.88	0	0.12	6090	0	0	$61	$3.7	$115675
0.5	A	0.14	0.57	0.29	750	0	2102	$72	$1.8	$61625
	W	0.85	0	0.15	5909	0	0	$62	$4.1	$118686
0.8	A	0.13	0.32	0.54	750	0	1882	$73	$5.2	$68738
	W	0.83	0	0.17	5764	0	0	$62	$4.4	$120849

Table 5.7: Revenue and cost table for Retailer A and Retailer W.

	$\gamma = 0$		$\gamma = 0.5$		$\gamma = 0.8$	
Revenue	Retailer A	Retailer W	Retailer A	Retailer W	Retailer A	Retailer W
Store sales revenue	$50517	$314253	$51297	$307552	$51280	$301857
Normal online sales	$0	$0	$38889	$17956	$117064	$33178
Membership channel sales	$226942	$0	$154478	$0	$87484	$0
Membership fee	$504	$0	$6179	$0	$10109	$0
Total sales revenue	$277963	$314253	$250842	$325508	$265939	$335034
Cost						
Inventory cost	$2250	$18268	$2250	$17726	$2250	$17291
Delivery cost	$9263	$0	$6304	$0	$5647	$0
Return cost	$1009	$0	$859	$269	$909	$498
Acquisition cost	$210461	$206068	$183391	$213750	$191932	$220662
Total Cost	$222983	$224336	$192805	$231745	$200738	$238451
Profit						
Cross-sale profit	$3614	$25758	$3587	$24923	$3537	$24265
Total profit	$58594	$115675	$61625	$118686	$68738	$120849
Ratio						
Gross profit ratio	21.1%	36.8%	24.6%	36.5%	25.8%	36.1%
Inventory cost ratio	1.0%	8.1%	1.2%	7.6%	1.1%	7.3%
Delivery cost ratio	4.2%	0%	3.3%	0%	2.8%	2%
Return cost ratio	0.5%	0%	0.4%	0.1%	0.5%	0.2%

the model setting $((p_0 - c_p)/p_0)$, which is lower than the profit ratio achieved by the other two channels when the fulfillment strategies are optimized. Thus, the greater the regular online sales, the lower the gross profit ratio.

As expected, increasing γ from 0.5 to 0.8 has exactly the opposite effect. It gives the retailers the incentive to scale back channel operations and raise prices. This would increase both the revenue and the profit. While the ratio of operating costs (inventory and delivery), falls slightly.

5.3.3 General case with general hassle cost distribution

In this section, we adopt a more general hassle cost distribution, whose PDF takes a quadratic form, i.e.,

$$f_h(h_s, h_l) = a \cdot h_s^2 + b \cdot h_s + c \cdot h_l^2 + d \cdot h_l + e. \tag{5.1}$$

To calibrate and solve the general model with such a hassle cost distribution, we must employ the specialized algorithm developed in Chapter 4. Table 5.8 provides the default values for the algorithmic parameters. Using the 87% in-store channel market share of Retailer W, we set $\underline{H_s} = 0$, $\overline{H_s} = 8$, $\underline{H_l} = 11$, and $\overline{H_l} = 30$, with $a =$-3.364868e-5, $b =$-1.345947e-4, $c =$-3.364868e-5, $d =$1.517556e-3, $e =$-8.121602e-3. The distribution is visualized in Figure 5.5. Other input parameters to the model remain the same.

Recall that, to avoid local maxima, we usually need to solve the general model multiple times from different initial points. A simple strategy is to choose these initial points randomly within the feasible set. However, we find a better strategy to
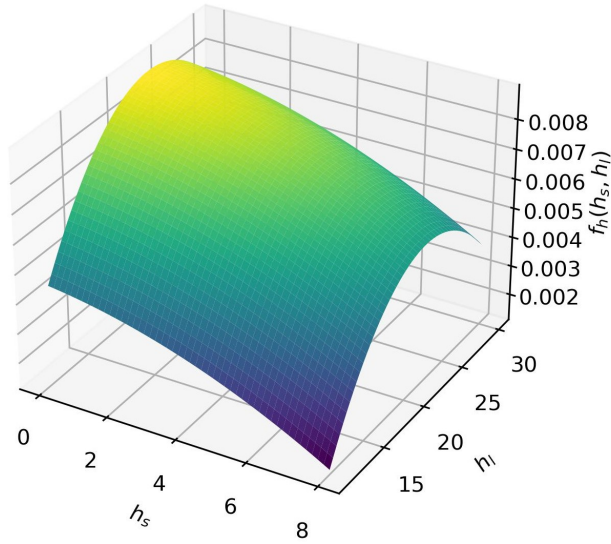
Figure 5.5: The distribution of the hassle cost with a quadratic form PDF.

Table 5.8: The algorithm parameters that used for the general case.

	IP	U	ϵ_1 in Alg 1	ϵ_2 in Alg 1	E	ϵ_1 in Alg 2	ϵ_2 in Alg 2
values	10	1000	10^{-3}	0.5^{30}	10	1.5	1000

Figure 5.6: The convergence pattern of the gradient ascent method applied to solve Model A.

guide the sampling of initial points based on certain heuristic rules identified through experiments. One example is to avoid starting from a solution where the share of the in-store channel is near zero for Model W. The opposite (e.g., regions where the share of membership channel is zero) is true for Model A.

Figure 5.6 illustrates the pattern of convergence when applying the algorithm for solving Model A. As seen in the plot, the objective function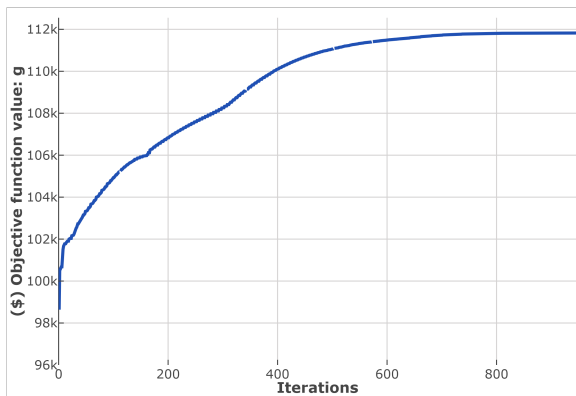 value climbs monotonically by more than 10% compared to the initial value, and the algorithm achieved it within 1000 iterations. Note that monotonicity is not a feature of the model, but baked in the design of the algorithm.

The main results are reported in Table 5.9. We see that the strategies for Retailer A (the second column) and Retailer W (the third column) are different with the general distribution. The membership channel is more profitable for Retailer A which has over 70% market share. Retailer A charges a much higher membership fee becomes, though it also provides a very high level of service (the same-day delivery probability y_m is close to 99%). Retailer A also charges a slightly higher (a difference of about \$1.2) price than Retailer W, employs nearly all store space for the in-store channel, and satisfies all online shopping demand from the distribution center. Again, this is largely dictated by the constraint of space.

From the third column, we can see that the membership fee charged by Retailer W is extremely low (less than \$1). Although almost all online shopping consumers choose to buy the membership (thanks to the low membership premium), only less than one-tenth of consumers' needs can be met on time. All these online orders are fulfilled from the distribution center. Since guaranteeing the minimum service quality is a more realistic choice for retailers, we set $y_m \geq 0.7$ and $y_m \geq 0.8$, and report the corresponding results in columns 4 and 5 of the table, respectively. As the minimum membership service quality increases, the membership fee rises, and all online shopping consumers migrate from the membership channel to the regular online shopping channel and the in-store channel. Yet, the profit decreases only 1% in this process. This suggests that, given the strength of the in-store channel, Retailer W's bottom line is not significantly affected by the online shopping channel strategies. Interestingly, the total profit for both retailers increases compared to the uniform distribution case Retailer A's profit rises by over 80%, and Retailer W gains more than 25%. The reason for this discrepancy is that, since fewer customers are located in the low waiting cost "areas" in Figure 4.3 and Figure 4.4, Retailers do not have to put too much effort into reducing the number of customers lost to the normal online channel, especially for Retailer A.

Finally, as a validation of the algorithm, we set $a = b = c = d = 0$, $\overline{h_s} = H_s$, $\overline{h_l} = H_l$, $\underline{h_l} = 0$, $q_w = 0$, $p = p_0$ and keep y_m as a constant. This degenerates the general case with quadratic PDF to the special case examined in Section 5.3.1. As expected, the optimal solution obtained by our algorithm in this case perfectly matches the results reported in Table 5.5.

Table 5.9: Optimal solutions of the general case with quadratic customer hetero-geneity.

Retailer	Retailer A	Retailer W	W with $y_m \geq 0.7$	W with $y_m \geq 0.8$
x_s	0.1102	0.8549	0.8733	0.8733
x_m	0.7087	0.1451	1.36E-06	0.0000
x_o	0.1812	6.12E-06	0.1267	0.1267
q_s	749.9976	6330.0884	6333.3325	6333.3320
q_w	0.0031	0.1451	0.0024	0.0000
q	5115.6813	64.0800	0.0033	0.0000
p	67.9298	66.7222	66.6622	66.6620
r	15.5450	0.9326	13.0557	15.0000
y_m	0.9873	0.0848	0.7000	0.8000
g	111824.7500	149462.6094	147786.6719	147786.6406

CHAPTER **6**

Conclusion

In this Chapter, we conclude the report by summarizing the main findings in Section 5.1 and touching upon several directions for future investigation in Section 5.2.

6.1 Summary of findings

We have created a model of omnichannel retail that allows us to explore the trade-offs involved in making fulfillment decisions. One of the most surprising findings from the base model – formulated as a Stackelberg game – is the lack of lower-level Nash equilibrium that mimics a typical routing game seen in the transportation literature. We had expected that customers would distribute between the channels in the same way travelers would between roads. This expectation, however, was not borne out, despite our channels being indeed as congestible as roads, at least from the mathematical point of view. The fundamental reason for this discrepancy is that the retailer controls both channels, and will always adjust the fulfillment decisions so that the channel with greater intrinsic profitability becomes the dominant channel. Thus, absence of hard physical constraints, the optimal solution is always an all-or-nothing distribution. This result sheds light on the important role that the physical store space plays in driving fulfillment decisions in the base model. The mechanism is straightforward: when this space becomes a scarce resource, the retailer is forced to expand the alternative channel even if they prefer to have everyone visit their store.

Another interesting analytical result offered by the base model has to do with the impact of providing inventory information to customers so that they can avoid stockout events. We found this information is a mixed blessing for the in-store channel, depending on the magnitude of the cross-sale profits. The information will increase the store's appeal if and only if the profit generated from cross-sales is *smaller* than from direct sales.

We calibrated the base model to roughly match the operations of Walmart and Amazon in the U.S. on a 1/3000 scale. Walmart is used as the benchmark, in the sense that its actual channel distribution is replicated by the model through the calibration process. With Walmart as the benchmark, we found Amazon, which has a more efficient delivery operation and a much smaller physical footprint, would prefer an e-commerce-only fulfillment strategy. Amazon is also slightly less profitable than Walmart, largely because it cannot leverage cross-sale benefits as much as Walmart

does. If our estimate of cross-sale benefit is accurate – a big IF for sure – brick-and-mortar stores are going to be seen as important assets in the future of e-commerce.

Our sensitivity analysis of the base model shows fulfillment strategies are highly sensitive to the cross-sale effect, the online order return rate, and the fuel price. Three findings from the analysis are especially noteworthy. First, it establishes a minimum cross-sale profit that would make the in-store channel an attractive option for the retailer. We estimate this profit amounts to roughly 10% of the revenues from direct sales. Second, for a retailer with a strong preference for the offline channel, a lower online order return rate can hurt its profit. This paradoxical phenomenon arises because the retailer is forced to lower the price in-store, hence taking a hit in profit, in order to compete with its own membership channel enhanced by a lower return rate. Third, a higher fuel price will reduce the appeal of the in-store channel because customers must bear a higher cost when visiting the store. Interestingly, the retailer will respond to a fuel price surge by charging *less*, not more, for the same-delivery membership. To offset the loss, the retailer will reduce the delivery capacity, which will in turn lower the level of service. Thus, customers will end up visiting stores less frequently, paying less for delivery but enduring a worse experience.

We have also extended the basic model to allow for user heterogeneity and developed a specialized algorithm for the general model. While heterogeneity affects the results considerably, it has not fundamentally altered the insights drawn from the above analysis.

6.2   Future work

In the near future, this research can be extended at least in two directions. The first, discussed in Section 6.2.1, concerns linking heterogeneity to where customers live, which in turn allows us to explore the traffic impact of omnichannel retail. Section 6.2.2 outlines a plan to build and analyze an omnichannel model with heterogeneous products.

6.2.1   Spatial heterogeneity and traffic impact

We propose to differentiate customers based on where they live relative to the store. As we have discussed in the numerical experiments in Chapter 5, the distance from the store is a dominating component in h_s (i.e., the hassle cost associated with the in-store channel). Since h_s increases with the distance required to travel from a customer's home to the store, those who live closer to the store naturally find the in-store channel more attractive. Thus, incorporating travel distance into the model links retail decisions with the spatial structure of the city, as well as the distribution of population. This feature allows us to examine not only the role of travel burden on fulfillment strategies but also their traffic implications.

6.2.2 Product heterogeneity

The models presented in previous chapters assume that there is only one generic product with an identical gross profit margin ($p_0 - c_p$), physical size (a_s, a_w), and return rate (θ). As a consequence, the retailer's fulfillment strategy is completely independent of what types of products they sell. In reality, however, the assortment and fulfillment decisions may vary with the product type. For example, bulky and low-value products (e.g. groceries) may be easier to sell in-store than to deliver to homes. On the other hand, the membership channel may be more efficient to fulfill lightweight and high-value products like consumer electronics. Customers' channel preference also depends on product types[1]. They would go to the physical store if they want to see and try the product in person, and this is reflected by the different return rates for different products[2]. As omnichannel retail allows customers to seamlessly migrate between channels, it is important for the retailer to anticipate customers' product-dependent channel preferences when making fulfillment decisions.

One may consider an integrated product assortment and fulfillment optimization problem that allocates different products to different channels while taking customers' preferences into consideration. A natural starting point would be to add another product into the base model, before extending it to include other features (e.g., spatial and other heterogeneity) and more types of products.

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[1]https://www.forbes.com/sites/blakemorgan/2019/10/17/15-reasons-customers-will-still-go-to-stores-in-the-future/?sh=4c11d5505b3b

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