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Abstract Operating Rooms (ORs) are one of the higher cost drivers of hospital budget and one of the highest source of income. Several performance criteria have been reported to lead and to evaluate the OR planning decisions. Usually, patient priority maximisation and OR utilisation maximisation are the most used objectives in literature. On the contrary, the workload balance criteria, which leads to a smooth stay bed occupancies, seems less used in literature. In this paper we propose a hierarchical multi-objective optimisation model for bed levelling and patient priority maximisation for the combined Master Surgical Scheduling and Surgical Cases Assignment problems. The aim of this work is to develop a methodology for OR planning and scheduling capable to take into account such different performance criteria.

Keywords: operating room planning, elective surgery, multi-objective optimisation.

1 Introduction

Operating Rooms (ORs) are one of the higher cost drivers of hospital budget and one of the highest source of income. The challenge that hospital management has to face is achieving a better and optimised use of hospital shared resources, able to reduce costs and increase profits. The adoption of decision methods as operations research can provide tools able to address the complexity of OR planning and scheduling [8, 13]. Such a complexity is given by different characteristics of the hos-

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pital organisation, where wards access to shared and limited resources such as ORs, ward beds, Intensive Care Unit (ICU) beds, Anaesthesiologists, Surgeons, Nurses. Other factors increase the complexity, such as the uncertainty of patient clinical conditions (e.g. the length of stay after the surgical intervention) and the patient surgery time in the OR session that can lead to cancellations or delays of surgery in the schedule.

OR scheduling and planning can be defined by three hierarchical decisions levels: strategic, tactical and operational that consider respectively the long, medium and short term objectives. The strategic level considers resource allocation problem, determining the number of surgeries, which staff to use for surgeries and defining the amount of the resources available. At tactical level the master surgical schedule, that is the assignment of OR blocks to surgical specialties, is defined together with the number of surgeons, the definition of ward and ICU use, and the need of equipment. Finally, at the operational decision level are defined two problems, that is (i) selecting elective patients usually from a long waiting list and assigning them to a specific OR time session (i.e., an operating room open on a specific day) over a planning horizon [18, 10], and (ii) determining the precise sequence of surgical procedures and the allocation of resources for each OR time session [15, 19]. Such problems are further challenged by the inherent stochasticity of their main parameters, such as the surgery duration, the length of stay and the arrival of non-elective patients [7, 2, 11, 1, 16, 22, 12].

Bed availability is a topic that recently received a particular attention. Ward bed availability inside a hospital with different surgical specialties is considered in [3, 16, 22] while other studies [23, 19] consider only the use of Intensive Care Unit (ICU) or Post-Anaesthesia Care Unit (PACU) [24], or both [6, 9].

Several performance criteria have been reported to lead and to evaluate the OR planning decisions [8]. Usually, patient priority maximisation [10] and OR utilisation maximisation [14] are the most used, but also minimise delays and cancellations [16], maximise patient satisfaction [20] and minimise fixed patient costs or societal costs [23] were considered as objective function for OR planning. On the contrary, the workload balance criteria leads to a smooth – without peaks – stay bed occupancies determining a smooth workload in the ward and, by consequence, an improved quality of care provided to patients [5, 21, 4].

In this paper we propose a hierarchical multi-objective optimisation model for bed levelling and patient priority maximisation for the combined Master Surgical Scheduling and Surgical Cases Assignment problems. The aim of this work is to develop a methodology for OR planning and scheduling able to take into account such different performance criteria. The problem description and the multi-objective optimisation model are reported in Section 2. Preliminary computational analysis is reported and discussed in Section 3 and Section 4 closes the paper.

2 Problem Statement and Mathematical Formulation

The problem addressed in this work can be formalised as follows. The goal is to simultaneously assign the OR blocks to a surgical specialty and to schedule patient surgeries in order to maximise a hierarchical objective function considering bed levelling and patients priority. More in details, the primary objective consists in maximising the number of beds occupied in a surgical specialty department, in the day in which the occupation is minimum, which represents the bottleneck of the problem. The secondary objective consists in maximising the global patients satisfaction. To each patient is assigned a score, which is computed as its priority level divided by the waiting time between the diagnosis and the surgery. The global patient satisfaction is defined as the sum of the scores related to the patients which are selected for surgery within the planning horizon.

For each patient are known the surgical specialty to which he/she is assigned, the priority level, the expected length of stay (LOS), the number of days elapsed from the diagnosis, the expected surgery duration. For each specialty is known the number of beds available on each day. Furthermore, the length of each OR block is supposed to be known. The objective is twofold. First, we try to maximise the minimum occupation of beds in a day in a department, secondly to maximise patients satisfaction, as described in the previous paragraph. A patient is be assigned to an OR block only if that block has been assigned to the surgical specialty which the patient belongs. The total expected duration of surgeries scheduled in a OR block can not exceed its length. Each scheduled patient occupies a bed in the day of his/her surgery and for a number of following days equal to his/her LOS.

Before reporting the mathematical model, we are required to introduce the following notation. Let I, J and K be respectively the sets of patients, surgical specialties and operating rooms, each indexed by i, j and k. Let $T = \{1, \dots, N_t\}$ be the set of days in the planning horizon, indexed by t. Let I_j be the subset of patients that belong to specialty j, $j \in J$. For each patient $i \in I$, we are given the expected duration of the surgery p_i, the priority coefficient π_i, and the expected Length of Stay μ_i, expressed in days. Let Φ_{it} be the number of elapsed day between diagnosis of patient i and day t. Note that each OR block in the planning horizon is uniquely defined by the pair of indices (k,t). We denote by s_{kt} the time capacity of the OR session (k,t). Let Λ_{jt} be the number of beds available for specialty j on day t. Finally, let P and M set to $\sum_i \pi_i$ and $\frac{1}{P+1}$, respectively.

Let us introduce the following decision variables: a binary variable X_{ikt} equals to 1 if patient i is assigned to block k on day t, and 0 otherwise; a binary variable Z_{jkt} equals to 1 if block k on day t has been assigned to specialty j, and 0 otherwise; a binary variable Y_{it} equals to 1 if patient i occupies a bed on day t, and 0 otherwise; a binary variable W_it equals to 1 if patient i surgery is scheduled on day t. Let be also O_1 and O_2 the primary and the secondary objective.

$$\max z = O_1 + MO_2 \tag{1a}$$

$$\text{s.t.} \quad \sum_{k\in K}\sum_{t\in T} X_{ikt} \leq 1\,, \qquad i \in I \tag{1b}$$

$$\sum_{i\in I_j} X_{ikt} \leq |I_j| Z_{jkt}\,, \qquad j \in J, k \in K, t \in T \tag{1c}$$

$$\sum_{j\in J} Z_{jkt} \leq 1\,, \qquad k \in K, t \in T \tag{1d}$$

$$\sum_{i\in I} p_i X_{ikt} \leq s_{kt}\,, \qquad k \in K, t \in T \tag{1e}$$

$$W_{it} = \sum_{k\in K} X_{ikt}\,, \qquad i \in I, t \in T \tag{1f}$$

$$\sum_{\tau=t}^{\min(t+\mu_i;N_t)} Y_{i\tau} \geq min(\mu_i + 1; N_t - t + 1) W_{it}\,, \quad t \in T \tag{1g}$$

$$\sum_{\tau=\max(t-\mu_i,1)}^{t} W_{i\tau} \geq Y_{i\tau}\,, \qquad t \in T \tag{1h}$$

$$\sum_{i\in I_j} Y_{it} \leq \Lambda_{jt}\,, \qquad t \in T, j \in J \tag{1i}$$

$$O_1 \leq \sum_{i\in I_j} Y_{it}\,, \qquad t \in T, j \in J \tag{1j}$$

$$O_2 = \sum_{i\in I}\sum_{t\in T}\sum_{k\in K} \frac{\pi_i}{\Phi_{it}} X_{ikt}\,. \tag{1k}$$

The hierarchical objective function is reported in (1a). The role of the multiplier M is to ensure that if a solution S_1 has a higher value of O_1 with respect to S_2 it would be preferred whichever the correspondent values of O_2. In other words, the secondary objective intervenes in the solutions comparison only when the value of O_1 is exactly the same. Constraint (1b) states that only a subset of patients can be selected from the long waiting list. A patient can be assigned to an OR block only if it is assigned to the surgery specialty to which he/she belongs, as stated in constraint (1c). Constraint (1e) implies that each block must be assigned to at most one specialty. Constraint (1d) imposes that the sum of the surgery times of the patients scheduled in each OR time block (k,t) may not exceed the time block capacity s_{kt}. Constraint (1f) allows to detect whether patient i surgery is scheduled on day t. Constraints (1g) and (1h) imply that, if a patient i is scheduled on day t, he/she will occupy a bed for the following μ_i days. Constraints (1i) limits for each specialty the number of beds occupied each day to the maximum number of available beds, given by the number of beds in the department, reduced by the number of beds occupied by patients from the previous planning assignment. The primary objective function (1j) concerns the maximisation of the number of beds used in the day and the specialty department with the minimal bed usage, which works as bottleneck approach. The max min bed occupation objective function tends also to implicitly fill as much as possible the OR blocks thus avoiding under utilisation of operating rooms. The secondary objective (1k) concerns the maximisation of the patient served multiplied by the relative corresponding patient priority and divided by the waiting days from the diagnosis.

We decided to analyse also what happens if we exchange the roles of the primary and secondary objective function. This can be obtained modifying the objective function

$$\max \rho O_2 + \frac{O_1}{\lambda_{min} + 1} \tag{2}$$

where ρ represents a multiplier constant such as, whichever value O_2 takes, ρO_2 is integer. We identify with $\lambda_{\min}$ a strict upper bound on O_1, computed as $\lambda_{min} = \min_{(i \in I, t \in T)} \Lambda(i,t)$. In this way, the second term of the objective function takes always values between 0 and 1. Therefore, a solution S_1 which has an higher value of O_2 with respect to another solution S_2, and a value null for O_1, would results always better than S_2, whichever is the value of O_1 for S_2.

3 Quantitative analysis

In this section we provide a quantitative analysis in order to evaluate the behaviour of the mathematical model (1b)-(1k) varying the two objective functions (1a) and (2). To this end, we generate three sets of benchmark instances B_1, B_2 and B_3 as follows: the operating time p_i case mix has been generated using the generator proposed in [17]; the elapsed time Φ_{it} is uniformly distributed in $[1, 365]$; the LOS μ_i is generated accordingly to the expected surgery times, basing on the consideration that longer surgeries usually require longer stays. Five different priority levels are defined, and for each patient the priority level π_i is randomly generated.

The first set, B_1, is composed of 8 instances, divided in 2 groups, (1a-1b-1c-1d) and (2a-2b-2c-2d), with 200 patients uniformly distributed among 4 different specialties, each specialty has an availability of 10 beds. For each specialty we consider that some of the beds can be occupied by patients operated before the planning horizon, therefore the actual availability of beds may results lower than the capacity of the department. Priorities are different for each instance, while waiting time from the diagnosis, expected surgery duration, and LOS are constant within the same instance group, but varying among the two groups. The number of ORs is equal to 5, each one with a single block for day with a duration of 480 minutes. The time horizon is equal to 5 days but each OR is open only for 4 days. The day on which each OR is closed changes among ORs, such that, on each day, 4 ORs result available. The second set, B_2, contains the same instances of B_1 but consider a bed capacity equal to 20 beds for each specialty. As in the previous set, actual bed availability may results lower than the capacity, due to bed occupation by patients operated before the planning horizon. Finally, the third set, B_3, is composed by 4 instances (a,b,c,d) with 400 patients and 8 specialties. Each instance is obtained merging the patients of two smaller instances in the B_1 in such a way that the first instance belongs to the group 1 while the second to the group 2. The number of ORs is 10 and their availability is defined with the same rule used for the other sets, hence 8 ORs are available on each day.

The computational results are obtained solving the model using Xpress 7.9. with a time limit of 3600 seconds. The computational test have been performed on a PC with a 4-core Intel i7-5500U with 2.4GHz CPU and 16 Gb of main memory.

Table 1 Comparing solutions varying the objective functions (1a) and (2): benchmark B_1.

	objective (1a)				objective (2)			
instance	O_1	O_2	gap	time	O_1	O_2	gap	time
I200J4B10-1a	8	5.0230	0.00%	64.84	7	5.4377	0.70%	3600
I200J4B10-1b	8	4.7447	0.00%	51.60	7	4.8893	3.86%	3600
I200J4B10-1c	8	4.4770	0.00%	30.13	7	4.7799	3.29%	3600
I200J4B10-1d	8	5.7638	0.00%	67.44	7	6.1313	2.31%	3600
I200J4B10-2a	8	5.6571	0.00%	95.74	7	5.9036	1.41%	3600
I200J4B10-2b	8	6.9856	0.00%	64.82	5	7.0913	0.35%	3600
I200J4B10-2c	8	7.5927	0.00%	42.48	5	7.7670	0.14%	3600
I200J4B10-2d	8	9.2648	0.00%	46.03	5	9.3825	1.70%	3600

Table 1 reports the results for benchmark B_1. It is worth noting that the primary objective could have an impact making more challenging the solution process: actually, the solutions with patient priority maximisation (2) have a larger average gap (1.72%) and running time (3600 secs.) than those obtained with (1a) (0.00% and 57.89 secs.).

Table 2 Comparing solutions varying the objective functions (1a) and (2): benchmark B_2.

	objective (1a)				objective (2)			
instance	O_1	O_2	gap	time	O_1	O_2	gap	time
I200J4B20-1a	13	5.5446	0.00%	480.28	6	5.9682	0.52%	3600
I200J4B20-1b	13	5.2572	0.00%	444.46	6	5.4323	1.26%	3600
I200J4B20-1c	13	4.6907	0.00%	955.57	6	5.2938	0.50%	3600
I200J4B20-1d	13	6.1827	0.00%	519.12	6	6.6528	0.56%	3600
I200J4B20-2a	12	6.2183	0.00%	53.16	5	6.4215	0.68%	3600
I200J4B20-2b	12	7.1698	0.00%	186.60	0	7.7247	0.57%	3600
I200J4B20-2c	12	7.8644	0.00%	74.10	5	8.3281	0.41%	3600
I200J4B20-2d	12	9.4352	0.00%	24.97	5	9.9327	0.44%	3600

Table 2 reports the results for benchmark B_2. The results confirm the remarks for B_1 even if the average gap for (2) is reduced to 0.62% and the average running time for (1a) increases up to 342.28 seconds.

Finally, Table 3 reports the results for benchmark B_3. We observe that for larger instances, the previous remarks are totally overturned. Although all the solutions

Table 3 Comparing solutions varying the objective functions (1a) and (2): benchmark B_3.

	objective (1a)				objective (2)			
instance	O_1	O_2	gap	time	O_1	O_2	gap	time
I400J4B20-a	11	11.7966	21.42%	3600	0	12.1370	2.79%	3600
I400J4B20-b	10	12.3142	28.55%	3600	0	12.5311	5.67%	3600
I400J4B20-c	10	12.9874	28.55%	3600	2	13.3342	2.64%	3600
I400J4B20-d	10	15.9563	28.55%	3600	0	16.3653	1.92%	3600

reach the time limit, the average gap for (2) (3.26%) is smaller than the gap for (1a) (26.77%). The difference in terms of average gap could depend on whether the instances in B_3 have a larger number of patients but the same number of stay beds than those, for instance, in B_2: indeed, this situation results in an increased number of the possible patient combinations that can be operated in compliance with the operating constraints.

In terms of system performances, we can observe that taking into account only O_2 the system is not able to guarantee a smooth bed occupancy. Further, considering O_1 as primary objective, the solutions in terms of O_2 are slightly worse than those obtained considering O_2 as primary objective, while the solutions in terms of O_1 are strongly better than those obtained considering O_2 as primary objective, as showed in Tables (1)-(3). By consequence, the use of O_1 as primary objective seems to lead to better global solutions.

4 Conclusions

In this paper we propose a hierarchical multi-objective optimisation model for bed levelling and patient priority maximisation for the combined Master Surgical Scheduling and Surgical Cases Assignment problems. The aim of this work is to develop a methodology for OR planning and scheduling capable to take into account such different performance criteria. The computational results prove the feasibility of our approach showing also a counter-intuitive result. In fact, on small instances the model where the primary objective function is the patient satisfaction results to be more difficult to solve respect to the model where the primary objective is the bed levelling, while on larger instances the model behaviour is totally overturned. This aspect will be deeply investigated investigated with further tests. Finally, the running time required to obtain a solution suggested the need of developing ad-hoc solution algorithms.

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