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# Assigning surgery cases to operating rooms: A VNS approach for leveling ward beds occupancies 

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#### Abstract

This paper deals with the problem of scheduling over a given planning horizon a set of elective surgery patients into a set of available operating room block times. The aim is to level the post-surgery ward bed occupancies during the days, thus allowing a smooth workload in the ward and, as a consequence, an improved quality of care provided to patients. Exploiting the flexibility of the Variable Neighbourhood Search, we provide a general solution framework which we show could be easily adapted to different operative contexts.


Keywords: Operating room planning, bed levelling, Variable Neighbourhood Search

## 1 Introduction

Operating Rooms (ORs) planning is a critical activity with important financial impacts for most hospital setting. In many publicly funded health care systems demand for surgery very often overwhelms supply therefore causing long waiting list and waiting times and impact on patients quality of life [9].

When planning ORs, one of the main questions hospital managers and surgeons are faced with is how should the available OR capacity be allocated in order to improve efficiency and productivity and how can efficiency be attained and measured. The review of the operations research and management science scientific literature clearly reveals an increasing interest of researchers towards OR planning and scheduling problems [2,4]. Researchers frequently distinguish between strategic (long term), tactical (medium term) and operational (short term) decisions in order to better characterize their planning or scheduling problem even if there are no clear and universally accepted definitions of these three decision levels [2]. The operational decisions concerning the short term period are generally distinguished into "advance scheduling" and "allocation scheduling" [7]. The first, usually referred as Surgical Case Assignment Problem (SCAP), consists in assigning a surgery date and OR block to each patient over the considered planning horizon, which can range from one week to one month (see, e.g., [8]). Whereas, the second deals with determining the sequencing of surgical procedures in each block, i.e., the starting time of each procedure on the specific day of surgery, and the identification of resources needed for each OR time block and day combination in order to implement it as efficiently as possible (see, e.g., [6]).

Two paradigms can be considered. When planning with the closed block scheduling setting, each specialty is assigned to a given number of OR time blocks (usually half-day or full day length) for each planning period, where it can schedules their surgical cases [10]. On the other hand, the open block scheduling paradigm assumes that the OR time blocks are shared among different specialties and each patient is assigned to a time slot within a given OR block [3].

In this paper we consider the planning decisions concerning the advance scheduling problem, that is SCAP, in a closed block scheduling setting under different operative contexts. Different performance criteria have been used to

[^0]evaluate an OR planning decision as reported in [2]. In this paper we consider the criteria dealing with the ward stay bed levelling. A planning leading to a smooth - without peaks - stay bed occupancies, will determine a smooth workload in the ward and, at the end, an improved quality of care provided to patients. The workload balance is a challenging problem as discussed in [1,11].

The paper is organized as follows. The problem is described and modelled as a $0-1$ linear programming model in Section 2. Section 3 describes the main elements of our VNS solution approach. Finally, the conclusions and future work directions reported in Section 4 close the paper.

## 2 Problem definition and formulation

Given a set of patients waiting to be operated for a set of surgical specialties and a number of OR time blocks assigned to each specialty, we face problem of determining for a given planning horizon the surgery date and operating room assigned to each patient, i.e., the SCAP.

The planning decisions have to satisfy many resource constraints related to OR time blocks length, number of OR time blocks assigned to each surgical specialty, number of ward stay beds available for each specialty and day.

Let us introduce some necessary notation. Let $I, J, K$ and $T$ be respectively the sets of patients, surgical specialties, operating room and days of the planning horizon, each indexed by the corresponding letter, $i, j, k$ and $t$. Each OR time block within the planning horizon is then uniquely defined by a pair of indices $(k, t)$ which give, respectively the OR and day of the planning horizon when the block is scheduled.

For each patient $i \in I$, the expected duration of the surgery $p_{i}$, expressed in minutes, and the expected Length of Stay (LOS) $\mu_{i}$, expressed in days, are given. In addition, let $I_{j}$ be the subset of patients that belong to specialty $j, j \in J$, and $I_{h}$ the subset of patients having LOS $\mu_{i}=h, h=1, \ldots, \mu_{\max }$, where $\mu_{\text {max }}$ represents the longest LOS. Clearly, subsets $I_{j}$ define a partition of $I$ as do subsets $I_{h}$. Each patient is also characterized by a given priority. Let $p=1, \ldots, P$ be the set of priorities that can be assigned to each patient and $I_{p}$ be the subset of patients having the same priority $p$ : a patient in $I_{p}$ has greater priority than a patient in $I_{p+1}, p=1, \ldots, P-1$.

We denote by $s_{k t}$ the time available for surgery in operating room $k \in K$ on day $t \in T$ and with $\Lambda_{t}^{j}$ the number of ward stay beds available for specialty $j \in J$ on day $t \in T$. Note that, the stay beds are managed as a specialized resource. This means that the number of ward beds $\Lambda_{t}^{j}$ available for specialty $j$ on day $t$ are not accessible to patients belonging to other surgical specialties.

We assume that the bed availability is homogeneous among specialties.
To formulate the problem, we assume as input data the cyclic timetable that gives, for each day of the planning horizon, the assignment of surgical specialties to OR time blocks (i.e., the master surgical schedule). Note that, this timetabling include the availability of surgeons and surgical staff resources in the ORs and days a priori assigned to each specialty $j$. We denote this assignment with the parameter $\tau_{k t}^{j}$ which is equal to 1 if specialty $j \in J$ is assigned to OR time block $(k, t), 0$ otherwise. We introduce the decision variable $x_{i k t}$ equal to 1 when a patient $i \in I$ is assigned to OR $k \in K$ on day $t \in T, 0$ otherwise.

The selection of the patients from the waiting list and their assignment to OR blocks is modelled by the following constraints:

$$
\begin{array}{ll}
\sum_{k \in K, t \in T} x_{i k t} \leq 1 & \forall i \in I \\
\sum_{i \in I_{j}} x_{i k t} \leq M \tau_{k t}^{j} & \forall j \in J, k \in K, t \in T \\
\sum_{i \in I} p_{i} x_{i k t} \leq s_{k t} & \forall k \in K, t \in T \tag{3}
\end{array}
$$

Constraints (1) state that a patient can be scheduled at most once. Constraints (2) ensure that each patient $i \in I_{j}$, i.e., belonging to a given specialty $j \in J$, can only be assigned to a compatible OR time block, that is one for which $\tau_{k t}^{j}=1$. Note that $M$ represents a suitably defined integer value large enough to make the constraint non binding whenever $\tau_{k t}^{j}=1$. Constraints (3) impose that the sum of the surgery times of the patients scheduled in each OR time block $(k, t)$ may not exceed the time block capacity $s_{k t}$.

The following constraints imposes a priority in the patient selection.

$$
\begin{array}{ll}
x_{i^{\prime} k t}\left|I_{p}\right| \leq \sum_{i \in I_{p}} \sum_{k \in K, t \in T} x_{i k t} & p=1, \ldots, P-1, \forall i^{\prime} \in I_{p+1} \\
\sum_{i \in I_{p}} \sum_{k \in K} x_{i k t} \geq \sum_{i \in I_{p+1}} \sum_{k \in K} x_{i k t} & p=1, \ldots, P-1, \forall t \tag{5}
\end{array}
$$

Constraints (4) imposes that in order to schedule a patient $i^{\prime}$ with a given priority, all the patients having a greater priority must be already scheduled in one of the days of the planning horizon. Constraints (5) state that for each day $t$ the number of scheduled patients with an higher priority must be greater or equal to the number of patients with a smaller one.

To smooth the bed occupation during the days of the planning horizon, we are required to count the number of beds used each day. If $x_{i k t}=1$, the
patient $i$ will occupy a bed from day $t$ until day $t+\mu_{i}$. Let us introduce a further set of decision variables $y_{i t}^{j}$ which is equal to 1 if the patient $i$ will occupy a bed of specialty $j$ on day $t, 0$ otherwise.

$$
\begin{array}{ll}
\sum_{t^{\prime}=t}^{t+\mu_{i}} y_{i t^{\prime}}^{j}=\mu_{i} x_{i k t} & \forall i \in I, j \in J, k \in K, t \in T \\
\sum_{i \in I} y_{i t}^{j} \leq \Lambda_{t}^{j}-\lambda_{t}^{j} & \forall j \in J, t \in T \tag{7}
\end{array}
$$

Constraints (6) implies that $y_{i t}^{j}=\ldots=y_{i\left(t+\mu_{i}\right)}^{j}=1$ when $x_{i k t}=1,0$ otherwise. Constraints (7) limit for each specialty the number of ward beds used each day to the maximum number $\Lambda_{t}^{j}$ of bed available for working days reduced by the number $\lambda_{t}^{j}$ of beds already resulting occupied from the previous planning assignment.

Recalling that we assume an homogeneous bed availability among specialties, the objective function seeks to maximize the number of beds used in the day with the minimal bed usage, which work as bottleneck approach. The whole model follows.

$$
\begin{array}{cll}
\mathcal{M}: \max & y \\
& \text { s.t. } & \sum_{i \in I} y_{i t}^{j} \geq y \\
& (1)-(7) \\
& x_{i k t}, y_{i t}^{j} \in\{0,1\}, \quad y \in \mathbb{R} .
\end{array} \quad \forall j \in J, t \in T
$$

Note that the max min stay bed utilisation objective function tends also to implicitly fill as much as possible the OR blocks thus avoiding under utilization of operating rooms.

Regarding the different operative contexts which could be adressed, we remark that constraints (5) could be removed from the formulation if the planning horizon is short, i.e., about a week, since constraints (4) are enougth to guarantee the precedence among patients with different priorities. On the contrary, constraints (5) become necessary for larger planning horizon, to avoid that patients with greater priority can be scheduled in the last days of the time horizon.

We would also want to outline that we are working under an operative scenario where demand for surgery overwhelms supply thus determining long waiting list. In many setting found in the literature, the authors assume that the whole set of the patients should be operated on in the planning horizon. We can simply address this quite common assumption by removing constraints (4) and modifying constraints (1) as equal to 1 assignment constraints.

## 3 VNS solution approach

This paper would like to exploit the inherent flexibility of the Variable Neighbourhood Search methodology [5] in order to explore the solution of the problem $\mathcal{M}$ under different operative contexts.

Neighbourhoods. We define three different neighbourhoods. Let us introduce the following values: $\Delta_{t}=\sum_{i \in I} y_{i t}, t_{\min }=\arg \min _{t} \Delta_{t}$ and $\Delta_{\min }=$ $\min _{t} \Delta_{t}$.

The first neighbourhood, p-swap(in,in,h), exchanges $h$ patients scheduled on day $t_{\min }$ with other $h$ patients scheduled on a day $t \neq t_{\min }$. The second, p -swap (in, out, h ), is similar to the first one but the other $h$ patients (denoted as out) are selected among those not yet scheduled. In both neighbourhoods, $h$ ranges from 1 to $\Delta_{\text {min }}$.
The third, p-shift(in,1), tries to add a patient not currently scheduled to fill as much as possible the OR time blocks without deteriorating the value of the objective function. On the contrary, p-shift (out,1) removes the patient having the longest $\mu_{i}$ from each OR time block in the schedule.
All neighbourhoods generate only feasible solutions, that is satisfying constraints (1)-(5).

VNS framework. We are ready to present the VNS solution framework to solve $\mathcal{M}_{1}$ as depicted in Algorithm 1. In the following we refer to the basic VNS scheme discussed in [5] (cf., algorithm 7). From a notational point of view, we use $S$ to denote a solution, $\ell$ instead of $k$ to denote the $k$ th neighbourhood and upper $T$ instead of $t$ for the computational time.

```
Algorithm 1 A VNS for bed levelling
BL-VNS ( \(S, T_{\text {max }}\) )
repeat
    \(\ell \longleftarrow 1 ; \ell_{\max } \longleftarrow 2 \Delta_{\min }-1 ;\)
    repeat
    \(S^{\prime} \longleftarrow\) Shake \((S, \ell)\);
    \(S^{\prime \prime} \longleftarrow\) FirstImprovement \(\left(S^{\prime}\right)\);
    if \(\left(f\left(S^{\prime \prime}\right)>f(S)\right)\) then
            \(S \longleftarrow S^{\prime \prime} ; \ell \longleftarrow 1 ; \ell_{\max } \longleftarrow 2 \Delta_{\text {min }}-1 ;\)
        else
            \(\ell \longleftarrow \ell+1 ;\)
        end;
```

until $\ell=\ell_{\text {max }}$;
${ }_{7} T \longleftarrow$ cpuTime();
until $T \geq T_{\max }$;
return $S$

In our solution framework, the FirstImprovement consists in a Local Search based on the exploration of the three neighbourhoods p -swap(in,in,1), p -swap(in,out,1) and p -shift(in, 1). The first improvement is justified by the fact that the waiting list is usually composed of different patients having the same values for the $p_{i}$ and $\mu_{i}$ parameters.

The Shake procedure exploits the neighbourhoods p-swap(in,in,h), p -swap (in, out, h ) and p -shift (out, 1 ) in sequence. For $\ell \in\left[1, \ldots, \Delta_{\text {min }}\right.$ $1]$, it applies p-swap (in, in, h) with $h=\ell+1$; for the next $\ell \in\left[\Delta_{\text {min }}, 2 \Delta_{\text {min }}-\right.$ 2], it applies p-swap(in, out,h) with $h=\ell-\Delta_{\min }+1$; finally, for the last $\ell$, it applies p-shift(out,1).

Computational Efficiency. From the basic framework discussed in 1, we derived two further versions of the algorithm in order to reduce the running time without decreasing the quality of the final solution. We developed rBLVNS which is a reduced version of the proposed algorithm: FirstImprovement adopts only p-swap(in,in,1) and p-shift(in,1) while Shake uses only p-swap(in,out,h) and p-shift (out,1). We also developed BL-AVNS which is the adaptive version of the Algorithm 1. It incorporates an adaptive mechanism for the intelligent selection of the value $h$ based on the ranking of the values determining the larger improvement in the objective function.

## 4 Conclusions

This paper deals with the problem of scheduling over a given planning horizon a set of elective surgery patients into a set of available operating room block times. The aim is to level the post-surgery ward bed occupancies during the days, thus allowing a smooth workload in the ward and, as a consequence, an improved quality of care provided to patients.

We considered two kind of resources: ORs and post-surgery stay beds. The objective function herein tested is aimed at levelling the ward bed occupancy rates over the days of the planning horizon. The problem and its operative contexts is reported as $0-1$ linear model.

Exploiting the inherent flexibility of the VNS methodology, we provide
a general solution framework which we show can be easily adapted to the different operative contexts and settings.

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