Structure Constants in $\mathcal{N}=2$ Superconformal Quiver Theories at Strong Coupling and Holography

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In a four-dimensional $\mathcal{N}=2$ superconformal quiver theory with gauge group $SU(N)\times SU(N)$ and bifundamental matter, we analytically obtain the exact strong-coupling behavior of the normalized 3-point correlators of single-trace scalar operators in the large-N limit using localization techniques. We then obtain the same strong-coupling behavior from the holographic dual using the AdS/CFT correspondence at the supergravity level. This agreement confirms the validity of the analytic strong-coupling results and of the holographic correspondence in a nonmaximally supersymmetric setup in four dimensions.

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Introduction.—We consider a supersymmetric quiver theory in four dimensions with gauge group $\mathrm{SU}(N)\times\mathrm{SU}(N)$ and bifundamental matter, schematically represented in Fig. 1. This model, which arises as a $\mathbb{Z}_2$ orbifold of $\mathcal{N}=4$ Supersymmetric Yang-Mills (SYM) theory has $\mathcal{N}=2$ supersymmetry, is conformally invariant at the quantum level, and admits a holographic dual description in terms of Type II B string theory on $\mathrm{AdS}_5\times S^5/\mathbb{Z}_2$ [1,2]. As such it is one of the simplest four-dimensional $\mathcal{N}=2$ theories to explore the strong-coupling regime and probe the holographic correspondence in a nonmaximally supersymmetric setup (see also [3]).

With this aim, we study 2- and 3-point functions of protected gauge-invariant operators defined as

$$U_k(x)=\frac{1}{\sqrt{2}}[\mathrm{tr}\phi_0(x)^k+\mathrm{tr}\phi_1(x)^k], \tag{1}$$

$$T_k(x)=\frac{1}{\sqrt{2}}[\mathrm{tr}\phi_0(x)^k-\mathrm{tr}\phi_1(x)^k]. \tag{2}$$

Here, k is an integer ≥ 2 and $\phi_{0,1}$ are the chiral scalar fields of the adjoint vector multiplets in the two nodes of the quiver. Replacing $\phi_{0,1}$ with their complex conjugates $\bar{\phi}_{0,1}$ yields the antichiral operators $\bar{U}_k$ and $\bar{T}_k$. All are primary operators with conformal dimension $\Delta=k$ and charge $Q=\pm k$ in the chiral or antichiral case. We call the operators, Eqs. (1) and (2), untwisted (U) and twisted (T) since they are, respectively, even and odd under the $\mathbb{Z}_2$ symmetry exchanging the nodes of the quiver.

Conformal invariance, charge conservation, and $\mathbb{Z}_2$ symmetry fix the form of the 2-point functions to be

$$\langle U_k(x)\bar{U}_k(y)\rangle=\frac{G_{U_k}}{|x-y|^{2k}}, \tag{3}$$

$$\langle T_k(x)\bar{T}_k(y)\rangle=\frac{G_{T_k}}{|x-y|^{2k}}, \tag{4}$$

where the coefficients G_{U_k} and G_{T_k} depend on k, N, and the 't Hooft coupling λ. Also, the 3-point functions are constrained by the symmetries of the theory. Here, we will consider the following correlators:

$$\langle U_k(x)U_\ell(y)\bar{U}_p(z)\rangle=\frac{G_{U_kU_\ell\bar{U}_p}}{|x-z|^{2k}|y-z|^{2\ell}}, \tag{5}$$

$$\langle U_k(x)T_\ell(y)\bar{T}_p(z)\rangle=\frac{G_{U_kT_\ell\bar{T}_p}}{|x-z|^{2k}|y-z|^{2\ell}}, \tag{6}$$

with the understanding that $p=k+\ell$ for charge conservation. Again, the coefficients in the numerators are functions of N, λ, and of the conformal dimensions. One could consider also the conjugate 3-point functions where

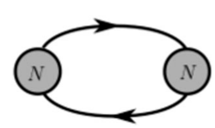

FIG. 1. The quiver diagram of the $\mathrm{SU}(N)\times\mathrm{SU}(N)$ superconformal theory. Each circle denotes a $\mathrm{SU}(N)$ factor and the oriented lines represent the bifundamental hypermultiplets.

chiral and antichiral operators are exchanged, as well as $\langle T_k(x) T_\ell(y) \bar{U}_p(z)\rangle$ and their conjugates. For simplicity, in this Letter we focus on the above cases.

The coefficients G in Eqs. (3)–(6) are sensitive to the normalization of the operators. To remove such dependence we define the structure constants

$$C_{U_k U_\ell \bar{U}_p} = \frac{G_{U_k U_\ell \bar{U}_p}}{\sqrt{G_{U_k} G_{U_\ell} G_{U_p}}}, \tag{7}$$

$$C_{U_k T_\ell \bar{T}_p} = \frac{G_{U_k T_\ell \bar{T}_p}}{\sqrt{G_{U_k} G_{T_\ell} G_{T_p}}}, \tag{8}$$

which, together with the spectrum of conformal operators, are part of the conformal field theory data. We study these structure constants in the large-N 't Hooft limit. Using supersymmetric localization, we analytically obtain the exact dependence on λ and predict the strong-coupling behavior, which is then obtained also with a holographic calculation using the AdS/CFT correspondence [4–7].

Localization results.—The flat-space correlators discussed above can be conformally mapped to correlators defined on a 4-sphere. These, in turn, can be evaluated [9–16] in terms of a matrix model determined by localization techniques [17].

The matrix model has the quiver structure of Fig. 1, with two Hermitian $N \times N$ matrices a_0 and a_1 defined in the two nodes. Neglecting instanton contributions that are exponentially suppressed in the planar limit, its partition function is

$$\mathcal{Z} = \int da_0 da_1 \, \mathrm{e}^{-\mathrm{tr} a_0^2 - \mathrm{tr} a_1^2 - S_{\mathrm{int}}} \equiv \langle \mathrm{e}^{-S_{\mathrm{int}}} \rangle_0, \tag{9}$$

where $\langle\rangle_0$ indicates the vacuum expectation value with respect to the Gaussian measure, and S_{int} is a perturbative series in λ given in Eq. (C.5) of [15]. The gauge theory operators U_k and T_k in Eqs. (1) and (2) are represented in the matrix model, respectively, by $\mathcal{O}_k^+$ and $\mathcal{O}_k^-$ with

$$\mathcal{O}_k^{\pm} = \frac{1}{\sqrt{2}} :(\mathrm{tr} a_0^k \pm \mathrm{tr} a_1^k):, \tag{10}$$

where the normal ordering $::$ means that one has to subtract the contractions with all operators of lower conformal dimension. As proven in [9,16], in the large-N limit this amounts to perform a Gram-Schmidt diagonalization within the set of single-trace operators only. Also the antichiral operators $\bar{U}_k$ and $\bar{T}_k$ are represented by $\mathcal{O}_k^{\pm}$, and thus the coefficients G in the 2- and 3-point functions can be computed by evaluating the vacuum expectation value of products of such operators. For example,

$$G_{U_k T_\ell \bar{T}_p} = \langle \mathcal{O}_k^+ \mathcal{O}_\ell^- \mathcal{O}_p^- \rangle \equiv \frac{1}{\mathcal{Z}} \langle \mathcal{O}_k^+ \mathcal{O}_\ell^- \mathcal{O}_p^- \mathrm{e}^{-S_{\mathrm{int}}} \rangle_0. \tag{11}$$

In [15], it has been shown that the interaction action S_{int} only contains the twisted operators $\mathcal{O}_k^-$. This allowed us to evaluate the partition function as

$$\mathcal{Z} = \det(1 - \mathsf{X})^{-\frac{1}{2}}, \tag{12}$$

where X is an infinite matrix whose elements are

$$\mathsf{X}_{i,j} = -8(-1)^{\frac{i+j+2ij}{2}} \sqrt{ij} \int_0^\infty \frac{dt \, \mathrm{e}^t}{t(\mathrm{e}^t - 1)^2} J_i\left(\frac{t\sqrt{\lambda}}{2\pi}\right) J_j\left(\frac{t\sqrt{\lambda}}{2\pi}\right). \tag{13}$$

Here, J_i is the Bessel function of the first kind and the indices i and j are both even or both odd. Remarkably, using the full Lie Algebra approach developed in [11,12], one can show that at large N also the expectation values as the one in Eq. (11), can be written in closed form in terms of the matrix X. This means that their full dependence on λ is known through integrals of Bessel functions.

The correlators involving only untwisted operators, which do not appear in S_{int}, are actually λ-independent. Indeed, at large N we find

$$G_{U_k} = k\left(\frac{N}{2}\right)^k \equiv \mathcal{G}_k, \tag{14}$$

$$G_{U_k, U_\ell, \bar{U}_p} = \frac{k\ell p}{2\sqrt{2}} \left(\frac{N}{2}\right)^{\frac{k+\ell+p}{2}-1} \equiv \mathcal{G}_{k,\ell,p} \tag{15}$$

for all values of λ. From this it easily follows that

$$C_{U_k U_\ell \bar{U}_p} = \frac{\sqrt{k\ell p}}{\sqrt{2}N}, \tag{16}$$

which, apart from the factor of $\sqrt{2}$ due to the $\mathbb{Z}_2$ orbifold, is the same expression of the $\mathcal{N} = 4$ SYM theory [18].

On the contrary, the correlators with twisted operators depend on λ, as one can see already at the first perturbative order by using Feynman diagrams (see [13,15,19]). Exploiting the matrix model formulation, one can go to very high orders in perturbation theory and, with limited computational effort, generate long series in λ. As shown in [15] for the 2-point correlators, these series can be resummed in closed form in terms of the matrix X, thus obtaining the λ-dependence beyond perturbation theory. From the asymptotic behavior of X for $\lambda \to \infty$ [14,20], we can then determine the coefficients G_{T_k} at strong coupling. In [16] these methods have been generalized to the 3-point correlators in an orientifold model. Building on these results, we have further extended these calculations to quiver theories [8] and managed to obtain an analytic expression of $G_{U_k T_\ell \bar{T}_p}$ for any value of λ by resumming the perturbative expansions in terms of the matrix X, which can be extrapolated to strong coupling. For the two-node quiver of Fig. 1, our findings can be summarized as

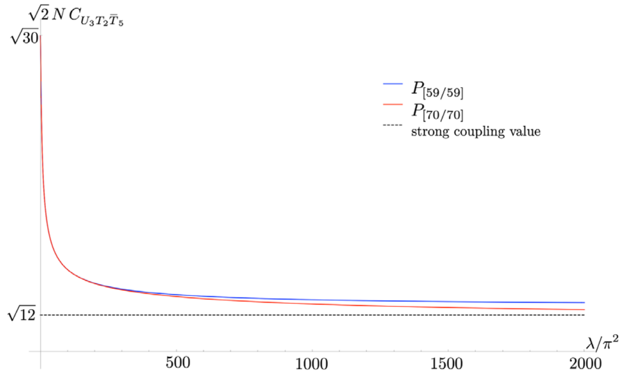

FIG. 2. Plot of the Padé approximations of degree 59 (blue line) and 70 (red line) of the structure constant $C_{U_3T_2\bar{T}_5}$. For larger values of λ, they tend toward the predicted strong-coupling value (dashed black line).

$$G_{T_k} = \begin{cases} \mathcal{G}_k & \text{for } \lambda \to 0, \\ \frac{4\pi^2 k(k-1)}{\lambda}\mathcal{G}_k & \text{for } \lambda \to \infty, \end{cases} \tag{17}$$

and

$$G_{U_kT_\ell\bar{T}_p} = \begin{cases} \mathcal{G}_{k,\ell,p} & \text{for } \lambda \to 0, \\ \frac{4\pi^2(\ell-1)(p-1)}{\lambda}\mathcal{G}_{k,\ell,p} & \text{for } \lambda \to \infty, \end{cases} \tag{18}$$

where $\mathcal{G}_k$ and $\mathcal{G}_{k,\ell,p}$ are defined in Eqs. (14) and (15). From these expressions, it follows that

$$C_{U_kT_\ell\bar{T}_p} = \begin{cases} \frac{\sqrt{k\ell p}}{\sqrt{2}N} & \text{for } \lambda \to 0, \\ \frac{\sqrt{k(\ell-1)(p-1)}}{\sqrt{2}N} & \text{for } \lambda \to \infty. \end{cases} \tag{19}$$

We emphasize that our methods allow one to evaluate the structure constants for *all* values of λ, and not only in the asymptotic regimes. As an example, in Fig. 2 we show how the structure constant $C_{U_3T_2\bar{T}_5}$ varies in a region of intermediate values of λ, where we have done two Padé resummations of the perturbative series.

Holographic derivation.—We now derive the structure constants Eqs. (7) and (8) at strong coupling using the AdS/CFT correspondence [4–6]. Since we are interested in the large-N results, we can work in the classical supergravity approximation. However, it is useful to start from the string setup.

We consider the $\mathbb{Z}_2$ orbifold projection from a stack of $2N$ regular D3 branes in Type II B string theory that engineer a $\mathcal{N}=4$ SYM theory with gauge group SU($2N$). Breaking this configuration into two stacks of N fractional D3-branes located at the orbifold fixed point [21], we obtain the quiver theory of Fig. 1.

The fractional D3-branes are soliton configurations emitting the metric and a 4-form potential C_4 with a self-dual field strength, together with the scalars b and c corresponding to the wrapping of the 2-forms B_2 and C_2 around the exceptional 2-cycle of the orbifold [22]. We therefore have an untwisted sector comprising the metric and the 4-form C_4, which propagate in 10 dimensions, and a twisted sector with the scalars b and c, which propagate in the six-dimensional space defined at the orbifold fixed point. In the near horizon limit these spaces become, respectively, $\mathrm{AdS}_5\times S^5/\mathbb{Z}_2$ and $\mathrm{AdS}_5\times S^1$ [1,2].

The dynamics of the untwisted fields is governed by the equations of Type II B supergravity in $\mathrm{AdS}_5\times S^5/\mathbb{Z}_2$ with the field strength of C_4 proportional to the volume form of the AdS_5 space. The fluctuations about this background are bulk fields dual to the untwisted operators of the quiver theory. We set [23]

$$G_{mn} = g_{mn} + h_{mn}, \qquad C_{4m_1\ldots m_4} = c_{m_1\ldots m_4} + a_{m_1\ldots m_4}, \tag{20}$$

where g_{mn} and $c_{m_1\ldots m_4}$ are the background fields, while the fluctuations are as in [18,24], namely

$$h_{\mu\nu} = h'_{(\mu\nu)} - \frac{3}{25}h_2 g_{\mu\nu} \;\text{ with }\; g^{\mu\nu}h'_{(\mu\nu)} = 0,$$
$$h_{\alpha\beta} = h'_{(\alpha\beta)} + \frac{1}{5}h_2 g_{\alpha\beta} \;\text{ with }\; g^{\alpha\beta}h'_{(\alpha\beta)} = 0,$$
$$a_{\mu_1\mu_2\mu_3\mu_4} = -\epsilon_{\mu_1\mu_2\mu_3\mu_4\nu}\partial^\nu a, \quad a_{\alpha_1\alpha_2\alpha_3\alpha_4} = \epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4\beta}\partial^\beta a. \tag{21}$$

We now expand the fluctuations in the spherical harmonics of S^5. Since we are interested in those Kaluza-Klein (KK) modes that can couple to the fields of the twisted sector that are localized at the $\mathbb{Z}_2$ orbifold fixed point, we can restrict our attention to harmonics of the form

$$Y^k = \frac{1}{2^{\frac{|k|}{2}}}\mathrm{e}^{\mathrm{i}k\theta}\cos^{|k|}(\phi) \quad \text{for } k\in\mathbb{Z}, \tag{22}$$

where $\theta\in[0,2\pi]$ parametrizes the circle S^1 transverse to AdS_5 at the orbifold fixed point, and ϕ is one of the S^5 coordinates selected in such a way that $\phi=0$ corresponds to the fixed point. The relevant expansions are

$$h_2 = \sum_k h_{2,k}Y^k, \qquad a = \sum_k a_k Y^k, \qquad h'_{(\mu\nu)} = \sum_k h'_{(\mu\nu),k}Y^k. \tag{23}$$

Doing the same analysis as in [18,24], one can show that the combinations

$$s_k = \frac{1}{20(k+2)}[h_{2,k} - 10(k+4)a_k] \tag{24}$$

for $k>0$, and the complex conjugates s_k^*, correspond to KK excitations in AdS_5 with mass squared $m^2 = k(k-4)$

that are dual to the untwisted operators U_k and $\bar{U}_k$ of the quiver theory. Furthermore, as in [18] we have

$$h'_{(\mu\nu),k} = \frac{4\nabla_{(\mu}\nabla_{\nu)}s_k}{k+1}, \qquad h'_{(\mu\nu),-k} = \frac{4\nabla_{(\mu}\nabla_{\nu)}s_k^*}{k+1} \tag{25}$$

for $k > 0$. The linearized equations for s_k and s_k^* can be derived from the quadratic action

$$S_{\rm untw} = \frac{4(2N)^2}{(2\pi)^5}\int_{\rm AdS_5} d^5z\sqrt{g}\sum_{k>0}A_k[\partial s_k^*\cdot\partial s_k + k(k-4)s_k^*s_k]\frac{\pi^3}{2}, \tag{26}$$

where the prefactor is the rewriting of the gravitational constant using the parameters of the quiver theory in units where the radius of $\rm AdS_5$ is set to 1. The last factor of $\pi^3/2$ is the volume of $S^5/\mathbb{Z}_2$, and finally

$$A_k = \left[\frac{32k(k-1)(k+2)}{k+1}\right]\left[\frac{2}{2^k(k+1)(k+2)}\right], \tag{27}$$

where the first bracket was derived in [18] and the second bracket comes from the normalization of the spherical harmonics, Eq. (22).

Let us now turn to the twisted sector. As shown in [2], the dynamics of the scalars b and c is described by

$$S_6 = \frac{1}{2\kappa_6^2}\left[\int d^6x\sqrt{G_6}\left(\frac{1}{2}\partial b\cdot\partial b + \frac{1}{2}\partial c\cdot\partial c\right) + 4\int C_4\wedge db\wedge dc\right], \tag{28}$$

where we have adopted the conventions of [15], with G_6 being the determinant of the metric and $2\kappa_6^2$ the gravitational constant. Assuming $ds^2 = ds^2_{\rm AdS_5} + d\theta^2$, and expanding b and c in harmonics of S^1, namely

$$b = \sum_k b_k e^{ik\theta}, \qquad c = \sum_k c_k e^{ik\theta}, \tag{29}$$

one can show [2,8,15] that the combination

$$\eta_k = c_k - ib_k \tag{30}$$

for $k > 0$, and its complex conjugate η_k^*, are KK excitations in $\rm AdS_5$ with mass-squared $m^2 = k(k-4)$, which are dual to the twisted operators T_k and $\bar{T}_k$ of the quiver theory. Their dynamics is governed by the action

$$S_{\rm tw} = \frac{4(2N)^2}{(2\pi)^3 2\lambda}\int_{\rm AdS_5} d^5z\sqrt{g}\sum_{k>0}\frac{1}{2}[\partial\eta_k^*\cdot\partial\eta_k + k(k-4)\eta_k^*\eta_k]2\pi, \tag{31}$$

where the prefactor comes from the gravitational constant $1/2\kappa_6^2$ of the orbifold using the AdS/CFT dictionary, and the last factor of 2π is just the length of S^1.

The actions Eqs. (26) and (31) can be used to obtain the 2-point functions Eqs. (3) and (4) at large N and at strong coupling with the AdS/CFT methods [6]. Using Eq. (17) of [25] and the correction factor in Eq. (95) from (26), we deduce that

$$G_{U_k} = \frac{4(2N)^2}{(2\pi)^5}A_k\frac{1}{\pi^2}\frac{\Gamma(k+1)}{\Gamma(k-2)}\frac{2(k-2)}{k}\frac{\pi^3}{2}. \tag{32}$$

In a similar fashion, from Eq. (31) we find

$$G_{T_k} = \frac{4(2N)^2}{(2\pi)^3 2\lambda}\frac{1}{2}\frac{1}{\pi^2}\frac{\Gamma(k+1)}{\Gamma(k-2)}\frac{2(k-2)}{k}2\pi. \tag{33}$$

In writing these formulas, we have not taken into account the possible presence of an arbitrary proportionality constant in the coupling between the quiver operators and the supergravity modes on the boundary of $\rm AdS_5$ since, as shown in [18], this constant drops out in the normalized structure constants.

To compute the 3-point correlators we have to work out the cubic interactions of the KK modes. For the untwisted ones, we can rely again on the analysis of [18], which, translated in our notations, leads to

$$S'_{\rm untw} = \frac{4(2N)^2}{(2\pi)^5}\int_{\rm AdS_5} d^5z\sqrt{g}\sum_{k,\ell,p>0}[V_{k\ell p}s_k s_\ell s_p^*\delta_{k+\ell-p,0} + {\rm c.c.}]\frac{\pi^3}{2}, \tag{34}$$

where the cubic coupling $V_{k\ell p}$ can be read from Eqs. (3.39) and (3.40) of [18]. From this action, using the AdS/CFT formulas of [25], we obtain

$$G_{U_kU_\ell\bar{U}_p} = \frac{N^2}{2^{\frac{k+\ell+p}{2}-7}\pi^6}\frac{k(k-1)(k-2)}{k+1}\times\frac{\ell(\ell-1)(\ell-2)}{\ell+1}\frac{p(p-1)(p-2)}{p+1}, \tag{35}$$

where the δ function imposing charge conservation is understood. Combining Eqs. (35) and (32), it follows that

$$C_{U_kU_\ell\bar{U}_p} = \frac{\sqrt{k\ell p}}{\sqrt{2}N}, \tag{36}$$

in agreement with the localization result, Eq. (16).

We now consider the twisted sector. In this case we have to expand the twisted action, Eq. (28), to first order in the fluctuations to obtain the couplings involving one untwisted mode and two twisted ones. Using Eq. (21) and the relations Eqs. (24) and (25), up to a boundary term we obtain

$$S'_{\rm tw} = \frac{4(2N)^2}{(2\pi)^3 2\lambda}\int_{\rm AdS_5} d^5z\sqrt{g}\sum_{k,\ell,p>0}\frac{1}{2}[W_{k\ell p}s_k\eta_\ell\eta_p^*\delta_{k+\ell-p,0} + {\rm c.c.}]2\pi, \tag{37}$$

where [8]

$$W_{k\ell p} = -(k+\ell-p)(k+p-\ell) \times \frac{(k+\ell+p-2)(k+\ell+p-4)}{2^{\frac{k}{2}}(k+1)}. \tag{38}$$

We observe that if one uses the δ function that imposes charge conservation, this cubic coupling vanishes. This is a well-known feature of all couplings related to extremal correlators [26,27], and is not in contradiction with the fact that the final correlators are nonvanishing. Indeed, the zero in the coupling coefficient is compensated by a pole in the cubic Witten diagram of the 3-point function, so that the product yields a finite result. This can be clearly seen [18,26,27] if one imposes the charge-conserving δ function only at the end, as we are going to do [28]. With this understanding, using the AdS/CFT formulas of [25], we obtain

$$G_{U_kT_\ell\bar{T}_p} = \frac{N^2}{2^{\frac{k}{2}-3}\pi^6\lambda}\frac{k(k-1)(k-2)}{k+1} \times (\ell-1)(\ell-2)(p-1)(p-2). \tag{39}$$

Then, from Eqs. (33) and (32), it follows that

$$C_{U_kT_\ell\bar{T}_p} = \frac{\sqrt{k(\ell-1)(p-1)}}{\sqrt{2}N}, \tag{40}$$

which confirms the localization result, Eq. (19), at strong coupling.

Conclusions.—By exploiting the power of supersymmetric localization we were able to obtain the exact λ dependence of the structure constants of single-trace operators in the 2-node quiver theory of Fig. 1 in the large-N limit. When all operators are untwisted, the structure constants are λ-independent, like in the $\mathcal{N}=4$ SYM theory, but when two of the operators are twisted they depend on λ in a highly nontrivial way. These twisted structure constants are therefore observables which, remarkably, can be followed from weak to strong coupling in an analytic way. The strong-coupling behavior of the structure constants predicted by localization is confirmed by a holographic calculation based on the AdS/CFT correspondence. This agreement can be seen either as a validation of the strong-coupling extrapolation of the localization results or, alternatively, as an explicit check of the AdS/CFT correspondence for a nonmaximally supersymmetric theory in four dimensions. We finally mention that these results can be generalized to quiver theories with more than two nodes, as well as to orientifold models [8].

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