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ADAPTIVE OUTPUT FEEDBACK STABILIZATION FOR NONLINEAR SYSTEMS WITH UNKNOWN POLYNOMIAL-OF-OUTPUT GROWTH RATE AND SENSOR UNCERTAINTY

Yanjun Shen and Lei Lin

In this paper, the problem of adaptive output feedback stabilization is investigated for a class of nonlinear systems with sensor uncertainty in measured output and a growth rate of polynomial-of-output multiplying an unknown constant in the nonlinear terms. By developing a dual-domination approach, an adaptive observer and an output feedback controller are designed to stabilize the nonlinear system by directly utilizing the measured output with uncertainty. Besides, two types of extension are made such that the proposed methods of adaptive output feedback stabilization can be applied for nonlinear systems with a large range of sensor uncertainty. Finally, numerical simulations are provided to illustrate the correctness of the theoretical results.

Keywords: adaptive stabilization, polynomial-of-output growth rate, measurement sensitivity, output feedback, observer

Classification: 93D15,93D21,93C10

1. INTRODUCTION

In the past decades, the problem of stabilization has been studied for nonlinear systems with lower-triangular form via output feedback. Various control methods have been developed for nonlinear systems with known growth rate [21, 25], unknown growth rate [13, 18], unknown polynomial-of-output growth rate [5, 6, 14, 15] and growth rate depending on input and output [1, 2]. For instance, in [15], an output feedback controller design method was presented for nonlinear systems with uncertain control coefficient and unknown polynomial-of-output growth rate. The authors in [6] considered output-feedback stabilization for nonlinear systems with input matching uncertainty and unknown polynomial-of-output growth rate. Global regulation was discussed for a class of nonlinear time-delay systems by output feedback [5]. In the above literatures, all results have been achieved by assuming that the output can be measured accurately.

Recently, researchers have paid a great deal of attention on output feedback stabilization for nonlinear systems with unknown measurement sensitivity or unknown output function. In [23], the problem of global output feedback stabilization was studied

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for upper-triangular systems with unknown output function by using the homogeneous domination approach. A design method of sampled-data output feedback control was proposed for a class of nonlinear systems with unknown output function [22]. The authors in [19] illustrated that nonlinear systems with unknown control coefficients could be transformed into nonlinear systems with unknown output function, and then designed output-feedback controllers for a class of uncertain nonlinear systems with unknown output function and unknown growth rate. There exists a critical assumption in [19, 22, 23], that is differentiability of the output function, which may not always hold in practice. There also exists another basic assumption, that is a limit range of the unknown measurement sensitivity. To cope with unknown measurement sensitivity, a dual-domination approach was proposed to stabilize the nonlinear systems [3]. Furthermore, the authors studied output feedback regulation for nonlinear systems with unknown measurement sensitivity and unknown linear growth rate [8]. The problem of output feedback stabilization was also considered for nonlinear systems with sensor uncertainty, unknown linear growth rate and stochastic disturbances in [12]. However, there are no results of output feedback stabilization for nonlinear systems with sensor uncertainty and unknown polynomial-of-output growth rate, which motivates this paper.

In this paper, the problem of adaptive output feedback stabilization is investigated for a class of nonlinear systems with sensor uncertainty and a growth rate of polynomial-of-output multiplying an unknown constant in the nonlinear terms. Our major contributions include: (I) We propose an adaptive output feedback stabilization with two variable gains and a constant gain to deal with the unknown constant, the polynomial-of-output, and the sensor uncertainty. (II) We also extend our methods such that the proposed adaptive output feedback stabilization can be applied for nonlinear systems with a larger range of sensor uncertainty.

The remainder of this paper is organized as follows. In Section 2, we present some useful lemmas and problem description. Our main results are given in Section 3, that is, adaptive output feedback stabilization for a class of nonlinear systems with growth rate depending on output and sensor uncertainty. In Section 4, we extend the proposed methods to nonlinear systems with a larger range of sensor uncertainty. Numerical simulations are provided to illustrate the validity of the proposed design methods in Section 5. Section 6 concludes this paper.

2. PRELIMINARIES AND PROBLEM DESCRIPTION

In this paper, we consider the following single-input single-output (SISO) uncertain nonlinear system:

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(t,x), i = 1, \dots, n-1, \\ \dot{x}_n = u + f_n(t,x), \\ y = \theta(t)x_1, \end{cases} \tag{1}$$

where $x = (x_1, \dots, x_n)^T \in \mathbb{R}^n$, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the system state, control input and measurement output. The functions f_i: $\mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}$ are continuous in the first argument and locally Lipschitz in the rest argument. The sensor sensitivity $\theta(t)$ ($t \in \mathbb{R}^+$) is an unknown continuous function.

We need the following assumptions.

Assumption 2.1. There exists a known integer $p \geq 1$ and an unknown constant $\varrho > 0$ such that

$$|f_i(t,x)| \leq \varrho(1+|x_1|^p)(|x_1|+\cdots+|x_i|), i=1,\ldots,n.$$

Assumption 2.2. The sensor sensitivity $\theta(t)$ is an unknown continuous function satisfying $\theta(t) \in [1-\bar{\theta}, 1+\bar{\theta}]$, where $\bar{\theta}$ is an allowable sensitivity error.

Remark 2.3. Unlike [24], the constant ϱ in the growth rate is assumed to be unknown in this paper. Compared with the Assumption 2 in [8], the growth rate in Assumption 2.1 is not an unknown constant, but a time-varying function related to the output. This undoubtedly increases the difficulty of designing an output feedback controller. In particular, since $(1+|x_1|^p) > 1$, the condition that satisfies Assumption 2 in [8] also satisfies Assumption 2.1. Thus, Assumption 2.1 has a wider range of applications.

The nonlinear system (1) with Assumption 2.1 is significant not only in control theory but also in engineering practice. Some models such as circuits with nonlinear resistance [17] and business cycles [4] can be described as,

$$\ddot{\vartheta} + \mu(1-\vartheta^2)\dot{\vartheta} + \vartheta = u, \tag{2}$$

where μ is an unknown constant. Suppose that only the variable ϑ is measurable. Coordinate transformation $x_1 = \vartheta$, $x_2 = \dot{\vartheta}$ transforms the system (2) into the following form

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = u - x_1 - \mu(1-x_1^2)x_2, \\ y = x_1. \end{cases} \tag{3}$$

Obviously, the condition in Assumption 2.1 holds with $\varrho = \max\{1, |\mu|\}$, $p = 2$. Therefore, the system (3) is in the form of the system (1).

Remark 2.4. Due to manufacturing reasons, there always exists a sensitivity error $\theta(t)$ in Assumption 2.2. For instance, as shown in [3], the displacement sensor of a magnetic bearing suspension system experiences $\pm 10\%$ sensitivity error, which means that the sensor sensitivity $\theta(t)$ satisfies $\theta(t) \in [1-0.1, 1+0.1]$.

The following inequalities are referred from [10, 20] and will be used later.

Lemma 2.5. (Krstic and Deng [10]) For $(x,y)^T \in \mathbb{R}^2$, the following Young's inequality holds,

$$xy \leq \frac{v^p}{p}|x|^p + \frac{1}{qv^q}|y|^q,$$

where $v > 0$, the constants $p > 1$ and $q > 1$ satisfy $(p-1)(q-1) = 1$.

Lemma 2.6. (Yang and Lin [20]) For $p \in [1, +\infty)$ and any $x_i \in \mathbb{R}, i = 1, \ldots, n$, the following inequality holds,

$$(|x_1|+\cdots+|x_n|)^p \leq n^{p-1}(|x_1|^p+\cdots+|x_n|^p).$$

3. MAIN RESULTS

In this section, we will construct an output feedback controller with two variable gains and a constant gain for the nonlinear system (1) with unknown sensor sensitivity $\theta(t)$ and the growth rate of polynomial-of-output $(1+|x_1|^p)$ multiplying the unknown constant ϱ.

For the system (1), we construct the following observer

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 - (L_1L_2)a_1\hat{x}_1, \\ \dot{\hat{x}}_2 = \hat{x}_3 - (L_1L_2)^2 a_2\hat{x}_1, \\ \quad\vdots \\ \dot{\hat{x}}_n = u - (L_1L_2)^n a_n\hat{x}_1, \end{cases} \tag{4}$$

where the dynamic gains L_1 and L_2 are updated by

$$\dot{L}_1 = L_2^{1-2\sigma}\Big(\Big(\frac{\hat{x}_1}{L_1^\sigma}\Big)^2 + \Big(\frac{y}{(1-\bar{\theta})L_1^\sigma}\Big)^2\Big), L_1(0) = 1, \tag{5}$$

$$\dot{L}_2 = -\alpha(L_2 - 1) + \beta\Big(1 + \Big(\frac{|y|}{1-\bar{\theta}}\Big)^p\Big)^2, L_2(0) = 1, \tag{6}$$

respectively, $\hat{x} = (\hat{x}_1, \ldots, \hat{x}_n)^T \in \mathbb{R}^n$ is the observer state. $a_i > 0$, for $i = 1, \ldots, n$ are coefficients of the Hurwitz polynomial $h_1(s) = s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n$. σ is a constant satisfying $0 < \sigma < 1/(2p)$. α, β are two positive constants to be designed.

Introduce the following change of coordinates,

$$e_i = \frac{x_i - \hat{x}_i}{(L_1L_2)^{i-1+\sigma}}, i = 1, \ldots, n, \tag{7}$$

$$z_1 = \frac{x_1}{(L_1L_2)^\sigma}, z_i = \frac{\hat{x}_i}{(L_1L_2)^{i-1+\sigma}L_3^{i-1}}, i = 2, \ldots, n, \tag{8}$$

where $L_3 \geq 1$ is a constant gain. The controller $u(t)$ is given by

$$\begin{aligned} u &= (L_1L_2)^{n+\sigma}L_3^n v, \\ v &= -b_n(L_1L_2)^{-\sigma}y - b_{n-1}z_2 - \ldots - b_1 z_n, \end{aligned} \tag{9}$$

where $b_1, \ldots, b_n$ are control gains.

From (1), (4), (7) and (8), we have

$$\dot{e} = L_1L_2Ae + L_1L_2az_1 - \Big(\frac{\dot{L}_1}{L_1} + \frac{\dot{L}_2}{L_2}\Big)D_\sigma e + F, \tag{10}$$

where $e = (e_1, \ldots, e_n)^T$, $D_\sigma = diag(\sigma, 1+\sigma, \ldots, n-1+\sigma)$,

$$A = \begin{pmatrix} -a_1 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -a_{n-1} & 0 & \ldots & 1 \\ -a_n & 0 & \ldots & 0 \end{pmatrix}, F = \begin{pmatrix} \frac{1}{(L_1L_2)^\sigma}f_1 \\ \frac{1}{(L_1L_2)^{1+\sigma}}f_2 \\ \vdots \\ \frac{1}{(L_1L_2)^{n-1+\sigma}}f_n \end{pmatrix}.$$

Consider the augmented system described by

$$\begin{cases} \dot{x}_1 = x_2 + f_1(t,x), \\ \dot{\hat{x}}_2 = \hat{x}_3 - (L_1L_2)^2 a_2 \hat{x}_1, \\ \quad \vdots \\ \dot{\hat{x}}_n = u - (L_1L_2)^n a_n \hat{x}_1. \end{cases} \tag{11}$$

From (7)- (9), the augmented system (11) can be rewritten as follows,

$$\begin{array}{l} \dot{z} = L_1L_2L_3Bz + L_1L_2L_3B_z b_n(1-\theta(t))z_1 + L_1L_2D_1e_2 \\ +\frac{L_1L_2}{L_3}D_2(e_1 - z_1) - \left(\frac{\dot{L}_1}{L_1} + \frac{\dot{L}_2}{L_2}\right)D_\sigma z + F_z, \end{array} \tag{12}$$

where $z = (z_1, \ldots, z_n)^T$, $B_z = (0, 0, \ldots, 1)^T$, $D_1 = (1, 0, \ldots, 0)^T$,

$$B = \begin{pmatrix} 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 \\ -b_n & -b_{n-1} & \ldots & -b_1 \end{pmatrix}, \; D_2 = \begin{pmatrix} 0 \\ a_2 \\ \frac{1}{L_3}a_3 \\ \vdots \\ \frac{1}{L_3^{n-2}}a_n \end{pmatrix}, \; F_z = \begin{pmatrix} \frac{1}{(L_1L_2)^\sigma}f_1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$$

We can suitably choose the coefficients b_i, $i = 1, \ldots, n-1$, such that the matrix B is Hurwitz. Then, there exist two positive definite matrices P and Q satisfying [9]

$$\begin{array}{l} A^TP + PA \le -2I, c_1I \le D_\sigma P + PD_\sigma \le c_2I, \\ B^TQ + QB \le -2I, c_3I \le D_\sigma Q + QD_\sigma \le c_4I, \end{array} \tag{13}$$

where $c_i > 0, i = 1, \ldots, 4$ are four real constants.

Then, we have the following results.

Proposition 3.1. For any $t_f \in (0, +\infty]$, there exists an unique solution to the closed-loop system $(10)-(12)$ on the maximal interval $[0, t_f)$. Moreover, if the following conditions are satisfied,

$$\alpha \le \min\left\{\frac{1}{c_2}, \frac{1}{c_4}\right\}, \tag{14}$$

and

$$\beta \ge \max\left\{\frac{1}{c_1}, \frac{1}{c_3}\right\}, \tag{15}$$

$$L_3 \ge \max\left\{1, \frac{1+4k_1}{8\rho}, 12\|P\|^2, 2\|Q\|\|a\|\right\}, \tag{16}$$

and

$$0 < \bar{\theta} < \min\left\{1, \frac{1}{b_n\|Q\|}\right\}, \tag{17}$$

where $k_1 = 3 + 2\|Q\|^2 + \|P\|^2\|a\|^2$, and $\rho = 1 - b_n\bar{\theta}\|Q\|$, then, the variable $L_1(t)$ is bounded on $[0, t_f)$.

P r o o f. Since the vector fields of the closed-loop system (1), (4), (5) and (6) are continuous and locally Lipschitz in $(x, \hat{x}, L_1, L_2)$, there exists an unique solution to the closed-loop system on the maximal interval $[0, t_f)$.

Next, consider the following Lyapunov function

$$V_1 = V_{11}(e) + V_{12}(z) = e^T Pe + z^T Qz, \tag{18}$$

where $V_{11}(e) = e^T Pe$, $V_{12}(z) = z^T Qz$.

The derivative of the function $V_{11}(e)$ is given by

$$\begin{aligned} &\dot{V}_{11}(e) = L_1L_2e^T(A^T P + PA)e + 2L_1L_2e^T Paz_1 \\ &-\Big(\tfrac{\dot{L}_1}{L_1} + \tfrac{\dot{L}_2}{L_2}\Big)e^T(D_\sigma P + PD_\sigma)e + 2e^T PF. \end{aligned} \tag{19}$$

From (5) and (6), we can deduce that $L_1 \geq 1$, $L_2 \geq 1$, and $\dot{L}_1 \geq 0$.

Due to $\frac{|\theta(t)|}{1-\bar{\theta}} \geq 1$, we have

$$\Big(1 + \Big(\frac{|y|}{1-\bar{\theta}}\Big)^p\Big)^2 \geq (1 + |x_1|^p)^2, \tag{20}$$

From (13), it follows that

$$\begin{aligned} &-\Big(\tfrac{\dot{L}_1}{L_1} + \tfrac{\dot{L}_2}{L_2})e^T(D_\sigma P + PD_\sigma\Big)e \\ &\leq -\tfrac{\dot{L}_1}{L_1}c_1\|e\|^2 - \tfrac{\dot{L}_2}{L_2}e^T(D_\sigma P + PD_\sigma)e \\ &\leq -\tfrac{\dot{L}_2}{L_2}e^T(D_\sigma P + PD_\sigma)e. \end{aligned} \tag{21}$$

Substituting (6) into (21), from (14) (15) and (20), we obtain

$$\begin{aligned} &-\tfrac{\dot{L}_2}{L_2}e^T(D_\sigma P + PD_\sigma)e \\ &= \alpha e^T(D_\sigma P + PD_\sigma)e - \tfrac{1}{L_2}\Big(\alpha + \beta\Big(1 + \Big(\tfrac{|y|}{1-\bar{\theta}}\Big)^p\Big)\Big)e^T(D_\sigma P + PD_\sigma)e \\ &\leq \alpha c_2\|e\|^2 - \tfrac{1}{L_2}\Big(\alpha + \beta\Big(1 + \Big(\tfrac{|y|}{1-\bar{\theta}}\Big)^p\Big)^2\Big)c_1\|e\|^2 \\ &\leq L_2\|e\|^2 - \tfrac{1}{L_2}(1 + |x_1|^p)^2\|e\|^2. \end{aligned} \tag{22}$$

It follows from Lemma 2.5 that

$$2L_1L_2e^T Paz_1 \leq L_1L_2\|e\|^2 + L_1L_2\|P\|^2\|a\|^2\|z\|^2 \tag{23}$$

From (7), (8), Lemma 2.6 and Assumption 2.1, we obtain

$$\begin{aligned} &\|F\| \leq \varrho(1 + |x_1|^p)\Big(\tfrac{|x_1|}{(L_1L_2)^\sigma} + \tfrac{|x_1|+|x_2|}{(L_1L_2)^{1+\sigma}} + \cdots + \tfrac{|x_1|+\cdots+|x_n|}{(L_1L_2)^{n-1+\sigma}}\Big) \\ &\leq \varrho(1 + |x_1|^p)\Big(\tfrac{n|x_1|}{(L_1L_2)^\sigma} + \tfrac{(n-1)|x_2|}{(L_1L_2)^{1+\sigma}} + \cdots + \tfrac{|x_n|}{(L_1L_2)^{n-1+\sigma}}\Big) \\ &\leq \varrho(1 + |x_1|^p)\Big(\tfrac{n|x_1|}{(L_1L_2)^\sigma} + \tfrac{(n-1)(|\hat{x}_2|+(L_1L_2)^{1+\sigma}|e_2|)}{(L_1L_2)^{1+\sigma}} + \cdots + \\ &\tfrac{(|\hat{x}_n|+(L_1L_2)^{n-1+\sigma}|e_n|)}{(L_1L_2)^{n-1+\sigma}}\Big) \\ &\leq \varrho(1 + |x_1|^p)\Big(n\sum_{i=1}^{n} L_3^{i-1}|z_i| + (n-1)\sqrt{n}\|e\|\Big). \end{aligned} \tag{24}$$

Lemma 2.5 and the inequality (24) imply that

$$
\begin{aligned}
&2e^TPF \le 2\|e\|\|P\|\varrho(1+|x_1|^p)\Big(n\sum_{i=1}^{n} L_3^{i-1}|z_i| + (n-1)\sqrt{n}\|e\|\Big)\\
&\le \tfrac{1}{L_2}(1+|x_1|^p)^2\|e\|^2 + L_2\|P\|^2\varrho^2\Big(n\sum_{i=1}^{n} L_3^{i-1}|z_i| + (n-1)\sqrt{n}\|e\|\Big)^2\\
&\le \tfrac{1}{L_2}(1+|x_1|^p)^2\|e\|^2 + 2L_2\|P\|^2\varrho^2(n^2\Big(\sum_{i=1}^{n} L_3^{i-1})^2\|z\|^2 + (n-1)^2 n\|e\|^2\Big).
\end{aligned} \tag{25}
$$

Substituting (22), (23) and (25) into (18), we have

$$
\begin{aligned}
&\dot V_{11}(e) \le -L_1L_2\|e\|^2 + L_1L_2\|P\|^2\|a\|^2\|z\|^2 + L_2\|e\|^2\\
&+2L_2\|P\|^2\varrho^2\Big(n^2\Big(\sum_{i=1}^{n} L_3^{i-1}\Big)^2\|z\|^2 + (n-1)^2n\|e\|^2\Big).
\end{aligned} \tag{26}
$$

The derivative of $V_{12}(z)$ along the system (12) is given as follows,

$$
\begin{aligned}
&\dot V_{12}(z) \le L_1L_2L_3z^T(B^TQ+QB)z + 2L_1L_2L_3z^TQB_zb_n\\
&(1-\theta(t))z_1 + 2L_1L_2z^TQD_1e_2 + 2\tfrac{L_1L_2}{L_3}z^TQD_2(e_1-z_1)\\
&-\Big(\tfrac{\dot L_1}{L_1}+\tfrac{\dot L_2}{L_2}\Big)z^T(D_\sigma Q+QD_\sigma)\Big)z + 2z^TQF_z.
\end{aligned} \tag{27}
$$

Similar to the inequality (22), we obtain

$$
\begin{aligned}
&-\Big(\tfrac{\dot L_1}{L_1}+\tfrac{\dot L_2}{L_2}\Big)z^T(D_\sigma Q+QD_\sigma)\Big)z\\
&\le \alpha c_4\|z\|^2 - \tfrac{1}{L_2}c_3(\alpha+\beta\Big(1+\Big(\tfrac{|y|}{1-\theta}\Big)^p\Big)^2)\|z\|^2\\
&\le L_1L_2\|z\|^2 - \tfrac{1}{L_2}(1+|x_1|^p)^2\|z\|^2.
\end{aligned} \tag{28}
$$

Note that $\|D_1\|=1$, $\|D_2\|\le\|a\|$. From (16) and Lemma 2.5, we have

$$
\begin{aligned}
&2L_1L_2z^TQD_1e_2 + 2\tfrac{L_1L_2}{L_3}z^TQD_2e_1\\
&\le \tfrac{L_1L_2}{2}\|e\|^2 + 2L_1L_2\|Q\|^2\|z\|^2\\
&+\tfrac{L_1L_2}{4}\|e\|^2 + \tfrac{4L_1L_2}{L_3^2}\|Q\|^2\|a\|^2\|z\|^2\\
&\le \tfrac{3L_1L_2}{4}\|e\|^2 + 2L_1L_2\|Q\|^2\|z\|^2 + L_1L_2\|z\|^2.
\end{aligned} \tag{29}
$$

Moreover,

$$
-2\tfrac{L_1L_2}{L_3}z^TQD_2z_1 \le 2\tfrac{L_1L_2}{L_3}\|Q\|\|a\|\|z\|^2 \le L_1L_2\|z\|^2. \tag{30}
$$

Assumption 2.1 and Lemma 2.5 imply that

$$
2z^TQF_z \le \tfrac{1}{L_2}(1+|x_1|^p)^2|\|z\|^2 + L_2\varrho^2\|Q\|^2\|z\|^2. \tag{31}
$$

Substituting (28)–(31) into (27), one can obtain that

$$
\begin{aligned}
&\dot V_{12}(z) \le -2(L_1L_2L_3)\|z\|^2 + 2(L_1L_2L_3)\|Q\|b_n|(1-\theta(t))|\|z\|^2\\
&+\tfrac{3L_1L_2}{4}\|e\|^2 + 3L_1L_2\|z\|^2 + 2L_1L_2\|Q\|^2\|z\|^2 + L_2\varrho^2\|Q\|^2\|z\|^2.
\end{aligned} \tag{32}
$$

It follows from (26) and (32) that

$$\begin{aligned}\dot{V}_1 &\le -\tfrac{1}{4}L_1L_2\|e\|^2 + L_1L_2\|P\|^2\|a\|^2\|z\|^2 + L_2\|e\|^2\\ &+2L_2\|P\|^2\varrho^2\Big(n^2\Big(\sum_{i=1}^{n} L_3^{i-1}\Big)^2\|z\|^2 + (n-1)^2n\|e\|^2\Big)\\ &-2(L_1L_2L_3)\|z\|^2 + 2(L_1L_2L_3)\|Q\|b_n|(1-\theta(t))|\|z\|^2\\ &+3L_1L_2\|z\|^2 + 2L_1L_2\|Q\|^2\|z\|^2 + L_2\varrho^2\|Q\|^2\|z\|^2.\end{aligned} \tag{33}$$

According to the condition (17), we have $1 > 1-b_n|1-\theta(t)|\|Q\| \ge 1-b_n\bar{\theta}\|Q\| = \rho > 0$. From the condition (16), the inequality (33) can be rewritten as follows,

$$\begin{aligned}\dot{V}_1 &\le -L_1L_2L_3\Big(2\rho - \tfrac{3+2\|Q\|^2+\|P\|^2\|a\|^2}{L_3} - \tfrac{2\|P\|^2\varrho^2n^2\left(\sum_{i=1}^{n} L_3^{i-1}\right)^2+\varrho^2\|Q\|^2}{L_1L_3}\Big)\|z\|^2\\ &-L_2\Big(\tfrac{1}{4}L_1 - (1+2\|P\|^2\varrho^2(n-1)^2n)\Big)\|e\|^2\\ &\le -L_2L_1L_3\Big(2\rho - \tfrac{k_1}{L_3} - \tfrac{k_2}{L_1L_3}\Big)\|z\|^2 - L_2\Big(\tfrac{1}{4}L_1 - k_3\Big)\|e\|^2\\ &\le -L_2\Big(\tfrac{1}{4}L_1 - k_2 - k_3)(\|e\|^2 + \|z\|^2\Big),\end{aligned} \tag{34}$$

where $k_1 = 3 + 2\|Q\|^2 + \|P\|^2\|a\|^2$, $k_2 = 2\|P\|^2\varrho^2n^2(\sum_{i=1}^{n} L_3^{i-1})^2 + \varrho^2\|Q\|^2$, $k_3 = 1 + 2\|P\|^2\varrho^2(n-1)^2n$.

Assume that $L_1(t)$ is not bounded on the interval $[0, t_f)$, then

$$\lim_{t\to t_f} L_1(t) = +\infty. \tag{35}$$

Since $\dot{L}_1 \ge 0$ and $L_1 \ge 1$, $L_1(t)$ is a monotone nondecreasing function. From (35), there exists a time $t^* > 0$ such that $L_1(t) \ge 4(k_2 + k_3 + 1), \forall t \in [t^*, t_f)$. Then, from the differential inequality (34), we have

$$\dot{V}_1(t) \le -L_2(\|e\|^2 + \|z\|^2), \forall t \in [t^*, t_f).$$

From (5) and Lemma 2.6, one can obtain that

$$\begin{aligned}\dot{L}_1 &= L_2^{1-2\sigma}\Big(\Big(\tfrac{\hat{x}_1}{L_1^\sigma}\Big)^2 + \Big(\tfrac{y}{(1-\bar{\theta})L_1^\sigma}\Big)^2\Big)\\ &= L_2(z_1-e_1)^2 + L_2\Big(\tfrac{\theta(t)}{1-\bar{\theta}}\Big)^2 z_1^2\\ &\le L_2\gamma(\|e\|^2 + \|z\|^2),\end{aligned}$$

where $\gamma = 2 + \left(\frac{1+\bar{\theta}}{1-\bar{\theta}}\right)^2$. Hence,

$$\int_{t^*}^{t_f} \dot{L}_1\,\mathrm{d}t \le \int_{t^*}^{t_f} L_2\gamma(\|e\|^2 + \|z\|^2)\,\mathrm{d}t \le \gamma V_1(\|e(t^*)\|, \|z(t^*)\|).$$

Therefore,

$$+\infty = L_1(t_f) - L_1(t^*) = \int_{t^*}^{t_f} \dot{L}_1\,\mathrm{d}t \le \gamma V_1(\|e(t^*)\|, \|z(t^*)\|),$$

which is impossible. Thus, the dynamic gain L_1 is bounded on $[t^*, t_f)$ and $\lim_{t\to t_f} L_1(t)$ is finite.

Now, consider the change of coordinates

$$\xi_i = \frac{x_i - \hat{x}_i}{(L_1^* L_2)^{i-1+\sigma}}, i = 1, \ldots, n, \tag{36}$$

and

$$\varepsilon_1 = \frac{x_1}{(L_1^* L_2)^{\sigma}}, \varepsilon_i = \frac{\hat{x}_i}{(L_1^* L_2)^{i-1+\sigma} L_3^{i-1}}, \tag{37}$$

where L_1^* is a constant satisfying

$$\begin{aligned}
&L_1^* > \max\Big\{L_1(+\infty), 12\sqrt{n-1}\|P\|\|a\|L_1, 6\varrho^2\|P\|^2\,,\\
&12L_1\sqrt{n}(b_n(1+\bar{\theta}) + \sum_{i=1}^{n-1} b_i)\|P\|, \frac{2+2\|P\|^2\varrho^2 n^2(\sum_{i=1}^{n} L_3^{i-1})^2}{L_3}\,,\\
&2+2\|P\|^2\varrho^2(n-1)^2 n\Big\}.
\end{aligned} \tag{38}$$

It can be deduced from $(1), (4), (36)$ and (37) that

$$\begin{aligned}
\dot{\xi} &= L_1^* L_2 A\xi + L_1^* L_2 a\varepsilon_1 - L_1 L_2 M a\xi_1\\
&+ L_1 L_2 M a\varepsilon_1 - \tfrac{\dot{L}_2}{L_2} D_\sigma \xi + F^*,
\end{aligned} \tag{39}$$

where $M = diag\Big[1, \frac{L_1}{L_1^*}, \ldots, \Big(\frac{L_1}{L_1^*}\Big)^{n-1}\Big]$, $F^* = \begin{pmatrix} \frac{1}{(L_1^* L_2)^{\sigma}} f_1 \\ \frac{1}{(L_1^* L_2)^{1+\sigma}} f_2 \\ \vdots \\ \frac{1}{(L_1^* L_2)^{n-1+\sigma}} f_n \end{pmatrix}.$

From $(11), (36)$ and (37), we have

$$\begin{aligned}
\dot{\varepsilon} &= L_1^* L_2 L_3 A\varepsilon + L_1^* L_2 L_3 a\varepsilon_1 + L_1 L_2 \Gamma a\xi_1 + L_1^* L_2 D_1 \xi_2\\
&- L_1 L_2 \Gamma a\varepsilon_1 + B_z \tfrac{u}{(L_1^* L_2)^{n-1+\sigma} L_3^{n-1}} - \tfrac{\dot{L}_2}{L_2} D_\sigma \varepsilon + F_\varepsilon^*,
\end{aligned} \tag{40}$$

where $\Gamma = diag\Big[0, \frac{L_1}{L_1^* L_3}, \ldots, \Big(\frac{L_1}{L_1^* L_3}\Big)^{n-1}\Big]$, $F_\varepsilon^* = \begin{pmatrix} \frac{1}{(L_1^* L_2)^{\sigma}} f_1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$

□

Proposition 3.2. If the conditions $(14)-(16)$ are satisfied, then, the variables $L_2(t)$, $\varepsilon(t), \xi(t)$ are bounded on $[0, t_f)$.

P r o o f. The derivative of $V_{21}(\varepsilon) = \varepsilon^T P\varepsilon$ along the system (40) is given as follows,

$$\begin{aligned}
\dot{V}_{21}(\varepsilon) &= L_1^* L_2 L_3 \varepsilon^T (A^T P + PA)\varepsilon + 2L_1 L_2 \varepsilon^T P\Gamma a\xi_1\\
&+ 2L_1^* L_2 L_3 \varepsilon^T P a\varepsilon_1 - 2L_1 L_2 \varepsilon^T P\Gamma a\varepsilon_1\\
&+ 2L_1^* L_2 \varepsilon^T P D_1 \xi_2 + 2\varepsilon^T P F_\varepsilon^* + 2\varepsilon^T P B_z \tfrac{u}{(L_1^* L_2)^{n-1+\sigma} L_3^{n-1}}\\
&- \tfrac{\dot{L}_2}{L_2} \varepsilon^T (D_\sigma P + P D_\sigma)\varepsilon.
\end{aligned} \tag{41}$$

Note that $\|D_1\| = 1$, $\|\Gamma\| \leq \sqrt{n-1}$. From (38) and Lemma (2.5), we have

$$\begin{aligned}&2L_1L_2\varepsilon^T P\Gamma a\xi_1 \leq 6(n-1)\tfrac{L_1L_2}{L_3}\|a\|^2\|P\|^2\xi_1^2 + \tfrac{1}{6}L_1L_2L_3\|\varepsilon\|^2\\&\leq 6(n-1)\tfrac{L_1L_2}{L_3}\|a\|^2\|P\|^2\xi_1^2 + \tfrac{1}{6}L_1^*L_2L_3\|\varepsilon\|^2,\end{aligned} \tag{42}$$

$$2L_1^*L_2L_3\varepsilon^T Pa\varepsilon_1 \leq 6L_1^*L_2L_3\|a\|^2\|P\|^2\varepsilon_1^2 + \tfrac{1}{6}L_1^*L_2L_3\|\varepsilon\|^2. \tag{43}$$

From (38), we obtain

$$\begin{aligned}&-2L_1L_2\varepsilon^T P\Gamma a\varepsilon_1 \leq 2\sqrt{n-1}L_1L_2\|P\|\|a\|\|\varepsilon\|^2\\&\leq \tfrac{1}{6}L_1^*L_2L_3\|\varepsilon\|^2.\end{aligned} \tag{44}$$

Since $\|D_1\| = 1$, from (16) and Lemma 2.5, we have

$$\begin{aligned}&2L_1^*L_2\varepsilon^T PD_1\xi_2 \leq 6\tfrac{L_1^*L_2}{L_3}\|P\|^2\xi_2^2 + \tfrac{1}{6}L_1^*L_2L_3\|\varepsilon\|^2\\&\leq \tfrac{1}{2}L_1^*L_2\|\xi\|^2 + \tfrac{1}{6}L_1^*L_2L_3\|\varepsilon\|^2.\end{aligned} \tag{45}$$

It follows from (38), Assumption 2.1 and Lemma 2.5 that

$$\begin{aligned}&2\varepsilon^T PF_\varepsilon^* \leq \tfrac{1}{L_2}(1+|x_1|^p)^2\|\varepsilon\|^2 + L_2\varrho^2\|P\|^2\|\varepsilon\|^2\\&\leq \tfrac{1}{L_2}(1+|x_1|^p)^2\|\varepsilon\|^2 + \tfrac{1}{6}L_1^*L_2L_3\|\varepsilon\|^2.\end{aligned} \tag{46}$$

Using the inequalities (6), (20), (14) and (15), we have

$$\begin{aligned}&-\tfrac{\dot{L}_2}{L_2}\varepsilon^T(D_\sigma P + PD_\sigma)\varepsilon\\&\leq \alpha c_2\|\varepsilon\|^2 - \tfrac{1}{L_2}c_1\Big(\alpha + \beta\Big(1+\Big(\tfrac{|y|}{1-\theta}\Big)^p\Big)^2\Big)\|\varepsilon\|^2\\&\leq L_2\|\varepsilon\|^2 - \tfrac{1}{L_2}(1+|x_1|^p)^2\|\varepsilon\|^2.\end{aligned} \tag{47}$$

From (9) and (37), it follows that

$$\begin{aligned}&\tfrac{u}{(L_1^*L_2)^{n-1+\sigma}L_3^{n-1}}\\&= L_2\tfrac{L_1^nL_3}{(L_1^*)^{n-1}}(-b_n\theta(t)\varepsilon_1 - \sum_{i=1}^{n-1}b_i\Big(\tfrac{L_1^*}{L_1}\Big)^{n-i}\varepsilon_{n+1-i}).\end{aligned}$$

Moreover, from (38), we have

$$\begin{aligned}&2\varepsilon^T PB_z\tfrac{u}{(L_1^*L_2)^{n-1+\sigma}L_3^{n-1}}\\&\leq 2L_1L_2L_3\|P\|\|\varepsilon\|\Big(b_n(1+\bar{\theta})\|\varepsilon_1\| + \sum_{i=1}^{n-1}b_i\|\varepsilon_{n+1-i}\|\Big)\\&\leq 2L_1L_2L_3\Big(b_n(1+\bar{\theta}) + \sum_{i=1}^{n-1}b_i\Big)\|P\|\|\varepsilon\|(|\varepsilon_1| + |\varepsilon_2| + \cdots + |\varepsilon_n|)\\&\leq 2L_1L_2L_3\sqrt{n}\Big((1+\bar{\theta}) + \sum_{i=1}^{n-1}b_i\Big)\|P\|\|\varepsilon\|^2\\&\leq \tfrac{1}{6}L_1^*L_2L_3\|\varepsilon\|^2.\end{aligned} \tag{48}$$

Substituting (42) – (48) into (41), one can obtain

$$\begin{aligned}&\dot{V}_{21}(\varepsilon) \leq -L_1^*L_2L_3\|\varepsilon\|^2 + 6(n-1)\tfrac{L_1L_2}{L_3}\|a\|^2\|P\|^2\xi_1^2\\&+6L_1^*L_2L_3\|a\|^2\|P\|^2\varepsilon_1^2 + \tfrac{1}{2}L_1^*L_2\|\xi\|^2 + L_2\|\varepsilon\|^2.\end{aligned} \tag{49}$$

The derivative of $V_{22}(\xi) = \xi^T P\xi$ along the system (39) is given by

$$\begin{array}{l}\dot{V}_{22}(\xi) = L_1^* L_2 \xi^T (A^T P + PA)\xi + 2L_1^* L_2 \xi^T Pa\varepsilon_1 \\ -2L_1 L_2 \xi^T PMa\xi_1 + 2L_1 L_2 \xi^T PMa\varepsilon_1 \\ -\frac{\dot{L}_2}{L_2}\xi^T (D_\sigma P + PD_\sigma)\xi + 2\xi^T PF^*.\end{array} \tag{50}$$

From Lemma 2.5, we can obtain

$$2L_1^* L_2 \xi^T Pa\varepsilon_1 \le 6L_1^* L_2 \|P\|^2 \|a\|^2 \varepsilon_1^2 + \tfrac{1}{6} L_1^* L_2 \|\xi\|^2. \tag{51}$$

Since $\|M\| \le \sqrt{n}$, the condition (38) and Lemma 2.5 imply

$$-2L_1 L_2 \xi^T PMa\xi_1 \le 6nL_1^* L_2 \|P\|^2 \|a\|^2 \xi_1^2 + \tfrac{1}{6} L_1^* L_2 \|\xi\|^2, \tag{52}$$

$$2L_1 L_2 \xi^T PMa\varepsilon_1 \le 6nL_1^* L_2 \|P\|^2 \|a\|^2 \varepsilon_1^2 + \tfrac{1}{6} L_1^* L_2 \|\xi\|^2. \tag{53}$$

Note that,

$$\|F^*\| \le (1 + |x_1|^p)\varrho\Big(n \sum_{i=1}^n L_3^{i-1} |\varepsilon_i| + (n-1)\sqrt{n}\|\xi\|\Big).$$

Thus, from Lemma 2.5 and Lemma 2.6, we have

$$\begin{array}{l}2\xi^T PF^* \le \frac{1}{L_2}(1 + |x_1|^p)^2 \|\xi\|^2 \\ +2L_2 \|P\|^2 \varrho^2 \Big(n^2 \Big(\sum_{i=1}^n L_3^{i-1}\Big)^2 \|\varepsilon\|^2 + (n-1)^2 n\|\xi\|^2\Big).\end{array} \tag{54}$$

From (6), (14), (15) and (20), it follows that

$$\begin{array}{l}-\frac{\dot{L}_2}{L_2}\xi^T (D_\sigma P + PD_\sigma)\xi \\ \le \alpha c_4 \|\xi\|^2 - \frac{1}{L_2} c_3 \Big(\alpha + \beta\Big(1 + \Big(\frac{|y|}{1-\theta}\Big)^p\Big)^2\Big)\|\xi\|^2 \\ \le L_2 \|\xi\|^2 - \frac{1}{L_2}(1 + |x_1|^p)^2 \|\xi\|^2.\end{array} \tag{55}$$

Substituting (51) – (55) into (50) yields

$$\begin{array}{l}\dot{V}_{22}(\xi) \le -\Big(\frac{3}{2} L_1^* L_2 - 2L_2 \|P\|^2 \varrho^2 (n-1)^2 n - L_2\Big)\|\xi\|^2 \\ +6L_1^* L_2 \|P\|^2 \|a\|^2 \varepsilon_1^2 + 6nL_1^* L_2 \|P\|^2 \|a\|^2 \xi_1^2 \\ +6nL_1^* L_2 \|P\|^2 \|a\|^2 \varepsilon_1^2 + 2L_2 \|P\|^2 \varrho^2 n^2 \Big(\sum_{i=1}^n L_3^{i-1}\Big)^2 \|\varepsilon\|^2\end{array} \tag{56}$$

Then, (49) and (56) imply that

$$\begin{array}{l}\dot{V}_2 = \dot{V}_{21}(\varepsilon) + \dot{V}_{22}(\xi) \\ \le -L_2(L_1^* L_3 - 2\|P\|^2 \varrho^2 n^2 (\sum_{i=1}^n L_3^{i-1})^2 - 1)\|\varepsilon\|^2 \\ -L_2(L_1^* - 2\|P\|^2 \varrho^2 (n-1)^2 n - 1)\|\xi\|^2 \\ +L_2(6(n-1)\frac{L_1^*}{L_3}\|a\|^2 \|P\|^2 + 6nL_1^* \|P\|^2 \|a\|^2)\xi_1^2 \\ +L_2(6L_1^* L_3 \|P\|^2 \|a\|^2 + 6L_1^* \|a\|^2 \|P\|^2 + 6nL_1^* \|P\|^2 \|a\|^2)\varepsilon_1^2 \\ \le -L_2(\|\varepsilon\|^2 + \|\xi\|^2) + m_2 L_2 \xi_1^2 + m_3 L_2 \varepsilon_1^2,\end{array} \tag{57}$$

where $m_2 = 6(n-1)\frac{L_1^*}{L_3}\|a\|^2\|P\|^2 + 6nL_1^*\|P\|^2\|a\|^2$, $m_3 = 6L_1^*L_3\|a\|^2\|P\|^2 + 6L_1^*\|P\|^2\|a\|^2 + 6nL_1^*\|P\|^2\|a\|^2$.

From (36), (37) and Lemma 2.6, we have

$$\begin{aligned}
& m_2L_2\xi_1^2 + m_3L_2\varepsilon_1^2 = L_2\Big(\tfrac{m_2(x_1-\hat{x}_1)^2}{(L_1^*L_2)^{2\sigma}} + \tfrac{m_3x_1^2}{(L_1^*L_2)^{2\sigma}}\Big) \\
& \le L_2\tfrac{(2m_2+m_3)(x_1^2+\hat{x}_1^2)}{(L_1^*L_2)^{2\sigma}} \\
& \le (2m_2+m_3)L_2^{1-2\sigma}\Big(\Big(\tfrac{\hat{x}_1}{L_1^\sigma}\Big)^2 + \Big(\tfrac{y}{(1-\bar{\theta})L_1^\sigma}\Big)^2\Big) \\
& \le (2m_2+m_3)\dot{L}_1.
\end{aligned}$$

Hence

$$\dot{V}_2 \le -L_2(\|\varepsilon\|^2 + \|\xi\|^2) + (2m_2+m_3)\dot{L}_1, \forall t \in [0, t_f). \tag{58}$$

From (58), it follows that

$$\begin{aligned}
& \lambda_{\min}(P)(\|\varepsilon\|^2 + \|\xi\|^2) - (\varepsilon(0)^TP\varepsilon(0) + \xi(0)^TP\xi(0)) \\
& \le -\int_0^t L_2(\|\varepsilon\|^2 + \|\xi\|^2)\,\mathrm{d}t \\
& +(2m_2+m_3)(L_1(t)-1), \forall t \in [0, t_f).
\end{aligned}$$

Then,

$$\begin{aligned}
& \|\varepsilon\|^2 + \|\xi\|^2 \le \tfrac{1}{\lambda_{\min}(P)}(\varepsilon(0)^TP\varepsilon(0) + \xi(0)^TP\xi(0) \\
& +(2m_2+m_3)(L_1(t)-1)), \forall t \in [0, t_f),
\end{aligned}$$

$$\begin{aligned}
& \int_0^t L_2(\|\varepsilon\|^2 + \|\xi\|^2)\,\mathrm{d}t \le \varepsilon(0)^TP\varepsilon(0) + \xi(0)^TP\xi(0) \\
& +(2m_2+m_3)(L_1(t)-1), \forall t \in [0, t_f).
\end{aligned}$$

From Proposition 1, we have L_1 is bounded on $[0, t_f)$, thus, $\int_0^t L_2\|\varepsilon\|\,\mathrm{d}t$, $\int_0^t L_2\|\xi\|\,\mathrm{d}t$, ε and ξ are bounded on $[0, t_f)$. Then, $e(t)$ and $z(t)$ are bounded on $[0, t_f)$.

Because L_1 and z_1 are bounded on $[0, t_f)$, we have

$$|\tfrac{x_1}{L_2^\sigma}| = L_1^\sigma|z_1| \le C, \forall t \in [0, t_f),$$

where C is a real constant. Then,

$$|x_1| \le CL_2^\sigma, \forall t \in [0, t_f).$$

Since $0 < \sigma < \frac{1}{2p}$, from Lemma (2.5), one can obtain

$$\begin{aligned}
& \dot{L}_2 = -\alpha(L_2-1) + \beta\Big(1 + \Big(\tfrac{|y|}{1-\bar{\theta}}\Big)^p\Big)^2 \\
& \le -\alpha L_2 + 2\beta\Big(\tfrac{1+\bar{\theta}}{1-\bar{\theta}}\Big)^{2p}C^{2p}L_2^{2p\sigma} + 2\beta + \alpha, \\
& \le -\tfrac{\alpha}{2}L_2 + b, \forall t \in [0, t_f),
\end{aligned}$$

where $b > 0$ is a suitable constant. Obviously, L_2 is bounded on $[0, t_f)$. □

Theorem 3.3. For the system (1) with the Assumptions 2.1 and 2.2 and a given allowable sensitivity error $\bar{\theta}$, if the conditions (14) – (17) hold, then, under the output feedback controller (4) – (6) and (9), the system (1) converges to the equilibrium at origin, which means that $\lim_{t\to+\infty} x(t) = \lim_{t\to+\infty} \hat{x}(t) = 0$.

P r o o f. From Proposition 1 and 2, we can obtain that L_1, L_2, ε, ξ are bounded on the interval $[0,+\infty)$ and $\int_0^{+\infty} L_2\|\varepsilon\| \,\mathrm{d}t \leq +\infty$, $\int_0^{+\infty} L_2\|\xi\| \,\mathrm{d}t \leq +\infty$. By the Barbalat's Lemma [7], we have $\lim_{t\to+\infty} \varepsilon(t) = 0$ and $\lim_{t\to+\infty} \xi(t) = 0$. Therefore, $\lim_{t\to+\infty} e(t) = 0$ and $\lim_{t\to+\infty} z(t) = 0$, which implies that $\lim_{t\to+\infty} x(t) = 0$ and$\lim_{t\to+\infty} \hat{x}(t) = 0$. Since L_1, L_2 are bounded and $\lim_{t\to+\infty} x(t) = 0$, $\lim_{t\to+\infty} \hat{x}(t) = 0$, we have $\lim_{t\to+\infty} u(t) = 0$. The proof is completed. □

Remark 3.4. Although $\theta(t)$ is unknown, the output with the measurement error is known and available. That is, the measurable output contains unknown disturbance. Moreover, the output with the measurement error $y(t)$ is directly applied to design the output feedback controller (9). In order to stabilize the nonlinear system (1), we use two variable gains L_1, L_2 and a constant gain L_3. The unknown constant ϱ, and polynomial-of-output $(1 + |x_1|^p)$ are coped with by the variable gains L_1, and L_2, respectively. The constant gain L_3 is used to deal with the sensor uncertainty $\theta(t)$.

4. FURTHER EXTENSIONS

From Theorem 3.3, we can obtain that the sensor uncertainty $\theta(t)$ is in a small neighborhood near 1, which implies that the measured output is very close to the actual output. However, in practice, it may occur that the measured output is very larger or very smaller than the actual output, such as $\theta(t) \in [1.04, 2.16]$ or $\theta(t) \in [0.455, 0.945]$. In what follows, we discuss the problem of adaptive output feedback stabilization under such cases.

In order to derive one of our extension, we need the following assumption.

Assumption 4.1. The sensor sensitivity $\theta(t)$ is an unknown continuous function satisfying $\theta(t) \in [R(1-\bar{\theta}), R(1+\bar{\theta})]$, where R is a known positive constant. Then, $R\bar{\theta}$ is an allowable sensitivity error.

Remark 4.2. Compared with Assumption 2.2 [12, 16], $\theta(t)$ has a wider range. In fact, let $R = 1$, Assumption 4.1 is reduced to Assumption 2.2. Therefore, Assumption 2.2 can be regarded as a special case of Assumption 4.1.

Consider the following coordinate transformation,

$$s_i = Rx_i, i = 1, \dots, n,$$

$$\delta(t) = \tfrac{\theta(t)}{R}, \omega = Ru,$$

and

$$g_i(t,x) = Rf_i(t,x), i = 1, \dots, n.$$

The system (1) can be written as,

$$\begin{cases} \dot{s}_i = s_{i+1} + g_i(t,s), i = 1, \dots, n-1, \\ \dot{s}_n = \omega + g_n(t,s), \\ y = \delta(t)s_1. \end{cases}$$

From Assumption 2.1, we can obtain

$$\begin{aligned}&|g_i(t,s)| = R|f_i(t,x)| \le \varrho(1+|x_1|^p)R(|x_1|+\cdots+|x_i|)\\&= \varrho(1+|\tfrac{s_1}{R}|^p)(|s_1|+\cdots+|s_i|),\end{aligned}$$

Then,

$$\delta(t) \in [1-\bar{\theta}, 1+\bar{\theta}].$$

Construct the following observer

$$\begin{cases}\dot{\hat{x}}_1 = \hat{x}_2 - (L_1L_2)a_1\hat{x}_1,\\ \dot{\hat{x}}_2 = \hat{x}_3 - (L_1L_2)^2a_2\hat{x}_1,\\ \quad\vdots\\ \dot{\hat{x}}_n = u - (L_1L_2)^n a_n\hat{x}_1,\end{cases} \tag{59}$$

where the dynamic gains L_1 and L_2 are updated by

$$\dot{L}_1 = L_2^{1-2\sigma}\Big(\Big(\tfrac{R\hat{x}_1}{L_1^\sigma}\Big)^2 + \Big(\tfrac{y}{(1-\bar{\theta})L_1^\sigma}\Big)^2\Big), L_1(0)=1, \tag{60}$$

and

$$\dot{L}_2 = -\alpha(L_2-1) + \beta\Big(1+\Big(\tfrac{|y|}{R(1-\bar{\theta})}\Big)^p\Big)^2, L_2(0)=1, \tag{61}$$

$\hat{x} = (\hat{x}_1,\ldots,\hat{x}_n)^T \in \mathbb{R}^n$ is observer state.

Consider the following coordinate transformation,

$$\hat{s}_i = R\hat{x}_i, i=1,\ldots,n.$$

$$\chi_i = \frac{s_i - \hat{s}_i}{(L_1L_2)^{i-1+\sigma}}, i=1,\ldots,n,$$

$$\varsigma_1 = \frac{s_1}{(L_1L_2)^\sigma}, \varsigma_i = \frac{\hat{s}_i}{(L_1L_2)^{i-1+\sigma}L_3^{i-1}}, i=2,\ldots,n.$$

The controller $u(t)$ is given by

$$\begin{aligned}&u(t) = \tfrac{\omega}{R}, \omega = (L_1L_2)^{n+\sigma}L_3^n\nu,\\&\nu = -b_n(L_1L_2)^{-\sigma}y - b_{n-1}\varsigma_2 - \ldots - b_1\varsigma_n.\end{aligned} \tag{62}$$

Then, we have the following results.

Theorem 4.3. For the system (1) with the Assumptions 2.1 and 4.1, if the parameters $\bar{\theta}$, α, β, L_3 satisfy the conditions (16) – (17), then, under the output feedback controller (59) – (61) and (62), the system (1) converges to the equilibrium at origin, which means that $\lim_{t\to+\infty} x(t) = \lim_{t\to+\infty}\hat{x}(t) = 0$.

P r o o f . Using the same methods to Theorem 3.3, we can easily obtain the conclusions. Detailed proofs are omitted.

In what follows, we make an other extension.

Consider the following system:

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(t,x), i = 1,\dots,n-1, \\ \dot{x}_n = u + f_n(t,x), \\ y = \theta(t)Rx_1, \end{cases} \tag{63}$$

where $x = (x_1,\dots,x_n)^T \in \mathbb{R}^n$, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the system state, control input and measurement output. The sensor sensitivity $\theta(t)$ $(t \in \mathbb{R}^+)$ is an unknown continuous function. $R \in \mathbb{R}$ is an unknown positive constant. □

Besides Assumptions 2.1, the system is required to meet the following assumption.

Assumption 4.4. There exist two positive constants R_1 and R_2 such that $R_1 \le R \le R_2$ and $\frac{R_2}{R_1} \le \frac{1+\bar{\theta}}{1-\bar{\theta}}$.

Construct the following observer

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 - (L_1L_2)a_1\hat{x}_1, \\ \dot{\hat{x}}_2 = \hat{x}_3 - (L_1L_2)^2a_2\hat{x}_1, \\ \quad\vdots \\ \dot{\hat{x}}_n = u - (L_1L_2)^n a_n\hat{x}_1, \end{cases} \tag{64}$$

where the dynamic gains L_1 and L_2 are given by

$$\dot{L}_1 = L_2^{1-2\sigma}\Big(\Big(\tfrac{R_2\hat{x}_1}{L_1^\sigma}\Big)^2 + \Big(\tfrac{y}{(1-\bar{\theta})L_1^\sigma}\Big)^2\Big), L_1(0) = 1, \tag{65}$$

and

$$\dot{L}_2 = -\alpha(L_2 - 1) + \beta\Big(1 + \Big(\tfrac{|y|}{R_1(1-\bar{\theta})}\Big)^p\Big)^2, L_2(0) = 1. \tag{66}$$

The controller $u(t)$ is given as

$$\begin{aligned} u(t) &= (L_1L_2)^{n+\sigma}L_3^n\nu, \\ \nu &= -b_n(L_1L_2)^{-\sigma}\tfrac{y}{R_2} - b_{n-1}\tfrac{\hat{x}_2}{(L_1L_2)^\sigma L_3} - \dots - b_1\tfrac{\hat{x}_n}{(L_1L_2)^{n-1+\sigma}L_3^{n-1}}. \end{aligned} \tag{67}$$

Theorem 4.5. For the system (63) with Assumptions 2.1 and Assumptions 4.4, if the sensor sensitivity $\theta(t)$ satisfies

$$\theta(t) \in \Big[\frac{R_2(1-\bar{\theta})}{R_1}, 1+\bar{\theta}\Big],$$

and the conditions (16)–(17) hold, then, under the output feedback controller (64)–(66) and (67), the system (63) converges to the equilibrium at origin, which means that $\lim_{t\to+\infty} x(t) = \lim_{t\to+\infty} \hat{x}(t) = 0$.

P r o o f . The proofs are similar to Theorem 3.3 and omitted. □

5. NUMERICAL SIMULATIONS

Case 1. Consider the following SISO nonlinear system with sensor uncertainty,

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = u - x_1 - \mu(1 - x_1^2)x_2, \\ y = \theta(t)x_1, \end{cases} \tag{68}$$

where μ is an unknown constant and $\theta(t)$ is an unknown function. It is easy to verify that system (68) satisfies Assumption 2.1 with $p = 2$, $\varrho = \max\{1, |\mu|\}$.

Based on Theorem 3.3, set $a_1 = 1.5$, $a_2 = 1.5$, $b_1 = 1$, $b_2 = 0.5$, $\sigma = 0.24$. From (13), we choose $P = \begin{pmatrix} 1.6667 & -1 \\ -1 & 2.1112 \end{pmatrix}$, $Q = \begin{pmatrix} 3.5 & 2 \\ 2 & 3 \end{pmatrix}$. Then, we obtain $c_1 = 0.3515$, $c_2 = 5.6842$, $c_3 = 0.4301$, $c_4 = 8.6900$ and $\frac{1}{b_n\|Q\|} = 0.3798$. Obviously, we have $\bar{\theta} \leq \min\{1, 0.3798\}$. According to (14) - (16), we construct the following controller for the system (68),

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 - 1.5L_1L_2\hat{x}_1, \\ \dot{\hat{x}}_2 = u - 1.5(L_1L_2)^2\hat{x}_1, \\ u = 150^2(L_1L_2)^{2.24}\Big(- 0.5(L_1L_2)^{-0.24}y - \frac{\hat{x}_2}{150(L_1L_2)^{1.24}}\Big), \\ \dot{L}_1 = L_2^{0.52}\Big(\frac{\hat{x}_1^2}{L_1^{0.48}} + \frac{y^2}{0.75^2L_1^{0.48}}\Big), L_1(0) = 1, \\ \dot{L}_2 = -0.115(L_2 - 1) + 2.85\Big(1 + \frac{y^2}{0.75^2}\Big)^2, L_2(0) = 1. \end{cases}$$

In the numerical simulation, we set the parameters $\mu = 3$, $\alpha = 0.115$, $\beta = 2.85$, $L_3 = 150$, and the measurement error $\theta(t) = 1 + 0.2\sin(10t)$ satisfying $\theta(t) \in [0.8, 1.2]$, and the initial conditions $x_1(0) = 0.35$, $x_2(0) = 1$, $\hat{x}_1(0) = 2$, $\hat{x}_2(0) = 5$. The simulation results are shown in Figures 1−5. It is observed that $\lim_{t\to+\infty} x_1(t) = \lim_{t\to+\infty} x_2(t) = \lim_{t\to+\infty} \hat{x}_1(t) = \lim_{t\to+\infty} \hat{x}_2(t) = 0$, $\lim_{t\to+\infty} u(t) = 0$, which demonstrates the effectiveness of the controller.

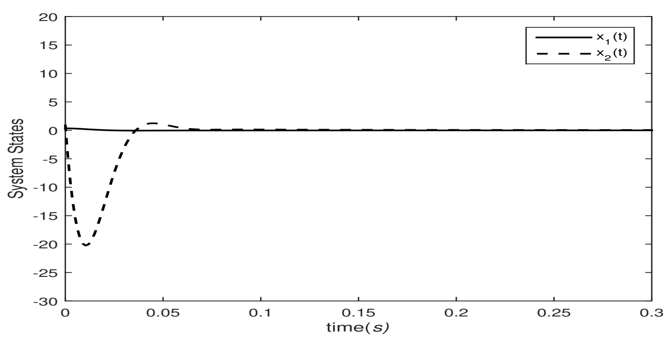

Fig. 1. The trajectories of the states of the closed-loop system.

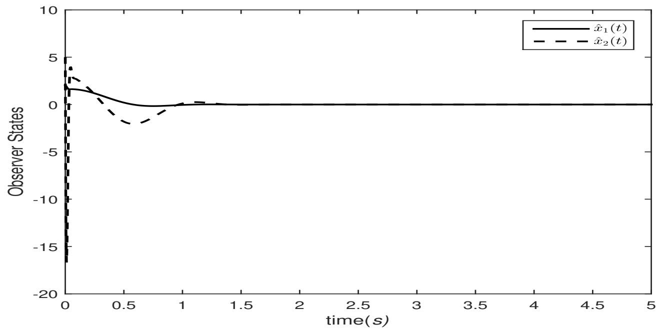

Fig. 2. The trajectories of the states of the observer.

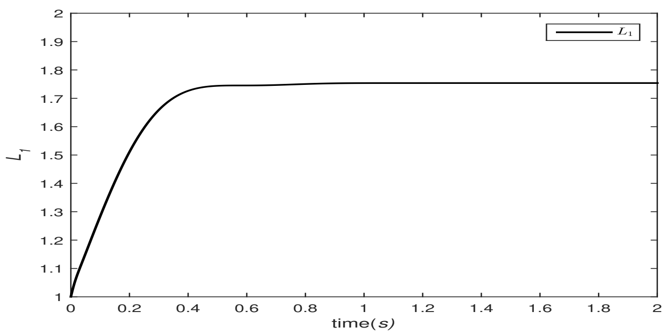

Fig. 3. The trajectory of the gain L_1.

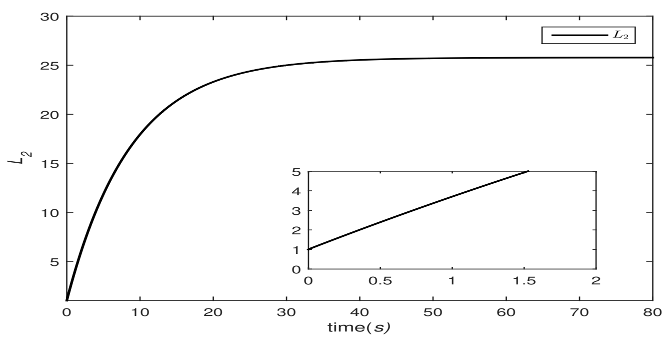

Fig. 4. The trajectory of the gain L_2.

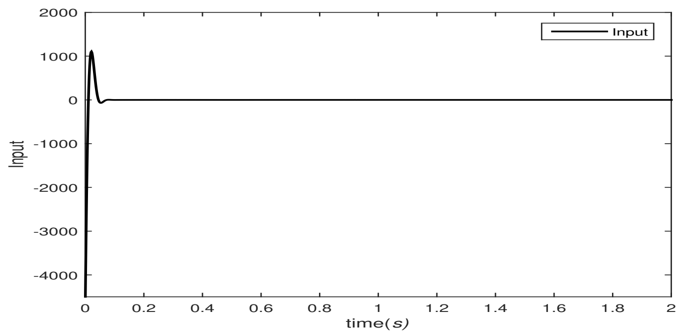

Fig. 5. The trajectory of the input u.

Case 2. Consider the following SISO nonlinear system (68) with measurement sensitivity $\theta(t) = R\delta(t)$, where R is a positive real number and $\delta(t)$ is an unknown function.

Based on Theorem 4.3, set the same values of the parameters a_1, a_2, b_1, b_2, σ to those in Case 1. We have $\bar{\theta} \leq \min\{1, 0.3798\}$. According to (14) - (16), we construct the following output feedback controller,

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 - 1.5L_1L_2\hat{x}_1, \\ \dot{\hat{x}}_2 = u - 1.5(L_1L_2)^2\hat{x}_1, \\ u = 150^2(L_1L_2)^{2.24}\Big(- 0.5(L_1L_2)^{-0.24}\frac{y}{R} - \frac{\hat{x}_2}{150(L_1L_2)^{1.24}}\Big), \\ \dot{L}_1 = L_2^{0.52}\Big(\frac{R^2\hat{x}_1^2}{L_1^{0.48}} + \frac{y^2}{0.75^2L_1^{0.48}}\Big), L_1(0) = 1, \\ \dot{L}_2 = -0.115(L_2 - 1) + 2.85\Big(1 + \frac{y^2}{R^2 0.75^2}\Big)^2, L_2(0) = 1. \end{cases}$$

In the numerical simulation, we set the parameters $\mu = 3$, $\alpha = 0.115$, $\beta = 2.85$, $L_3 = 150$, $R = 100$, $\delta(t) = 1 + 0.2\sin(10t)$, and the initial conditions $x_1(0) = 0.5$, $x_2(0) = 2$, $\hat{x}_1(0) = 1$, $\hat{x}_2(0) = 1$. The simulation results are shown in Figures 6−10. It is observed that $\lim_{t\to+\infty} x_1(t) = \lim_{t\to+\infty} x_2(t) = \lim_{t\to+\infty} \hat{x}_1(t) = \lim_{t\to+\infty} \hat{x}_2(t) = 0$, $\lim_{t\to+\infty} u(t) = 0$, which demonstrates the effectiveness of the controller.

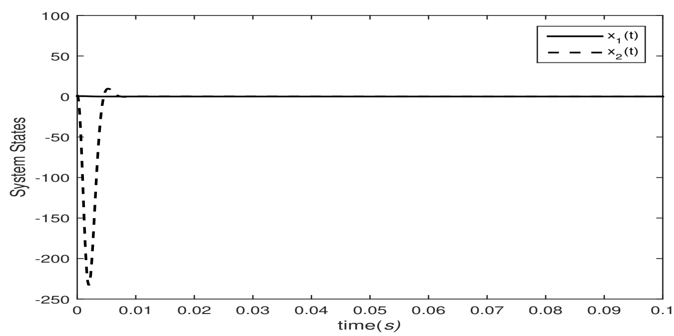

Fig. 6. The trajectories of the states of the closed-loop system.

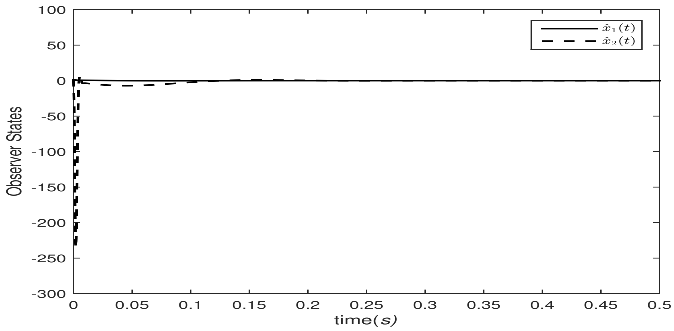

Fig. 7. The trajectories of the states of the observer.

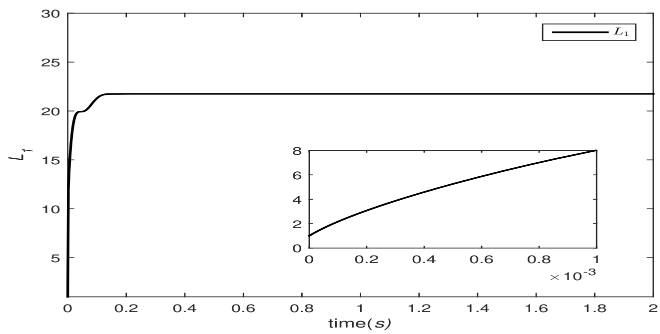

Fig. 8. The trajectory of the gain L_1.

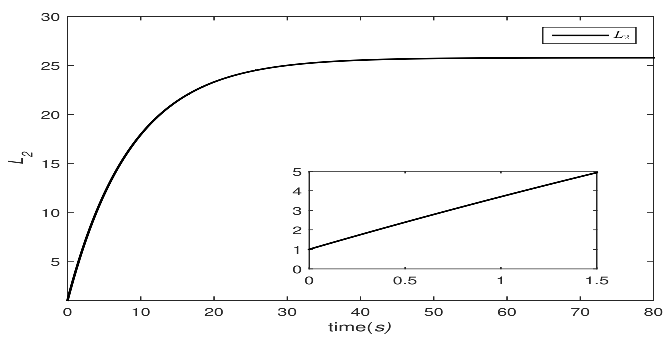

Fig. 9. The trajectory of the gain L_2.

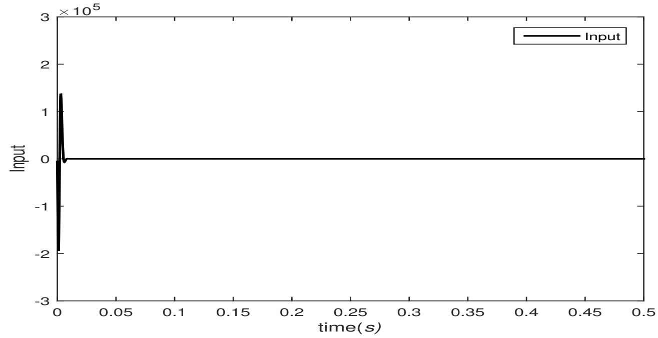

Fig. 10. The trajectory of the input u.

Case 3. Consider the following SISO nonlinear system (68) with $\theta(t) = R\delta(t)$, where R is an unknown positive real number and the measurement sensitivity $\delta(t)$ is an unknown function.

Based on Theorem 4.5, set the same values of the parameters a_1, a_2, b_1, b_2, σ to those in Case 1. We have $\bar{\theta} \leq \min\{1, 0.3798\}$. According to (14)−(16), we construct the following output feedback controller,

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 - 1.5L_1L_2\hat{x}_1, \\ \dot{\hat{x}}_2 = u - 1.5(L_1L_2)^2\hat{x}_1, \\ u = 150^2(L_1L_2)^{2.24}(-0.5(L_1L_2)^{-0.24}\frac{y}{R_1} - \frac{\hat{x}_2}{150(L_1L_2)^{1.24}}), \\ \dot{L}_1 = L_2^{0.52}\left(\frac{R_1^2\hat{x}_1^2}{L_1^{0.48}} + \frac{y^2}{0.75^2L_1^{0.48}}\right), L_1(0) = 1, \\ \dot{L}_2 = -0.115(L_2 - 1) + 2.85(1 + \frac{y^2}{R_2^2 0.75^2})^2, L_2(0) = 1. \end{cases}$$

In the numerical simulation, we set the parameters $R = 50$, $R_1 = 45$, $R_2 = 60$, $\mu = 3$, $\alpha = 0.115$, $\beta = 2.85$, $L_3 = 150$, and $\delta(t) = 1 + 0.2|\sin(10t)|$ satisfying $\delta(t) \in [1, 1.2]$, and the initial conditions $x_1(0) = 0.35$,$x_2(0) = 1$, $\hat{x}_1(0) = 1$, $\hat{x}_2(0) = 1$. The simulation results are shown in Figures 11−15. It is observed that $\lim_{t\to+\infty} x_1(t) = \lim_{t\to+\infty} x_2(t) = \lim_{t\to+\infty} \hat{x}_1(t) = \lim_{t\to+\infty} \hat{x}_2(t) = 0$, $\lim_{t\to+\infty} u(t) = 0$, which demonstrates the effectiveness of the controller.

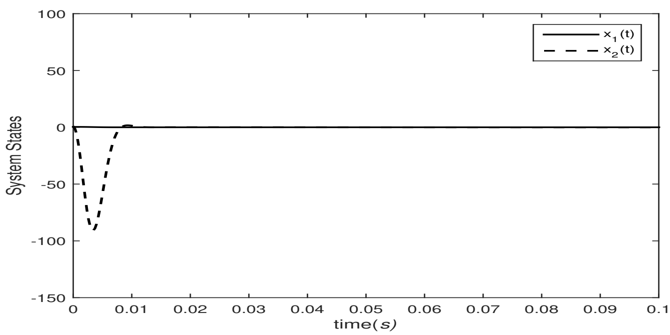

Fig. 11. The trajectories of the states of the closed-loop system.

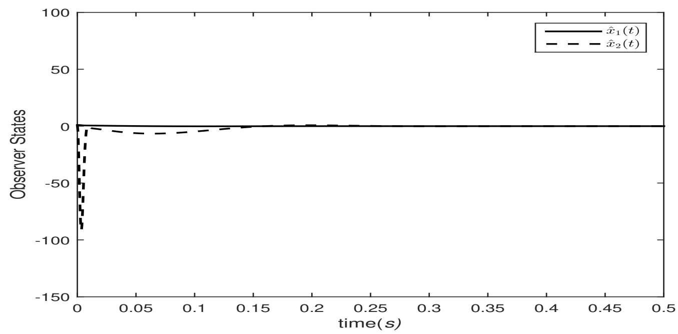

Fig. 12. The trajectories of the states of the observer.

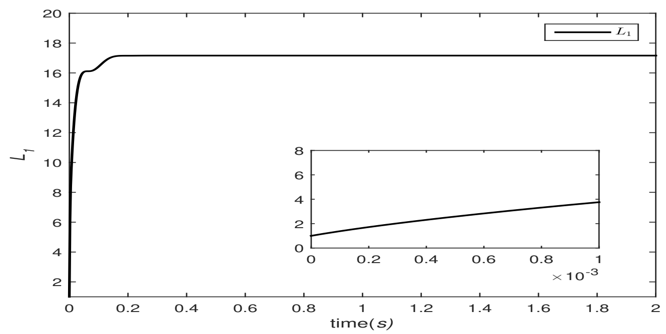

Fig. 13. The trajectory of the gain L_1.

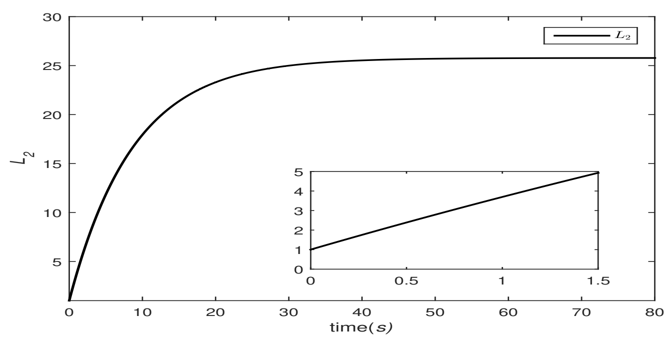

Fig. 14. The trajectory of the gain L_2.

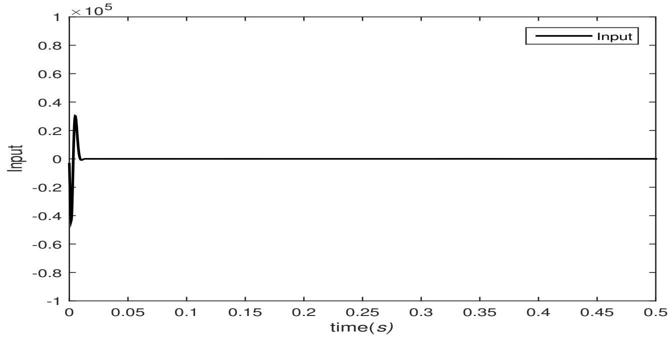

Fig. 15. The trajectory of the input u.

6. CONCLUSION

The problem of adaptive output-feedback stabilization was investigated for a class of uncertain nonlinear systems in this paper. The growth rate of the nonlinear systems was unknown and could be described as polynomial-of-output multiplying an unknown constant. Because of the existence of sensor uncertainty, the measurement output was not accurate. By developing the dual-domination approach, an adaptive output-feedback controller was designed to deal with the problem. Then, the methods were extended to nonlinear systems with larger sensor uncertainty. Finally, numerical simulations were provided to illustrate the effectiveness of the theoretical results.

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