Risk Sensitive Decision Making Using Type Reduction Methods

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1 Introduction

Decision making processes (e.g. in recommender systems) are often based on data containing co–occurences (e.g., which items are often purchased together) [6], ratings (e.g., zero stars to five stars) [1], or preferences (e.g., which option is preferred over which other options) [10]. In this paper we consider decision making based on ratings. Normalized ratings can be considered as membership values in the fuzzy set of suitable options. For example, n of five stars may be mapped to a membership of $u = n/5$. Based on such memberships a fuzzy decision making process [2] will choose the option with maximum membership.

Often experts are not willing or not able to specify exact ratings but only to specify intervals of ratings (e.g., between three and four stars, or satisfaction between 60% and 80%), so the ratings contain uncertainty. Intervals of ratings may be represented as intervals of memberships (e.g., $u \in [0.6, 0.8]$). Such interval ratings can be considered as interval–valued fuzzy sets [4, 5] or interval type–2 fuzzy sets [7, 8, 14]. Interval type–2 fuzzy decision making [12] will choose the most suitable option based on the individual membership intervals, taking into account the degree of risk that the decision maker is willing to take.

Here we consider an interval type–2 fuzzy decision making approach that employs a type reduction process [9] which maps each membership interval (type–2) to a single membership value (type–1) and then chooses the option with maximum membership. Several different methods for type reduction have been proposed in the literature, for example the Nie–Tan method (NT) [9], consistent linear type reduction (CLTR) [11], the uncertainty weight method (UW) [13], and consistent quadratic type reduction (CQTR) [11].

This paper presents an experimental study comparing these four type reduction methods to a decision making process using four alternatives.

2 Type Reduction Methods

Given an upper membership $\overline{u} \in [0,1]$ and a lower membership $\underline{u} \in [0,1]$, where $\underline{u} \leq \overline{u}$, the Nie–Tan method (NT) [9] is defined by the type reduction formula that yields the membership

$$u_{\mathrm{NT}} = \frac{\underline{u} + \overline{u}}{2} \tag{1}$$

The consistent linear type reduction (CLTR) [11] is defined by

$$u_{\mathrm{CLTR}} = a \cdot \underline{u} + (1-a) \cdot \overline{u} \tag{2}$$

with a parameter $a \in [0,1]$ that quantifies the degree of caution in the decision making process. The uncertainty weight method (UW) [13] is defined as

$$u_{\mathrm{UW}} = \frac{1}{2}(\underline{u} + \overline{u}) \cdot (1 + \underline{u}(x) - \overline{u}(x))^{\alpha} \tag{3}$$

with the parameter $\alpha \geq 0$. For easier comparison of the different methods we set

$$\alpha = \mathrm{atanh}(a) \tag{4}$$

so we can keep the parameter $a \in [0,1]$ and for $a = 0$ we have $\alpha = 0$ and for $a \to 1$ we have $\alpha \to \infty$. The consistent quadratic type reduction (CQTR) [11] is defined as

$$u_{\mathrm{CQTR}} = a \cdot \underline{u} + (1-a) \cdot \overline{u} - (1-a) \cdot (\overline{u} - \underline{u})^2 \tag{5}$$

3 Experiments

In this section we will compare the four type reduction methods presented in the previous section in experiments with a decision making process for four alternatives with the following membership values:

$$u_1 \in [0,1] \tag{6}$$
$$u_2 \in [0.55, 0.95] \tag{7}$$
$$u_3 \in [0.7, 0.75] \tag{8}$$
$$u_4 = 0.5 \tag{9}$$

We want to emphasize at this point that the numerical values in these experiments have a fuzzy and not a probabilistic character, so they represent memberships, not probabilities [3]. So u_1, an interval between zero and one does not represent a lack of information that may be changed after an experiment when further information is provided (e.g. about the director of a movie or the recipe of a drink). Instead, an interval between zero and one indicates a large uncertainty in the rating that is between zero and one. And u_4, a singleton membership of 0.5 does not represent a 50% probability whether this option is desirable or not but a 50% degree of desirability. The membership intervals u_2 and u_3 are both higher than 0.5, u_2 has a higher uncertainty than u_3, but u_2 has a higher average than u_3.

Fig. 1 shows the results obtained for these data with the Nie–Tan (NT) method. The three grey vertical bars in this plot represent the member-

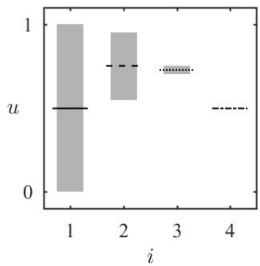

Figure 1: Decision memberships for the ratings u_i, $i = 1, \ldots, 4$, obtained by the Nie–Tan (NT) method.

ship intervals of the first three options, and the dash–dotted horizontal line represents the membership of the fourth option. The four horizontal lines (solid, dashed, dotted, and dash–dotted) represent the output of the NT method. For the fourth option, the NT method yields the original membership value. In this case, the input is type–1, so type reduction is not necessary. A type reduction operator with this property is called type–1 consistent [11]. For the other three options, NT yields the average of the intervals. The maximum of these averages is for u_3, so the decision process will prefer option 3. In this decision the degree of uncertainty of the different options is not taken into account. A cautious decision maker may feel incomfortable that u_2 has a lower membership of only 0.55 and might prefer u_3 with a lower membership of 0.7. On the other hand, a very risky decision maker may not be satisfied that u_2 has an upper membership of only 0.95 and might even prefer u_1 with an upper membership of 1. The degree of risk that the decsion maker is willing to take is taken into account in the three other type reduction methods.

Fig. 2 shows the results obtained with the consistent linear type reduction (CLTR) method. CLTR is a parametric method with the parameter

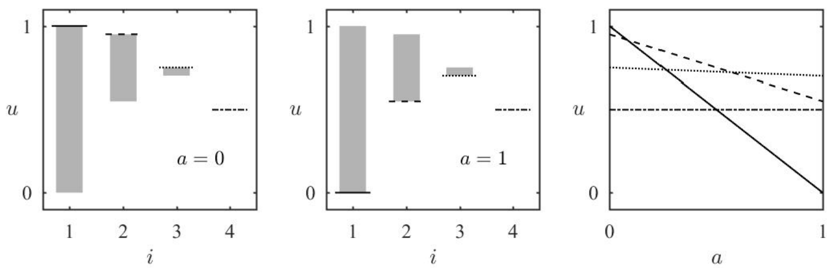

Figure 2: Decision memberships for the consistent linear type reduction (CLTR) method.

$a \in [0, 1]$. The left plot of Fig. 2 shows the results for $a = 0$, where CLTR always yields the maximum of each interval, with a maximum for option 1. The middle plot of Fig. 2 shows the results for $a = 1$, where CLTR always yields the minimum of each interval, with a maximum for option 3. The right plot of Fig. 2 shows the results for all $a \in [0, 1]$ that are plotted along the horizontal axis. So, in contrast to the left and middle plots, the right plot does not display the four different options on the horizontal axis but a. The four different options are represented by the four different curves: solid for option 1, dashed for option 2, dotted for option 3, and dash–dotted for option 4. For very small values of $a < 0.0833$ the solid curve is on top, so the risky decision is option 1. For $a \in [0.0833, 0.5714]$ the dashed curve is on top, so the medium risk decision is option 2. And for large $a > 0.5714$ the dotted curve is on top, so the cautious decision is option 3. Option 1, the option with complete uncertainty is chosen in the risky decision here. In many applications this is not desirable. Instead, options with complete uncertainty should often be completely ignored. A type reduction operator with this property ignores indifference [11]. The two remaining type reduction operators, UW and CQTR both ignore indifference.

Fig. 3 shows the results obtained with the uncertainty weight (UW) method. Again, the left plot shows the results for $a = 0$, the middle plot

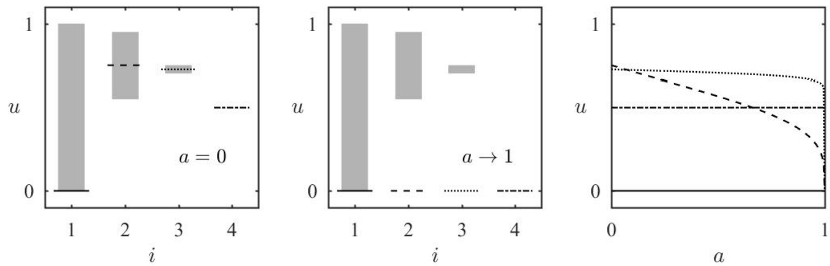

Figure 3: Decision memberships for the uncertainty weight (UW) method.

shows the results for $a = 1$, and the right plot shows the results for all $a \in [0, 1]$. For option 1 (solid curve) we always obtain zero membership, so complete indifference is ignored, as pointed out above. For $a = 0$ we obtain memberships at the mean of each interval. In the limit for $a \to 1$ all memberships approach zero, which appears counter–intuitive. For small $a < 0.074$ the dashed curve is on top (option 2), and for larger $a > 0.074$ the dotted curve is on top (option 3). For large $a > 0.661$ the dashed curve (option 2) falls below the dash–dotted curve (option 4) so the option $u_4 = 0.5$ would be preferred over $u_2 \in [0.55, 0.95]$ although all values of u_2 are higher than u_4. Also this behaviour seems counter–intuitive.

Fig. 4 shows the results obtained with the consistent quadratic type reduction (CQTR) method. Again we always obtain membership zero for option 1 (solid curve), so complete indifference is ignored. For $a = 1$ we obtain the memberships of the bottom of each interval. For $a < 0.2208$ the dashed curve is on top and for $a > 0.2208$ the dotted curve is on top, so for low degrees of caution option 2 is chosen, and for high degrees of caution option 3 is chosen. The dash–dotted curve (option 4) is

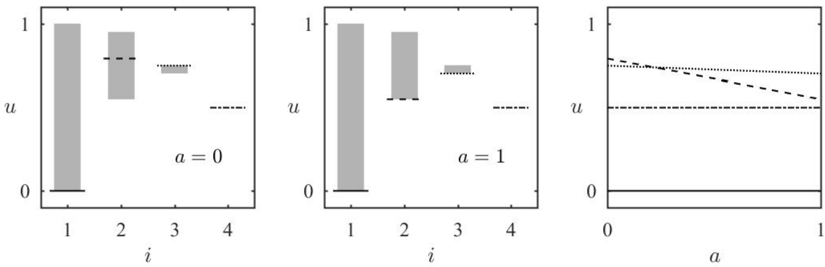

Figure 4: Decision memberships for the consistent quadratic type reduction (CQTR) method.

always below the dotted curve (option 3), which matches the intuitive expectation.

4 Conclusions

In this paper we have considered decision making processes based on interval ratings. Each rating is represented as an interval of memberships in the set of suitable options. Type reduction methods are used to convert the membership intervals to crisp memberships, and then the option with the highest membership is chosen.

We have considered four different type reduction operators from the literature: the Nie–Tan method (NT), consistent linear type reduction (CLTR), the uncertainty weight method (UW), and consistent quadratic type reduction (CQTR). All four type reduction operators were applied in experiments with four alternatives with different degrees of utility and different degrees of uncertainty.

The experiments have shown that

- NT does not consider the degree of uncertainty and is not able to take into account the degree of risk that the decision maker is willing to take.
- CLTR prefers an option with maximum uncertainty, if the risk level is high enough.

- UW prefers an option with a completely certain rating over a more uncertain option although the uncertain rating is always better than the certain one.
- CQTR is the only one of these four type reduction operators that is able to take into account the risk in the decision process, that ignores indifference, and that does not prefer a crisp membership over an interval of higher memberships.

Based on these results we would recommend CQTR as a type reduction method for risk sensitive decision makers.

References

[1] R. M. Bell and Y. Koren. Lessons from the Netflix prize challenge. *ACM SIGKDD Explorations Newsletter*, 9(2):75–79, 2007.

[2] R. Bellman and L. Zadeh. Decision making in a fuzzy environment. *Management Science*, 17(4):141–164, 1970.

[3] J. C. Bezdek. Fuzzy models — what are they, and why? *IEEE Transactions on Fuzzy Systems*, 1(1):1–6, 1993.

[4] M. Gehrke, C. Walker, and E. Walker. Some comments on interval valued fuzzy sets. *International Journal of Intelligent Systems*, 11(10):751–759, 1996.

[5] M. B. Gorzałczany. A method of inference in approximate reasoning based on interval–valued fuzzy sets. *Fuzzy Sets and Systems*, 21(1):1–17, 1987.

[6] M. Hildebrandt, S. S. Sunder, S. Mogoreanu, I. Thon, V. Tresp, and T. Runkler. Configuration of industrial automation solutions using multi–relational recommender systems. In *European Conference on Machine Learning and Principles and Practice of Knowledge Discovery in Databases*, Dublin, Ireland, September 2018.

[7] Q. Liang and J. M. Mendel. Interval type–2 fuzzy logic systems: Theory and design. *IEEE Transactions on Fuzzy Systems*, 8(5):535–550, 2000.

[8] J. M. Mendel, R. I. John, and F. Liu. Interval type–2 fuzzy logic systems made simple. *IEEE Transactions on Fuzzy Systems*, 14(6):808–821, 2006.

[9] M. Nie and W. W. Tan. Towards an efficient type–reduction method for interval type–2 fuzzy logic systems. In *IEEE International Conference on Fuzzy Systems*, pages 1425–1432, Hong Kong, 2008.

[10] T. A. Runkler. Mapping utilities to transitive preferences. In Jesús Medina, Manuel Ojeda-Aciego, José Luis Verdegay, David A. Pelta, Inma P. Cabrera, Bernadette Bouchon-Meunier, and Ronald R. Yager, editors, *Information Processing and Management of Uncertainty in Knowledge–Based Systems. Theory and Foundations*, volume CCIS 853, pages 127–139. Springer, 2018.

[11] T. A. Runkler, C. Chen, and R. John. Type reduction operators for interval type–2 defuzzification. *Information Sciences*, 467C:464–476, 2018.

[12] T. A. Runkler, S. Coupland, and R. John. Interval type–2 fuzzy decision making. *International Journal of Approximate Reasoning*, 80:217–224, 2017.

[13] T. A. Runkler, S. Coupland, R. John, and C. Chen. Interval type–2 defuzzification using uncertainty weights. In S. Mostaghim, C. Borgelt, and A. Nürnberger, editors, *Frontiers in Computational Intelligence*, pages 47–59. Springer, 2017.

[14] L. A. Zadeh. The concept of a linguistic variable and its application to approximate reasoning. *Information Sciences*, 8:199–249, 9:43–80, 1975.