The Cosmological Constant Problem and Gravity in the Infrared

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Abstract

In this thesis we explore low energy extensions of Einstein's theory of General Relativity (GR). Initially our motivation will be the cosmological constant problem where large radiative corrections due to quantum field theories minimally coupled to the dynamical metric in GR leads to unacceptably large space-time curvatures. We will discuss the cosmological constant problem in detail, paying special attention to how it affects the *global* structure of a space-time whose dynamics are dictated by GR. With this in mind we will present and discuss recently proposed global modifications of GR which in the semi-classical limit sequester the radiatively unstable loop corrections to the cosmological constant from the space-time curvature. This is achieved by supplementing the local dynamics of GR with highly non-trivial global constraints, and we demonstrate how this can be achieved in a theory which is manifestly local. In this theory we will also consider the effects of an early universe phase transition on the late time dynamics.

Away from global modifications of GR we will also consider local modifications which necessarily involve the propagation of new degrees of freedom. We outline the possible *screening mechanisms* which, since no new gravitational degrees of freedom have been observed in local environments, are an important feature of any local modification of GR. For one of these mechanisms, namely, the Vainshtein mechanism, we will consider the regime of validity of theories which make use of the Vainshtein mechanism and assess suggestions that one can trust these theories beyond the scale we would naively expect them to become strongly coupled. Following this we will move onto the chameleon mechanism, another example of a screening mechanism, and present a high energy extension motivated by the breakdown in the original chameleon theory in the early universe. The interactions of the resulting *DBI chameleon* theory will be motivated by our discussion of the Vainshtein mechanism.

List of papers

This thesis contains material from the following five papers:

'Sequestering effects on and of vacuum decay'
N. Kaloper, A. Padilla and D. Stefanyszyn
Phys. Rev. D 94 (2), 025022 (2016)
arXiv:1604.04000

'How to Avoid a Swift Kick in the Chameleons'
A. Padilla, E. Platts, D. Stefanyszyn, A. Walters, A. Weltman and T. Wilson
JCAP 1603 (3), 058 (2016)
arXiv:1511.05761

'A Manifestly Local Theory of Vacuum Energy Sequestering'
N. Kaloper, A. Padilla, D. Stefanyszyn and G. Zahariade,
Phys. Rev. Lett. 116 (5), 051302 (2016)
arXiv:1505.01492

'Unitarity and the Vainshtein Mechanism'
N. Kaloper, A. Padilla, P. Saffin and D. Stefanyszyn,
Phys. Rev. D 91 (4), 045017 (2015)
arXiv:1409.3243

'Generalised Scale Invariant Theories'
A. Padilla, D. Stefanyszyn and M. Tsoukalas
Phys. Rev. D 89 (6), 065009 (2013)
arXiv:1312.0975

Deceleration

The work presented in this thesis is based on research conducted at the Particle Theory Group, School of Physics and Astronomy, University of Nottingham, United Kingdom.

Chapter 1 and 2 of this thesis contain background material and introductions to a number of topics which the remaining chapters are based on. In chapter 3, section 3.1 is a review of known results while section 3.2 is original work done in collaboration with Nemanja Kaloper, Antonio Padilla and George Zahariade [1].

Chapter 4 initially contains a short review of some know results but section 4.2 onwards is original work done in collaboration with Nemanja Kaloper and Antonio Padilla [2]. Chapter 5 begins with a detailed review of background material but section 5.3 onwards is original work done in collaboration with Nemanja Kaloper, Antonio Padilla and Paul Saffin [3].

Chapter 6 again begins with a detailed review of known results. Section 6.2 contains new methods for reproducing some of these results and section 6.3 onwards is original work. This chapter is based on work done in collaboration with Antonio Padilla, Emma Platts, Anthony Walters, Amanda Weltman and Toby Wilson [4]. Figures (6.4), (6.5)-(6.7) were produced by Toby Wilson and figures (6.9) - (6.13) were produced by Emma Platts and Anthony Walters.

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Notation and conventions

Throughout this thesis we will use Natural units such that $c = \hbar = 1$. For example, in these units we write the Planck mass as $M_{\rm pl}^2 = 1/8\pi G_N \approx 10^{18}$GeV where G_N is Newton's gravitational constant.

We will use the mostly-positive signature for the metric $(-,+,+,+)$. For example, we express the Minksowski metric as $ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$.

Greek indices $\mu, \nu, ... = (0, 1, 2, 3)$ are space-time indices, we denote a partial derivative as ∂_μ and a covariant derivative as ∇_μ. We denote symmetrisation and anti-symmetrisation over enclosed indices respectively as

$$S_{(\mu\nu)} = \frac{1}{2}(S_{\mu\nu} + S_{\nu\mu}), \quad S_{[\mu\nu]} = \frac{1}{2}(S_{\mu\nu} - S_{\nu\mu}). \tag{1}$$

A number of times during this thesis we will make use of the Friedmann-Lemaître-Robertson-Walker (FLRM) ansatz for the metric which is given by

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega_2^2 \right) \tag{2}$$

where we have used spherical polar coordinates and $d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the metric on a 2-sphere.

Contents

Chapter 1

Introduction

At long distances gravity is by far the weakest of the four fundamental forces in Nature. Indeed, the gravitational force between two electrons is approximately 10^{-42} in units of the corresponding electromagnetic force. Similar sized hierarchies exist between gravity and the strong and weak nuclear forces. Even so, for cosmology gravity is often the only relevant interaction and striving for a consistent description of our observable universe is strongly correlated with efforts to understand the gravitational interaction. As a consequence, major developments in observational cosmology can have a huge impact on the way we think about gravity.

1.1 General Relativity

Our best description of gravity at long distances, or low energies, is Einstein's theory of General Relativity (GR) presented in 1915. In Einstein's eyes, GR offered a geometric description of a dynamical space-time motivated by solving inconsistencies between special relativity and Newtonian gravity. Many experiments have verified that the relativistic corrections to Newtonian gravity provided by GR are indeed required to match observations. These include; the perihelion precession of Mercury, the bending of light by the sun, and gravitational red-shift. Tests of gravity on Earth and more widely in the solar system continue to confirm that GR is the gold standard theory of gravity, see e.g. [5–10] and references therein. More recently, gravitational waves, another prediction of GR, have been observed due to the inspiral and merger of a pair of black holes [11].

The Einstein-Hilbert action, including a bare cosmological constant Λ_c, is given by

$$S_{\rm GR} = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \Lambda_c\right] \tag{1.1}$$

where $R = g^{\mu\nu}R_{\mu\nu}$ is the Ricci scalar, $g_{\mu\nu}$ is the metric tensor, $R_{\mu\nu}$ is the Ricci tensor and $g = \det g_{\mu\nu}$. Other than the cosmological constant, the only other dimensionful scale is the Planck scale $M_{\rm pl}$. If we couple matter fields to gravity in the minimal way, then the total action is a linear combination of the gravitational action and the action of the field theory sector

$$S = S_{\rm GR} + S_m(g^{\mu\nu}, \Phi) \tag{1.2}$$

where Φ denotes all fields that couple minimally to the metric. The resulting field equations after varying the action with respect to $g_{\mu\nu}$ are a set of ten partial differential equations given by

$$M_{\rm pl}^2 G_{\mu\nu} = T_{\mu\nu} - g_{\mu\nu}\Lambda_c \tag{1.3}$$

where $G_{\mu\nu}$ is the Einstein tensor defined as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R \tag{1.4}$$

and is divergence-free thanks to the Bianchi identity $\nabla_\mu G^\mu{}_\nu = 0$. We note that this is an off-shell statement and is not a consequence of the equations of motion. $T_{\mu\nu}$ is the energy-momentum tensor of the field theory sector defined as

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{\mu\nu}}. \tag{1.5}$$

As long as the field theory is diffeomorphism invariant, the energy-momentum tensor is also covariantly conserved $\nabla_\mu T^\mu{}_\nu = 0$ which provides a neat consistency check of Einstein's equations. The total cosmological constant which sources curvature is a combination of the bare value Λ_c, which is a purely classical contribution, and a contribution from the field theory sector $V_{\rm vac}$ which is stored in the energy-momentum tensor as $T^{\mu}{}_{\nu} = -\delta^{\mu}{}_{\nu} V_{\rm vac}$. Gravity has no way of distinguishing between the two and any observations constrain only the linear combination $\Lambda = \Lambda_c + V_{\rm vac}$.

1.2 An accelerating universe

In contrast to 1915, precision measurements in cosmology now present a brand new arena where we can study gravity on enormous distance scales. The most intriguing development in this direction in recent years is the overwhelming evidence in support of a period of late time acceleration of the universe. The first observations of this sort came from Type 1a supernovae which appeared dimmer, and therefore further away, than expected if the universe was dominated by non-relativistic particles at late times [12, 13]. Many other experiments, including precise measurements of the Cosmic Microwave Background (CMB) [14–17], verified that the energy density stored in non-relativistic fields at late times is a sub-dominate component of the total energy budget of the universe and that a form of *dark energy* was causing the universe to accelerate.

In principle it is very simple to account for a period of late time acceleration within GR by virtue of the cosmological constant $\Lambda = \Lambda_c + V_{\rm vac}$. Since the energy density of a cosmological constant does not red-shift, yet all other forms of energy density satisfying null energy conditions do, it will eventually dominate the energy density sourcing gravity and drive the universe into a de Sitter phase where it expands exponentially [18]. The magnitude of the de Sitter curvature is then set by the size of the cosmological constant. In fact, all cosmological data to date are consistent with a cosmological constant being responsible for the observed late time acceleration with a magnitude $\Lambda_{\rm obs} \sim ({\rm meV})^4$ [17].

However, in practice there are huge complications. The energy density of the cosmological constant is expected to receive contributions from vacuum fluctuations of each massive particle coupled to gravity. Thanks to the equivalence principle of GR, these vacuum fluctuations, just like all other forms of matter and energy, gravitate. Yet for the standard model of particle physics the energy density stored in these fluctuations is extremely sensitive to high energy physics and its natural value even when we only consider the effects of the electron is many orders of magnitude larger than the observed value. Radiative corrections due to heavier particles only makes the situation worse, each contributing the fourth power of their mass to the overall cosmological constant. In GR, if we are to match the observation of a small cosmological constant, we must *repeatedly* fine tune a classical parameter in the gravitational action every time the effective description of the field theory sector changes. This could be due to changing the order of the loop expansion to which the vacuum energy is calculated, or due to a change in the Wilsonian cut-off of the effective action. This is the infamous *cosmological constant problem* [19–25] and is the focus of a large part of this thesis.

One's initial reaction is to tackle the problem at the heart of the field theory since the root cause of the problem is large quantum corrections from massive particles. A very elegant example in this regard is supersymmetry (see e.g. [26]). In a supersymmetric theory every boson is accompanied by a fermion with the same mass and interaction strengths. Since the radiative corrections to the cosmological constant from bosons and fermions have opposite signs, they cancel and the net cosmological constant is zero. However, when supersymmetry is broken at some scale M^2_{susy} the masses of the boson and its supersymmetric fermionic partner differ by $|M^2_{\text{boson}} - M^2_{\text{fermion}}| \sim gM^2_{\text{susy}}$ where g is the coupling strength to the supersymmetric breaking sector [24]. The result is that the failure in the cancellation of vacuum energy allows quantum corrections to drive the cosmological constant to values $\Lambda \sim gM^4_{\text{susy}}$. In our universe supersymmetry is broken at least up to the TeV scale [27] and therefore the natural value for the cosmological constant is $\Lambda \gtrsim (\text{TeV})^4 \gg (\text{meV})^4$. Supersymmetry therefore offers little in the way of a solution to the cosmological constant problem in our universe.

The lack of a solution from the field theory sector has prompted efforts to tackle the problem from the gravitational side by modifying gravity and violating the equivalence principle in the process, see [28, 29] for reviews. In other words, allowing the cosmological constant to be radiatively unstable and large but to prevent it from sourcing curvature. In this thesis we will discuss in detail the pre-requisites of a gravity theory whose goal is to have something non-trivial to say about the cosmological constant problem. Much effort will be devoted to studying the most extreme long distance modifications of gravity, namely, *global* modifications which only affect the gravitational equations of GR in the infinite wavelength limit. We will argue that this is a natural thing to do since the vacuum energy does not vary in space-time and can therefore be considered as an infinite wavelength source. In this regard, following a detailed discussion of the cosmological constant problem in chapter 2, we will focus on the *vacuum energy sequestering scenario* in chapter 3. We will present and discuss the original form of this theory [30] and outline its very non-trivial predictions [31, 32]. Emphasis will be placed on how the theory successfully modifies gravity *only* globally such that all sources other than the cosmological constant gravitate as in GR. In doing so we will discuss the models inability to describe a local quantum field theory (QFT) and the possible obstruction to UV completion. With this in mind we will then present a manifestly local version of the sequestering scenario [1] which retains the positive features of the original model but crucially is better placed to be realised as the low energy limit of a UV complete theory which satisfies the axioms of quantum mechanics.

Following on from presenting the model and discussing its features in detail, in chapter 4 we will discuss the important issue of phase transitions where the local value of the cosmological constant can change. This has the potential to spoil any cancellation of the vacuum energy so we will examine how they source curvature in the sequestering theory. We will consider a field theory with two minima and investigate the nucleation and growth of bubbles of true vacuum. In doing so we will place mild constraints on the local sequestering theory which ensure that there are no catastrophic instabilities with respect to tunnelling between maximally symmetric vacua. In this chapter we follow the work presented in [2].

There are two potential outcomes of a non-anthropic solution[1] to the cosmological constant problem. The first is that the cosmological constant which

[1] We will briefly touch on an anthropic approach to the cosmological constant in chapter 2.

sources curvature is stabilised with respect to changes in the field theory sector such that if it is set to be small, it stays small even after quantum corrections. We can then readily fix its re-normalised value to match observations resulting in a simple explanation for the acceleration of the universe as long as observations remain consistent with a cosmological constant being the driving force. This would be completely comparable to many situations in particle physics where parameters are protected from large radiative corrections and is known as *technical naturalness* [33] and is usually attributed to the appearance of a symmetry when the parameter is taken to zero. More generally, a parameter is technically natural if the quantum corrections to the parameter are proportional to the parameter itself. Most notably, the electron would naively receive large mass re-normalisations due to its coupling to heavier particles. However, thanks to the chirality of massless fermions, to all orders in perturbation theory these corrections are only ever proportional to the electron mass itself and not the masses of particles running around the loops [33]. Unlike the cosmological constant, the electron mass is not strongly sensitive to the UV.

The other possibility is that the cosmological constant is forced to be exactly zero. This is of course more restrictive than the previous case and in such a theory one still requires an explanation for the late time acceleration. A popular route in this regard are quintessence models where a canonical scalar field with a potential couples minimally to gravity, see [34] for many examples. When the potential dominates the scalar field dynamics, it can drive the universe into an accelerating phase. This scenario is also often quoted as another motivation for studying long distance modifications of gravity where the aim is to provide an $\mathcal{O}(1)$ deviation from GR on Hubble scales $H_0 \sim 10^{-33}$eV to realise an approximate de Sitter phase at late times in the absence of a cosmological constant. We note that modifications of gravity employed to solve the cosmological constant problem and those introduced to provide late time acceleration do not always coincide. Indeed, as we shall discuss in detail, there are certain features of modifications of gravity used to solve the cosmological constant problem that are often absent when the goal is to accelerate the universe.

1.3 Screening mechanisms

The primary issue with modifying gravity to accelerate the universe is not realising the late time acceleration, but to conform with local tests of gravity. Any *local* long distance modification of GR necessarily requires new degrees of freedom which mediate long range forces. In almost all examples, modifications of gravity make use of light scalar fields which couple to matter degrees of freedom. Away from motivations by dark energy, attempts at UV completing gravity can also result in the presence of a number of light scalar fields with gravitational strength couplings. This is the case in string theory after string compactifications (see e.g. [35]). In any case, a fluctuating scalar field will mediate a new force and any such deviations from GR must be shut down to $\mathcal{O}(10^{-5})$ in the solar system for the theory to not be ruled out by local experiments [5].

For example, consider the following action which describes a massive scalar field ϕ which couples to the trace of the energy-momentum tensor

$$S = \int d^4x \left[-\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{g\phi T}{M_{\rm pl}} \right] \tag{1.6}$$

where g is a dimensionless coupling constant. When $g \sim \mathcal{O}(1)$ the scalar field couples to the matter sector with gravitational strength. The dynamics of ϕ is governed by the usual Klein-Gordan equation

$$\Box\phi - m^2\phi + \frac{gT}{M_{\rm pl}} = 0. \tag{1.7}$$

Most of the mass in the solar system is due to the Sun which we can model as a static and spherically symmetric non-relativistic point source of mass M with $T^{\mu\nu} = M\delta^\mu_0\delta^\nu_0\delta^3(\vec{x})$ such that $T = -M\delta^3(\vec{x})$. In this case the static, spherically symmetric scalar field potential is (e.g. [29, 36])

$$\phi(r) = -\frac{gM}{4\pi M_{\rm pl}}\frac{e^{-mr}}{r} \tag{1.8}$$

where we assume that ϕ vanishes asymptotically. As expected for a massive boson we recover the usual potential with a Yukawa suppression[2]. The force per unit mass experienced by a test particle due to the scalar field is $\vec{F} = g\vec{\nabla}\phi/M_{\rm pl}$ (see e.g. [36]) and is therefore only suppressed relative to GR for large m ($mr \gg 1$), in which case the cosmological effects of the scalar are also suppressed, or for small g in which case the effects of the scalar field are again limited and this fine tuning may also be undesirable since it generates a hierarchy of scales. We will also see that in modifications of gravity one usually has $g \sim \mathcal{O}(1)$. In an attempt to evade these issues, modifications of gravity are accompanied by a *screening mechanism* whose purpose is to dynamically suppress any unwanted gravitational strength forces in the well tested regimes.

Screening mechanisms are best understood by studying the dynamics of a scalar field fluctuation $\pi(x)$ about some background solution $\bar{\phi}(x)$. At quadratic order in fluctuations the action of a theory with a screening mechanism typically takes the form

$$S = \int d^4x \left[-\frac{1}{2} Z^{\mu\nu}[\bar{\phi}]\partial_\mu\pi\partial_\nu\pi - \frac{1}{2}M^2[\bar{\phi}]\pi^2 + \frac{g[\bar{\phi}]\pi\delta T}{M_{\rm pl}} \right] \tag{1.9}$$

where we refer to $Z^{\mu\nu}$ as the Z factor, M is an environmental dependent mass, g is an environmental dependent coupling, and δT is the fluctuation in the trace of the energy-momentum tensor. When $Z^{\mu\nu}, g \sim \mathcal{O}(1)$, the scalar field fluctuation couples to δT with gravitational strength. Broadly speaking, the three most popular screening mechanisms found in the literature can each be understood as relying on one of $Z^{\mu\nu}$, M or g responding to the presence of a massive source. Let us discuss each in turn.

1.3.1 $Z^{\mu\nu}$: the Vainshtein mechanism

Our first example of a screening mechanism is the Vainshtein mechanism [38], see [39] for a review, where derivative self-interactions of scalar fields make them weakly coupled to matter degrees of freedom. Theories with Vainshtein screening always have at least a minimal shift symmetry on the scalar, $\phi \to$

[2] One can also derive the Yukawa form of this potential by calculating the S-matrix of $2 \to 2$ fermionic particle scattering where a canonical massive fermion ψ couples to the scalar via $g\bar{\psi}\psi\phi$ (see e.g. [37]).

$\phi+c$, such that $M[\bar{\phi}]=0$ and $g[\bar{\phi}]\sim\mathcal{O}(1)$. When the interactions dominate the equations of motion we have $Z^{\mu\nu}\gg 1$ such that upon canonically normalising, the fluctuation couples to the source with a strength $\frac{1}{M_{\rm pl}\sqrt{Z}}\ll\frac{1}{M_{\rm pl}}$. The result is that the scalar force is much weaker than the spin-2 force of GR and unobservable in local experiments. A common feature of this mechanism is that one must introduce a new energy scale, which controls the strength of the derivative interactions, which must be very small in units of the Planck scale to ensure that the screening is effective. This generically reduces the regime of validity of the theory compared to GR.

In chapter 5 we will introduce the Vainshtein mechanism in much more detail and discuss how it found its origins in attempts to give the graviton a mass. We will then discuss the ability of theories which incorporate the Vainshtein mechanism to act as consistent effective descriptions of an unknown UV completion, with particular emphasis paid to suggestions as to how the classical theories can be trusted beyond the scale we would expect quantum corrections to dominate the dynamics [40]. In doing so we will study the classical dynamics of UV complete field theories as avatars of modifications of gravity and investigate the effects of higher order corrections to GR on its classical solutions. In this chapter we follow [3].

1.3.2 M: the chameleon mechanism

Another popular screening mechanism is the chameleon mechanism introduced by Khoury and Weltman in 2003 [41, 42]. Here the scalar field has a canonical kinetic term such that $Z^{\mu\nu}[\bar{\phi}]=\eta^{\mu\nu}$ and again couples to matter with gravitational strength with $g[\bar{\phi}]\sim\mathcal{O}(1)$. However, now the mass of the scalar field fluctuation is environment dependent. In high density regions like the solar system, the mass is large such that the corresponding scalar force is short range thanks to the Yukawa suppression in equation (1.8). Whereas at large distances where the densities are much smaller, the field is light and mediates a long range force. A conformal coupling between the scalar field and matter degrees of freedom is crucial to the effectiveness of the fifth force suppression. However, it was recently shown that this coupling caused issues for the chameleon in the early universe where a classical treatment of the chameleon's evolution could not be trusted prior to Big Bang Nucleosynthesis [43,44]. This is the basis of chapter 6. There we will begin by reviewing the chameleon mechanism in detail followed by a discussion of how the chameleon's classical description cannot be trusted at energy scales where we would expect to make predictions. Motivated by these issues, we then present a modification of the chameleon model which retains its ability to screen unwanted forces in local regions but has a much larger regime of validity in the early universe. In doing so we will follow the work in [4]. The structure of this modification is motivated by the results of chapter 5.

1.3.3 g: the symmetron mechanism

Finally, although we won't come across it in the main body of this thesis, the final screening mechanism is the symmetron mechanism [45]. This screening mechanism shares similarities with the chameleon mechanism inasmuch as screening takes place in regions of large matter density. There the scalar force becomes very weak, whereas in regions of low matter density the scalar field meditates a long range gravitational strength force. However, in contrast to the chameleon mechanism, it is the vacuum expectation value (VEV) of the scalar field which depends on the matter density. With a linear $g[\bar{\phi}]$, the coupling strength between the scalar fluctuation and the matter fields is proportional to the VEV, and screening works by ensuring that when the matter density is large, the VEV of the scalar field is attracted to $\bar{\phi} = 0$ where the coupling to matter vanishes. This is also the VEV where the symmetry $\bar{\phi} \to -\bar{\phi}$ is restored, hence the name. In regions of low density the symmetry is broken since there $\bar{\phi} \neq 0$ and the fluctuation couples to matter with gravitational strength. The effects of 1-loop quantum corrections to the symmetron potential on screening was recently investigated in [46].

1.4 GR as an effective field theory

A common focus of this thesis will be the ability of modifications of gravity, and more generally scalar field theories with screening mechanisms, to act as consistent effective field theories (EFTs). Since GR is always the theory of gravity we compare to, before moving onto the main body of this thesis let us first describe why it is such a good EFT. This exercise will also be very useful when we compare theories with a Vainshtein mechanism to GR in chapter 5.

Einstein's route to GR was a geometric one guided by the equivalence principle and general co-ordinate invariance. However, with the advent of quantum field theory we now understand the interactions of GR as an inevitable consequence of a few basic principles without any mention of the equivalence principle or general coordinate invariance. GR is simply the unique result of constructing a low energy description of interacting massless spin-2 particles, consistent with Lorentz invariance and locality [47–53]. This is completely comparable to Yang-Mills theory which is simply the unique result of writing down low energy interactions between massless spin-1 particles [54].

The associated gauge invariances for spin-1 and spin-2 (general coordinate invariance) are a consequence of demanding Lorentz invariance and are not fundamental properties in their own right. Indeed, these gauge 'symmetries' do not yield transformations in a particle's Hilbert space. They are really just redundancies required to write down Lorentz invariant self-interactions between massless particles. In addition, the only way to write down Lorentz invariant interactions between the graviton and other fields is if all coupling constants are equivalent. In other words, Lorentz invariance implies the equivalence principle too [55, 56]. This field theoretic approach to constructing theories also allows one to show that no self-interacting theories of massless particles with spin $\geqslant 3$ can exist [56, 57].

Given the complexity of Einstein's equations it is very difficult to find exact solutions for $g_{\mu\nu}$. A very powerful way of making predictions is perturbation theory where we reduce the equations of motion to a set linear equations which are much easier to solve and compute small corrections to these solutions. Perturbation theory also makes manifest the statement that GR is a theory of interacting massless spin-2 particles. As an example, in the absence of matter fields and with $\Lambda_c = 0$ consider expanding the metric around Minkowski space $\eta_{\mu\nu}$ by

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{1.10}$$

where $h_{\mu\nu}$ is the graviton fluctuation. This exercise will also allow us to extract the most important distance scales in GR. Truncating the Einstein-Hilbert action at quadratic order in $h_{\mu\nu}$ yields

$$S = \frac{M_{\rm pl}^2}{4}\int d^4x \left[-\frac{1}{2}\partial_\sigma h_{\mu\nu}\partial^\sigma h^{\mu\nu} + \partial_\mu h_{\nu\sigma}\partial^\nu h^{\mu\sigma} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\sigma h \partial^\sigma h\right] \tag{1.11}$$

where indices are raised and lowered with $\eta_{\mu\nu}$ e.g. $h = \eta^{\mu\nu}h_{\mu\nu}$. This is a theory of a spin-2 field propagating on a flat background. The field is clearly massless since there are only kinetic energy terms[3]. The tuning between each operator is extremely important and ensures that the theory describes the correct number of two helicity degrees of freedom of a Lorentz invariant massless spin-2 particle[4]. The tuning can be understood from the fact that the action is invariant under linearised diffeomorphisms

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \tag{1.12}$$

where ξ_μ is the gauge parameter. Indeed, as we have said, the theory is only manifestly Lorentz invariant if it inherits this redundancy. In this sense we could have arrived at a low energy Lorentz invariant theory of interacting massless spin-2 particles by writing down all possible contractions of $h_{\mu\nu}$ including at most two derivatives and demand that it is invariant under (1.12). This would have lead us uniquely to the terms in (1.11).

At some point this linearised approximation will break down. When we

[3]We will come back to the issue of adding a mass to the graviton in chapter 5.

[4]Any massless boson with a non-zero spin has only the two helicity degrees of freedom since in a Lorentz invariant theory one can never boost to the rest frame of a massless particle. A consequence of this statement is that there is a discontinuity between the number of degrees of freedom of a massive non-zero spin boson and the corresponding massless particle. This is why the massless nature of the photon and graviton is stable under quantum corrections.

expanded the action we dropped terms of the schematic form

$$M_{\rm pl}^2 h\partial^2 h^{n+1} \tag{1.13}$$

with $n \geqslant 1$, which are the derivative self-interactions of the graviton. Upon canonical normalisation $\hat{h}_{\mu\nu} = M_{\rm pl}h_{\mu\nu}$ these operators enter the action as

$$\frac{\hat{h}\partial^2\hat{h}^{n+1}}{M_{\rm pl}^n} \tag{1.14}$$

and due to the presence of the inverse metric, there are an infinite tower of these irrelevant operators with each coupling constant fixed by the non-linear diffeomorphism invariance. We note that the cubic interaction has been measured by virtue of observations of the perhelion precession of Mercury. These operators differ to the ones in (1.11) by increasing powers of $\hat{h}/M_{\rm pl}$ such that the linearised approximation breaks down when $\hat{h} \sim M_{\rm pl}$. When this happens the operators in the $n \to \infty$ limit dominate. This is *classical strong coupling* and signals the break down of *tree level* perturbation theory.

In the presence of a heavy source of mass M, such as the Sun, we can approximate the energy scale when these higher order operators become important and signal the break down of the quadratic truncation. If we supplement the canonically normalised quadratic action with the matter coupling $\hat{h}_{\mu\nu}T^{\mu\nu}/M_{\rm pl}$[5], then the linearised equations of motion in the presence of the massive source are schematically

$$\partial^2\hat{h} \sim \frac{M}{M_{\rm pl}}\frac{1}{r^3} \tag{1.15}$$

where we have assumed that the source is a non-relativistic, static and spherically symmetric point source with $T^{\mu\nu} = M\delta_0^\mu\delta_0^\nu\delta^3(\vec{x})$, and r is the radial co-ordinate. By dimensional analysis, the solution to this equation takes the form

$$\hat{h} \sim \frac{M}{M_{\rm pl}r}. \tag{1.16}$$

[5] If $T^{\mu\nu}$ is conserved then this matter coupling does not break the gauge symmetry (1.12) and so is Lorentz invariant.

The radius at which $\hat{h} \sim M_{\rm pl}$ is therefore

$$r_s \sim \frac{M}{M_{\rm pl}^2} \tag{1.17}$$

and is precisely the Schwarzschild radius for the massive body. For all macroscopic sources $r_s \gg 1/M_{\rm pl} \sim 10^{-33}$cm. For example, for the Sun we have $r_s \sim 1$km and for the Earth $r_s \sim 1$cm. When $r \gg r_s$ we can reproduce Newtonian gravity and analytically calculate small relativistic corrections to Newton's law of gravitation. The perturbative nature of this energy regime means it encompasses the scales where we know most about gravity. Fortunately, given the smallness of r_s for the Sun, we can make amazingly accurate predictions for the motion of planets in the solar system. Similarly for gravitational experiments on Earth. This was imperative to the fact that GR enjoyed experimental success only a few years after it was introduced and is really thanks to the weakness of gravity at these energy scales. As we approach the source, the linear approximation breaks down at $r \sim r_s$ and to calculate the gravitational potential one is required to solve the full non-linear field equations by including all operators of the form (1.14) which amounts to solving (1.3). We refer the reader to [58] for a more rigorous approach to solving the linear equations of GR.

It is well known that GR is not a complete description of gravity at all energy scales. The presence of the irrelevant operators (1.14) ensures that GR cannot be re-normalised with a finite number of counter terms [59] and the interactions can only be trusted at energies below a certain threshold. The uniqueness of GR as a theory of interacting massless spin-2 particles is only valid at low energies, or in other words, if we restrict the operators to involve no more than two derivatives. We saw this explicitly when we expanded the action. The irrelevant operators coming from the interactions of the Ricci scalar only ever involved two powers of momenta and the higher order operators were only higher order in $\hat{h}_{\mu\nu}$. The cosmological constant term is simply the zero-th order term in a gauge invariant derivative expansion.

There are of course more operators we can write down which are consistent with gauge invariance and we should indeed include them all since there is nothing stopping them from being generated quantum mechanically. These can be built out of the Riemann curvature, the metric and the covariant derivative. For example, all allowed terms involving four derivatives are

$$R^2, \quad R_{\mu\nu}R^{\mu\nu}, \quad R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}, \quad \Box R \tag{1.18}$$

and each enter the gravitational action with arbitrary coefficients. Note that gauge invariance fixes the coefficients of the $\hat{h}_{\mu\nu}$ operators such that they re-sum into the relevant curvature invariants but the coefficients of these is *a priori* arbitrary. Since $\Box R$ and the Gauss Bonnet combination $\mathcal{G} = R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} - 4R^{\mu\nu}R_{\mu\nu} + R^2$ are total derivatives, there are only two non-trivial operators with four derivatives. Therefore, the effective field theory of gravity in four space-time dimensions, including operators with no more than four derivatives, is

$$S = \int d^4x\sqrt{-g}\left[-\Lambda_c + \frac{M_{\rm pl}^2}{2}R + c_1R^2 + c_2R^{\mu\nu}R_{\mu\nu} + \ldots\right]. \tag{1.19}$$

The ellipses denote higher order curvature terms carrying at least six derivatives. This action has received much attention e.g. [60–63]. Given a UV completion of gravity one could perform a matching procedure to fix the values of the dimensionless coupling constants c_1, c_2. However, in the absence of a full UV theory they should be fixed by observation, just like the coupling constants in any EFT. Before discussing these interactions in more detail, it is worth mentioning that this truncation has been shown to be a re-normalisable theory thanks to the improved behaviour of the graviton propagator $\sim 1/k^4$ [60]. However, it is not a consistent theory of quantum gravity at all energy scales since the higher order equations of motion ensures that around Minkowski space the theory propagates ghostly degrees of freedom. The ghosts can only be kept under control when the higher order corrections are treated perturbatively.

Again, working in the absence of matter fields and setting $\Lambda_c = 0$, after expanding around flat space these higher order corrections correspond to irrelevant operators of the schematic form

$$\frac{\hat{h}\partial^4\hat{h}^{m+1}}{M_{\rm pl}^{m+2}} \tag{1.20}$$

where $m \geqslant 0$ and we have assumed that the Einstein-Hilbert terms are canonical. At a given order in $\hat{h}$, they differ from the Einstein-Hilbert terms by a single power of $\partial^2/M_{\rm pl}^2$ which is small as long as $\partial \ll M_{\rm pl}$. In other words, the higher order corrections are small if we restrict ourselves to gravitational process where momentum transfer is smaller than $M_{\rm pl} \sim 10^{18}$GeV. At energies far below the Planck scale we expect the effects of the higher order terms to be negligible. This is emphasised beautifully by the experimental bounds on c_1, c_2 which given the weakness of gravity in the experimentally probed regions are $c_1, c_2 \leq 10^{74}$ [61, 64].

After writing down an EFT for gravity there is nothing stopping us from doing quantum gravity, albeit perturbatively, since we know what the leading order quantum corrections should look like. For example, the 1-loop correction to the Newtonian potential $\delta V(r)$ between two massive bodies, in units of the classical non-relativistic potential $V(r)$, is [64–67]

$$\frac{\delta V(r)}{V(r)} = \frac{41}{10\pi}\frac{1}{r^2 M_{\rm pl}^2} \tag{1.21}$$

which is a purely quantum mechanical effect! It happens to be, however, that these corrections are incredibly small at long distances. At distances of order 1μm which is at the forefront of tests of the inverse square law, see e.g. [68], this correction is $\sim \mathcal{O}(10^{-58})$. Even if we could perform precise tests of gravity on the energy scales at the forefront of particle physics $\sim 10^{-17}$cm, the correction would still only be $\sim \mathcal{O}(10^{-32})$. In fact, we would have expected the leading order quantum correction to have this form without performing any explicit calculations. Indeed, graviton loop diagrams will come with at least an extra power of $1/M_{\rm pl}^2$ relative to the tree level diagrams and then by dimensional analysis the leading order correction must be of the form $a/(M_{\rm pl}r)^2$. The explicit calculation is then required to calculate the dimensionless constant a.

Therefore we can happily do quantum gravity at large distances, issues only arises when we start to reach the Planck scale. At this point we reach *quantum strong coupling*; loop perturbation theory breaks down and the amplitudes of graviton scattering processes become non-unitary since they grow as $(E/M_{\rm pl})^2$ where E is the energy scale of the corresponding process. Beyond this scale we require a fully fledged UV completion of gravity that can take care of the infinite tower of curvature invariants. String theory is the most promising candidate in this direction (see e.g. [69, 70]).

Here we have seen that the EFT of gravity has two expansion parameters representing three distinct energy regimes. The first is the classical expansion parameter $\hat{h}/M_{\rm pl}$ which controls the effects of classical non-linearities within GR. The second is a quantum expansion parameter $\partial/M_{\rm pl}$ which controls the effects of quantum corrections to GR. The linear regime is defined as the scale of energies for which $\hat{h} \ll M_{\rm pl}$ *and* $\partial \ll M_{\rm pl}$. The classical non-linear regime, for example inside a black hole but far from the would be singularity, is when $\hat{h} \gtrsim M_{\rm pl}$ but $\partial \ll M_{\rm pl}$. The quantum mechanical regime is when $\partial \sim M_{\rm pl}$. It is highly non-trivial that a large hierarchy exists where we can trust the non-linear theory of GR while keeping quantum corrections small. As we shall discuss in chapter 5, this is rarely true for scalar field theories and modifications of gravity. We refer the reader to [64, 65] for more details on the EFT structure of GR.

Chapter 2

The Cosmological Constant Problem

This chapter is devoted to a detailed discussion of the cosmological constant problem. We begin by arguing that the problem can be traced back to two very rigorous foundations, both consequences of 20th century physics, which have found experimental verification in a number of physical processes. We then explain how the vacuum energy contributions to the cosmological constant are calculated in quantum field theory (QFT) and in the framework of semi-classical gravity. Using an interacting scalar field as an example, we explain why we must *repeatedly* fine tune a classical parameter in General Relativity (GR) to match observations of a small cosmological constant, emphasising that the issue is one of radiative instability. We then show how the cosmological constant affects curvature in GR, and in doing so we will discuss how one could tackle the problem with a modification of gravity. We end this chapter by going through Weinberg's no-go theorem which argues that the cosmological constant problem cannot be solved using new dynamical fields without some level of fine tuning [20]. We refer the reader to the number of available reviews on the cosmological constant problem for more details [20–25].

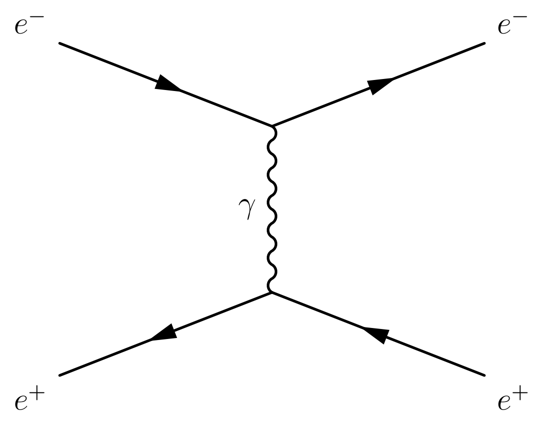
2.1 Two rigorous foundations

The cosmological constant problem is a result of combining our two best descriptions of fundamental physics: the standard model of particle physics and Einstein's theory of General Relativity. The primary causes of the problem can be traced back to two very rigorous foundations which we deal with in turn.

The first foundation is that *the vacuum has energy*. In QFT, the energy of the vacuum contributes to physical processes via loop corrections to tree level scattering amplitudes. For example, consider the coulomb potential between an electron e^- and a positron e^+ resulting from Bhabha scattering $e^-e^+ \to e^-e^+$ (see e.g. [37] for more details than what will be presented here). In quantum electrodynamics (QED) the coulomb potential is due to the exchange of a virtual photon as dictated by the following QED Lagrangian

$$\mathcal{L}_{\rm QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu\partial_\mu - m_e)\psi - e\bar{\psi}\gamma^\mu A_\mu\psi \tag{2.1}$$

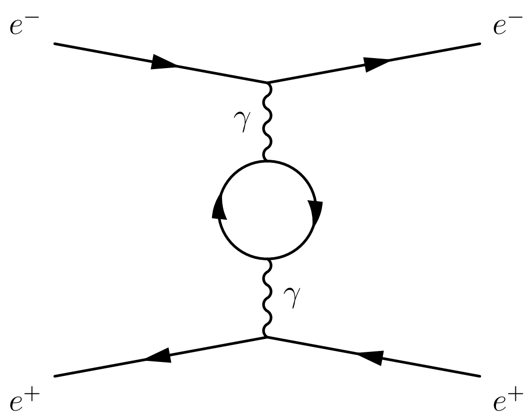
where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor for the vector potential A_μ, ψ is the relevant charged fermion of mass m_e, γ^μ are the gamma matrices and $e^2 = 4\pi\alpha$ where $\alpha \approx 1/137$ is the fine structure constant at low energies. At tree level, the contributions to this process come from s and t channel Feynman diagrams with the dominate contribution coming from the following t-channel diagram



where γ denotes a virtual photon. The resulting potential in the non-relativistic limit is the usual one of classical electrodynamics

$$V(r) = -\frac{e^2}{4\pi r} \tag{2.2}$$

yielding the expected attractive force between opposite charges. However, this is not the full story in QED and we can calculate a 1-loop correction to the tree level coulomb potential thanks to the creation of a virtual e^+e^- pair which corrects the virtual photon propagator. This process is known as vacuum polarisation and is pictorially expressed as



This diagram involves two extra cubic interaction vertices relative to the tree level one meaning that the resulting amplitude will have two extra powers of the coupling constant $e \ll 1$. It is therefore a small perturbative correction. The non-relativistic potential described by a sum of the tree level and 1-loop diagrams is the Uehling potential [71] and its form depends on the energy scale of interest. On distance scales larger than the Compton wavelength of the electron $r \gg 1/m_e$, the Uehling potential is [37, 71]

$$V(r) = -\frac{e^2}{4\pi r}\left[1 + \frac{e^2}{16\pi^{3/2}}\frac{1}{(m_e r)^{3/2}}e^{-2m_e r}\right] \tag{2.3}$$

and the loop correction is exponentially suppressed. On distance scales smaller than the Compton wavelength of the electron $r \ll 1/m_e$, the Uehling potential is [37, 71]

$$V(r) = -\frac{e^2}{4\pi r}\left[1 + \frac{e^2}{6\pi^2}\left(\log\frac{1}{m_e r} - \frac{5}{6} - \gamma_E\right)\right] \tag{2.4}$$

where $\gamma_E \approx 0.577$ is the Euler-Mascheroni constant. Here we see the logarithmic running of the fine structure constant at 1-loop. In each case, this correction to the tree level potential energy is solely due to the interactions between the virtual photon and the vacuum fluctuations of the virtual electron and positron pair. It is an example of where theory predicts that vacuum fluctuations affect physical process. However, do we see these effects in experiments? Indeed we do, and the most famous example is the Lamb shift [72].

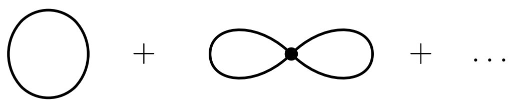
In 1947 Willis Lamb observed a small splitting between energy levels of the Hydrogen atom which are predicted to be degenerate in tree level QED. This observation made physicists realise that loop corrections are important in describing physical processes and paved the way for modern re-normalisation techniques. Indeed, the QED 1-loop correction to the atomic energy levels of the Hydrogen atom are in excellent agreement with observations of the Lamb shift [73–76] and the vacuum polarisation diagram is one of the contributors [37].

Having argued that vacuum fluctuations exist, we now come to the second important foundation: *the principle of equivalence* which tells us that all forms of matter and energy gravitate and that they all gravitate with the same strength. Therefore in a theory obeying the equivalence principle, as is the case in GR, we expect the vacuum energy fluctuations which contribute to the Lamb shift to gravitate. Again, at least for the processes discussed above, this second principle is realised in Nature. Indeed, it is observed that these loop corrections not only affect the inertial energy of an atom but they also affect its gravitational energy. For example, the loop corrections to the inertial energy of both Aluminium and Platinum are of order 1 part in 10^3 but differ by a factor of 3, yet the ratio between their gravitational energy and inertial energy are equivalent to at least 1 part in 10^{12} [77]. In this sense the vacuum energy contributions to the Lamb shift obey the equivalence principle to at least 1 part in 10^9.

In conclusion, vacuum fluctuations exist in Nature, have a non-trivial impact on physical processes, and at least in certain situations have been seen to gravitate. Following this logic to its conclusions leads to the cosmological constant problem.

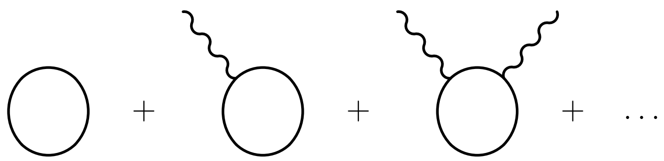
2.2 Fine tuning, fine tuning, fine tuning...

In this thesis we are interested in the effects of vacuum energy on the cosmological constant. The processes we have discussed so far have been employed to argue that vacuum fluctuations exist, but they do not contribute to the cosmological constant since they include standard model particles on external legs. The quantum mechanical contributions to the cosmological constant are pure Feynman bubble diagrams which only involve virtual particles running around loops. For example, consider a scalar field of mass m_ϕ and quartic self-interaction $\lambda\phi^4$, the relevant Feynman diagrams expressed in a loop expansion are[1]



(2.5)

where the ellipses denote higher order loop diagrams. Thanks to the equivalence principle, these diagrams must also couple to external gravitons and at a given loop order we can add an arbitrary high number of them. For example, at the 1-loop level we have



(2.6)

where now the ellipses denote further 1-loop diagrams with an increasing number of external gravitons. Fortunately, gauge invariance ensures that the infinite series of diagrams sums into a contribution to the gravitational action of

[1] The following results and conclusions are qualitatively the same for other massive particles with different spin.

the form

$$- V_{\text{vac}}^{\phi,\text{1-loop}} \int d^4x\sqrt{-g}. \tag{2.7}$$

We could also have argued this based on Lorentz invariance. Indeed, the vacuum of space-time is observed to be Lorentz invariant to a very high order of accuracy [78] and the only way the vacuum energy could enter the energy-momentum tensor in a Lorentz invariance way is $T_{\mu\nu} = -V_{\text{vac}}g_{\mu\nu}$. It follows that the vacuum energy must enter the action as in (2.7).

To calculate the vacuum energy we use a semi-classical framework where we treat the gravitational degrees of freedom classically but treat the field theory sector quantum mechanically. In practice, we first expand the action around Minkowski space with the metric fluctuation canonically normalised. We then calculate the loop corrections in the limit where we decouple gravity i.e. by sending $M_{\text{pl}} \to \infty$. This leaves us with a field theory defined in Minkowski space where we can use the standard techniques of QFT. Once we have calculated the vacuum energy contributions to a given order in perturbation theory, we then relax M_{pl} back to its finite value and allow gravity to classically probe the resulting vacuum energy in the manner dictated by gauge invariance. This ensures that we ignore all Feynman diagrams with virtual graviton exchanges. There are two motivations for doing this. First of all, the matter loops are already very problematic when we match our calculation of the vacuum energy to observations, as we shall see in a moment. Also, the contributions involving virtual gravitons are sensitive to quantum gravity effects and are therefore not as well understood. However, even with Planck suppression, it is still expected that relative to the observed value of the cosmological constant these contributions are not negligible [79] and these would need to be dealt with in a fully fledged solution to the cosmological constant problem.

When we calculate the loop contributions to the cosmological constant we have to deal with divergent momentum integrals and we therefore have to implement a regularisation procedure. One way to do so is by using dimensional regularisation where we work in $d = 4 - \epsilon$ dimensions (here we closely follow

[25]). In this case the 1-loop result for our massive scalar field example is [23]

$$V_{\rm vac}^{\phi,\text{1-loop}} = -\frac{m_\phi^4}{(8\pi)^2}\left[\frac{2}{\epsilon} + \log\left(\frac{\mu^2}{m_\phi^2}\right) + \text{finite}\right] \tag{2.8}$$

where μ is an arbitrary mass scale which we must introduce by dimensional analysis when we regulate and is therefore regulator dependent. The arbitrariness of μ ensures that any finite contributions are also arbitrary since we can always absorb them into a re-definition of μ. It is simple to see that by sending $\epsilon \to 0$ the result is divergent. As is the case in any QFT when we calculate loops, we remove the divergence by adding a counter term $\Lambda_{\rm c}$. The counter term required here is

$$\Lambda_{\rm c}^{\phi,\text{1-loop}} = \frac{m_\phi^4}{(8\pi)^2}\left[\frac{2}{\epsilon} + \log\left(\frac{\mu^2}{M^2}\right) + \text{finite}\right] \tag{2.9}$$

where M is the re-normalisation scale. The re-normalised vacuum energy $\Lambda_{\rm ren}^{\phi,\text{1-loop}}$ is the linear combination of the two and at the 1-loop level is

$$\Lambda_{\rm ren}^{\phi,\text{1-loop}} = \frac{m_\phi^4}{(8\pi)^2}\left[\log\left(\frac{m_\phi^2}{M^2}\right) + \text{finite}\right]. \tag{2.10}$$

Again, the finite contributions are completely arbitrary since they can always be absorbed into a re-definition of the re-normalisation scale M. This emphasises the important fact that we have no way of predicting the magnitude of the vacuum energy contributions to the cosmological constant from massive particles in EFT. After removing the divergences the result will *always* depend on an arbitrary mass scale. In comparison to $V_{\rm vac}^{\phi,1-\text{loop}}$, the counter term $\Lambda_{\rm c}^{\phi,1-\text{loop}}$ has a divergent and finite part. We can consider the finite part as simply the bare cosmological constant that we argued we were free to add to the Einstein-Hilbert action since it will enter the gravitational action in exactly the same way (this explains our choice of notation).

Since we cannot predict the size of the cosmological constant we have to measure it and adjust the finite part of Λ_c appropriately such that our theory matches observations. For example, if we assume that our scalar field is the Higgs boson of the standard model then it has a mass $m_\phi = 126\text{GeV}$ [80, 81]. Given that observations place an upper bound on the total cosmological constant of (meV)^4, the finite contributions to the vacuum energy at the 1-loop level must cancel to an accuracy of 1 part in 10^{60}.

Let us now consider the 2-loop correction to the vacuum energy from the massive scalar field. Since this Feynman diagram includes a 4-point interaction, the result will depend on the coupling constant λ. The precise form of the result is not important and dimensional analysis tell us it takes the form

$$V_{\text{vac}}^{\phi,\text{2-loop}} \sim \lambda m_\phi^4. \tag{2.11}$$

Importantly, this is an *additive* correction to the 1-loop result. Again, if we assume that this scalar is the Higgs boson, then even with $\lambda \sim 0.1$, this is a huge contribution to the cosmological constant relative to the observed value (meV)^4. Now having already fixed the bare cosmological constant to match observations at the 1-loop level, we have to *re-tune* its value to a high order of accuracy to cancel the unwanted contributions at the 2-loop level. Similarly we have to re-tune as we go to 3-loops, 4-loops and so on. At every order in perturbation theory, the vacuum energy contribution from the Higgs boson is not significantly suppressed relative to either the observed value of the cosmological constant or the previous loop correction. In other words, it is *radiatively unstable* and we have to re-tune the bare cosmological constant at every order in loop perturbation theory to deal with this instability. We are seeing that the cosmological constant is very sensitive to high energy physics and therefore we say that its observed small value is unnatural. Also, the fact that we cannot predict the size of the cosmological constant in EFT is an inevitable consequence of re-normalisable field theory and is indeed the case for parameters in the standard model such as the electron mass or charge. However, in these cases there is no radiative instability and no need to repeatedly fine tune.

We can also describe the problem using Wilsonian effective actions where it is clear that the vacuum energy is extremely sensitive to high energy physics and changes in the Wilsonian cut-off, see e.g. [82, 83]. We begin by assuming that we have a microscopic action $S[\phi_i]$ built out of some fields ϕ_i which is valid at high energy scales. The idea of Wilsonian actions is that we do not need to know the details of the high energy physics if we are only interested in low energy processes. In other words, the high energy modes and low energy modes *decouple* and each energy scale in Nature can be understood on its own without detailed knowledge of other scales. The fact that decoupling is respected by Nature in a variety of situations is imperative to our current knowledge of physics. It would be a disaster if we had to know about Planckian scale gravitational interactions to understand why apples fall from trees. Similarly, we can gain a very good understanding of atoms without detailed knowledge of quarks and gluons.

A sufficient description of physics described by $S[\phi_i]$ below some energy scale μ is given by integrating out the high energy degrees of freedom from the path integral. To do so we split the degrees of freedom in the microscopic action into low energy modes ϕ_l and high energy modes ϕ_h with the distinction that the high energy modes cannot be excited with energies below μ. The Wilsonian effective action for ϕ_l is then

$$\exp(iS_{\text{eff}}[\phi_l]) = \int D\phi_h \exp(iS[\phi_l, \phi_h]) \tag{2.12}$$

and can be expressed as an expansion of ϕ_l and their derivatives.

The vacuum energy contributions from the low energy particles in this effective action are dominated by the heaviest particle in the spectrum and is therefore $V_{\text{vac}} \sim \mu^4$ i.e. it is given by the fourth power of the Wilsonian cut-off. Given an observation of the total cosmological constant we then have to adjust Λ_c appropriately. However, there is nothing special about the energy scale μ and the associated Wilsonian action. Indeed, if a parameter in a theory is set to be small relative to the cut-off, we would like it to remain so for any other effective action defined at a new scale. This is however not the case for the vacuum energy since if we further integrate out modes with energies in the range $\hat{\mu} < E < \mu$ such that we are left with an effective action valid at energy scales below $\hat{\mu}$, the dominant contribution to the vacuum energy is now $\hat{\mu}^4$. If there is a hierarchy between μ and $\hat{\mu}$ we have to choose a new Λ_c to match observations. Every time we change the cut-off of the Wilsonian action we have to choose a completely new value for Λ_c.

In principle there is nothing wrong with the vacuum energy being dependent on the cut-off. The real issue is that its dependence is power law rather than logarithmic and again we must repeatedly fine tune Λ_c to match observations when our effective description of the field theory changes. This is what really makes the vacuum energy problematic compared to many parameters in the standard model which receive logarithmic re-normalisations. The exception is the Higgs mass. Since the mass of scalar fields is not protected by a discontinuity in the number of degrees of freedom between massless and massive fields, or by chiral symmetry, it is also the subject of large radiative corrections. However, the observed Higgs mass by yet be rendered natural with the discovery of low scale supersymmetry, see e.g. [26]. For more details regarding this approach to describing the cosmological constant problem we refer the reader to [24, 83].

A discussion of the cosmological constant problem is somewhat incomplete without mentioning anthropics. Indeed, there is a 'solution' to the cosmological constant problem which does not involve rendering the vacuum energy corrections to it stable against large radiative corrections, or by preventing the unstable contributions from sourcing curvature. The anthropic approach to the cosmological constant is to assume that naturalness plays no role and that large fine tunings are fundamental in our ability to describe Nature. The vacuum we find ourselves performing observations in is simply the one where we were able to evolve and there are many other vacua with vastly different values of the cosmological constant where structure is not able to form. In this thesis we will take the view that it is not too late to look for more desirable solutions and refer the reader to [21, 22] for more details.

Finally, we note that the cosmological constant problem can be rephrased by asking why the universe has evolved into a macroscopic state? If the cosmological constant had taken on its natural value in our universe, then with gravity described by GR, the universe would have evolved into a de Sitter phase at a much earlier stage in its life time and with a very large de Sitter curvature. This would have prevented the universe from having the large cosmological horizon which we observe today and would have not allowed for all its rich structure. Our incomplete knowledge about the cosmological constant and how it affects dynamics truly represents a huge gap in our understanding of the universe.

2.3 Vacuum energy and cosmology

The most obvious fact about the cosmological constant is that its energy density does not red-shift. Understanding the implications of this is very important if we are to begin tackling the cosmological constant problem. Let us define the cosmological constant in a more rigorous way by defining the space-time average of a scalar quantity Q (it must be a scalar by general covariance) as

$$\langle Q\rangle = \frac{\int d^4x\sqrt{-g}Q}{\int d^4x\sqrt{-g}} \tag{2.13}$$

where the integrals are taken over the *entire* space-time volume. The cosmological constant is the *only* part of the energy-momentum tensor which at a given epoch in the history of the universe is equal to its space-time averaged value[2]

$$\Lambda = \langle\Lambda\rangle. \tag{2.14}$$

With this in mind let us see exactly how the cosmological constant sources curvature in GR. We begin by decomposing Einstein's equations (1.3) into a set of traceless equations and a pure trace equation

$$M_{\text{pl}}^2\left(R^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu R\right) = T^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu T^\alpha{}_\alpha \tag{2.15}$$

$$M_{\text{pl}}^2 R = 4\Lambda_c - T^\alpha{}_\alpha \tag{2.16}$$

where $T^\alpha{}_\alpha = g^{\mu\nu}T_{\mu\nu}$ is the trace of the energy-momentum tensor. This is a completely equivalent description of on-shell GR as (1.3). The cosmological constant drops out from the traceless equations and its only affect on curvature

[2]This is true if we ignore phase transitions. For the moment this is a sufficient assumption and we will discuss phase transitions later on in chapter 4.

is by virtue of the trace equation. At this point we remind the reader that the quantum mechanical corrections to the total cosmological constant are packaged inside $T^\mu{}_\nu = -\delta^\mu{}_\nu V_{\rm vac}$ which is absent in the right hand side of equation (2.15). We can further decompose the trace equation by taking its space-time average

$$M_{\rm pl}^2 \langle R \rangle = 4\Lambda_c - \langle T^\alpha{}_\alpha \rangle \tag{2.17}$$

where we have used $\Lambda_c = \langle \Lambda_c \rangle$. The total cosmological constant will then also drop out from the difference between the trace equation and its space-time average. We can therefore describe the dynamics of GR, over the full space-time, via the following equations

$$M_{\rm pl}^2 \left(R^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu R \right) = T^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu T^\alpha{}_\alpha \tag{2.18}$$

$$M_{\rm pl}^2 (R - \langle R \rangle) = \langle T^\alpha{}_\alpha \rangle - T^\alpha{}_\alpha \tag{2.19}$$

$$M_{\rm pl}^2 \langle R \rangle = 4\Lambda_c - \langle T^\alpha{}_\alpha \rangle. \tag{2.20}$$

At this point let us comment on the space-time average of the Ricci scalar since one may worry about potential diverges. These are avoided in the numerator by only including sub-Planckian curvatures. This is a natural thing to do in classical gravity since for larger curvatures our classical solutions would be anyway untrustworthy due to quantum corrections. There can also be a divergence in the denominator if the space-time volume is infinite in which case we either have $\langle R \rangle = 0$, which is the case in a radiation or matter dominated cosmology, or $\langle R \rangle =$ constant for asymptotically de Sitter or anti-de Sitter space-times. In this case the value of $\langle R \rangle$ is given by the value of the Ricci scalar on the relevant maximally symmetric solution since the space-time integrals cancel. We refer the reader to [31, 84] for more details and in particular [31] for a discussion of black holes.

It is instructive to from now on extract the vacuum energy piece from the full energy-momentum tensor by defining

$$T^\mu{}_\nu = \tau^\mu{}_\nu - \delta^\mu{}_\nu V_{\rm vac} \tag{2.21}$$

where $\tau^\mu{}_\nu$ is the energy-momentum tensor of all energy sources other than the vacuum energy i.e. all localised sources. Making this replacement in the above equations leads to

$$M_{\rm pl}^2\left(R^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu R\right) = \tau^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu\tau^\alpha{}_\alpha \tag{2.22}$$

$$M_{\rm pl}^2(R - \langle R\rangle) = \langle\tau^\alpha{}_\alpha\rangle - \tau^\alpha{}_\alpha \tag{2.23}$$

$$M_{\rm pl}^2\langle R\rangle = 4(\Lambda_c + V_{\rm vac}) - \langle\tau^\alpha{}_\alpha\rangle. \tag{2.24}$$

Since this decomposition is rarely seen in the literature, let us again emphasise that here we are doing nothing more than showing how the cosmological constant sources curvature in GR. We now see how it only directly sources the *space-time averaged* value of the Ricci scalar $\langle R\rangle$. Its affect on local curvature terms is secondary and due to equations (2.22), (2.23). We are required to repeatedly fine tune Λ_c to take care of the radiative instability of $V_{\rm vac}$, to keep $\langle R\rangle$ radiatively stable. The final equation (2.24) only depends on space-time averaged quantities and in this sense it is a *global* equation since it depends on the entire space-time. If we are to prevent the radiatively unstable contributions to the cosmological constant from sourcing curvature then we should have something non-trivial to say about this equation. This motivates us to consider modifying gravity globally to tackle the cosmological constant problem.

Let us now give a qualitative and complementary argument as to why we would want to modify gravity globally to tackle the cosmological constant problem by answering the following question: given a box of energy-momentum $T_{\mu\nu}^{\rm box}$, how does a modification of gravity whose single job is to prevent the vacuum energy from sourcing curvature, know which part of $T_{\mu\nu}^{\rm box}$ is the vacuum energy? The only way that a gravity theory can know which form of energy density is due to vacuum energy is with knowledge of the future evolution. Indeed, as we have said, the vacuum energy only differs from other sources by the fact that it does not evolve. At a given time shot of the universe, the vacuum energy is indistinguishable from other all forms of energy density. How much about the future does the gravity theory need to know? Let $T_{\mu\nu}^{\rm box}$ contain a scalar field whose potential and initial conditions are such that its configuration remains constant until asymptotically large distances and times where it begins to evolve. A modification of gravity therefore has to wait until asymptotically far in the future to know that this was just a scalar field in extreme slow roll rather than the vacuum energy. In other words, the *only* way of extracting the vacuum energy from the full energy-momentum tensor is to scan *all* of space and *all* of time. It follows that if we are to prevent *only* the vacuum energy from sourcing curvature, which we can model as an infinite wavelength source, we have to modify gravity globally. This link between the cosmological constant and causality was first discussed in [84].

This can be relaxed somewhat if one allows the modification of gravity to also affect sources with a finite wavelength. Indeed, in the original degravitation papers [84, 85] the goal was to prevent all sources with a wavelength larger than some characteristic scale L_{critical} from sourcing gravity. In the $L_{\text{critical}} \to \infty$ limit this must reduce to a global modification as we have just argued. For example, in [84] the idea was to promote Newton's constant to a high-pass filter such that long wavelength sources $L > L_{\text{critical}}$ coupled to gravity very weakly and short wavelength sources $L < L_{\text{critical}}$ coupled to gravity as in GR. This was realised with the following field equations

$$M_{\text{pl}}^2(1 + \mathcal{F}(L^2\nabla^2))G_{\mu\nu} = T_{\mu\nu} - g_{\mu\nu}\Lambda_c \tag{2.25}$$

where the filter function satisfies $\mathcal{F}(\alpha) \to 0$ for $\alpha \gg 1$ and $\mathcal{F}(\alpha) \gg 1$ for $\alpha \ll 1$. These equations lead to asymptotically small curvatures even in the presence of a large cosmological constant. In the limit $L_{\text{critical}} \to \infty$ the equations reduce to

$$M_{\text{pl}}^2 G_{\mu\nu} - \frac{\mathcal{F}(0)M_{\text{pl}}^2}{4} g_{\mu\nu}\langle R\rangle = T_{\mu\nu} - g_{\mu\nu}\Lambda_c \tag{2.26}$$

which we can fully decompose as we did above for GR yielding

$$M_{\text{pl}}^2\left(R^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu R\right) = \tau^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu\tau^\alpha{}_\alpha \tag{2.27}$$

$$M_{\text{pl}}^2(R - \langle R\rangle) = \langle\tau^\alpha{}_\alpha\rangle - \tau^\alpha{}_\alpha \tag{2.28}$$

$$(1 + \mathcal{F}(0))M_{\text{pl}}^2\langle R\rangle = 4(\Lambda_c + V_{\text{vac}}) - \langle\tau^\alpha{}_\alpha\rangle. \tag{2.29}$$

The modification of gravity is clearly only apparent in the global equation of GR (2.24). The main downfall of this work is that the equations (2.25) are not covariant[3] and no action or variational principle was presented from which these equations of motion followed. This makes it difficult to analyse the viability of the model. In the next chapter where we present the sequestering proposal, we shall see how one can successfully modify the global equation of GR in a theory with an action and a variational principle.

2.4 Weinberg's no-go theorem

A question one can ask is if we can add new fields to the matter sector in such a way that they cancel the large vacuum energy contributions to the cosmological constant by *self-adjusting* thereby offering a dynamical solution to the problem. However, Weinberg's no-go theorem [20] states that this is impossible without simply transferring the fine tuning of the bare cosmological constant to the potential of the self-adjusting fields while maintaining mass hierarchies in the matter sector. Here we review the no-go theorem and refer the reader to [25, 29, 31] for more details.

We begin by assuming that the gravitational action is built out of the space-time metric $g_{\mu\nu}$ and a bunch of scalar fields ϕ_i employed to self-adjust and eat up the vacuum energy contributions to the cosmological constant. Here we allow the index i to run from $1...N$ and we concentrate on scalars for simplicity. More generally, Weinberg's argument is valid for self-adjusting fields of any tensor rank. The action for this theory is

$$S = \int d^4x \mathcal{L}(g_{\mu\nu}, \phi_i) \tag{2.30}$$

where in $\mathcal{L}$ we allow for any interactions between the scalars and the metric. We assume that a solution exists which corresponds to a translationally invariant

[3] In the $L_{\text{critical}} \to \infty$ limit the equations of motion are covariant as can be seen from equation (2.26).

vacuum with $g_{\mu\nu} = \eta_{\mu\nu}, \phi_i =$ constant. In this case the field equations are

$$\frac{\partial \mathcal{L}}{\partial g^{\mu\nu}} = 0, \qquad \frac{\partial \mathcal{L}}{\partial \phi_i} = 0. \tag{2.31}$$

Now for these solutions to not be fine tuned Weinberg assumes that the trace of the gravitational field equations is related to the scalar field equations of motion via

$$2g^{\mu\nu}\frac{\partial \mathcal{L}}{\partial g^{\mu\nu}} + \sum_i f_i(\phi_i)\frac{\partial \mathcal{L}}{\partial \phi_i} = 0 \tag{2.32}$$

where $f_i(\phi_i)$ are generic functions of the scalar fields. Given that the variation of the action (2.30) is

$$\delta S = \int d^4x \left[\frac{\delta \mathcal{L}}{\delta g^{\mu\nu}}\delta g^{\mu\nu} + \sum_i \frac{\delta \mathcal{L}}{\delta \phi_i}\delta\phi_i\right] \tag{2.33}$$

the relation (2.32) is guaranteed to be realised with constant fields if the action is invariant under the infinitesimal transformations $\delta g^{\mu\nu} = 2\alpha g^{\mu\nu}$, $\delta\phi_i = \alpha f_i(\phi_i)$. Performing field redefinitions on the scalars $\phi_i \to \tilde{\phi}_i$ allows us to write the transformations as

$$\delta g^{\mu\nu} = 2\alpha g^{\mu\nu}, \quad \delta\tilde{\phi}_1 = -\alpha, \quad \delta\tilde{\phi}_j = 0 \tag{2.34}$$

where j runs from $2...N$. This is equivalent to a conformal transformation so the action will be invariant if it is constructed out of $\hat{g}_{\mu\nu} = e^{2\tilde{\phi}_1}g_{\mu\nu}$ and $\tilde{\phi}_j$. Therefore, gauge invariance dictates that the on-shell action is

$$S = \int d^4x\sqrt{-g}e^{4\tilde{\phi}_1}V(\tilde{\phi}_j) \tag{2.35}$$

and the gravity equation $\partial\mathcal{L}/\partial g^{\mu\nu} = 0$ yields

$$e^{4\tilde{\phi}_1}V(\tilde{\phi}_j) = 0. \tag{2.36}$$

We have to therefore either take $V(\tilde{\phi}_j) = 0$ which corresponds to fine tuning or $e^{4\tilde{\phi}_1} \to 0$. The physical masses in the field theory sector all scale as $m^2_{\text{phys}} = e^{2\tilde{\phi}_1}m^2$ and therefore this second option corresponds to taking the conformal limit. Our universe is not conformally invariant so this option is ruled out by observations.

This no-go theorem is very general and attempts at solving the cosmological constant problem should have a good answer as to how it is avoided. One possibility is to break translational invariance on the background by allowing the fields to admit homogeneous solutions see e.g. [86–88]. We will come across other ways in the following chapter when we discuss the sequestering proposal.

Chapter 3

Vacuum Energy Sequestering

This chapter is based on the vacuum energy sequestering proposal introduced by Kaloper and Padilla in 2012. Here we will present the original sequestering theory [30–32] and explain how it successfully restricts the radiatively unstable contributions to the cosmological constant from sourcing classical gravity, while ensuring that all finite wavelength sources gravitate in exactly the same way as in General Relativity (GR). This feature of the theory guarantees that the proposal passes all the stringent tests of solar system gravitational physics without any form of screening mechanism. We will explain how the proposal manages to get around Weinberg's no-go theorem by enforcing global constraints on space-time averaged parameters without introducing any new local propagating degrees of freedom. We will also discuss obstacles one is likely to face when looking for high energy completions of the model. Motivated by these issues, we will then present a more recent version of the sequestering proposal [1] which is qualitatively very similar to the original one, inasmuch as Einstein's equations are only modified globally to prevent only the vacuum energy from gravitating, but crucially the theory is better placed to be realised as the low energy description of a more fundamental theory.

3.1 The original sequester

In the previous chapter we saw how the radiatively unstable contributions to the cosmological constant from massive standard model particles minimally coupled to gravity affect space-time. In particular, we emphasised that from a gravitational point of view one has to scan the full space-time to extract the vacuum energy from the full $T_{\mu\nu}$. With this in mind, we asked how could one modify gravity in such a way that *only* the vacuum energy is restricted from sourcing curvature and all other sources gravitate in the usual way. We concluded that we must modify gravity *globally* by yielding new information about the global component of Einstein's equations (2.24). The sequestering proposal does just that.

The primary difference between sequestering and GR is presence of the global variables Λ_c and λ. These are global variables in the sense that they do not vary in space-time but are varied over in the action. They do not yield any new local degrees of freedom, rather they enforce highly non-trivial global constraints. As we shall see, these global constraints are vital in the model's ability to sequester the vacuum energy.

The original sequestering theory is based on the following action [30]

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \Lambda_c - \lambda^4\mathcal{L}_m(\lambda^{-2}g^{\mu\nu},\Phi)\right] + \sigma\left(\frac{\Lambda_c}{\lambda^4\mu^4}\right) \tag{3.1}$$

where σ is a smooth function and μ is a mass scale introduced on dimensional grounds. Its value is expected to be fixed phenomenologically. We emphasise that the function σ sits *outside* of the space-time integral and is therefore a truly global addition to the Einstein-Hilbert action. The matter fields Φ couple minimally to the conformally rescaled metric $\tilde{g}_{\mu\nu} = \lambda^2 g_{\mu\nu}$, and as a consequence λ fixes the hierarchy between the physical masses of fields $m_{\rm phys}$ and the bare Lagrangian mass parameters m via

$$m_{\rm phys} = \lambda m. \tag{3.2}$$

To see this consider the action of a free massive scalar field ϕ coupled to $\tilde{g}_{\mu\nu}$

$$\begin{aligned} S_\phi &= \int d^4x\sqrt{-\tilde{g}}\left[-\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{m^2}{2}\phi^2\right] \\ &= \int d^4x\sqrt{-g}\left[-\frac{\lambda^2}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\lambda^4m^2}{2}\phi^2\right] \end{aligned} \tag{3.3}$$

and upon canonically normalising ϕ we find that the pole of the scalar propagator is at $m_{\rm phys} = \lambda m$.

Throughout this chapter we will only be interested in classical gravity. As we explained in the previous chapter, we will work in a semi-classical framework and simply ask how the loop corrections to the vacuum energy affect the space-time curvature having calculated them in the $M_{\rm pl} \to \infty$ limit then relaxed $M_{\rm pl}$ back to its finite value of $\sim 10^{18}$GeV. This means that we only consider matter loop corrections to the cosmological constant and ignore graviton loops. In this sense all the gravitational information we require is given by the equations of motion. For the action (3.1), the full set of equations are given by varying with respect to $g_{\mu\nu}$, Λ_c and λ. Variation with respect to $g_{\mu\nu}$ yields

$$M_{\rm pl}^2 G^\mu{}_\nu = \lambda^4 \tilde{T}^\mu{}_\nu - \delta^\mu{}_\nu \Lambda_c \tag{3.4}$$

where $\tilde{T}^\mu{}_\nu$ is the energy momentum tensor with respect to the conformally rescaled metric $\tilde{g}_{\mu\nu}$. If we associate the global parameter Λ_c with the bare cosmological constant of GR and use $\lambda^4 \tilde{T}^\mu{}_\nu = T^\mu{}_\nu$, then these are precisely Einstein's field equations. However, in sequestering we have more information given by varying with respect to the global variables. Variation with respect to Λ_c yields the first global constraint where the space-time four volume is tied to the global variables via

$$\frac{\sigma'}{\lambda^4 \mu^4} = \int d^4x \sqrt{-g} \tag{3.5}$$

where a prime denotes differentiation with respect to the argument. An immediate consequence of this equation is that if the four volume of the universe is to be infinite, we are forced to fix $\lambda = 0$ on-shell since μ^4 and σ' are assumed to be finite. Note that we require a differentiable σ such that the variational principle is well defined. Vanishing λ is equivalent to the vanishing of all particle masses in the field theory sector by virtue of equation (3.2). This is not a valid description of our universe and therefore the only consistent sequestering vacua are those where the universe has a finite space-time volume and realises a future collapse. We will discuss this prediction in more detail later on. Finally, variation with respect to λ yields our final constraint by constraining the space-time integral of the traced energy-momentum tensor in terms of the global variables via

$$4\Lambda_c \frac{\sigma'}{\lambda^4 \mu^4} = \int d^4x \sqrt{-g} \lambda^4 \tilde{T}^\alpha{}_\alpha. \tag{3.6}$$

The full sequestering system is therefore a set of twelve equations. This final equation also tells us that σ cannot be a pure logarithm since in this case we have

$$4 = \int d^4x \sqrt{-g} \lambda^4 \tilde{T}^\alpha{}_\alpha \tag{3.7}$$

which is simply an artificial constraint placed on the matter sector. To zoom in on the gravitational dynamics it is constructive to eliminate the global variables in favour of a clear modification of Einstein's equations. To do so, we take the ratio between equations (3.6) and (3.5) which eliminates all λ dependence and results in the single global constraint

$$4\Lambda_c = \langle T^\alpha{}_\alpha \rangle. \tag{3.8}$$

Therefore, the variation of the global variables has fixed the bare value of the cosmological constant in terms of the space-time average of the traced energy-momentum tensor. This is very different to GR where the bare cosmological constant is a completely free parameter. Here its value is fixed on-shell and this constraint is vital to the cancellation of vacuum energy. We can use equation (3.8) to now eliminate the remaining global variable Λ_c from (3.4) leaving us with the final gravitational field equations

$$M_{\rm pl}^2 G^\mu{}_\nu = T^\mu{}_\nu - \frac{1}{4} \delta^\mu{}_\nu \langle T^\alpha{}_\alpha \rangle \tag{3.9}$$

or equivalently, after making the replacement $T^\mu{}_\nu = \tau^\mu{}_\nu - \delta^\mu{}_\nu V_{\rm vac}$ to extract the quantum corrections to the cosmological constant,

$$M_{\rm pl}^2 G^\mu{}_\nu = \tau^\mu{}_\nu - \frac{1}{4} \delta^\mu{}_\nu \langle \tau^\alpha{}_\alpha \rangle. \tag{3.10}$$

The vacuum energy has dropped out from the right hand side of equation

(3.10) and is thereby restricted from sourcing curvature. The only gravitational sources are due to local excitations $\tau^{\mu}{}_{\nu}$ which source curvature in exactly the same way as in GR. Importantly, this cancellation is effective at *all* orders in perturbation theory and whenever the Wilsonian cut-off of the effective matter action is altered. The sequestering constraint (3.8) has forced the bare cosmological constant Λ_c to absorb *all* quantum mechanical contributions V_{vac} to the overall cosmological constant without *any* fine tuning. The process of repeatedly fine tuning the bare cosmological constant in GR is done automatically in sequestering thanks to the global constraint.

There remains a residual cosmological constant $\langle\tau^{\alpha}{}_{\alpha}\rangle/4$ which is radiatively stable. It is given by the space-time average of the traced energy-momentum tensor of the local excitations. Its value can now be fixed by observations which place an upper bound of $(\text{meV})^4$. However, we do not want this residual cosmological constant to be responsible for the observed acceleration of the universe since if it is dominating the matter sector today, it will continue to do so and the universe will never collapse. As we discussed above this would only be a consistent sequestering solution if the field theory sector was conformally invariant. With this in mind we ask: is $\langle\tau^{\alpha}{}_{\alpha}\rangle/4 < (\text{meV})^4$ consistent with the model such that the late time acceleration can be realised by another source? If we assume that the sources which contribute to $\tau^{\alpha}{}_{\alpha}$ obey standard energy conditions, then in a universe which collapses in the future the magnitude of $\langle\tau^{\alpha}{}_{\alpha}\rangle$ is dominated by the epoch when the universe transitions from acceleration to deceleration [31] i.e. just before the required collapse. We can therefore approximate $\langle\tau^{\alpha}{}_{\alpha}\rangle \sim \rho_{\text{tran}} = M_{\text{pl}}^2 H_{\text{tran}}^2$ where ρ_{tran} and H_{tran} are the energy density and Hubble parameter respectively at the transition. Given that the universe is yet to reach the transition, the Hubble parameter there must be smaller than the Hubble parameter today $H_0 \gtrsim H_{\text{tran}}$ given that we are assuming the present acceleration does not have $w = -1$. It follows that $\langle\tau^{\alpha}{}_{\alpha}\rangle < (\text{meV})^4$ and the model is therefore consistent with a sub-dominant residual cosmological constant today. We refer the reader to [32] for a discussion of how the model can realise a late time collapse by employing a scalar field with a linear potential. There it is also shown how this scalar can provide acceleration prior to the collapse with the magnitude of the acceleration naturally small by virtue of a shift symmetry.

At this stage we have not really made connection with the global equation of GR (2.24) which we argued we had to focus on to tackle the cosmological constant problem. Let us now decompose the sequestering equations of motion in the same way we did for GR in equations (2.22-2.24) and make the global nature of this modification of gravity transparent. We again split the gravitational equations (3.4) into a set of traceless equations and a trace equation. We then take the difference between the trace equation and the space-time average of the trace equation. This, as expected, yields exactly the same equations as in GR but now we have the addition of the sequestering constraint. So the decomposed equations of motion for sequestering are

$$M_{\rm pl}^2\left(R^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu R\right) = \tau^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu \tau^\alpha{}_\alpha \tag{3.11}$$

$$M_{\rm pl}^2(R - \langle R\rangle) = \langle \tau^\alpha{}_\alpha\rangle - \tau^\alpha{}_\alpha \tag{3.12}$$

$$M_{\rm pl}^2\langle R\rangle = 4(\Lambda_c + V_{\rm vac}) - \langle \tau^\alpha{}_\alpha\rangle \tag{3.13}$$

$$4(\Lambda_c + V_{\rm vac}) = \langle \tau^\alpha{}_\alpha\rangle \tag{3.14}$$

and the final equation (3.14) forces the cancellation of the vacuum energy.

A completely equivalent way of expressing the sequestering constraint is realised if we combine equation (3.14) and (3.13) yielding

$$\langle R\rangle = 0. \tag{3.15}$$

This tells us much more about the allowed nature of sequestering vacua. Indeed, not only must the universe collapse in the future, but in addition the integral of the Ricci scalar over the full finite space-time must vanish. This further restricts the space of GR solutions which are consistent sequestering solutions.

We could have arrived at this constraint more directly by transforming the action (3.1) into the *Jordan frame*. Under the rescalings

$$g_{\mu\nu} \to \hat{g}_{\mu\nu} = \lambda^2 g_{\mu\nu} \tag{3.16}$$

$$\Lambda_c \to \tilde{\Lambda}_c = \frac{\Lambda_c}{\lambda^4} \tag{3.17}$$

the action reads

$$S = \int d^4x\sqrt{-\tilde{g}}\left[\frac{M_{\rm pl}^2}{2\lambda^2}\tilde{R} - \tilde{\Lambda}_c - \mathcal{L}_m(\tilde{g}^{\mu\nu}, \Phi)\right] + \sigma\left(\frac{\tilde{\Lambda}_c}{\mu^4}\right) \tag{3.18}$$

where $\tilde{R}$ is the Ricci scalar defined with respect to $\tilde{g}_{\mu\nu}$. We now see that the equation of motion with respect to λ leads to a direct constraint on the space-time integral of the Ricci scalar forcing it to vanish

$$\frac{M_{\rm pl}^2}{\lambda^3}\int d^4x\sqrt{-\tilde{g}}\tilde{R} = 0. \tag{3.19}$$

Given that we require a finite volume to accommodate mass hierarchies, this is equivalent $\langle R\rangle = 0$. For completeness, variation with respect to $\tilde{g}_{\mu\nu}$ and $\tilde{\Lambda}_c$ leads respectively to

$$M_{\rm pl}^2\tilde{G}^\mu{}_\nu = \lambda^2\tilde{T}^\mu{}_\nu - \delta^\mu{}_\nu\lambda^2\tilde{\Lambda}_c \tag{3.20}$$

$$\int d^4x\sqrt{-\tilde{g}} = \frac{\sigma'}{\mu^4}. \tag{3.21}$$

Let us briefly conclude. We have shown how by enforcing global constraints in the gravitational field equations of GR, we can arrange for the radiatively unstable loop corrections to the cosmological constant to not source classical gravity. The way the original sequestering proposal realises these constraints is to introduce global parameters in addition to the dynamical metric, and a function which relates the two which is not integrated over in the action. The mechanism then works by allowing λ to have knowledge of the field theory sector, by coupling to all bare mass parameters in the field theory Lagrangian with the vacuum energy scaling as λ^4 to all orders in perturbation theory, which is then passed on to Λ_c via their interaction in the global function. Λ_c then absorbs the radiative corrections.

We note that the sequestering proposal shares some similarities with Tseytlin's approach to the cosmological constant problem [89]. Tseytlin proposed that the combination of the Einstein-Hilbert action and the matter action is to be divided by the four volume of the universe such that the action is

$$S_{\text{tseytlin}} = \frac{\int d^4x\sqrt{-g}\left[\frac{M_{\text{pl}}^2}{2}R - \mathcal{L}_m(g^{\mu\nu},\Phi)\right]}{\mu^4\int d^4x\sqrt{-g}} \tag{3.22}$$

where in comparison to (3.1), μ is a mass scale introduced on dimensional grounds. To see the main issue with Tseytlin's proposal, we follow [31] and express this theory using the global variables of sequestering, but with slightly different couplings, by the action

$$S_{\text{tseytlin}} = \int d^4x\sqrt{-g}\left[\frac{\lambda^4 M_{\text{pl}}^2}{2}R - \Lambda_c - \lambda^4\mathcal{L}_m(g^{\mu\nu},\Phi)\right] + \frac{\Lambda_c}{\lambda^4\mu^4}. \tag{3.23}$$

Now variation with respect to the global variable Λ_c yields

$$\lambda^4 = \frac{1}{\mu^4\int d^4x\sqrt{-g}} \tag{3.24}$$

and shows the equivalence between the two formulations. We now transform (3.23) into the Einstein frame by taking $g^{\mu\nu} \to \lambda^4 g^{\mu\nu}$ yielding

$$S_{\text{tseytlin}} = \int d^4x\sqrt{-g}\left[\frac{M_{\text{pl}}^2}{2}R - \frac{\Lambda_c}{\lambda^8} - \frac{1}{\lambda^4}\mathcal{L}_m(\lambda^4 g^{\mu\nu},\Phi)\right] + \frac{\Lambda_c}{\lambda^4\mu^4}. \tag{3.25}$$

Now we see that the tree level contributions to the cosmological constant coming from the field theory sector scale as $1/\lambda^4$ since they are packaged into the matter Lagrangian by $\mathcal{L}_m = V_{\text{vac}}^{\text{tree}}$. However, the loop corrections to the cosmological constant scale as $1/\lambda^8$ after we canonically normalise the matter degrees of freedom since the physical masses are related to the mass parameters in the Lagrangian by $m_{\text{phys}}^2 = m^2/\lambda^4$. This means that the tree level contributions are indeed decoupled from gravity but the radiative corrections gravitate in the usual way [31]. In [89] it was also realised that Tseytlin's model suffers from large radiative corrections to the Planck mass rendering gravity much weaker than in GR. This is not the case in the sequestering proposal as long as the field theory cut off is not larger than the Planck scale [30].

The presence of global variables is what allows the theory to evade Weinberg's no-go theorem. As we discussed in the previous chapter, Weinberg's theorem is very general but it assumes all fields which are varied over in the action are functions of the space-time co-ordinates. That is not the case here. However, the global variables are somewhat unusual and it is not obvious how one should treat them quantum mechanically. Indeed, the action (3.1) does not describe a local field theory with a standard Hamiltonian and path integral, and one may worry about whether it can be UV completed. We do note, however, that allowing would be fundamental constants to vary in the action draws links with string compactifications [90] and wormhole corrections in Euclidean quantum gravity [91]. It has taught us something very important though; it is possible to make progress on the cosmological constant problem using global constraints, and these constraints can come from a variational principle. In the following section we will build on these ideas and introduce a manifestly local sequestering action [1], which still yields the important global constraints allowing for the cancellation of vacuum energy, but where all fields are functions of the space-time co-ordinates. For more information on the original sequestering proposal we refer the reader to the original sequestering papers [30–32] and [25].

3.2 The local sequester

To help us understand the structure of the manifestly local theory of vacuum energy sequestering, it is first instructive to discuss the gauge invariant formulation of unimodular gravity introduced by Henneaux and Teitelboim [92]. To do this, however, we must first discuss unimodular gravity and why it does not solve the cosmological constant problem. More details on unimodular gravity in the context of the cosmological constant problem can be found in [93].

Unimodular gravity is equivalent to GR, but with the constraint $\sqrt{-g} = 1$ imposed on the dynamics i.e. the variational principle is restricted by assuming a fixed metric determinant such that

$$\frac{\delta}{\delta g^{\mu\nu}}\sqrt{-g} = 0. \tag{3.26}$$

The resulting variation of the Einstein-Hilbert action (1.1) yields

$$M_{\rm pl}^2\left(R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R\right) = T_{\mu\nu} - \frac{1}{4}g_{\mu\nu}T \tag{3.27}$$

which are the traceless Einstein equations. Since the bare cosmological constant enters the action as $\sqrt{-g}\Lambda_c$, it drops out of these equations and for the same reason the quantum corrections to the cosmological constant $V_{\rm vac}$ also drop out from the right hand side. One may then conclude that this solves the cosmological constant problem, see e.g. [94]. However, taking the divergence of (3.27) and using the Bianchi identity we find

$$\nabla_\mu(M_{\rm pl}^2 R + T) = 0 \tag{3.28}$$

or equivalently

$$M_{\rm pl}^2 R + T = 4\Lambda_c. \tag{3.29}$$

This is simply the trace of the Einstein equations and the bare cosmological constant has returned as an integration constant. If we combine equations (3.27) and (3.29) we recover the full Einstein equations

$$M_{\rm pl}^2 G_{\mu\nu} = T_{\mu\nu} - g_{\mu\nu}\Lambda_c \tag{3.30}$$

and in the process, the cosmological constant problem. The only difference between GR and unimodular gravity is that the bare cosmological constant appears as an integration constant in unimodular gravity rather than a constant term in the action as in GR. So in unimodular gravity we must *repeatedly* choose a new integration constant to deal with the radiative corrections from the field theory sector. This is no better or worse than having to repeatedly re-tune a constant piece in the action.

One can enforce the unimodular constraint at the level of the action by virtue of a Lagrange multiplier $\lambda(x)$ such that the unimodular action is

$$S_{\text{unimodular}} = S_{\text{GR}} + \int d^4x \lambda(x)(\sqrt{-g} - 1). \tag{3.31}$$

This action breaks gauge invariance, making it clear that the unimodular condition $\sqrt{-g} = 1$ is really just a local choice of gauge. Indeed, in unimodular gravity the full diffeomorphism invariance of GR, which infinitesimally is $\delta g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$, is only retained with $\nabla_\mu \xi^\mu = 0$ which corresponds to transverse diffeomorphisms. For example, consider the FLRW metric of cosmology with flat spatial slices

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2) \tag{3.32}$$

where $a(t)$ is the scale factor. In cosmology we often makes use of the gauge symmetry of GR by rescaling the time coordinate $dt \to a(t)dt$ such that the FLRW metric is conformally related to the Minkowski metric $g_{\mu\nu} = a(t)^2\eta_{\mu\nu}$. However, there was nothing stopping us from performing a different rescaling $dt \to a(t)^{-3}dt$ such that the metric reads

$$ds^2 = -a(t)^{-6}dt^2 + a(t)^2(dx^2 + dy^2 + dz^2) \tag{3.33}$$

which satisfies $\sqrt{-g} = 1$. In a theory with full diffeomorphsim invariance we can always choose a co-ordinate system which satisfies the unimodular constraint, at least locally. As we have discussed in detail, this gauge symmetry of GR is merely a redundancy which we make use of to write down a Lorentz invariant theory of interacting massless spin-2 particles. In this sense fixing a gauge cannot tell us anything about physics and certainly not about the cosmological constant problem.

We now come to the Henneaux and Teitelboim gauge invariant formulation of unimodular gravity [92] which makes the tiny difference between GR and unimodular gravity manifest. To enforce the unimodular constraint in a gauge invariant way, one can use a *different measure* in the gravitational action. The standard covariant four volume is a 4-form and can be expressed as

$$\int d^4x\sqrt{-g} = \frac{1}{4!}\int \sqrt{-g}\epsilon_{\mu\nu\lambda\rho}dx^\mu dx^\nu dx^\lambda dx^\rho \tag{3.34}$$

where $\epsilon_{\mu\nu\lambda\rho}$ is the Levi-Civita symbol which transforms as a tensor density under a change of co-ordinates. But $\sqrt{-g}\epsilon_{\mu\nu\lambda\rho}$ is by no means a unique 4-form. Any 4-form $F_{\mu\nu\lambda\rho}$ can be used instead without breaking diffeomorphism invariance. Henneaux and Teitelboim exploited this fact and expressed unimodular gravity as

$$S = \frac{M_{\rm pl}^2}{2}\int d^4x\sqrt{-g}R - \int \Lambda_c(x)\left(\sqrt{-g}d^4x - \frac{1}{4!}F_{\mu\nu\lambda\rho}dx^\mu dx^\nu dx^\lambda dx^\rho\right) \tag{3.35}$$

where we define the 4-form $F_{\mu\nu\lambda\rho}$ as the exterior derivative of a 3-form $A_{\nu\lambda\rho}$ via

$$F_{\mu\nu\lambda\rho} = 4\partial_{[\mu}A_{\nu\lambda\rho]}. \tag{3.36}$$

It is important to realise that even though this theory maintains the full diffeomorphism invariance, $F_{\mu\nu\lambda\rho}$ is totally independent of the metric off-shell. It has the desired transformation properties thanks to the anti-symmetrisation over all its indices. It is therefore absent in the gravitational field equations. Variation with respect to $\Lambda_c(x)$ leads to a constraint on the metric determinant and variation with respect to the 3-form leads to $\partial_\mu\Lambda_c(x) = 0$ and therefore fixes the Lagrange multiplier to be constant on-shell. The integration constant found by solving this equation is precisely the bare cosmological constant we add to the action in GR. In fact, we can still express a gauge invariant formulation of unimodular gravity with a slightly different $\Lambda_c(x)$ dependence in the $F_{\mu\nu\lambda\rho}$ sector by

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \Lambda_c(x)\right] + \frac{1}{4!}\int \sigma\left(\frac{\Lambda_c(x)}{\mu^4}\right)F_{\mu\nu\lambda\rho}dx^\mu dx^\nu dx^\lambda dx^\rho \tag{3.37}$$

where μ has dimensions of mass and is introduced on dimensional grounds, and σ is an arbitrary smooth function. In line with our previous discussions of global constraints in sequestering, let us further explain how unimodular gravity fails to solve the cosmological constant problem by now coupling matter fields to the metric in the minimal way and studying the equations of motion. Variation with respect to $g^{\mu\nu}$ yields

$$M_{\text{pl}}^2 G^{\mu}{}_{\nu} = T^{\mu}{}_{\nu} - \delta^{\mu}{}_{\nu}\Lambda_c(x) \tag{3.38}$$

which are the Einstein equations with a dynamical 'cosmological constant'. As we discussed above, variation with respect to $\Lambda_c(x)$ yields a constraint on the metric determinant

$$\frac{\sigma'}{\mu^4}F_{\mu\nu\lambda\rho} = \sqrt{-g}\epsilon_{\mu\nu\lambda\rho} \tag{3.39}$$

and finally, variation with respect to $A_{\nu\lambda\rho}$ leads to

$$\frac{\sigma'}{\mu^4}\partial_\mu\Lambda_c(x) = 0 \tag{3.40}$$

which fixes Λ_c to be constant on-shell. If we plug this solution into the gravity equations (3.38) then on-shell the system reduces to GR with the addition of the constraint (3.39). With Λ_c fixed to be constant, this constraint is really understood as a constraint on the space-time volume since we can readily integrate the 4-form field strength without introducing any dependence on the metric. This yields

$$\frac{1}{4!}\frac{\sigma'}{\mu^4}\int F = \int d^4x\sqrt{-g}. \tag{3.41}$$

Now, unlike the original sequestering proposal this constraint does not provide us with any information about $\langle R\rangle$ or $\langle T^{\alpha}{}_{\alpha}\rangle$. In other words, it does not tell us anything about the global equation of GR (2.24) which we argued was an important feature of a theory whose aim is to tackle the cosmological constant problem. To see this a little clearer let us decompose these equations of motion in the same way we have done before for GR and for the original sequestering theory. The result is

$$M_{\rm pl}^2\left(R^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu R\right) = \tau^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu \tau^\alpha{}_\alpha \tag{3.42}$$

$$M_{\rm pl}^2(R - \langle R\rangle) = \langle \tau^\alpha{}_\alpha\rangle - \tau^\alpha{}_\alpha \tag{3.43}$$

$$M_{\rm pl}^2\langle R\rangle = 4(\Lambda_c + V_{\rm vac}) - \langle \tau^\alpha{}_\alpha\rangle \tag{3.44}$$

$$\star F_4 - \langle \star F_4\rangle = 0, \qquad \langle \star F_4\rangle = \frac{\mu^4}{\sigma'} \tag{3.45}$$

where we have used $T^\mu{}_\nu = \tau^\mu{}_\nu - \delta^\mu{}_\nu V_{\rm vac}$ and $\star$ denotes the Hodge dual of a form which on a n dimensional manifold maps a p-form to a $(n-p)$-form via

$$(\star F)_{\mu_1\dots\mu_{n-p}} = \frac{1}{p!}\tilde{\epsilon}^{\nu_1\dots\nu_p}{}_{\mu_1\dots\mu_{n-p}} F_{\nu_1\dots\nu_p} \tag{3.46}$$

where $\tilde{\epsilon}^{\nu_1\dots\nu_p}{}_{\mu_1\dots\mu_{n-p}}$ is the Levi Civita *tensor* related to the Levi Civita symbol by $\tilde{\epsilon}_{\mu_1\dots\mu_n} = \sqrt{-g}\epsilon_{\mu_1\dots\mu_n}$. For example, we have

$$\star F_4 = \frac{1}{4!}\frac{1}{\sqrt{-g}}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu\lambda\rho}. \tag{3.47}$$

With the equations of motion in this form we see that the unimodular gravity constraint is not enough to help with the cosmological constant problem since the constraint equations (3.45) completely decouple from the decomposed GR equations which are equivalent to (3.42-3.44). We are still required to repeatedly re-tune Λ_c to cope with the loop corrections in $V_{\rm vac}$.

What have we learned from this? We have seen how one of the global parameters of the original sequestering theory, namely Λ_c, can arise as an integration constant from a local equation of motion, and we have seen how we can enforce global constraints from a gravity theory constructed from local fields by introducing a new covariant measure. However, we have also seen that in the context of unimodular gravity this is not enough to tackle the cosmological constant problem. In the following, we will show how this can be remedied to yield qualitatively similar equations of motion to the original sequestering theory and therefore have something non-trivial to say about the cosmological constant problem.

Let us begin by revisiting the original sequestering theory in the Jordan frame which we remind the reader is described by the following action

$$S = \int d^4x\sqrt{-\tilde{g}}\left[\frac{M_{\rm pl}^2}{2\lambda^2}\tilde{R} - \tilde{\Lambda}_c - \mathcal{L}_m(\tilde{g}^{\mu\nu}, \Phi)\right] + \sigma\left(\frac{\tilde{\Lambda}_c}{\mu^4}\right) \tag{3.48}$$

where the variation with respect to the global parameter λ gave us the desired constraint on the space-time integral of the Ricci scalar. Our aim is to now realise a similar constraint but in a theory with only local fields. We have just seen in our discussion of unimodular gravity how the role of $\tilde{\Lambda}_c$ can be played by a scalar field which is fixed to be constant on-shell by virtue of its coupling to an exact 3-form. Since Λ_c was able to fluctuate off-shell, its variation yielded a global constant. Let us now do a similar thing for the remaining global parameter by promoting $M_{\rm pl}^2/\lambda^2$ to a scalar field $\kappa^2(x)$ which we want to be constant on-shell but whose variation allows us to realise a constraint on the space-time average of the Ricci scalar. To do so we introduce a second 4-form, in addition to the one used in the gauge invariant form of unimodular gravity, which couples to $\kappa^2(x)$, but is independent of the metric off-shell without breaking gauge invariance. All these properties are realised by the following action [1]

$$\begin{aligned} S = &\int d^4x\sqrt{-g}\left[\frac{\kappa^2(x)}{2}R - \Lambda_c(x) - \mathcal{L}_m(g^{\mu\nu}, \Phi)\right] \\ &+ \frac{1}{4!}\int dx^\mu dx^\nu dx^\lambda dx^\rho\left[\sigma\left(\frac{\Lambda_c(x)}{\mu^4}\right)F_{\mu\nu\lambda\rho} + \hat{\sigma}\left(\frac{\kappa^2(x)}{M_{\rm pl}^2}\right)\hat{F}_{\mu\nu\lambda\rho}\right] \end{aligned} \tag{3.49}$$

where σ and $\hat{\sigma}$ are independent smooth functions, both 4-forms are the exterior derivative of a 3-form, and $\mu \lesssim M_{\rm pl}$ are the field theory and gravitational cut-offs respectively. Given our discussion for the original version of sequestering around equation (3.7), again σ cannot be a logarithm. This is our manifestly local version of vacuum energy sequestering.

In comparison to all of our previous studies, we remind the reader that in the following we will work in a semi-classical framework where gravity simply probes the vacuum energy contributions to the cosmological constant, and in the absence of fine tuning we ask how much they affect space-time. Here we have five fields with which we vary with respect to. Initially, variation with respect to $g^{\mu\nu}$ yields

$$\kappa^2(x)G^\mu{}_\nu = (\nabla^\mu\nabla_\nu - \delta^\mu{}_\nu\nabla^2)\kappa^2(x) + T^\mu{}_\nu - \delta^\mu{}_\nu\Lambda_c(x). \tag{3.50}$$

Variation with respect to the two scalars $\Lambda_c(x)$ and $\kappa^2(x)$ yields respectively

$$\frac{\sigma'}{\mu^4}F_{\mu\nu\lambda\rho} = \sqrt{-g}\epsilon_{\mu\nu\lambda\rho} \tag{3.51}$$

$$\frac{\hat{\sigma}'}{M_{\rm pl}^2}\hat{F}_{\mu\nu\lambda\rho} = -\frac{1}{2}\sqrt{-g}R\epsilon_{\mu\nu\lambda\rho} \tag{3.52}$$

which tells us that neither σ nor $\hat{\sigma}$ can be linear functions. In this case the 4-forms would be completely specified by the geometry. Finally, variation with respect to the 3-forms fixes the scalars to be constant on-shell

$$\frac{\sigma'}{\mu^4}\partial_\mu\Lambda_c(x) = 0 \tag{3.53}$$

$$\frac{\hat{\sigma}'}{M_{\rm pl}^2}\partial_\mu\kappa^2(x) = 0. \tag{3.54}$$

This ensures that there are no new long range gravitational forces in addition to GR and means that we do not rely on any screening mechanisms to conform with local tests of gravity. Indeed, gravity is again only modified globally since these equations describe GR with the addition of global constraints which can be understood as a constraint on $\langle R\rangle$, in comparison to the original sequestering model, and in contrast to unimodular gravity. To see this we now work with constant scalars, denoting these integration constants simply as Λ_c and κ^2, such that the gravity equations (3.50) reduce to

$$\kappa^2 G^\mu{}_\nu = T^\mu{}_\nu - \delta^\mu{}_\nu\Lambda_c \tag{3.55}$$

where $\kappa \sim 10^{18}$GeV is the bare Planck mass. Note that having fixed κ to match observations, such that there is a hierarchy between it and the matter sector scales, it will be radiatively stable with respect to matter loops as long as

$\mu \lesssim M_{\rm pl}$. Now, we integrate the constraints (3.51), (3.52) yielding respectively

$$\frac{1}{4!}\frac{\sigma'}{\mu^4}\int F = \int d^4x\sqrt{-g} \tag{3.56}$$

$$\frac{1}{4!}\frac{\hat{\sigma}'}{M_{\rm pl}^2}\int \hat{F} = -\frac{1}{2}\int d^4x\sqrt{-g}R. \tag{3.57}$$

If we take the ratio of the two we find

$$\kappa^2\langle R\rangle = -2\frac{\hat{\sigma}'}{\sigma'}\frac{\mu^4\kappa^2}{M_{\rm pl}^2}\frac{\int \hat{F}}{\int F} \tag{3.58}$$

and so the space-time average of the Ricci scalar is fixed in terms of the ratio between a pair of 4-form fluxes. In the interest of clarity, in what follows we will simply denote this constraint as $\kappa^2\langle R\rangle = 4\Delta\Lambda$ with $\Delta\Lambda$ defined appropriately by (3.58). To see the effect of this constraint let us go back to the gravity equations and extract the $\langle R\rangle$ dependence by taking the trace and space-time average yielding

$$4\Lambda_c = \langle T^\alpha{}_\alpha\rangle + \kappa^2\langle R\rangle \tag{3.59}$$

where we have used $\Lambda_c = \langle\Lambda_c\rangle$, $\kappa^2 = \langle\kappa^2\rangle$. We can now use the constraint (3.58) to eliminate $\langle R\rangle$ from equation (3.59) in favour of the integrated 4-forms, packaged inside $\Delta\Lambda$, followed by eliminating Λ_c from the gravity equations. This results in the following gravitational field equations

$$\kappa^2 G^\mu{}_\nu = T^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu\langle T^\alpha{}_\alpha\rangle - \delta^\mu{}_\nu\Delta\Lambda \tag{3.60}$$

which are very similar to the final gravitational equations of the original sequestering model. If we make the substitution $T^\mu{}_\nu = \tau^\mu{}_\nu - \delta^\mu{}_\nu V_{\rm vac}$ to extract the radiatively unstable contributions to the cosmological constant, we see that they drop out from the right hand side of these equations and are restricted from souring curvature. The only gravitational sources are $\tau^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu\langle\tau^\alpha{}_\alpha\rangle - \delta^\mu{}_\nu\Delta\Lambda$ and again all finite wavelength sources gravitate as in GR. It is only the infinite wavelength vacuum energy which is decoupled.

Relative to the original sequestering theory, the local version has an added contribution to the residual cosmological constant, namely, $\Delta\Lambda$ which accompanies $\langle \tau^\alpha{}_\alpha \rangle/4$. Both of these contributions are radiatively stable and can therefore be fixed by observations. However, unlike in the original sequestering theory, this residual cosmological constant can be responsible for the observed late time acceleration of the universe since here we do not require a finite space-time volume to accommodate non-zero mass scales in the field theory sector. If we allow this residual cosmological constant to drive the acceleration then the space-time volume will be infinite. In this case the only contribution is $\Delta\Lambda$ since in an infinite universe with matter fields obeying null energy conditions we have $\langle \tau^\alpha{}_\alpha \rangle \to 0$ because the denominator of the space-time average diverges, while the numerator dilutes as the universe evolves. Observations then tell us that $\Delta\Lambda \sim (\text{meV})^4$. Radiative stability of $\Delta\Lambda$ follows from the fact that it is constructed out of the ratio of two 4-form fluxes and the ratio between the normalised derivatives of the functions σ and $\hat{\sigma}$. The 4-form integrals are infrared quantities and therefore insensitive to changes in the field theory cut-off. σ and $\hat{\sigma}$ will respond to matter loops by virtue of their respective Λ_c and κ dependence, but if they are sufficiently smooth then these corrections can only ever be $\mathcal{O}(1)$ since they enter via Λ_c/μ^4 and κ^2/M_{pl}^2.

For a final time, let us now decompose the equations of motion as we have done previously to emphasise the importance of the global constraints. For the local theory of sequestering we find

$$\kappa^2 \left(R^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu R \right) = \tau^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu \tau^\alpha{}_\alpha \tag{3.61}$$

$$\kappa^2 (R - \langle R \rangle) = \langle \tau^\alpha{}_\alpha \rangle - \tau^\alpha{}_\alpha \tag{3.62}$$

$$\kappa^2 \langle R \rangle + \langle \tau^\alpha{}_\alpha \rangle = 4(\Lambda_c + V_{\text{vac}}) \tag{3.63}$$

$$\star F_4 - \langle \star F_4 \rangle = 0, \qquad \langle \star F_4 \rangle = \frac{\mu^4}{\sigma'} \tag{3.64}$$

$$\star \hat{F}_4 - \langle \star \hat{F}_4 \rangle = \frac{M_{\text{pl}}^2}{2\kappa^2 \hat{\sigma}'} (\tau^\alpha{}_\alpha - \langle \tau^\alpha{}_\alpha \rangle) \tag{3.65}$$

$$\Delta\Lambda = \frac{1}{4}\kappa^2 \langle R \rangle = -\frac{\kappa^2 \hat{\sigma}'}{2M_{\text{pl}}^2} \langle \star \hat{F}_4 \rangle \tag{3.66}$$

where we have presented these equations in three sets. The first set (3.61 - 3.63) are precisely the equations we found in GR and reinforce that locally the theories are equivalent. The next equations, (3.64), are the constraints we found in unimodular gravity. It is not surprising that these equations appear here since if we set $\hat{\sigma} = 0$ in the local sequestering action (7.3) we recover the gauge invariant formulation of unimodular gravity. Finally, the very important additions to GR, and unimodular gravity, are given by equations (3.65, 3.66) and as we desired yields new information about the space-time average of the Ricci scalar. Equation (3.65) shows how the hatted 4-form is radiatively stable. The vacuum energy contributions drop out from the right hand side of this equation and loop corrections only affect $\kappa^2\sigma'/M_{\rm pl}^2$ which as we discussed above can only be $\mathcal{O}(1)$. In turn, equation (3.66) shows that the space-time average of the Ricci scalar is now rendered radiatively stable since its value is fixed by the radiatively stable quantity $\Delta\Lambda$. This means that in equation (3.63), since the left hand side is now radiatively stable, Λ_c has to adjust to absorb the loop corrections coming from $V_{\rm vac}$. Let us emphasise here that even though in a finite space-time volume $\langle \tau^\alpha{}_\alpha \rangle$ does not vanish, this equation does not lead to the cancellation of any local sources since $\langle \tau^\alpha{}_\alpha \rangle$ is a global quantity and therefore not observable in a casual manner, consistent with our previous statements that the model is locally equivalent to GR. In comparison to the original sequestering theory, the dynamics has constrained the space-time average of the Ricci scalar in such a way that the bare cosmological constant is forced to cancel the large loop corrections without any fine tuning. We note that it is the presence of the 4-forms which allows the theory to evade Weinberg's no-go theorem. This is because they offer non-gravitating measures which enter the action in a diffeomorphism invariant manner without coupling to the metric off-shell. This enables us to realise the global constraints.

Finally, let us comment on the ability to have a well defined Cauchy problem with a standard $3+1$ decomposition in both sequestering models. In the original sequestering theory, a solution to Einstein's equations is also a solution to the sequestering equations if the space-time average of the Ricci scalar vanishes on the solution. In practice this means that GR solutions will only exist for certain initial conditions and choices in parameter space. For example, in GR coupled to a scalar field with a linear potential, a cosmological solution with a future collapse is generic due to the instability of the scalar field. However, only a subclass of these theories will also satisfy the sequestering constraint. So to find sequestering solutions one would need to scan a range of initial conditions. The same is true for the local sequestering model, however in this case the current form of the local theory does not yield a fixed numerical value for the space-time average of the Ricci scalar. Knowledge of this would require a deeper understanding of the origin of the four-forms and the associated fluxes with the possibility that the ratio of the fluxes is fixed by physics in the UV.

3.3 Discussion

Following on from our discussion of the cosmological constant problem and the global nature of the vacuum energy in chapter 2, in this chapter we have shown how the radiatively unstable matter loop corrections[1] to the overall cosmological constant can be decoupled from gravity by enforcing global constraints on the dynamics of GR. We initially showed how this can be achieved through global variables whose variation forces the bare cosmological constant to absorb the loop corrections without fine tuning. We then saw how similar constraints can be realised in a manifestly local theory which has a standard Hamiltonian and path integral. This theory again only decouples infinite wavelength sources and therefore offers an alternative to GR only differing in the global structure of classical solutions. In chapter 7 we will discuss future work and other applications of the interactions used here to sequester the matter loop contributions to the vacuum energy.

[1]As we discussed in the main body of this chapter, and the previous chapter, here we are ignoring graviton loops.

Chapter 4

Sequestering and Phase Transitions

Until now we have been concentrating on the most pressing aspect of the cosmological constant which is its radiative instability. However, another well defined issue with the cosmological constant is how it is influenced by phase transitions [95–98]. If the field theory sector undergoes a phase transition then the vacuum energy can change by $\mathcal{O}(\mathcal{M}^4)$ where $\mathcal{M}$ is the energy scale of the transition. For standard model phase transitions, e.g. the confining phase transition in QCD, this change in the vacuum energy is vastly larger than the observed cosmological constant. For QCD we expect the jump in vacuum energy to be $\sim \mathcal{O}(\mathrm{GeV}^4)$ which is 48 orders of magnitude too large (see e.g. [99]). This means that any tuning or cancellation of the vacuum energy before the transition would be ruined after. There are also similar problems due to electroweak phase transitions, GUT scale phase transitions, etc. In this chapter our aim is to study the effects of an early universe phase transition on the late time curvature in the local sequestering theory. We will first recover the results presented in [30, 31] where it is argued that at late times an early universe phase transition has little impact on the space-time curvature, followed by a more rigorous analysis involving the nucleation and growth of bubbles of true vacuum. We will calculate the tunnelling rates between maximally symmetric vacua in the local sequestering theory and determine how sensitive the curvature of the universe after a phase transition is to the jump in vacuum energy. We do so following [2].

4.1 A minimal approach

Following [30,31], we concentrate on a single phase transition and assume that it occurs instantaneously over space-like surfaces. We model it by the simple step function

$$V = V_1(1 - \Theta(t - t_{\rm PT})) + V_2\Theta(t - t_{\rm PT}) \tag{4.1}$$

where V_1 is the vacuum energy before the phase transition which occurs at $t_{\rm PT}$, and V_2 is the vacuum energy after the phase transition with $V_1 > V_2$. In this case, according to the gravitational field equations of the original sequestering theory (3.10), space-time curvature is sourced by

$$\tau^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu\langle\tau^\alpha{}_\alpha\rangle = -\delta^\mu{}_\nu(V - \langle V\rangle) \tag{4.2}$$

since $\tau^\mu{}_\nu = -\delta^\mu{}_\nu V$. As expected, given the global nature of the equations of motion, the size of the effective cosmological constant at a given time is sensitive to the vacuum energy both before and after the phase transition through its dependence on

$$\langle V\rangle = \frac{V_1\int_{t_i}^{t_{\rm PT}} dta^3}{\int_{t_i}^{t_f} dta^3} + \frac{V_2\int_{t_{\rm PT}}^{t_f} dta^3}{\int_{t_i}^{t_f} dta^3} \tag{4.3}$$

where t_i and t_f are the time at the big bang and the big crunch respectively, and we are working on a cosmological background where $\sqrt{-g} = a^3$ with $a = a(t)$ the scale factor. It follows that before the phase transition we have

$$\tau^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu\langle\tau^\alpha{}_\alpha\rangle = -\delta^\mu{}_\nu|\Delta V|(1 - \Omega_{\rm PT}) \tag{4.4}$$

where $\Delta V = V_2 - V_1$ is the jump in vacuum energy induced by the transition and we have defined

$$\Omega_{\rm PT} = \frac{\int_{t_i}^{t_{\rm PT}} dta^3}{\int_{t_i}^{t_f} dta^3}. \tag{4.5}$$

While after the phase transition we have

$$\tau^\mu{}_\nu - \frac{1}{4}\delta^\mu{}_\nu\langle\tau^\alpha{}_\alpha\rangle = \delta^\mu{}_\nu|\Delta V|\Omega_{\rm PT}. \tag{4.6}$$

In the original sequestering theory the full space-time integral is dominated by the period where the evolution of the universe changes from expansion to contraction [31][1] and we can therefore approximate

$$\Omega_{\rm PT} \sim \left(\frac{a_{\rm PT}}{a_{\rm tran}}\right)^3 \frac{H_{\rm tran}}{H_{\rm PT}}. \tag{4.7}$$

Since we are interested in standard model phase transitions which occurred prior to the present acceleration and therefore prior to the turn around of the scale factor, we have $\Omega_{\rm PT} \ll 1$. The jump in vacuum energy is set by the energy scale of the transition with $|\Delta V| \sim M_{\rm pl}^2 H_{\rm PT}^2$ and therefore the space-time curvature after the transition is sourced by

$$\delta^\mu{}_\nu |\Delta V| \Omega_{\rm PT} \sim \rho_{\rm tran}\left(\frac{a_{\rm PT}^3}{a_{\rm tran}^3}\frac{H_{\rm PT}}{H_{\rm tran}}\right) \lesssim \rho_{\rm tran} \lesssim ({\rm meV})^4. \tag{4.8}$$

In conclusion, the effects of standard model phase transitions are suppressed at late times and only offer a small contribution to the effective cosmological constant. For the local sequestering theory, (4.2) is accompanied by $\Delta\Lambda$ which as we showed in the previous section is another radiatively stable contribution to the residual cosmological constant. It's value is insensitive to a phase transition and is therefore the same before and after the transition. It follows that the above calculation also holds for the local version of sequestering. However, there we have no reason to demand a universe with a future collapse so there is not necessarily a transition from acceleration to deceleration. In this case we still have $\Omega_{\rm PT} \ll 1$ for early universe phase transitions such that the curvature after the phase transition is insensitive to the jump in the vacuum energy by equation (4.6). The reason why this works out so nicely is simply because the space-time volumes are dominated by the space-time regions after the transition since these occurred in the early universe. So at late times the volume suppression becomes very effective. For more details we refer the reader to [25, 31].

[1]We recall that as explained in chapter 3, the only consistent solutions in the original sequestering theory have a future collapse.

4.2 False vacuum decay

In this section we will again consider the effects of a phase transition but we do so in the context of bubble nucleation. This will allow us to calculate the tunnelling rates between maximally symmetric vacua in the local sequestering theory (7.3). To do so, let us first be more rigorous about the variational principle since in the presence of a boundary we must include extra terms, like the Gibbons-Hawking term in General Relativity (GR) [100], to ensure that it is well defined. For the two 3-forms we simply impose Dirichlet boundary conditions but for the metric and the scalars the situation is slightly more subtle. As in GR, in addition to the terms in (7.3) we add to the sequestering action the boundary term

$$\int d^3x\sqrt{h}\kappa^2(x)K \tag{4.9}$$

where $K = h^{ij}K_{ij}$ is the trace of the extrinsic curvature on the boundary K_{ij}, and h_{ij} is the induced metric. Given that $\kappa^2(x)$ is fixed to be constant on-shell by the bulk equations of motion, variation of the full sequestering action with (4.9) included yields the following boundary terms

$$\frac{1}{2}\int d^3x\sqrt{h}[-\kappa^2(K^{ij} - Kh^{ij})]\delta h_{ij} + \int d^3x\sqrt{h}K\delta\kappa^2. \tag{4.10}$$

Usually one would now impose Dirichlet boundary conditions on both the metric and the scalar such that the action is stationary under this variation. However, Dirichlet boundary conditions on the scalar κ^2 (and on the other scalar Λ) would interfere with the crucial sequestering global constraints which come from the bulk equations of motion. Instead we impose Neumann boundary conditions on both scalars which ensures that there is no momentum loss across the boundary[2], and the following boundary condition on the metric

$$\delta h_{ij} = -\frac{\delta\kappa^2}{\kappa^2}h_{ij} \tag{4.11}$$

[2]This is somewhat trivial since it is guaranteed by the bulk equations of motion.

which although looks somewhat unusual at first is equivalent to imposing Dirichlet boundary conditions on the Einstein frame metric. This now ensures that the variational principle is well defined and puts us in a position where we can investigate tunnelling between maximally symmetric vacua in the local sequestering theory.

4.2.1 Materialisation of the bubble

As for the simplified calculation above, we specialise to a matter sector with two vacua with differing vacuum energies. In line with our previous studies we will work in a semi-classical limit where gravity is treated classically yet we allow the matter sector to undergo quantum tunnelling between the two vacua. In this semi-classical picture the vacuum of highest energy density is unstable due to barrier penetration i.e. it is a false vacuum. In the absence of gravity this process in field theory and the corresponding tunnelling rates were described in [101,102] by Coleman and, Callan and Coleman respectively. Gravitational effects were then included in [103] by Coleman and de Luccia. In all cases, to calculate the tunnelling rates between the two vacua one must calculate the bounce which is a solution of the Euclidean field equations which interpolates between the two vacua. In the following we must therefore make use of Euclidean signature by virtue of the Wick rotation

$$t \to -it_E, \qquad S \to iS_E \tag{4.12}$$

from Lorentzian signature. We will also work in the thin wall limit [103] where we assume that the difference in vacuum energy in the two vacua is small compared to the height of the barrier which separates them.

Set-up and Euclidean solutions

It has been shown in [104, 105] that the tunnelling rates are dominated by Euclidean configurations with maximal symmetries i.e. ones which are $O(4)$

invariant. Therefore our bounce geometry throughout will be of the form [103]

$$ds^2 = dr^2 + \rho^2(r)d\chi^2 \tag{4.13}$$

where $d\chi^2 = \gamma_{ij}dx^i dx^j$ is the metric of a unit 3-sphere in Euclidean signature. This ensures that in what follows all fields will only be a function of the radial co-ordinate r. Following [103], we set up a co-ordinate system with the wall separating the two vacua at $r = 0$, the exterior of a bubble at $r > 0$, and the interior of a bubble at $r < 0$. We denote the exterior and interior of a bubble as $\mathcal{M}_+$ and $\mathcal{M}_-$ respectively and in what follows we shall refer to this co-ordinate system as 'Coleman's co-ordinates'. We model the wall as a delta function weighted by its tension σ_w and the global vacuum energy as a step function with contributions

$$V(r) = \begin{cases} V_+ & r > 0 \\ V_- & r < 0. \end{cases} \tag{4.14}$$

With the rotational symmetry of our system the 3-forms take the form

$$A_{ijk} = A(r)\sqrt{\gamma}\epsilon_{ijk} \tag{4.15}$$
$$\hat{A}_{ijk} = \hat{A}(r)\sqrt{\gamma}\epsilon_{ijk} \tag{4.16}$$

and by virtue of the 3-form equations of motion given by (3.53) and (3.54), the scalars Λ and κ^2 are constant. The remaining field equations coming from (3.50),(3.51) and (3.52) reduce to

$$3\kappa^2\left(\frac{\rho'^2}{\rho^2} - \frac{1}{\rho^2}\right) = -(\Lambda_c + V(r)) \tag{4.17}$$
$$\kappa^2\left(\frac{\rho'^2}{\rho^2} - \frac{1}{\rho^2} + 2\frac{\rho''}{\rho}\right) = -(\Lambda_c + V(r) + \sigma_w\delta(r)) \tag{4.18}$$
$$\frac{\sigma'}{\mu^4}A'(r) = \rho^3 \tag{4.19}$$
$$\frac{\hat{\sigma}'}{M_{\rm pl}^2}\hat{A}'(r) = -3\left(\frac{1}{\rho^2} - \frac{\rho'^2}{\rho^2} - \frac{\rho''}{\rho}\right)\rho^3 \tag{4.20}$$

where we have used the following expression for the Ricci scalar calculated on the $O(4)$ invariant metric

$$R = 6\left[\frac{1}{\rho^2} - \left(\frac{\rho'}{\rho}\right)^2 - \frac{\rho''}{\rho}\right]. \tag{4.21}$$

Ignoring the bubble wall i.e. far away from $r = 0$, the geometry is dictated by the following field equation

$$\rho'^2 = 1 - q^2\rho^2 \tag{4.22}$$

where we have defined the local vacuum curvature as

$$q^2 = \frac{\Lambda_c + V}{3\kappa^2}. \tag{4.23}$$

As is the case in GR, the local cosmological constant receives a classical contribution Λ_c which in sequestering is simply an integration constant, and the field theory contribution V. Equation (4.22) can be easily solved to give

$$\rho(r) = \frac{1}{q}\sin q(r_0 + \epsilon r) \tag{4.24}$$

where $\epsilon = \pm 1$ and r_0 is a constant of integration whose value can differ on either side of the wall. This solution holds for q real, imaginary and when $q \to 0$. These three possibilities represent the three maximally symmetric configurations. In Euclidean signature they correspond to the sphere, hyperboloid and plane respectively, whereas in Lorentzian signature, they would correspond to de Sitter space, anti-de Sitter space and Minkowski space respectively. From equations (4.19) and (4.20), the solutions for the 3-forms, given the solution for $\rho(r)$, are

$$A(r) = A_0 + \frac{\mu^4}{\sigma'}\int_0^r \rho^3 dr \tag{4.25}$$

$$\hat{A}(r) = \hat{A}_0 - \frac{6M_{\text{pl}}^2}{\hat{\sigma}'}\int_0^r q^2\rho^3 dr \tag{4.26}$$

where A_0 and $\hat{A}_0$ are integration constants whose value could also differ on either side of the wall.

Matching conditions and allowed configurations

To impose matching conditions on the fields across the bubble wall, in GR one usually considers continuity conditions on the dynamical fields and the Israel junction conditions [106]. However, in sequestering not all fields are dynamical, representing the fact that the theory is locally equivalent to GR as discussed in the previous chapter, so here to find the relevant matching conditions we integrate the field equations given in (4.17) - (4.20) across the bubble wall. Equations (4.17) and (4.19) yield the conditions

$$\frac{1}{q_+}\sin q_+ r_0^+ = \frac{1}{q_-}\sin q_- r_0^- \tag{4.27}$$

$$A_0^+ = A_0^- \tag{4.28}$$

where the first of these is simply continuity in the radius of the 3-sphere $\rho(r)$ on either side of the wall. Equations (4.18) and (4.20) yield the following discontinuities supported by the delta function source

$$2\kappa^2 \frac{\Delta\rho'}{\rho_0} = -\sigma_w \tag{4.29}$$

$$\frac{\hat{\sigma}'}{M_{\rm pl}^2}\Delta\hat{A} = 3\rho_0^2\Delta\rho'. \tag{4.30}$$

Here and throughout the remainder of this section we have used the notation $\rho_0 = \rho(0)$ and $\Delta Q = Q(0^+) - Q(0^-)$ is the jump in a quantity Q across the wall. The second of these discontinuities is equivalent to a jump in the hatted 3-form given by

$$\hat{A}_0^+ - \hat{A}_0^- = -\frac{3}{2}\frac{M_{\rm pl}^2}{\kappa^2\hat{\sigma}'}\rho_0^3\sigma_w. \tag{4.31}$$

Now, equation (4.29) tells us that not all configurations can be realised with a non-negative tension wall ($\sigma_w \geq 0$). Instead we have to satisfy the condition

$$\Delta\rho' = \Delta(\epsilon\cos q r_0) \leq 0 \tag{4.32}$$

if the bubble wall is supported by realistic matter. Unsurprisingly, this is precisely the constraint one finds in GR. To extract useful information from this condition we also define $\langle Q\rangle = \frac{Q_+ + Q_-}{2}$ for some Q such that the condition

(4.32) becomes

$$\Delta\epsilon \cos\langle qr_0\rangle \cos\frac{\Delta(qr_0)}{2} - 2\langle\epsilon\rangle \sin\langle qr_0\rangle \sin\frac{\Delta(qr_0)}{2} \le 0. \tag{4.33}$$

From this we can deduce the conditions upon which tunnelling can happen. Let us focus on two examples. Initially consider tunnelling to and from regions with $q^2 > 0$ i.e. to and from de Sitter space in Lorentzian signature. To remain within the validity of Coleman's co-ordinate system, i.e. to avoid co-ordinate singularities, the solution for $\rho(r)$ given by equation (4.24) at $r = 0$ tells us that we have $qr_0 \in [0, \pi]$ from which we can infer

$$\langle qr_0\rangle \in [0, \pi], \qquad \frac{|\Delta(qr_0)|}{2} \in [0, \pi/2]. \tag{4.34}$$

In every case one of either $\langle\epsilon\rangle$ or $\Delta\epsilon$ will vanish and the other will be non-zero. First consider the case where $\langle\epsilon\rangle \neq 0$ such that we have $\epsilon_+ = \epsilon_- = \pm 1$ with the condition (4.32) reducing to

$$\langle\epsilon\rangle \sin\frac{\Delta(qr_0)}{2} \ge 0. \tag{4.35}$$

It follows that we require $\langle\epsilon\rangle$ and $\Delta(qr_0)$ to have the same sign, therefore for $\epsilon_+ = \epsilon_- = 1$ tunnelling is allowed if $(qr_0)_+ \ge (qr_0)_-$, and for $\epsilon_+ = \epsilon_- = -1$ tunnelling is allowed if $(qr_0)_+ \le (qr_0)_-$. Now consider the case where $\langle\epsilon\rangle = 0$ and $\Delta\epsilon \neq 0$ such that we have $\epsilon_+ = -\epsilon_-$ with the condition (4.32) reducing to

$$\Delta\epsilon \cos\langle qr_0\rangle \le 0. \tag{4.36}$$

It follows that for $\epsilon_+ = 1$, $\epsilon_- = -1$ tunnelling is allowed for $\langle qr_0\rangle \in [\pi/2, \pi]$ and for $\epsilon_+ = -1$, $\epsilon_- = 1$, tunnelling is allowed for $\langle qr_0\rangle \in [0, \pi/2]$.

Next consider tunnelling from a region with positive curvature $q_+^2 > 0$ to a region with negative curvature $q_-^2 < 0$. In Lorentzian signature this would represent tunnelling from de Sitter space to anti-de Sitter space. Here the condition (4.32) becomes

$$\epsilon_+ \cos(qr_0)_+ - \epsilon_- \cosh(|q|r_0)_- \le 0 \tag{4.37}$$

	$S_+ - S_-$	$S_+ - H_-$	$H_+ - S_-$	$H_+ - H_-$
$\epsilon_\pm = 1$	$(qr_0)_+ \geq (qr_0)_-$	allowed	not allowed	$\lvert q\rvert_+ \leq \lvert q\rvert_-$
$\epsilon_\pm = -1$	$(qr_0)_+ \leq (qr_0)_-$	not allowed	allowed	$\lvert q\rvert_+ \geq \lvert q\rvert_-$
$\epsilon_+ = 1, \epsilon_- = -1$	$\langle qr_0\rangle \in [\pi/2, \pi]$	not allowed	not allowed	not allowed
$\epsilon_+ = -1, \epsilon_- = 1$	$\langle qr_0\rangle \in [0, \pi/2]$	allowed	allowed	allowed

Table 4.1: Summary of allowed configurations after we demand the tension σ_w is non-negative. S denotes the sphere, H the hyperboloid, and planar limits can be extracted from the table by taking $q_+ \to 0$ or $q_- \to 0$.

which is satisfied for any $(qr_0)_+$, $(|q|r_0)_-$ and ϵ_+ as long as $\epsilon_- = 1$. We summarise these results in table 4.1 and also present the results considering the other possible configurations we have not calculated explicitly here. In all cases these are equivalent to what one finds in GR [103] since the extra sectors leading to the global sequestering constraints play no role.

Tunnelling rates

In the presence of gravity, and in the semi-classical limit, the probability of decay of the false vacuum per unit volume per unit time is given by [101–103]

$$\frac{\Gamma}{V} \sim \exp^{-B/\hbar} \tag{4.38}$$

where $B = S_{\mathrm{E}}^{\mathrm{bounce}} - S_{\mathrm{E}}^{\infty}$ is the difference between the on-shell Euclidean action calculated on the bounce solution and calculated on the initial false vacuum solution. Quantum corrections to this tunnelling rate are $\mathcal{O}(\hbar)$ and so are ignored in the semi-classical limit [101]. The bounce solutions are the Euclidean bubble configurations we calculated above and therefore have knowledge of the curvature in both the false and true vacuum either side of the wall. The initial vacuum solution is covered by the same co-ordinate range as the bounce solution but with no jump in curvature i.e. it is only dependent on the value of the curvature in the false vacuum. For the bounce solution the radial co-ordinate ranges from its minimum value in the interior r^-_{min} to its maximum value in the exterior r^+_{max}. These limits correspond to values where this choice of co-ordinate system breaks down and can be extracted from the $\rho(r)$ solution. Generically they depend on the choice of ϵ and whether we have $q^2 > 0$, $q^2 < 0$, or $q^2 \to 0$. First of all consider a vacuum with $q^2 > 0$ such that we have

$\rho(r) = \frac{1}{q}\sin q(\epsilon r + r_0)$. The limits of the co-ordinate system exist at $\rho(r) = 0$ which is a co-ordinate singularity and corresponds to solutions with

$$r = \epsilon\left(\frac{n\pi}{q} - r_0\right) \tag{4.39}$$

where $n \in \mathbb{R}$. So, when $\epsilon = 1$ we have $r_{\text{max}} = \frac{\pi}{q} - r_0$ and $r_{\text{min}} = -r_0$, and when $\epsilon = -1$ we have $r_{\text{max}} = r_0$ and $r_{\text{min}} = r_0 - \frac{\pi}{q}$. For a Minkowski vacuum we have $\rho(r) = \epsilon r + r_0$ and so the limits of the co-ordinate system exist at $\rho(r) = 0$ and $\rho(r) \to \infty$. Therefore if $\epsilon = 1$ we have $r_{\text{max}} = \infty$ and $r_{\text{min}} = -r_0$, and if $\epsilon = -1$ we have $r_{\text{max}} = r_0$ and $r_{\text{min}} = -\infty$. Finally, for a vacuum with $q^2 < 0$ we have $\rho(r) = \frac{1}{|q|}\sinh|q|(\epsilon r + r_0)$ leading to equivalent conclusions as for the Minkowski case. These results can be summarised as

$$r_{\text{min}} = \begin{cases} -r_0\,, & \epsilon = +1 \\ r_0 - \frac{\pi}{q}\,, & \epsilon = -1,\ q^2 > 0 \\ -\infty\,, & \epsilon = -1,\ q^2 \leq 0 \end{cases} \tag{4.40}$$

$$r_{\text{max}} = \begin{cases} \frac{\pi}{q} - r_0\,, & \epsilon = +1,\ q^2 > 0 \\ \infty\,, & \epsilon = +1,\ q^2 \leq 0 \\ r_0\,, & \epsilon = -1 \end{cases} \tag{4.41}$$

and are important for calculating the tunnelling rates.

On-shell, the sequestering Euclidean action is very similar to the GR one since the scalar fields are fixed to be constant. The only difference is given by the dependence on the 4-forms. Specifically, for sequestering we have

$$B = B_{\text{GR}} - \sigma\Delta c - \hat{\sigma}\Delta\hat{c} \tag{4.42}$$

where

$$\Delta c = \int_{\text{bounce}} F_4 - \int_{\text{initial vac}} F_4 \tag{4.43}$$

$$\Delta\hat{c} = \int_{\text{bounce}} \hat{F}_4 - \int_{\text{initial vac}} \hat{F}_4. \tag{4.44}$$

Let us now calculate each of the three contributions to (4.42) in turn. In all cases B is independent of $r_{\rm max}$ since it enters $S_{\rm E}^{\rm bounce}$ and $S_{\rm E}^{\infty}$ in exactly the same way and therefore cancels. Using the expression for the Ricci scalar on the $O(4)$ symmetric background geometry (4.21), the tunnelling exponent in GR is given by

$$B_{\rm GR} = \Omega_3\Delta\left[\int_{r_{\rm min}}^{0} dr[3\kappa^2(\rho+\rho\rho'^2) - 3\kappa^2 q^2\rho^3]\right] + \sigma_w\Omega_3\rho_0^3 \tag{4.45}$$

where Ω_3 is the volume of the unit 3-sphere and we explicitly see the boundary term contribution due to the non-negative tension wall. On-shell we have the following relationship by virtue of equation (4.22)

$$3\kappa^2(\rho+\rho\rho'^2) - 3\kappa^2 q^2\rho^3 = 6\kappa^2\rho\rho'^2 \tag{4.46}$$

which allows us to perform the integral very simply and write the GR exponent as

$$B_{\rm GR} = -2\kappa^2\Omega_3\Delta\left[\frac{1}{q^2}[\rho'^3]_{r_{\rm min}}^0\right] + \sigma_w\Omega_3\rho_0^3. \tag{4.47}$$

The contributions to the tunnelling rates from the flux terms can be calculated given the solutions for the 3-forms (4.25), and the matching condition for the hatted 3-form (4.31). We find

$$\Delta c = -\frac{\mu^4}{\sigma'}\Omega_3\Delta\left[\int_{r_{\rm min}}^{0} dr\rho^3\right] \tag{4.48}$$

$$\Delta\hat{c} = \frac{M_{\rm pl}^2}{2\hat{\sigma}'}\Omega_3\left(12\Delta\left[\int_{r_{\rm min}}^{0} dr q^2\rho^3\right] - 3\frac{\sigma_w}{\kappa^2}\rho_0^3\right). \tag{4.49}$$

Again we can simplify these expressions using the fact that on-shell we have

$$\int_{r_{\rm min}}^{0} dr\rho^3 = -\frac{1}{3q^4}[\rho'(3-\rho'^2)]_{r_{\rm min}}^0 \tag{4.50}$$

such that the flux contributions to the tunnelling exponent reduce to

$$\Delta c = \frac{\mu^4}{3\sigma'}\Omega_3\Delta\left[\frac{1}{q^4}[\rho'(3-\rho'^2)]_{r_{\rm min}}^0\right] \tag{4.51}$$

$$\Delta\hat{c} = -\frac{M_{\rm pl}^2}{2\hat{\sigma}'}\Omega_3\left(4\Delta\left[\frac{1}{q^2}[\rho'(3-\rho'^2)]_{r_{\rm min}}^0\right] + 3\frac{\sigma_w}{\kappa^2}\rho_0^3\right). \tag{4.52}$$

These two equations along with equation (4.47) gives us the full expression for the tunnelling exponent B in sequestering. We can see that divergences, which can lead to infinitely enhanced tunnelling rates with $B \to -\infty$ or infinitely suppressed tunnelling rates with $B \to \infty$ by equation (4.38), only occur when $r_{\rm min} = -\infty$. From equations (4.40) this is realised when $\epsilon = -1$ and $q^2 \leq 0$, and corresponds to either hyperbolic or planar geometries. Given the results summarised in table 4.1, the corresponding configurations with a non-negative tension wall have $\epsilon_+ = -1$ and $q_+^2 \leq 0$. We can see from the form of $B_{\rm GR}$ that in GR these tunnelling rates are infinitely suppressed since $B_{\rm GR} \to \infty$ and there is no possibility for infinitely enhanced rates. However, in sequestering we have the addition of the flux terms which can vastly alter this conclusion. To zoom in on the relevant terms we note that the divergent contributions to the full tunnelling exponent in sequestering goes as

$$\sim \frac{\Omega_3}{8}\left[\frac{2\kappa^2}{|q|_+^2}\left(1+\frac{M_{\rm pl}^2\hat{\sigma}}{\kappa^2\hat{\sigma}'}\right)+\frac{\mu^4}{3|q|_+^4}\frac{\sigma}{\sigma'}\right]e^{-3|q|_+(r^+_{\rm min}-r^+_0)} \tag{4.53}$$

which can diverge to $+\infty$ or $-\infty$ depending on the form of the sequestering functions σ and $\hat{\sigma}$. Having $B \to -\infty$ would lead to catastrophic vacuum decay with the semi-classical approximation becoming untrustworthy. To avoid this we now impose the following conditions on the sequestering functions

$$1+\frac{M_{\rm pl}^2\hat{\sigma}}{\kappa^2\hat{\sigma}'}>0, \qquad \frac{\mu^4\sigma}{\sigma'}>0 \tag{4.54}$$

which ensure that the only divergent behaviour is $B \to \infty$ as is in GR. When these conditions are satisfied, the spectrum of allowed configurations for tunnelling between vacua in sequestering are equivalent to those of GR. With this in mind we update table 4.1 to take this into account such that the spectrum of allowed configurations are now given in table 4.2.

We now consider the other possible tunnelling configurations with walls of non-negative tension but where the tunnelling exponent does not diverge. Again in each case the most important information is encoded in $\rho'(r_{\rm min})$ and $\rho'(0)$ on either side of the wall. As an example consider tunnelling between

	$\mathrm{S}_+ - \mathrm{S}_-$	$\mathrm{S}_+ - \mathrm{H}_-$	$\mathrm{H}_+ - \mathrm{S}_-$	$\mathrm{H}_+ - \mathrm{H}_-$
$\epsilon_\pm = 1$	$(qr_0)_+ \geq (qr_0)_-$	allowed	not allowed	$\lvert q\rvert_+ \leq \lvert q\rvert_-$
$\epsilon_\pm = -1$	$(qr_0)_+ \leq (qr_0)_-$	not allowed	$B \to \infty$	$B \to \infty$
$\epsilon_+ = 1, \epsilon_- = -1$	$\langle qr_0 \rangle \in [\pi/2, \pi]$	not allowed	not allowed	not allowed
$\epsilon_+ = -1, \epsilon_- = 1$	$\langle qr_0 \rangle \in [0, \pi/2]$	allowed	$B \to \infty$	$B \to \infty$

Table 4.2: Summary of allowed configurations after we demand the tension σ_w is non-negative and imposing the conditions (4.54) on the sequestering functions. Again, S denotes the sphere, H the hyperboloid, and planar limits can be extracted from the table by taking $q_+ \to 0$ or $q_- \to 0$.

two vacua each with $q^2 > 0$ such that on each side of the wall we have

$$\rho'(r) = \epsilon \cos q(\epsilon r + r_0). \tag{4.55}$$

Given the results summarised in (4.40), for $\epsilon = 1$ we have $\rho'(r_{\min}) = \cos q(-r_0 + r_0) = 1$, and for $\epsilon = -1$, we have $\rho'(r_{\min}) = -\cos q(-r_0 + \frac{\pi}{q} + r_0) = 1$. We also have $\rho'(0^\pm) = \epsilon_\pm \cos(qr_0)_\pm \geq -1$ which given the constraints imposed in table 4.2 leads to $\rho'(0^-) \geq \rho'(0^+)$. In fact, these conclusions are not unique to this example and for all of the remaining tunnelling configurations with a non-negative tension wall we have

$$\rho'(r_{\min}) = 1, \qquad -1 \leq \rho'(0^+) \leq \rho'(0^-). \tag{4.56}$$

In these cases we can write the tunnelling rates as a function of $\rho'(0)$ on either side of the wall and by explicit calculation equations (4.47) and (4.51) become

$$B_{\mathrm{GR}} = 2\Omega_3 \kappa^2 \rho_0^2 \Delta \left[\frac{1}{1+\rho'(0)} \right] \geq 0 \tag{4.57}$$

$$-\sigma \Delta c = \Omega_3 \frac{\mu^4 \rho_0^4}{3} \frac{\sigma}{\sigma'} \Delta \left[\frac{1}{1+\rho'(0)} + \left(\frac{1}{1+\rho'(0)} \right)^2 \right] \tag{4.58}$$

$$-\hat{\sigma} \Delta \hat{c} = -\Omega_3 M_{\mathrm{pl}}^2 \rho_0^2 \frac{\hat{\sigma}}{\hat{\sigma}'} \Delta \left[\rho'(0) + \frac{4}{1+\rho'(0)} \right] \tag{4.59}$$

where we have eliminated the dependence on σ_w by virtue of the condition (4.29), and the dependence on q^2 by the field equation (4.22). Again, unlike in GR where all possibilities have exponentially suppressed rates since $B_{\mathrm{GR}} > 0$ by equation (4.57), in sequestering the flux terms allow for unsuppressed rates since the overall sign of these contributions is not fixed. To ensure that all the remaining tunnelling rates are exponentially suppressed we therefore have to further reduce the allowed space of sequestering theories parametrised by the functions σ and $\hat{\sigma}$. Demanding that the sum of terms in (4.57) - (4.59) yields a positive tunnelling exponent, and also taking into the account the conditions imposed to avoid $B \to -\infty$, we must satisfy the following conditions

$$\frac{\kappa^2 \hat{\sigma}'}{M_{\text{pl}}^2 \hat{\sigma}} > 2, \qquad \frac{\sigma'}{\mu^4 \hat{\sigma}'} > 0. \tag{4.60}$$

When these conditions are satisfied, not only is the spectrum of allowed configurations in sequestering equivalent to GR as summarised in table 4.2, but all of the tunnelling rates in the sequestering model are exponentially suppressed. These conditions are therefore very important and in any future work in the context of the local sequestering theory they should always be satisfied to avoid instabilities with respect to quantum tunnelling.

Although the generic form of the tunnelling rates for a given process will be equivalent to GR, the exact rates are still expected to differ due to the presence of the flux contributions. To see this let us consider two special cases as was done for GR in [103]. Initially consider decay from a false vacuum with $q^2 > 0$ into a vacuum with $q^2 \to 0$. In this case we have $\rho'(0^-) = 1$ and $\rho'(0^+) \in [-1, -1]$ and the tunnelling exponent is given by

$$B = B_{\text{GR}} \left[1 + \frac{\mu^4}{12 q^2 \kappa^2} \frac{\sigma}{\sigma'} s(8 - 3s) - \frac{M_{\text{pl}}^2}{\kappa^2} \frac{\hat{\sigma}}{\hat{\sigma}'} s \right] \tag{4.61}$$

where $B_{\text{GR}} = \Omega_3 \frac{\kappa^2}{q^2} s^2$ and

$$s = 1 - \rho'(0^+) = \frac{\sigma_w^2}{2\kappa^4 q^2} \left(\frac{1}{1 + \sigma_w^2 / 4\kappa^2 q^2} \right). \tag{4.62}$$

Even with the conditions (4.60) satisfied, this tunnelling rate can be either further suppressed or enhanced by the presence of the flux terms. In comparison to GR, the dominant processes have $0 \leq s \ll 1$ in which case the exponent reduces to

$$B \approx B_{\text{GR}} \left[1 + \frac{2\mu^4}{3 q^2 \kappa^2} \frac{\sigma}{\sigma'} s - \frac{M_{\text{pl}}^2}{\kappa^2} \frac{\hat{\sigma}}{\hat{\sigma}'} s \right]. \tag{4.63}$$

The relative minus sign between the contributions from the hatted and un-hatted fluxes is very important. We see that for a large jump in curvature due to the phase transition, the hatted fluxes dominate over the un-hatted ones and therefore enhance the tunnelling rate to a vacuum with zero curvature relative to GR i.e. make tunnelling into Minkowski space easier. However, when the jump in vacuum energy induced by the transition is small, the un-hatted fluxes win out and the tunnelling rates are further suppressed in sequestering relative to GR making it more difficult to tunnel.

Next we consider decay from a vacuum with zero curvature to one with negative curvature i.e. $0 \to -|q|^2$. In this case we have $\rho'(0^+) = 1$ and $\rho'(0^-) \geq 1$ and the tunnelling exponent is given by

$$B = B_{\rm GR}\left[1 - \frac{\mu^4}{12|q|^2\kappa^2}\frac{\sigma}{\sigma'}s(8-3s) - \frac{M_{\rm pl}^2}{\kappa^2}\frac{\hat{\sigma}}{\hat{\sigma}'}s\right] \tag{4.64}$$

where here we have $B_{\rm GR} = \Omega_3 \frac{\kappa^2}{|q|^2}s^2$ and

$$s = 1 - \rho'(0^-) = -\frac{\sigma_w^2}{2\kappa^4|q|^2}\left(\frac{1}{1-\sigma_w^2/4\kappa^2|q|^2}\right). \tag{4.65}$$

Again we work in the limit with $0 \leq s \ll 1$ such that we have

$$B \approx B_{\rm GR}\left[1 - \frac{2\mu^4}{3|q|^2\kappa^2}\frac{\sigma}{\sigma'}s - \frac{M_{\rm pl}^2}{\kappa^2}\frac{\hat{\sigma}}{\hat{\sigma}'}s\right]. \tag{4.66}$$

As shown in [103], and more recently beyond the thin wall limit in [107], there are no sensible solutions with $|q|^2 < \sigma_w^2/4\kappa^2$ meaning that we have $s < 0$. The result of this is that when the conditions (4.60) are satisfied, the effect of the flux terms is to further stabilise the Minkowski false vacuum relative to GR.

Before moving on to calculating the vacuum curvature in each vacua, let us briefly conclude this section. Here we have seen that if we impose some mild conditions on the sequestering functions σ and $\hat{\sigma}$ the tunnelling rates in sequestering are very similar to GR without any unsuppressed rates and avoiding all possible rapid instabilities. The theory allows for tunnelling between a pair of vacua with positive curvature in either direction, yet tunnelling upwards is heavily suppressed as in GR. Tunnelling can proceed from a region of positive curvature to one with vanishing or negative curvature but the opposite transition is infinitely suppressed. More specifically, by studying two illuminating examples, we have seen that sequestering makes a near Minkowski vacuum more favourable compared to GR.

4.2.2 Growth of the bubble

So far we have described the materialisation of the bubble of true vacuum. We now can study its evolution by Wick rotating back to Lorentzian signature where in the neighbourhood of the wall the geometry is described by the metric [103]

$$ds^2 = dr^2 + \rho(r)^2(-d\tau^2 + \cosh^2\tau d\Omega_2^2) \tag{4.67}$$

where

$$\rho(r) = \begin{cases} \frac{1}{q_+}\sin q_+(\epsilon_+ r + r_0^+)\,, & r > 0 \\ \frac{1}{q_-}\sin q_-(\epsilon_- r + r_0^-)\,, & r < 0 \end{cases} \tag{4.68}$$

with the wall still located at $r = 0$. As we mentioned above, now the interior or exterior of the bubble corresponds to either de Sitter space with $q^2 > 0$, Minkowski space with $q^2 \to 0$ or anti-de Sitter space with $q^2 < 0$.

As described in [103], in this co-ordinate system we can see that $\tau = 0$ is a special point. Specifically, it is a minimal space-like surface with vanishing extrinsic curvature. It is a stationary point in the geometry where we can consistently perform a Wick rotation into Euclidean signature where we calculated the bounce solutions. So from now on we shall refer to $\tau = 0$ as the nucleation time. For $\tau < 0$ there is no wall and the full geometry lives the initial vacuum, but for $\tau > 0$ the bubble has nucleated and we have a space-time with two regions separated by the bubble wall.

We are now in a position to calculate the vacuum energy contributions in each vacua. This will enable us to see the effects of the phase transition both before and after the bubble has nucleated i.e. in both the true and false vacuum. These are controlled by the following pair of integrated fluxes

$$c = \int F_4 = \frac{\mu^4}{\sigma'} \int d^4x\sqrt{-g} \tag{4.69}$$

$$\hat{c} = \int \hat{F}_4 = -\frac{M_{\rm pl}^2}{2\hat{\sigma}'} \int d^4x\sqrt{-g}R \tag{4.70}$$

since these equations lead to the crucial global constraint on the space-time average of the Ricci scalar $\langle R \rangle$. For all of the possible solutions we have considered the full space-time volume is split into three components. There is the volume prior to bubble nucleation which we denote as $\mathcal{V}_{\rm b}^+$, the volume of the exterior after bubble nucleation denoted as $\mathcal{V}_{\rm a}^+$ and the volume in the interior after bubble nucleation denoted as $\mathcal{V}_{\rm a}^-$. In the following we shall ignore the bubble wall since its contribution will always be negligible. We therefore have $\mathcal{V}_{\rm total} = \mathcal{V}_{\rm b}^+ + \mathcal{V}_{\rm a}^+ + \mathcal{V}_{\rm a}^-$. With this notation the integrated fluxes are simply given by

$$c = \frac{\mu^4}{\sigma'}(\mathcal{V}_{\rm b}^+ + \mathcal{V}_{\rm a}^+ + \mathcal{V}_{\rm a}^-) \tag{4.71}$$

$$\hat{c} = -\frac{6M_{\rm pl}^2}{\hat{\sigma}'}[(q^2\mathcal{V}_{\rm b})^+ + (q^2\mathcal{V}_{\rm a})^+ + (q^2\mathcal{V}_{\rm a})^-]. \tag{4.72}$$

Writing the ratio between the space-time volume of either the exterior or interior of the bubble and the space-time volume before bubble nucleation as

$$R^{\pm} = \frac{\mathcal{V}_{\rm a}^{\pm}}{\mathcal{V}_{\rm b}^+} \tag{4.73}$$

we can express the fluxes as

$$\frac{c}{\mathcal{V}_{\rm b}^+} = \frac{\mu^4}{\sigma'}(1 + R^+ + R^-) \tag{4.74}$$

$$\frac{\hat{c}}{\mathcal{V}_{\rm b}^+} = -\frac{6M_{\rm pl}^2}{\hat{\sigma}'}[q_+^2(1 + R^+) + q_-^2 R^-]. \tag{4.75}$$

We are now in a position to see how the curvature in each vacua depends on the three individual space-time volumes. We again make use of the notation we have throughout by defining

$$\langle q^2 \rangle = \frac{q_+^2 + q_-^2}{2}, \qquad \Delta q^2 = q_+^2 - q_-^2 \tag{4.76}$$

such that we can write equation (4.75) as

$$\langle q^2 \rangle = \frac{-1}{1 + R_+ + R_-} \left[\frac{\hat{\sigma}' \hat{c}}{6M_{\rm pl}^2 \mathcal{V}_{\rm b}^+} + \frac{\Delta q^2}{2}(1 + R_+ - R_-) \right] \tag{4.77}$$

or equivalently after using equation (4.74) we find

$$\langle q^2 \rangle = -\frac{\mu^4 \hat{\sigma}'}{6M_{\rm pl}^2 \sigma'} \frac{\hat{c}}{c} - \frac{\Delta q^2}{2} \frac{1 + R_+ - R_-}{1 + R_+ + R_-}. \tag{4.78}$$

In terms of the two individual cosmological constants we can then write

$$q_+^2 = -\frac{\mu^4 \hat{\sigma}'}{6M_{\rm pl}^2 \sigma'} \frac{\hat{c}}{c} + \frac{\Delta q^2}{1 + \mathcal{I}} \tag{4.79}$$

$$q_-^2 = -\frac{\mu^4 \hat{\sigma}'}{6M_{\rm pl}^2 \sigma'} \frac{\hat{c}}{c} - \frac{\Delta q^2}{1 + \mathcal{I}^{-1}} \tag{4.80}$$

where we have introduced the ratio

$$\mathcal{I} = \frac{1 + R_+}{R_-} = \frac{\mathcal{V}_{\rm b}^+ + \mathcal{V}_{\rm a}^+}{\mathcal{V}_{\rm a}^-}. \tag{4.81}$$

We therefore infer from equations (4.79) and (4.80) that if $\mathcal{I} \gg 1$, or in other words if the space-time volume in the false vacuum dominates over the volume in the true vacuum, the false vacuum is insensitive to the jump in the vacuum energy induced by the phase transition while the true vacuum is indeed sensitive. However, if $\mathcal{I} \ll 1$ such that the true vacuum is the dominate source of the total volume, it is the true vacuum which is insensitive to the jump. Therefore our understanding of the effects of phase transitions requires us to compute $\mathcal{I}$ for each of the possible transitions described in the previous section. We note that this is qualitatively similar to what we found in section 4.1 where for an early universe phase transition, which here would correspond to the false vacuum having a sub dominant volume, it is the space-time after the transition which is insensitive to the jump.

As we have seen here, the dynamics in sequestering is sensitive to the full space-time and to calculate the ratio $\mathcal{I}$, Coleman's co-ordinates which we have employed so far are not adequate since they do not cover the full space-times in each case. Instead we must use global co-ordinates. Full details of the global co-ordinates and their mapping to Coleman's can be found in the appendix in chapter 8, but let us summarise here (see e.g. [108]). De Sitter space in global co-ordinates is described by

$$ds^2 = -dt^2 + \frac{\cosh^2 qt}{q^2}(d\theta^2 + \sin^2\theta d\Omega_2^2) \tag{4.82}$$

where $t \in (-\infty, \infty)$ and $\theta \in [0, \pi]$. Tunnelling between vacua can occur at any of the minimal spacelike surfaces

$$\cos\theta = \frac{\tanh qt}{\tanh\alpha} \tag{4.83}$$

where α is a constant. As we show and discuss in the appendix, this surface locally maps into the $\tau = 0$ surface in Coleman's co-ordinate patch. The α dependence arises since we make use of the de Sitter symmetries and perform a Lorentz transformation prior to defining the mapping, with α corresponding to the rapidity. For Minkowski space, the globally defined metric is given by

$$ds^2 = -dt^2 + du^2 + u^2 d\Omega_2^2 \tag{4.84}$$

where $t \in (-\infty, \infty)$ and $u \in [0, \infty)$. In this case the minimal spacelike surfaces where the bubble nucleates occur at $t = t_0 =$ constant. Again, we show in the appendix that one can locally map these surfaces to the tunnelling surface $\tau = 0$ in Coleman's coordinates. Finally, for anti-de Sitter space in global coordinates we have

$$ds^2 = -\frac{\cosh^2|q|u}{|q|^2}dt^2 + du^2 + \frac{\sinh^2|q|u}{|q|^2}d\Omega_2^2 \tag{4.85}$$

where $t, u \in (-\infty, \infty)$. As was the case in Minkowski space, the minimal spacelike surfaces that map to $\tau = 0$ correspond to $t = t_0 =$ constant. For each case we can also calculate the surface where the bubble wall exists. In

Coleman's co-ordinates this was given by $r = 0$ and in global co-ordinates we have

$$
\begin{aligned}
\text{dS} &: \quad \cos qr_0 = \cosh\alpha\cosh qt\cos\theta - \sinh\alpha\sinh qt \\
\text{flat} &: \quad r_0^2 = u^2 - (t-t_0)^2 \\
\text{AdS} &: \quad \cosh|q|r_0 = \cosh|q|u\cos(t-t_0)
\end{aligned}
$$

which again is shown explicitly in the appendix. The expressions shown here in global co-ordinates for both the nucleation time and the position of the wall are important when calculating the volume ratios since they form the boundaries of the three volumes. There is a further complication with an anti-de Sitter interior since as shown in [103, 109] it suffers from a curvature singularity. As we show in the appendix, in global co-ordinates this surface is

$$\cosh|q|u\cos(t-t_0) = -1 \tag{4.86}$$

with the relevant mapping to Colemans's co-ordinates also given in the appendix. We are now in a position to calculate the volume ratios. Even though the individual volumes will have divergences, the ratios can be computed reliably and to do so we use a time reversal symmetric cut-off. This ensures that in the absence of the bubble in each case the regulators respect the space-time symmetries. The explicit calculations are presented in the appendix, here we summarise the results as

$$\mathcal{I}_{\text{dS}\to\text{dS}} \sim \frac{q_-}{q_+} \tag{4.87}$$

$$\mathcal{I}_{\text{dS}\to\text{M}} = 0 \tag{4.88}$$

$$\mathcal{I}_{\text{dS}\to\text{AdS}} = \infty \tag{4.89}$$

$$\mathcal{I}_{\text{M}\to\text{AdS}} = \infty \tag{4.90}$$

$$\mathcal{I}_{\text{AdS}\to\text{AdS}} = \infty \tag{4.91}$$

where $\mathcal{I}_{\text{X}\to\text{Y}}$ denotes tunnelling from X to Y, where X, Y are dS (de Sitter), M (Minkowski) and AdS (anti-de Sitter).

First consider the case which is of most phenomenological interest. From equation (4.87) we see that if we tunnel from a region of high de Sitter curvature to a region of low de Sitter curvature such that $q_- < q_+$, we have $\mathcal{I} < 1$ meaning that the dependence on the jump in the vacuum energy due to the phase transition is suppressed in the true vacuum. It only has a non-trivial effect in the false vacuum. This is telling us that the sequestering mechanism is most efficient after an early universe phase transition. Again this is due to the large volume suppression which occurs in the true vacuum. Intuitively the reason a de Sitter space-time has a larger volume the smaller its curvature is because it is causally connected to a larger region of space-time. One may have expected that the residual cosmological constant i.e. the contribution to $q_\pm^2$ which is independent of Δq^2, would have to be tuned against the jump in vacuum energy induced by the phase transition in order to realise a small de Sitter curvature as dictated by observations. This is not the case in sequestering. The residual cosmological constant can be set to be small to match observations and we do not have to pick another value for it after the field theory experiences a phase transition. As we see from equation (4.88), this conclusion extends to the case where we take $q_- \to 0$, in other words when the true vacuum is Minkowski space. In contrast, there is a discontinuity in the volume ratios as shown in equation (4.89). Here we see that regardless of the size of the curvature in each region, it is the de Sitter exterior which dominates the total volume. This means that for tunnelling from a region of large de Sitter curvature to a region of small absolute anti-de Sitter curvature, it is the false vacuum which is insensitive to the jump. The reason this happens is that the gravitational collapse in the anti-de Sitter interior redcues the number of divergent directions relative to the de Sitter exterior.

Now consider equation (4.90) which tells us that if the false vacuum is Minkowski space then with respect to tunnelling to anti-de Sitter space it is the false vacuum which is insensitive to the phase transition. Again, the volume suppression in the exterior arises from the fact that in an anti-de Sitter bubble the space-time is cut-off due to a curvature singularity. Finally, since we cannot tunnel up from an anti-de Sitter true vacuum to an anti-de Sitter false vacuum, we also see from equation (4.91) that again it is the region of lowest absolute curvature where the volume suppression makes the vacuum energy insensitive to the jump.

Therefore, barring one exception, it is the vacuum with the lowest absolute curvature which sequestering makes insensitive to the phase transition without any tuning of the residual cosmological constant.

4.3 Discussion

In this chapter we have considered the effects of an early universe phase transition on the dynamics in the local sequestering theory. Importantly, we have found that the interior of a bubble with small absolute curvature relative to the absolute curvature in the exterior is insensitive to the jump in the vacuum energy induced by the transition. So the local sequestering model allows one to avoid the need to repeatedly fine tune the cosmological constant as one calculates matter loop corrections to it, as we discussed in chapter 3, but also one does not need to worry about standard model phase transitions since, compatible with observations, they have very little effect on the late time curvature we observe. This is in contrast to GR, where to guarantee a small de Sitter curvature one must tune the bare cosmological constant against the energy scale of the phase transition. Here we have considered bubble nucleation when drawing these conclusions and to avoid catastrophic instabilities, and more generally tunnelling rates which are exponentially favoured, we have had to place mild constraints on the sequestering parameter space which should be satisfied in all future considerations.

In some sense the phase transitions we have considered can be thought of as local sources and we may therefore expect their gravitational effects to be equivalent in both sequestering and GR since the two theories have locally equivalent dynamics. However, here we asked how the phase transitions contribute to the cosmological constant which required us to consider the full space-time geometries. As such the global dynamics of sequestering plays an important role and since its global structure differs to that of GR, the gravitational effects of the phase transitions also differs.

Finally, let us remind the reader that these results are due to the global nature of the sequestering equations of motion and the volume suppression that comes with this. We realised this global modification of GR without introducing any new local degrees of freedom. This meant that we did not require any form of screening mechanism to hide unwanted forces in local environments. For other modifications of gravity this is not the case and we will discuss two of the possible screening mechanisms these theories make use of in the remainder of this thesis.

Chapter 5

Unitarity and the Vainshtein Mechanism

This chapter is devoted to a detailed study of the Vainshtein mechanism with particular attention paid to the regime of validity of theories which incorporate the Vainshtein mechanism. As we briefly discussed in the introduction, the Vainshtein mechanism is a way for long distance modifications of gravity to conform with local gravitational tests, while deviating from General Relativity (GR) on Hubble scales. It is one of a number of possible *screening mechanisms*. We will begin this chapter with an introduction to the Vainshtein mechanism; explaining how it successfully shuts down unwanted forces in local environments and how it was born out of attempts to give the graviton a mass. We will discuss the main downfall of modifications of gravity which make use of the Vainshtein mechanism, namely, their very low strong coupling scales which ensure that predictions cannot be made on the scales at the forefront of gravitational tests. In doing so we will discuss suggestions as to how theories with a Vainshtein mechanism can be trusted beyond these very low energy scales to a higher, environmentally dependent strong coupling scale [40]. The remainder of this chapter contains a critical assessment of the validity of this *environmental strong coupling scale* where we study the low energy dynamics of UV complete field theories as avatars for long distance modifications of gravity, following the work of [3]. At various steps along the way we will compare and contrast with GR and before concluding this chapter with a discussion of our results, we will compliment our field theory examples by studying higher order corrections to GR in D dimensions.

5.1 Introduction to the Vainshtein mechanism

For a complete discussion of the Vainshtein mechanism we must first discuss the challenges faced when trying to construct a consistent theory of interacting *massive* spin-2 particles. We follow the treatment of Hinterbichler in [58] and refer the reader there and to the reviews [28, 29, 39] for more details.

5.1.1 Linear massive spin-2

We begin at the linearised level, where the most general Lorentz invariant theory of a massive spin-2 particle propagating on a flat background with Einstein-Hilbert kinetic terms is

$$S = \int d^4x \left[\mathcal{L}_{GR} - \frac{m^2}{2}(h_{\mu\nu}h^{\mu\nu} + ah^2) + \frac{h_{\mu\nu}T^{\mu\nu}}{M_{\rm pl}} \right] \tag{5.1}$$

where $\mathcal{L}_{GR}$ is the linearised kinetic structure of the Einstein-Hilbert action given by

$$\mathcal{L}_{GR} = -\frac{1}{2}\partial_\sigma h_{\mu\nu}\partial^\sigma h^{\mu\nu} + \partial_\mu h_{\nu\sigma}\partial^\nu h^{\mu\sigma} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\sigma h \partial^\sigma h \tag{5.2}$$

and m is the mass of the graviton. We are raising and lowering indices with the Minkowski metric $\eta_{\mu\nu}$ and $h = \eta^{\mu\nu}h_{\mu\nu}$ is the trace of the graviton fluctuation. The mass term breaks the linearised gauge invariance of GR (1.12) indicating that (5.1) describes more degrees of freedom than the two helicity modes of a massless spin-2 field. Indeed, we expect a massive spin-2 particle to have five degrees of freedom and in 1939 Fierz and Pauli showed that this action only describes the correct number of polarisations if we impose the *Fierz-Pauli tuning* of $a = -1$ [110]. For any other choice of a the theory will inherit a massive scalar ghost in addition to the massive graviton with a mass

$$m^2_{\rm ghost} = -\frac{4a+1}{2(a+1)}m^2. \tag{5.3}$$

Clearly the mass of the ghost becomes infinite in the limit of the Fierz-Pauli tuning such that it is non-dynamical at all energies and can be ignored.

In 1970, van Dam, Veltman and Zakharov showed that in the presence of a massive source like the Sun, the predictions of the linear theory of massive gravity differed to those of linearised GR even in the limit where $m \to 0$ [111, 112]. For example, when the mass of the graviton is asymptotically small, the prediction for light bending by the Sun is 25% different to the GR prediction. Predictions related to the perihelion precession of Mercury also differed. This is known as the *vDVZ discontinuity* and its origin can be traced to the presence of a scalar degree of freedom which does not decouple from the trace of the energy-momentum tensor even in the limit of vanishing mass. This scalar degree of freedom is simply the longitudinal mode of the massive graviton.

To see this explicitly we must first use the Stükelberg trick to restore the gauge symmetry broken by the graviton mass term. The Stükelberg trick is a useful way of making the degrees of freedom of a theory manifest by introducing new fields and redundancies while maintaining the spectrum of the original theory. By restoring the gauge symmetry we ensure that no degrees of freedom are lost in the $m \to 0$ limit which would otherwise be discontinuous. This trick was first utilised in massive gravity in [113]. To restore the linear gauge symmetry of GR (1.12) we make the following replacement in the action (5.1)

$$h_{\mu\nu} \to h_{\mu\nu} + 2\partial_{(\mu}A_{\nu)} + 2\partial_\mu\partial_\nu\phi \tag{5.4}$$

where A_μ is a vector field and ϕ is a scalar field. This replacement has the same structure as the linearised gauge symmetry we are aiming to restore and therefore $\mathcal{L}_{GR}$ is invariant. The matter coupling is also invariant after integrating by parts and assuming that the energy-momentum tensor is conserved $\partial_\mu T^{\mu\nu} = 0$. The mass term does transform yielding

$$\begin{aligned} S = \int d^4x \Big[&\mathcal{L}_{GR} - \frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) - \frac{1}{2}m^2 F_{\mu\nu}F^{\mu\nu} \\ &- 2m^2(h_{\mu\nu}\partial^\mu A^\nu - h\partial_\mu A^\mu) - 2m^2(h_{\mu\nu}\partial^\mu\partial^\nu\phi - h\partial^2\phi) + \frac{h_{\mu\nu}T^{\mu\nu}}{M_{\rm pl}} \Big] \end{aligned} \tag{5.5}$$

where $F_{\mu\nu} = \partial_\mu A_\nu + \partial_\nu A_\mu$ and we have set $a = -1$ to avoid the scalar ghost. This action is invariant under the two gauge transformations

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \qquad A_\mu \to A_\mu - \xi_\mu \tag{5.6}$$

$$A_\mu \to A_\mu + \partial_\mu \Lambda, \qquad \phi \to \phi - \Lambda \tag{5.7}$$

and the linearised gauge symmetry of GR has been restored by virtue of (5.6).

Having restored the gauge redundancy we can readily take the massless limit by first making the rescalings $A_\mu \to A_\mu/m$ and $\phi \to \phi/m^2$ followed by $m \to 0$. The resulting action is

$$S = \int d^4x \left[\mathcal{L}_{GR} - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} - 2h_{\mu\nu} \partial^\mu \partial^\nu \phi + 2h\partial^2\phi + \frac{h_{\mu\nu} T^{\mu\nu}}{M_{\text{pl}}} \right]. \tag{5.8}$$

However, at this stage the relevant couplings are not completely transparent because of the kinetic mixing between $h_{\mu\nu}$ and ϕ so we diagonalise by virtue of the field re-definition $h_{\mu\nu} = \tilde{h}_{\mu\nu} + \eta_{\mu\nu}\phi$ yielding

$$S = \int d^4x \left[\tilde{\mathcal{L}}_{GR} - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (\partial\phi)^2 + \frac{\tilde{h}_{\mu\nu} T^{\mu\nu}}{M_{pl}} + \frac{\phi T}{\sqrt{6} M_{\text{pl}}} \right] \tag{5.9}$$

where $\tilde{\mathcal{L}}_{GR}$ is the linearised kinetic structure of GR for the metric fluctuation $\tilde{h}_{\mu\nu}$ and we have canonically normalised ϕ. The five degrees of freedom of the massive graviton are now manifest even in the limit of vanishing mass. We have the two helicity degrees of freedom of a massless graviton which couples to the energy-momentum tensor as in GR, the two helicity degrees of freedom of a massless vector field which has decoupled from the energy-momentum tensor, and the single degree of freedom of a massless scalar field which, crucially, couples to the trace of the energy-momentum tensor with a coupling only a factor of $\sqrt{6}$ weaker than gravitational strength. This coupling is the origin of the vDVZ discontinuity and explains why the massless limit of a linearised massive graviton results in different predictions compared to linearised GR.

To see this more clearly take a static point source of mass M such that $T = -M\delta^3(\vec{x})$, and assume a static and spherically symmetric solution for longitudinal mode $\phi = \phi(r)$, then we have (e.g. [29])

$$\phi(r) = -\frac{M}{\sqrt{96\pi^2}M_{\rm pl}}\frac{1}{r}. \tag{5.10}$$

This solution has exactly the same form as the Newtonian potential and therefore the ratio between the scalar force due to the longitudinal mode and the Newtonian one is

$$\frac{F_\phi}{F_N} \sim \mathcal{O}(1) \tag{5.11}$$

as we discussed in the introduction.

The vDVZ discontinuity is actually unique for massive spin-2 particles being absent for spin-0 and spin-1. Its absence for spin-0 is obvious given that there is no discontinuity in the number of degrees of freedom for a massless and a massive scalar field, but it is less trivial for the spin-1 case as we shall now show.

Take the theory for a massive abelian vector field A_μ coupled to a source J^μ

$$S = \int d^4x \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu + A_\mu J^\mu\right] \tag{5.12}$$

which describes the correct number of three degrees of freedom. Two of these are the transverse modes of A_μ and the other is the scalar longitudinal mode. The gauge symmetry associated with the massless spin-1 field is

$$A_\mu \to A_\mu + \partial_\mu \Lambda \tag{5.13}$$

which is broken only by the mass term if we assume that the source is conserved $\partial_\mu J^\mu = 0$. We now restore this symmetry using the Stükelberg trick as we did for the spin-2 case by making the following replacement in (5.12)

$$A_\mu \to A_\mu + \partial_\mu \phi \tag{5.14}$$

where again ϕ is a scalar field. The resulting action after canonically normalising ϕ is

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - \frac{1}{2} m^2 A_\mu A^\mu - \frac{1}{2}(\partial\phi)^2 - m\partial_\mu \phi A^\mu + A_\mu J^\mu \right] \quad (5.15)$$

where we have integrated by parts and used the fact that the source is conserved. This action is now invariant under the gauge symmetry

$$A_\mu \to A_\mu + \partial_\mu \Lambda, \quad \phi \to \phi - m\Lambda \quad (5.16)$$

and we can readily take the $m \to 0$ limit yielding

$$S = \int d^4x \left[-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\partial\phi)^2 + A_\mu J^\mu \right]. \quad (5.17)$$

The three degrees of freedom of the massive spin-1 field are manifest and split into the two helicity degrees of freedom of a massless vector and the single degree of freedom of a massless scalar, but in contrast to the spin-2 case, the scalar degree of freedom has completely decoupled and there is no vDVZ discontinuity.

5.1.2 Non-linear massive spin-2 part I

In any case, the linearised action is not the full story and the effective field theory (EFT) of a massive graviton will contain non-linear corrections to (5.1). These interactions can correct both the kinetic structure as is the case in GR, and the non-derivative sector by virtue of a more general potential in addition to the Fierz-Pauli mass term. We shall continue to assume an Einstein-Hilbert structure in the kinetic sector such that the non-linear completion there is given by the Ricci scalar (see e.g. [114, 115] for a discussion on other possibilities), if we restrict ourselves to two derivatives. There are also infinitely many non-derivative operators we can write down. Our only requirement of the non-linear theory at this stage is that it reduces to (5.1) in the linearised limit.

For the moment we will ignore higher order potential terms and concentrate on non-linear interactions in the kinetic sector such that our non-linear theory is GR with the Fierz-Pauli mass term and is given by

$$S = \frac{M_{\rm pl}^2}{2} \int d^4x \sqrt{-g} \left[R - \frac{m^2}{4} g^{\mu\alpha} g^{\nu\beta} (h_{\mu\nu} h_{\alpha\beta} - h_{\mu\alpha} h_{\nu\beta}) \right] + S_m(g_{\mu\nu}, \Phi) \quad (5.18)$$

where $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. In the presence of a source of mass M and with a spherically symmetrical geometry, Vainshtein found that the non-linearities of (5.18) became important at a scale [38]

$$r_V \sim \left(\frac{M}{m^4 M_{\rm pl}^2} \right)^{1/5} \quad (5.19)$$

which we refer to as the *Vainshtein radius*. At distance scales below r_V, the linearised truncation (5.1) is no longer valid and we must consider the full non-linear structure to make predictions. Vainshtein's primary observation was that in the limit $m \to 0$ we have $r_V \to \infty$ such that we can *never* trust the linearised approximation for asymptotically small graviton masses. In other words, the calculation which leads to the vDVZ discontinuity is outside the regime of validity of the linearised theory and we should therefore revisit the issue in a full non-linear completion of massive gravity.

To understand the origin of the Vainshtein radius let us again restore the gauge invariance broken by the mass term to make the degrees of freedom and their interactions manifest. However, here the gauge invariance we wish to restore is the full non-linear diffeomorphism invariance of GR which is a metric transformation of the form

$$g_{\mu\nu}(x) \to \frac{\partial f^\alpha}{\partial x^\mu} \frac{\partial f^\beta}{\partial x^\nu} g_{\alpha\beta}(f(x)) \quad (5.20)$$

where $f(x)$ are arbitrary functions. We refer the reader to [58,113] for a detailed discussion as to how this redundancy is restored, here we merely state that we must make the following replacement in the action (5.18)

$$\begin{aligned} h_{\mu\nu} \to H_{\mu\nu} = h_{\mu\nu} + 2\partial_{(\mu} A_{\nu)} + 2\partial_\mu \partial_\nu \phi - \partial_\mu A^\alpha \partial_\nu A_\alpha \\ - \partial_\mu A^\alpha \partial_\nu \partial_\alpha \phi - \partial_\mu \partial^\alpha \phi \partial_\nu A_\alpha - \partial_\mu \partial^\alpha \phi \partial_\nu \partial_\alpha \phi. \quad (5.21) \end{aligned}$$

This leads to a bunch of interaction terms involving $\hat{h}_{\mu\nu}$, $\hat{A}_\mu$ and $\hat{\phi}$ which are canonically normalised fields related to the fields in (5.21) by

$$\hat{h}_{\mu\nu} = \frac{M_{\rm pl}}{2} h_{\mu\nu}, \quad \hat{A}_\mu = \frac{m M_{\rm pl}}{2} A_\mu, \quad \hat{\phi} = \frac{m^2 M_{\rm pl}}{2\sqrt{6}} \phi. \tag{5.22}$$

The generic form of these interactions is schematically [113]

$$\Lambda_{(n)}^{4-a-2b-3c} \hat{h}^a (\partial \hat{A})^b (\partial^2 \hat{\phi})^c \tag{5.23}$$

where $a + b + c \geqslant 3$. The scales $\Lambda_{(n)}$ are given by

$$\Lambda_{(n)} = (m^{n-1} M_{\rm pl})^{1/n} \tag{5.24}$$

where

$$n = \frac{4 - a - 2b - 3c}{2 - a - b - c}. \tag{5.25}$$

If we assume $m < M_{\rm pl}$, then the smallest scale is given by the largest value of n which turns out to correspond to a cubic $\hat{\phi}$ interaction suppressed by

$$\Lambda_{(5)} = (m^4 M_{\rm pl})^{1/5}. \tag{5.26}$$

This cubic interaction is therefore the leading order non-linearity and since the Vainshtein radius (5.19) we are trying to understand was related to the onset of non-linearities, let us zoom in on the related dynamics. To do so we take the following decoupling limit

$$m \to 0, \quad M_{\rm pl} \to \infty, \quad T \to \infty, \quad \Lambda_5, \frac{T}{M_{\rm pl}} \ \text{ fixed} \tag{5.27}$$

which ensures that all interactions between $\hat{h}_{\mu\nu}$, $\hat{A}_\mu$ and $\hat{\phi}$ other than the $\hat{\phi}$ cubic interaction vanish. The resulting action, in comparison to the linear case, contains kinetic mixing between $\hat{h}_{\mu\nu}$ and $\hat{\phi}$ which again we diagonalise by the field re-definition $\hat{h}_{\mu\nu} \to \tilde{h}_{\mu\nu} + \eta_{\mu\nu} m^2 \hat{\phi}$ yielding

$$S = \int d^4x \left[\tilde{\mathcal{L}}_{GR} - \frac{1}{4}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} + \frac{\tilde{h}_{\mu\nu}T^{\mu\nu}}{M_{\rm pl}} \right. \\ \left. - \frac{1}{2}(\partial\hat{\phi})^2 + \frac{c}{\Lambda_{(5)}^5}\left((\Box\hat{\phi})^3 - \Box\hat{\phi}\partial_\mu\partial_\nu\hat{\phi}\partial^\mu\partial^\nu\hat{\phi}\right) + \frac{\hat{\phi}T}{\sqrt{6}M_{\rm pl}} \right] \quad (5.28)$$

where $c \sim \mathcal{O}(1)$ is a constant. It is easy to see that in the limit $\Lambda_{(5)} \to \infty$ the action reduces to (5.9). However, with finite $\Lambda_{(5)}$ the non-linearities play a non-trivial role and will become important at some scale spoiling the $\hat{\phi}$ linearised solution (5.10). This scale can be estimated by comparing the linear and non-linear terms in (5.28) and asking when they become comparable i.e. at what scale does

$$\frac{\partial^4\hat{\phi}}{\Lambda_{(5)}^5} \sim \mathcal{O}(1)? \quad (5.29)$$

If we plug in the solution (5.10) then we find that the linearised truncation breaks down at

$$r_V \sim \frac{1}{\Lambda_{(5)}}\left(\frac{M}{M_{\rm pl}}\right)^{1/5} = \left(\frac{M}{m^4 M_{\rm pl}^2}\right)^{1/5} \quad (5.30)$$

where we have used $\Lambda_{(5)} = (m^4 M_{\rm pl})^{1/5}$. This is precisely the Vainshtein radius we quoted above. We now understand it as the scale where the non-linearities of the longitudinal mode of the massive graviton become important. This makes it clear that the Vainshtein radius is nothing more than the largest distance scale at which *tree level* perturbation theory breaks down. For GR, the Vainstein radius is therefore the Schwarzschild radius since as we discussed in the introduction, this is simply the scale where the non-linearities of GR become important.

This exercise has also taught us that this non-linear completion of (5.1) is pathological and contains a scalar ghost. We can see this explicitly in the equation of motion for $\hat{\phi}$

$$\Box\hat{\phi} + \frac{6c}{\Lambda_5^5}\left[\Box\hat{\phi}\Box^2\hat{\phi} - \partial^\mu\partial^\nu\hat{\phi}\partial_\mu\partial_\nu\Box\hat{\phi}\right] + \\ \frac{3c}{\Lambda_5^5}\left[\partial_\mu\Box\hat{\phi}\partial^\mu\Box\hat{\phi} - \partial_\mu\partial^\alpha\partial^\beta\hat{\phi}\partial^\mu\partial_\alpha\partial_\beta\hat{\phi}\right] = -\frac{T}{6M_{\rm pl}} \quad (5.31)$$

which is fourth order and therefore by Ostrogradski's theorem [116] describes at least one ghostly degree of freedom. Indeed, a full Hamiltonian analysis of (5.18) confirms that this theory propagates six degrees of freedom rather than the five modes of a health massive graviton. The extra ghostly degree of freedom is known as the *Boulware-Deser ghost* [117] and to avoid it we must include non-derivative interactions to (5.18). We shall discuss these now followed by a discussion of the importance of the Vainshtein radius in the context of suppressing scalar forces.

5.1.3 Non-linear massive spin-2 part II

We now include non-derivative interactions which form a potential for the graviton. There is no known symmetry that fixes the co-efficients between the non-linear operators in the graviton's potential so we include them all with arbitrary couplings. If we assume that the potential reduces to the Fierz-Pauli mass term in the linearised limit then the generic non-linear theory with at most two derivatives is

$$S = \frac{M_{\rm pl}^2}{2}\int d^4x\sqrt{-g}\left[R - \frac{m^2}{4}V(g,h)\right] \tag{5.32}$$

where $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and $V(g,h) = V_2(g,h) + V_3(g,h) + V_4(g,h) + ...$ is the potential of which there are an infinite number of terms. The first five are

$$V_2(g,h) = [h^2] - [h]^2 \tag{5.33}$$

$$V_3(g,h) = c_1[h^3] + c_2[h]^2[h] + c_3[h]^3 \tag{5.34}$$

$$V_4(g,h) = d_1[h^4] + d_2[h^3][h] + d_3[h^2]^2 + d_4[h^2][h]^2 + d_5[h]^4 \tag{5.35}$$

$$V_5(g,h) = f_1[h^5] + f_2[h^4][h] + f_3[h^3][h]^2 + f_4[h^3][h^2] \tag{5.36}$$

$$+ f_5[h^2]^2[h] + f_6[h^2][h]^3 + f_7[h]^5 \tag{5.37}$$

where square brackets denote a trace with respect to the full metric $g_{\mu\nu}$. For example, $[h]^2 = g^{\mu\nu}h_{\mu\nu}g^{\alpha\beta}h_{\alpha\beta}$ and $[h^2] = g^{\mu\alpha}g^{\nu\beta}h_{\alpha\beta}h_{\mu\nu}$. Again, if we restore the non-linear gauge symmetry of GR by making the replacement (5.21), we get a bunch of interactions between the canonically normalised fields $\hat{h}_{\mu\nu}$, $\hat{A}_\mu$

and $\hat{\phi}$ suppressed by one of the scales $\Lambda_{(n)}$.

For generic co-efficients in the potential we still have an issue since the Boulware-Deser ghost remains thanks to derivative $\hat{\phi}$ interactions as was the case for (5.18). However, it was shown in [118,119] by de Rham, Gabadadze and Tolley that with a particular tuning between the co-efficients in the potential, the lowest scale in the theory can be raised from $\Lambda_{(5)} = (m^4 M_{\rm pl})^{1/5}$ to $\Lambda_{(3)} = (m^2 M_{\rm pl})^{1/3}$ and at the same time, at least in the decoupling limit, the Boulware-Deser ghost is removed. It was then shown in [120] by Hassan and Rosen, using a full Hamiltonian analysis, that with this tuning the full non-linear theory describes the correct number of five degrees of freedom confirming that the Boulware-Deser ghost is absent at all orders. In [121], Hassan and Rosen then presented a ghost free bi-gravity theory where the reference metric in massive gravity has its own dynamics. This theory has a number of neat features, not least a smooth limit to GR which does not rely on the Vainshtein mechanism. One can simply decouple the massive modes, leaving only the massless ones (see [122, 123] for more details).

The full structure of this potential can be found in [118–120] and again we zoom in on the leading order interactions responsible for the break down of linearity. These are again due to the longitudinal mode $\hat{\phi}$ and become manifest in the following decoupling limit

$$m \to 0, \quad M_{\rm pl} \to \infty, \quad \Lambda_3 \ \text{ fixed.} \tag{5.38}$$

The resulting action retains some $\hat{h}_{\mu\nu}$, $\hat{\phi}$ mixing and all but one of these interactions can be removed with local field re-definitions yielding

$$\begin{aligned} S = \int d^4x \Big[&\hat{\mathcal{L}}_{GR} + \frac{\hat{h}_{\mu\nu}T^{\mu\nu}}{M_{\rm pl}} - \frac{1}{2}(\partial\hat{\phi})^2 + \frac{\alpha_1}{\Lambda^3_{(3)}}(\partial\hat{\phi})^2\Box\hat{\phi} + \frac{\alpha_2}{\Lambda^6_{(3)}}(\partial\hat{\phi})^2\left([\hat{\Pi}]^2 - [\hat{\Pi}^2]\right) \\ &+ \frac{\alpha_3}{\Lambda^9_{(3)}}(\partial\hat{\phi})^2\left([\hat{\Pi}]^3 - 3[\hat{\Pi}^2][\hat{\Pi}] + 2[\hat{\Pi}^3]\right) - \frac{\alpha_4}{\Lambda^6_{(3)}}\hat{h}^{\mu\nu}\hat{X}_{\mu\nu} + \frac{\hat{\phi}T}{\sqrt{6}M_{\rm pl}} + \frac{\alpha_5}{\Lambda^3_{(3)}}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi}T^{\mu\nu}\Big] \end{aligned} \tag{5.39}$$

where $\hat{\Pi}_{\mu\nu} = \partial_\mu\partial_\nu\hat{\phi}$ and square brackets again denote a trace e.g. $[\hat{\Pi}] =$

$\eta^{\mu\nu}\partial_\mu\partial_\nu\hat{\phi}$, $[\hat{\Pi}^2] = \partial^\mu\partial^\nu\hat{\phi}\partial_\mu\partial_\nu\hat{\phi}$. We have also defined

$$\hat{X}_{\mu\nu} = ([\hat{\Pi}]^3 - 3[\hat{\Pi}][\hat{\Pi}^3] + 2[\hat{\Pi}^3])\eta_{\mu\nu} - 3([\hat{\Pi}]^2 - [\hat{\Pi}^2])\Pi_{\mu\nu} + 6[\hat{\Pi}]\hat{\Pi}^2_{\mu\nu} - 6\Pi^3_{\mu\nu} \quad (5.40)$$

which satisfies $\partial_\mu \hat{X}^{\mu\nu} = 0$. The dimensionless co-efficients α_i depend on the tuned co-efficients in the potential and their exact form can be found in [58]. The special structure of the derivative self-interactions of $\hat{\phi}$ in (5.39) ensures that its equation of motion is second order, representing the fact that there is no Boulware-Deser ghost. Now that we have a ghost free theory we shall study the Vainshtein mechanism in more detail.

Again, in the limit $\Lambda_{(3)} \to \infty$, the linear solution for $\hat{\phi}$ is given by (5.10), but as long as $\Lambda_{(3)}$ is finite the non-linear $\hat{\phi}$ terms will spoil this solution at the Vainshtein radius. Since the structure of the these interactions differs to those of (5.28), the Vainshtein radius will also differ and to extract it let us study a subset of (5.39) by setting $\alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0$ such that action for the longitudinal mode is

$$S = \int d^4x \left[-\frac{1}{2}(\partial\hat{\phi})^2 + \frac{\alpha_1}{\Lambda^3_{(3)}}(\partial\hat{\phi})^2\Box\hat{\phi} + \frac{\hat{\phi}T}{\sqrt{6}M_{\rm pl}} \right]. \quad (5.41)$$

To extract the Vainshtein radius we again compare the linear and non-linear terms such that the linearised solution breaks down when

$$\frac{\partial^2\hat{\phi}}{\Lambda^3_{(3)}} \sim \mathcal{O}(1) \quad (5.42)$$

which corresponds to a scale

$$r_V \sim \frac{1}{\Lambda_{(3)}}\left(\frac{M}{M_{\rm pl}}\right)^{1/3} = \left(\frac{M}{m^2M^2_{\rm pl}}\right)^{1/3} \quad (5.43)$$

where we have used $\Lambda_{(3)} = (m^2M_{\rm pl})^{1/3}$. This is the Vainshtein radius for dRGT massive gravity and in comparison to the ghostly case above, it is simply the largest distance scale at which tree level perturbation theory breaks down in the presence of a massive source.

We shall now show how below the Vainshtein radius the force due to the longitudinal mode is suppressed relative to the Newtonian one. Within the Vainshtein radius we must consider the full non-linear field equation for $\hat{\phi}(r)$. For $r \ll r_V$ the form of the solution can be extracted by dimensional analysis and is given by

$$\hat{\phi}(r) \sim \Lambda_{(3)}^{3/2} \left(\frac{M}{M_{\rm pl}}\right)^{1/2} \sqrt{r} \tag{5.44}$$

which no longer looks like the Newtonian potential. Indeed, the ratio between the scalar force and the Newtonian one is now

$$\frac{F_\phi}{F_{\rm N}} \sim \left(\frac{r}{r_V}\right)^{3/2} \tag{5.45}$$

which is small as long as $r \ll r_V$. This is the *Vainshtein mechansim.* The onset of non-linearities causes the scalar force to become much smaller than the Newtonian one, thereby suppressing the effects of deviations from GR in well tested regimes.

For a clearer picture of how the mechanism works let us expand the action (5.41) around some background solution $\hat{\phi}(x) = \bar{\phi}(x) + \pi(x)$, $T = \bar{T} + \delta T$ thereby making connection with our discussion of screening mechanisms in the introduction. At quadratic order in fluctuations the action reduces to

$$S = \int d^4x \left[-\frac{1}{2} Z^{\mu\nu}[\bar{\phi}] \partial_\mu \pi \partial_\nu \pi + \frac{\pi \delta T}{\sqrt{6} M_{\rm pl}} \right] \tag{5.46}$$

where

$$Z^{\mu\nu}[\bar{\phi}] = \eta^{\mu\nu} \left[1 - \frac{4\alpha_1}{\Lambda_{(3)}^3} \Box \bar{\phi} \right] + \frac{4\alpha_1}{\Lambda_{(3)}^3} \partial^\mu \partial^\nu \bar{\phi} \tag{5.47}$$

and again we refer to Z as the Z-factor. When $Z \sim \mathcal{O}(1)$ i.e. when the non-linearities are suppressed, the scalar fluctuation couples to the trace of the energy-momentum tensor with gravitational strength providing a force comparable to the Newtonian one. However, when $\partial\partial\hat{\phi} \gg \Lambda_{(3)}^3$ the Z factor becomes large, and upon canonically normalising, the scalar fluctuation couples to the trace of the energy-momentum tensor with a strength $\frac{1}{\sqrt{Z} M_{\rm pl}} \ll \frac{1}{M_{\rm pl}}$. This weakening of the π coupling to matter ensures that the scalar force becomes negligible and allows the theory to mimic GR.

5.1.4 Galileons and beyond

So far we have discussed the Vainshtein mechanism in the context of massive gravity since this was its origin, but it is by no means special to massive gravity. Indeed, there are other non-linear scalar field theories which inherit the Vainshtein mechanism which can never appear in the effective theory of a massive graviton. Recall the replacement in equation (5.21) which we were required to make to restore the non-linear gauge invariance in massive gravity. The longitudinal mode only ever appeared with at least two derivatives and the resulting gauge invariant action was therefore also guaranteed to be invariant under the *global* symmetry

$$\phi(x) \to \phi(x) + b_\mu x^\mu + c \tag{5.48}$$

where b_μ and c are constants. In fact, this symmetry along with the requirement of ghost freedom uniquely fixes the ϕ derivative self-interactions of (5.39). The only scalar field theory which is invariant under (5.48) and has second order field equations in four space-time dimensions is

$$\mathcal{L}_{\rm gal} = \sum_{m=1}^{5} c_m \Lambda^{6-3m} \mathcal{L}_m \tag{5.49}$$

where

$$\mathcal{L}_1 = \phi \tag{5.50}$$

$$\mathcal{L}_2 = (\partial\phi)^2 \tag{5.51}$$

$$\mathcal{L}_3 = (\partial\phi)^2[\Pi] \tag{5.52}$$

$$\mathcal{L}_4 = (\partial\phi)^2([\Pi]^2 - [\Pi^2]) \tag{5.53}$$

$$\mathcal{L}_5 = (\partial\phi)^2([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3]). \tag{5.54}$$

$$\tag{5.55}$$

Here ϕ is known as the *galileon* and the symmetry (5.48) is the *galileon symmetry*. More generally, in D space-time dimensions there are $D+1$ terms invariant under (5.48) with second order field equations. The galileon symmetry first arose in the decoupling of the Dvali-Gabadadze-Porrati (DGP) brane-world model [124]. In DGP, gravity behaves like four dimensional GR below some length scale $L_{\rm DGP}$, but at distances larger than $L_{\rm DGP}$ gravity becomes five dimensional and weaker. The critical length scale is usually taken to be of order the current Hubble scale $L_{\rm DGP} \sim 1/H_0$ motivated by the current acceleration of the universe. After integrating out the bulk degrees of freedom one finds an effective four dimensional action which consists of the usual massless graviton of GR, but also a scalar degree of freedom with the *cubic* galileon derivative self-interaction (5.52) suppressed by a scale $\Lambda_{\rm DGP} = (M_{\rm pl}/L^2_{\rm DGP})^{1/3}$ [125]. This is precisely the $\Lambda_{(3)}$ scale in massive gravity if we identify $L_{\rm DGP}$ with the Compton wavelength of the graviton. The remaining terms were presented in [126] and the model was extended to a bi-galileon theory in [127].

The galileons have received much attention as field theories in their own right given their neat properties. Most interestingly, a large class of scattering amplitudes were studied in [128] and it was concluded that the quartic interaction (5.53) was special among the full class of galileon interactions. It was shown that in the limit where the momentum of one external leg is taken soft, the quartic galileon theory had an enhanced soft limit such that scattering amplitudes begin at a higher order in z where the soft particle has momentum zp and the soft limit is $z \to 0$. This phenomena was explained in [129] and attributed to an extended symmetry of the quartic galileon which is quadratic in the space-time co-ordinates. Another interesting property of the galileons is that the co-efficients c_m are not re-normalised at any order in perturbation theory [125,130]. Only operators with at least two derivatives on each field are generated quantum mechanically.

In any case, the galileon symmetry ensures that no operators of the form $(\partial\phi)^{2n}$ with $n > 1$ enter the effective field theory of a massive graviton. These operators are invariant under the minimal shift symmetry $\phi(x) \to \phi(x) + c$, where c is a constant, but not the galileon symmetry. Yet a theory with these interactions will have a Vainshtein mechanism due to the presence of non-linear derivative interactions. To see this let us consider the most simple example of this type

$$S = \int d^4x \left[-\frac{1}{2}(\partial\phi)^2 + \frac{c_1}{\Lambda^4}(\partial\phi)^4 + \frac{g\phi T}{M_{\rm pl}} \right] \tag{5.56}$$

where $c_1, g \sim \mathcal{O}(1)$ are dimensionless coupling constants and ϕ can be thought of as the goldstone boson of a broken U(1) symmetry. The coupling to matter does not break the shift symmetry since with $\partial_\mu T^\mu{}_\nu = 0$ we can write

$$\frac{g\phi T}{M_{\rm pl}} = \frac{g\phi}{M_{\rm pl}} \partial_\mu (x^\nu T^\mu{}_\nu) \tag{5.57}$$

which is a total derivative under the shift symmetry. In the linearised limit, the static and spherically symmetric solution for ϕ in the presence of a static point source of mass M is again given by (5.10). The Vainshtein radius is now the scale at which

$$\frac{(\partial\phi)^2}{\Lambda^4} \sim \mathcal{O}(1) \tag{5.58}$$

which occurs at

$$r_V = \frac{1}{\Lambda} \left(\frac{M}{M_{\rm pl}} \right)^{1/2} . \tag{5.59}$$

The difference between this Vainshtein radius and the one associated with galileons and dRGT massive gravity, is the $\sim M^{1/2}$ behaviour instead of $\sim M^{1/3}$. Inside r_V, the non-linear solution is, by dimensional analysis,

$$\phi(r) \sim \Lambda^{4/3} \left(\frac{M}{M_{\rm pl}} \right)^{1/3} r^{1/3} \tag{5.60}$$

and the ratio between the scalar force and the Newtonian one is

$$\frac{F_\phi}{F_N} \sim \left(\frac{r}{r_V} \right)^{4/3} . \tag{5.61}$$

Therefore, as was the case for non-linear massive gravity, on distance scales $r \ll r_V$, the scalar force is suppressed relative to the Newtonian. However here the suppression is faster and the Vainshtein mechanism is more efficient.

5.1.5 Positivity conditions

An important feature of the Vainshtein mechanism is the intricate link between the sign of the co-efficient c_1 in (5.56) and the nature of the background solution where we wish to screen. From an effective field theory point of view, and in the absence of details about the UV completion, the co-efficients of irrelevant operators are arbitrary and should be fixed by observations. However, we can only realise the Vainshtein mechanism with a particular choice. To see this for (5.56) we expand the action to quadratic order $\phi(x) \to \bar{\phi}(x) + \pi(x)$, $T + \bar{T} + \delta T$ yielding

$$S = \int d^4x \left[-\frac{1}{2} Z^{\mu\nu}[\bar{\phi}] \partial_\mu \pi \partial_\nu \pi + \frac{g\pi\delta T}{M_{\rm pl}} \right] \tag{5.62}$$

with

$$Z^{\mu\nu}[\bar{\phi}] = \eta^{\mu\nu} \left[1 - \frac{4c_1}{\Lambda^4} (\partial\bar{\phi})^2 \right] - \frac{8c_1}{\Lambda^4} \partial^\mu \bar{\phi} \partial^\nu \bar{\phi}. \tag{5.63}$$

If we initially specialise to static and spherically symmetric solutions $\bar{\phi} = \bar{\phi}(r)$ then the components of the Z factor are

$$Z^{tt} = -1 + \frac{4c_1}{\Lambda^4} \bar{\phi}'^2 \tag{5.64}$$

$$Z^{ti} = 0 \tag{5.65}$$

$$Z^{ij} = \delta^{ij} \left(1 - \frac{4c_1}{\Lambda^4} \bar{\phi}'^2 \right) - \frac{8c_1}{\Lambda^4} \partial^i \bar{\phi} \partial^j \bar{\phi}. \tag{5.66}$$

To retain three positive and one negative eigenvalue of Z throughout evolution we therefore require $c_1 < 0$. If we have $c_1 > 0$, near the Vainshtein scale the fluctuation π becomes infinitely strongly coupled. A solution where $\bar{\phi}(r)$ dies off at infinity only exists for all $r > 0$ if we set $c_1 < 0$. The opposite is true for time-dependent, homogeneous configurations. Indeed, if we set $\bar{\phi} = \bar{\phi}(t)$ then the components of the Z factor are

$$Z^{tt} = -1 - \frac{12c_1}{\Lambda^4} \dot{\bar{\phi}}^2 \tag{5.67}$$

$$Z^{ti} = 0 \tag{5.68}$$

$$Z^{ij} = \delta^{ij} \left(1 + \frac{4c_1}{\Lambda^4} \dot{\bar{\phi}}^2 \right) \tag{5.69}$$

and we therefore require $c_1 > 0$ to retain three positive and one negative eigenvalue and to realise the Vainshtein mechanism. So the sign of c_1 is crucial in determining whether the Vainshtein mechanism can be employed or not. This turns out to be an important result since there are more fundamental constraints one can place on the coupling constants of leading order irrelevant operators as discussed in [131]. By studying $2 \to 2$ scattering amplitudes at tree level, here we use the standard definition of the Mandelstam s, t, u variables, it was concluded in [131] that in the forward limit ($t \to 0$), the co-efficient of s^2 must be *strictly positive* if the UV completion is Lorentz invariant with an analytic S-matrix. Therefore, this positivity condition must be satisfied if the high energy theory is a local QFT or weakly coupled string theory. The $2 \to 2$ scattering amplitude for (5.56) is (e.g. [29])

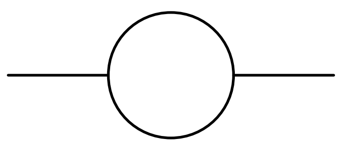
$$\mathcal{A}(s,t) = \frac{2c_1}{\Lambda^4}(s^2+t^2+u^2) = \frac{4c_1}{\Lambda^4}(s^2+t^2+st) \tag{5.70}$$

where we have used $s + t + u = 0$ since the shift symmetry ensures ϕ is massless. We therefore require $c_1 > 0$ if the theory has a hope of being UV completed in a standard way. This is very interesting from the point of view of the Vainshtein mechanism. As we have just discussed, we can only reliably screen scalar forces on static backgrounds in (5.56) if the co-efficient of the leading order irrelevant operator is negative! This is in stark violation of the positivity condition coming from scattering amplitudes. This suggests one cannot implement the Vainshtein mechanism in (5.56) to suppress scalar forces in the solar system while describing the low energy limit of a local, Lorentz invariant UV completion. However, we also saw above that we require $c_1 > 0$ to screen on homogeneous backgrounds, in which case the screening condition coincides with the positivity condition from scattering amplitudes.

The situation is even worse for galileons since the positivity conditions can *never* be satisfied. This is a consequence of the galileon symmetry (5.48) and the fact that the operator $(\partial\phi)^4/\Lambda^4$ is absent from the galileon effective field theory. Indeed, the $2 \to 2$ scattering amplitude for galileons begins at s^3 rather than s^2 such that the co-efficient of s^2 vanishes. The galileon theory therefore violates the positivity conditions for all choices of the coupling constants and the results of [131] suggest that it cannot be UV completed into a local QFT or weakly coupled string theory. This is one of the main issues facing theories with a Vainshtein mechanism. However, interestingly it has been argued in [132] that even though massive gravity is strongly associated with galileons, these results do not extend to the full non-linear theory beyond the decoupling limit and that a small parameter space of dRGT massive gravity is consistent with analyticity constraints.

5.2 Strong coupling scales

By dimensional analysis, the irrelevant operators required to realise the Vainshtein mechanism are suppressed by a dimensionful energy scale. In the decoupling limit of massive gravity this scale is a particular combination of the graviton mass and the Planck scale, depending on the type of interaction. In the decoupling limit of DGP gravity it depends on the critical length L_{DGP} and the Planck scale. In other theories it is a mass scale in its own right, at least in effective field theory. In any case, this scale is the quantum strong coupling scale where *loop perturbation theory* breaks down. For example, consider the galileon theory (5.49) where the ϕ propagator goes like $1/p^2$ with p the 4-momentum. The cubic interaction (5.52) corrects this tree level propagator at 1-loop via the diagram



The correction is of the form

$$\frac{1}{p^2}\frac{p^6}{\Lambda^6} \tag{5.71}$$

and is therefore only perturbative for processes with $p < \Lambda$. As energies reach the strong coupling scale, the 1-loop correction becomes important and loop perturbation theory breaks down. It is also where perturbative unitarity breaks down given that the $2 \to 2$ scattering amplitude in the cubic galileon model is (see e.g. [131])

$$\mathcal{A}(s,t) \sim \frac{s^3+t^3+u^3}{\Lambda^6} + \mathcal{O}(s^4,t^4,u^4) \tag{5.72}$$

and therefore the S-matrix is only exactly unitary if $p < \Lambda$. Λ plays the same role for the scalar interactions as $M_{\rm pl}$ does for the spin-2 interactions in GR. This behaviour usually points to the need for a UV completion where new degrees of freedom extend the theory beyond the scale Λ by restoring unitarity. This is the primary role of the Higgs in the standard model where it preserves unitarity in the scattering of W-bosons.

Let us stick with massive gravity for now and explore the size of the scales $\Lambda_{(n)}$. Massive gravity is most often employed as a long distance modification of gravity and one takes $m \sim 10^{-33}$eV such that the Compton wavelength of the graviton is of order the current Hubble scale H_0. Inserting this value and that of the Planck mass $M_{\rm pl} \sim 10^{27}$eV into $\Lambda_{(n)} = (m^{n-1}M_{\rm pl})^{1/n}$ yields

$$\Lambda_{(n)} = 10^{(60-33n)/n}\text{eV}. \tag{5.73}$$

The lowest scale in ghost free massive gravity is $\Lambda_{(3)}$ which corresponds to

$$\Lambda_{(3)} \sim 10^{-13}\text{eV} = 10^{-40}M_{\rm pl}. \tag{5.74}$$

More illustratively, this corresponds to a distance scale of 1000km and ensures that we cannot use the dRGT theory of massive gravity to make predictions for Earth based tests of gravity. Below 1000km the gravitational potential is swamped by quantum effects, and we require a (partial) UV completion to keep control on these distance scales. As we discussed above, this is also the scale where interactions become strongly coupled in DGP if we take $L_{\rm DGP} = 1/H_0$. In both cases it is the interactions of a scalar field which become strong and therefore our desire to realise interesting gravitational effects on Hubble scales, by introducing new scalar degrees of freedom, restricts our ability to make predictions for local experiments.

We are now in a position to calculate the Vainshtein radius (5.43) associated with dRGT massive gravity. To do so we also need to specify the mass of the source M and since our aim is to suppress forces in the solar system, we use the mass of the Sun such that $M \sim 10^{66}\text{eV}$. The Vainshtein radius is therefore

$$r_V = \frac{1}{\Lambda_{(3)}} \left(\frac{M}{M_{\text{pl}}} \right)^{1/3} \sim 10^{21}\text{cm} \tag{5.75}$$

which is way beyond the size of the solar system. This is also much larger than the Schwarzschild radius, or the Vainhstein radius, of GR because of the huge hierarchy between $\Lambda_{(3)}$ and M_{pl}. To calculate the gravitational potential due to the Sun in dRGT massive gravity at a distance 10^{21}cm away, we must therefore include non-linear effects!

In the previous section we understood the Vainshtein mechanism as the weakening of the coupling between matter and the scalar fluctuation π thanks to derivative non-linearities becoming large. We saw this explicitly for the cubic galileon theory, but in (5.46) we ignored self-interactions of the fluctuation. The full action for π is

$$S = \int d^4x \left[-\frac{1}{2} Z^{\mu\nu}[\bar{\phi}] \partial_\mu \pi \partial_\nu \pi + \frac{\alpha_1}{\Lambda^3_{(3)}} (\partial\pi)^2 \Box \pi + \frac{\pi \delta T}{\sqrt{6} M_{\text{pl}}} \right]. \tag{5.76}$$

Evaluating the Z factor (5.47) inside the Vainshtein radius using the solution (5.44) yields

$$S \sim \int d^4x \left[\left(\frac{r_V}{r} \right)^{3/2} \left(\dot{\pi}^2 - \frac{4}{3} \pi'^2 - (\partial_\Omega \pi)^2 \right) + \frac{\alpha_1}{\Lambda^3_{(3)}} (\partial\pi)^2 \Box \pi + \frac{\pi \delta T}{\sqrt{6} M_{\text{pl}}} \right] \tag{5.77}$$

where Ω denotes the angular co-ordinates. The kinetic term has a large enhancement inside r_V so we canonically normalise. The resulting coupling to matter is weakened as we saw earlier, but the strong coupling scale of the π interaction is also altered to

$$\tilde{\Lambda}_{(3)}(x) = \left(\frac{r_V}{r} \right)^{3/4} \Lambda_{(3)} \gg \Lambda_{(3)} \tag{5.78}$$

which we refer to as the *environmental strong coupling scale*. Qualitatively, the derivative interactions correct the π propagator on non-trivial backgrounds and after canonical normalisation, π scattering becomes strong at a scale dependent on the environmental parameters. The strong coupling scale $\Lambda_{(3)}$ we have been discussing up to this point can be considered as a vacuum scale associated with scattering π's on a background with $\bar{\phi} = 0$.

It was argued in [40] that quantum effects can be ignored until this new strong coupling scale. Their results were guided by the fact that even when the derivative interactions become large, the classical theory is well behaved. For a generic theory with higher order derivatives, ghost instabilities would be problematic and one would need quantum effects to cure the instability before the non-linearities kick in. However, when ghosts do not appear, one can trust the classical non-linear effects and compute quantum corrections on a non-trivial background $\bar{\phi} \neq 0$ with a large Z factor. Quantum corrections then also receive Z factor suppression and the theory remains perturbative down to distances $\tilde{\Lambda}^{-1}$. It was suggested that no new degrees of freedom enter the effective action until we reach this environmental strong coupling scale.

For DGP with $L_{\rm DGP} \sim H_0^{-1}$ at the surface of the Earth, the strong coupling scale is increased by 8 orders of magnitude to $\tilde{\Lambda}_{\oplus}^{-1} \sim 1$cm [40], whereas for dRGT massive gravity with a Hubble scale mass, it is increased by 3 orders of magnitude to $\tilde{\Lambda}_{\oplus}^{-1} \sim 1$km [133]. Here we have taken the mass and radius of the Earth to be $M_{\oplus} \sim 10^{33} M_{\rm pl}$ and $r_{\oplus} \sim 10^4$km respectively. The difference is due to the scalar-graviton mixing in massive gravity (see equation (5.39)) which is absent in DGP. One can increase the environmental scale for massive gravity but only at the expense of increasing the graviton mass, as is the case for the vacuum scale, thereby decreasing its affect on cosmology. In any case, this environmental enhancement of strong coupling is desirable since it potentially increases the regime of validity of a theory by many orders of magnitude thanks to the Vainshtein mechanism. The remainder of this chapter is devoted studying the consistency of this environmental enhancement following [3].

5.3 The Vainshtein mechanism and Higgs completions

Our aim in this section is to investigate the effectiveness of the Vainshtein mechanism, and the corresponding enhancement of strong coupling, by studying fully UV complete theories which realise the Vainshtein mechanism at low energies. The question we would like to ask is if the low energy models can be trusted as consistent effective descriptions of the corresponding UV theory inside the Vainshtein regime and beyond the vacuum strong coupling scale of the truncated theory, as is assumed in modifications of gravity e.g. [40, 133]. In the absence of a UV theory of gravity, we will study field theories as avatars for what one may expect in models of modified gravity. At first sight this would seem to hamper our ability to draw conclusions about gravity theories. However, as we have seen in the preceding sections of this chapter, the most interesting dynamics, and issues with strong coupling, in these models can be understood by studying interacting scalar fields propagating in flat space. In this sense we will argue that reliable connections can be made. However, to compliment this section we shall also study the effects of UV corrections in gravity in the following section.

As we have already discussed in section (5.1.5), it is by no means straight forward to find a UV complete theory which will allow us to study the Vainshtein mechanism at low energies. Indeed, it has been argued that galileons can never appear at low energies after integrating out weakly coupled physics [131]. This also holds for our goldstone example (5.56) if we choose to screen on static backgrounds. However, if we choose the co-efficient of the irrelevant operator in (5.56) such that screening is realised on time-dependent backgrounds then the self interactions there do have a standard Wilsonian UV completion. By associating the scalar ϕ with the helicity-0 mode of a massive Abelian gauge field, it enjoys a UV completion into a U(1) Higgs model. The bulk of this section will be devoted to studying the classical dynamics of this theory in the IR and UV. We shall also extend our analysis to a non-Abelian gauge theory which can be UV completed into an SU(2) Higgs theory. In both cases our aim is to investigate the effects of the heavy physics required for the UV completion on the low energy theory where the Vainshtein mechanism is at work.

5.3.1 U(1)

We begin with the most simple of our examples and the one whose low energy dynamics we have already discussed. Indeed, the self-interactions of the goldstone boson in (5.56) can be UV completed into the U(1) Higgs model

$$S_{\rm UV} = \int d^4x \left[-\partial_\mu \Phi^\dagger \partial^\mu \Phi - \lambda \left(|\Phi|^2 - \eta^2\right)^2\right] \tag{5.79}$$

where Φ is a complex scalar, λ is the quartic coupling constant, and the vacuum $|\Phi|^2 = \eta^2$ spontaneously breaks the U(1) symmetry $\Phi \to e^{i\theta}\Phi$. The resulting degrees of freedom are made manifest with the parametrisation

$$\Phi = \rho e^{i\alpha} \tag{5.80}$$

which leads to

$$S_{\rm UV} = \int d^4x \left[-(\partial\rho)^2 - \rho^2(\partial\alpha)^2 - \lambda(\rho^2-\eta^2)^2\right]. \tag{5.81}$$

Here ρ is the radial mode with a mass $m^2 = 4\lambda\eta^2$ and α is the massless angular mode. The equations of motion are

$$\Box\rho - \rho(\partial\alpha)^2 - 2\lambda\rho(\rho^2-\eta^2) = 0 \tag{5.82}$$

$$\partial_\mu(\rho^2\partial^\mu\alpha) = 0. \tag{5.83}$$

At energies below the mass of the radial mode, we can integrate it out at tree level using its equation of motion leaving us with a low energy effective description in terms of the angular mode given by [134]

$$S_{IR} = \int d^4x \;\; \eta^2 \left[-(\partial\alpha)^2 + \frac{(\partial\alpha)^4}{m^2} + \frac{c}{m^4}\partial_\sigma\alpha\partial^\nu\alpha\partial_\mu\partial_\nu\alpha\partial^\mu\partial^\sigma\alpha + \dots\right] \tag{5.84}$$

where $c \sim \mathcal{O}(1)$ is a dimensionless constant and the ellipses denotes operators of the form

$$\mathcal{O}_n \sim \eta^2 m^2 \left(\frac{\partial}{m}\right)^n \tag{5.85}$$

with $n > 6$ and even by Lorentz invariance. Here α is dimensionless and if we canonically normalise then the vacuum strong coupling scale is $\Lambda \sim \lambda^{1/4}\eta$ such that at weak coupling $m < \Lambda$. As promised, we recognise the canonical kinetic term and the leading irrelevant operator as the structure of (5.56) in vacuum $T = 0$.

If $\lambda > 0$, which we require for a stable potential in the UV, then $m^2 > 0$ and the co-efficient of the leading order irrelevant operator is *positive* as expected from the results of [131]. This is probably the most simple example where one can see the positivity conditions in action. If we wish to realise Vainshtein screening with (5.84) on a static background, we must choose $\lambda < 0$ given our discussion in section (5.1.5). This corresponds to an unstable U(1) Higgs theory. For the remainder of this section we will therefore study Vainshtein screening on time-dependent backgrounds, where the scalar is excited from its trivial vacuum state to one with a non-zero energy density $\mathcal{E}$, such that the UV completion is stable.

We now ask the following question: what is the maximum amount of energy we can put into the low energy system until it becomes a bad approximation to the UV theory? In other words, for which energy densities do the classical homogeneous solutions for $\dot{\alpha}_{IR}$ in the low energy theory significantly differ to $\dot{\alpha}_{UV}$ in the full UV complete model given the same initial data? Whereby same initial data, we mean same initial velocity for the angular mode in both cases and the same total energy densities for the IR and UV solutions. To perform calculations in the low energy theory we must truncate since there are an infinite number of operators organised in a derivative expansion. We choose to truncate at the first term which allows for the Vainshtein mechanism such that the low energy effective theory we study is

$$S_{IR} = \int d^4x \;\; \eta^2 \left[-(\partial\alpha)^2 + \frac{(\partial\alpha)^4}{m^2}\right] \tag{5.86}$$

and we therefore compare the dynamics of α in (5.86) and (5.81).

Since we are assuming $\alpha = \alpha(t)$, the dynamics of the IR theory is simply $\dot{\alpha}_{IR} = \omega$ where ω is a constant and fixed as an initial condition. The conserved energy density of the solution is

$$\mathcal{E}_{IR} = \eta^2\omega^2 + \frac{3\omega^4}{4\lambda}. \tag{5.87}$$

The dynamics of the UV theory is

$$\dot{\rho}^2 + \rho^2\dot{\alpha}_{UV}^2 + \lambda(\rho^2 - \eta^2)^2 = \mathcal{E}_{UV} \tag{5.88}$$

$$\rho^2\dot{\alpha}_{UV} = J \tag{5.89}$$

where J is the constant angular momentum which depends on the initial data via

$$J = \rho_0^2\omega. \tag{5.90}$$

The complete dynamics and energy density of the IR theory are given once we fix ω, λ and η. To evolve the UV theory we must also specify three extra pieces of initial data: $\dot{\alpha}_{UV}(0)$, $\dot{\rho}(0) = v$ and $\rho(0) = \rho_0$. Given that we are matching the initial velocity of the angular mode in the IR and UV, we have $\dot{\alpha}_{UV}(0) = \omega$. However, a low energy observer is completely ignorant to the full details of the UV theory and therefore has no way of knowing the initial data of the radial mode. Matching the energy densities of the solutions $\mathcal{E} = \mathcal{E}_{IR} = \mathcal{E}_{UV}$ yields a constraint ensuring that only one of the two initial conditions for the radial mode is free. With this in mind we choose to scan over *all* consistent choices of v^2, fixing ρ_0 accordingly.

Let us parametrise the range of initial velocities by $v^2 \in [0, v_{\max}^2]$ where we identify $v_{\max}$ shortly. To scan over all possibilities we introduce a parameter $\theta \in [0, \pi/2]$ and set $v^2 = v_{\max}^2 \sin^2\theta$. It follows from (5.88) that the initial velocity of the radial mode is

$$v^2 = \mathcal{E} - \rho_0^2\omega^2 - \lambda(\rho_0^2 - \eta^2)^2 \tag{5.91}$$

which allows us to extract ρ_0 for a given v^2 upon inserting (5.87) and specifying the data required to evolve the IR theory. Since (5.91) is quadratic in ρ_0 there are two solutions given by

$$\rho_0^2 = \left(\eta^2 - \frac{\omega^2}{2\lambda}\right) \pm \sqrt{\frac{\omega^4}{\lambda^2} - \frac{v_{\rm max}^2}{\lambda}\sin^2\theta}. \tag{5.92}$$

The value of $v_{\rm max}^2$ is given by maximising the RHS of (5.91) as a function of ρ_0 yielding

$$v_{\rm max}^2 = \mathcal{E} - \eta^2\omega^2 + \frac{\omega^4}{4\lambda} = \frac{\omega^4}{\lambda}. \tag{5.93}$$

Plugging this into (5.92) and taking the positive root to ensure ρ_0 is real at all energies gives

$$\rho_0 = \eta\sqrt{1 + \frac{\omega^2}{\lambda\eta^2}\left(\cos\theta - \frac{1}{2}\right)}. \tag{5.94}$$

With all the initial data fixed we solve for $\dot{\alpha}_{\rm UV}(t)$ and compare it to $\dot{\alpha}_{\rm IR}(t) = \omega$. For a given energy density $\mathcal{E}$ we propose the following measure for comparing the solutions

$$\chi(\mathcal{E}) = \max\left\{\left|\frac{\dot{\alpha}_{UV}^2(t) - \dot{\alpha}_{IR}^2(t)}{\dot{\alpha}_{UV}^2(t) + \dot{\alpha}_{IR}^2(t)}\right|, t \geq 0, 0 \leq \theta \leq \pi/2\right\} \tag{5.95}$$

which respects the shift symmetry of α in the IR and UV and has well defined limits. When the IR solution is a good approximation of the UV one, $\chi(\mathcal{E}) \to 0$ and as the solutions diverge $\chi(\mathcal{E}) \to 1$. There is no well defined value of χ for which the low energy theory is a bad approximation to the UV theory but we take a moderate approach and deem that as $\chi(\mathcal{E}) \gtrsim \mathcal{O}(0.1)$ the low energy effective description has broken down. This is only a single order of magnitude below its maximum value and seems to be the obvious choice.

Qualitatively, we specify initial values for ω, $\mathcal{E} = \mathcal{E}(\lambda, \eta)$ and then evolve the IR and UV systems for a large range of $\theta \in [0, \pi/2]$ which fixes v^2 and ρ_0 for each run. For a given $\mathcal{E}$ we calculate the maximum value of χ over the full range of θ and over the full trajectory in each case. This process ensures that we cover a wide range of initial data when we compare the IR and UV. We repeat for a range of values of $\mathcal{E}$ either side of the vacuum strong coupling scale $\Lambda \sim \lambda^{1/4}\eta$.

For this U(1) case we do not need to solve any differential equations since

(5.95) can be calculated algebraically. Using equation (5.89) and $\dot{\alpha}_{IR} = \omega$ we can write

$$\chi(\mathcal{E}) = \left| \frac{\rho^4 - \rho_0^4}{\rho^4 + \rho_0^4} \right| \tag{5.96}$$

whose maximum value corresponds to the maximum or minimum value of ρ. Equation (5.88) tells us that the radial mode experiences an effective potential of the form

$$V_{\text{eff}}(\rho) = \frac{\omega^2 \rho_0^4}{\rho^2} + \lambda(\rho^2 - \eta^2)^2 \tag{5.97}$$

where the angular momentum creates a potential barrier at small values of ρ. The general form of this potential is plotted below in figure (5.1).

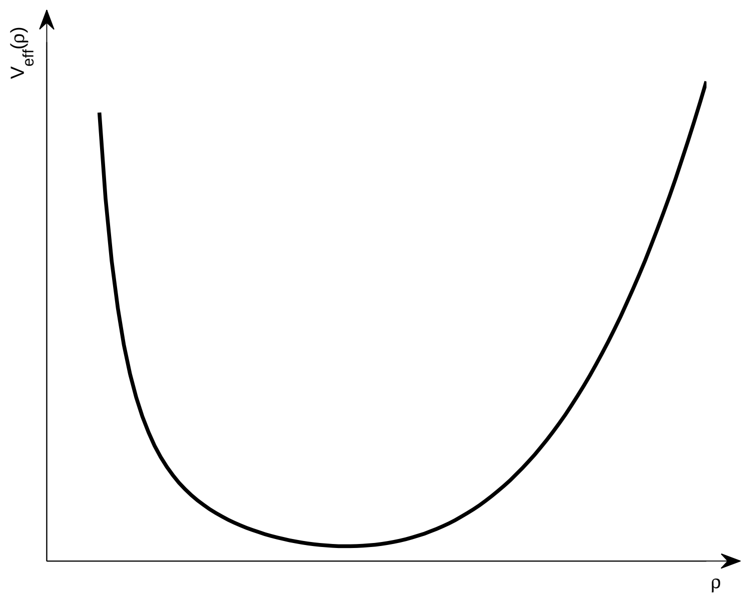


Figure 5.1: The generic form of $V_{\text{eff}}(\rho)$

It is clear that the maximum and minimum value of ρ is realised when $\dot{\rho} = 0$ or equivalently when ρ solves

$$\frac{\omega^2 \rho_0^4}{\rho^2} + \lambda(\rho^2 - \eta^2)^2 = \mathcal{E}. \tag{5.98}$$

Solving this equation and taking the largest real solution for ρ allows us to calculate $\chi(\mathcal{E})$. We repeat for a range of initial conditions to calculate the maximum value of $\chi(\mathcal{E})$. The results are displayed in figure (5.2) where we plot $\chi(\mathcal{E})$ as a function of $\mathcal{E}/\Lambda^4$.

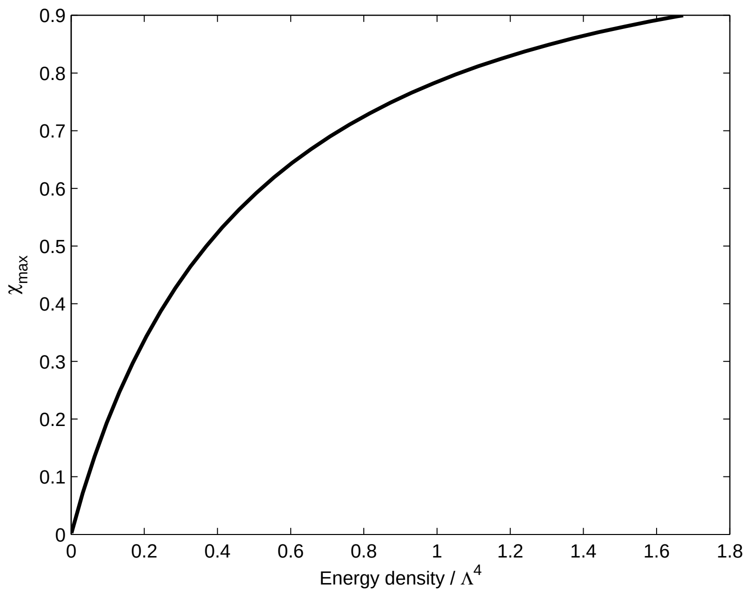


Figure 5.2: The maximum value of χ, considering a wide range of initial data for the radial mode, vs the total energy density in units of the vacuum strong coupling scale

It is clear that as the energy density of the solutions becomes comparable to the vacuum strong coupling scale, the low energy effective description becomes a poor approximation to the full UV theory. To extend our solution for $\dot{\alpha}_{\rm IR}(t)$ to energies beyond the vacuum strong coupling scale we must take into account the effects of the UV completion i.e. the radial mode. We note that this has nothing to do with quantum mechanics; we are comparing the classical dynamics. When one calculates the environmental strong coupling scale in modifications of gravity, it is assumed that the classical dynamics of the truncated theory remains reliable until the environmental strong coupling scale. This is explicit in the calculation of the enhanced strong coupling scale since one considers the scattering of the fluctuation on a background solution of the truncated theory. At least for this example, this is not the case and any calculation leading to an environmental strong coupling scale would be untrustworthy.

5.3.2 SU(2)

Let us now consider a slightly more complicated example where again the low energy dynamics realise the Vainshtein mechanism, thanks to the presence of irrelevant operators, with a known UV completion. The UV theory is again (5.79) but to make the degrees of freedom manifest we now use the parametrisation

$$\Phi = \rho \begin{pmatrix} \sin(\alpha)e^{i\beta_1} \\ \cos(\alpha)e^{i\beta_2} \end{pmatrix} \tag{5.99}$$

which leads to

$$S_{UV} = \int d^4x \left[-(\partial\rho)^2 - \rho^2 \left((\partial\alpha)^2 + \sin^2\alpha(\partial\beta_1)^2 + \cos^2\alpha(\partial\beta_2)^2\right) - \lambda(\rho^2 - \eta^2)^2\right]. \tag{5.100}$$

This theory has four scalar degrees of freedom; three angular modes α, β_1 and β_2, and one massive radial mode ρ with the same mass $m^2 = 4\lambda\eta^2$ as for the U(1) case. The equations of motion are

$$\Box\rho - \rho\left[(\partial\alpha)^2 + \sin^2\alpha(\partial\beta_1)^2 + \cos^2\alpha(\partial\beta_2)^2\right] - 2\lambda\rho(\rho^2 - \eta^2) = 0 \tag{5.101}$$

$$\rho^2\Box\alpha + 2\rho\partial_\mu\rho\partial^\mu\alpha - \rho^2\sin\alpha\cos\alpha\left[(\partial\beta_1)^2 - (\partial\beta_2)^2\right] = 0 \tag{5.102}$$

$$\partial_\mu(\rho^2\sin^2\alpha\partial^\mu\beta_1) = 0 \tag{5.103}$$

$$\partial_\mu(\rho^2\cos^2\alpha\partial^\mu\beta_2) = 0 \tag{5.104}$$

and the low energy theory is again given by integrating out the radial mode at tree level by virtue of its equation of motion (5.101). The resulting theory is built out of an infinite tower of operators constructed out of the angular modes. Truncated to no more than four derivatives i.e. including only the leading order terms required for the Vainshtein mechanism, the low energy theory we will study is

$$S_{IR} = \int d^4x \;\; \eta^2 \Bigg\{ - \left[(\partial\alpha)^2 + \sin^2\alpha(\partial\beta_1)^2 + \cos^2\alpha(\partial\beta_2)^2\right] + \frac{\left[(\partial\alpha)^2 + \sin^2\alpha(\partial\beta_1)^2 + \cos^2\alpha(\partial\beta_2)^2\right]^2}{m^2} \Bigg\} \tag{5.105}$$

with interactions becoming strong at $\Lambda \sim \lambda^{1/4}\eta$. We follow the U(1) example above and compare the dynamics of the IR theory (5.105) and the UV theory (5.100). If $\beta_1 = \beta_2 =$ constant, then the low energy theory is equivalent to (5.86) in the U(1) example, therefore to realise the Vainshtein mechanism we must work with time-dependent solutions. In this case the UV dynamics is

$$\ddot{\rho} - \rho\left(\dot{\alpha}^2 + \sin^2\alpha\dot{\beta_1}^2 + \cos^2\alpha\dot{\beta_2}^2\right) + 2\lambda\rho(\rho^2 - \eta^2) = 0 \tag{5.106}$$

$$\rho\ddot{\alpha} + 2\dot{\rho}\dot{\alpha} - \rho\sin\alpha\cos\alpha(\dot{\beta}_1^2 - \dot{\beta}_2^2) = 0 \tag{5.107}$$

$$\partial_t\left[\rho^2\sin^2\alpha\dot{\beta}_1^2\right] = 0 \tag{5.108}$$

$$\partial_t\left[\rho^2\cos^2\alpha\dot{\beta}_2^2\right] = 0 \tag{5.109}$$

and the constant energy density is

$$\mathcal{E}_{\rm UV} = \dot{\rho}^2 + \rho^2\left(\dot{\alpha}^2 + \sin^2\alpha\dot{\beta_1}^2 + \cos^2\alpha\dot{\beta_2}^2\right) + \lambda(\rho^2 - \eta^2)^2. \tag{5.110}$$

The dynamics in the IR is

$$\begin{aligned}\sin\alpha\cos\alpha\left(\dot{\beta_1}^2 - \dot{\beta_2}^2\right)&\left[1 + \frac{2}{m^2}\left(\dot{\alpha}^2 + \sin^2\alpha\dot{\beta_1}^2 + \cos^2\alpha\dot{\beta_2}^2\right)\right] \\ &-\partial_t\left[\dot{\alpha} + \frac{2\dot{\alpha}}{m^2}\left(\dot{\alpha}^2 + \sin^2\alpha\dot{\beta_1}^2 + \cos^2\alpha\dot{\beta_2}^2\right)\right] = 0\end{aligned} \tag{5.111}$$

$$\partial_t\left[\sin^2\alpha\dot{\beta}_1 + \frac{2\sin^2\alpha\dot{\beta}_1}{m^2}\left(\dot{\alpha}^2 + \sin^2\alpha\dot{\beta_1}^2 + \cos^2\alpha\dot{\beta_2}^2\right)\right] = 0 \tag{5.112}$$

$$\partial_t\left[\cos^2\alpha\dot{\beta}_2 + \frac{2\cos^2\alpha\dot{\beta}_2}{m^2}\left(\dot{\alpha}^2 + \sin^2\alpha\dot{\beta_1}^2 + \cos^2\alpha\dot{\beta_2}^2\right)\right] = 0 \tag{5.113}$$

and the constant energy density is

$$\mathcal{E}_{\rm IR} = \eta^2\left(\dot{\alpha}^2 + \sin^2\alpha\dot{\beta_1}^2 + \cos^2\alpha\dot{\beta_2}^2\right) + \frac{3}{4\lambda}\left(\dot{\alpha}^2 + \sin^2\alpha\dot{\beta_1}^2 + \cos^2\alpha\dot{\beta_2}^2\right)^2. \tag{5.114}$$

To compare the theories we follow exactly the same process as we did above for the U(1) case. We match the initial conditions for the angular modes and match the energy densities of the solutions in the IR and UV. This still leaves the initial conditions for the radial mode in the UV unspecified. Again we scan over a wide range of initial velocities and use the constraint from matching the energy densities to fix the initial value of the radial mode. In the interest of clarity, we concentrate on the dynamics of α and β_1 only, and use the same measure to compare the solutions as we did above such that there are two different values of $\chi(\mathcal{E})$ whose maximum values we calculate. The results are plotted below in figure (5.3) and are very similar to the U(1) case. Again we see that as the energy density of the solutions becomes comparable to the vacuum strong coupling scale, the theories diverge and the IR dynamics for both α and β_1 are no longer a consistent description of the UV theory. We are seeing the effects of the UV completion before we reach energies of order the vacuum strong coupling scale, thereby rendering the calculation of the environmental strong coupling scale very dubious.

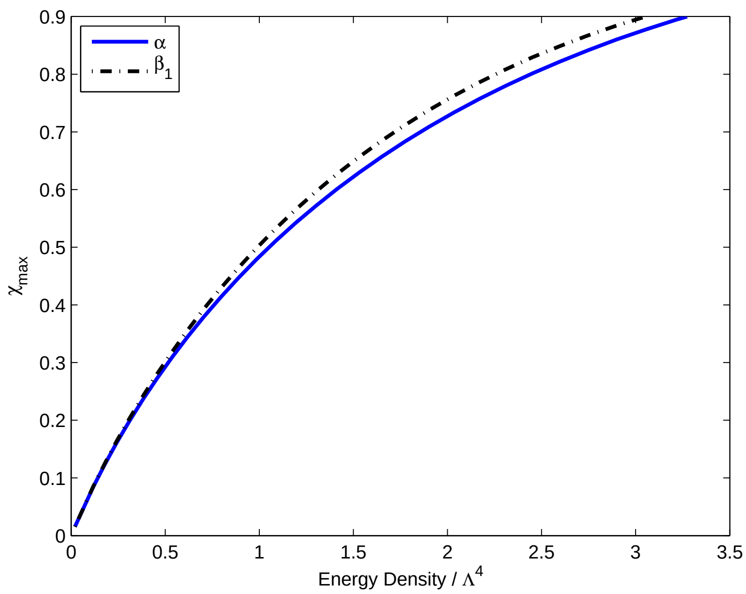


Figure 5.3: The maximum value of χ for α and β_1, considering a wide range of initial data for the radial mode, vs the total energy density in units of the vacuum strong coupling scale

5.4 The Vainshtein mechanism and Einstein Gauss-Bonnet gravity

Let us now discuss the effects of UV completions in gravity. In the introduction we briefly touched upon these issues in four dimensional GR, where we showed how higher order curvature terms become important at distances of order $1/M_{\rm pl}$. We also explained how the Vainshtein mechanism is realised in GR with the Vainshtein radius being equivalent to the Schwarzschild radius. Here we shall go through these statements in D dimensions. We shall study the effects of higher order corrections, expected to be required for a UV completion of gravity, on the classical solutions of GR in the presence of a massive source.

Expanding the D dimensional Einstein-Hilbert action on a flat background $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ leads to an infinite tower of irrelevant operators each with two derivatives but with increasing powers of the fluctuation h. Schematically we have

$$\frac{M_D^{D-2}}{2}\int d^Dx\sqrt{-g}R \sim \int d^DxM_D^{D-2}\left(h\partial^2 h + h^2\partial^2 h + h^3\partial^2 h + \ldots\right). \quad (5.115)$$

If we couple a source to the fluctuation in the way dictated by Lorentz invariance $\int d^Dx h_{\mu\nu}T^{\mu\nu}$, then for a point source of mass M the solution to the linearised theory is

$$\hat{h}_{\rm lin} \sim \frac{M}{M_D^{(D-2)/2}} r^{-(D-3)} \quad (5.116)$$

where we have canonically normalised by $h_{\mu\nu} \to M_D^{(2-D)/2}\hat{h}_{\mu\nu}$. The linearised solution breaks down as $\hat{h} \sim M_D^{(D-2)/2}$ which corresponds to the D-dimensional Schwarzschild radius which is, by definition, the Vainshtein radius

$$r_V = r_s \sim \frac{1}{M_D}\left(\frac{M}{M_D}\right)^{1/(D-3)}. \quad (5.117)$$

Below this scale one must consider all the two derivative non-linear corrections to (5.116) which sum up into the full Schwarzschild geometry [135].

Let us now consider a UV correction to the GR action. In the absence of a full UV completion of gravity, we shall consider the most simple four derivative correction, motivated by string theory, which allows us to study classical solutions without instabilities. This is the Gauss-Bonnet combination. We shall use this as a toy model for a UV completion of gravity in the sense that it includes higher order operators. The 'UV' theory is therefore

$$S_{UV} = \frac{M_D^{D-2}}{2} \int d^D x \sqrt{-g} \left[R + \alpha' \left(R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \right) \right] \quad (5.118)$$

where $\alpha' > 0$ is the slope parameter and of order the string length $l_s^2 > 1/M_D^2$. Black hole solutions to (5.118) have been studied [136,137] and are given by a Schwarzschild like metric of the form

$$ds^2 = -V(r) dt^2 + \frac{dr^2}{V(r)} + r^2 d\Omega_{D-2} \quad (5.119)$$

where

$$V(r) = 1 + \frac{r^2}{2\tilde{\alpha}'} \left(1 - \sqrt{1 + \frac{8\tilde{\alpha}' M}{(D-2)\Omega_{D-2} M_D^{D-2} r^{D-1}}} \right). \quad (5.120)$$

Here we have defined $\tilde{\alpha}' = (D-3)(D-4)\alpha'$ and Ω_{D-2} is the volume of a unit $D-2$ sphere. As expected, we can recover the Schwarzschild solution of D dimensional GR by expanding the square root to leading order yielding

$$V(r) \approx 1 - \frac{2M}{(D-2)\Omega_{D-2} M_D^{D-2} r^{D-3}} + \dots \quad (5.121)$$

which is a valid expansion as long as

$$\frac{8\tilde{\alpha}' M}{(D-2)\Omega_{D-2} M_D^{D-2} r^{D-1}} < 1. \quad (5.122)$$

When this expansion breaks down the Schwarzschild solution is no longer valid and we must consider the higher order corrections to describe the Black hole geometry in Einstein Gauss-Bonnet gravity. The radius when this occurs is

$$r_{GB} \sim \frac{1}{M_D} \left(\frac{M}{M_D} \right)^{1/(D-1)} (\alpha' M_D^2)^{1/(D-1)} \gtrsim \frac{1}{M_D} \left(\frac{M}{M_D} \right)^{1/(D-1)}. \quad (5.123)$$

For macroscopic sources such as the Sun, we have $M \gg M_D$ and therefore

$r_{GB} < r_s$. In other words, the effects of the UV completion manifest themselves at a radius inside the Schwarzschild radius but, crucially, way beyond the vacuum cut off $1/M_D$. So the operators required to restore unitarity become relevant for classical solutions at much smaller energy scales then where unitarity is violated in the truncated theory. This is comparable to the behaviour we saw in our field theory examples with a Higgs completion in section (5.3).

We note that here we have treated the correction to the Einstein-Hilbert action non-perturbatively. In a fully fledged UV completion of gravity this truncation may not be valid, but here our aim is to merely show how high energy corrections can manifest themselves on classical solutions at larger distance scales than we would naively expect.

5.5 Discussion

In this chapter we have seen how the UV completions required to restore perturbative unitarity in non-renormalisable field theories, can have non-trivial effects on the classical dynamics of the low energy degrees of freedom. The dynamics of the UV modes become important at some scale between the Vainshtein scale and the vacuum strong coupling scale. This immediately renders the environmental strong coupling scale untrustworthy, at least for the examples we have studied. This is because to calculate this enhanced strong coupling scale one considers the scattering of fluctuations at energies above the vacuum strong coupling scale, but calculated on a background solution of the truncated low energy theory. For the examples we have discussed, this background solution is no longer a good description of the UV completion beyond the vacuum strong coupling scale.

The scale at which the low energy dynamics becomes a poor effective description of the UV dynamics is model dependent, and the presence of a hierarchy between the Vainshtein scale and the break down of the effective description is highly non-trivial. Indeed, since the Vainshtein mechanism requires irrelevant operators to become classical relevant, we would expect an infinite tower to become important too. This would signal the break down of calculability since in the absence of knowledge of the UV theory, each would generically come with arbitrary co-efficients. For example, quantum mechanically we can generate operators of the form

$$\mathcal{O}_Q \sim \frac{(\partial^2\phi)^n}{\Lambda^{3n-4}} \tag{5.124}$$

as corrections to the galileon model (5.49) (see e.g. [40]). Relative to the canonical kinetic term, in the presence of a massive source, these operators become important at a scale

$$r_Q \sim \frac{1}{\Lambda}\left(\frac{M}{M_{\rm pl}}\right)^{\frac{2-n}{4-3n}}. \tag{5.125}$$

In the limit $n \to \infty$, we have $r_Q \to r_V$ where r_V is the Vainshtein radius for galileons and dRGT massive gravity. Therefore we cannot consider the non-linear terms in the galileon model non-perturbatively without including the effects of an infinite number of quantum corrections. This is also true for our goldstone example, since we would expect that as the operator $(\partial\phi)^4$ becomes important, an infinite tower of $(\partial\phi)^{2n}$ would too, dominated by the $n \to \infty$ limit. In both cases we can understand this qualitatively. For galileons the onset of Vainshtein screening occurs when the *acceleration* of the scalar field becomes large relative to the strong coupling scale i.e. when $\partial\partial\phi \sim \Lambda^3$. It is then clear that in the Vainshtein regime, the dominant term will be the operator with largest power of $\partial\partial\phi/\Lambda^3$. Similarly for the goldstone example, here the onset of screening is due to large *velocities* i.e. when $(\partial\phi)^2 \sim \Lambda^4$ and the largest power of $(\partial\phi)^2/\Lambda^4$ will dominate.

One way to keep control of these operators is if the low energy theory possesses a powerful symmetry. This is exactly what happens in GR where we saw that a hierarchy exists where we can trust the interactions in the Einstein-Hilbert action non-perturbatively, while keeping the higher order operators suppressed. This is because Lorentz invariance ensures that the co-efficient of each operator with two derivatives sums into the Ricci scalar. We can then organise the quantum corrections in a derivative expansion. However, enforcing Lorentz invariance in scalar field theories still leaves lots of room for a model builder to construct self-interactions. We therefore look for extra symmetries which fix the co-efficients of an infinite tower of operators. For example, consider the scalar sector of the DBI action [138], which has been used in the context of screening scalar forces in the solar system in [139]. It is built from an infinite tower of irrelevant operators of the form $c_n(\partial\phi)^{2n}$ but unlike a generic theory of this type it is invariant under

$$\phi \to \gamma(\phi + \Lambda^2 v_\mu x^\mu) \tag{5.126}$$

$$x^\mu \to x^\mu + \frac{\gamma - 1}{v^2} v^\mu v_\sigma x^\sigma + \gamma v^\mu \frac{\phi}{\Lambda^2} \tag{5.127}$$

where $\gamma = 1/\sqrt{1 - v^2}$. This symmetry ensures that the operators sum into

$$\Lambda^4 \sum_n c_n \frac{(\partial\phi)^{2n}}{\Lambda^{4n}} = -g\Lambda^4 \sqrt{1 + \frac{(\partial\phi)^2}{\Lambda^4}} \tag{5.128}$$

where $c_0 = g > 0$. The internal symmetry on ϕ has reduced the infinite number of coupling constants to a single overall constant. Now when the first irrelevant operator i.e. $(\partial\phi)^4$ becomes important, we do not lose control since we can consider the full square root structure. Also, the uniqueness of (5.128) with respect to the DBI symmetry ensures that this structure is not spoilt by quantum corrections. This is completely analogous to GR where the square root structure is replaced by the Ricci scalar which is protected from quantum corrections thanks to its uniqueness with respect to Lorentz invariance. This is the only known example of a non-linear symmetry, for scalar field theories with one derivative per field in flat space, which fixes the relative co-efficients between the operators. It may also be unique given that it is the only example where the theory enjoys an enhanced soft limit [128]. Such a symmetry is unknown for galileons.

This DBI structure can be thought of as a partial UV completion of (5.56), but as is the case in GR where we can generate higher order curvature invariants, we still expect quantum corrections to become important at some scale at which we should no longer trust the truncation to the square root. These quantum corrections will also be invariant under (5.126) but will involve at least one field with two derivatives. We shall discuss this in more detail in the next chapter where we exploit the time-dependent Vainshtein screening in this theory in the early universe. There we shall also study the structure of the DBI interactions and the symmetry (5.126) in more detail. Our purpose here is to merely emphasise the importance of symmetries in our ability to keep control when the Vainshtein mechanism is active.

Chapter 6

DBI Chameleon and the Early Universe

This chapter is based on an extension of the chameleon theory. The chameleon mechanism is a way to shut down scalar forces in local environments introduced by Khoury and Weltman in [41, 42]. However, it was shown in [43, 44] that the coupling between the chameleon and matter degrees of freedom, which is required to realise successful screening, causes a break down in the classical description of the chameleon around the period of Big Bang Nucleosynthesis (BBN) due to standard model fields transitioning from relativistic to non-relativistic. If the chameleon model is to describe a credible alternative theory of gravity, then one would expect to have control over the theory on these energy scales. The extension of the chameleon model described in this chapter is motivated by this issue and includes new interactions which extend the regime of validity of the theory.

We begin this chapter with an introduction to the chameleon mechanism followed by a detailed discussion of the issues raised in [43, 44], complimenting their numerical analysis with a dynamical systems approach. We will then show how these problems can be cured by correcting the chameleon model at high energies where we follow the work of [4]. The main idea is to combine the chameleon mechanism with the Vainshtein mechanism. The introduction of Vainshtein screening ensures that in the early universe the coupling between the scalar and matter fields is suppressed thereby allowing the model to evade the constraints on the matter coupling imposed in [43, 44]. The structure of the non-linear terms required for the Vainshtein screening is motivated by the results of chapter 5 and the resulting theory reduces to the original chameleon model when the Vainshtein mechanism is not active. However, when the derivative interactions become important we are able to trust the classical description of the model deep into the early universe, in contrast to the standard chameleon theory.

6.1 Introduction to the chameleon mechanism

In comparison to the Vainshtein mechanism which we have discussed in the preceding chapter, the chameleon mechanism is another way to suppress scalar forces in the solar system. The chameleon mechanism still relies on the presence of non-linearities but here the minimal shift symmetry on the scalar field used for Vainshtein screening is broken, and interactions appear in the scalar's potential rather than in the kinetic sector. Imperative to the effectiveness of the screening is a conformal coupling between the scalar field and the degrees of freedom in the matter sector which contributes to the effective potential experienced by the scalar.

When we discussed the Vainshtein mechanism in chapter 5 it was clear that the derivative interactions required there had a non-trivial origin. For example, we saw how one of the galileon interactions (5.49) appeared in the four dimensional effective action of DGP gravity [124] after integrating out the bulk degrees of freedom, and how together with the remaining ones controlled the behaviour of the longitudinal mode of the ghost free massive graviton [118,119]. We also saw how the Vainshtein mechanism can be realised in a theory of a goldstone boson associated with a broken U(1) symmetry. However, the chameleon mechanism has a different origin. Its introduction is much more phenomenological where it is assumed that light scalar fields with gravitational strength couplings to matter fields exist in Nature and we therefore require an explanation of why they are not observed locally[1].

In any case, the aim of the chameleon mechanism is to ensure that in regions of high density, such as in the solar system where no scalar forces are

[1]See [140] for attempts to embed chameleon theories within string theory.

observed, the effective mass of the chameleon is large such that the corresponding scalar force between massive bodies is short range. While in low density environments, the mass is small and the corresponding force long range. In principle, this would mean that the scalar field can play a non-trivial role on large distances but remain invisible to local gravitational experiments. However, it has been shown that under certain assumptions the chameleon cannot explain dark energy as a genuine modification of gravity [141, 142]. In [142] it was argued that one can place an upper bound on its Compton wavelength of order 1Mpc, three orders of magnitude below the current Hubble scale, and the corresponding Yukawa suppression means that its effects are limited to non-linear scales. Even with this in mind, the mechanism is still interesting from the point of view of suppressing scalar forces. This is especially true given the fact that string theory, our most promising candidate of high energy physics, is often plagued by many scalar fields with gravitational strength couplings after string compactifications (see e.g. [35]). In this sense the model warrants further study and scrutiny.

To understand the chameleon mechanism we start with the following action

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right] + S_m[\tilde{g}_{\mu\nu}, \Phi] \tag{6.1}$$

where matter fields are denoted collectively as Φ and move on geodesics of the conformally rescaled metric $\tilde{g}_{\mu\nu} = A^2(\phi)g_{\mu\nu}$ with $A(\phi)$ an *a priori* arbitrary function which we refer to as the *conformal factor*. $V(\phi)$ is the bare scalar potential. The resulting equation of motion for the scalar is

$$\Box\phi = V_{,\phi} - A^3(\phi)A_{,\phi}\tilde{T} \tag{6.2}$$

where $\tilde{T} = \tilde{g}^{\mu\nu}\tilde{T}_{\mu\nu}$ and

$$\tilde{T}_{\mu\nu} = -\frac{2}{\sqrt{-\tilde{g}}}\frac{\delta S_m}{\delta\tilde{g}^{\mu\nu}} \tag{6.3}$$

is the *Jordan frame* energy-momentum tensor. Diffeomorphism invariance tells us that it is covariantly conserved $\tilde{\nabla}^\mu\tilde{T}_{\mu\nu} = 0$ since matter fields couple minimally to $\tilde{g}_{\mu\nu}$. We have also used the notation $V_{,\phi} = \partial V/\partial\phi$. Variation with respect to the metric yields

$$M_{\rm pl}^2 G_{\mu\nu} = T^m_{\mu\nu} + T^\phi_{\mu\nu} \tag{6.4}$$

where $T^m_{\mu\nu}$ and $T^\phi_{\mu\nu}$ are respectively the energy-momentum tensor for the matter fields and the scalar field defined with respect to the Einstein frame metric. The Bianchi identity $\nabla_\mu G^{\mu\nu} = 0$ tells us that the Einstein frame energy-momentum tensor is not conserved and instead satisfies

$$\nabla_\mu T^{\mu\nu}_m = \frac{A_{,\phi}}{A} T_m \partial^\nu \phi \tag{6.5}$$

where we have used the scalar equation of motion (6.2) and $T_m = A^4 \tilde{T}$. If we now specialise to non-relativistic matter sources with $T_m = -\hat{\rho}$ and define $\rho = A^{-1}\hat{\rho}$, then on an FLRW background i.e. equation (2) with zero spatial curvature, and with a homogeneous configuration for the scalar, the 0-component of (6.5) is

$$\dot{\rho} + 3H\rho = 0 \tag{6.6}$$

where $H = \dot{a}/a$. In other words, the density ρ is conserved in the Einstein frame. In terms of ρ, the scalar equation of motion tells us that the scalar experiences an effective potential of the form

$$V_{\rm eff} = V(\phi) + A(\phi)\rho \tag{6.7}$$

and therefore depends on the bare potential $V(\phi)$ and on the matter energy density ρ. If the bare potential and the conformal factor slope in opposite directions, the effective potential will have an environmental minimum $\phi_{\rm min}$. The resulting effective mass of fluctuations about this minimum is therefore also ρ dependent and given by

$$m^2_{\rm eff}(\phi_{\rm min}) = V_{,\phi\phi}(\phi_{\rm min}) + A_{,\phi\phi}(\phi_{\rm min})\rho. \tag{6.8}$$

It is common in the literature to choose $V_{,\phi} < 0$ and $A_{,\phi} > 0$ such that the effective potential is of the form shown in figure (6.1) where we have chosen a linear conformal factor. The effective potential is the solid line while $V(\phi)$ and $A(\phi)$ are represented as dashed lines. The small gradient of the conformal factor corresponds to a small matter energy density. If we now increase the matter energy density then the effective potential is shown in figure (6.2) and it is clear that the result is to increase the curvature of the effective potential thereby increasing the effective mass of the field. This is the foundation of the chameleon mechanism.

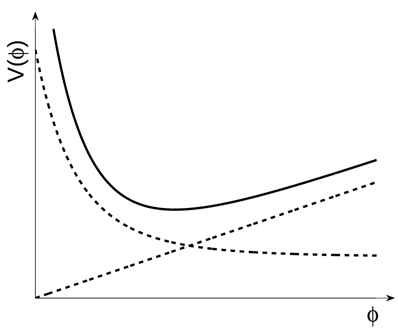


Figure 6.1: Low Density

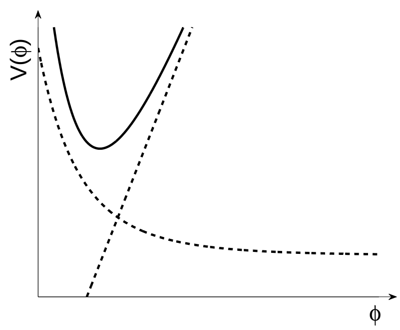


Figure 6.2: High Density

Let us illustrate the mechanism with an example. We consider a bare potential with an inverse power law behaviour

$$V(\phi) = \frac{M^{4+n}}{\phi^n} \tag{6.9}$$

with $n > 0$ and M is a dimensionful energy scale which is constrained to be $M \lesssim$ meV [41, 42]. This upper bound comes from ensuring that the model is consistent with all constraints on deviations from General Relativity (GR), in particular from the Eöt-Wash tests of the inverse square law [143] and (the slightly less constraining) Casimir force measurements [144,145]. This potential was the first to be considered in the context of the chameleon mechanism [41, 42]. It is common in the literature to choose $A(\phi) = \exp(\beta\phi/M_{\rm pl})$ where β is a dimensionless coupling constant such that when $\beta \sim \mathcal{O}(1)$, the scalar field couples to the matter degrees of freedom with gravitational strength mediating a force comparable to the spin-2 one in GR. For small field excursions relative to the Planck mass, as is required by constraints on the variation of particle masses from BBN until today [146], we can approximate the conformal factor as

$$A(\phi) \approx 1 + \frac{\beta\phi}{M_{\rm pl}}. \tag{6.10}$$

It follows that the minimum of the effective potential is

$$\phi_{\rm min} = \left(\frac{nM^{4+n}M_{\rm pl}}{\beta\rho}\right)^{\frac{1}{n+1}} \tag{6.11}$$

and the resulting effective mass of fluctuations about this minimum is

$$m_{\rm eff}^2 = n(n+1)M^{-\frac{n+4}{n+1}}\left(\frac{\beta\rho}{nM_{\rm pl}}\right)^{\frac{n+2}{n+1}}. \tag{6.12}$$

With $n > 0$ it is clear that the effective mass increases with ρ causing the fluctuations to decouple in dense environments even when $\beta \sim \mathcal{O}(1)$. We note that one can realise the chameleon mechanism with more familiar scalar field potentials which are analytic around $\phi = 0$ by reversing the sign of the matter coupling $\beta \to -\beta$ e.g. [147]. For a more detailed discussion of the chameleon mechanism we refer the reader to [28, 29, 41, 42].

6.2 Kicking the chameleon

We now come to the issues raised in [43, 44]. On-shell, the chameleon's coupling to matter degrees of freedom is via the trace of the energy-momentum tensor. In a radiation dominated universe we can model the matter degrees of freedom as a classical conformal field theory for which the trace of the energy-momentum tensor vanishes. Therefore one would naively expect that any interactions between the chameleon and matter degrees of freedom in a radiation dominated universe would only be realised through the mediation of virtual gravitons. Indeed, on a flat FLRW background the Einstein frame equations of motion from the chameleon action (6.1) are

$$3M_{\rm pl}^2 H^2 = \rho_\phi + \rho \tag{6.13}$$

$$M_{\rm pl}^2\left(2\dot{H} + 3H^2\right) = -p_\phi - p \tag{6.14}$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = -\frac{\beta}{M_{\rm pl}}\rho\Sigma \tag{6.15}$$

where $H = \dot{a}/a$ is the Hubble parameter, ρ and p are respectively the energy density and pressure of matter, and ρ_ϕ and p_ϕ are the energy density and pressure of the scalar field given respectively by

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{6.16}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{6.17}$$

We have also defined $\Sigma = (\rho - 3p)/\rho$. If the matter sector is dominated by a perfect fluid, we can write $\Sigma = 1 - 3w$ where w is the equation of state parameter. In a radiation dominated universe we have $w = 1/3$ and therefore $\Sigma = 0$ and the right hand side of equation (6.15) vanishes.

In the majority of situations this is a reasonable approximation. However, whenever a massive relativistic particle in the standard model becomes non-relativistic, Σ can become *temporarily* non-zero in a radiation dominated era. If we account for all standard model particles then Σ receives a number of *kicks* as a function of Jordan frame temperature $T_J \sim 1/a_J = e^{-\beta\phi/M_{\rm pl}}/a$ as shown in figure (6.3) extracted from [43].

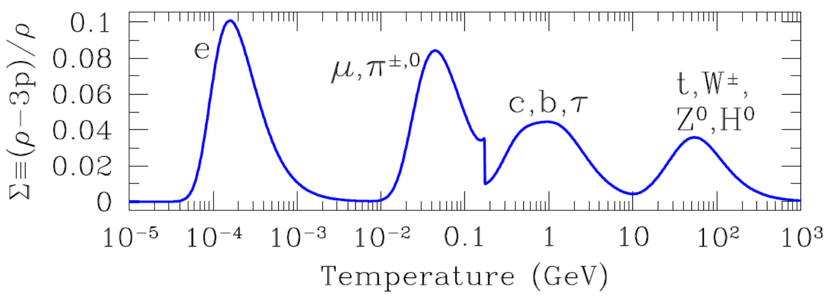


Figure 6.3: Plot of $\Sigma(T_J)$ showing four distinct kicks

These kicks occur as the temperature of the radiation bath is of order the mass of a given particle. There are four distinct kicks with Jordan frame temperatures varying from approximately 10^{-4}GeV, where the kick is due to the electron, to approximately 10^2GeV where the kick is due to the heaviest particles in the standard model e.g. the top quark. At temperatures larger than the scale of a given kick there is no contribution to Σ since the corresponding particles are relativistic. The contribution is also small at temperatures below the scale of the kick thanks to Boltzman suppression.

As shown in (6.3), the deviation of Σ from zero can be $\mathcal{O}(0.1)$ and can therefore play a non-trivial role on the chameleon's evolution. This was initially investigated in [148–150] and in [150] it was concluded to be a *positive* contribution since the primary effect of the kicks is to push the chameleon towards smaller values helping to satisfy BBN constraints in the process. A more thorough investigation was given in [43, 44] where quantum mechanical effects of the subsequent evolution after the kicks were considered. There it was concluded that the kicks have a *negative* impact on the chameleon cosmology rendering the classical treatment of the chameleon at BBN untrustworthy. This conclusion is due to the existence of a *surfer solution* with $\dot{\phi} = -M_{\rm pl}H/\beta$, which exists at constant Jordan frame temperature, forcing the chameleon to the minimum of its effective potential with a very large velocity relative to the energy scale M which controls the strength of the interactions in the bare potential. This is because the surfer solution is associated with standard model kicks in the early universe where the temperature was much larger than M which is set by late universe physics. Also, as we saw in the previous section it is common that $M \lesssim$ meV to satisfy fifth-force constraints. Generically, the velocity of the chameleon when it arrives at the minimum is set by the Jordan frame temperature of the most significant kick and has $|\dot{\phi}|^{1/2} > 0.07$MeV in the absence of fine tuning [43, 44]. The issue of having a very large velocity by the time the scalar reaches the minimum of the effective potential can be traced to the fact that the field is undergoing extremely large field excursions $\Delta\phi$ on the surfer solution.

After reaching the minimum, the chameleon rolls up the steep side of its effective potential until its kinetic energy runs out at the reflection point when it begins to roll back down. This behaviour results in rapid variations in the chameleon's mass over a very short time scale generating highly energetic fluctuations. The energy density of these fluctuations can become $\mathcal{O}(1)$ in units of the energy density stored in the background field thereby rendering any classical treatment untrustworthy and signalling the break down of the effective field theory (EFT) in the early universe. It was shown in [43, 44] that this surfer solution, and the resulting issues, existed for generic initial conditions provided $\beta > 1.82$. To avoid it one must therefore tune the coupling between the chameleon and matter fields which is undesirable for a number of reasons. Particularly, one would like to keep the theoretical constraints on the parameters of an effective theory minimal to maximise the chances of finding a UV completion. Also, fine tuning β would make one question the relevance of the many experimental tests of the chameleon parameter space which concentrate on the strongly coupled region (see e.g. [143,151–156]). This motivates looking for a dynamical solution to these issues that does not require a fine tuning of β.

Let us now show the existence of the surfer solution as a fixed line using a dynamical systems analysis. For the moment we shall ignore the scalar's potential as is done in [43, 44] and assume that in the early universe the trace of the energy-momentum tensor is the dominate source of the scalar's evolution. We assume that the potential decays at large field values such that the scale controlling the strength of these interactions enters $V(\phi)$ with a positive power. It is consistent to ignore the potential when $\phi \gg M$. This approximation will break down after the subsequent evolution when the chameleon approaches the minimum of its effective potential. Initially we eliminate all ρ dependence from the equations of motion which does not enter via Σ such that the cosmological evolution is described by

$$M_{\rm pl}^2 H^2(4-\Sigma) + 2M_{\rm pl}^2\dot{H} + \frac{(2+\Sigma)}{3}\rho_\phi = 0 \tag{6.18}$$

$$\ddot{\phi} + 3H\dot{\phi} + 3M_{\rm pl}\beta\Sigma H^2\left(1 - \frac{\rho_\phi}{3M_{\rm pl}^2H^2}\right) = 0. \tag{6.19}$$

This allows us to identify the surfer solution using an autonomous dynamical system. To do so we define the following variables

$$x = \beta\phi + M_{\rm pl}\ln a \tag{6.20}$$

$$y = \frac{\rho_\phi}{3M_{\rm pl}^2 H^2} \tag{6.21}$$

$$z = \beta\dot{\phi} + M_{\rm pl}H \tag{6.22}$$

such that the autonomous system is given by

$$\dot{x} = z \tag{6.23}$$

$$\dot{y} = \frac{1}{H_y}\left[3H_z H z - \frac{1}{2}M_{\rm pl}H_z H^2(1-y)(2+\Sigma(1-6\beta^2)) + \frac{H^2}{2}(\Sigma - 4 - y(\Sigma+2)\right] \tag{6.24}$$

$$\dot{z} = -3Hz + \frac{1}{2}M_{\rm pl}H^2(1-y)\left(2+\Sigma(1-6\beta^2)\right) \tag{6.25}$$

where e.g. $H_y = \partial H/\partial y$. Our choice of x is because $\dot{x} = z = 0$ corresponds to a constant Jordan frame temperature which characterises the surfer solution [43, 44]. y is the ratio between the energy density of the chameleon and the critical energy density. The Hubble parameter $H(y,z)$ is given implicitly by

$$M_{\rm pl}^2 H^2 y = \frac{(z - M_{\rm pl}H)^2}{6\beta^2} \tag{6.26}$$

which can be solved to give

$$H(y,z) = \frac{z}{M_{\rm pl}(1 \pm \sqrt{6\beta^2 y})}. \tag{6.27}$$

Assuming the conformal coupling to matter decreases over time, equation (6.25) tells us that $\frac{z}{M_{\rm pl}} < H$ which is only satisfied for all β, y if we take the lower root of (6.27).

There is a trivial set of fixed points with $y \neq 1/6\beta^2$, x an arbitrary constant and $z = H = 0$. This corresponds to an empty universe and is therefore of little interest to cosmology and the chameleon kicks. However, there is also a non-trivial set of fixed points which corresponds to $\Sigma = \Sigma_c = 2/(6\beta^2 - 1)$ with $y = 1/6\beta^2$, x again arbitrary and $z = 0$. In this case $H(y,z)$ is *unconstrained.*

This line of fixed points has a constant Jordan frame temperature and can exist at any scale as long as Σ passes through the critical value Σ_c. These fixed points correspond to the surfer solution. The form of Σ_c makes it clear that by making β sufficiently small, the solution will not exist with a matter sector constructed from standard model fields since in figure (6.3) we have $\Sigma \lesssim \mathcal{O}(0.1)$ for all temperatures.

To see this more clearly we solve the dynamical system numerically by setting $\Sigma = \Sigma_c$ and considering a range of initial conditions. We show the (H, z) plane of the trajectories below in figure (6.4) where we can see that the line $z = 0$ is an attractor for all $H(y, z)$.

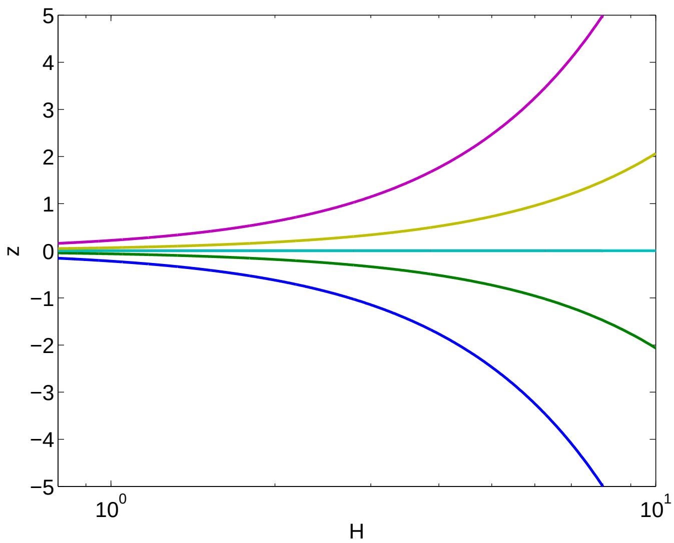


Figure 6.4: Plot of z against H for a range of initial conditions with $\Sigma = \Sigma_c$. Figure produced by Toby Wilson.

In a more realistic model Σ is not a constant and will vary from the critical value as the Jordan frame temperature varies as shown in figure (6.3). Given our definition of dynamical variables we have $\Sigma = \Sigma(T_J = T_{J0}e^{-x/M_{\rm pl}})$. Let us now confirm the attractive behaviour shown in figure (6.4) analytically by expanding around the surfer solution ($x = x_0,\ y = 1/6\beta^2,\ z = 0$) and allowing $\Sigma(x)$ to deviate from the critical value. We set $x = x_0 + \delta x$, $y = 1/6\beta^2 + \delta y$, $z = \delta z$ and $\Sigma(x) = 2/(6\beta^2 - 1) + \delta\Sigma = 2/(6\beta^2 - 1) + \Sigma'(x_0)\delta x$ and expand the dynamical system to leading order yielding

$$\delta\dot{x} \approx \delta z \tag{6.28}$$

$$\delta\dot{y} \approx \frac{1}{3M_{\rm pl}\beta^2}\delta z + H\left(\frac{1}{6\beta^2}-1\right)^2\Sigma'(x_0)\delta x \tag{6.29}$$

$$\delta\dot{z} \approx -3H\delta z - 3\beta^2 M_{\rm pl}H^2\left(\frac{1}{6\beta^2}-1\right)^2\Sigma'(x_0)\delta x \tag{6.30}$$

where the constraint (6.26) gives $\delta z = -3M_{\rm pl}\beta^2 H\delta y$. The form of each solution to $\delta x, \delta y, \delta z$ is $\mathcal{A}_+\exp(m_+t) + \mathcal{A}_-\exp(m_-t)$ where

$$m_\pm = -\frac{3H}{2}\left(1 \pm \sqrt{1-\frac{4}{3}\beta^2 M_{\rm pl}\left(\frac{1}{6\beta^2}-1\right)^2\Sigma'(x_0)}\right). \tag{6.31}$$

For a given kick, Σ will generically pass through the critical value required for the surfer solution to exist at two different temperatures $T_J^{c1,c2}$ either side of the kick's peak. At the larger temperature T_J^{c1}, Σ is rising from zero towards its peak with $d\Sigma/dT_J < 0$ while at the smaller temperature T_J^{c2} it is decreasing with $d\Sigma/dT_J > 0$. Even if the solution with the larger temperature is unstable and not attracted to the $z = 0$ line, the smaller one could be and since this is experienced last, it is the most important. At this temperature $\Sigma'(x_0) = -\frac{T_J^{c2}}{M_{\rm pl}}d\Sigma/dT_J < 0$ and therefore m_+ is real and negative while m_- is real and positive. So for m_+ the fluctuations around the surfer solution *decay*. This attracts the solution towards to the surfer reinforcing the numerical results in figure (6.4). The solution with m_- represents an instability for the surfer but this instability is very mild because for physically realistic kicks we have $M_{\rm pl}|\Sigma'(x_0)| \sim \Sigma(x_0) = 2/(6\beta^2-1)$ and so $m_- \lesssim H$. This means that even over a Hubble time the fluctuation is approximately constant and can be absorbed into a re-definition of x_0.

6.3 Introduction to the DBI chameleon

The primary cause of the break down in the chameleon's classical description is its conformal coupling to matter with $\beta \sim \mathcal{O}(1)$. If we tuned β to much smaller values then the effect of the kicks on the chameleon's evolution would be suppressed and the troublesome surfer solution would no longer exist [43,44]. As we have mentioned this is not a satisfactory solution so we seek to find another route where we can suppress the effects of the kicks but maintain $\beta \sim \mathcal{O}(1)$. In other words, we want to weaken the coupling to the matter degrees of freedom *dynamically*. As we saw in the previous chapter, this is precisely what the Vainshtein mechanism does.

With this motivation, we correct the original chameleon theory (6.1) such that it can realise the Vainshtein mechanism in the early universe when standard model particles become non-relativistic, while retaining its screening features in the solar system. To do so we must include derivative self interactions. We argued in the previous chapter that a generic theory with $(\partial\phi)^{2n}$ operators loses control at the scale where the non-linearities kick in. We also argued that the galileon interactions (5.49) are just as untrustworthy in the non-linear regime. In this sense corrections of either of these forms would not allow us to extend the regime of validity of the chameleon theory. We therefore choose to correct (6.1) using the square root structure of the DBI interactions (5.128). The non-trivial symmetry (5.126) associated with these interactions ensures that we can trust the theory when the Vainshtein mechanism is active since it fixes each co-efficient in an expansion of $(\partial\phi)^{2n}$ operators. Also, in a pure DBI theory the co-efficients are not renormalised by loop corrections. Our corrected chameleon action is therefore

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R + \Lambda^4 - \Lambda^4\sqrt{1+\frac{(\partial\phi)^2}{\Lambda^4}} - V(\phi)\right] + S_m[\tilde{g}_{\mu\nu},\Phi] \quad (6.32)$$

which we refer to as the *DBI chameleon*. We recover the original chameleon action (6.1) to leading order when $(\partial\phi)^2 \ll \Lambda^4$ i.e. at small field gradients. The next to leading order operator coming from the square root is $(\partial\phi)^4$ with a *positive* coefficient. The fact that this co-efficient is positive ensures that the derivative self-interactions can be realised after integrating out heavy physics in a Lorentz invariant UV completion [131]. As we discussed in the previous chapter, this simultaneously ensures that the Z factor becomes large, as required for the Vainshtein mechanism, on a homogeneous background. This is exactly the behaviour we are after since we wish to screen in a cosmological setting in the early universe.

Let us discuss the DBI structure in more detail by sending $M_{\rm pl} \to \infty$ to decouple gravity and dropping the potential $V(\phi)$. The action is now equivalent to (5.128) with a cosmological constant. To see the growing Z factor on homogeneous backgrounds we expand the action to quadratic order around a time dependent solution $\phi(x) = \bar{\phi}(t) + \pi(x)$ yielding $\delta\mathcal{L} \sim \frac{1}{2}Z^{\mu\nu}\partial_\mu\pi\partial_\nu\pi$ where

$$Z^{tt} = \left(1 - \frac{\dot{\bar{\phi}}^2}{\Lambda^4}\right)^{-3/2}, \qquad Z^{ti} = 0, \qquad Z^{ij} = \left(1 - \frac{\dot{\bar{\phi}}^2}{\Lambda^4}\right)^{-1/2}\delta^{ij}. \tag{6.33}$$

As $\dot{\bar{\phi}}^2 \to \Lambda^4$ the Z factor grows rendering the coupling between the fluctuation and matter degrees of freedom weaker than gravity without tuning β, as desired. In a cosmological setting the DBI symmetry (5.126) is

$$t \to \frac{1}{\sqrt{1-v^2}}\left(t - v\frac{\phi}{\Lambda^2}\right), \qquad \phi \to \frac{1}{\sqrt{1-v^2}}(\phi - \Lambda^2 vt). \tag{6.34}$$

Although this symmetry ensures that loop corrections do not spoil the square root structure, loops will generate other operators also invariant under (6.34). To see the structure of these operators it is instructive to understand the origin of the DBI interactions and the symmetry from a higher dimensional picture [157]. Consider a probe brane localised at $y = \phi(x^\mu)/\Lambda^2$ in a 5D Minkowski bulk with $ds^2 = \eta^{\mu\nu}dx^\mu dx^\nu + dy^2$ such that y is the coordinate of the fifth dimension and x^μ are four dimensional. Given that $dy = \partial_\mu\phi\, dx^\mu/\Lambda^2$, the induced metric on the brane is

$$\hat{g}_{\mu\nu} = \eta_{\mu\nu} + \frac{\partial_\mu\phi\partial_\nu\phi}{\Lambda^4} \tag{6.35}$$

with an inverse

$$\hat{g}^{\mu\nu} = \eta^{\mu\nu} - \frac{\partial^\mu\phi\partial^\nu\phi}{\Lambda^4 + (\partial\phi)^2}. \tag{6.36}$$

The resulting brane action can be constructed out of $\hat{g}_{\mu\nu}$ and its derivatives. To zero-th order in a derivative expansion diffeomorphism invariance on the brane dictates that the action is

$$\hat{S} = -\Lambda^4 \int d^4x \sqrt{-\hat{g}} = -\Lambda^4 \int d^4x \sqrt{1 + \frac{(\partial\phi)^2}{\Lambda^4}} \tag{6.37}$$

where Λ^4 is the brane tension and we have recovered the DBI interactions. We can now understand the origin of the DBI symmetry (5.126) as coming from the five dimensional Lorentz invariance of the higher dimensional theory.

If we wanted to screen on a static background with this theory we would require the $(\partial\phi)^4$ term to have a negative co-efficient which is realised if we take $\phi \to i\phi$. In the higher dimensional theory this corresponds to a flat bulk with an $SO(2,3)$ rather than $SO(1,4)$ symmetry, or in other words, a space-time with *two* time directions. Again, we are seeing how screening on static backgrounds seems to be incompatible with Lorentz invariant completions [131] as we discussed in chapter 5. In this case screening has been investigated in [139]. At this point let us emphasis that in our case we do not require the Vainshtein mechanism to suppress the scalar force in the solar system. In local environments the chameleon limit will be a good approximation and the chameleon mechanism will be active there. In the presence of an extended object with a constant density profile, static and spherically symmetric solutions to theories constructed from ϕ and $\partial_\mu\phi$ have been studied in [36]. It is shown which regions of parameter space admit sensible power law solutions without sharp spatial gradients, at least in the limits where analytical results can be found. The simple monomial solution considered in [36] is not realised by the DBI chameleon theory when the potential is chosen to allow for the chameleon mechanism. It would be interesting to perform a more in depth analysis, for example looking for polynomial solutions and/or relaxing the assumption of a constant density profile for the extended source, to see if well behaved solutions exist[2].

The higher order corrections in the effective action for ϕ correspond to operators in the probe brane action with derivatives on the induced metric. These can be encoded in increasing powers of the extrinsic curvature on the

[2]As emphasised in [139], the DBI symmetry means that a constant gradient profile can be removed by a boost and is therefore unobservable. So in our case any observable constant gradient profile must be proportional to the symmetry breaking parameters.

brane and covariant derivatives of the extrinsic curvature. For the induced metric (6.35) the extrinsic curvature is

$$\hat{K}_{\mu\nu} = \frac{\gamma \nabla_\mu \nabla_\nu \phi}{\Lambda^2} \tag{6.38}$$

where

$$\gamma = \frac{1}{\sqrt{1 + \frac{(\partial\phi)^2}{\Lambda^4}}} \tag{6.39}$$

is the gamma factor. Given the equations in (6.33), an increasing Z factor and decreasing matter coupling corresponds to an increasing gamma factor. We note that more generally the action can contain terms built of the Riemann curvature of $\hat{g}_{\mu\nu}$ but these can be eliminated in favour of extrinsic curvature terms by the Gauss-Codazzi relation $R_{\mu\nu\sigma\rho} = K_{\mu\sigma}K_{\nu\rho} - K_{\mu\rho}K_{\nu\sigma}$. The general effective theory on the brane is therefore

$$\hat{S} = -\Lambda^4 \int d^4x \sqrt{-\hat{g}} \left[1 + \frac{a}{\Lambda}\hat{K} + \frac{b_1}{\Lambda^2}\hat{K}^2 + \frac{b_2}{\Lambda^2}\hat{K}_{\mu\nu}\hat{K}^{\mu\nu} + \dots \right] \tag{6.40}$$

where indices are raised and lowered with $\hat{g}_{\mu\nu}$ e.g. $\hat{K} = \hat{g}^{\mu\nu}\hat{K}_{\mu\nu}$, and $a, b_1, b_2, \dots$ are arbitrary dimensionless coupling constants. The ellipses denote operators with at least three powers of the extrinsic curvature and all powers which involve derivatives of the extrinsic curvature. Since all these operators are also invariant under the DBI symmetry we include them all in our correction to the chameleon action. Assuming that the dimensionless coupling constants are $\mathcal{O}(1)$, the truncation to (6.37) which is sufficient to realise the Vainshtein mechanism is therefore valid as long as $K^\mu{}_\nu \ll \Lambda$ where

$$K^\mu{}_\nu = \frac{\gamma}{\Lambda^2}\nabla^\mu\nabla_\nu\phi - \frac{\gamma^3}{\Lambda^6}\nabla^\sigma\phi\nabla^\mu\phi\nabla_\nu\nabla_\sigma\phi. \tag{6.41}$$

On a cosmological background, the DBI invariant quantum corrections to the (6.37) are small if

$$\frac{\gamma H\dot{\phi}}{\Lambda^3} \ll 1 \qquad \frac{\gamma^3\ddot{\phi}}{\Lambda^3} \ll 1. \tag{6.42}$$

On a background with $K \sim \Lambda$, an infinite tower of irrelevant operators becomes important and the effective theory on the brane runs out of control. In the following section where we study the evolution of the DBI chameleon action in the presence of the standard model kicks, we will check that these conditions are satisfied on the scales of interest. One may worry that the positive powers of γ in (6.42) makes it difficult to satisfy these constraints since the growing Z factor required for screening corresponds to growing γ. However, we can still realise the Vainshtein mechanism while maintaining $\gamma \sim \mathcal{O}(1)$. In this case the higher order operators are suppressed if $H \ll \Lambda$ where we have taken $\dot{\phi}^2 \sim \Lambda^4$ in the DBI limit and assumed $\ddot{\phi} \sim H\dot{\phi}$.

For arbitrary co-efficients in the brane action, operators with two or more powers of the extrinsic curvature lead to higher order equations of motion for ϕ. However, if one tunes the coefficients in (6.40) such that the equations of motion for ϕ are second order, in the small field limit one recovers the galileon interactions (5.49) [157,158]. This makes the connection between the structure of the galileon terms and the second order nature of their equations of motion manifest since this tuning corresponds to a brane action constructed from the Lovelock invariants [159]. For example, at quadratic order in the extrinsic curvature we require $b_1 = -b_2$ for the resulting ϕ operators to have second order equations of motion and by the Gauss-Codazzi relation, this tuning leads to the Ricci scalar in the brane action. In the small field limit this operator corresponds to the quartic galileon. This also emphasises the fact that to trust the truncation to the galileon interactions (5.49) we must tune the co-efficients of operators which would otherwise lead to higher order equations of motion. We do not have any reason to do this unless the tuning is protected by some symmetry which in this case it is not. Other maximally symmetric bulk geometries were also considered in [157].

Here we have argued that the neat properties of the DBI interactions are vital in our ability to extend the validity of the chameleon theory and we have calculated the conditions we need to satisfy to keep the higher order operators also invariant under the DBI symmetry negligible. However, in the full DBI chameleon action (6.32) the symmetry is broken by the potential $V(\phi)$ and at finite $M_{\rm pl}$. One might expect that the neat properties are lost in the full theory since loop corrections associated with these interactions can indeed spoil the square root structure and the structure of the higher order operators. However, quantum gravity corrections induced at finite $M_{\rm pl}$ are Planck suppressed and can be consistently ignored with $\Lambda \ll M_{\rm pl}$. Similarly, corrections due to the scalar field's conformal coupling to matter can be ignored with $\Lambda \ll M_{\rm pl}/\beta$. Both of these will be trivially satisfied, in the second case even for a wide range of β values, since the scale of the standard model kicks is at least 15 orders of magnitude below the Planck scale. We can also ignore corrections due to the potential. Indeed, if we assume that the potential is analytic at large values of ϕ, as is usually the case in chameleon theories e.g. the potential (6.9), the mass scale M which controls the strength of these interactions enters the action with positive powers such that they become infinitely weak when $M \to 0$. Therefore any breaking of the square root structure associated with the potential comes with a factor of $\left(\frac{M}{\Lambda}\right)^n$, with $n > 0$, relative to the tree level operators. We can therefore neglect these terms when $M \ll \Lambda$. This is in fact also a natural hierarchy for the theory to posses since M is controlled by physics in the late universe while the scale Λ is set by scale of the standard model kicks in the early universe.

In conclusion, with $M \ll \Lambda \ll M_{\rm pl}$, while satisfying the conditions (6.42), we can trust the truncation to, and the structure of, the DBI interactions in the full theory even in the limit $\dot{\phi}^2 \to \Lambda^4$ where the Vainshtein mechanism is active. We also note that the DBI structure imposes a speed limit on the scalar ensuring that $\dot{\phi} < \Lambda^2$ throughout evolution. This gives us a sound basis for nullifying the effects of the standard model kicks.

6.4 Kicking the DBI chameleon

We now return to the standard model kicks. In this section we again study the consequences of the kicks on the evolution of the scalar but now we do so in the DBI chameleon theory (6.32) and show how their adverse effects on the chameleon cosmology found in [43, 44] is nullified with a judicious choice of Λ. We do so using a combination of analytic and numerical results.

We saw in section 6.2 that the surfer solution $\dot{\phi} = -M_{\rm pl}H/\beta$, which corresponds to a constant Jordan frame temperature, could exist at any scale $M_{\rm pl}H$ when the kicks are present in the field theory sector and with Σ passing through $\Sigma_c = 2/(6\beta^2 - 1)$. With the inclusion of the DBI interactions we expect the surfing behaviour to be spoiled when $\dot{\phi}^2 \sim \Lambda^4$ i.e. when the DBI interactions are strong and the Vainshtein mechanism is active. Given the form of the surfer solution we must therefore choose $\Lambda^2 \lesssim M_{\rm pl}H/\beta$ to invalidate it. We note that with a hierarchy between Λ and $M_{\rm pl}$ which is required to avoid quantum gravity corrections to the DBI chameleon action, we can satisfy this bound while maintaining $H \ll \Lambda$ which ensures that the higher order operators are suppressed.

We begin by first setting $\Sigma = \Sigma_c$ and studying the resulting dynamics. This allows us to see the positive contributions from the DBI interactions at work. In this section we will not be interested in modelling kicks at realistic energy scales our only aim is to prove the effectiveness of the DBI interactions in simple scenarios. We will then allow Σ to vary as a function of Jordan frame temperature representing a more realistic description of the field theory sector. Here we will work with energy scales corresponding to the standard model kicks. We will concentrate on a single kick and approximate it as a Gaussian function which for our purposes is a good approximation to the true functions in figure (6.3).

6.4.1 Constant Σ

Let us define $P(X) = \Lambda^4 - \Lambda^4\sqrt{1 - \frac{2X}{\Lambda^4}}$ where $X = -(\partial\phi)^2/2$ such that the cosmological equations of motion coming from the DBI chameleon action (6.32) are

$$3M_{\rm pl}^2 H^2 = \rho_\phi + \rho \tag{6.43}$$

$$M_{\rm pl}^2\left(2\dot{H} + 3H^2\right) = -p_\phi - p \tag{6.44}$$

$$(P_{,X} + 2XP_{,XX})\ddot{\phi} + 3HP_{,X}\dot{\phi} + V'(\phi) = -\frac{\beta}{M_{\rm pl}}\rho\Sigma \tag{6.45}$$

where as before p and ρ are the pressure and energy density of the matter degrees of freedom respectively, $\Sigma = 1 - 3w$ when the matter sector is dominated by a perfect fluid with $p = w\rho$, and $P_{,X} = \partial P/\partial X$. The energy density of the scalar field is now $\rho_\phi = 2XP_{,X} - P(X)$ and the pressure is $p_\phi = P(X)$. As a consistency check we recover the corresponding equations for the chameleon when we decouple the DBI interactions by sending $\Lambda \to \infty$. Following section 6.2, we drop the potential $V(\phi)$ and eliminate all ρ dependence which does not enter the equations of motion via Σ yielding the following two equations

$$M_{\rm pl}^2 H^2(4-\Sigma) + 2M_{\rm pl}^2\dot{H} + p_\phi - \rho_\phi + \frac{(2+\Sigma)}{3}\rho_\phi = 0 \tag{6.46}$$

$$(P_{,X} + 2XP_{,XX})\ddot{\phi} + 3HP_{,X}\dot{\phi} + 3M_{\rm pl}\beta\Sigma H^2\left(1 - \frac{\rho_\phi}{3M_{\rm pl}^2 H^2}\right) = 0. \tag{6.47}$$

To clearly see the effect of the DBI interactions with finite Λ we use the same set of dynamical variables (6.20,6.21,6.22) as we did for the chameleon case such that these equations of motion can be expressed as the following autonomous system

$$\dot{x} = z \tag{6.48}$$

$$\begin{aligned}\dot{y} =& \frac{1}{H_y}\Big[3H_z H s^2 z - \frac{1}{2}M_{\rm pl}H_z H^2[(1-y)(2+\Sigma(1-6\beta^2 s^3))\\ &+ 3y(1-s) - 6(1-s^2)] + \frac{H^2}{2}(\Sigma - 4 - y(\Sigma+2) + 3y(1-s))\Big]\end{aligned} \tag{6.49}$$

$$\dot{z} = -3Hs^2 z + \frac{1}{2}M_{\rm pl}H^2\left[(1-y)\left(2+\Sigma(1-6\beta^2 s^3)\right) + 3y(1-s) - 6(1-s^2)\right] \tag{6.50}$$

where we have defined

$$s = \sqrt{1 - \frac{(z - M_{\rm pl}H)^2}{\beta^2\Lambda^4}} \tag{6.51}$$

and e.g. $H_y = \partial H/\partial y$. We remind the reader that our choice of x is such that $z = \dot{x} = 0$ represents a constant Jordan frame temperature which characterises the surfer solution. The Hubble parameter $H(y,z)$ is now given implicitly by the equation

$$\left(\Lambda^4 - \frac{(z - M_{\rm pl}H)^2}{\beta^2}\right)\left(1 + \frac{3M_{\rm pl}^2 H^2 y}{\Lambda^4}\right)^2 = \Lambda^4. \tag{6.52}$$

In the chameleon limit we found two sets of fixed points. The first set was trivial and the surfer solution with $z = 0$ only existed in an empty universe with $H = 0$. However, the interesting line of fixed points had $z = 0$ but with y fixed such that H remained unconstrained. This ensured that the surfer could exist at any energy scale. To see when this behaviour is realised in the presence of the DBI interactions we set $z = 0$ in equation (6.52) yielding a polynomial in $M_{\rm pl}H$ given by

$$\left(\frac{9y^2}{\beta^2\Lambda^8}\right)(M_{\rm pl}H)^6 + \left(\frac{6y}{\beta^2\Lambda^4} - \frac{9y^2}{\Lambda^4}\right)(M_{\rm pl}H)^4 + \left(\frac{1}{\beta^2} - 6y\right)(M_{\rm pl}H)^2 = 0. \tag{6.53}$$

When does this equation fail to constrain the Hubble parameter? For this to happen we require the co-efficient of each power of $M_{\rm pl}H$ to vanish which can only happen in two different ways. The first is by taking $\Lambda \to \infty$ and setting $y = 1/6\beta^2$. This limit simply reduces the theory to the original chameleon model and this choice of y is precisely the one we found in section 6.2. It is therefore not surprising that $H(y, z)$ is unconstrained in this case. The only other way is to take $\beta \to \infty$ and set $y = 0$. It is also not surprising that these choices allow the surfer to appear at any energy scale since this limit makes the scalar field strongly coupled to the matter sector and the conformal coupling dominates over the suppression coming from the DBI interactions for all finite Λ. Therefore, there is no way for the surfer to exist in the DBI chameleon theory at an arbitrary energy scale when we have finite Λ and $\beta \sim \mathcal{O}(1)$.

We can see this behaviour numerically by solving the dynamical system for a range of initial conditions. To do so we set $\Sigma = \Sigma_c = 2/(6\beta^2 - 1)$ and initially work with finite Λ and $\beta \sim \mathcal{O}(1)$. The resulting trajectories in the (H, z) plane are shown below in figure (6.5). This plot confirms that when the DBI interactions are strong at high energies, the surfer solution is no longer an attractor. The stable $z = 0$ line is only present at low energies when $H \lesssim \Lambda^2/M_{\rm pl}$ which is what we expect since there the DBI interactions are weak. This is because we are working with a constant Σ so as the temperature cools the surfer will inevitably appear when the theory becomes a good approximation to the original chameleon theory. For these trajectories we have set $\Lambda/M_{\rm pl} \sim 5\times10^{-3}$ such that $H \ll \Lambda$ throughout ensuring that the higher order corrections are suppressed relative to the DBI interactions. In figure (6.6) we plot the trajectories with the same initial conditions and the same finite Λ but now with β increased by an order of magnitude. As expected we see the surfing behaviour reappearing at high energies due to the strong coupling between the scalar and matter fields. We see similar results in figure (6.7) where we again we use the same initial conditions, $\beta \sim \mathcal{O}(1)$ but have increased the value of Λ by an order of magnitude such that the DBI interactions are weak over the full trajectories with $H \lesssim \Lambda^2/M_{\rm pl}$ at all times.

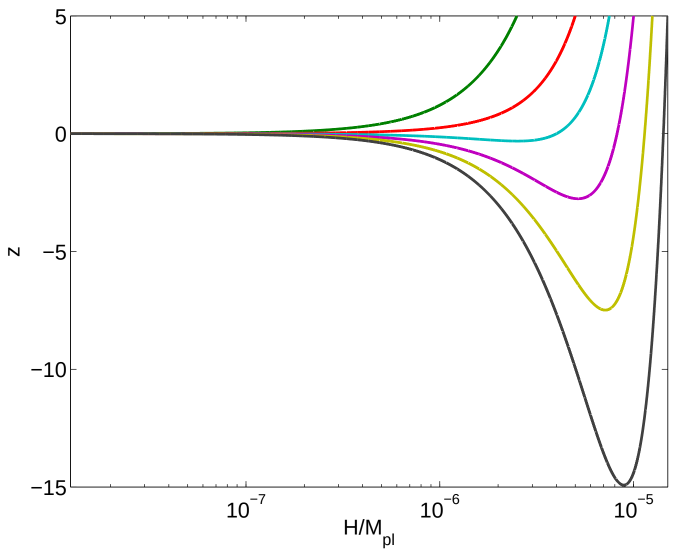


Figure 6.5: Plot of z against H, with finite Λ and $\beta \sim \mathcal{O}(1)$, for a range of initial conditions with $\Sigma = \Sigma_c$. Figure produced by Toby Wilson.

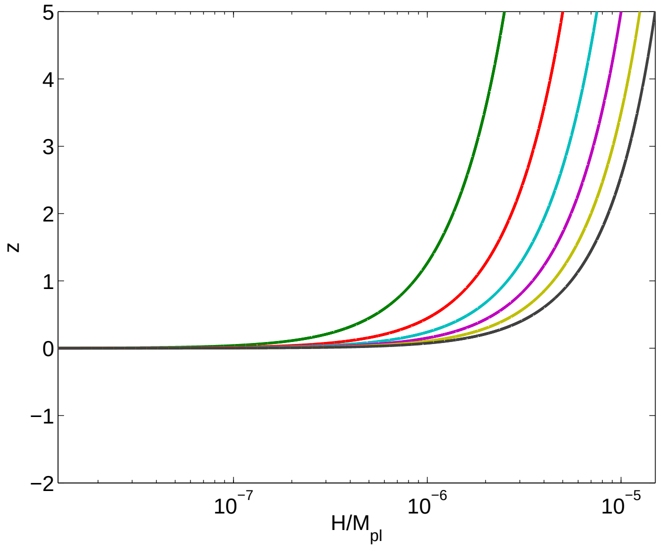


Figure 6.6: Plot of z against H, with finite Λ and $\beta \gg 1$, for a range of initial conditions with $\Sigma = \Sigma_c$. Figure produced by Toby Wilson.

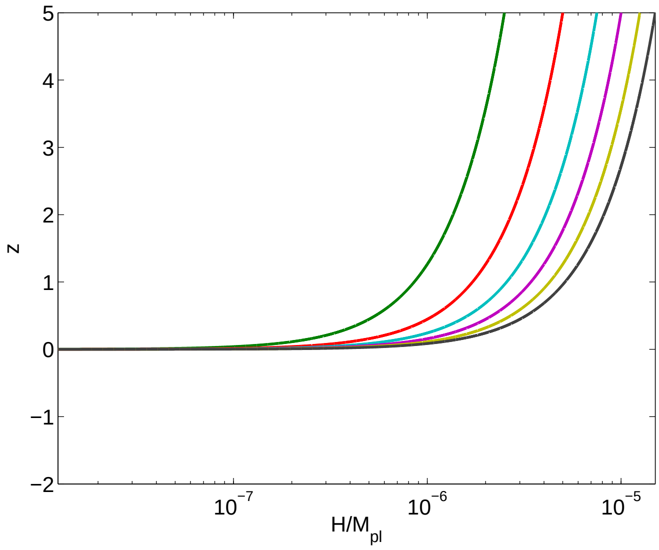


Figure 6.7: Plot of z against H, with $\Lambda^2 > M_{\rm pl}H$ and $\beta \sim \mathcal{O}(1)$ for a range of initial conditions with $\Sigma = \Sigma_c$. Figure produced by Toby Wilson.

We remind the reader that in these plots we have set $\Sigma = \Sigma_c$ which is not a realistic model. In reality Σ will vary from the critical value as the Jordan frame temperature varies as shown in figure (6.3). Our aim here is to merely prove that for a given energy scale, the surfing behaviour can be nullified for a particular choice of Λ without tuning β. For the standard model kicks, the last and most significant kick is due to the electron with $M_{\rm pl}H \sim (\mathrm{MeV})^2$. We must therefore set $\Lambda \lesssim \mathrm{MeV}/\beta$ to suppress the effects of the kicks in a realistic scenario.

Having shown that the DBI interactions do indeed have the desired effect, we must still check that the scalar is prevented from undergoing large field excursions. It was the large field excursions in the chameleon theory which caused the chameleon to reach the minimum of its effective potential with a high velocity resulting in the break down in its classical description. The field excursions are given by

$$\Delta\phi = \int_{t_i}^{t_i+\Delta t} \dot{\phi}\,\mathrm{d}t \tag{6.54}$$

where Σ is well approximated by Σ_c between times t_i and $t_i+\Delta t$. The size of the field excursions will generically depend on the initial value of z and since we expect the maximum variations to come from the surfer solution we set $z(t_i)=0$ such that $\dot{\phi}(t_i) = -M_{\rm pl}H/\beta$. It follows from equation (6.54) that

$$\frac{\Delta\phi}{M_{\rm pl}} \approx -\frac{H\Delta t}{\beta}. \tag{6.55}$$

Small variations in Σ corresponds to small variations in the Jordan frame temperature which are given by

$$\frac{\Delta T_J}{T_J} = -\int_{t_i}^{t_i+\Delta t} \frac{z}{M_{\rm pl}}\,\mathrm{d}t \tag{6.56}$$

which we can approximate as

$$\frac{\Delta T_J}{T_J} \approx -\frac{\dot{z}(t_i)}{M_{\rm pl}}\frac{\Delta t^2}{2} \tag{6.57}$$

since $z(t) \approx \dot{z}(t_i)(t-t_i)$. We can use equation (6.50) to extract $\dot{z}(t_i)$ by setting $z(t_i)=0$, $y(t_i)\sim\mathcal{O}(1)$ and $\Sigma=\Sigma_c$ yielding

$$\dot{z}(t_i) \sim \mathcal{O}(1)\frac{M_{\rm pl}^3H^4}{\beta^2\Lambda^4}. \tag{6.58}$$

Plugging this into (6.57) and combining the result with (6.55) leads to

$$\frac{\Delta\phi}{M_{\rm pl}} \approx \mathcal{O}(1)\frac{\Lambda^2}{M_{\rm pl}H}\sqrt{\left|\frac{\Delta T_J}{T_J}\right|}. \tag{6.59}$$

The assumption of $\Sigma \approx \Sigma_c$ is only valid when $\left|\frac{\Delta T_J}{T_J}\right| \lesssim 1$ therefore we have

$$\frac{\Delta\phi}{M_{\rm pl}} \lesssim \frac{\Lambda^2}{M_{\rm pl}H}. \tag{6.60}$$

This shows that as we decrease the value of Λ, thereby increasing the strength of the DBI interactions, we decrease the field excursions of the scalar, as desired. We shall confirm this heuristic estimate numerically when we consider non constant Σ in the next section.

6.4.2 Varying Σ

We now study the evolution of the DBI chameleon in the presence of a non-constant kick where $\Sigma = \Sigma(T_J)$. In this section we shall use slightly different variables by defining a dimensionless field $\varphi = \phi/M_{\rm pl}$ and using Einstein frame e-folds $N = \ln a$ instead of cosmological time as the evolution variable. These were the variables used in [43, 44] when surfer was first identified. In this case it corresponds to $\varphi' = -1/\beta$ where a prime denotes differentiation with respect to N. The equations of motion for the DBI chameleon (6.46,6.47) are now

$$2\frac{H'}{H} + (4-\Sigma) + \frac{1}{M_{\rm pl}^2H^2}\left[p_\phi - \rho_\phi + \left(\frac{2+\Sigma}{3}\right)\rho_\phi\right] = 0 \tag{6.61}$$

$$\left(P_{,X} + P_{,XX}\, M_{\rm pl}^2H'^2\varphi'^2\right)\left(\varphi'' + \frac{H'}{H}\varphi'\right) + 3P_{,X}\,\varphi' + 3\beta\Sigma\left(1 - \frac{\rho_\phi}{3M_{\rm pl}^2H^2}\right) = 0. \tag{6.62}$$

We recover the chameleon equations of motion in the limit $\Lambda \to \infty$ which in terms of these variables are

$$2\frac{H'}{H} + (4-\Sigma) + \left(\frac{2+\Sigma}{3}\right)\frac{\varphi'^2}{2} = 0 \tag{6.63}$$

$$\varphi'' + \left(\frac{H'}{H} + 3\right)\varphi' + 3\beta\Sigma\left(1 - \frac{\varphi'^2}{6}\right) = 0 \tag{6.64}$$

and we remind the reader that in each case we have neglected the potential.

In this section we model the standard model kicks as Gaussian functions and concentrate on a single kick for simplicity. This will be sufficient in our aim to show the effectiveness of the DBI interactions as long as we allow Σ to pass through Σ_c. We will also model this single kick as the one due to the electron since this is the most significant kick and occurs at the smallest energy scale. In this sense if the DBI interactions are able to suppress the effects of this kick on the scalar's evolution, the preceding kicks will also be harmless because at those scales the DBI interactions will be even stronger. In any case we set

$$\Sigma(T_J) = A\exp\left[-\frac{(\log T_J - \log T_{peak})^2}{\sigma^2}\right] \tag{6.65}$$

where A, T_{peak} and σ are free parameters which we can fix to model the desired kick. For the electron we set $A = 0.1$, $T_{peak} = 2\times 10^{-4}$GeV and $\sigma = 0.3$ such that our simplified kick is shown below in figure (6.8). However, to evolve the system we require $\Sigma(N)$. We can easily derive this given that the Jordan frame temperature is $T_J \sim e^{-\beta\phi/M_{\rm Pl}}/a$ which leads to

$$T_J\left(N\right) = T_{J,i}\exp\left[-N - \beta\Delta\varphi\left(N\right)\right] \tag{6.66}$$

where at $N = 0$ the initial Jordan frame temperature is $T_{J,i}$ and $\Delta\varphi(N) = \varphi(N) - \varphi_i$. Plugging this expression into (6.65) yields $\Sigma(N)$. We set $T_{J,i} = 10^{-2}$GeV for the electron as dictated by figure (6.3).

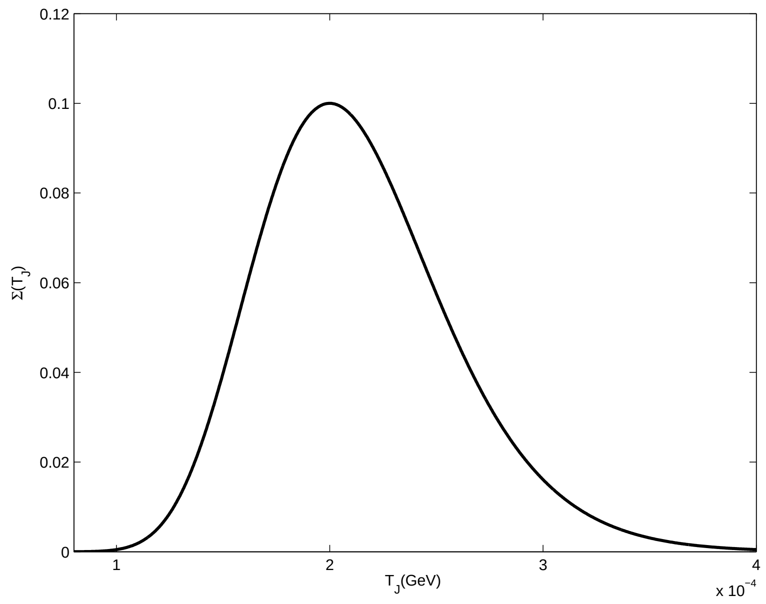


Figure 6.8: Plot of $\Sigma(T_J)$ for the simplified kick we will consider to show the effects of the DBI corrections. The parameters are chosen to model the standard model kick due to the electron.

In the presence of this kick we shall evolve the equations of motion for the chameleon and for the DBI chameleon. We choose a range of values for Λ in the DBI chameleon runs initially concentrating on the parameter space where Λ varies from Λ_k to $10\Lambda_k$ where Λ_k is the scale of the kick. This will allow us to emphasis the effects of the DBI correction on the resulting evolution. We will consider smaller values of Λ, and therefore stronger DBI interactions (assuming the same initial $\dot{\phi}$), later on to illustrate the suppression in the scalar's field excursion. To close the system of equations we also have to specify the initial energy density ρ_i which is given by [44]

$$\rho_{r,i} = \frac{\pi^2}{30} g_* \left(T_{J,i}\right) T_{J,i}^4 e^{4\beta\varphi_i} \tag{6.67}$$

with $g_*(T_{J,i}) = 10.75$. Here we have neglected any contributions to the matter energy density from non-relativistic particles.

To compare the chameleon and the DBI chameleon we choose to match the initial energy densities in each run. Given that we will always have $\gamma \sim \mathcal{O}(1)$ to help keep the higher order corrections under control, this amounts to matching the initial values of ϕ and $\dot{\phi}$ from which we can infer φ_i, φ_i' and H_i to evolve the systems. We use the same initial velocity in each run and choose its value such that when $\Lambda = \Lambda_k$ we have $\gamma \approx 1.7$ which is strongly in the DBI regime. Since this is the smallest value of Λ we shall consider it is the largest value of γ and therefore yields the largest suppression to the matter coupling.

In figure (6.9) we plot the $\varphi' - \varphi$ plane. It is clear that the surfer behaviour does not exist for the two runs with the smallest values of Λ (solid lines). In these cases the scalar velocity decays to zero while in the other cases it asymptotes to $-1/\beta$ as we would expect on the surfer solution. For these trajectories we have set $\beta = 3$ and plotted the constant $\varphi' = -1/\beta$ line for ease of comparison. The run with the largest value of Λ is a very good approximation to the chameleon run (black line) while the other two DBI chameleon runs which do surf (dashed and dashed dotted lines) do not exactly follow the chameleon but the DBI interactions are not strong enough to prevent the surfer from existing. The behaviour shown in this figure is exactly what we expect from our earlier results and discussions. In figure (6.10) we plot Σ as a function of Einstein frame e-folds N with the critical value $\Sigma_c = 2/(6\beta^2 - 1)$ also plotted at all N. With $\beta = 3$ it corresponds to $\Sigma_c \approx 0.038$. We can see how the runs at finite Λ which do not surf see the expected form of the kick function over a finite number of e-folds and Σ does not get caught at the critical value. Whereas the DBI chameleon runs which do surf, and the chameleon run, see a kick function which asymptotes to the critical value where the Jordan frame temperature ceases to evolve. Recall that Σ crosses the critical value at two different values of N. However, the surfing behaviour is experienced at the larger N, or similarly, the smallest Jordan frame temperature. This is precisely what we expected based on our perturbative analysis in section 6.2 where we saw that the fluctuations around the surfer solution only decayed at the smaller temperature.

Finally for these runs, we plot the field excursions experienced by the field over the full trajectories in figure (6.11). It is clear that for the runs where surfing exists the Jordan frame temperature does not evolve once it has reached the critical value whereas for the two runs with sufficiently small Λ the surfing behaviour is non-existent and the Jordan frame temperature decreases as the system evolves. We also see that the smaller values of Λ yield smaller field excursions. However, each of these runs has $\Delta\phi \sim M_{\rm pl}$ which is problematic for the reasons discussed above, but they can be decreased by further lowering Λ. Indeed, the lowest value we have considered so far is a worse case scenario where $\Lambda = \Lambda_k$. Let us now consider a range of Λ values below Λ_k and demonstrate how the field excursions decrease with decreasing Λ as we estimated in equation (6.60).

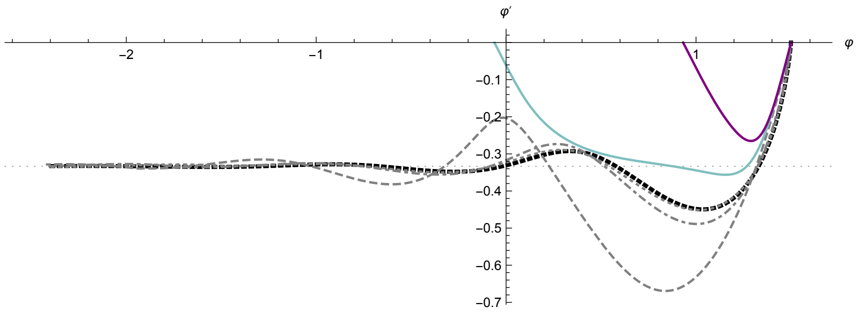


Figure 6.9: Plot of the trajectories in the $\varphi' - \varphi$ plane for a range of Λ values with $\beta = 3$ and evolved for 15 Einstein frame e-folds. See figure (6.11) for legend. Figure produced by Emma Platts and Anthony Walters.

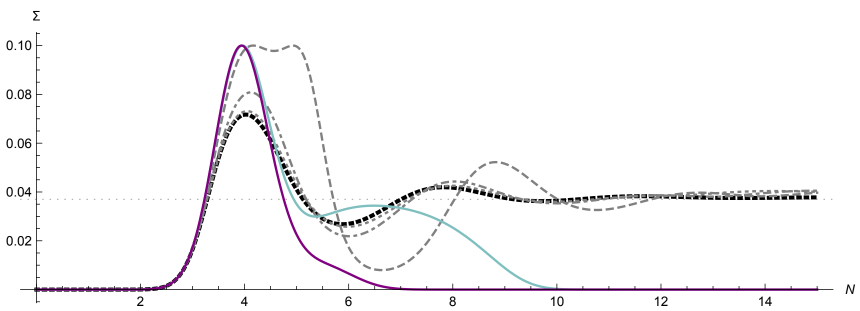


Figure 6.10: Plot of $\Sigma(N)$ for each run with the critical value $\Sigma_c = 2/(6\beta^2 - 1)$ plotted for comparison and $\beta = 3$. Figure produced by Emma Platts and Anthony Walters.

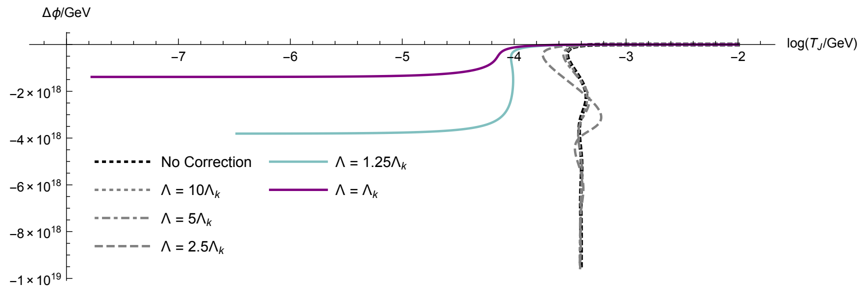


Figure 6.11: Plot of the field excursion in units of GeV against Jordan frame temperature for $\Lambda \geq \Lambda_k$ with $\beta = 3$. Figure produced by Emma Platts and Anthony Walters.

We can do this in two ways. The first is by decreasing the size of Λ but keeping the initial velocity of the field the same as for the previous runs. The effect of this is to raise the value of γ and to further weaken the coupling between the scalar and matter fields. We plot the resulting field excursions as a function of Jordan frame temperature in figure (6.12) for values of Λ between Λ_k and $5 \times 10^{-3}\Lambda_k$. We can clearly see that the field excursions decrease as we decrease Λ. The other possibility is to decrease Λ and the initial velocity at the same rate such that the γ factor and the matter coupling strength remains the same as before. We plot the resulting field excursions in figure (6.13) and again we see them decreasing as Λ is lowered. In this case the suppression is due to the smaller initial velocity as dictated by the DBI speed limit. In both cases we see how the excursions can be decreased by many orders of magnitude below the Planck scale, as desired.

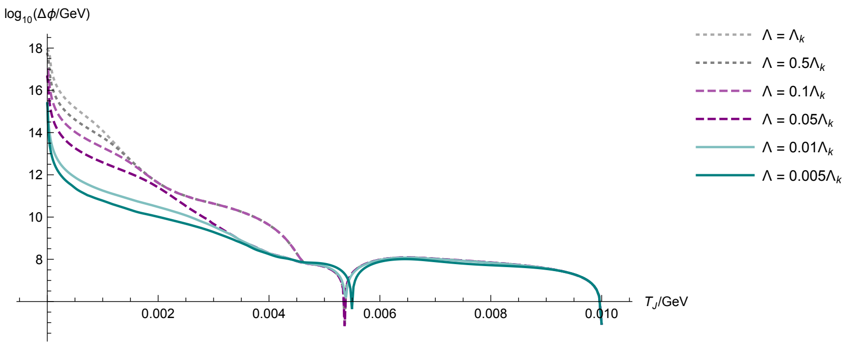


Figure 6.12: Plot of the field excursion in units of GeV against Jordan frame temperature for $\Lambda \leq \Lambda_k$ and $\beta = 3$. Here the initial velocity is the same for each run such that the γ is increased as Λ is decreased. Figure produced by Emma Platts and Anthony Walters.

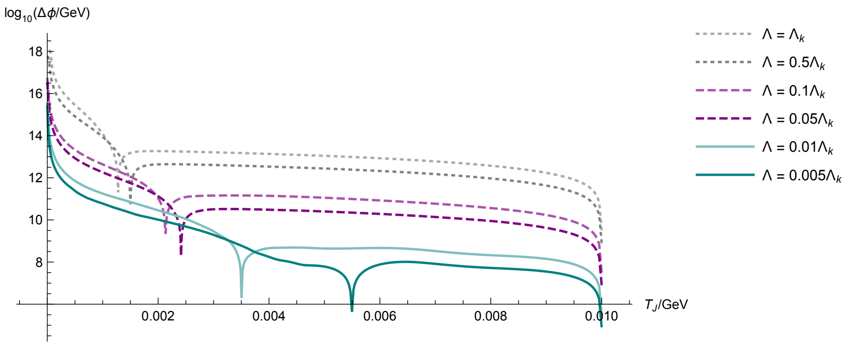


Figure 6.13: Plot of the field excursion in units of GeV against Jordan frame temperature for $\Lambda \leq \Lambda_k$ and $\beta = 3$. Here the initial velocity is decreased with decreasing Λ such that γ is the same in each run. Figure produced by Emma Platts and Anthony Walters.

6.5 Discussion

In this chapter we have introduced a UV extension of the chameleon theory. This was motivated by the break down in the original model's classical description in the early universe due to standard model particles becoming non-relativistic. The conformal coupling between the chameleon and matter degrees of freedom in a radiation dominated universe drives the chameleon towards the minimum of its effective potential with a huge velocity. This results in rapid variations in the effective mass of fluctuations. Consequently, highly energetic modes are excited rendering the effective description of the chameleon uncontrollable.

These conclusions are due to a surfer solution which exists at a constant Jordan frame temperature and allows the chameleon to undergo extremely large field excursions. The existence of the surfer solution and the resulting problems can only be avoided in the original chameleon theory by tuning the matter coupling such that the chameleon is insensitive to the standard model kicks shown in figure (6.3). Here we have shown that this fate can be avoided in the DBI chameleon theory without tuning the matter coupling.

Relative to the original chameleon theory, the DBI chameleon includes derivative self interactions of the scalar field which weaken the effective coupling between the scalar and matter fields in a dynamical way in the early universe due to cosmological Vainshtein screening. We saw that the suppression is effective in eliminating the troublesome surfer solution in the presence of a given kick if we choose $\Lambda \lesssim \sqrt{M_{\rm pl}H/\beta}$ where $M_{\rm pl}H$ is the scale of the kick. Concentrating on the most significant kick, which is due to the electron, we required $\Lambda \lesssim \mathrm{MeV}/\sqrt{\beta}$.

We also considered the effects of quantum corrections to the kinetic structure of the DBI chameleon action and showed that the theory is stable with respect to these corrections with $M \ll \Lambda \ll M_{\rm pl}$ and $H \ll \Lambda$ throughout evolution. The hierarchy of scales between the various coupling constants is naturally satisfied, and for the electron kick we must therefore also set $\Lambda \gg (\mathrm{MeV})^2/M_{\rm pl}$, given that $M_{\rm pl}H = (\mathrm{MeV})^2$, to keep the quantum corrections suppressed. Our ability to realise the Vainshtein mechanism on a homogeneous background while keeping higher order operators under control is due to the DBI structure enjoying a non-linearly realised symmetry. This ensures that we did not lose calculability even in the $n \to \infty$ limit of a theory constructed from a tower of $(\partial\phi)^{2n}$ operators. We understood this symmetry as coming from the five dimensional Lorentz invariance of a probe brane theory embedded in a 5D Minkowski bulk.

We remind the reader that the infrared dynamics of the chameleon remain unaltered and in the solar system the chameleon mechanism is effective, suppressing the scalar force in well tested environments. Finally, it would be interesting to investigate signatures of the DBI chameleon which differ from the original chameleon on other aspects of early universe cosmology.

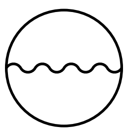
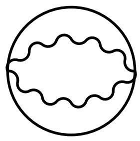
Chapter 7

Discussion and Future Work

A large part of this thesis is motivated by the cosmological constant problem and new approaches to tackling it. This problem is one of the major open questions in gravity and particle physics and a solution to it, or indeed the lack of a solution, is bound to teach us something about fundamental physics. In chapter 2 we described the cosmological constant problem in detail, explaining how the radiative instability of vacuum energy loops associated with massive particles in the standard model requires us to repeatedly tune a classical parameter in the action of General Relativity (GR). We have to do this such that the dynamics allows the universe to evolve into the macroscopic state we find it in. Indeed, without a compelling solution to the cosmological constant problem we do not have an answer to the simple question: why is the universe big?

We emphasised that since the cosmological constant is really a global parameter, i.e. it does not vary in space-time, if one is to prevent only the vacuum energy from sourcing curvature then one should seek to modify the global structure of Einstein's equations. We showed how this can be done, with the field equations coming from a local action with a well defined variational principle, in chapter 3. There we discussed the vacuum energy sequestering scenario [1, 30, 31] where global constraints restricted the vacuum energy from sourcing space-time curvature.

However, there remains many open questions which both the original and local sequestering models face. In particular, how could the mechanism also deal with loop corrections to the cosmological constant which involve internal gravitons? These are ignored in the current framework since we treat gravity classically and compute all loop corrections in the limit where it is decoupled. Beyond this limit, we would expect contributions to the overall cosmological constant by virtue of diagrams of the form

 +  + … (7.1)

where here the corrections are due to a massive particle coupled to internal gravitons. By dimensional analysis one would expect that the first diagram contributes [79]

$$\delta\Lambda = \mathcal{O}\left(\frac{M^6}{M_{\rm pl}^2}\right) + \cdots \tag{7.2}$$

where M is the field theory cut-off. If we model this as a standard model particle then we may well expect that $M \sim$ TeV in which case the first diagram yields a vacuum energy $\sim 10^{30}(\text{meV})^4$. This is a substantial contribution to the cosmological constant which must be cancelled[1]. However, beyond simple dimensional analysis, the calculation of this diagram would be somewhat speculative since it would be sensitive to quantum gravity effects. Also, we have only directly tested gravity at much smaller energy scales than we have in particle physics. It may be that gravity is supersymmetric way below the TeV scale in which case these diagrams would be far from the full story and their contributions may be cancelled at energies not much larger than the observed cosmological constant by the graviton's super-partner. These are sufficient reasons to initially focus on pure matter loop corrections, as we have done in this thesis, but it is certainly a worthwhile exercise to investigate the effects of these diagrams too. A way to do this has been presented in [160] where the space-time average of the Gauss-Bonnett combination is held fixed by virtue

[1]Of course, as we include diagrams with more internal gravitons, the Planck suppression becomes more effective and for a standard model particle coupled to gravity those diagrams will eventually become harmless.

of the four forms rather than the Ricci scalar. Another possibility would be to study the interplay between the local sequestering theory and supergravity.

One may expect that this would be aided with the introduction of kinetic terms for the scalars and 4-forms which in the current form of the local sequestering action (7.3) are absent. Whether they are generated quantum mechanically, and more generally if the full structure of the sequestering action is stable against quantum corrections, remains an open question which again progress on would be made by considering embeddings of the local sequestering model in supergravity. An important question in this regard is if kinetic energy terms can be included without spoiling the cancellation of pure matter loops? As a first pass, consider the new action

$$S = \int d^4x\sqrt{-g}\left[\frac{\kappa^2(x)}{2}R - \frac{a}{2}(\partial\kappa)^2 - \Lambda_c(x) - \frac{b}{2}(\partial\Lambda_c)^2 - \frac{c}{4!}F^2 - \frac{d}{4!}\hat{F}^2\right]$$
$$\frac{1}{4!}\int dx^\mu dx^\nu dx^\lambda dx^\rho\left[\sigma\left(\frac{\Lambda_c(x)}{\mu^4}\right)F_{\mu\nu\lambda\rho} + \hat{\sigma}\left(\frac{\kappa^2(x)}{M_{\rm pl}^2}\right)\hat{F}_{\mu\nu\lambda\rho}\right] + S_m(g_{\mu\nu},\Phi) \tag{7.3}$$

where a, b, c, d are constants which in some cases are dimensionful, and e.g. $F^2 = F_{\mu\nu\lambda\rho}F^{\mu\nu\lambda\rho}$. This reduces to the local sequestering theory when $a = b = c = d = 0$. The first thing to note is that the 4-forms now appear in the gravitational sector since their kinetic terms couple to the metric. Also, the variation with respect to the corresponding 3-forms does not fix the scalars to be constant on-shell. Instead they fix $\sigma - 2c \star F$ and $\hat{\sigma} - 2d \star \hat{F}$ to be constant. A consequence of this is that the would be global constraint on the Ricci scalar from varying with respect to the scalars, which is vital to the cancellation of matter loops, now depends on the non-constant scalars and the metric. To retain the exact calculation with the kinetic terms, we would therefore require another mechanism to render the scalars constant on-shell, or at least fluctuating very slowly.

It may be the case that with the inclusion of kinetic terms it is not possible to only sequester the vacuum energy. There may also be effects on finite wavelength sources thereby opening up avenues for observational tests of the model. In the current form of the local sequestering theory, it is difficult to distinguish it from GR in a low energy experiment since they are locally equivalent theories[2]. It is only the global structure which is very different. This further motivates looking for high energy completions of sequestering since even in the absence of an experimental test, this will enable one to learn more about the validity of the model.

Away from the cosmological constant problem, it would be interesting to implement the structure of the sequestering models in other areas of cosmology and particle physics. For example, in the local sequestering theory, the 4-forms were a vital ingredient since they provided non-gravitating measures without breaking gauge invariance. We saw how this allowed us to decouple the vacuum energy from curvature. It would be interesting to apply these techniques to gravitationally decouple other energy density sources in a similar way to screening mechanisms in modifications of gravity. We argued in the introduction that the acceleration of the universe offers motivation for modifying GR on large distances to realise an accelerating cosmology in the absence of a cosmological constant. However, the very stringent tests of gravity in local environments necessitates a screening mechanism for these models which shuts down new long range forces in solar system [5]. In fact, regardless of late time cosmology, one may expect that light scalar fields with gravitational strength couplings exist in Nature in attempts to UV complete gravity and so we must explain why they have not been seen. In this regard, one could couple a long range scalar to a 4-form with the strength of the coupling sensitive to the environment, much like the scalar mass in the chameleon mechanism.

One could also use 4-forms to impose interesting constraints on other Lorentz scalars, rather than merely fixing a scalar to have a constant profile. For example, consider the following coupling between the kinetic term of a scalar field and a 4-form

$$\int dx^\mu dx^\nu dx^\lambda dx^\rho (\partial\phi)^2 F_{\mu\nu\lambda\rho} \tag{7.4}$$

where $F_{\mu\nu\lambda\rho} = 4\partial_{[\mu} A_{\nu\lambda\rho]}$. Now variation with respect to the 3-form fixes the covariant kinetic term of the scalar field to be equal to an integration constant. This is reminiscent of the constraint which appears in the theory of mimetic

[2]We will touch on a potential way in a moment.

dark matter [161, 162], however here the constraint would be more flexible due to the arbitrariness of the integration constant. In each case, however, it is initially more important to understand how the non-trivial interactions between the scalars, which do couple to gravity, and the 4-forms, which do not always couple to gravity, can be realised in a more fundamental framework.

In chapter 4 we investigated another aspect of the cosmological constant problem in the context of the local sequestering theory, namely, what are the effects of an early universe phase transition on the late time dynamics? In doing so we considered the nucleation and growth of bubbles of true vacuum and calculated tunnelling rates between maximally symmetric vacua. In order to match the spectrum of rates found in GR we placed constraints on the sequestering functions σ and $\hat{\sigma}$ which must always be satisfied to avoid instabilities. Most notably, we found that a true vacuum with small de Sitter curvature which has tunnelled from a false vacuum with a larger de Sitter curvature is insensitive to the jump induced by the phase transition and we do not have tune the residual cosmological constant against this jump. Of course, unlike for a pure constant source in the energy-momentum tensor, the phase transition contributions are not completely sequestered but are harmless in our ability to match observations. However, the fact they are not completely sequestered may open up an avenue for testing the theory, specifically, it was recently argued in [98] that phase transitions, and their effect on the gravitational dynamics, could affect the mass-radius relationship of a Neutron star. Given that the sequestering theory is locally equivalent to GR, the possibility of an observational test in the theory's current form is very intriguing!

In chapter 5 we introduced the Vainshtein mechanism which is one of most popular screening mechanisms in the literature employed to suppress the effects of light scalar fields in local environments and argued that the main issue facing theories with Vainshtein screening is their very low cut-offs. We concentrated on suggestions that these theories can be trusted beyond the energy scale where the scalar sector becomes strongly coupled in the absence of sources, where instead one computes quantum corrections on non-trivial backgrounds. This can widen the regime of validity of these theories by many orders of magnitude and is therefore a very desirable property. However, for the examples we have studied in this thesis, we saw that the UV completions manifest themselves somewhere between the Vainshtein scale, where the non-linearities kick in, and the vacuum strong coupling scale rendering the calculation which leads to this enhancement of strong coupling very dubious. We emphasised that prior to worrying about where the theory breaks down because of loop corrections, one should also worry about at which scale the operators which are ultimately included to restore perturbative unitarity become classically important. We argued that being able to trust a theory inside the Vainshtein mechanism is aided by the presence of a symmetry as is the case in the DBI theory, and in GR. This motivates one to construct other scalar field theories which have similar non-trivial symmetries.

The very low strong coupling scales certainly represent a challenge for these theories since without knowledge of a partial UV completion, one cannot make predictions for local gravitational experiments. In some sense this should be treated as an opportunity since learning about possible UV completions for the non-trivial scalar field theories which are at the heart of the Vainshtein mechanism, may even teach us something about UV completions for GR, and more widely about aspects of field theory. Indeed, we saw in chapter 5 that there is a link between the structure of galileons and GR when we studied the theory of a probe brane. Similar conclusions have been drawn in [163] where it is argued that galileons are the scalar anologue of GR. However, as we also discussed in chapter 5, there are obstacles one faces when trying to UV complete these models as emphasised in [131]. Work has begun in this regard e.g. [164], and given that the conflict between galileons and S-matrix analyticity does not extend to the full theory of massive gravity [132], there is reason to be optimistic that one could find a local, Lorentz invariant partial UV completion of dRGT massive gravity [118,119] or Hassan and Rosen bi-gravity [121] - both which rely on the Vainhstein mechanism to pass local tests.

In the final part of this thesis, chapter 6, we presented an extension of the chameleon theory [4]. The chameleon [41,42] was shown to suffer from issues in the early universe where its classical description could not be trusted prior to Big Bang Nucleosynthesis [43, 44]. This is because as relativstic particles become non-relativstic in a radiation dominated universe, they impart a kick on the chameleon which sends it towards the minimum of its effective potential with a very large velocity. The DBI chameleon presented in this thesis avoids this fate by combining the chameleon mechanism with the Vainshtein mechanism and therefore extends the validity of the chameleon deep into the early universe. The structure of the extension of the chameleon theory was motivated by our discussions in chapter 5. By using the DBI derivative interactions we were able to render the chameleon weakly coupled to matter fields in the early universe without running out of control of the effective description thanks to the DBI symmetry. As we discussed in detail in this chapter, our ability to weaken the coupling between the chameleon and matter degrees of freedom in the presence of the kicks is by no means unique to the DBI interactions, but the majority of field theories which are used to realise the Vainshtein mechanism lack the protection from loop corrections which the DBI symmetry helps with here.

In the late universe and within the solar system, the DBI interactions are very weak and any deviations in observational predictions between the chameleon and the DBI chameleon will be heavily suppressed. However, in the early universe the theories are desirably different enabling the DBI chameleon to be scrutinised experimentally versus the chameleon.

Chapter 8

Appendix

8.1 Maximally symmetric space-times

The maximally symmetric space-times are solutions to Einstein's equations in the presence of a cosmological constant, where the Riemann tensor satisfies

$$R_{\mu\nu\alpha\beta} = q^2(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha}) \tag{8.1}$$

with q^2 the vacuum curvature. There are three such solutions corresponding to de Sitter space with $q^2 > 0$, anti-de Sitter space with $q^2 < 0$, and Minkowski space with $q^2 = 0$.

De Sitter space and co-ordinates

De Sitter space-time in four dimensions is best described as a hyperboloid embedded in a five dimensional Minkowski space. The embedding space-time has the following metric

$$ds_5^2 = -dT^2 + dW^2 + dR^2 + R^2 d\Omega_2^2 \tag{8.2}$$

where $d\Omega_2^2 = d\chi^2 + \sin^2\chi d\phi^2$ is the metric of a 2-sphere and the embedded surface is

$$W^2 + R^2 - T^2 = 1/q^2 \tag{8.3}$$

where $1/q$ is the de Sitter radius of curvature. This makes the $SO(1,4)$ symmetry group of four dimensional de Sitter space manifest. The five dimensional Minkowski space is invariant under the full five dimensional Poincaré group but the hyperboloid breaks translational invariance leaving only the $SO(1,4)$ Lorentz symmetry in tact. In global co-ordinates (t, θ, χ, ϕ) the de Sitter metric is

$$ds^2 = -dt^2 + \frac{\cosh^2 qt}{q^2}(d\theta^2 + \sin^2\theta d\Omega_2^2) \tag{8.4}$$

where $t \in (-\infty, \infty)$ and $\theta \in [0, \pi]$. The mapping from the embedding co-ordinates to the global ones is

$$W = \frac{1}{q}\cosh qt \cos\theta \tag{8.5}$$

$$R = \frac{1}{q}\cosh qt \sin\theta \tag{8.6}$$

$$T = \frac{1}{q}\sinh qt. \tag{8.7}$$

In Coleman's co-ordinates, which only cover a patch of the full de Sitter space, the metric is

$$ds^2 = dr^2 + \rho^2(r)(-d\tau^2 + \cosh^2\tau d\Omega_2^2) \tag{8.8}$$

where

$$\rho(r) = \frac{\sin q(\epsilon r + r_0)}{q} = \frac{\sin Q(r)}{q} \tag{8.9}$$

with $\epsilon = \pm 1$, $Q(r) \in [0, \pi]$, $\tau \in (-\infty, \infty)$, and where we have defined $Q(r) = q(\epsilon r + r_0)$. The mapping from the embedding co-ordinates to Coleman's co-ordinates is

$$W = \frac{1}{q}\cos Q(r) \tag{8.10}$$

$$T = \rho(r)\sinh\tau \tag{8.11}$$

$$R = \rho(r)\cosh\tau. \tag{8.12}$$

However, if we now combine the two co-ordinate mappings then this tells us that a point on the waist of the hyperboloid at $\tau = 0$ is not an arbitrary point in global co-ordinates. It corresponds to $t = 0$. So to map Coleman's co-ordinates to the global ones let us first make use of the Lorentz symmetry on the hyperboloid by performing a boost along the W direction with rapidity α such that the mapping between the embedding co-ordinates and Coleman's is

$$W' = \frac{1}{q}\cos Q(r) \tag{8.13}$$

$$T' = \rho(r)\sinh\tau \tag{8.14}$$

$$R = \rho(r)\cosh\tau \tag{8.15}$$

where

$$W' = W\cosh\alpha - T\sinh\alpha \tag{8.16}$$

$$T' = T\cosh\alpha - W\sinh\alpha. \tag{8.17}$$

Now the mapping between Coleman's co-ordinates and the global ones is

$$\cos Q(r) = \cosh\alpha\cosh qt\cos\theta - \sinh\alpha\sinh qt \tag{8.18}$$

$$\sin Q(r)\sinh\tau = \cosh\alpha\sinh qt - \sinh\alpha\cosh qt\cos\theta \tag{8.19}$$

$$\sin Q(r)\cosh\tau = \cosh qt\sin\theta \tag{8.20}$$

and $\tau = 0$ in Coleman's co-ordinates is mapped to an arbitrary time in global co-ordinates.

Anti-de Sitter space and co-ordinates

We now come to anti-de Sitter space-time where a similar description exists. Here the $SO(1,4)$ invariance of the embedding space-time is replaced by an $SO(2,3)$ symmetry such that the five dimensional embedding metric is

$$ds_5^2 = -dT^2 - dW^2 + dR^2 + R^2 d\Omega_2^2 \tag{8.21}$$

and the embedded surface is

$$W^2 + T^2 - R^2 = \frac{1}{|q|^2}. \tag{8.22}$$

Again this surface breaks the translational invariance of the embedding space-time such that only the $SO(2,3)$ invariance remains. In global co-ordinates (t, u, χ, ϕ) the anti-de Sitter metric is

$$ds^2 = -\frac{\cosh^2 |q|u}{|q|^2}dt^2 + du^2 + \frac{\sinh^2 |q|u}{|q|^2}d\Omega_2^2 \tag{8.23}$$

where $t \in (-\infty, \infty)$ and $u \in [0, \infty]$. The mapping from the embedding co-ordinates to the global ones is

$$W = \frac{1}{|q|}\cosh |q|u \cos t \tag{8.24}$$

$$T = \frac{1}{|q|}\cosh |q|u \sin t \tag{8.25}$$

$$R = \frac{1}{|q|}\sinh |q|u. \tag{8.26}$$

In Coleman's co-ordinates the metric is again given by (8.8) with

$$\rho(r) = \frac{\sinh |q|(\epsilon r + r_0)}{|q|} = \frac{\sinh Q(r)}{|q|} \tag{8.27}$$

where now $Q(r) = |q|(\epsilon r + r_0)$. The mapping from the embedding co-ordinates to Coleman's ones is

$$W = \frac{1}{|q|}\cosh Q(r) \tag{8.28}$$

$$T = \frac{1}{|q|}\sinh Q(r) \sinh \tau \tag{8.29}$$

$$R = \frac{1}{|q|}\sinh Q(r) \cosh \tau. \tag{8.30}$$

If we now use this mapping to map the global co-ordinates to Coleman's ones then again we see that $\tau = 0$ does not map to an arbitrary time in global co-ordinates. So before doing this let us again use the anti-de Sitter symmetry and first perform a rotation by an angle t_0 in the $T - W$ plane on the hyperboloid such that the embedding co-ordinates are mapped to Coleman's by

$$W' = \frac{1}{|q|} \cosh Q(r) \tag{8.31}$$

$$T' = \frac{1}{|q|} \sinh Q(r) \sinh \tau \tag{8.32}$$

$$R = \frac{1}{|q|} \sinh Q(r) \cosh \tau \tag{8.33}$$

where

$$W' = W \cos t_0 + T \sin t_0 \tag{8.34}$$

$$T' = T \cos t_0 - W \sin t_0. \tag{8.35}$$

Now the mapping between the global co-ordinates and Coleman's is

$$\cosh Q(r) = \cosh |q| u \cos(t - t_0) \tag{8.36}$$

$$\sinh Q(r) \sinh \tau = \cosh |q| u \sin(t - t_0) \tag{8.37}$$

$$\sinh Q(r) \cosh \tau = \sinh |q| u \tag{8.38}$$

and $\tau = 0$ maps to an arbitrary time in global co-ordinates.

Minkowski space and co-ordinates

Finally we have four dimensional Minkowski space and in global co-ordinates the metric is

$$ds^2 = -dt^2 + du^2 + u^2 d\Omega_2^2. \tag{8.39}$$

In Coleman's co-ordinates the metric is again given by (8.8) with

$$\rho(r) = \epsilon r + r_0. \tag{8.40}$$

The canonical mapping from global co-ordinates to Coleman's is given by

$$u = \rho(r) \cosh \tau \tag{8.41}$$

$$t = \rho(r) \sinh \tau \tag{8.42}$$

however, again with this mapping $\tau = 0$ corresponds to $t = 0$. So now we make use of the translational invariance of Minkowski space to shift the co-ordinate t by an arbitrary amount $-t_0$ such that the mapping is

$$u = \rho(r)\cosh\tau \tag{8.43}$$

$$t = \rho(r)\sinh\tau + t_0. \tag{8.44}$$

Now $\tau = 0$ is mapped to an arbitrary time in global co-ordinates.

8.2 Anti-de Sitter singularity

It was shown in [109] that an anti-de Sitter interior suffers from a curvature singularity. It turns out that the singularity corresponds to $Q(r) = 0$ in Coleman's co-ordinates. However, this is also a co-ordinate singularity so to find the surface corresponding to the curvature singularity in global co-ordinates, we make use of another co-ordinate system, namely, cosmological co-ordinates where the anti-de Sitter metric is

$$ds^2 = -d\eta^2 + \bar{\rho}^2(\eta)(d\lambda^2 + \sinh^2\lambda d\Omega_2^2) \tag{8.45}$$

with

$$\bar{\rho}(\eta) = \frac{\sin|q|\eta}{|q|}. \tag{8.46}$$

In these co-ordinates the curvature singularity forms at $\eta = \pi/|q|$ [109]. The mapping from the five dimensional embedding co-ordinates to these co-ordinates is

$$W = \frac{\cos|q|\eta}{|q|} \tag{8.47}$$

$$T = \frac{\sin|q|\eta}{|q|}\cosh\lambda \tag{8.48}$$

$$R = \frac{\sin|q|\eta}{|q|}\sinh\lambda \tag{8.49}$$

and therefore the mapping between the cosmological co-ordinates and Coleman's is

$$\cosh Q(r) = \cos|q|\eta \tag{8.50}$$

$$\sinh Q(r)\sinh\tau = \sin|q|\eta\cosh\lambda \tag{8.51}$$

$$\sinh Q(r)\cosh\tau = \sin|q|\eta\sinh\lambda. \tag{8.52}$$

So the singularity occurs at $\cosh Q(r) = -1$ and by using the mapping (8.36) this corresponds to the surface

$$\cosh|q|u\cos(t-t_0) = -1 \tag{8.53}$$

in global co-ordinates.

8.3 Space-time volumes

Each of the maximally symmetric space-time volumes we need to calculate in order to calculate $\mathcal{I}$ are divergent. We must therefore regulate on the relevant surface. We only need to consider the ratios of volumes to infer the effects of phase transitions so it is only the divergent behaviour in each case which is important. For each possible space-time there are three volumes of interest, namely, prior to bubble nucleation, in the exterior after bubble nucleation, and in the interior after bubble nucleation. Which volumes are important depends on the particular solution we are considering. In any case, in this section of the appendix we compute all three for de Sitter, Minkowski, and anti-de Sitter.

De Sitter Volumes

The volume of the total de Sitter space in global co-ordinates is

$$\frac{V^{\mathrm{dS}}_{\mathrm{total}}}{\Omega_2} = \int_{-\infty}^{\infty} dt \frac{\cosh^3 qt}{q^3}\int_0^{\pi} d\theta\sin^2\theta. \tag{8.54}$$

We split up the full de Sitter space-time into the three sections described above and shown in figure (8.1) where we have labelled them as A,B,C.

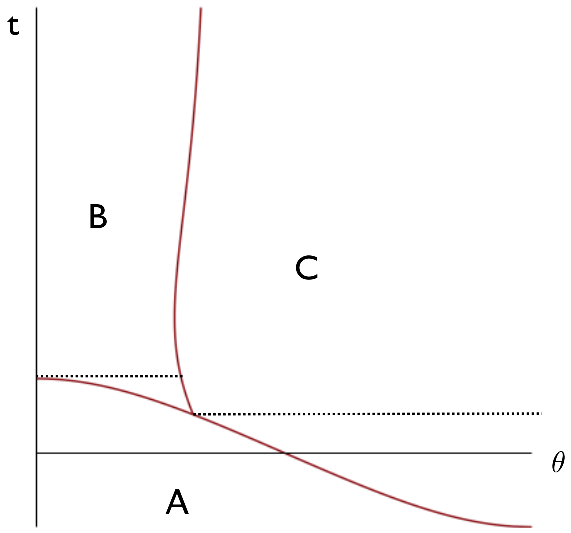


Figure 8.1: The three relevant volumes for de Sitter space

Region A will always be the volume before bubble nucleation, however, regions B and C can be either the true or false vacuum after bubble nucleation depending on configuration we are interested in. The boundary of region A is the nucleation surface which corresponds to $\tau = 0$ in Coleman's co-ordinates and by the mapping given in equation (8.19), the surface

$$t_{\rm tun}(\theta) = \frac{1}{q}\tanh^{-1}(\tanh\alpha\cos\theta) \tag{8.55}$$

in global co-ordinates. The wall separating regions B and C is at $r = 0$ in Coleman's co-ordinates which in global co-ordinates is the surface

$$\theta_{\rm wall}(t) = \cos^{-1}\left(\frac{\cos qr_0 + \sinh\alpha\sinh qt}{\cosh\alpha\cosh qt}\right) \tag{8.56}$$

by the mapping given in equation (8.18). We are now in a position to calculate each volume which are given by

$$\frac{V_{\rm X}^{\rm dS}}{\Omega_2} = \int_{\rm X} dtd\theta\frac{\cosh^3 qt\sin^2\theta}{q^3} \tag{8.57}$$

where X is either A,B or C.

Given that we are assuming finite α, region A is only divergent for $t \to -\infty$ so to regulate we cut-off the space-time at $t = -t_{\rm reg}$ in the past. The upper integration limit is simply $t_{\rm tun}(\theta)$ and the volume is therefore

$$\frac{V_{\rm A}^{\rm dS}}{\Omega_2} = \int_0^{\pi} d\theta \sin^2\theta \int_{-t_{\rm reg}}^{t_{\rm tun}(\theta)} dt \frac{\cosh^3 qt}{q^3} \tag{8.58}$$

which after performing the t integral is

$$\frac{V_{\rm A}^{\rm dS}}{\Omega_2} = \int_0^{\pi} d\theta \frac{\sin^2\theta}{3q^4} [\sinh qt(\cosh^2 qt + 2)]_{-t_{\rm reg}}^{t_{\rm tun}(\theta)}. \tag{8.59}$$

Given that $t_{\rm tun}$ is finite, the volume of region A is dominated by the regulated surface so we can approximate it by

$$\frac{V_{\rm A}^{\rm dS}}{\Omega_2} \sim \frac{\pi}{48q^4} e^{3qt_{\rm reg}}. \tag{8.60}$$

The volume for region B is

$$\frac{V_{\rm B}^{\rm dS}}{\Omega_2} = \int_0^{\theta_{\rm wall}(t)} d\theta \sin^2\theta \int_{t_\dagger}^{t_{\rm reg}} dt \frac{\cosh^3 qt}{q^3}. \tag{8.61}$$

The upper limit on the θ integral is at $\theta_{\rm wall}$ as shown in figure (8.1). We have again cut-off the space-time on the divergent surface using the regulator $t = t_{\rm reg}$. We assume that the regulator in the future and the past are the same such that in the absence of the wall the volumes for $t \geqslant 0$ and $t \leqslant 0$ are equivalent in line with the symmetries of de Sitter space. The lower limit on the t integral is chosen for simplicity. The region below the dotted line which we do not include is finite so would not affect our results if we included it. By performing the integrals we find

$$\frac{V_{\rm B}^{\rm dS}}{\Omega_2} \sim \left(\cos^{-1}(\tanh\alpha) - \frac{\tanh\alpha}{\cosh\alpha}\right) \frac{1}{48q^4} e^{3qt_{\rm reg}}. \tag{8.62}$$

Finally for de Sitter space we have region C whose volume is

$$\frac{V_{\rm C}^{\rm dS}}{\Omega_2} = \int_{\theta_{\rm wall}(t)}^{\pi} d\theta \sin^2\theta \int_{t_\star}^{treg} dt \frac{\cosh^3 qt}{q^3}. \tag{8.63}$$

The choice of integration limits follows simply from our discussion for region B, and performing the integrals yields

$$\frac{V_C^{\mathrm{dS}}}{\Omega_2} \sim \left(\pi - \cos^{-1}(\tanh\alpha) + \frac{\tanh\alpha}{\cosh\alpha}\right)\frac{1}{48q^4}e^{3qt_{\mathrm{reg}}}. \tag{8.64}$$

Anti-de Sitter Volumes

For anti-de Sitter space the total volume in global co-ordinates is

$$\frac{V_{\mathrm{total}}^{\mathrm{AdS}}}{\Omega_2} = \int_{-\infty}^{\infty} dt \int_0^{\infty} du \frac{\cosh|q|u \sinh^2|q|u}{|q|^3}. \tag{8.65}$$

We again consider the three relevant regions of anti-de Sitter space as shown below in figure (8.2) where region A is prior to bubble nucleation, region B is in the interior after bubble nucleation and region C is in the exterior after bubble nucleation.

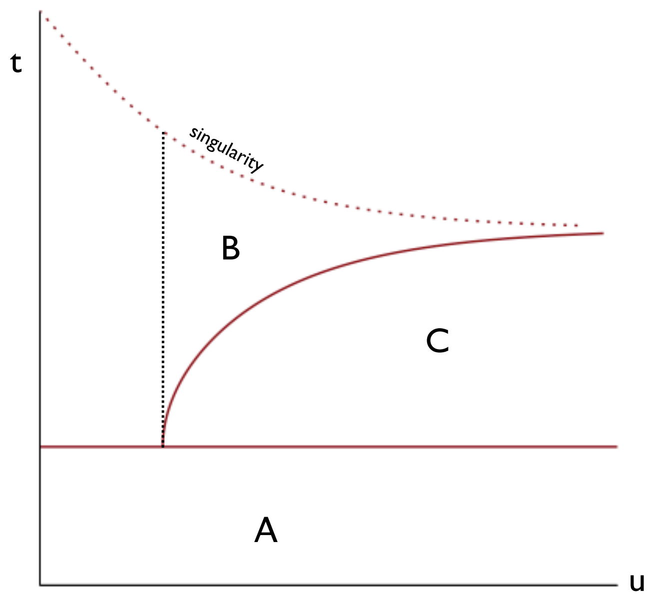


Figure 8.2: The three relevant volumes in anti-de Sitter space with region B cut-off at the curvature singularity.

The boundary of region A is again the nucleation surface which now in global co-ordinates, given the mapping in equation (8.37), corresponds to

$$\cosh|q|u \sin(t - t_0) = 0 \tag{8.66}$$

and we take the principle root of this equation ($t = t_0$) when calculating the volumes. In global co-ordinates the wall separating regions B and C is at the surface

$$\cosh|q|r_0 = \cosh|q|u\cos(t - t_0) \tag{8.67}$$

or equivalently

$$t_{\mathrm{wall}}(u) = t_0 + \cos^{-1}\left(\frac{\cosh|q|r_0}{\cosh|q|u}\right) \in (t_0, t_0 + \frac{\pi}{2}) \tag{8.68}$$

by the mapping in equation (8.36). As described above, for an anti-de Sitter interior we also have to take into account the curvature singularity which is at

$$t_{\mathrm{sing}}(u) = t_0^- + \cos^{-1}\left(\frac{-1}{\cosh|q|_-u}\right) \in (t_0^- + \frac{\pi}{2}, t_0^- + \pi) \tag{8.69}$$

and cut-off the interior appropriately. We now compute each volume which are given by

$$\frac{V_{\mathrm{X}}^{\mathrm{AdS}}}{\Omega_2} = \int_{\mathrm{X}} dtdu\frac{\cosh|q|u\sinh^2|q|u}{|q|^3}. \tag{8.70}$$

For region A we have

$$\frac{V_{\mathrm{A}}^{\mathrm{AdS}}}{\Omega_2} = \int_{-\infty}^{t_0} dt \int_0^{u_{reg}} du\frac{\cosh|q|u\sinh^2|q|u}{|q|^3} \tag{8.71}$$

which diverges in both the t and u direction. For now we only regulate in the u direction by cutting off the integral at $u = u_{\mathrm{reg}}$ such that

$$\frac{V_{\mathrm{A}}^{\mathrm{AdS}}}{\Omega_2} \sim \frac{1}{24|q|^4}e^{3|q|u_{\mathrm{reg}}}\int_{-\infty}^{t_0} dt. \tag{8.72}$$

Region B is the interior of an anti-de Sitter bubble so cutting of the t integral appropriately yields the volume

$$\frac{V_{\mathrm{B}}^{\mathrm{AdS}}}{\Omega_2} = \int_{u_\star}^{u_{\mathrm{reg}}} du\frac{\cosh|q|u\sinh^2 q_-u}{|q|^3}\int_{t_{\mathrm{wall}}(u)}^{t_{sing}(u)} dt \tag{8.73}$$

where we have ignored the finite part of the volume which lies to the left of the dotted line ($u = u_\star$). Again having regulated in the u direction, the volume to leading order is

$$\frac{V_{\rm B}^{\rm AdS}}{\Omega_2} \sim \frac{(1+\cosh|q|r_0)}{8|q|^4} e^{2|q|u_{\rm reg}}. \tag{8.74}$$

Finally, we have the exterior for an anti-de Sitter bubble where the volume is given by

$$\frac{V_{\rm C}^{\rm AdS}}{\Omega_2} = \int_{u_{\rm wall}(t_0)}^{u_{\rm reg}} du \frac{\cosh|q|u \sinh^2|q|u}{|q|^3} \int_{t_0}^{t_{wall}(u)} dt. \tag{8.75}$$

The limits of integration should be clear by virtue of previous discussions and again we have regulated in the u direction yielding

$$\frac{V_{\rm C}^{\rm AdS}}{\Omega_2} \sim \frac{\pi}{48|q|^4} e^{3|q|u_{\rm reg}}. \tag{8.76}$$

Minkowski Volumes

The final three volumes we need to calculate are for the Minkowski space-times. The volume of the total Minkowski space in global co-ordinates is

$$\frac{V_{\rm total}^{\rm M}}{\Omega_2} = \int_{-\infty}^{\infty} dt \int_0^{\infty} du u^2 \tag{8.77}$$

and again we shown the three relevant regions below in figure (8.3) where region A is prior to bubble nucleation, and regions B and C can be either the exterior or interior of the bubble after nucleation.

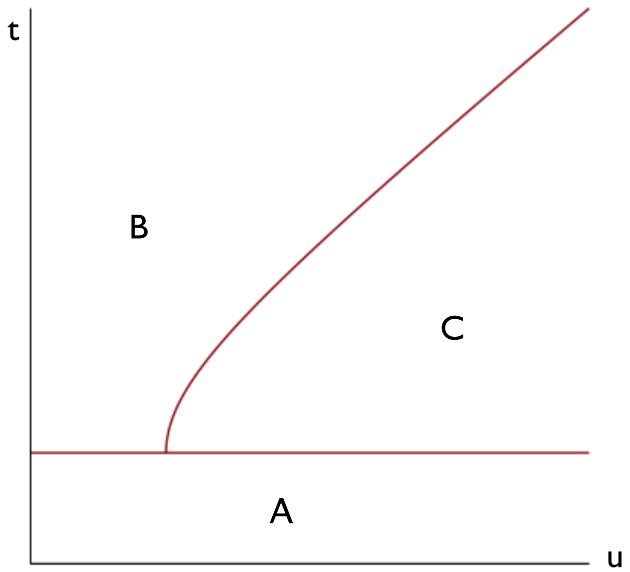


Figure 8.3: The three relevant volumes for a Minkowski space-time.

The nucleation surface is at $t = t_0$ and in global co-ordinates the wall separating regions B and C is at

$$u_{\text{wall}}(t) = \sqrt{r_0^2 + (t - t_0)^2} \tag{8.78}$$

or equivalently

$$t_{\text{wall}}(u) = t_0 + \sqrt{u^2 - r_0^2}. \tag{8.79}$$

The volume of each region is given by

$$\frac{V_{\text{X}}^{\text{M}}}{\Omega_2} = \int_{\text{X}} dt du u^2. \tag{8.80}$$

For region A we have

$$\frac{V_{\text{A}}^{\text{M}}}{\Omega_2} = \int_{-\infty}^{t_0} dt \int_0^{u_{\text{reg}}} du u^2 \tag{8.81}$$

which is divergent in both the t and u direction. Cutting of the space-time in the u direction at u_{reg} yields

$$\frac{V_{\text{A}}^{\text{M}}}{\Omega_2} \sim \frac{u_{\text{reg}}^3}{3} \int_{-\infty}^{t_0} dt. \tag{8.82}$$

The volume for region B is

$$\frac{V_{\rm B}^{\rm M}}{\Omega_2} = \int_{t_0}^{t_{\rm reg}} dt \int_0^{u_{\rm wall}(t)} du u^2 \sim \frac{t_{\rm reg}^4}{12} \tag{8.83}$$

where we have regulated by cutting off the t integral at $t = t_{\rm reg}$. Finally we have region C whose volume is given

$$\frac{V_{\rm C}^{\rm M}}{\Omega_2} = \int_{u_{\rm wall}(t_0)}^{u_{\rm reg}} du u^2 \int_{t_0}^{t_{\rm wall}(u)} dt \sim \frac{u_{\rm reg}^4}{4} \tag{8.84}$$

where we have regulated in the u direction.

8.4 Calculating the ratio $\mathcal{I}$

With the calculations of the space-time volumes in hand, we can now compute the ratio $\mathcal{I} = \frac{\mathcal{V}_{\rm b}^+ + \mathcal{V}_{\rm a}^+}{\mathcal{V}_{\rm a}^-}$ for the different possible configurations. Initially consider tunnelling between de Sitter vacua. There are four different possible configurations which correspond to the four possible arrangements of regions B and C after nucleation i.e. BB,BC,CB,CC. Since it is only the divergent behaviour which is important we ignore the explicit α dependence and express the volumes we have just computed as

$$\mathcal{V}_{\rm b}^+ \sim \frac{\pi}{48q_+^4} e^{3q_+ t_{\rm reg}^+} \tag{8.85}$$

$$\mathcal{V}_{\rm a}^+ \sim f_+(\alpha_+) \frac{\pi}{48q_+^4} e^{3q_+ t_{\rm reg}^+} \tag{8.86}$$

$$\mathcal{V}_{\rm a}^- \sim f_-(\alpha_-) \frac{\pi}{48q_-^4} e^{3q_- t_{\rm reg}^-}. \tag{8.87}$$

The form of $f_\pm$ depends on the chosen configuration. Now to eliminate the regulator dependence in the ratios we match the geometries at their point of intersection at the wall. This amounts to matching the radii of the 2-spheres at this intersection. Doing so yields

$$\left(\frac{e^{qt_{\rm reg}}}{q\cosh\alpha}\right)_+ = \left(\frac{e^{qt_{\rm reg}}}{q\cosh\alpha}\right)_- \tag{8.88}$$

in the large t_{reg} limit. We can now readily compute the volume ratio yielding

$$\mathcal{I}_{\text{dS}\to\text{dS}} \sim \frac{\cosh^3\alpha_+}{\cosh^3\alpha_-}\left(\frac{1+f_+(\alpha_+)}{f_-(\alpha_-)}\right)\frac{q_-}{q_+}. \tag{8.89}$$

For very specific choices of parameters the co-efficient of $\frac{q_-}{q_+}$ could be zero or singular. These will not be generic so we will ignore them such that we have

$$\mathcal{I}_{\text{dS}\to\text{dS}} \sim \frac{q_-}{q_+}. \tag{8.90}$$

For tunnelling between anti-de Sitter vacua the relevant volumes are

$$\mathcal{V}_{\text{b}}^{+} \sim \frac{1}{24|q|_+^4}e^{3|q|_+u_{\text{reg}}^+}\int_{-\infty}^{t_0^+}dt \tag{8.91}$$

$$\mathcal{V}_{\text{a}}^{+} \sim \frac{\pi}{48|q|_+^4}e^{3|q|_+u_{\text{reg}}^+} \tag{8.92}$$

$$\mathcal{V}_{\text{a}}^{-} \sim \frac{(1+\cosh|q|_-r_0^-)}{8|q|_-^4}e^{2|q|_-u_{\text{reg}}^-}. \tag{8.93}$$

We again match the radii of the 2-spheres at the intersection point which for large u_{reg} gives

$$\left(\frac{e^{|q|u_{\text{reg}}}}{|q|}\right)_+ = \left(\frac{e^{|q|u_{\text{reg}}}}{|q|}\right)_- . \tag{8.94}$$

This leads to the following volume ratio

$$\mathcal{I}_{\text{AdS}\to\text{AdS}} \to \infty. \tag{8.95}$$

This result is primarily due to the fact that the anti-de Sitter interior is cut-off thereby reducing the number of divergent directions in the interior volume. For tunnelling from a de Sitter space-time to an anti-de Sitter space-time the volumes are given by

$$\mathcal{V}_{\text{b}}^{+} \sim \frac{\pi}{48q_+^4}e^{3q_+t_{\text{reg}}^+} \tag{8.96}$$

$$\mathcal{V}_{\text{a}}^{+} \sim f_+(\alpha_+)\frac{\pi}{48q_+^4}e^{3q_+t_{\text{reg}}^+} \tag{8.97}$$

$$\mathcal{V}_{\text{a}}^{-} \sim \frac{(1+\cosh|q|_-r_0^-)}{8|q|_-^4}e^{2|q|_-u_{\text{reg}}^-}. \tag{8.98}$$

Again to eliminate the regulator dependence from the ratios we match the radii of the 2-spheres at the intersection point yielding

$$\left(\frac{e^{qt_{\mathrm{reg}}}}{q\cosh\alpha}\right)_{+}=\left(\frac{e^{|q|u_{\mathrm{reg}}}}{q}\right)_{-}. \tag{8.99}$$

It is then straight forward to see that the volume ratio reduces to

$$\mathcal{I}_{\mathrm{dS}\to\mathrm{AdS}}\to\infty. \tag{8.100}$$

Again, by the previous discussion this result is intuitive since by cutting off the interior of the anti-de Sitter bubble we have reduced the number of divergent directions making the exterior the dominant contribution to the total volume. For tunnelling from a de Sitter space-time to a Minkowski one, the volumes from the previous section are

$$\mathcal{V}_{\mathrm{b}}^{+}\sim\frac{\pi}{48q_{+}^{4}}e^{3q_{+}t_{\mathrm{reg}}^{+}} \tag{8.101}$$

$$\mathcal{V}_{\mathrm{a}}^{+}\sim f_{+}(\alpha_{+})\frac{\pi}{48q_{+}^{4}}e^{3q_{+}t_{\mathrm{reg}}^{+}} \tag{8.102}$$

$$\mathcal{V}_{\mathrm{a}}^{-}\sim\frac{(t_{\mathrm{reg}}^{-})^{4}}{12}. \tag{8.103}$$

We again match the geometries at the intersection point yielding

$$\left(\frac{e^{qt_{\mathrm{reg}}}}{2q\cosh\alpha}\right)_{+}=t_{\mathrm{reg}}^{-} \tag{8.104}$$

and consequently the following volume ratio

$$\mathcal{I}_{\mathrm{dS}\to\mathrm{M}}=0. \tag{8.105}$$

Again this result is intuitive since by comparing the expressions for the volumes in global co-ordinates, Minkowski space has more divergent directions than de Sitter space. This is also what we would expect by taking the $q_{-}\to 0$ limit of equation (8.90). Finally, for tunnelling from a Minkowksi space-time to an anti-de Sitter one the relevant volumes are

$$\mathcal{V}_{\mathrm{b}}^{+} \sim \frac{(u_{\mathrm{reg}}^{+})^{3}}{3} \int_{-\infty}^{t_0^+} dt \tag{8.106}$$

$$\mathcal{V}_{\mathrm{a}}^{+} \sim \frac{(u_{\mathrm{reg}}^{+})^{4}}{4} \tag{8.107}$$

$$\mathcal{V}_{\mathrm{a}}^{-} \sim \frac{(1+\cosh |q|_{-} r_0^{-})}{8|q|_{-}^{4}} e^{2|q|_{-} u_{\mathrm{reg}}^{-}}. \tag{8.108}$$

Now matching the geometries gives

$$u_{\mathrm{reg}}^{+} = \left(\frac{e^{|q| u_{\mathrm{reg}}}}{2|q|} \right)_{-} \tag{8.109}$$

and therefore yields the following volume ratio

$$\mathcal{I}_{\mathrm{M} \to \mathrm{AdS}} \to \infty \tag{8.110}$$

as expected with an anti-de Sitter bubble being cut-off at the singular surface.

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