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ПРИМЕНА АНАЛИТИЧКОГ ХИЈЕРАРХИЈСКОГ ПРОЦЕСА ЗА ИЗБОР НАЈПРИХВАТЉИВИЈЕГ КОНСТРУКЦИЈСКОГ СИСТЕМА

APLICATION OF ANALITIC HIERACHICAL PROCESS FOR CHOICE OF MOST ACCEPTABLE STRUCTURAL SYSTEM

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Резиме

У овоме раду је разматран вишекритеријумски избор конструкцијског система. Прво је објашњен Аналитички хирархијски процес (АХП), који је предложио Саати, за вишекритеријумско доношење одлука (МКДО) са прописаним алтернативама и критеријумима. У раду је предложен поступак за трансформацију матрице одлучивања, која је карактеристична за ТОПСИС, ВИКОР и друге вишекритеријумске методе, у матрицу компарације АХП методе. Према описаној процедури развијен је одговарајући рачунарски програм. У раду је приказана студија случаја која се односи на избор најприхватљивијег конструкцијског система за једну индустријску халу саразличитим типовима конструкције као алтернативама и задатим критеријумима.

Кључне речи: АХП метод, вишекритеријумско одлучивање, конструкцијски систем.

Summary

The multi criteria choice of a structural system is considered in this paper. The AHP method, proposed and developed by Saaty, for multi criteria decision making (MCDM) with possible alternatives and prescribed criteria is elaborated firstly. One procedure for the transformation of the decision matrix, that is characteristic for the TOPSIS, VICOR and other methods of MCDM to the comparison matrix of AHP method is proposed in the paper. According to the described procedure, is developed corresponding computer program. One case study related to the most acceptable choice of the structural system, for one industrial hall with different types of structure as alternatives and with given criteria, is presented.

Key words: AHP method, Multi criteria decision making, Structural system.

1. INTRODUCTION

The Analytic Hierarchy Process (AHP) for choosing factors that are important for decision making (DM) was proposed by Thomas Saaty [1,2]. This is one of the useful methods in multi criteria decision making (MCDM). In this process factors are selected and arranged in a hierarchy structure descending from one overall goal to criteria, sub criteria and alternatives, as it shown in Fig. 1.

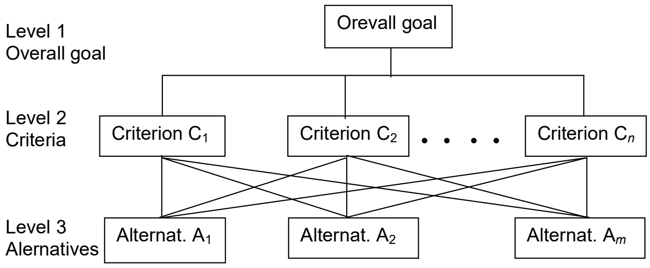

Figure 1. Hierarchical levels

Each level may represent different factors (economical, technical, social, etc.) that have to be evaluated by experts. It helps the decision maker to assess whether the issues in each level are the same order of magnitude, so he can compare such homogeneous elements accurately.

A decision maker can insert or eliminate levels and elements as necessary to the task of setting priorities or to focus on one or more parts of the system. Elements that have a global character can be represented at the higher levels of the hierarchy. The fundamental approach of AHP is to break down a big problem into several smaller problems that are solved separately to determine their priority vectors at each level [2]. According to these values of the separate priority vectors, the final priority vector of the alternatives is calculated taking into account relationships between hierarchy levels. Level 1 is related to the overall goal, which includes ranking of alternatives and determination of the best or most appropriate alternative. Level 2 includes prescribed criteria, while level 3 contains alternatives that are related to these criteria.

In the literature and practice, as it known, exist several other methods (TOPSIS, VIKOR, ELECTRE, PROMETHEE and others) that are used in practice for multi criteria decision making. These methods are described in the literature (see for example Opricovic's book [3]. This problem of application of the modified Fuzzy TOPSIS method for multiple criteria choice of objects of reconstruction and maintenance is considered and solved in the works with fuzzy numbers [4,5] and with stochastic numbers [6].

2. DESCRIPTION OF THE AHP PROCEDURE

In this paper is considered the problem of multi criteria decision making in which are ranked given alternatives A_1, A_2,…,A_m for prescribed criteria C_1, C_2,...,C_n. Some of criteria should be maximized and they belong to the set Ω_b (benefits) and others, that belong to the set Ω_c (costs), should be minimized. Unlike other mentioned methods of multi criteria decision making, relative weights w_i of factors F_i (i = 1,2,...,k), which in this case are criteria or alternatives, are compared in dependence on corresponding level. These weights are assessed usually by the decision making team. According to these values is determined the priority matrix $\mathbf{F} = [f_{ij}]_{k\times k}$.

Elements of the priority matrix **F** are

$$f_{ij} = w_i/w_j, i=1,2,...,k; j=1,2,...,k. \quad (1)$$

where w_i and w_j are weights of corresponding criteria C_i and C_j. This matrix is known as a *reciprocal matrix*, since it has positive entries everywhere and satisfies the reciprocal property

$$f_{ij} = 1/f_{ji}, i = 1,2,...,k; j = 1,2,..., k. \quad (2)$$

so it has following form

$$\mathbf{F} = \begin{bmatrix} 1 & f_{12} & \cdots & f_{1k} \\ 1/f_{12} & 1 & \cdots & f_{2k} \\ \cdot & \cdot & \cdots & \cdot \\ 1/f_{1k} & 1/f_{2k} & \cdots & 1 \end{bmatrix} \quad (3)$$

This matrix is *consistent*, because the following conditions are satisfied

$$f_{jp} = f_{ip}/f_{ij}, i=1,2,...,k; j=1,2,...,k, p=1,2,...,k. \quad (4)$$

The value f_{ij}, according to Saaty [1,2], represents the par-wise comparison or importance of the factor F_i over factor F_j at the certain level of the hierarchy. Because of that, matrix **F** is colled *pair-*

wise matrix. He proposed fundamental scale for making judgments of thise values, that are given in the Table 1.

Table 1. The fundametal scale of values f_{ij}.[1,2]

Value f_{ij}	Definition of impotance	Explanation
1	Equal importance	Two activities contribute equaly to the objective
3	Moderate importance	One activity is moderate favored over another
5	Esential or strong importance	One activity is strongly favored over another
7	Very strong importance	Activity is very strongly dominated in practice
9	Extreme importance	Dominance has highest order of affirmation
2, 4, 6, 8	Intermid. values between two adjanced judgements	These values are used when compromise is needed

According to Saaty [2] , necessary and sufficient condition for consistency is that the principal *eigenvalue* λ_{max} *of matrix* **F** is

$$\lambda_{\max} = k. \quad (5)$$

To make elements of vector **w** unique, it is necessary to normalize them by dividing each element by their sum, i. e.

$$\overline{w}_i = w_i / (w_1 + w_2 + ... + w_k), i=1,2,\ldots,k. \quad (6)$$

The values f_{ij}, according to Saaty [2], represents the pair-wise *comparison or importance* of the factor F_i in comparison to the factor F_j at a certain level of the hierarchy. As Saaty [2] emphasizes in a general decision making it is impossible to give precise values of the elements f_{ij} according to formula (1), but only estimate them. The difference between estimated the values f_{ij} and quotient w_i/w_j $\Delta_{ij} = f_{ij} - w_i/w_j$ cause inconsistency of the matrix **F,** so that equations (4) are not valid. The principal eigenvalue of the matrix **F** is [1]

$$\lambda_{\max} \geq k. \quad (7)$$

The problem of finding vector of priorities (weight vector) of considered factors **w** leads to an eigenvalue problem of the form

$$\mathbf{Fw} = \lambda \mathbf{w} \quad (8)$$

To every eigenvalue λ_i corresponds eigenvector $\mathbf{w}_i$ that represents solution of this system of k homogenous linear equations. Maximal positive real eigenvalue $\lambda_{\max}$ and corresponding eigenvector **w** is accepted as *priority vector* or *weight vector*. Since matrix **F** is not consistent one, Saaty [1] introduced *consistency index CI* for this matrix, which is calculated by the formula

$$CI = (\lambda_{\max} - k)/(k - 1), \quad (9)$$

and *consistency ratio CR*

$$CR = CI/RI \quad (10)$$

where *RI* is *random consistency*, which depends on the size of matrix k, and its values are given in Table 2, proposed by Saaty [1].

Table 2. Average random consistency *RI* [1,2]

Size of matrix k	Random consistency *RI*
1	0
2	0
3	0.58
4	0.90
5	1.12
6	1.24
7	1.32
8	1.41
9	1.45
10	1.49

If $CR \leq 0.10$, the estimates of the vector **w** are acceptable. Otherwise, the consistency of the matrix **F** has to be improved changing values of some its elements taking into account that this matrix must be reciprocal one. Saaty's method is based on calculation of the maximal eigenvalues and corresponding eigenvectors, and hence is known as the *eigenvector AHP method*. The problem of multi criteria decision making by AHP method solves in the next steps.

First step. Define the problem, overall goal that have to be attained, criteria and alternatives.

Second step. Define the hierarchy structure from the top level thru intermediate levels that contain criteria and sub criteria to the lowest level, which usually contains alternatives as it shown in Fig. 1.

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Third step. Formulate the pair-wise comparison reciprocal matrix **C** for the criteria C_1, C_2, $\dots C_n$, where n is number of criteria, by assessing priority values c_{ij} (i=1,2,…, n; j =1,2,…, n) using the fundamental comparison scale

$$\mathbf{C} = \begin{array}{c} \\ C_1 \\ C_2 \\ \dots \\ C_n \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{bmatrix} 1 & c_{12} & \dots & c_{1n} \\ 1/c_{12} & 1 & \dots & c_{2n} \\ . & . & \dots & . \\ 1/c_{1n} & 1/c_{2n} & \dots & 1 \end{bmatrix} \end{array} . \quad (11)$$

Solve the eigenvalue problem $\mathbf{Cw}=\lambda\mathbf{w}$ to determine the principal eigenvalue λ_{max}, corresponding eigenvector $\mathbf{w} = [w_1, w_2, \dots, w_n]^T$ and normalized eigenvector $\overline{\mathbf{w}} = [\overline{w}_1, \overline{w}_2, \dots, \overline{w}_n]^T$ according to formula (6). This normalized vector is the *priority vector of criteria*. Calculate, according to formulas (9) and (10) and Table 2, consistency index CI and consistency ratio CR. If $CR \leq 0.10$, accept assessed elements of pair-wise matrix C and obtained values of the principal eigenvelue λ_{max} and priority vector of criteria **w**. If $CR > 0.10$, improve consistency of the matrix **C** by changing some of its elements and repeat procedure until this condition is not satisfied.

Fourth step. Formulate pair-wise comparison matrices $\mathbf{A}^{(i)}$ (j=1,2,…,m) related to the criterion C_i i=1,2,...,n

$$\mathbf{A}^{(i)} = \begin{array}{c} (C_i) \\ A_1 \\ A_2 \\ \dots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & \dots & A_m \end{array} \\ \begin{bmatrix} 1 & a_{12}^{(i)} & \dots & a_{1m}^{(i)} \\ 1/a_{12}^{(i)} & 1 & \dots & a_{2m}^{(i)} \\ . & . & \dots & . \\ 1/a_{1m}^{(i)} & 1/a_{2m}^{(i)} & \dots & 1 \end{bmatrix} \end{array} , \quad (12)$$

Solve eigenvalue problems $\mathbf{A}^{(j)}\mathbf{p}^{(j)} = \lambda^{(j)}\mathbf{p}^{(j)}$ and find the principal eigenvalue $\lambda^{(j)}_{max}$, corresponding eigenvector $\mathbf{p}^{(j)}$, consistency indices $CI^{(j)}$ and consistency ratios $CR^{(j)}$ (j = 1,2,…, m). Normalize vector $\mathbf{p}^{(j)}$ to obtain local priority vector $\overline{\mathbf{p}}^{(j)} = [\overline{p}_1^{(j)}, \overline{p}_2^{(j)}, \dots, \overline{p}_m^{(j)}]^T$ for the alternative A_j (j=1,2,...,m). This procedure is the same as in the step 3.

Fifth step. Formulate local priority matrix **P** that contains normalized local priority vectors

$$\mathbf{P} = [\overline{\mathbf{p}}^{(1)}\ \overline{\mathbf{p}}^{(2)} \dots \overline{\mathbf{p}}^{(n)}], \text{ or}$$

$$\mathbf{P} = \begin{bmatrix} \overline{p}_1^{(1)} & \overline{p}_1^{(2)} & \dots & \overline{p}_1^{(n)} \\ \overline{p}_2^{(1)} & \overline{p}_2^{(2)} & \dots & \overline{p}_2^{(n)} \\ . & . & \dots & . \\ \overline{p}_m^{(1)} & \overline{p}_m^{(2)} & \dots & \overline{p}_m^{(n)} \end{bmatrix} \quad (13)$$

Multiply this matrix from the right by the priority vector of criteria $\overline{\mathbf{w}} = [\overline{w}_1, \overline{w}_2, \dots, \overline{w}_n]^T$ and obtain *vector of global priorities* **g**

$$\mathbf{g} = \mathbf{P}\overline{\mathbf{w}} = [g_1, g_2, \dots, g_n]^T . \quad (14)$$

According to entries of this vector, rank alternatives, in such a way, that alternatives with higher value are better ranked. The best ranked alternative is A^* which has maximal value g_j.

3. TRANSFORMATION OF DECISION MATRIX D TO PRIORITY MATRIX C

In many situations, the problem of MCDM, to find the best or most preferable alternative A^* among given alternatives A_1, A_2,…,A_m taking into account n prescribed criteria C_1,C_2,…,C_n, is solved by the TOPSIS, VIKOR or other mentioned methods with previously determined decision matrix **D**

$$\mathbf{D} = \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1n} \\ d_{21} & d_{22} & \dots & d_{2n} \\ \dots & \dots & \dots & \dots \\ d_{m1} & d_{m2} & \dots & d_{mn} \end{bmatrix} \qquad (15)$$

and the vector of weights of criteria $\mathbf{w} = [w_1, w_2, ..., w_n]$.

Elements d_{ij} are ratings or corresponding values of alternative A_i with respect to criterion C_j (i=1,2,…,m; j=1,2,…,n). In this paper is proposed one method for determination of the priority matrices **C** and $\mathbf{A}^{(j)}$ (j=1,2,…,n), taking into account expressions (1) – (3) and Saaty's Table 1.

To use these elements, calculated or assessed by experts for formulation of comparison matrices that corresponds to the application of AHP procedure, here is proposed next method.

Elements c_{ij} of the matrix **C** should calculate by the next formulas

$$c_{ij} = round\,(w_i / w_j), \quad i = 1,2,..,n;\ j = 1,2,..,m. \qquad (16)$$

If $c_{ij} < 1/9$, then $c_{ij} = 1/9$ and

If $c_{ij} > 9$, then $c_{ij} = 9$; (17)

where w_{pi} are previously proposed values of weighting coefficients for the criteria, while c_{ij} are integer values, and operator *round* means the rounding of obtained values to integer ones.

In a similar way, elements $a_{ij}^{(k)}$ of matrix $A^{(k)}$ related to the criterion C_k (k=1,2,…,n) may be calculated by the formulas

$a_{ij}^{(k)} = round\,(d_{ik} / d_{jk})$ if criterion C_k is maximized (18)

$a_{ij}^{(k)} = round\,(d_k / d_{jk})$ if criterion C_k is minimized, (19)

If $a_{ij}^{(k)} < 1/9$, then $a_{ij}^{(k)} = 1/9$ and if $a_{ij}^{(k)} > 9$, then $a_{ij}^{(k)} = 9$; (i=1,2,…,n; j=1,2,…n;k=1,2,…n).

These values may be calculated without rounding and further used in the AHP. In this case all priority matrices are consistent ones and have, according to (9) and (10) *consistency index CI* = 0 and *consistency ratio CR* = 0.

According to this procedure the third author have written corresponding computer program in the programming system MATLAB.

4. CASE STUDY

The proposed method of the AHP concerns to the choice of the optimal structural system of an industrial hall with dimensions at the base of 50.00 m×120,00 m. The building in the construction in Sevojno is shown in Figure 3. This hall has been designed and constructed by the construction enterprise "Zalatibor", Užice 1986. First author of this work took part in the design of this structural system together with professor Milorad Ivković, dipl. gradj. inž. (main designer of the structural system) and B. Furtula, Dipl. Civ. Eng. responible structural designer. Responsible architectural designer was architect Stevan Ljubičic. Design and construction team (project management) was headed by M. Ranđelović, Dipl. Civ. Eng.

Figure 3. Industrial hall under construction

For the choice of the structural system four alternatives are considered:

- Alternative A_1 – Two chord reinforced concrete and steel girders supported by the reinforced concrete columns, shown in Figure 3,
- Alternative A_2 - Reinforced concrete structure with classical prestressed concrete girders,
- Alternative A_3 – Classical frame structure of reinforced concrete.
- Alternatve A_4 - Steel structure with steel lattices.

Five criteria are taken into account:

- C_1 - Summary costs of the design and the construction of the hall converted in thousands of euros,
- C_2 - Costs of annual maintenance of the building in euros,,
- C_3 - Time necessary for the construction works in weeks,
- C_4 - Technological possibilities of the contractor firm to construct this industrial hall in the chosen system,
- C_5 Functional and aesthetic suitability of the building.

This analysis has been performed several years ago with new input data, that are assessed by authors of this work. According to these data authors of this work , have assessed for this example next input data: decision matrix **D,** weight vector of criteria **w** and sets of criteria Ω_b and Ω_c

$$\mathbf{D} = \begin{array}{c} \begin{matrix} C_1 & C_2 & C_3 & C_4 & C_5 \end{matrix} \\ \begin{bmatrix} 1200 & 8.0 & 38.0 & 9.0 & 8.5 \\ 1320 & 8.0 & 40.0 & 8.5 & 6.5 \\ 1350 & 8.2 & 42.0 & 7.0 & 7.0 \\ 1250 & 8.5 & 43.0 & 8.8 & 8.5 \end{bmatrix} \end{array} \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix},$$

$$\Omega_b = (C_1, \quad C_2 \quad C_3), \quad \Omega_c = (C_4 \quad C_5),$$

$$\mathbf{w} = [0.35 \quad 0.25 \quad 0.15 \quad 0.20 \quad 0.05 \,],$$

Using mentioned computer program, developed by authors of this work, are obtained local priority matrix **P,** vector of global priorities of alternatives **g** and rank of alternatives **r**

$$\mathbf{P} = \begin{array}{c} \begin{matrix} C_1 & C_2 & C_3 & C_4 & C_5 \end{matrix} \\ \begin{bmatrix} 0.2661 & 0.2553 & 0.2675 & 0.2707 & 0.2787 \\ 0.2419 & 0.2553 & 0.2541 & 0.2556 & 0.2131 \\ 0.2369 & 0.2491 & 0.2420 & 0.2105 & 0.2295 \\ 0.2555 & 0.2409 & 0.2364 & 0.2632 & 0.2787 \end{bmatrix} \end{array}$$

$$\mathbf{g} = \begin{bmatrix} 0.2658 \\ 0.2441 \\ 0.2384 \\ 0.2517 \end{bmatrix} \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix}, \quad \mathbf{r} = \begin{bmatrix} 1 \\ 4 \\ 2 \\ 3 \end{bmatrix}.$$

According these results alternatives are ranked (A_1, A_4, A_2, A_3). The best ranked alternative is $A_1 = A^*$. This alternatives has been accepted for realization and the hall successfully finished before contracted term.

5. CONCLUSION

Compared with other methods of MCDM, AHP method is based on different approach in which factors are arranged in a hierarchic structure. A decision maker can insert or eliminate levels and elements as necessary to the task of setting priorities or to sharpen the focus on one or more parts of the system. Elements that have a global character can be represented at the higher levels of the hierarchy.

Proposed procedure for transformation of the decision matrix to the comparison matrix enables much easier formulation of these matrices, especially when number of the criteria or sub criteria is large. This method can be successfully applied to decision making for solving various problems of construction industry, choice of the structural systems, reconstruction and maintenance of different objects and settlements and in many other situations. Proposed eigen values procedure, based on starting decision matrix **D** enables easier formulation of priorities matrices **C** and **A** on corresponding hierarchical levels of the analysis.

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