Activation Sites Estimation using a Fast Algorithm Based on an Eikonal Equation

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Context and objectives

- Classical algorithms for ECGI solve at each time instant an static and extremely ill-posed inverse problem, requiring an adequate regularization strategy (or instance Tikhonov)
- An alternative way of regularizing the inverse problem is to **consider information from other time steps, taking into account that electric potentials in the heart are obtained by the propagation of an electric wave**.
- Our objectives: design a fast algorithm based on a simple activation front model to retrieve significant features of the ECGI inverse problem

Propagation model

- We consider in the present work only the surface of the heart
- **Anisotropic eikonal equation**: simplified model of electrical activation front in asymptotic regime:

$$\begin{cases} \|\nabla T\|^2_{D(x)} = 1, \\ T(x_i^0) = \tau_i, i = 1 \dots s, \end{cases}$$

with
  - T depolarization time of heart surface,
  - $D(x)$ tensor quantity describing the anisotropic conduction
  - τ_i activation times at sources x_i^0 (earliest activation sites)

  $\Rightarrow$ parameters elevant in the context of EGCI
- $T(x)$ is also the length of the shortest path joining the sources to x

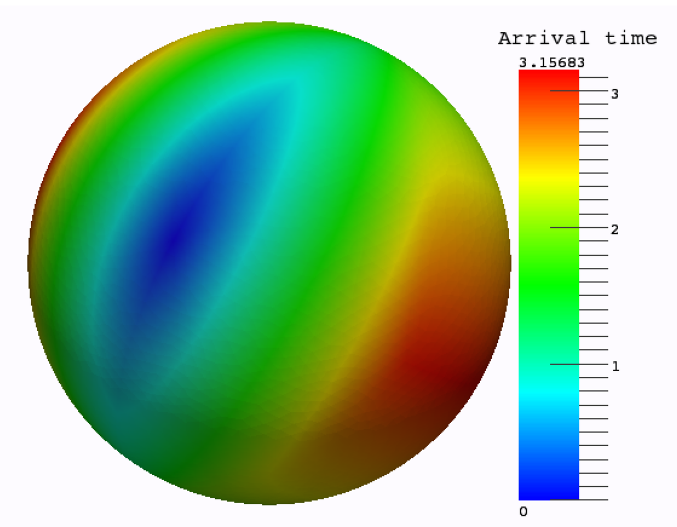

Figure: Example of solution of eikonal equation with anisotropic conduction

Methods

- Iterative algorithm to minimize misfit between observations and predictions of the model
- **Cost function**

$$J(x,\tau) = \frac{1}{2}\|G(\tau + \phi_x) - g^{\mathrm{OBS}}\|^2_Y.$$

with G observation operator (on heart or torso surface), x sources location, g^{OBS} observations with noise
- Least square minimization with Gauss Newton algorithm
- Sensitivity of cost function with respect to source locations and activation times depends on **Logarithmic map**:

  if γ is a unique shortest geodesic path from point x to point y then

$$Log_x(y) = \gamma'(0) = \text{ initial direction of } \gamma$$

- **Vector Heat Method**: efficient computational method to compute Logarithmic map, avoiding to compute all geodesics

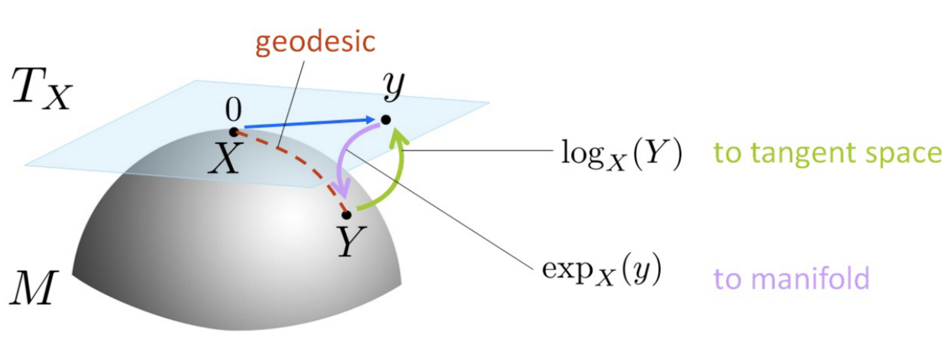

Figure: Illustration of Log map from et al

Test case: Localization of 2 sources and metric estimation

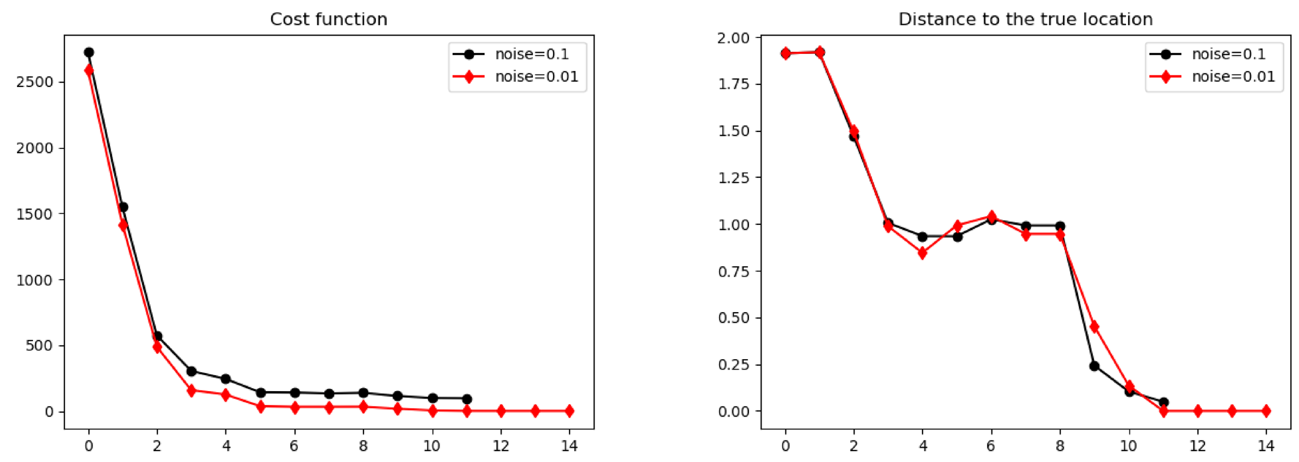

Figure: Left: evolution of the cost function J. Right: evolution of the distance to the solution $\|x^k - x^*\|$.

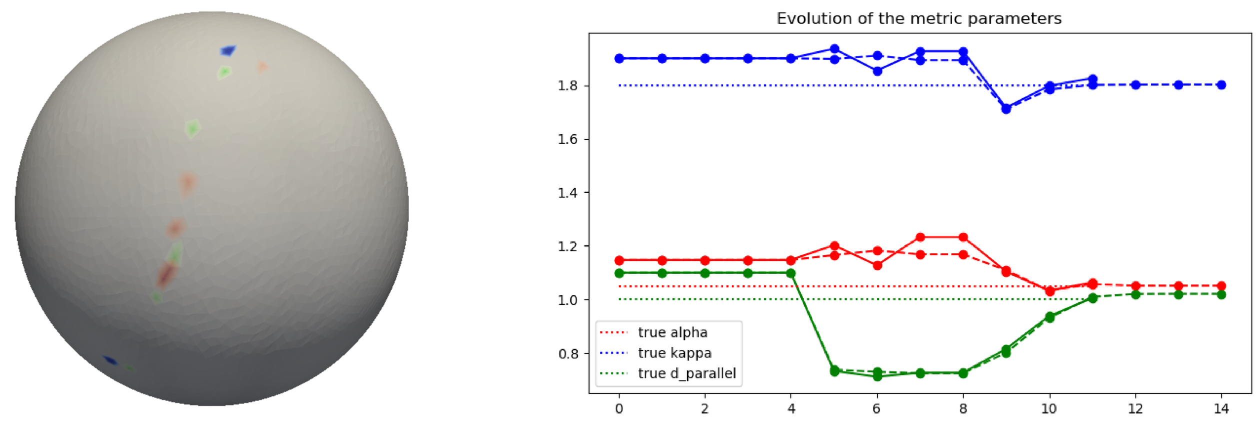

Figure: Left: location of the successive iterates x^k (red) and the true source point x^* (blue). Right: evolution of the parameters of the metric.

Test case: mimicking an ECG inversion

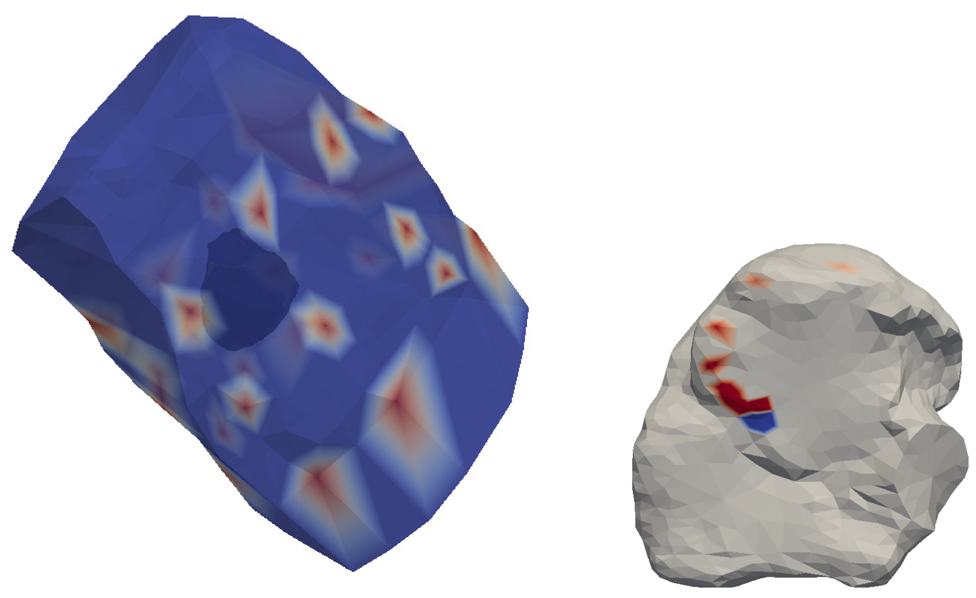

Figure: Left: configuration of the torso (electrodes in red) and the heart surface (visible by transparency). Right: location of the successive iterates x^k (red) and the true source point x^* (blue)

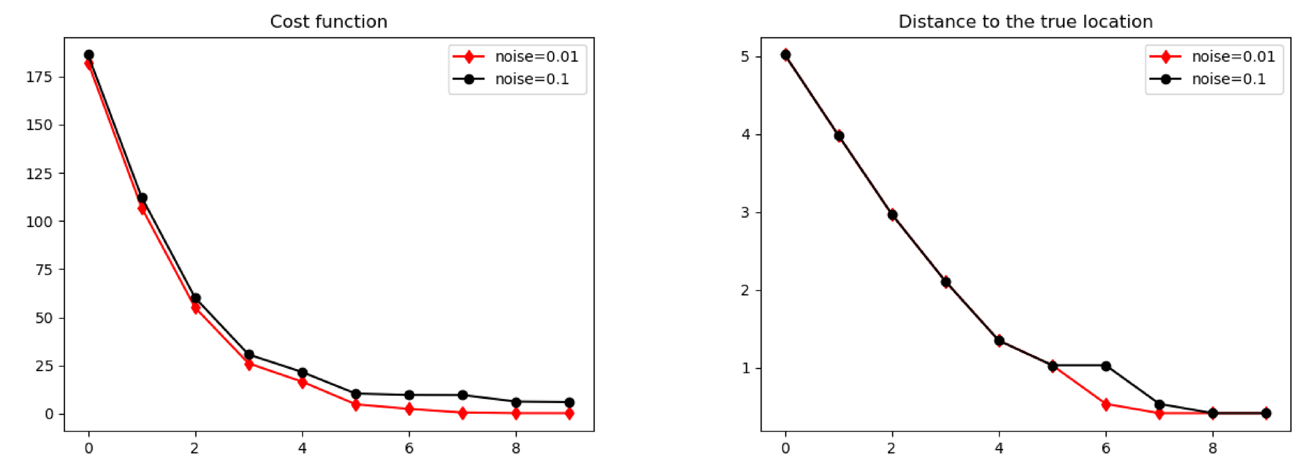

Figure: Left: evolution of the cost function J. Right: evolution of the distance to the solution $\|x^k - x^*\|$.

Conclusion

- Efficient new method to estimate sources and activation times in the eikonal equation on a surface $\Rightarrow$ applicable to ECGI.
- In case of complex or pathological propagation patterns, need to extend the method to a cardiac volume.

References

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