

Celestial Mechanics and Dynamical Astronomy

Electronic Supplementary Material to

“On Optimal Two-Impulse Earth–Moon Transfers in a Four-Body Model”

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The initial conditions of the points highlighted in Figure 7 are reported in Table A. These numerical values are given with high accuracy (15 decimals) to allow an independent reproduction of the results. The Pareto-efficient solutions can be reconstructed by simply integrating the initial conditions under the PBRFBP dynamics; i.e. the i -th transfer or bit is $\gamma^{(i)} = \{\varphi(\mathbf{x}_i, t_i; t), t \leq t_f\}$ where φ is the flow of (5).

Table A: Initial condition $\mathbf{x}_i = (x_i, y_i, \dot{x}_i, \dot{y}_i)$ and initial, final time, t_i, t_f of solutions (i)–(x) in Figure 7, 8 and Table 4.

	(i)	(ii)	(iii)	(iv)	(v)
x_i	-0.019710793706630	0.004430665316507	-0.022223338375692	-0.022469248813271	-0.020509279478864
y_i	-0.015255813698632	0.003867133727872	-0.013727223510871	-0.013543346852322	-0.014833373417121
\dot{x}_i	9.554404402969761	-2.421772563013686	8.593048071999794	8.477163558651014	9.289091958818513
\dot{y}_i	-4.734752068884238	10.383974704885071	-6.305349228506921	-6.458691168284453	-5.234406611705439
t_i	-1.803851247295145	-4.242568609277442	28.854324263774792	5.183780363655846	-2.783372600934104
t_f	-0.745915731505332	-0.928062659449309	35.904880040219062	12.518449758919608	7.256777745724602
	(vi)	(vii)	(viii)	(ix)	(x)
x_i	-0.020796119659883	-0.021300676720177	-0.019048265996746	-0.019022730972677	-0.020288471040515
y_i	-0.014668044128481	-0.014358714903565	-0.015566582598150	-0.015577872203673	-0.014955651584857
\dot{x}_i	9.185639864842763	8.996820692867965	9.751858086858906	9.759101553325923	9.365880586172640
\dot{y}_i	-5.414082611512081	-5.733172198727265	-4.321076476149917	-4.305155198773667	-5.096246610790654
t_i	-2.825197110547974	3.020458855979923	2.932062269968396	6.480457482011631	0.068219944002269
t_f	8.989837035669957	17.277056927521521	17.819572739337175	21.838729667546875	19.078513034970545

Table B: Data of transfer samples belonging to family a (Δv are in m/s).

Sol	α	β	θ_i	δ	$h_{2,f}$	$H_{2,f}$	J_f	Δv_f	Δv
a_1^+	4.252290888376	1.402157100606	1.667383569559	1.057924428519	0.0114	0.3056	2.3752	810.4	3944.8
a_2^+	5.633879787539	1.404945878425	6.144515170400	2.615962829677	0.0118	0.5158	1.9557	898.9	4055.0
a_3^+	0.842979367789	1.406764463306	1.356688768053	4.218742460358	0.0120	0.6268	1.7340	944.4	4114.7
a_4^+	3.348939227571	1.408514488040	3.842249354110	6.903698075803	0.0121	0.6608	1.6662	958.2	4142.2
a_5^+	4.208338472731	1.402044205156	2.577489648304	10.000795545595	0.0110	0.0928	2.8000	717.3	3850.9
a_1^-	4.065178639568	1.403442468457	1.353198236767	0.805366820770	-0.0116	0.3894	2.1616	846.0	3990.5
a_2^-	5.361124260595	1.404539474349	2.709286366639	2.325548051238	-0.0117	0.4637	2.0126	877.3	4030.2
a_3^-	1.192584194881	1.407111594526	4.839299650398	4.600129475444	-0.0120	0.6182	1.7030	940.9	4114.0
a_4^-	3.648464977227	1.408184543861	4.184825367605	7.361643034256	-0.0120	0.5875	1.7645	928.4	4109.8
a_5^-	4.204117236811	1.402044483564	2.577354362392	10.046535968859	-0.0109	0.0682	2.8053	706.4	3839.9

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Table C: Data of transfer samples belonging to family b (Δv are in m/s).

Sol	α	β	θ_i	δ	$h_{2,f}$	$H_{2,f}$	J_f	Δv_f	Δv
b_1^+	0.215107085265	1.402558265613	3.849790162865	3.259791073632	0.0115	0.3273	2.3318	819.7	3957.2
b_2^+	3.274914018198	1.405452575015	3.756543962351	6.607868389938	0.0118	0.5180	1.9512	899.8	4059.9
b_3^+	4.165806662212	1.401846765827	1.886983805417	8.274468720509	0.0112	0.2230	2.5400	774.7	3906.7
b_4^+	3.259404571570	1.407406549835	3.750174623792	6.136442170987	0.0121	0.6506	1.6865	954.1	4129.4
b_5^+	4.157563742559	1.401965337718	5.358624840898	7.156365100673	0.0113	0.2271	2.5319	776.5	3909.4
b_1^-	0.369715173230	1.402447258122	4.111746882921	3.447508808550	-0.0114	0.2929	2.3551	804.9	3941.6
b_2^-	2.448166521256	1.404884548712	2.928808142085	5.709718695759	-0.0117	0.4635	2.0130	877.2	4032.8
b_3^-	4.174580210102	1.401849858010	5.034553918006	8.296390114353	-0.0112	0.2009	2.5393	765.1	3897.1
b_4^-	3.046686676441	1.407291052413	0.394041543680	5.917404290307	-0.0120	0.6239	1.6918	943.2	4117.6
b_5^-	4.009503069577	1.402538262443	5.335357166595	6.939241208037	-0.0113	0.2510	2.4390	786.8	3924.2

Table D: Data of transfer samples belonging to families c , d (Δv are in m/s).

Sol	α	β	θ_i	δ	$h_{2,f}$	$H_{2,f}$	J_f	Δv_f	Δv
c_1^+	4.063783494297	1.401522012103	4.729454191036	7.033898841089	0.0112	0.2210	2.5441	773.8	3903.3
c_2^+	3.427285099082	1.403560818424	3.960945550035	6.432889714852	0.0116	0.3848	2.2171	844.1	3989.4
c_3^+	2.480707696443	1.402212056844	3.058819808299	5.568006948406	0.0114	0.2872	2.4118	802.5	3937.3
c_4^+	3.611131345279	1.402530026335	0.973402713645	6.835050408181	0.0114	0.3034	2.3797	809.4	3946.7
c_5^+	4.058039114112	1.401354929668	4.627277971792	7.351584618527	0.0112	0.2084	2.5693	768.3	3896.5
c_1^-	2.754201657972	1.402549179605	0.140469901096	5.845327519691	-0.0114	0.2892	2.3623	803.3	3940.8
c_2^-	3.345726014275	1.402767798775	3.858325285979	6.517058277713	-0.0114	0.3017	2.3374	808.7	3947.8
c_3^-	3.644279711940	1.402538537507	4.214717313285	6.864330024760	-0.0114	0.2805	2.3799	799.6	3936.9
c_4^-	4.059331479473	1.401372370370	1.489986039842	7.332369547055	-0.0112	0.1872	2.5668	759.0	3887.3
c_5^-	4.317064117351	1.401795637660	1.626612380287	7.603967576489	-0.0112	0.2195	2.5021	773.1	3904.7
d_1^-	3.636565150564	1.403294333379	4.238298754471	6.627383705975	-0.0115	0.3384	2.2639	824.4	3967.7
d_2^-	3.830120656954	1.402475710481	4.599398609960	6.806771866283	-0.0113	0.2740	2.3928	796.8	3933.7
d_3^-	3.912445608858	1.401972200392	4.719249053231	6.882666857182	-0.0113	0.2352	2.4706	780.0	3913.0
d_4^-	4.081266358623	1.401499904826	4.715879986486	7.052855624653	-0.0112	0.1971	2.5470	763.4	3892.7
d_5^-	4.319515000948	1.401856714554	4.814428742190	7.308138775860	-0.0112	0.2246	2.4918	775.4	3907.5

Table E: Data of transfer samples belonging to families e , f (Δv are in m/s).

Sol	α	β	θ_i	δ	$h_{2,f}$	$H_{2,f}$	J_f	Δv_f	Δv
e_1^+	4.147723781636	1.402092137706	2.605803338547	11.927916322073	0.0109	0.0654	2.8546	705.1	3839.0
e_2^+	4.195381970611	1.402051060212	2.610120027190	11.993419445390	0.0109	0.0622	2.8609	703.7	3837.3
e_3^+	4.260192508069	1.402077533643	2.583250061439	12.189159262373	0.0109	0.0665	2.8524	705.6	3839.4
e_1^-	4.157159925241	1.402078177710	2.606801550042	11.792074367447	-0.0109	0.0444	2.8531	695.6	3829.4
e_2^-	4.198832470368	1.402050110977	2.615526630965	11.845042863911	-0.0109	0.0422	2.8575	694.7	3828.2
e_3^-	4.253525586153	1.402071132405	2.580695323732	12.028283844219	-0.0109	0.0457	2.8505	696.2	3830.0
f_1^+	4.151891843626	1.402086002986	5.754845919871	11.947569714772	0.0109	0.0665	2.8525	705.6	3839.4
f_2^+	4.198799517273	1.402050417800	5.749491822325	12.022361413507	0.0109	0.0637	2.8580	704.3	3837.9
f_3^+	4.261357040927	1.402078716812	5.725135179832	12.214621956991	0.0109	0.0683	2.8488	706.4	3840.2
f_1^-	4.167548260354	1.402066910825	5.760425792268	11.819314675018	-0.0109	0.0451	2.8517	696.0	3829.7
f_2^-	4.204197741898	1.402049631276	5.753494375038	11.881742813297	-0.0109	0.0436	2.8547	695.3	3828.9
f_3^-	4.253697429252	1.402071225331	5.715732940351	12.066575362480	-0.0109	0.0478	2.8462	697.2	3830.9

Table F: Data of transfer samples belonging to families g, h, i (Δv are in m/s).

Sol	α	β	θ_i	δ	$h_{2,f}$	$H_{2,f}$	J_f	Δv_f	Δv
g_1^-	4.182373037335	1.402684711341	0.363550323766	14.269053511658	-0.0108	0.0282	2.8854	688.4	3826.9
g_2^-	4.193694329121	1.402659006474	0.348200642583	14.480639546892	-0.0108	0.0029	2.9362	676.9	3815.2
g_3^-	4.210707138843	1.402626439171	0.341622063212	14.775020117653	-0.0107	-0.0156	2.9733	668.5	3806.6
h_1^-	4.146820623243	1.402811711822	3.484711717309	14.293395642887	-0.0108	0.0237	2.8945	686.3	3825.9
h_2^-	4.158855464025	1.402766619679	3.461436386250	14.550978642303	-0.0108	-0.0044	2.9508	673.6	3812.8
h_3^-	4.156352594944	1.402778112755	3.446111907596	14.799362248882	-0.0107	-0.0160	2.9741	668.3	3807.6
i_1^-	4.142242238369	1.402864925471	0.147362458330	14.999495864920	-0.0107	-0.0139	2.9699	669.3	3809.2
i_2^-	4.276480721750	1.402588579355	0.264434008643	15.321474614190	-0.0107	-0.0262	2.9946	663.7	3801.4
i_3^-	4.515321794137	1.402847722785	0.586057434959	15.735447291823	-0.0107	-0.0231	2.9884	665.1	3804.9

Table G: Data of transfer samples belonging to families j, k (Δv are in m/s).

Sol	α	β	θ_i	δ	$h_{2,f}$	$H_{2,f}$	J_f	Δv_f	Δv
j_1^+	3.964544365380	1.403677338743	2.566384589700	16.726246530120	0.0108	-0.0123	3.0096	670.0	3816.3
j_2^+	4.224630330202	1.402077884057	3.124301761209	17.343745112358	0.0107	-0.0155	3.0161	668.5	3802.3
j_3^+	4.379504445319	1.402261932010	3.265483278329	17.776482729342	0.0108	0.0160	2.9531	682.9	3818.1
k_1^-	3.998645459380	1.402942049099	5.082145382774	17.750632495335	-0.0107	-0.0233	2.9886	665.0	3805.5
k_2^-	4.185108110638	1.402063044418	5.424761343788	18.340788665943	-0.0107	-0.0248	2.9916	664.3	3798.0
k_3^-	4.415821829794	1.402347743273	5.601695298837	18.691070596272	-0.0107	-0.0153	2.9726	668.6	3804.5

Table H: Data of transfer samples belonging to families l, m, n (Δv are in m/s).

Sol	α	β	θ_i	δ	$h_{2,f}$	$H_{2,f}$	J_f	Δv_f	Δv
l_1^+	3.930005399924	1.404474334702	2.325367081777	18.019421332231	0.0106	-0.0811	3.1470	638.5	3791.0
l_2^+	4.220630050535	1.402077201559	3.079433652998	18.995187228240	0.0106	-0.0875	3.1597	635.5	3769.3
l_3^+	4.356872352938	1.402222234881	3.194009044816	19.608480083992	0.0106	-0.0830	3.1508	637.6	3772.5
m_1^+	4.104575443108	1.402214480627	2.326603682628	19.307322661917	0.0106	-0.0731	3.1309	642.2	3777.0
m_2^+	4.223508200180	1.402068654459	2.488499275868	19.986664215269	0.0106	-0.0799	3.1445	639.1	3798.0
m_3^+	4.257932789284	1.402089731019	2.519022246012	20.501608427470	0.0106	-0.0786	3.1420	639.6	3773.5
n_1^+	4.179792908771	1.402073278208	2.415786468262	19.273871487267	0.0106	-0.0676	3.1200	644.7	3778.5
n_2^+	4.228410728757	1.402069510648	2.468556607257	19.540862454225	0.0106	-0.0685	3.1219	644.3	3778.0
n_3^+	4.290172468528	1.402124322768	2.516364378097	19.811434667184	0.0106	-0.0648	3.1146	646.0	3780.1

Table I: Data of transfer samples belonging to families o, p (Δv are in m/s).

Sol	α	β	θ_i	δ	$h_{2,f}$	$H_{2,f}$	J_f	Δv_f	Δv
o_1^-	4.879245051664	1.410883844595	2.161746749933	16.363634525136	-0.0109	0.0445	2.8529	695.7	3898.1
o_2^-	5.184028756452	1.410782605499	2.452415468264	17.073559777403	-0.0108	-0.0019	2.9459	674.7	3876.3
o_3^-	6.010810818630	1.410297239534	3.098975200907	18.136052888078	-0.0108	-0.0111	2.9643	670.5	3868.4
o_4^-	0.930095035965	1.409995191083	4.164633566715	19.501102334758	-0.0107	-0.0142	2.9705	669.1	3864.6
o_5^-	2.639014031620	1.410096258819	5.704212537244	21.246362344147	-0.0107	-0.0149	2.9719	668.8	3865.1
o_1^+	5.155853349247	1.410841292153	2.457175409153	16.737697460259	0.0109	0.0337	2.9179	690.8	3892.9
o_2^+	5.582512158897	1.410784101535	2.907677514655	17.641774656389	0.0107	-0.0269	3.0389	663.3	3865.0
o_3^+	0.200953325157	1.410243144276	3.610927074222	18.944143114293	0.0107	-0.0430	3.0710	656.0	3853.4
o_4^+	1.583248031688	1.410049492793	4.843168338916	20.564358230666	0.0107	-0.0484	3.0818	653.5	3849.4
o_5^+	2.992640834820	1.410138804114	6.124220359858	22.104190300919	0.0107	-0.0502	3.0853	652.7	3849.3
p_1^-	5.022229171492	1.410854603396	5.450049207204	16.557256512153	-0.0109	0.0373	2.8672	692.5	3894.7
p_2^-	5.497639357010	1.410589260809	5.837144088936	17.515628439560	-0.0108	-0.0068	2.9556	672.5	3872.6
p_3^-	0.020486154561	1.410222523303	0.228748281106	18.435337099453	-0.0108	-0.0110	2.9640	670.6	3867.9
p_4^-	1.519672362685	1.410008239478	1.562386938068	20.049687489479	-0.0108	-0.0134	2.9688	669.5	3865.1
p_5^-	3.058423279463	1.410119885948	2.958114992217	21.634523489859	-0.0107	-0.0139	2.9699	669.3	3865.7
p_1^+	5.291490796795	1.410828177263	5.744478012088	16.911536728630	0.0108	0.0280	2.9292	688.3	3890.3
p_2^+	5.709785601599	1.410724644300	6.161021805049	17.859885956506	0.0107	-0.0310	3.0469	661.5	3862.7
p_3^+	0.445786440664	1.410197120338	0.694406155661	19.242562023524	0.0107	-0.0427	3.0703	656.1	3853.2
p_4^+	2.497306252429	1.410152727146	2.529928887739	21.526887933769	0.0107	-0.0477	3.0802	653.9	3850.6
p_5^+	3.342284641749	1.410052131810	3.321547105013	22.477024424883	0.0107	-0.0491	3.0830	653.2	3849.2

Table J: Data of transfer samples belonging to families q, r, s, t (Δv are in m/s).

Sol	α	β	θ_i	δ	$h_{2,f}$	$H_{2,f}$	J_f	Δv_f	Δv
q_1^+	0.033811938038	1.410659119648	3.674130519906	19.741812297022	0.0106	-0.0797	3.1442	639.1	3839.8
q_2^+	0.108791841830	1.410562400439	3.710222767135	19.968241671325	0.0106	-0.0842	3.1532	637.0	3837.0
q_3^+	0.491223999802	1.410319147095	3.992195057834	20.656893741883	0.0106	-0.0844	3.1535	637.0	3835.0
r_1^+	0.134835807387	1.410654308228	0.638851039185	19.848727670143	0.0106	-0.0799	3.1446	639.1	3839.7
r_2^+	0.223467001790	1.410536615589	0.677798266138	20.061061770190	0.0106	-0.0837	3.1522	637.3	3837.0
r_3^+	0.436505534285	1.410398783888	0.833540037572	20.584679097046	0.0106	-0.0849	3.1546	636.7	3835.4
s_1^-	4.170159232525	1.402721869479	6.239280173013	20.127265918812	-0.0109	0.0606	2.8205	703.0	3841.8
s_2^-	4.194204735702	1.402656967129	6.244249804099	20.387915382506	-0.0108	0.0258	2.8903	687.3	3825.6
s_3^-	4.252748833570	1.402575402277	0.003397203349	20.541238596444	-0.0108	0.0207	2.9006	685.0	3822.6
t_1^-	4.393669118705	1.402638082275	3.341235442267	20.349343117732	-0.0109	0.0671	2.8076	705.8	3844.0
t_2^-	4.406123815897	1.402654623089	3.356592324586	20.540997135154	-0.0109	0.0375	2.8668	692.6	3830.8
t_3^-	4.419973474774	1.402674069227	3.374381594258	20.732651152577	-0.0108	0.0223	2.8972	685.7	3824.2