Islamic Profit and Loss Sharing Contracting versus Regular Equity in Entrepreneurial Finance: Risk Sharing and Managerial Incentives

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Abstract: An entrepreneur shares business risk with the investors providing capital for her firm. Risk sharing is *per se* beneficial, but also results in an agency problem from diminished incentives for the entrepreneur. This classical trade-off depends on the financial contracting between the entrepreneur and the financier. As an alternative to debt or equity, we consider *musharaka* financing, an Islamic profit and loss sharing contract. First, we show that debt is inferior to equity or *musharaka* even though debt financing ensures first best efforts in our model. Whether financing with equity or by use of *musharaka* results in higher utility for the entrepreneur depends on how the firm's risks are related and on the structure of the costs the entrepreneur has to bear when extending effort.

Keywords: Entrepreneurial finance; Islamic finance; Equity; *Musharaka*; Partnership; Profit and loss sharing; Risk sharing; Managerial incentives

JEL Classification: D25; D81; D82; G32; L26

1. Introduction

An entrepreneur needs capital and has to provide effort to advance the success of their business. Capital can be raised primarily as debt or equity, however, hybrid instruments might also be available. In this paper, we focus on a near-equity financial instrument known as *musharaka* in Islamic finance.[1] A *musharaka* contract results in profit and loss sharing not unlike a partnership with an equity investor. *Musharaka* is, however, more similar to project financing in which the investor provides capital to be invested in some particular tangible assets or a joint venture (Jobst, 2017), and the profits and losses from this particular project are shared. In contrast, providing equity capital results in co-ownership of the entire business, and the equity holders share all profits or losses and any increase or decrease in total value of the firm.

Therefore, *musharaka* and equity financing differ with respect to risk sharing and result in different incentives for the entrepreneur. We compare the alternative financial contracts in an agency-theoretic model with risk sharing and managerial incentives for the entrepreneur as the main determinants of the entrepreneur's utility. We start with debt financing and then consider *musharaka* and equity financing from the perspective of an entrepreneur in a principal-agent relationship.

In our model, although (risk-free) debt financing results in first best effort of the entrepreneur, this alternative of raising capital turns out to be always inferior to *musharaka* and equity financing; with debt financing, the entrepreneur foregoes the benefits of risk sharing, which cannot be compensated for by the effect of higher incentives. The direct comparison of *musharaka* and equity financing reveals no definite ranking. Which of the alternative financial instruments results in higher utility for the entrepreneur depends on the structure of the risks from the entrepreneurial business and on the structure of the costs the entrepreneur has to bear when spending effort. The partial profit and loss sharing via *musharaka* financing, that is, the sharing of profits and losses arising not from the whole business, but from a particular project or certain assets only, results in suboptimal risk sharing. On the other hand, *musharaka* comes with better incentives for the entrepreneur and results in a higher increase in expected firm value net of (private) costs under certain conditions. We consider this trade-off analytically and present a simple numerical example for illustration.

The rest of this paper is structured as follows. In the next section, we provide a short introduction into Islamic (corporate) finance and compare *musharaka* and equity financing with respect to contractual matters. Section 3 provides the review of the literature on agency relationships and risk sharing pertaining to Islamic mode of

[1] Prohibition of activities deemed sinful by Islam governs Islamic banking and finance. From a financial perspective, such prohibitions apply to interest, speculation, excessive uncertainty, and short selling (Iqbal and Mirakhor, 2011).

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financing. In section 4, we present the theoretical model used for our analyses. Section 5 explores debt financing in this theoretical framework while *musharaka* financing is analyzed and compared to debt in section 6. In section 7, we apply our model to equity financing. We then compare equity with *musharaka* in section 8. The insights of this theoretical comparison are illustrated by means of a numerical analysis in section 9. Section 10 provides concluding remarks.

2. Profit and Loss Sharing in Islamic Finance

Islamic methods of financing include "synthetic loans", lease contracts, and profit sharing contracts (Jobst, 2017). The profit sharing contracts adhere the most to the three principles underlying Islamic finance: principle of equity (wealth distribution), principle of participation (risk sharing), and principle of ownership (asset-based financing) (Hussain et al., 2015). Two types of profit sharing contracts are "profit sharing and loss bearing" *mudaraba* contracts and "profit and loss sharing" (PLS) *musharaka* contracts (Hussain et al., 2015).

In a *mudaraba* transaction, an investor provides capital for a project or an investment whereas an entrepreneur plays the role of a manager. The investor is not entitled to take part in the management process, and the profits generated from the investment are distributed between the investor and the entrepreneur based on a pre-agreed ratio while only the investor bears any losses unless they result from the negligence of the entrepreneur (Jobst, 2017). *Mudaraba* financing is used mostly by banks, where the bank can act both as an investor (on the assets side) and as an entrepreneur (on the liabilities side).

Musharaka can be defined as "an equity partnership agreement with one or more partners to jointly finance an investment project" (Jobst, 2017). In a *musharaka* transaction, both the investor and the entrepreneur provide capital to finance a project or an investment. *Musharaka* financing can be used for corporate financing by non-financial companies, and it is a preferred way of Islamic financing due to its features of profit-and-loss and risk sharing. Profit and loss sharing with a *musharaka* contract is between an entrepreneur starting or managing a small or medium-sized business and an investor/partner as a financier, who is not necessarily a bank.

Financing with regular equity, as well as by means of a *musharaka* contract, results in a partnership relationship between an entrepreneur and investor(s) (for *musharaka* as partnership, see, e.g., Usmani (1999), Mirakhor and Zaidi (2007), or Jobst (2017)). One of the parties to the partnership, the entrepreneur, is running a business or planning to start one. The other party invests in the business and shares profits and losses. With regular equity, the investment is open-ended, and all the partners hold a share in the whole business and participate in the firm's profits and the increase (or decrease) in its value. Unlike equity, *musharaka* contracts are not

necessarily open-ended or can be terminated by notice of one partner. However, if one of the partners wishes to terminate his/her participation, the remaining partners may buy him/her out of the partnership (Usmani, 1999). Both types of partnership entail the right, but not the obligation of the investor(s) to take part in the management process.

The main difference between equity and *musharaka* financing is, or at least can be, in the extent of profit and loss sharing. Equity capital is used for financing the entire business of a firm, and each of the equity partners holds a share in the success of that business in total. On the other hand, *musharaka* financing can be used for financing a specific project or investment, fixed assets, real estate, commodities, working capital, or other tangible assets to be used in production or trade (see Usmani (1999), Habib (2018), Rahman et al. (2020)). Yildirim (2021) identifies *musharaka* as a project specific instrument that does not entitle the investors as the owners of the entire firm. Jobst (2017) considers a *musharaka* contract being similar to a joint venture. While a share in the firm's equity entitles the financier to a claim on all of the firm's assets, a *musharaka* contract would allow a claim only on the profit generated by the investment being financed (Bacha et al., 2015). Then a *musharaka* investor does not profit from an increase in firm value in total, but only from the revenues and the increase in value of the financed assets or projects.

This kind of partnership with sharing of profits and losses from specific assets requires identifiability and measurability, which are least problematic for tangible assets. Therefore, in our analysis, we make a distinction between tangible and intangible assets, wherein the *musharaka* contract involves an investment in tangible assets only and, consequently, sharing the profits and losses from only those assets. The profits from the increase in the value of intangible assets or growth opportunities are not shared between the *musharaka* investor and the entrepreneur.

3. Literature on Agency Aspects and Risk Sharing in (Islamic) Finance

We consider a principal-agent relationship between a risk-averse entrepreneur and a risk-averse external financier (investor/partner). The entrepreneur as the manager of the firm can extend effort to increase its future value. The effort comes at private costs for the entrepreneur. In that case, the contract between the principal and the agent determines not only the allocation of risk, but also the managerial incentives and hence the agent's effort. This principal-agent setting is similar to those in the seminal papers by Holmström (1979), Shavell (1979), and Grossman and Hart (1983).

In corporate finance, principal-agent relationships between managers (agents) and shareholders or bondholders as principals have been extensively explored, with Jensen and Meckling (1976) as one of the most influential papers. In that literature,

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external shareholders and bondholders are assumed to be largely diversified, hence interested in the market value of their claims. Then the optimal capital structure mainly has to trade off the agency costs of debt and equity, tax benefits, and bankruptcy costs. The allocation of risk by financial contracting and the firm's capital structure are *per se* not important if all investors manage their portfolio risk via buy-or-sell transactions on a broad capital market.[2]

This is different for small and medium sized enterprises (SME), i.e., in entrepreneurial context. The entrepreneur often has a considerable share of their wealth invested in their firm and, therefore, cares for the direct impact of the risk on their utility and not only for the (risk-dependent) market value of the firm. Similarly, the investor's utility depends directly on the risk he/she has to bear as financier if his/her financial claims are not traded on a frictionless capital market. This kind of financial relation between an individual entrepreneur and a financier, both risk-averse and both with a future uncertain wealth that depends primarily on the firm's performance, is what we are going to analyze.

Such a principal-agent relationship within the framework of Islamic financial transactions is seldom represented in literature even though there has been an extensive amount of research in the field of Islamic finance and banking. Most of this research, however, consists of various ways of comparing Islamic and conventional banks with respect to their financial characteristics and performance using econometric methods (e.g., Čihák and Hesse (2010), Beck et al. (2013), Johnes et al. (2014), Pappas et al. (2016), Alabbad and Schertler (2022)). The theoretical framework of Islamic financing instruments, especially, PLS contracts, with consideration for risk sharing and managerial incentives is hardly explored, with a few notable exceptions outlined below.

Mirakhor et al. (2007) classify and describe Islamic financial instruments, including the profit sharing contracts of *mudaraba* and *musharaka*. They define *mudaraba* financing as "financing by way of trust" and *musharaka* financing as a "partnership". They also refer to moral hazard problems in the context of *musharaka* and hint to the issue that risk aversion of individual entrepreneurs may reduce the efficiency of outcome-based incentive systems, which force them to absorb the risk that their income may vary owing to factors beyond their control. The authors qualitatively infer that high-powered incentives and efficient allocation of risk are, therefore, in conflict. In support of such conflict, Dar (2007) claims that the profit and loss sharing contracts in Islamic financing such as *mudaraba* and *musharaka* contracts lead to an incentive incompatibility between the entrepreneur and the

[2] With investor's unlimited access to frictionless secondary markets, a firm's capital structure is irrelevant for the ultimate allocation of its business risk. Therefore, the assumption of unlimited access to frictionless secondary markets is one of the building blocks of the Modigliani and Miller (1958) theorem.

investor. Our model thoroughly analyzes this conflict and compares *musharaka* financing with debt or equity with respect to the entrepreneur's utility.

Formal models of *musharaka* financing with agency conflicts can be found in, e.g., Yousfi (2013), Elfakir and Tkiouat (2015), and Arbi (2021). Yousfi (2013) considers PLS financing in a double-sided moral hazard problem with risk neutrality. Hence, risk sharing is not an issue. Nevertheless, the first best solution is not reachable under *musharaka* financing since PLS implies partially externalizing the marginal returns of each agent's effort. Elfakir et al. (2015) examine *musharaka* financing in an agency model, where either effort is assumed to be observable or the assumed distribution of output allows for a forcing contract. With all parties assumed to be risk neutral, Arbi (2021) analyzes diminishing *musharaka* contracts, which resemble leasing contracts. He considers a moral hazard problem when asset maintenance is not observable and concludes that if the lessor plans to buy out the lessee at the end of the contract, he/she purposefully neglects the maintenance of the asset.

Different from our problem of unobservable effort in combination with risk sharing is the moral hazard problem that may result if output or the state of nature under which production takes place is unobservable for the principal (costly state verification). That kind of moral hazard problems with contracts as revelation mechanisms are considered by, e.g., Haque and Mirakhor (1986), Presley and Sessions (1994), Yustiardhi et al. (2020), and Ajmi et al. (2020).

In overview, the existing theoretical literature mostly focuses on *mudaraba* financing when it comes to Islamic profit sharing contracts and the informational asymmetries inherent in such contracts. *Musharaka* financing has not been compared to other forms of financing with a consideration for risk sharing and effort-based moral hazard problem in a theoretical framework. Our analysis contributes to the existing literature by theoretically deriving the implications of risk sharing and incentives in a partnership relationship between an entrepreneur and a *musharaka* investor.

4. The Model

We employ the LEN (Linear-Exponential-Normal) model, which is well-known from agency theoretic frameworks dating back to seminal papers by Holmstrom and Milgrom (1987), Spremann (1988), and Holmstrom and Milgrom (1991). For the entrepreneur E (she) seeking financing of her business, as well as for potential financiers, we assume an exponential utility function:

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Assumption 1:

$$U\left(W_{1,i}\right) = -e^{\left(-\theta_i W_{1,i}\right)}, \tag{1}$$

where $W_{1,i}$ is the future uncertain wealth of individual i (i.e., the entrepreneur or a financier) in $t=1$, and $\theta_i > 0$ is i's (constant) absolute risk aversion (Pratt, 1964).

The future wealth $W_{1,E}$ of the entrepreneur depends on the outcome of her business and on how that outcome is divided between herself and an outside investor/partner (he) as a financier. This division depends on the financial contract between the entrepreneur and the financier. The financier's future wealth also depends on the firm's success and how it is shared.

The entrepreneur's business needs an initial investment of capital C. We assume that this investment is in tangible assets and intangible assets or provides for growth opportunities. The tangible assets are of value $\tilde{a}$ in $t=1$, and the value of the intangible assets or growth opportunities in $t=1$ is $\tilde{b}$. Both values depend on the entrepreneur's unobservable effort. With efforts e_a and e_b, respectively, she can shift the distributions of $\tilde{a}$ and $\tilde{b}$ to the right:

Assumption 2:

Asset values depend on the entrepreneur's effort and are jointly normal distributed:

$$\tilde{a} = x + e_a + \tilde{\varepsilon}_a \tag{2}$$

and

$$\tilde{b} = y + e_b + \tilde{\varepsilon}_b, \tag{3}$$

where $x > 0$ and $y > 0$ are constants, e_a and e_b indicate the entrepreneur's effort, and $\tilde{\varepsilon}_a$ and $\tilde{\varepsilon}_b$ are jointly normal distributed random variables with $E\left(\tilde{\varepsilon}_a\right) = 0$, $E\left(\tilde{\varepsilon}_b\right) = 0$, $Var\left(\tilde{\varepsilon}_a\right) > 0$, and $Var\left(\tilde{\varepsilon}_b\right) > 0$. The constants, x and y, and the distributions of the error terms, $\tilde{\varepsilon}_a$ and $\tilde{\varepsilon}_b$, are common knowledge to all the involved parties.

The entrepreneur not only chooses the total effort $(e_a + e_b)$, but also has to decide how to allocate her effort between different tasks, i.e., providing for the value $\tilde{a}$ of tangible assets or for the value of intangible assets or growth opportunities, $\tilde{b}$. With her effort, the entrepreneur increases the expected value of the firm, but that comes at private costs.

Assumption 3:

e_a and e_b are unobservable for anyone but the entrepreneur. The entrepreneur's private cost of exerting effort is

$$c(e_a, e_b) = \frac{e_a^2 + e_b^2}{2} + \gamma e_a e_b, \tag{4}$$

where $-1 < \gamma < 1$ represents the interdependence between the efforts e_a and e_b.[3] This cost function is common knowledge, but the actually chosen efforts are known only to the entrepreneur.

The non-observability of the efforts results in a moral hazard problem with the entrepreneur as the agent and the outside financier as the principal. The moral hazard problem might be alleviated to some extent by monitoring effort from the principal; however, monitoring at any reasonable cost is not a perfect revelation mechanism. At least some informational asymmetry remains in spite of monitoring. Therefore, in our model we focus on that effort which remains hidden information even after due monitoring by the financier.

With the cost function in (4), we assume increasing marginal costs and a positive or negative cross derivative, $\frac{d^2 c}{de_a de_b} = \gamma$. For $\gamma > 0$, a higher effort e_a to increase the value $\tilde{a}$ of tangible assets increases the marginal cost of effort e_b and *vice versa*. Efforts are independent when $\gamma = 0$. For $\gamma < 0$, a higher effort e_a decreases the marginal cost of effort e_b to increase the value $\tilde{b}$ of intangible assets, and *vice versa*.

Limited management capacity of the entrepreneur probably results in a negative relation between e_a and e_b, i.e., a higher effort in one area goes hand in hand with a lower effort in the other. We find that for $\gamma > 0$ in our model. But also a

[3] We restrict γ to the interval $]-1;1[$ to rule out nonsensical results: see appendix A.

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positive relation between e_a and e_b may prevail, i.e., a higher effort in one area results in lower effort costs and hence in a higher effort in the other, e.g., due to complementarities or learning effects. We find that for $\gamma < 0$ in our model.

Given the assumptions 1 and 2 above, the certainty equivalent of the entrepreneur's uncertain future wealth is (Freund, 1956)

$$CE_E = E(W_{1,E}) - \frac{1}{2}\theta_E Var(W_{1,E}), \tag{5}$$

where the expected value $E(W_{1,E})$ and the variance $Var(W_{1,E})$ depend on the financial contract entered by the entrepreneur and her financier. This financial contract directly affects the entrepreneur's utility by allocating the expected value of the firm and the business risk. Furthermore, the financial contract also has incentive effects on the entrepreneur's effort decision, hence indirectly determining her utility.

Assumption 4:

The entrepreneur chooses the financial structure that maximizes the certainty equivalent (5) of her future uncertain wealth, i.e., her expected utility, by deciding between alternative financial contracts, and subsequently, she chooses the effort that maximizes her utility. Financiers are also risk-averse and rational and anticipate the entrepreneur's effort decision correctly. As a premise, the financier must know the entrepreneur's relevant characteristics and abilities (Spremann, 1988). (In our model, this is ensured by the common knowledge components in assumptions 2 and 3.) If the financier is not aware of or does not have trust in the entrepreneur's ability to run the business sufficiently successfully, he will refrain from investing either via *musharaka* or equity.

In the following, we consider three variants of financial structure for the entrepreneurial firm: (i) debt financing; (ii) financing through a *musharaka* contract with an investor I who will hold a share α_I in $\tilde{a}$ (but not a share in $\tilde{b}$ due to the nature of the *musharaka* contract as discussed in section 2); (iii) financing with equity, i.e., a partnership with an equity partner P who will hold a share α_P in the firm, that is, in both asset classes.

The entrepreneur's initial wealth $W_{0,E}$ in $t = 0$ may or may not be sufficient to finance the investment in her business, i.e., $W_{0,E} \geq C$ or $W_{0,E} < C$. In case of sufficient capital, $W_{0,E} \geq C$, she does not need an outside financier, but as we will see, will nevertheless profit from contracting with an outside financier because of the merits

of risk sharing. In case of insufficient capital, i.e., when $W_{0,E} < C$, the entrepreneur needs external financing to start her business. The necessary funds, $\left(C - W_{0,E}\right)$, may be raised by debt financing, through a *musharaka* contract, or by equity financing. If the entrepreneur refrains from starting her business, she invests all her capital in risk-free assets on the market, earning a return rate of r_f. Even with investing in the firm after raising external capital with *musharaka* contract or as equity, she may have some money left to invest outside her firm at the rate of r_f. We assume that the same return rate is charged on any debt of the entrepreneur's firm. This implies that the debt is risk-free, and no transaction costs or other market imperfections result in differences between lending and borrowing rates. We state this assumption to concentrate on risk sharing benefits and adverse incentive effects instead of other implications of market imperfections.

Assumption 5:

In case the entrepreneur raises any debt to finance her firm, this debt is free of default risk, and the interest rate r_f on debt is the same as the rate of return on alternative (risk-free) investments outside the firm.

5. Debt Financing

We first consider debt financing for the entrepreneur with initial wealth $W_{0,E} < C$. In this case, the entrepreneur's wealth in $t = 1$ is

$$\tilde{W}_{1,E} = \tilde{a} + \tilde{b} - c\left(e_a, e_b\right) - \left(1 + r_f\right)\left(C - W_{0,E}\right), \tag{6}$$

with $\left(C - W_{0,E}\right)$ being the debt raised. $\tilde{a} + \tilde{b} - \left(1 + r_f\right)C$ could be identified as the firm's economic profit or loss. The expected value and risk thereof, together with the cost of effort, determine the entrepreneur's certainty equivalent of her future wealth. This certainty equivalent of her future wealth from (6) is

$$CE_E = E\left(\tilde{a} + \tilde{b}\right) - c\left(e_a, e_b\right) - \left(1 + r_f\right)\left(C - W_{0,E}\right) - \frac{1}{2}\theta_E Var\left(\tilde{a} + \tilde{b}\right). \tag{7}$$

With (2) and (3) for the asset values from assumption 2 and (4) for the cost function from assumption 3, we see how this certainty equivalent depends on the entrepreneur's effort:

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$$CE_E = x + y + e_a + e_b - \frac{1}{2}\left(e_a^2 + e_b^2\right) - \gamma e_a e_b - \left(1 + r_f\right)\left(C - W_{0,E}\right) - \frac{1}{2}\theta_E Var\left(\tilde{a} + \tilde{b}\right). \quad (8)$$

After the business is initiated with debt financing, the entrepreneur chooses the efforts e_a and e_b that maximize her certainty equivalent from (8). This optimization leads to

$$e_a = \frac{1}{1+\gamma} \quad , \quad e_b = \frac{1}{1+\gamma}, \quad (9)$$

which are the first best efforts, since with risk free debt, the entrepreneur not only bears the total marginal costs, but also the total marginal benefit of her efforts. Both efforts from (9) decrease in the parameter γ, which measures the interdependence of the marginal costs. The more expensive one effort makes (additional) effort in the other part of the firm, the lower are the optimal efforts in both parts. Note that both efforts have the same value under debt financing. With the optimal efforts from (9), the certainty equivalent from (8) is

$$CE_E = x + y + \frac{1}{1+\gamma} - \left(1 + r_f\right)\left(C - W_{0,E}\right) - \frac{1}{2}\theta_E Var\left(\tilde{a} + \tilde{b}\right). \quad (10)$$

The entrepreneur will start her business with debt financing only if this certainty equivalent (10) exceeds her future wealth in case she refrains from it:

$$x + y + \frac{1}{1+\gamma} - \left(1 + r_f\right)\left(C - W_{0,E}\right) - \frac{1}{2}\theta_E Var\left(\tilde{a} + \tilde{b}\right) > \left(1 + r_f\right)\left(W_{0,E} + w\right), \quad (11)$$

with w for the present value of the alternative wage she could earn elsewhere if not managing the firm. From (11) follows

$$\frac{1}{1+r_f}\left[x + y + \frac{1}{1+\gamma} - \frac{1}{2}\theta_E Var\left(\tilde{a} + \tilde{b}\right)\right] > C + w. \quad (12)$$

This condition (12) is an NPV criterion: The present value of expected future value of the firm minus deduction for risk (on the LHS) must exceed the invested capital plus the present value of the alternative wage (on the RHS). With debt financing, no risk sharing is provided, so the deduction for risk (i.e., the risk premium)

depends on the total risk $Var\left(\tilde{a}+\tilde{b}\right)$ and on the entrepreneur's individual risk preference θ_E.

In the following sections, we will consider how alternative financing contracts affect the entrepreneur's incentives and risk sharing and what certainty equivalent results for the entrepreneur.

6. *Musharaka* Financing

The entrepreneur is now facing the opportunity to finance tangible assets with capital contribution from an investor through a *musharaka* profit and loss sharing contract. Based on this contract, the *musharaka* investor I provides capital in amount of S_I and is entitled to a share α_I in the value $\tilde{a}$ of tangible assets. Since the assets in which the investor has a stake are tangible, their value is relatively easy to observe and hence to be divided based on the contractual share α_I, separately from the firm's total value.

The investor's certainty equivalent CE_I of his future wealth depending on the share α_I and the capital S_I invested in the entrepreneurial firm is

$$CE_I = \alpha_I E\left(\tilde{a}\right) - \left(1+r_f\right)\left(\mathrm{S}_I - W_{0,I}\right) - \frac{1}{2}\theta_I \alpha_I^2 Var\left(\tilde{a}\right), \tag{13}$$

with $W_{0,I}$ for the investor's initial wealth in $t=0$ and $\theta_I > 0$ for his risk aversion. $\alpha_I E\left(\tilde{a}\right) - \left(1+r_f\right)\mathrm{S}_I$ is the investor's expected economic profit or loss.

We assume that the *musharaka* contract results in a certainty equivalent for the investor exactly as high as with his outside option.

Assumption 6:

$$\begin{aligned} & \alpha_I E\left(\tilde{a}\right) - \left(1+r_f\right)\left(\mathrm{S}_I - W_{0,I}\right) - \frac{1}{2}\theta_I \alpha_I^2 Var\left(\tilde{a}\right) = \left(1+r_f\right)W_{0,I} \\ \Leftrightarrow \quad & \alpha_I E\left(\tilde{a}\right) - \frac{1}{2}\theta_I \alpha_I^2 Var\left(\tilde{a}\right) = \left(1+r_f\right)\mathrm{S}_I \end{aligned}. \tag{14}$$

This assumption is consistent with the assumption 5 that has the same implication for the firm's lender. Neither the lender nor the now considered investor I earns more (on a risk-adjusted basis) than with their outside option. Without this

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similarity, differences in risk sharing and incentive effects would be contaminated with differences in distribution of wealth.

Sharing the value $\tilde{a}$ in *musharaka* financing and with the investor's capital provision S_I, the entrepreneur's certainty equivalent is

$$\begin{aligned} CE_E = &(1-\alpha_I)E(\tilde{a}) + E(\tilde{b}) - c(e_a, e_b) - (1+r_f)(C - W_{0,E} - S_I) \\ &- \frac{1}{2}\theta_E(1-\alpha_I)^2 Var(\tilde{a}) - \frac{1}{2}\theta_E Var(\tilde{b}) - \theta_E(1-\alpha_I)Cov(\tilde{a},\tilde{b}) \end{aligned} . \quad (15)$$

If $(C - W_{0,E} - S_I) > 0$, the entrepreneur additionally requires some debt financing. On the other hand, if $(C - W_{0,E} - S_I) < 0$, she invests the surplus on the market. As stated in assumption 5, the interest rate r_f on debt is the same as the rate of return on alternative (risk-free) investments outside the firm. Hence, we do not have to differentiate cases with respect to the sign of $(C - W_{0,E} - S_I)$.

With (2) and (3) for the asset values from assumption 2 and (4) for the cost function from assumption 3, the entrepreneur's certainty equivalent becomes

$$\begin{aligned} CE_E = &(1-\alpha_I)(x + e_a) + (y + e_b) - \frac{1}{2}(e_a^2 + e_b^2) - \gamma e_a e_b - (1+r_f)(C - W_{0,E} - S_I) \\ &- \frac{1}{2}\theta_E(1-\alpha_I)^2 Var(\tilde{a}) - \frac{1}{2}\theta_E Var(\tilde{b}) - \theta_E(1-\alpha_I)Cov(\tilde{a},\tilde{b}) \end{aligned} . \quad (16)$$

This is the entrepreneur's certainty equivalent to be maximized by the choice of efforts e_a and e_b after the *musharaka* financing contract has been signed, that is, with given α_I and S_I. For the entrepreneur, the following efforts are optimal:

$$e_a = \frac{1-\gamma-\alpha_I}{1-\gamma^2} \quad , \quad e_b = \frac{1-\gamma+\gamma\alpha_I}{1-\gamma^2} . \quad (17)$$

Note that financing the firm with the considered *musharaka* contract results in lower effort e_a compared to the case of debt financing (see (9)) for all $\alpha_I > 0$. Since the entrepreneur has to share the fruits of her effort in raising the value $\tilde{a}$ of the tangible assets, her incentives to do so are diluted. And a larger share α_I of the *musharaka* investor results in higher dilution of incentives and hence in lower effort e_a. This is the classical moral hazard problem in a principal-agent relationship. On

the other hand, e_b is higher than with debt financing if $\gamma > 0$ (*vice versa* for $\gamma < 0$). But the total effort $(e_a + e_b)$ is less than under debt financing for all $\alpha_I > 0$:

$$e_a + e_b = \frac{2-\alpha_I}{1+\gamma} < \frac{2}{1+\gamma}. \tag{18}$$

In this respect, a dilution of overall incentives by *musharaka* financing is to be observed.

If the investor correctly anticipates the entrepreneur's efforts based on his share α_I, we can substitute for S_I from the investor's participation constraint (14) and the efforts from (17) in (16):

$$\begin{aligned} CE_E = {} & x + y + \frac{1}{1+\gamma} - \frac{\alpha_I^2}{2\left(1-\gamma^2\right)} - \left(1+r_f\right)\left(C - W_{0,E}\right) \\ & - \frac{1}{2}\left[\theta_E\left(1-\alpha_I\right)^2 + \theta_I\alpha_I^2\right]Var\left(\tilde{a}\right) - \frac{1}{2}\theta_E Var\left(\tilde{b}\right) - \theta_E\left(1-\alpha_I\right)Cov\left(\tilde{a},\tilde{b}\right) \end{aligned}. \tag{19}$$

Now our last step (in fact, the entrepreneur's first decision) is to find the optimal share α_I that maximizes her certainty equivalent in (19). This maximization leads to

$$\alpha_I^* = \frac{\theta_E\left[Var\left(\tilde{a}\right) + Cov\left(\tilde{a},\tilde{b}\right)\right]}{\frac{1}{1-\gamma^2} + \left(\theta_E + \theta_I\right)Var\left(\tilde{a}\right)}. \tag{20}$$

Since we want to make sure that this inner solution lies in the economically sensible interval $]0;1[$, i.e., $0 < \alpha_I^* < 1$, we state[4]

Assumption 7:

$$Cov\left(\tilde{a},\tilde{b}\right) > -Var\left(\tilde{a}\right) \tag{21}$$

and

[4] Otherwise, we would have to continue with the boundary solution of either $\alpha_I = 0$ or $\alpha_I = 1$.

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$$Cov\left(\tilde{a},\tilde{b}\right) < \frac{\dfrac{1}{1-\gamma^2} + \theta_I Var\left(\tilde{a}\right)}{\theta_E}. \tag{22}$$

From condition (21) follows $\alpha_I^* > 0$. This condition holds for any positive covariance and even with a negative covariance if the variance $Var\left(\tilde{a}\right)$ of the value of the tangible assets is not "small" compared to the variance $Var\left(\tilde{b}\right)$ of the value of the other (intangible) assets.[5] From condition (22) follows $\alpha_I^* < 1$.

Note that α_I^* takes the maximum value for $\gamma = 0$. In that case, the effort e_b (from (17)) is independent of α_I. Hence, there is no effort distortion in that area. That allows for more risk sharing, that is, a higher value of α_I. But since the total effort $\left(e_a + e_b\right)$ decreases in α_I (see (18)), the choice of α_I always has to trade off this incentive effect against the benefits of risk sharing.

With the optimal share α_I^* from (20), the entrepreneur's maximized certainty equivalent from (19) is

$$\begin{aligned} CE_E = {} & x + y + \frac{1}{1+\gamma} - \left(1+r_f\right)\left(C - W_{0,E}\right) - \frac{1}{2}\theta_E Var\left(\tilde{a}+\tilde{b}\right) \\ & + \frac{1}{2}\theta_E \left[Var\left(\tilde{a}\right) + Cov\left(\tilde{a},\tilde{b}\right)\right] \underbrace{\frac{\theta_E\left[Var\left(\tilde{a}\right) + Cov\left(\tilde{a},\tilde{b}\right)\right]}{\dfrac{1}{1-\gamma^2} + \left(\theta_E + \theta_I\right)Var\left(\tilde{a}\right)}}_{\alpha_I^*}. \end{aligned} \tag{23}$$

As in (11), we now compare this certainty equivalent from (23) with her future wealth in case she refrains from the business. Under the *musharaka* financing, the entrepreneur gains if

[5] If $Var\left(\tilde{a}\right) > \rho_{\left(\tilde{a},\tilde{b}\right)}^2 Var\left(\tilde{b}\right)$, then $\sigma_{\tilde{a}} > \left|\rho_{\left(\tilde{a},\tilde{b}\right)}\right|\sigma_{\tilde{b}} \Leftrightarrow \sigma_{\tilde{a}}^2 > \left|\rho_{\left(\tilde{a},\tilde{b}\right)}\right|\sigma_{\tilde{a}}\sigma_{\tilde{b}}$. Adding the covariance to both sides results in $\sigma_{\tilde{a}}^2 + \rho_{\left(\tilde{a},\tilde{b}\right)}\sigma_{\tilde{a}}\sigma_{\tilde{b}} = Var\left(\tilde{a}\right) + Cov\left(\tilde{a},\tilde{b}\right) > \left[\left|\rho_{\left(\tilde{a},\tilde{b}\right)}\right| + \rho_{\left(\tilde{a},\tilde{b}\right)}\right]\sigma_{\tilde{a}}\sigma_{\tilde{b}} \geq 0$. Therefore, $Var\left(\tilde{a}\right) > \rho_{\left(\tilde{a},\tilde{b}\right)}^2 Var\left(\tilde{b}\right)$ suffices for $Var\left(\tilde{a}\right) + Cov\left(\tilde{a},\tilde{b}\right) > 0 \Leftrightarrow Cov\left(\tilde{a},\tilde{b}\right) > -Var\left(\tilde{a}\right)$.

$$x+y+\frac{1}{1+\gamma}-\left(1+r_f\right)\left(C-W_{0,E}\right)-\frac{1}{2}\theta_E Var\left(\tilde{a}+\tilde{b}\right)$$
$$+\frac{1}{2}\theta_E\left[Var\left(\tilde{a}\right)+Cov\left(\tilde{a},\tilde{b}\right)\right]\frac{\theta_E\left[Var\left(\tilde{a}\right)+Cov\left(\tilde{a},\tilde{b}\right)\right]}{\frac{1}{1-\gamma^2}+\left(\theta_E+\theta_I\right)Var\left(\tilde{a}\right)}>\left(1+r_f\right)\left(W_{0,E}+w\right) \quad (24)$$

$$\Leftrightarrow \quad \frac{1}{1+r_f}\left[x+y+\frac{1}{1+\gamma}-\left(1+r_f\right)\left(C-W_{0,E}\right)-\frac{1}{2}\theta_E Var\left(\tilde{a}+\tilde{b}\right)\right.$$
$$\left.+\frac{1}{2}\frac{\theta_E^2\left[Var\left(\tilde{a}\right)+Cov\left(\tilde{a},\tilde{b}\right)\right]^2}{\frac{1}{1-\gamma^2}+\left(\theta_E+\theta_I\right)Var\left(\tilde{a}\right)}\right]>C+w \quad . \quad (25)$$

The LHS of condition (25) is again a risk-adjusted present value, given that the entrepreneur optimally chooses the terms of the *musharaka* contract and individually optimizes her effort. If this present value exceeds the total investment needed plus the present value of her alternative wage, the business has a positive NPV adjusted for risk.

More importantly, we address the question which of the so far considered financing contracts results in a higher utility for the entrepreneur. To answer this question, we have to compare certainty equivalent in (23) under the optimal *musharaka* contract with the certainty equivalent in (10) in case of debt financing. This comparison reveals

Proposition 1:

Musharaka financing is always more advantageous for the entrepreneur compared to debt since the certainty equivalent in (23) exceeds the certainty equivalent in (10) if

$$\frac{\theta_E^2\left[Var\left(\tilde{a}\right)+Cov\left(\tilde{a},\tilde{b}\right)\right]^2}{\frac{1}{1-\gamma^2}+\left(\theta_E+\theta_I\right)Var\left(\tilde{a}\right)}>0, \quad (26)$$

which holds for all parameters compatible with our assumptions.

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Therefore, despite diluted incentives to extend effort, *musharaka* financing always results in a higher certainty equivalent for the entrepreneur than debt financing due to the benefits from risk sharing.

It might even be the case that the business is not advantageous for the entrepreneur if financed with debt (i.e., condition (12) does not hold), but it is advantageous with *musharaka* financing (i.e. condition (25) holds). In this case, risk sharing increases investment activity.

7. Equity Financing

Instead of financing with capital from an investor by means of a *musharaka* profit and loss sharing contract as considered above, we now analyze financing with regular equity capital. Equity capital comes from a partner P who provides capital in amount of S_P and then holds a share α_P in the firm, i.e., he becomes a regular shareholder. Therefore, as opposed to the *musharaka* investor, the partner P does not profit from the returns of some (tangible) assets only, but from the overall increase or decrease in firm value. This difference affects the incentives for the entrepreneur to spend effort and results in an optimal value for the partner's share α_P that differs from the *musharaka* investor's share α_I (in (20)) and, consequently, different allocation of risk.

We will now analyze how financing with equity capital affects the entrepreneur's utility and then compare the results with those from debt financing and *musharaka* profit and loss sharing.

For the partner P, we assume that he is also risk-averse with an exponential utility function and a risk aversion factor of $\theta_P > 0$ (see assumption 1). Therefore, the certainty equivalent CE_P of his future wealth depending on the share α_P and the capital S_P invested in the firm is

$$CE_P = \alpha_P E\left(\tilde{a}+\tilde{b}\right) - \left(1+r_f\right)\left(\mathrm{S}_P - W_{0,P}\right) - \frac{1}{2}\theta_P \alpha_P^2 Var\left(\tilde{a}+\tilde{b}\right), \tag{27}$$

with $W_{0,P}$ for the partner's initial wealth in $t=0$, and $\alpha_P E\left(\tilde{a}+\tilde{b}\right) - \left(1+r_f\right)\mathrm{S}_P$ for his expected economic profit or loss.

We assume (similarly to assumption 6) that the equity partnership results in a certainty equivalent for the partner exactly as high as with his outside option.

Assumption 8:

$$\begin{aligned} &\alpha_P E\left(\tilde{a}+\tilde{b}\right)-\left(1+r_f\right)\left(\mathrm{S}_P-W_{0,P}\right)-\frac{1}{2}\theta_P\alpha_P^2 Var\left(\tilde{a}+\tilde{b}\right)=\left(1+r_f\right)W_{0,P} \\ \Leftrightarrow \quad &\alpha_P E\left(\tilde{a}+\tilde{b}\right)-\frac{1}{2}\theta_P\alpha_P^2 Var\left(\tilde{a}+\tilde{b}\right)=\left(1+r_f\right)\mathrm{S}_P \end{aligned} . \tag{28}$$

The reason for this assumption is again that none of the considered financiers should earn more (on a risk-adjusted basis) than with their outside option. Without this similarity, differences in risk sharing and incentive effects would be contaminated with differences in distribution of wealth.

With the partner P providing capital S_P and holding a share α_P in the firm, the entrepreneur's certainty equivalent is

$$\begin{aligned} CE_E = &\left(1-\alpha_P\right)E\left(\tilde{a}+\tilde{b}\right)-c\left(e_a,e_b\right)-\left(1+r_f\right)\left(C-W_{0,E}-S_P\right) \\ &-\frac{1}{2}\theta_E\left(1-\alpha_P\right)^2 Var\left(\tilde{a}+\tilde{b}\right) \end{aligned} . \tag{29}$$

Again, for $\left(C-W_{0,E}-S_P\right)>0$, the entrepreneur additionally requires some debt financing with an interest rate r_f. On the other hand, if $\left(C-W_{0,E}-S_P\right)<0$, she invests the remaining capital at the risk-free rate, r_f (see the discussion of (15) above).

With (2) and (3) for the asset values from assumption 2 and (4) for the cost function from assumption 3 follows:

$$\begin{aligned} CE_E = &\left(1-\alpha_P\right)\left(x+e_a+y+e_b\right)-\frac{1}{2}\left(e_a^2+e_b^2\right)-\gamma e_a e_b-\left(1+r_f\right)\left(C-W_{0,E}-S_P\right) \\ &-\frac{1}{2}\theta_E\left(1-\alpha_P\right)^2 Var\left(\tilde{a}+\tilde{b}\right) \end{aligned} . \tag{30}$$

This is the entrepreneur's certainty equivalent to be maximized by the choice of efforts e_a and e_b after the partnership contract has been signed, that is, with given α_P and S_P. For the entrepreneur, the following efforts are optimal:

$$e_a=\frac{1-\alpha_P}{1+\gamma} \quad , \quad e_b=\frac{1-\alpha_P}{1+\gamma} . \tag{31}$$

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Note that the equity financing results in the same effort ($e_a = e_b$) in both of the entrepreneur's tasks. This is the result of the fact that the entrepreneur evenly shares the fruits of all her efforts with the partner, so that the incentives for providing effort e_a and effort e_b are evenly diluted. And a larger share α_P of the equity partner results in higher dilution of incentives and hence in lower effort in both tasks. These efforts in (31) fall short of the first best efforts (see (9)) for any $\alpha_P > 0$.

If the partner correctly anticipates the entrepreneur's efforts based on his share α_P, we can substitute S_P from the partner's participation constraint (28) and the efforts from (31) in (30):

$$CE_E = x + y + \frac{1}{1+\gamma} - \frac{\alpha_P^2}{1+\gamma} - \frac{1}{2}\left[\theta_E\left(1-\alpha_P\right)^2 + \theta_P\alpha_P^2\right]Var\left(\tilde{a}+\tilde{b}\right). \tag{32}$$

The first-order condition for the maximization of (32) determines the optimal share α_P:

$$\alpha_P^* = \frac{\theta_E Var\left(\tilde{a}+\tilde{b}\right)}{\frac{2}{1+\gamma} + \left(\theta_E + \theta_P\right)Var\left(\tilde{a}+\tilde{b}\right)}. \tag{33}$$

Note that α_P^* increases in γ. The reason is that α_P^* has a negative impact on effort; $\frac{de_a}{d\alpha_P} = \frac{de_b}{d\alpha_P} = -\frac{1}{1+\gamma}$, but the larger γ, the smaller this negative marginal effect in absolute terms. Consequently, with larger values of γ, it becomes marginally less costly to choose higher values for α_P^* to improve on risk sharing.

With this optimal share α_P^* from (33), the entrepreneur's certainty equivalent from (32) is

$$\begin{aligned} CE_E = {} & x + y + \frac{1}{1+\gamma} - \left(1+r_f\right)\left(C - W_{0,E}\right) - \frac{1}{2}\theta_E Var\left(\tilde{a}+\tilde{b}\right) \\ & + \frac{1}{2}\theta_E Var\left(\tilde{a}+\tilde{b}\right)\underbrace{\frac{\theta_E Var\left(\tilde{a}+\tilde{b}\right)}{\frac{2}{1+\gamma} + \left(\theta_E + \theta_P\right)Var\left(\tilde{a}+\tilde{b}\right)}}_{\alpha_P^*}. \end{aligned} \tag{34}$$

In order to render the business with equity partnership advantageous for the entrepreneur, this certainty equivalent (34) must exceed her future wealth in case she refrains from the business:

$$x+y+\frac{1}{1+\gamma}-\left(1+r_f\right)\left(C-W_{0,E}\right)-\frac{1}{2}\theta_E Var\left(\tilde{a}+\tilde{b}\right)$$
$$+\frac{1}{2}\theta_E Var\left(\tilde{a}+\tilde{b}\right)\frac{\theta_E Var\left(\tilde{a}+\tilde{b}\right)}{\frac{2}{1+\gamma}+\left(\theta_E+\theta_P\right)Var\left(\tilde{a}+\tilde{b}\right)}>\left(1+r_f\right)\left(W_{0,E}+w\right) \tag{35}$$

$$\Leftrightarrow \quad \frac{1}{1+r_f}\left[x+y+\frac{1}{1+\gamma}-\frac{1}{2}\theta_E Var\left(\tilde{a}+\tilde{b}\right)\right.$$
$$\left.+\frac{1}{2}\frac{\theta_E^2\left[Var\left(\tilde{a}+\tilde{b}\right)\right]^2}{\frac{2}{1+\gamma}+\left(\theta_E+\theta_P\right)Var\left(\tilde{a}+\tilde{b}\right)}\right]>C+w \tag{36}$$

The LHS of (36) is again a risk-adjusted present value, given that the entrepreneur optimally chooses the terms of the partnership and individually optimizes her effort. If this present value exceeds the total investment needed plus the present value of her alternative wage, the business has a positive NPV adjusted for risk.

By comparing the certainty equivalent of the entrepreneur under equity partnership with a shareholder from (34) with the result for debt (10), we find

Proposition 2:

Equity financing results in a higher utility for the entrepreneur compared to debt since the certainty equivalent in (34) exceeds the certainty equivalent in (10) if

$$\frac{\theta_E^2\left[Var\left(\tilde{a}+\tilde{b}\right)\right]^2}{\frac{2}{1+\gamma}+\left(\theta_E+\theta_P\right)Var\left(\tilde{a}+\tilde{b}\right)}>0\,, \tag{37}$$

which holds for any $\gamma>-1$.

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Therefore, as in the case with *musharaka* financing, despite diminished incentive to exert effort, equity financing dominates debt financing. Again, it might even be the case that the business is not advantageous for the entrepreneur if financed with debt (i.e., condition (12) does not hold), but it is advantageous with equity financing (i.e., condition (36) holds).

Musharaka and equity financing differ in how risk is shared. But with both financial alternatives, the entrepreneur benefits from the risk sharing to an extent that compensates for the losses from diminished effort incentives.

8. *Musharaka* versus Equity

Both *musharaka* and equity financing are variants of profit and loss sharing contracts and, consequently, result in risk sharing. But the terms of risk sharing and incentive effects are different. For the optimal value of α_I in *musharaka* financing (see (20)), we calculated the entrepreneur's certainty equivalent in (23). For the optimal value of α_P in partnership with equity financing (see (33)), we calculated the entrepreneur's certainty equivalent in (34). For the sake of comparability, we further assume

Assumption 9:

The equity partner and the *musharaka* investor are equally risk-averse: $\theta_P = \theta_I \equiv \theta$.

This assumption ensures that none of the alternative financial contracts leads to more (or less) benefits from risk sharing only because of lower (or higher) risk aversion of the financier with whom the entrepreneur shares risk.

Comparing the certainty equivalent of the entrepreneur under partnership with an equity shareholder from (34) with the certainty equivalent under the *musharaka* contract (23) by applying assumption 9, we find

Proposition 3:

Musharaka financing may or may not result in higher utility for the entrepreneur compared to financing with regular equity depending on the structure of risks and the effort cost function. *Musharaka* results in a higher certainty equivalent for the entrepreneur than financing with equity, i.e., *musharaka* is more advantageous, iff

$$\left(1-\rho^2_{(\tilde{a},\tilde{b})}\right)\left\{(1+\gamma)(\theta_E+\theta)Var\left(\tilde{a}+\tilde{b}\right)+2\right\} < \frac{Var\left(\tilde{a}+\tilde{b}\right)}{Var\left(\tilde{b}\right)}\left[2-\frac{Var\left(\tilde{a}+\tilde{b}\right)}{(1-\gamma)Var\left(\tilde{a}\right)}\right], \quad (38)$$

with $\rho_{(\tilde{a},\tilde{b})}$ for the correlation between the two components of the firm's value.[6]

If condition (38) holds, that is, if *musharaka* financing results in a higher utility for the entrepreneur than equity financing, it must be because of better incentive effects overcompensating for the disadvantage in how risk is shared.

If incentives are not relevant and moral hazard problems do not exist, equity financing with linear sharing of total business risk[7] dominates *musharaka* financing as considered here, with sharing of risk in only $\tilde{a}$:

Lemma 1:

If the financial structure does not have an impact on the entrepreneur's effort (i.e., in the absence of agency problems), and the first best efforts, $e_a = e_b = \frac{1}{1+\gamma}$, are implemented, risk sharing with equity is superior to risk sharing under *musharaka* financing: $CE_E(equity) > CE_E(musharaka)$ for all $\left|\rho_{(\tilde{a},\tilde{b})}\right| < 1$. Proof: see appendix B.

In that case, only for $\left|\rho_{(\tilde{a},\tilde{b})}\right| = 1$, both alternatives of raising capital result in the same certainty equivalent for the entrepreneur since with perfect positive or negative correlation, there is, in fact, only one risk. The entrepreneur can, therefore, optimally share total business risk with the *musharaka* investor via his share in $\tilde{a}$. A certain share of the investor in $\tilde{a}$ only results in the same risk for the entrepreneur as the optimal share in both $\tilde{a}$ and $\tilde{b}$ of an equity partner if the risks are perfectly correlated. Then no advantage of equity financing with respect to risk sharing remains.

For all non-perfectly correlated risks in $\tilde{a}$ and $\tilde{b}$, i.e., $\left|\rho_{(\tilde{a},\tilde{b})}\right| < 1$, sharing total business risk $Var\left(\tilde{a}+\tilde{b}\right)$ with an equity partner, *ceteris paribus*, dominates partial risk sharing with *musharaka* financing. And the relative advantage of equity with respect

[6] The formal comparison of the certainty equivalents from (34) and (23) to deduce (38) is a straightforward, but lengthy task. A step-by-step illustration by the authors is available upon request.

[7] The optimal division of the sum of two risky assets, independently of the specific distributions, was already determined in Borch (1960). For the optimality of linear sharing rules, see also Borch (1968), Wilson (1968), Drèze (1990), and Lemaire (1990).

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to risk sharing increases in the ratio $\frac{Var(\tilde{a}+\tilde{b})}{Var(\tilde{a})}$. The higher $Var(\tilde{a}+\tilde{b})$ is compared to $Var(\tilde{a})$, the more important it is for the entrepreneur to share total risk instead of only $Var(\tilde{a})$ (see appendix C). The ratio $\frac{Var(\tilde{a}+\tilde{b})}{Var(\tilde{a})}$ of total business risk to the variance $Var(\tilde{a})$ is lowest if the risk in $\tilde{b}$ is negligible. If the risk in $\tilde{b}$, i.e., $Var(\tilde{b})$, is negligible (and so is the covariance), the entrepreneur does not benefit much from sharing this risk with a partner instead of bearing it alone under *musharaka* financing under which only the risk in $\tilde{a}$ is shared.

As a counterbalance to the suboptimal risk sharing, *musharaka* financing comes with superior incentive effects under certain conditions, which can be determined through the comparison of the profits from effort net of the entrepreneur's costs under *musharaka* and equity financing.

$$\begin{aligned} NPM &= \frac{1-\gamma-\alpha_I}{1-\gamma^2}+\frac{1-\gamma+\gamma\alpha_I}{1-\gamma^2} \\ &\quad -\frac{1}{2}\left[\left(\frac{1-\gamma-\alpha_I}{1-\gamma^2}\right)^2+\left(\frac{1-\gamma+\gamma\alpha_I}{1-\gamma^2}\right)^2\right]-\gamma\left(\frac{1-\gamma-\alpha_I}{1-\gamma^2}\right)\left(\frac{1-\gamma+\gamma\alpha_I}{1-\gamma^2}\right) \\ &= \frac{1}{1+\gamma}-\frac{\alpha_I^2}{2\left(1-\gamma^2\right)} \end{aligned} \tag{39}$$

is the net profit from extending effort under *musharaka,* and

$$\begin{aligned} NPE &= \frac{1-\alpha_P}{1+\gamma}+\frac{1-\alpha_P}{1+\gamma} \\ &\quad -\frac{1}{2}\left[\left(\frac{1-\alpha_P}{1+\gamma}\right)^2+\left(\frac{1-\alpha_P}{1+\gamma}\right)^2\right]-\gamma\left(\frac{1-\alpha_P}{1+\gamma}\right)\left(\frac{1-\alpha_P}{1+\gamma}\right) \\ &= \frac{1}{1+\gamma}-\frac{\alpha_p^2}{1+\gamma} \end{aligned} \tag{40}$$

is the net profit from extending effort under equity financing. Comparing those net profits from effort in (39) and (40), we find

Lemma 2:

$$NPM > NPE \quad \text{for} \quad \frac{\alpha_I^2}{\alpha_P^2} < 2(1-\gamma). \tag{41}$$

Proof: $NPM > NPE \Leftrightarrow \frac{\alpha_I^2}{2(1-\gamma^2)} < \frac{\alpha_p^2}{1+\gamma} \Leftrightarrow \frac{\alpha_I^2}{\alpha_p^2} < 2(1-\gamma)$.

A higher net profit from effort under *musharaka* financing compared to equity, i.e., $NPM > NPE$, is a necessary (but not sufficient) condition for *musharaka* to dominate equity. A higher net profit under *musharaka* is necessary to compensate for the disadvantage in risk sharing. However, even if the net profit under *musharaka* is higher, the entrepreneur's utility might nevertheless be lower due to the advantage in risk sharing of equity. Generally, there is a trade-off between risk sharing benefits and net profit of effort. For given (first best) efforts, equity financing results in a higher utility because of better risk sharing (see lemma 1). On the other hand, with the agency problem at work, *musharaka* financing results in a higher net profit of effort under the condition specified in lemma 2.

The advantage of *musharaka* financing with respect to net profit of effort (see lemma 2) depends (*inter alia*) on γ. For *musharaka* to result in a certainty equivalent at least as high as equity financing, *musharaka* must come with an advantage with respect to net profit: $NPM > NPE$. The higher γ gets, the smaller this advantage becomes; the difference in net profits (from (39) and (40)), *ceteris paribus*, decreases in γ:

$$\frac{d(NPM - NPE)}{d\gamma} = -\frac{\alpha_P^2}{(1+\gamma)^2} - \frac{\gamma\alpha_I^2}{(1-\gamma^2)^2} < 0 \tag{42}$$

for all γ.[8] The relative advantage of *musharaka* with respect to net profit, *ceteris paribus*, decreases in γ since a higher γ implies such a change of marginal cost of effort (see

[8] Proof: The partial derivative in (42) is obviously negative for all $\gamma \geq 0$. For $\gamma < 0$, we consider the equivalent condition $-\frac{\gamma\alpha_I^2}{\alpha_P^2} - (1-\gamma)^2 < 0$. From lemma 2, we know that $NPM > NPE$ requires $\frac{\alpha_I^2}{\alpha_P^2} < 2(1-\gamma)$. This latter condition implies $-\frac{\gamma\alpha_I^2}{\alpha_P^2} < -2(1-\gamma)\gamma$ if $\gamma < 0$. Therefore, we find $-\frac{\gamma\alpha_I^2}{\alpha_P^2} - (1-\gamma)^2 < -2(1-\gamma)\gamma - (1-\gamma)^2 < 0$ for all $-1 < \gamma < 0$. Thus, $\frac{d(NPM - NPE)}{d\gamma} < 0$ when $NPM > NPE$.

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(4)) that for given efforts, costs would increase. As a reaction, the entrepreneur adjusts her effort, and the net profit from effort changes. This adjustment is more detrimental for net profits in case of *musharaka* financing.

To take a closer look at the trade-off between risk sharing and incentive-dependent net profits from effort by means of a comparative statics analysis, we consider the case of indifference, that is, a constellation in which *musharaka* and equity financing result in the same utility for the entrepreneur and, therefore, (38) holds as an equation:

$$\left(1-\rho^2_{(\tilde{a},\tilde{b})}\right)\left\{(1+\gamma)(\theta_E+\theta)Var\left(\tilde{a}+\tilde{b}\right)+2\right\}=\frac{Var\left(\tilde{a}+\tilde{b}\right)}{Var\left(\tilde{b}\right)}\left[2-\frac{Var\left(\tilde{a}+\tilde{b}\right)}{(1-\gamma)Var\left(\tilde{a}\right)}\right]. \quad (43)$$

As discussed above, an increase in γ decreases the relative advantage of *musharaka* with respect to net profit. This is reflected in the fact that the LHS of (43) increases in γ while the RHS decreases in γ. As a counterbalance, the relative advantage of equity with respect to risk sharing must decrease to restore indifference. This relative risk sharing advantage of equity depends on the ratio $\frac{Var\left(\tilde{a}+\tilde{b}\right)}{Var\left(\tilde{a}\right)}$ (see the discussion to lemma 1). So, if we consider $Var\left(\tilde{a}\right)$ and $\rho^2_{(\tilde{a},\tilde{b})}$ as given, a larger value of γ calls for a smaller $Var\left(\tilde{b}\right)$ to restore indifference. If we instead consider both variances as given, a larger value of γ calls for a smaller $\rho^2_{(\tilde{a},\tilde{b})}$.

These comparative statics with respect to the parameter γ from the cost function and the relation between the risk $Var\left(\tilde{b}\right)$ in the intangible assets or growth opportunities and the risk $Var\left(\tilde{a}\right)$ in the tangible assets are now to be illustrated in the next section by means of a numerical analysis.

9. Numerical Analysis

Proposition 3 states the condition for *musharaka* financing to result in a higher utility for the entrepreneur:

$$\left(1-\rho^2_{(\tilde{a},\tilde{b})}\right)\left\{(1+\gamma)(\theta_E+\theta)Var\left(\tilde{a}+\tilde{b}\right)+2\right\}<\frac{Var\left(\tilde{a}+\tilde{b}\right)}{Var\left(\tilde{b}\right)}\left[2-\frac{Var\left(\tilde{a}+\tilde{b}\right)}{(1-\gamma)Var\left(\tilde{a}\right)}\right]. \quad (38)$$

In the following numerical analysis, we want to illustrate constellations of the variance ratio $\frac{Var(\tilde{b})}{Var(\tilde{a})}$, the correlation coefficient $\rho_{(\tilde{a},\tilde{b})}$, and the parameter γ from the entrepreneur's cost function for which this condition (38) holds. We consider the risk $Var(\tilde{b})$ in intangible assets or growth opportunities relative to the risk $Var(\tilde{a})$ in tangible assets instead of the total risk ratio $\frac{Var(\tilde{a}+\tilde{b})}{Var(\tilde{a})}$ or $\frac{Var(\tilde{a}+\tilde{b})}{Var(\tilde{b})}$ (which enter condition (38)) to disentangle the effects of the individual risks in intangible assets or growth opportunities, $Var(\tilde{b})$, and tangible assets, $Var(\tilde{a})$, and the correlation between those risks, measured by $\rho_{(\tilde{a},\tilde{b})}$.

Since the risk aversion factors θ_E and θ do not enter condition (38) separately, we only need to specify the sum of both. We assume, $\theta_E + \theta = 1$. For further simplification, we can assume a normalized $Var(\tilde{a}) = 1$ since our main focus is the variance ratio $\frac{Var(\tilde{b})}{Var(\tilde{a})}$.

Figures 1-3 illustrate the entrepreneur's indifference curves with γ on the horizontal axis and $\frac{Var(\tilde{b})}{Var(\tilde{a})}$ on vertical axis, for three different values of $\rho_{(\tilde{a},\tilde{b})}$, i.e., those constellations for which (43) holds. We start with the case of uncorrelated risks: $\rho_{(\tilde{a},\tilde{b})} = 0$. For this case, the curve in figure 1 represents all combinations of γ and $\frac{Var(\tilde{b})}{Var(\tilde{a})}$ which result in the entrepreneur being indifferent between *musharaka* and equity financing. Below (above) this indifference curve *musharaka* leads to a higher (lower) utility for the entrepreneur, given the respectively optimal contractual design.

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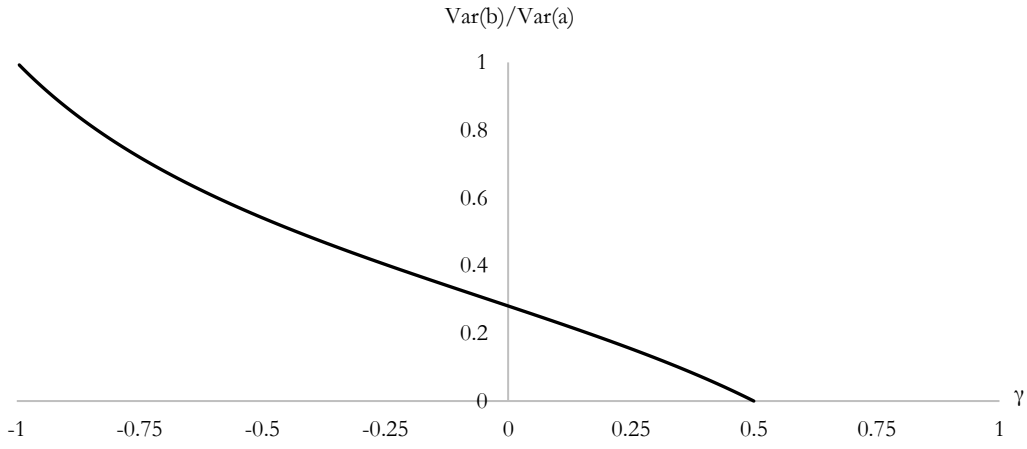

Figure 1. Indifference Curve ($\rho_{(\tilde{a},\tilde{b})} = 0$)

For *musharaka* to be advantageous, the risk $Var(\tilde{b})$ in the value $\tilde{b}$ of intangible assets or growth opportunities must be small relative to the risk $Var(\tilde{a})$ in the tangible assets since only the latter is shared with the *musharaka* investor. That observation is consistent with our discussion in section 8; for *musharaka* to be advantageous, total business risk $Var(\tilde{a}+\tilde{b})$ must not be much higher relative to the risk in $\tilde{a}$, i.e., $Var(\tilde{a})$. That is due to the relative importance of risk sharing. Only if the risk $Var(\tilde{b})$ in intangible assets or growth opportunities is relatively small (and so is $Var(\tilde{a}+\tilde{b})$), the renunciation of sharing this particular risk in case of *musharaka* financing results in a disadvantage sufficiently small to be compensated for by better incentives. If the risk $Var(\tilde{b})$ in intangible assets or growth opportunities is relatively large (and so is $Var(\tilde{a}+\tilde{b})$), the entrepreneur prefers equity financing because of the higher importance of comprehensive risk sharing ensured by equity.

In figure 1, we observe that the indifference curve is downward-sloping. This is due to the trade-off between risk sharing and incentives as already discussed in section 8. A higher γ decreases the advantage of *musharaka* with respect to net profits from effort and hence calls for a lower risk $Var(\tilde{b})$ in intangible assets or growth

opportunities relative to the risk $Var(\tilde{a})$ in tangible assets, that is, a lower $\frac{Var(\tilde{b})}{Var(\tilde{a})}$ ratio, for any given correlation. A lower $Var(\tilde{b})$ makes sharing of total risk $Var(\tilde{a}+\tilde{b})$ with equity less important, *ceteris paribus*, relative to the sharing of only $Var(\tilde{a})$ with *musharaka*.

Similar observations can be made for the case of a positive correlation: $\rho_{(\tilde{a},\tilde{b})}=0.5$. In figure 2, the indifference curve is also downward-sloping. Again, below (above) this indifference curve *musharaka* leads to higher (lower) utility for the entrepreneur, given the respectively optimal contractual design.

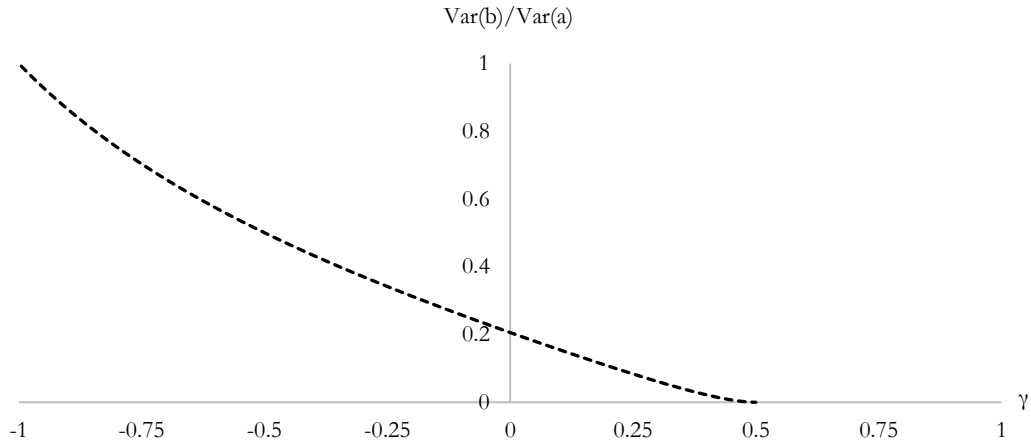

Figure 2. Indifference Curve ($\rho_{(\tilde{a},\tilde{b})}=0.5$)

Note that the indifference curve in figure 2 lies below the one in figure 1, that is, the indifference curve shifts downwards when $\rho_{(\tilde{a},\tilde{b})}$ increases. The reason is that, for any given combination of individual risks $Var(\tilde{a})$ and $Var(\tilde{b})$, a higher correlation results in a higher total risk $Var(\tilde{a}+\tilde{b})$, and a higher total risk implies that the comprehensive risk sharing ensured by equity is more important for the entrepreneur. Therefore, the relative risk sharing disadvantage of *musharaka* can only be compensated for in a smaller range of $\frac{Var(\tilde{b})}{Var(\tilde{a})}$.

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Lastly, we consider the case of a negative correlation: $\rho_{(\tilde{a},\tilde{b})} = -0.5$. Under this assumption, we identify the combinations of γ and $\frac{Var(\tilde{b})}{Var(\tilde{a})}$ which lead to the indifference curve depicted in figure 3.

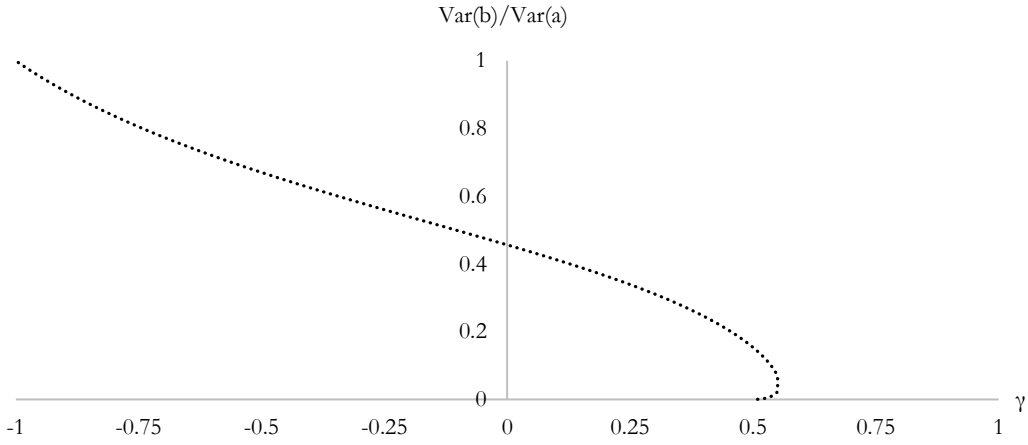

Figure 3. Indifference Curve ($\rho_{(\tilde{a},\tilde{b})} = -0.5$)

The fundamental observations we made regarding the indifference curves in figures 1 and 2 apply again. Note that in this case with $\rho_{(\tilde{a},\tilde{b})} = -0.5$, the indifference curve lies above the one in figure 1 for the case of non-correlated risks. The reasoning is the converse of the one given in the comparison of figure 2 with figure 1. Here, with $\rho_{(\tilde{a},\tilde{b})} = -0.5$, for any given combination of individual risks $Var(\tilde{a})$ and $Var(\tilde{b})$, this lower (negative) correlation results in a smaller total risk $Var(\tilde{a}+\tilde{b})$, and a smaller total risk implies that comprehensive risk sharing ensured by equity is less important for the entrepreneur. Therefore, the relative risk sharing disadvantage of *musharaka* is smaller and hence can be compensated for in a larger range of $\frac{Var(\tilde{b})}{Var(\tilde{a})}$.

Furthermore, it is noticeable that for relatively large values of γ, the indifference curve in figure 3 bends backwards. That has a mere technical reason. The

larger γ is, the smaller the ratio $\frac{Var(\tilde{a}+\tilde{b})}{Var(\tilde{a})}$ should be to result in indifference for reasons explained above. But in the range of very small values of $Var(\tilde{b})$, total risk $Var(\tilde{a}+\tilde{b})$ is decreasing in $Var(\tilde{b})$ if $\rho_{(\tilde{a},\tilde{b})}<0$. So, a decrease in $Var(\tilde{b})$ and, consequently, a decrease in $\frac{Var(\tilde{b})}{Var(\tilde{a})}$ results, *ceteris paribus,* in a higher total risk $Var(\tilde{a}+\tilde{b})$ which makes equity financing more preferable. Therefore, for large values of γ and very small $Var(\tilde{b})$, below the branch of the indifference curve bending backwards, equity financing dominates *musharaka*.

To sum up, we observe that *musharaka* contracting is more advantageous for the entrepreneur if the risk $Var(\tilde{b})$ from intangible assets or growth opportunities is small relative to the risk $Var(\tilde{a})$ in tangible assets. The higher the cross derivative γ of the entrepreneur's effort costs, the smaller the range of $\frac{Var(\tilde{b})}{Var(\tilde{a})}$ for which *musharaka* contracting is more advantageous. Furthermore, the correlation $\rho_{(\tilde{a},\tilde{b})}$ between the risks has an impact on the relative advantage of *musharaka* over equity. For given individual risks $Var(\tilde{a})$ and $Var(\tilde{b})$, the total risk $Var(\tilde{a}+\tilde{b})$ increases in $\rho_{(\tilde{a},\tilde{b})}$. And a higher total risk implies a higher importance of comprehensive risk sharing. Hence, equity becomes, *ceteris paribus*, more favorable.

10. Conclusion

The principal-agent relationship between an entrepreneur (the agent) and a financier (the principal) is explored for different financial contracts. The type of financial contract used has an impact on the extent of risk sharing, and the benefits thereof, and on managerial incentives.

We considered a model where all the parties involved are assumed to be risk-averse, and the entrepreneur can influence the outcome of her business by extending managerial effort at a private cost. An entrepreneur seeking funding for a business opportunity is faced with three alternatives: (i) taking out a loan; (ii) financing a project via *musharaka*, an Islamic profit and loss sharing (PLS) contract; (iii) raising regular equity form a financier who subsequently becomes a co-owner in the firm.

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A *musharaka* contract can be used to finance a specific project or investment, the acquisition of some real estate, commodities, working capital, or other tangible assets to be used in production or trade. In that case, only the profits and the risk of this particular project or investment are shared by the parties to the contract. In this paper, the above-mentioned investment is assumed to be in tangible assets. Financing with regular equity results in co-ownership of the whole firm instead. Therefore, all profits and losses from all tangible and intangible assets are shared, and so is the total business risk.

The theoretical analyses in this paper show that the entrepreneur can achieve a higher utility if she uses equity or *musharaka* financing instead of debt only. That is, both *musharaka* and equity financing are superior to debt financing for the entrepreneur. The benefits from risk sharing with an equity partner or a *musharaka* investor compensate for the losses from lower managerial incentives compared to debt financing.

Comparison of *musharaka* and equity financing shows no definite ranking. Both alternative financial contracts result in profit and loss sharing and hence in risk sharing, even though in various modes. The entrepreneur shares either total business risk with an equity partner or some particular risk with a *musharaka* investor. In both cases, the entrepreneur benefits from risk sharing, yet to varying degrees. On the other hand, both considered alternative financial contracts negatively affect the entrepreneur's incentives to work, but also differently. We analyzed the trade-offs between risk sharing and incentives for both financial contracts. As the main determinants, we identified the risks of the firm's tangible and intangible assets or growth opportunities, the correlation between those risks, and the entrepreneur's effort cost function.

Equity financing is preferable with respect to risk sharing, considering the entrepreneur's utility. *Musharaka* also results in risk sharing, although to a lesser extent. This disadvantage is compensated for by better incentives for the entrepreneur in case of *musharaka* financing, albeit only under certain conditions. Particularly, the risk in the firm's intangible assets or growth opportunities must not be high compared to the risk in tangible assets. Furthermore, the smaller the correlation between those risks, the better with the alternative of *musharaka* financing. And the entrepreneur's marginal costs of spending effort aiming to raise the value of tangible (intangible) assets must not increase much in the cost of effort for intangible (tangible) assets for *musharaka* financing to be more beneficial (i.e., the cross derivative of the cost function must not be large).

However, raising capital by means of regular equity is definitely more advantageous if the risk in the firm's tangible assets is smaller than the risk in intangible assets or growth opportunities. If the latter risk is not shared with an outside investor as with, e.g., the *musharaka* financing considered in our model, this disadvantage in risk sharing cannot be compensated for by better incentives.

The considerations in this paper can be extended in various ways. Since a *musharaka* investor, as well as an equity partner, is entitled to also take part in the management process, the effects of them also extending effort on managing either one particular project or the firm in general, respectively, could also be explored within the theoretical framework considered in this paper. Another possible extension would be applying the same assumptions and theoretical model in case the entrepreneur has the opportunity to raise capital from both an equity partner and a *musharaka* investor. In this case, the entrepreneur could, for example, finance tangible assets with *musharaka* besides external equity financing of the firm in total. Furthermore, in a more general setting, a scenario in which the entrepreneur finances a project with funding from two or more investors with different levels of risk aversion could also be analyzed within the theoretical framework applied in this paper. In such a scenario, the extent of profit-and-loss and risk sharing with different investors would depend on their individual risk preferences. The theoretical approach of this paper can be applied to analyze the use of multiple external sources of funding that are not limited to Islamic financial instruments. Other hybrid financial instruments could be considered in a similar manner. The model might also be modified and extended to account for other informational asymmetries, e.g., when the financier is unaware of the prospects of the project the entrepreneur wishes to undertake, or of the ability of the entrepreneur to manage the project. Adverse selection in such scenarios involving financing with a *musharaka* or a *mudaraba* contract could be explored.

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Appendices

Appendix A

In this appendix, we illustrate the reasoning behind restricting γ to the interval $]-1;1[$. When the entrepreneur's efforts are e_a and e_b, the net return from spending effort can be expressed as $e_a + e_b - \frac{e_a^2}{2} - \frac{e_b^2}{2} - \gamma e_a e_b$. From the first-order condition, we find the first best levels of effort that maximize this return to be $e_a = e_b = \frac{1}{1+\gamma}$, which leads to a net return of $\frac{1}{1+\gamma}$.

If $\gamma = -1$, the net return from extending effort is $e_a + e_b - \frac{e_a^2}{2} - \frac{e_b^2}{2} + e_a e_b$, and the optimal levels of e_a and e_b do not exist simultaneously. Only after arbitrarily deciding on e_a (or e_b) can the entrepreneur determine the optimal effort e_b (or e_a).

For $\gamma = 1$, the net return from extending effort is $(e_a + e_b) - \frac{1}{2}(e_a + e_b)^2$. The entrepreneur can choose any efforts e_a and e_b that add up to 1 to maximize the net return.

If $|\gamma| > 1$, the entrepreneur could instead choose arbitrarily to spend either no effort e_a or no effort e_b and yet earn a higher (or equal) net return in optimum than if the efforts $e_a = e_b = \frac{1}{1+\gamma}$ are chosen. With $|\gamma| > 1$, the optimal efforts are $e_a = 1$ and $e_b = 0$ or $e_b = 1$ and $e_a = 0$. Then the net return is $\frac{1}{2}$.

Only for $-1 < \gamma < 1$, we find that the net return in optimum, $\frac{1}{1+\gamma}$, exceeds the net return under the arbitrary constraint $e_b = 0$ (or $e_a = 0$), i.e., $\frac{1}{1+\gamma} > \frac{1}{2}$. Therefore, we restrict γ to the interval $]-1;1[$ to rule out nonsensical results.

Appendix B

Here we prove that if the financial structure does not affect the entrepreneur's effort, and the first best efforts are implemented, risk sharing with equity is superior to risk sharing under *musharaka* financing.

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In the absence of agency problems, that is, with the first best efforts, the certainty equivalent of the entrepreneur is

$$CE_E(equity) = x + y + \frac{1}{1+\gamma} - (1+r_f)(C - W_{0,E}) - \frac{1}{2}\left[\theta_E(1-\alpha_P)^2 + \theta_P\alpha_P^2\right]Var(\tilde{a}+\tilde{b}) \quad (44)$$

and

$$\begin{aligned} CE_E(musharaka) = x + y + \frac{1}{1+\gamma} - (1+r_f)(C - W_{0,E}) - \frac{1}{2}\left[\theta_E(1-\alpha_I)^2 + \theta_I\alpha_I^2\right]Var(\tilde{a}) \\ -\frac{1}{2}\theta_E Var(\tilde{b}) - \theta_E(1-\alpha_I)Cov(\tilde{a},\tilde{b}) \end{aligned} \quad (45)$$

under equity and *musharaka* financing, respectively.

With given (first best) efforts, the optimal shares are $\alpha_P^* = \frac{\theta_E}{\theta_E + \theta_P}$ for equity financing and $\alpha_I^* = \frac{\theta_E\left[Var(\tilde{a}) + Cov(\tilde{a},\tilde{b})\right]}{(\theta_E + \theta_I)Var(\tilde{a})}$ with *musharaka.*

Inserting these shares in (44) and (45), respectively, and applying assumption 9, $\theta_P = \theta_I \equiv \theta$, we find

$$\begin{aligned} & CE_E(equity) > CE_E(musharaka) \\ \Leftrightarrow \quad & \frac{1}{2}\frac{\theta_E^2}{\theta_E + \theta}Var(\tilde{a}+\tilde{b}) > \frac{1}{2}\frac{\theta_E^2\left[Var(\tilde{a}) + Cov(\tilde{a},\tilde{b})\right]^2}{(\theta_E + \theta)Var(\tilde{a})} \\ \Leftrightarrow \quad & Var(\tilde{a}+\tilde{b})Var(\tilde{a}) > \left[Var(\tilde{a}) + Cov(\tilde{a},\tilde{b})\right]^2 \\ \Leftrightarrow \quad & \left(Cov(\tilde{a},\tilde{b})\right)^2 < Var(\tilde{a})Var(\tilde{b}) \Leftrightarrow \rho^2_{(\tilde{a},\tilde{b})} < 1 \Leftrightarrow \left|\rho_{(\tilde{a},\tilde{b})}\right| < 1 \end{aligned} \quad . \quad (46)$$

Q.e.d.

Appendix C

Here we show that the higher $\frac{Var(\tilde{a}+\tilde{b})}{Var(\tilde{a})}$ is, the more important it is for the entrepreneur to share total risk instead of only $Var(\tilde{a})$.

The risk premium borne by the entrepreneur is

$$RPM = \frac{1}{2}\theta_E Var\left(\tilde{a}+\tilde{b}\right) - \frac{1}{2}\theta_E\left[Var\left(\tilde{a}\right)+Cov\left(\tilde{a},\tilde{b}\right)\right]\frac{\theta_E\left[Var\left(\tilde{a}\right)+Cov\left(\tilde{a},\tilde{b}\right)\right]}{\frac{1}{1-\gamma^2}+\left(\theta_E+\theta\right)Var\left(\tilde{a}\right)} \quad (47)$$

and

$$RPE = \frac{1}{2}\theta_E Var\left(\tilde{a}+\tilde{b}\right) - \frac{1}{2}\theta_E Var\left(\tilde{a}+\tilde{b}\right)\frac{\theta_E Var\left(\tilde{a}+\tilde{b}\right)}{\frac{2}{1+\gamma}+\left(\theta_E+\theta\right)Var\left(\tilde{a}+\tilde{b}\right)} \quad (48)$$

under *musharaka* and equity financing, respectively. The relative advantage of risk sharing under equity financing can be represented by the difference between these risk premia:

$$RPM - RPE = \frac{1}{2}\frac{\theta_E^2\left(Var\left(\tilde{a}+\tilde{b}\right)\right)^2}{\frac{2}{1+\gamma}+\left(\theta_E+\theta\right)Var\left(\tilde{a}+\tilde{b}\right)} - \frac{1}{2}\frac{\theta_E^2\left[Var\left(\tilde{a}\right)+Cov\left(\tilde{a},\tilde{b}\right)\right]^2}{\frac{1}{1-\gamma^2}+\left(\theta_E+\theta\right)Var\left(\tilde{a}\right)} > 0. \quad (49)$$

With given levels of $Var\left(\tilde{a}\right)$ and $\rho_{\left(\tilde{a},\tilde{b}\right)}$, the relationship between the risks being shared under the two alternatives depends on $Var\left(\tilde{b}\right)$. We can show that $\left(RPM - RPE\right)$ increases in $Var\left(\tilde{b}\right)$ and hence in $\frac{Var\left(\tilde{a}+\tilde{b}\right)}{Var\left(\tilde{b}\right)}$ for any given values of $Var\left(\tilde{a}\right)$ and $\rho_{\left(\tilde{a},\tilde{b}\right)}$:

$$\frac{d\left(RPM - RPE\right)}{dVar\left(\tilde{b}\right)} = \frac{\left(1+\gamma\right)\theta_E^2\left\{4+\left(1+\gamma\right)\left(\theta_E+\theta\right)Var\left(\tilde{a}+\tilde{b}\right)\right\}}{2\left\{2+\left(1+\gamma\right)\left(\theta_E+\theta\right)Var\left(\tilde{a}+\tilde{b}\right)\right\}^2} > 0. \quad (50)$$

Therefore, as $\frac{Var\left(\tilde{a}+\tilde{b}\right)}{Var\left(\tilde{a}\right)}$ increases, it becomes more important for the entrepreneur to share the total risk instead of only $Var\left(\tilde{a}\right)$.

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