Limits on the use of inertial actuators in active vibration control

Simone CINQUEMANI, Ferruccio RESTA, Massimo MONGUZZI

Dipartimento di Meccanica, Politecnico di Milano,
Campus Bovisa Sud, via La Masa 34, 20156, Milano, Italy

ABSTRACT

Inertial actuators are widely used device in active vibration control applications. The aim of the paper is to analyze the limits on the use of such devices in terms of stability when fully active skyhook damping control is implemented. Different control strategies are investigated and numerical simulation are led considering different vibrating mechanical systems.

1. INTRODUCTION

Undesirable vibrations can induce noise, bad performance or even severe damage. Passive damping materials have been used effectively for a very long time, but especially for micro-amplitude and low frequency vibration, passive damping materials are inadequate and not effective. Hence active vibration control (AVC) techniques with feedback control feature have begun to be used to meet stringent accuracy and performance requirements. Direct velocity feedback control is one of the most commonly used strategy. It consists on a secondary force, which is designed to be proportional to the absolute structure velocity, which acts on the structure with the effect of increasing the damping of the structure itself. As it is known [7], the system is unconditionally stable for any positive real feedback gain.

A common way to generate the control force is by using inertial actuators. An inertial actuator consists on a suspended mass on a fixed base through an elastic element which is put in vibration by an internal force. This force can be generated through different physical principles, for example, electromagnetically. When the base is bound to the structure to be controlled, the force exerted by the actuator to the structure is equal to the force of inertia resulting from the acceleration of the suspended mass. Knowing the dynamics of the actuator, it is possible to control the exerted force acting on the internal force. The advantage of inertial actuators is they are mounted directly on the vibrating structure and do not need a fixed external base to react. On the other side, the dynamics of these actuators is strongly coupled with the one of the structure. Moreover, control techniques that are unconditionally stable in the case of ideal actuators (as the Skyhook logic), can become unstable when the force is generated by inertial actuators. For this reason, the dynamics of the actuators must necessarily be considered in the design of an active vibration controller.

Many controllers have been developed for the use of inertial actuators in AVC applications: an overview of linear controllers is described in [2]. Some nonlinear controls considering the saturation of the devices are described in [5], while a deep study on the stability can be found in [3], [6]. Benassi, Elliott and Gardonio have done several numerical and experimental studies on the stability and on the performance of inertial actuators used to control vibrations of a thin plate [9], [8], also considering the opportunity to use the feedback of the transmitted force [1] or the relative displacement of the inertial mass [4], in order to increase the stability margin.

The aim of this paper is to analyze the opportunity of using inertial actuators for the practical arrangement of a fully active skyhook control, highlighting its limits in terms of stability. After a brief description of the problem, the paper introduces the use of a compensator filter to delete the causes of instability in order to make the controlled system unconditionally stable.

The paper is structured as follows.
Section 2 recalls the basis of the functioning principle of inertial actuators. Section 3 shows the traditional approach in control design highlighting the limits on the use of inertial actuators in terms of system stability. The use of a compensating filter to consider the dynamic of the actuator is investigated showing the improvements obtained in terms of vibration reduction. Finally conclusions are drawn in Section 4.

2. FUNCTIONING PRINCIPLE OF INERTIAL ACTUATORS

An inertial actuator consists on a mass, free to move, connected to a fixed base through a spring-damper element. The base is connected to the vibrating structure and a force, whose origin can be different depending on the type of actuator, acts respectively on the inertial mass and on the structure itself. The type of actuators to be used depends on the maximum force to be transmitted and, above all, on the operating range of frequencies. Most commonly used actuators are electromagnetic shaker or magnetostrictive devices.

Considering the actuator as an inertial mass mounted on a vibrating structure through a spring-damper element

(Fig.1), its equation of motion is:

$$m_a \ddot{z}_a + r_a \left(\dot{z}_a - \dot{z}_e \right) + k_a \left(z_a - z_e \right) = -f_a \qquad (1)$$

where:

- $\dot{z}_a$ is the suspended mass velocity;
- $\dot{z}_e$ is the velocity of the vibrating structure;
- f_a is the force generated by the actuator;
- f_t is the force transmitted to the structure.

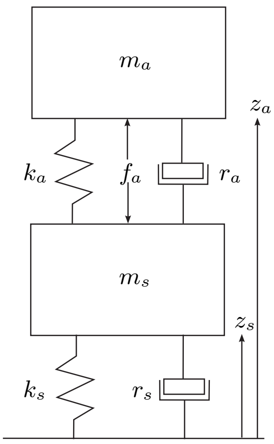

Figure 1: Mechanical scheme of an inertial actuator mounted on a single d.o.f. vibrating structure.

The force transmitted to the structure is equal to the force of inertia related to the actuator suspended mass:

$$f_t = T_a\, f_a - Z_{aa}\, \dot{z}_e = -m_a \ddot{z}_a \qquad (2)$$

where:

- $T_a = \dfrac{f_t}{f_a}$ is the actuator blocked response transfer function between the internal force generated by the actuator and the force transmitted to the structure, when the structure is still;
- $Z_{aa} = \dfrac{f_t}{\dot{z}_e}$ is the actuator mechanical impedance that is the transfer function between the structure velocity and the force transmitted when the actuator is not powered.

The two transfer functions can be obtained from the equations of motion (1). In the Laplace domain it results:

$$\begin{aligned} T_a(s) = \frac{F_t(s)}{F_a(s)} &= \frac{m_a s^2}{m_a s^2 + r_a s + k_a} = \\ &= \frac{s^2}{s^2 + 2 h_a \omega_a s + \omega_a^2} \end{aligned} \qquad (3)$$

where h_a and ω_a are respectively the system adimensional dumping and natural frequency.

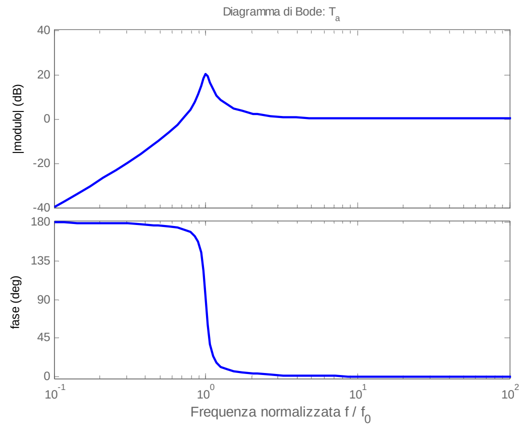

Figure 2: Actuator blocked response

Similarly, the mechanical impedance can be obtained from eq.(2), neglecting the term related to the internal force $f_a(s)$:

$$\begin{aligned} Z_{aa}(s) = \frac{F_t(s)}{v_e(s)} &= m_a s \cdot \frac{r_a s + k_a}{m_a s^2 + r_a s + k_a} = \\ &= m_a s \cdot \frac{2 h_a \omega_a s + \omega_a^2}{s^2 + 2 h_a \omega_a s + \omega_a^2} \end{aligned} \qquad (4)$$

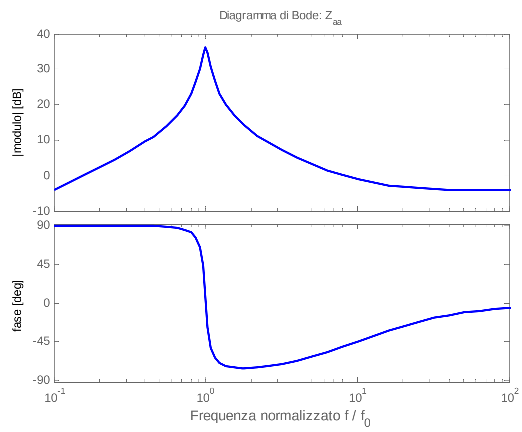

Figure 3: Actuator mechanical impedance

For frequencies higher than the actuator resonance, (Fig. 2, 3), both the two functions T_a, Z_{aa} tend to have a constant magnitude and no phase shift. In this condition the actuator can be suitably considered as an ideal generator of force. Vice versa, for lower frequencies, function T_a shows a $+180^o$ phase shift, while its magnitude significantly changes with frequency. In the same range, the mechanical impedance has a 90^o phase shift.

3. DIRECT VELOCITY FEEDBACK CONTROL

In AVC applications, the amplitude of vibration of a structure van be reduced designing the control force to be proportional and in phase with the structure velocity and thus to increase the damping of the structure itself.

The skyhook control is always unconditionally stable if the actuator can be considered as ideal [7]. This condition is not verified when the control force is generated through an inertial actuator linked to the structure [3], [6], [9], [8].

This limit is essentially due to the hypothesis, done by many researchers. They are:

1. to damp vibrations only in structures whose natural frequencies are higher than the one of the actuator,
2. to neglect the dynamic of the actuator and to consider the internal force generated by the actuator equal to the force actually transmitted to the structure.

Due to these assumptions the system can not be unconditionally stable. In the following the limits of this approach are shown. The paper introduces a compensator filter that, considering the dynamics of the actuator, allows to design the control force in order to be proportional to the structure velocity and exactly in phase with it. The problem of stability is analyzed in detail, firstly considering a single degree of freedom structure and, subsequently, extending the results to n d.o.f. vibrating systems.

Single d.o.f. vibrating structure - Traditional approach

Consider a single d.o.f. vibrating structure whose natural frequency (ω_s) is higher than the one (ω_0) of the inertial actuator used to suppress vibrations.

Considering the system as shown in Fig.1, the equations of motion can be written as:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_a \end{bmatrix} \begin{Bmatrix} \ddot{z}_1 \\ \ddot{z}_a \end{Bmatrix} + \begin{bmatrix} r_1 + r_a & -r_a \\ -r_a & r_a \end{bmatrix} \begin{Bmatrix} \dot{z}_1 \\ \dot{z}_a \end{Bmatrix} + \\ + \begin{bmatrix} k_1 + k_a & -k_a \\ -k_a & k_a \end{bmatrix} \begin{Bmatrix} z_1 \\ z_a \end{Bmatrix} = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} f_a \tag{5}$$

$$[M]\,\underline{\ddot{z}} + [R]\,\underline{\dot{z}} + [K]\,\underline{z} = [\Lambda_C]\,f_a \tag{6}$$

To increase the damping of the structure the control force is designed to be proportional and in phase with the structure absolute velocity:

$$f_a = -g_v \dot{z}_1 \tag{7}$$

Substituting eq. (7) in eq. (5) the effect of the control can be summarized considering the damping matrix of the controlled system.

$$([R] + [R_C]) = \begin{bmatrix} r_1 + r_a + g_v & -r_a \\ -r_a - g_v & r_a \end{bmatrix} \tag{8}$$

Since the resulting matrix is non symmetric, the controlled system has a stability limit as a function of the control gain g_v:

The frequency response of the system to a disturbing force is shown in Fig. 4. The solid line represents the behavior of the uncontrolled system, while the dashed line represents the behavior of the controlled one. It is noted that the control is able to effectively increase the damping of the structure, while it acts as a disturbance on the actuator inertial mass. The same conclusion can be drawn by observing the root locus of the coupled system.

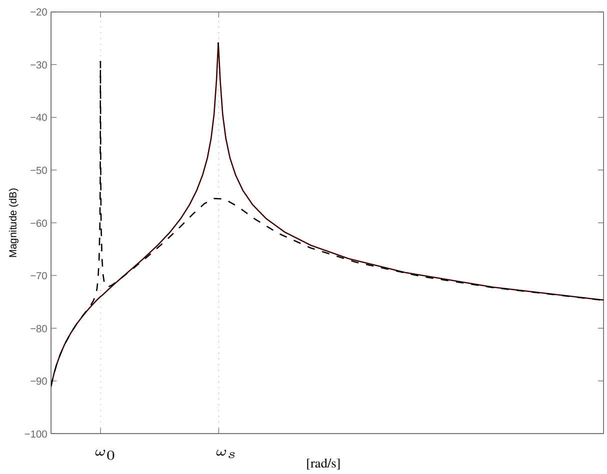

Figure 4: System frequency response: solid line - uncontrolled system; dashed line - controlled system

Same results can be achieved applying the criterion of Nyquist to the open loop transfer function:

$$L_1(s) = g_v \cdot \frac{\dot{z}_1}{f_a} = \\ = g_v \cdot \frac{m_a s^3}{(s-p_1)(s+p_1)(s-p_2)(s+p_2)} \tag{9}$$

The shape of the open loop transfer function shows the controlled system is only conditionally stable. The Nyquist diagram presents a ring in the left-half plane (due to the dynamics of the inertial actuator), which tends to enclose the point (- 1, $0j$) when the gain of the control is increased.

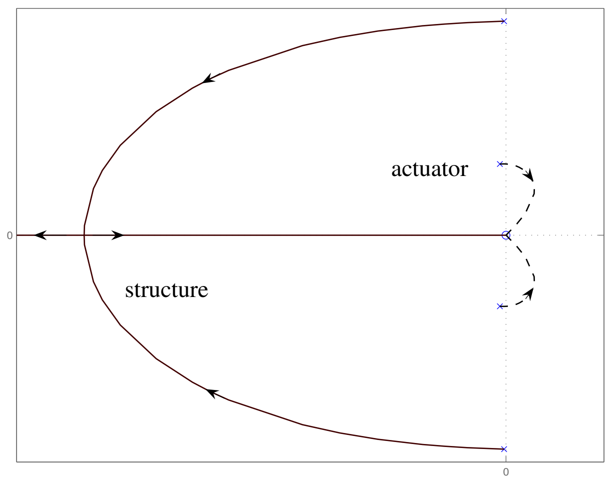

Figure 5: Root locus of the controlled system

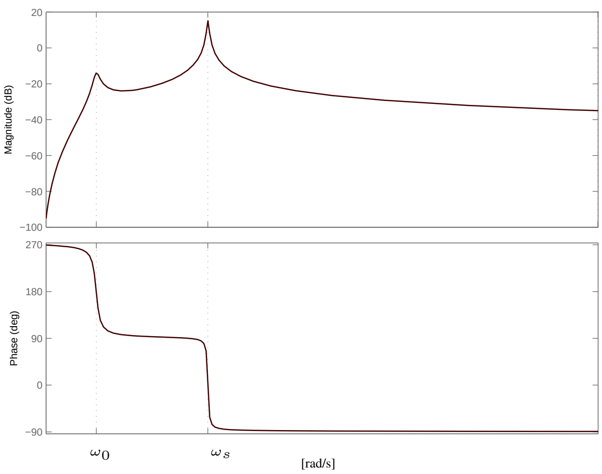

Figure 6: Open loop transfer function

If the vibrating structure were characterized by a natural frequency lower than the one of the inertial actuator ($\omega_s < \omega_0$) the problem remains unchanged. Since in the range of frequencies lower than ω_0 the force transmitted has a 180^o phase shift with respect to internal force generated by the actuator, the equation (7) becomes:

$$f_a = +g_v \dot{z}_1 \tag{10}$$

Single d.o.f. vibrating structure - Implementation of a compensator filter

Limits on stability can be theoretically overcame by using a compensator filter that takes into account the dynamics of the actuator. The simplest filter can be designed as:

$$K(s) = T_a^{-1}(s) \tag{11}$$

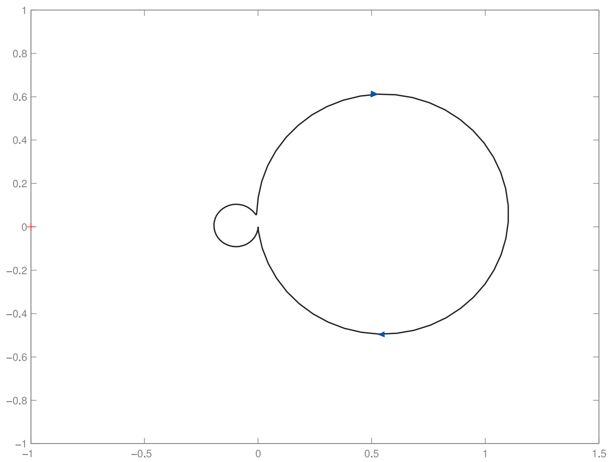

Figure 7: Nyquist plot of $L_1(s)$

Internal force generated by the actuator is:

$$f_a = -g_v \cdot \frac{1}{T_a} \dot{z}_1 = -k_p z_1 - k_d \dot{z}_1 - k_i \int z_1 dt \tag{12}$$

where:

$$\begin{cases} k_p = g_v \cdot 2h_a\omega_0 \\ k_d = g_v \\ k_i = g_v \cdot \omega_0^2 \end{cases} \tag{13}$$

To write the equation of motion of the controlled system in the time domain, the state of the variable integrated with respect to time has to be considered:

$$x_I = \int z_1 dt \rightarrow z_1 = \dot{x}_I \tag{14}$$

It results:

$$\left\{ \begin{array}{c} \underline{\dot{x}} \\ \dot{x}_I \end{array} \right\} = \left[\begin{array}{cc} [A] & [0] \\ \left[\tilde{C}\right] & [0] \end{array} \right] \left\{ \begin{array}{c} \underline{x} \\ x_I \end{array} \right\} + \left[\begin{array}{c} [B] \\ 0 \end{array} \right] f_a \tag{15}$$

$$f_a = -\underline{g}^T [C] \left\{ \begin{array}{c} \underline{x} \\ x_I \end{array} \right\} \tag{16}$$

where:

$$\left[\tilde{C}\right] = \left[\begin{array}{cccc} 0 & 0 & 1 & 0 \end{array} \right]$$

$$[C] = \left[\begin{array}{ccccc} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right] \tag{17}$$

are suitable matrix to extract corresponding quantities from the state and:

$$\underline{g}^T = \left[\begin{array}{ccc} k_p & k_d & k_i \end{array} \right]$$

collects the gains of the controller.

The frequency response of the coupled system (actuator + structure) is shown in Fig. 8. The solid line represents the case of uncontrolled system, while the dashed line represents the behavior of the controlled system using the filter compensator described in (11) to cancel the dynamics of the actuator. It is noted that the control force is able to effectively increase the damping of the structure, without significantly changing the amplitude of oscillations of the vibrating mass.

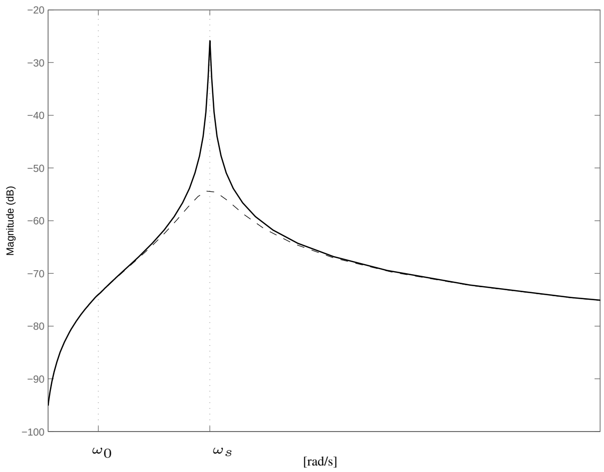

Figure 8: System frequency response: solid line - uncontrolled system; dashed line - controlled system

The stability of the controlled system can be assessed by the eigenvalues of the matrix of state equation (15), or by applying the Nyquist criterion to the open loop transfer function:

$$L_2(s) = = \frac{g_v \cdot m_a s^3}{(s-p_1)(s+p_1)(s-p_2)(s+p_2)} \cdot \frac{(s-p_a)(s+p_a)}{s^2} \quad (18)$$

Differently to the previous case, the control results to be always stable. Analyzing the open loop transfer function it is evident that the zeros of the filter should compensate the poles associated with the inertial actuator resonance. At the same time the two poles in the origin require that the phase at zero frequency is +90^o. For this reason the Nyquist diagram has no loops in the left-half plane, ensuring the unconditional stability.

Extension to n d.o.f. structures

Vibration control of a n degrees of freedom structure summarizes the cases previously seen. The need to consider the dynamics of the suspended mass of the inertial actuator still remains, while adding the problem concerning the possibility that the vibrating structure may have modes of vibration at frequencies even lower the resonance of the actuator.

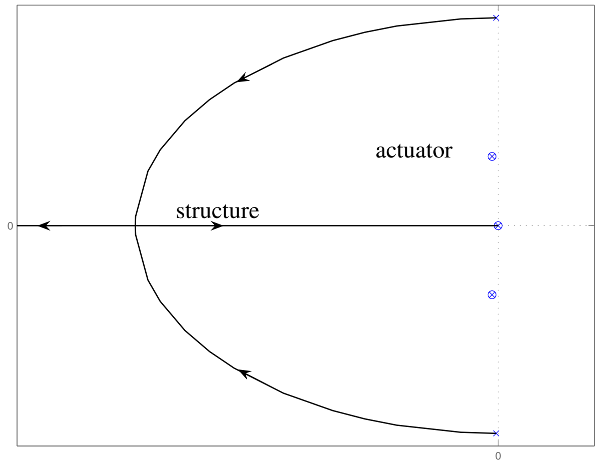

Figure 9: Root locus of the controlled system

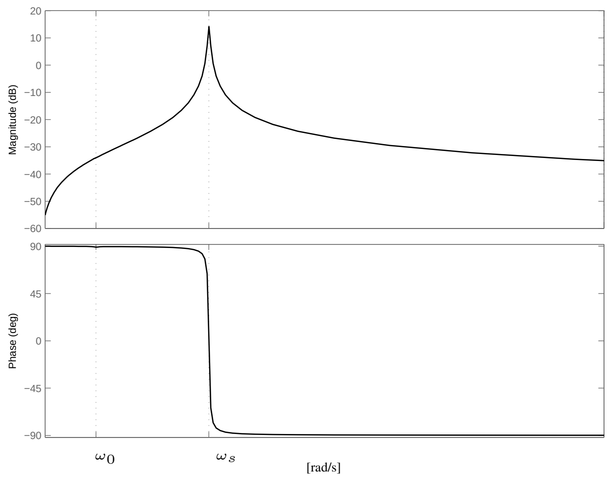

Figure 10: Open loop transfer function

Due to the transition of 180^o in the phase of the response blocked inertial actuator it is evident that it is not possible to have a control force in phase with the structure velocity simply designing the internal force to be proportional to it. In particular, while the traditional control increases the damping of structural modes whose frequencies are higher than the natural frequency of the actuator, at the same time it decreases the damping of structural modes with lower frequencies (Fig.12).

On the contrary, properly filtering the control signal using a compensator filter, the controlled system becomes unconditionally stable. All the structural modes can be effectively damped (Fig.13) and the oscillations of the suspended mass are not increasing.

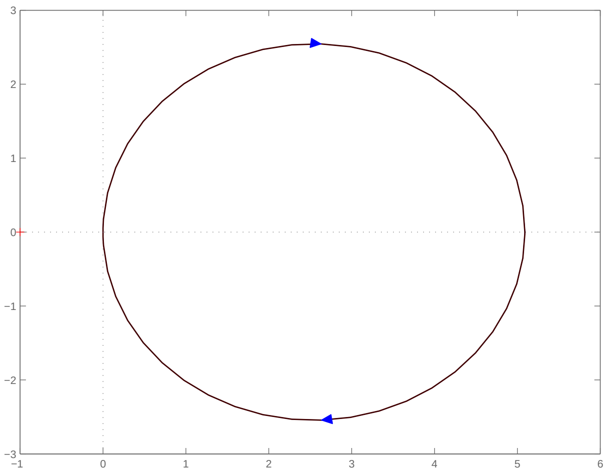

Figure 11: Nyquist plot of $L_2(s)$

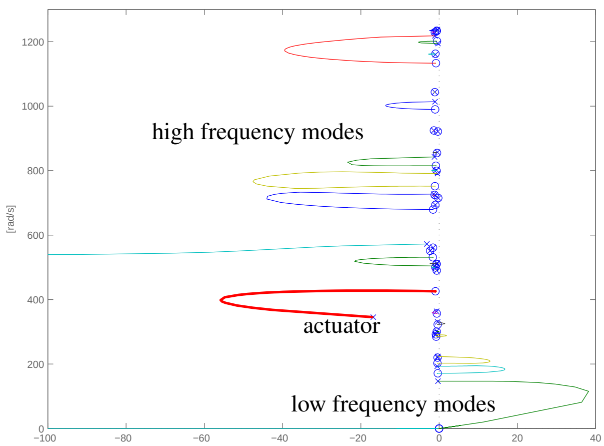

Figure 12: Root locus of a controlled n d.o.f. structure with traditional approach

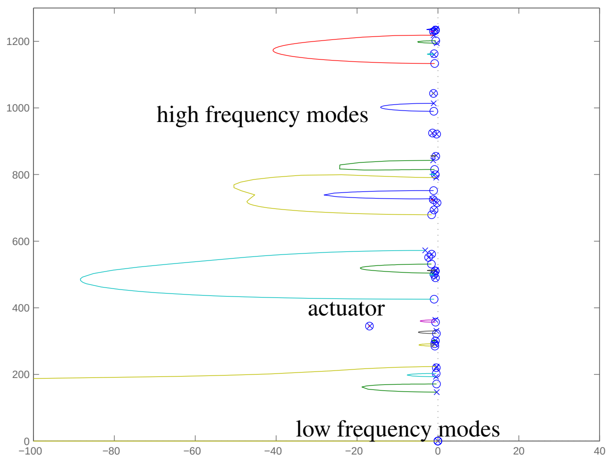

Figure 13: Root locus of a controlled n d.o.f. using the compensator filter

4. Conclusion

The use of inertial actuators in AVC application, using techniques known in the literature, implies a stability limit of the controlled system. It is due to the assumptions of neglecting the dynamics of the actuator considering it as ideal. The use of a compensator filter, describing the actuator dynamics, allows to overcome this limit and to make the controlled system unconditionally stable.

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