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Historical development of the Regime Geometry Equations, Part-I

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ABSTRACT

The present study intended to review the historical development of the regime geometry equations proposed by many researchers since 1895. Kennedy started the search for the attributes of stable channels on the Upper Bari Doab Canal of Punjab in 1980-1984. The regime concept was presented by Lindley (1919) that later on Lacey (1930) modified to a set of most widely used regime relations. Later on, Blench (1941-1957), and Simons and Albertson (1960) presented their equations. This period is called 'canal regime period' because most of the research focused on canal design. The concept of regime was extended to river systems by Leopold and Maddock in 1953. In 1963, Henderson proposed some regime equations that can be applied for predicting canal and river geometry. This period was the transition from canal regime period to river regime period and from an empirical approach to analytical approach and is called as 'Transitional Period'. Many other researchers written the information based on analytical approach, or a combination between analytical and empirical approaches, and more focused to river regime especially gravel bed river regime. This period is, called 'River regime period'. Flow discharge, sediment size, and bed-load sediment are the three main factors that influence the geometry of alluvial channels. The hydraulic geometry relationships indicate that sediment size and bed-load sediment strongly influence the channel slope, moderately influence the flow velocity, and slightly influence the channel depth and the channel width. This study also shows that in terms of Shields parameter, the exponents of discharge, sediment size, and sediment transport are dependent on the relative submergence (D/d_{50}).

1. Introduction

A river that has formed its channel in the sediment that is being transported or has been transported by the river is termed as 'Alluvial River'. In other words, a channel whose perimeter is made of the material transported by the channel is an alluvial channel. Alluvial rivers are less permanent than most other aspects of the landscape due to their readily erodible banks and beds. Researchers have given some definitions of a regime or stable channel. Four of them are presented as follows.

1.2 Kennedy (1895)

'A regime channel is a non-silting and non-scouring channel' [1].

2.3 Lane (1955)

'A stable channel is an earth channel which carries water, the banks and bed of which are not scoured objectionably by moving water, and in which objectionable deposits of sediment do not occur' [2].

1.3 Vanoni, Brooks, and Kennedy (1961)

'A channel is defined as being in regime if its mean measurable behavior during a given time period does not differ significantly from its mean measurable behavior during similar time periods before and after the given time period' [3].

1.4 Chorley and Kennedy (1971)

'A channel in regime is a channel in a steady-state equilibrium with a time span of 10-102 years' [4]. The morphological, hydraulic, and sedimentation characteristics of alluvial channels are determined by a large variety of factors, the mechanics of which is not fully understood [5]. However, there is agreement among researchers that water and sediment discharges are the two most important factors that form channel geometry. Sediment size and bed-load sediment transport have an important role in developing channel geometry; they represent the quality and the quantity of sediment discharge, respectively.

2. Review of the Development of the Regime Equations

According to the literature review conducted in the current study, the development of the Regime equations can be divided into three periods of time as follows.

2.1 Canal Regime Period

Period between 1900's-1950, called 'canal regime period'. In this period, regime theory was pioneered by Kennedy (1895) [1], and then followed by Lindley [6], Lacey (1930-1958) [7], Blench (1941-1957) [8], and Simons and Albertson [5]. It is called canal regime period because most of the research focused on canal design.

2.2 Transitional Period

Period between 1950's-1960 called 'Transitional Period'. In 1953, Leopold and Maddock [9] extended the regime concept to river systems. And in 1963, Henderson proposed some regime equations that can be applied for predicting canal and river geometry. This period was transition from canal regime period to river regime period and from empirical approach to analytical approach.

2.3 River Regime Period

Period between 1960's – 1988, called 'River regime period'. Most information written during this period is based on analytical approach, or combination between analytical and empirical approaches, and more focused to river regime especially gravel bed river regime. The main research contributors are given in the references from [10-16].

3. Historical Development

3.1 *Kennedy's Development*

Kennedy (Executive Engineer Punjab Irrigation) during 1890-1894 selected a number of sites on Upper Bari Doab Canal (Pakistan) for carrying out investigations about velocity and depth of the channel. He considered these reaches to be stable because no maintenance work was required. For designing canal, he recommended to assume a flow depth and calculate the design flow velocity [17].

According to Kennedy 'a regime channel is one, which neither silts nor scour'. He concluded that eddies generated from the bed, support the silt in suspension and therefore the silt supporting power of the stream is proportional to the bed width and not the perimeter of the channel. He in (1895) proposed a single relationship between mean flow depth (*D*, ft) and mean flow velocity (*V*, ft/s) as follow.

$$V = 0.84\, D^{0.64} \text{ (ft/sec)} \qquad (1)$$

This equation was applicable for only channels that are flowing in sandy silt of the same quality. Another factor m, critical velocity ratio, was introduced in the above equation to account for the variation of silt grade.

$$V = 0.84\, mD^{0.64} \qquad (2)$$

He used the Chezy's formula with Kutter's *C* for fixing the slopes.

3.2 Lindley's Development

As has already been emphasized that depending upon a slope available, many channels can be designed which have different cross-sections. However, whether channels will suit the requirement cannot be found out from Kennedy's theory. In order to make channel design more definite, Lindley proposed his formulae in 1919 [6].

Lindley stated the 'regime' concept of channels as 'When artificial channel is used to carry silty water, both the bed and banks scour or fill, changing depth, gradient and width, until a state of balance is attained, at which the channel is said to be in regime'. He used about 786 hydraulic survey observations made during 1915 to 1917 on the Lower Chenab Canal in Pakistan and proposed two hydraulic relationships as follows.

$$W = 3.8\, D^{1.61} \text{ (ft)} \qquad (3)$$

$$V = 0.95\ D^{0.57}\ \text{(ft/s)} \quad (4)$$

Where W is the channel width.

Both equations together with continuity equation can be used for designing. It was noticed for the first time that bed width and depths were introduced as regime variables. In his reply to the discussion on his paper he stated that 'The existence of these relations mean that the dimensions width, depth, and gradient of a channel to carry a given supply loaded with a given silt discharge were all fixed by nature, that is, uniquely determined'. The variables bed width, depth, and slope were observed, velocities, however, were not observed. They were computed by means of the Kutter and Chezy equations, the Kutter equation is given as Eq. 5.

$$C = \frac{41.65 + \frac{0.00281}{S} + \frac{1.811}{n}}{1 + \frac{n}{\sqrt{R}}\left(41.65 + \frac{0.00281}{S}\right)} \quad (5)$$

The magnitude of n, absolute rugosity coefficient, in Eq. 5 was assumed as constant at 0.0225. There was a serious defect in that, the water discharges were inferred only from a resistance function, the accuracy of which was unproven. Although Lindley's conceptual approach was an important development, his sets of equations were insufficient to define the regime condition. The width/depth ratio implicit in Eq. 3 was perhaps the most important feature.

3.3 Lacey's Development

In 1930, Lacey introduced one of the most important contributions to the design of regime canals [7]. He substituted the hydraulic radius (R, ft) for the depth (D, ft) and the wetted perimeter (P, ft) for the width (W, ft). He also introduced a silt factor (f_1) in his equations [5]. According to Lacey the regime conditions i.e. stable conditions, no change in bed width, depth and slope over a hydrological cycle shall be established when;

a. discharge is constant,

b. the alluvium in which the channel is flowing is incoherent, unlimited and of the same characteristics as the sediment charge carried by the water, and

c. silt grade and charge are constant.

According to Lacey, if the above three conditions exists, then the channel is said to be in true regime. However, it is seldom that the above conditions are realized in field. Regarding the shape of the stable channel he states that 'natural silt-transporting channels have a tendency to assume a semi-elliptical section which is confirmed by an inspection of large number of channels in final regime and an examination of cross-sections of discharge sites of rivers in well-defined straight reaches of known stability'. Hence Lacey gave the idea of initial and final regime for actual channel.

3.3.1 Initial regime

Those channels which are excavated with defective slopes and with narrow dimensions immediately throw down incoherent silt on the bed which further increase a non-silting equilibrium due to increase in slope which is termed as initial regime. These channels are subjected to some lateral restraint because the scouring of the banks is not allowed. The cross section of the channel is narrower and slopes and velocities of these channels are higher than they would have if the sides were not rigid.

3.3.2 Final regime

The Final regime conditions are said to have been achieved if (a) the continuous action of the current has overcome the resistance of the banks (b) the channel has adjusted its parameters like depth and slope according to the silt grade and discharge. Lacey concluded that 'stable channels would be semi-elliptical, with the major axis horizontal, and with the ratio of the major axis to the semi-minor axis depending on the nature of the silt carried, being greater for coarser silt'. Lacey initial equations were Eqs. 6, 7 and 8.

$$V = 1.17\ (Rf_1)^{1/2}\ \text{(ft/s)} \quad (6)$$

$$Af_1^2 = 3.8\ V^5 \quad (7)$$

$$P = 2.667\ Q^{1/2}\ \text{(ft)} \quad (8)$$

Later (during 1935-1958), he revised the first equation, i.e. Eq. 6, as Eq. 9.

$$V = 1.155\ (Rf_1)^{1/2} \quad (9)$$

And proposed some other relationships as Eq. 10 and 11.

$$S = 0.00055\ f_1^{5/3}\ Q^{-1/6} \quad (10)$$

$$V = (1.3458/Na)\ R^{3/4}\ S^{1/2} \quad (11)$$

Where $Na = 0.0225\ f_1^{1/4}$ is absolute rugosity coefficient.

$$f_1 = 1.59\ d_{50}^{1/2} \quad \text{(Lacey's silt factor)} \quad (12)$$

Where d_{50} is in mm.

The above equations can be restated in term of discharge as Eqs. 13-16.

$$V = 0.794\ Q^{1/6}\ f_1^{1/3}\ \text{(ft/s)} \quad (13)$$

$$P = 2.667\ Q^{1/2}\ \text{(ft)} \quad (14)$$

$$R = 0.473\ (Q/f_1)^{1/3}\ \text{(ft)} \quad (15)$$

$$A = 1.26\ Q^{5/6}\ f_1^{-1/3}\ \text{(ft}^2\text{)} \quad (16)$$

3.4 Inglis, Lacey and Ning Chin Development

Inglis and Lacey during 1948-1963 have made two important contributions to the regime equations. First, discussing the disparity between f_{vr} and f_{rs}. Here f_{vr} is the silt factor computed based on the relationship with velocity (V, ft/s) and hydraulic radius (R, ft), and f_{rs} is the silt factor computed based on relationship with hydraulic radius and the slope (S) [18-20]. They suggested that if final regime was achieved, the two values of the silt factor would be equal. The second important contribution is in the form of Inglis-Lacey equations in which the sediment discharge is included. The final Inglis-Lacey equations are given in term of P and R [17].

$$P = 2.668\, Q^{1/2}\, I^{1/4}\, (gd)^{-1/4}\ \text{(ft)} \quad (17)$$

$$R = 0.473\, Q^{1/3}\, (d/g)^{1/6}\, \mathrm{I}^{-1/3}\ \text{(ft)} \quad (18)$$

$$V = 0.794\, Q^{1/6}\, \mathrm{g}^{5/12}\, d^{1/12}\, I^{1/12}\ \text{(ft/s)} \quad (19)$$

$$S = 0.00054\, g^{1/12}\, d^{5/12}\, I^{5/12}\, Q^{-1/6} \quad (20)$$

Where; $I = C_v V_s/(gv)^{1/3}$, $C_v = Q_s Q$ (ppm), $d = d_{50}$ (mm), g = acceleration due to gravity (ft/sec^2) and Q_s = downstream sediment discharge rate.

Q is the design discharge (cfs), V_s (ft/s) is the weighted mean terminal fall velocity of sediment and v is the kinematic viscosity of water.

Related to the silt factor, Ning Chien (1957) [20] in his study about f_{vr} and f_{rs} shows that f_{vr} increases greatly as the ratio Q_s/Q is increased, but that f_{vr} remained relatively constant. Chien's interpretation was that f_{vr} depends on the sediment concentration of the flow whereas f_{rs} varies primarily with the bed material size. The results of his study are as Eqs. 21 and 22.

$$f_{vr} = 0.75\, (V^2/R) = 0.061\, (Q_s/Q)^{0.715} \quad (21)$$

$$f_{rs} = 192\, (RS^2)^{1/.3} = 2.2\, d^{0.45}\, (Q_s/Q)^{0.052} \quad (22)$$

He recommended to use Lacey's equation, $P = 2.667\ Q^{1/2}$ (ft).

3.5 Lane and Bose Development

In 1935, Lane [37] reviewed cross-sectional shapes, but its main significance lies perhaps in the statement: 'The quantity of slides in motion is an important factor in the shape of stable channels in alluvium and has not received the attention that its importance warrants'. This was a valid observation, as sediment charge (concentration in transport) did not appear in the Lacey formulation, although it was clear that there was a relatively narrow hand of concentration that would be consistent with the stability and low maintenance of irrigation systems.

Lane has defined the stable channel as 'A stable channel is an unlined earth channel (a) which carries water, (b) the banks and bed of which are not scoured objectionably by moving water, and (c) in which objectionable deposits of sediments do not occur'.

Bose in 1936 [38] used a painstaking analysis of Punjab canal data to derive an alternative slope function that depended on $Q^{0.21}$ rather than $Q^{1/6}$. He and the Staff of the Punjab Irrigation Research Institute, presented the following formulas.

$$P = 2.8\, Q^{1/2} \quad (23)$$

$$S \times 10 = 2.09\, d^{0.86}/Q^{0.21} \quad (24)$$

$$R = 0.47\, Q^{1/3} \quad (25)$$

Eqs. 23, 24, and 25 represent the results of several years of painstaking collection and statistical analysis of the data.

3.6 Blench's Development

In 1939, Blench another Engineer with years of experience in irrigation hydraulics, separated the influence of bed material from the composition of the alluvium forming the banks by introducing separate bed and side factors, F_b and F_s [8, 21]. Also, the slope function was normalized as a friction factor, a function of a width-based Reynolds Number. Blench considered a channel in regime to be one 'in which the average values that constitute regime such as discharge, width, depth, slope and meander pattern do not show any definite trend of variation over some time interval'. The basic Blench's regime equations are given as Eqs. 26 and 27 [8].

$$F_b = V^2/D,\ \text{the bed factor} \quad (26)$$

$$F_s = V^3/W,\ \text{the side factor} \quad (27)$$

From these definitions, it follows the Eqs. 28-30.

$$D = (F_s Q/F_b^{\,2})^{1/3}\ \text{(ft)} \quad (28)$$

$$W = (F_b Q/F_s)^{1/2}\ \text{(ft)} \quad (29)$$

$$V = (F_\mathrm{b}\, F_s\, Q)^{1/6}\ \text{(ft/s)} \quad (30)$$

He presents another equation that gives the slope, $V^2/gDS = 3.63\, (VW/\mathrm{v})^{1/4}\, (1{+}c/2330)$ or $S = F_b^{5/6}\, F_s^{1/12}\, v^{1/4}$ / $(3.63\, gQ^{1/6}\, (1{+}c/2330))$.

He recommends the values of F_s as 0.1, 0.2, and 0.3 for bank material of very slight, medium, and high cohesion, respectively, and the value of bed factor for sub-critical flow, $F_b = 9.6\, d^{1/2}\, (1 + 0.012\, c)$.

Where d is the median bed size in inches and c is the sediment-load concentration in ppm.

These equations provide the dimensions of a stable canal under a given discharge, sediment concentration, sediment size, and bank cohesiveness. Through the use of the side factor, this method accounts for the effect of bank cohesiveness on the channel geometry. It can be seen from these equations that the stable width decreases while the depth increases with an increase in bank cohesiveness. Since the bank slope of the canal is in direct relation to bank cohesiveness, canals with more cohesive banks have smaller widths and greater depths.

3.7 Simons and Albertson's Development

Simons and Albertson [5] extended this scope to canals of different characteristics, using data of the India-Pakistan canals and additional ones collected in Colorado, Wyoming, and Nebraska. Based on these data, they classified canals into the following five types; (1) Sand bed and banks, (2) sand bed and cohesive banks, (3) cohesive bed and banks, (4) coarse non-cohesive material, and (5) same as (2) but with heavy sediment loads, 2000-8000 ppm. The hydraulic geometry is distinguished according to the canal types so classified. Their data separated into groups according to bed and bank materials, each described by the Eqs. 31, 32 and 33.

$$P = k_1\, Q^{0.512} \text{ (ft)} \quad (31)$$

$$R = k_2\, Q^{0.361} \text{ (ft)} \quad (32)$$

$$V = k_3\, (R^2\, S)^{\mathrm{m}} \text{ (ft/s)} \quad (33)$$

Where Q (cfs); k_1, k_2, k_3 and m are constants depending on whether the canal had a coarse noncohesive bed and banks, sand bed and banks, sand bed and cohesive banks, or cohesive bed and banks. They illustrated that Lacey's regime equation fits well into the sand bed and cohesive banks classification.

In discussion with Simons and Albertson, Kansoh (1953) proposed two regime Eqs. 34-36.

$$W = 2.383\, Q^{0.5} \text{ (ft)} \quad (34)$$

$$D = 0.531\, Q^{0.361} \text{ (ft), sand bed and cohesive banks} \quad (35)$$

$$D = 0.303\, Q^{0.361} \text{ (ft), coarse and non-cohesive banks} \quad (36)$$

Where, Q is in cfs.

This method reflects the effects of bed and bank materials on the stable channel geometry in addition to the discharge. For the same canal type, the width and depth are in direct relation to the discharge, and they are essentially independent of the slope. For the same discharge, canals with cohesive banks are smaller in width, deeper in depth, and flatter in slope than those with noncohesive banks. This comparison is attributed to the fact that cohesive banks are generally steeper than noncohesive ones.

In the application of these equations, the design discharge and the canal type need to be specified beforehand; then the width, depth, and slope may be computed using the regime formulas. For the data used in establishing the regime relationships, sand bed canals have bed materials in the medium to fine sand range, cohesive-bed canals have bed materials finer than the sand size, and the coarse materials, or gravel, for the fourth type of canals are between 20 and 82 mm in median size.

3.7.1 Leopold and Maddock's development

Leopold and Maddock [22] extended the regime concept of alluvial channels to the rivers in the United States. They considered that the water and sediment discharges in a river as independent variables and the geometry of the channels as dependent variables. The approach used by Leopold and Maddock was essentially the same as that of previous authors (the empirical approach), but they made two very important contributions to the evolution of the regime concept. First, they differentiated between the hydraulic geometry relations applicable to a station and those applicable to different stations on a river. Secondly, in deriving, the relations for different stations in a river system, they used discharges of equal frequency. In their study, they used the stream gauging data available on about 119 gauging sites in nine different basins in the United States, with discharge varying from 13 to 554,699 cfs. They showed that the hydraulic geometry of alluvial channels could be expressed by the simple power function of the discharge.

The analysis was restricted to bankfull discharge only. The relations traditionally take the form of power functions given as Eqs. 37-39.

$$W = a\, Q^b \quad (37)$$

$$D = c\, Q^f \quad (38)$$

$$V = k\, Q^m \quad (39)$$

Note that, $Q = A \times V$ where $A = W \times D$ and $Q = W \times D \times V = aQ^b \times cQ^f \times kQ^m = ackQ^{b+f+m}$.

Therefore, $ack = 1$ and $b + f + m = 1$.

The hydraulic geometry of a river may be studied at a station (cross section) or downstream. In their study of 20 channel cross sections on the Great Plains and in the Southwest, Leopold and Maddock found b = 0.26, f = 0.40 and m = 0.34.

Downstream hydraulic geometry is studied at a constant frequency or duration of flow. Leopold and Maddock (1953) found that for Great Plains and Southwestern rivers at the mean annual discharge, b = 0.5, f = 0.4 and m = 0.1

Note that the average width exponent is 0.5, width increases faster than depth, and m is positive indicating that velocity tends to increase downstream.

3.8 Henderson's Development

In 1963, Henderson [23] combined the tractive force theory proposed by Lane with Strickler's formula that gave P-Q relationship similar in form to the Lacey equation. He considers the channel perimeter (P, ft) to be stable against movement (fixed) while the regime theory relates to channels formed in self-transported alluvium (live). For small or narrow channels, he proposed some equations as follows:

$$P = 1.03\, d^{-0.15}\, Q^{0.46} \text{ (ft)} \quad (40)$$

$$R = 0.188\, d^{-0.15}\, Q^{0.46} \text{ (ft)} \quad (41)$$

Q is design discharge (cfs) and $d = d_{50}$ (ft).

For wide channels, he used criterion equation for predicting meandered or braided rivers proposed by Leopold [9] and Wolman [39].

$$S = 0.06\, Q^{-0.44} \quad (43)$$

To find the following relationship by substituting the median bed sizes as given by Leopold and Wolman,

$$S = 0.64\, d^{1.14}\, Q^{-0.44} \quad (44)$$

Q is bankfull discharge (cfs) and $d = d_{50}$ (ft)

3.9 Langbein's Development

Langbein [10] applied statistical concept to study the geometry of river channels. He considered that a river system in humid regions and the channel geometry is responsive to some dominant discharge. By using power equations proposed by Leopold and Maddock, continuity equation and resistance law of Manning's type, he found the following values, b = 0.53, f = 0.37 and m = 0.10.

3.10 Chitale's Development

Chitale in 1966 [24, 25] studied the observed data of stable canals to examine the fitness of Lacey three equations (formulas). It was found that considerable deviation could occur in dimensions given by Lacey formulas in respect of all the three parameters, P, R, and S. The data used and deviations obtained are given in Table 1.

Table 1

Percentages of Deviations with Lacey Formulae

Region	No. of Sites	Period of Observations	% Error P	R	S
Punjab (India and Pakistan)	42 24 27	1933-1939 1933-1936 1962	11.27	11.57	32.23
Sindh (Pakistan)	86	1941, 1942, 1945	19.90	18.19	75.71
Uttar Pradesh (India)	73	1959-1962	42.55	20.00	47.99

A question arose as to whether the deviation in respect to P, R and S were independent of each other or were interdependent. Chitale observed that if some correlation between P, R and S is created, it could also be used as an alternative to the Lacey formula for working out the design parameters. Out of P, R and S, the two parameters, P and R define both shape and size of cross-section. The value of (P/R) equal to (W/D) approximately and provides a measures of shape characteristics while the size is given by PR. Study of relationship between shape, size and the parameters, Q, m, and S was therefore considered to hold promise in gauging insight into the physical phenomenon of interdependence of the canal parameter. Chitale's work is tabulated in Table 2.

After comparison with the Lacey formulas Chitale proposed the Eqs. 45-48.

$$P = 2.187\, Q^{0.523} \text{ (ft)} \quad (45)$$

$$R = 0.486\, Q^{0.341} \text{ (ft)} \quad (46)$$

$$V = 0.09\, Q^{0.136} \text{ (ft/s)} \quad (47)$$

$$S = 0.0005\, Q^{-0.165} \quad (48)$$

3.10.1 Chitale's conclusion

Lacey formulas give [25] expressions in terms of Q and m for P, R and S, which are independent of each other. However, study of comprehensive data on alluvial stable canals in India, Pakistan, U.S., and Egypt, comprising 252 observations showed that the canal dimensions, P, R, and S are interdependent. When Q and m are stipulated, the Lacey formulas give a unique set of design values of P, R, and S. This set is, however, one of the many combinations of P, R, and S, all of them being stable.

Since there are many such combinations, there is at least one degree of freedom to choose the value of one of the three parameters, P, R, and S. Once this is done, there are two independent constraints by way of shape relationships that uniquely determine the values of the remaining two parameters. Shape and size relationships reveal the nature of interdependence between the parameters, P, R, S, Q and m, and also provide alternate design formula many of which are found to be superior to the Lacey equations.

3.11 Engelund & Hansen's Development

Engelund and Hansen [12, 26] used the resistance law and sediment transport law to find hydraulic geometry relationships. By using available experimental data and principles of similarity, they found Eq. 49.

$$W = 6.97\, Q^{0.525}\, d^{-0.316}\ \text{(m)} \qquad (49)$$

Where Q is in m^3/s, and d is in mm.

3.12 Kellerhals's Development

In 1967, Kellerhals [11] proposed the following equations for straight gravel rivers. In his equations, he involved Nikuradse's equivalent sand grain roughness, k_s. If the boundary roughness consists of uniform sand with diameter d, k_s can be expected to be closely similar to d. He found that $k_s = d_{90}$ (ft).

$$W = 1.80\, Q_d^{0.5}\ \text{(ft)} \qquad (50)$$

$$D = 0.166\, Q_d^{0.4}\, Ks^{-0.12}\ \text{(ft)} \qquad (51)$$

$$S = 0.12\, Q_d^{-0.4}\, Ks^{0.92} \qquad (52)$$

Where, Q_d is a dominant discharge (cfs).

3.13 Bhowmik's Development

Bhowmik (1968) [27] studied the stability of alluvial channels in coarse material. In this study, he defined Eqs. 53-55.

$$W = 1.70\, Q^{0.52}\ \text{(ft)} \qquad (53)$$

$$D = 0.277\, Q^{0.352}\ \text{(ft)} \qquad (54)$$

$$V = 10\, (D^{2.5} S)^{0.167}\ \text{(ft/s)} \qquad (55)$$

3.14 Maddock's Development

Maddock [22] involved sediment transport and bed form in his study. He stated that the response of a dependent variable to a change in an independent variable was related to the constraints, natural or artificial, and placed on the system. As a result of his study, he concluded following equations.

$$V = B_{fv}\, q^{1/12}\, q_s^{1/4} \qquad (56)$$

$$S = B_{fs}\, q^{-5/6}\, q_s^{1/2} \qquad (57)$$

And, related to regime concepts, he considered the relation given in following equation.

$$D = C\, Q^{1/3} \qquad (58)$$

To be characteristic of streams with similar behavior with respect to the rate of energy consumption and to a constant size of sediment. B_{fv} and B_{fs} are coefficients related to bed form, c is a coefficient, q is unit discharge, and q_s is unit sediment discharge.

3.15 Alvarez and Villanueva's Development

Alvarez and Villanueva (1973) [28] proposed the following regime equations based on observation and measurements in Mexican rivers. To find these equations, they used three basic formulas, flow resistance, sediment transport capacity, and width to depth ratio.

$$W = T_w\, Q^{0.627}\, Q_s^{-0.118} \qquad (59)$$

$$D = T_d\, Q^{0.439}\, Q_s^{-0.083} \qquad (60)$$

$$S = T_s\, Q^{-0.768}\, Q_s^{0.56} \qquad (61)$$

Where T_w, T_d, and T_s are constants.

3.16 Griffiths's Development

Griffiths [13, 29] by using 186 field data sets from gravel-bed rivers in New-Zealand, obtained a resistance equation in the domain of interest $4 < (R/d_{50})$ as follows.

$$RS/((G-1)d_{50}) = \tau_{s*} = 0.056 \qquad (62)$$

He found that,

$$W = 5.28\, QS^{1.26}\, d_{50}^{-1.50} \qquad (63)$$

Where Q is in m^3/s, and d_{50} is in mm.

Table 2

Different Forms of General Velocity Equation and Corresponding Lacey Equations

Values of exponent *x* in general equation	Reduced form of general equation for velocity, in feet per second	Value of *m* from general equation, in millimetres	Original Lacey equation for velocity, in feet per second	Reduced form of Lacey equation for velocity, in feet per second	Value of m in millimetres, suggested by Lacey for this equation
0	V=1.54 $R^{1/2}$ $m^{1/4}$	-	V = 1.17 $f^{1/2}$ $R^{1/2}$	V = 1.55 $R^{1/2}$ $m^{1/4}$	-
1/4	V = 8.9 $R^{3/5}$ $S^{1/4}$ $m^{1/2}$	2.95	V = 9.5 $R^{1/2}$ $S^{1/2}$	V = 9.5 $R^{1/8}$ $S^{1/4}$	$m > 2.0$
1/3	V = 16.0 $R^{2/3}$ $S^{1/3}$	0.6	V = 16.0 $R^{2/3}$ $S^{1/2}$	V = 16.0 $R^{2/4}$ $S^{1/2}$	$0.6 < m < 2.0$
1/2	V = 51.4 $R^{1/4}$ $S^{1/2}$ $m^{1/2}$	-	V = 1.345 $R^{3/4}$ $S^{1/2}/Na$	V = 51.85 $R^{3/4}$ $S^{1/2}$ $m^{1/2}$	-
1/2	V = 51.4 $R^{1/4}$ $S^{1./2}$ $m^{1/2}$	0.29	V = 60.0 $R^{1/4}$ $S^{1/2}$	V = 60.0 $R^{3/4}$ $S^{1/2}$	$0.2 < m < 0.6$
1	V=1.717$RSm^{1/2}$	0.15	V= 4.500 RS	V = 4.500 R S	m< 0.2

Table 3

Percentages of Errors and Standard Deviations

Size of bed material, in millimetres	Velocity equation [a]	Percentage of error [b]	Standard deviation [c]
0.20 – 0.60	V = 60 $R^{3/4}$ $S^{1/2}$	23.67	0.67
0.05 – 0.20	V = 4.500 RS	49.41	1.26

Table 4

Showing Improved Equations

Size of bed material, in millimetres	Velocity equation [a]	Percentage of error [b]	Standard deviation [c]
0.20 – 0.60	$V = 19.94\ \tau^{-0.32} R^{0.75} S^{0.5} m^{-0.13}$	12.34	0.35
0.05 – 0.20	$V = 457\ \tau^{0.30} RS\ m^{-0.50}$	23.00	0.59

Table 5

Comparison of Statistical and Lacey Equations

Statistical Equations				Lacey Formulas		
	Percentage of error [a]	Correlation coefficient [b]	Standard deviation [c]	Formulas [a]	Percentage of error [b]	Standard deviation [c]
$P = 1.011\,\tau_r^{-0.139} (P/R)_r^{-0.46}$ or $P = 1.743\ R^{0.209\ S}\text{-}S^{0.017} Q^{0.414} m^{0.115}$	9.6	0.87	0.094	$P = 2.67\ Q^{1/2}$	20.47	33.41
$R_R = 1.013\ \tau_r^{-0.141} (P/R)_r^{-0.564}$ or $R = 0.2097\ p^{-0.978} S^{-0.244} Q^{0.761} m^{0.037}$	8.1	0.92	0.037	$R = 0.47\ Q^{1/3} f^{-1/3}$	24.51	1.828
$S_r = 0.987\ \tau^{-1.141} (P/R)^{0.564}$ or $S = P^{-4.00} R^{-4.02} Q^{3.197} m^{0.151}/594$	9.9	0.99	0.161	$S = 1/1.788\ pQ^{-5/6}$	148.77	2.2410^{-4}

[a] Equations are in foot-pound system.

[b] Percentage of error in estimated characteristics is same as coefficient of variation.

[c] Standard deviation (σ) has the unit of the characteristic.

Table 6

Comparison of Derived and Lacey Formulas

Statistical Equations				Lacey Formulas		
	Percentage of error [a]	Correlation coefficient [b]	Standard deviation [c]	Formulas [a]	Percentage of error [b]	Standard deviation [c]
(a) for 0.20 mm, 0.60 mm size of Bed Material						
$P = 1.304\ Q^{0.4880}\ S^{-0.1093}\ m^{0.1165}$	24	17.15	27.52	$P = 2.67\ Q^{1/2}$	12.00	19.26
$R = 0.2494\ Q^{0.3535}\ S^{-0.0614}\ m^{0.0059}$	25	12.06	1.032	$R = 0.47\ Q^{1/3}\ f^{-1/3}$	19.28	1.561
$P = 0.9610\ Q^{0.4783}\ S^{-0.1623}\ m^{-.1575}$	26	13.78	20.19	$P = 2.67\ Q^{1/2}$	13.68	20.04
$R = 0.0580\ Q^{0.3074}\ S^{-0.3149}\ m^{0.2019}$	27	15.29	1.103	$R = 0.47\ Q^{1/3}\ f^{-1/3}$	9.43	0.6801

[a] Equations are in foot-pound system.

[b] Percentage of error in estimated characteristics is same as coefficient of variation.

[c] Standard deviation (σ) has the unit of the characteristic.

3.17 Bray's Development

Bray [14] developed regime equations for gravel-bed rivers by using a data set of 70 gravel bed river reaches in Alberta, Canada. In these equations, he uses the 2 years flood as a characteristic discharge. Under the assumption that a gravel-bed channel is free to adjust, he used four different methods to develop regime equations for width, depth, velocity, and slope as power functions of the two years flood flow and a characteristic bed material size. The first method was linear regression, the second was a threshold method, the third was a dimensionless method, and the last method was multiple regression analysis. The equations derived from the last method are as follows.

$$W = 2.08\ Q^{0.528}\ d_{50}^{-0.07} \quad (64)$$

$$D = 0.256\ Q^{0.331}\ d_{50}^{-0.025} \quad (65)$$

$$V = 1.87\ Q^{0.140}\ d_{50}^{0.095} \quad (66)$$

$$S = 0.097\ Q^{-0.334}\ d_{50}^{0.586} \quad (67)$$

Where Q is 2 years discharge (cfs) and d_{50} (ft).

3.18 Parker's Development

Parker [30] on discussion with Bray [14] gave three sets of regime equations, width, depth, and slope, as the result of his study with data from Britain, Canadian rivers, and laboratory model. Parker explained that a gravel-bed stream with an imposed water discharge selects a width, which allows for gravel transport without global bank erosion. His regime equations are as under.

$$W = K_1\ Q_b^{m1} \quad \text{(m)} \quad (68)$$

$$D = K_2\ Q_b^{m2} \quad \text{(m)} \quad (69)$$

$$S = K_3\ Q_b^{m3} \quad (70)$$

$$V = (K_1 K_2)^{-1}\ Q_b^{1-(m1+m2)} \quad \text{(m/s)} \quad (71)$$

Where K_1 = 3.73 – 7.08, K_2 = 0.188 – 0.363, K_3 = 0.000623 – 0.0281, m_1 = 0.382 – 0.446, m_2 = 0.331 – 0.499, m_3 = (-0.02) – (0.46), and Q_b is the bankfull discharge (m^3/s).

3.19 Cartlon and Chang's Development

Charlton (1982) [31] separated regime equations for gravel-bed rivers into two categories. For channels with sediment transport rates are small and the D/d_{90} ratio between approximately 3 and 80 (deep channels), he recommends to use the following equations.

$$W = 3.74\ K\ Q_b^{0.45} \quad \text{(m)} \quad (72)$$

$$D = 0.114\ K^{-1.82}\ Q_b^{0.42}\ d_{65}^{-0.38}\ d_{90}^{0.24} \quad \text{(m)} \quad (73)$$

$$S = 0.409\ K^{-0.24}\ Q_b^{-0.418}\ d_{65}^{1.38}\ d_{90}^{-0.24} \quad (74)$$

Where $(D/d_{90}) < 3$, for shallow channels.

$$W = 3.74\ K\ Q_b^{0.45} \quad \text{(m)} \quad (75)$$

$$D = 0.477\ K^{-1.82}\ Q_b^{0.25}\ d_{65}^{1.22}\ d_{90}^{-0.55} \quad \text{(m)} \quad (76)$$

$$S = 0.123\ K^{-0.55}\ Q_b^{-0.25}\ d_{65}^{1.22}\ d_{90}^{-0.55} \quad (77)$$

Where Q_b is the bankfull discharge (m^3/s), d_{65} and d_{90} are in meters, K is the bank vegetation coefficient, $1.3 > K > 0.9$ for grass and light vegetation, and $1.1 > K > 0.7$ for trees and heavy vegetation.

Chang [15] used a quantitative approach, by using the existing relationships governing the flow and sediment transport processes, to show the equilibrium geometry of river channels as function of bankfull discharge, slope and depth. For rivers that have slopes ranging from moderately steep to fairly steep, he proposed the following equation.

$$S = 0.00763\, Q_b^{-0.51}\, d^{0.5} \tag{78}$$

Where Q_b is the bankfull discharge (m^3/s), and $d = d_{50}$ (mm).

3.20 Hey and Thorne's Development

Hey and Thorne [32, 33] introduced regime type equations for mobile gravel-bed rivers in the United Kingdom. The effect of vegetation on the bank is considered in their equations. They used standard multiple regression procedures to derive the equations for average bankfull channel values of width (W, m), wetted perimeter (P, m), mean depth (D, m), hydraulic radius (R, m), slope (S), and velocity (V, m/s), in term of the independent variables; bankfull discharge (Q, m^3/s), an independent estimate of bed load transport (Q_s, kg/s), bed material size (d, m) and variability, bank shear strength, vegetation density and valley slope.

The proposed equations are given as under.

$$D = 0.22\, Q_b^{0.37}\, d_{50}^{-0.1}\ \text{(m)} \tag{79}$$

$$S = 0.087\, Q_b^{-0.43}\, d_{50}^{-0.09}\, d_{84}^{0.84}\, Q_5^{6.10} \tag{80}$$

$$W = k_1\, Q_b^{0.50}\ \text{(m)} \tag{81}$$

$$P = k_2\, Q_b^{0.49}\, Q_s^{-0.01}\ \text{(m)} \tag{82}$$

$$R = k_3\, Q_b^{0.41}\, Q_s^{-0.02}\, d_{50}^{-0.14}\ \text{(m)} \tag{83}$$

$$V = k_4\, Q_b^{0.10}\, Q_s^{0.03}\, d_{50}^{0.18}\ \text{(m/s)} \tag{84}$$

Where the values of k_1, k_2, k_3 and k_4 depend on the following vegetation types.

1. grassy bank with no trees or shrubs
2. 1-5% tree/shrub cover
3. 5-50% tree/shrub cover
4. 5-50% shrub cover or incised into flood plain

3.21 Neill's Development

Neill (1968) [34] proposed the following regime equation for gravel-bed channels in low transport condition by combining the Shields function, a Lacey-type width equation and the Manning-strickler formula, and assuming hydraulic radius was approximated as 0.9 of the depth.

$$S = 0.854\, d_{50}^{1.29}\, Q_b^{-0.43} \tag{85}$$

Where Q_b is the bankfull discharge (m^3/s), and d_{50} (m).

3.22 Thorne, Chang and Hey's Development

Thorne, Chang and Hey [15, 32, 33] used the minimum stream power concept to find regime relationships for gravel-bed streams as follows:

$$S_{cr} = 0.00009892\, d_{50}^{-1.15}\, Q_b^{-0.42} \tag{86}$$

$$W = a\, Q_b^{0.47}\quad \text{(m)} \tag{87}$$

$$D = b\, Q_b^{0.42}\quad \text{(m)} \tag{88}$$

Where, S_{cr} is threshold channel slope, a and b are coefficients, Q_b is bankfull discharge (m^3/s), and d_{50} in mm.

3.23 Nouh, Clark and Davies's Development

Nouh (1988) [35] developed regime formulas relating the channel dimensions and pattern to the characteristics of flash flood and sediment flow in the channel. He collected field observations from 37 ephemeral channels of an extremely arid zone in Saudi Arabia. The results of his study are given in Eqs. 89, 90 and 91.

$$W = 28.30(Q_{p50}/Q)^{0.83} + 0.018(1+d)^{0.93}\, C^{1.25}\ \text{(m)} \tag{89}$$

$$D = 1.29\, (Q_{p50}/Q)^{0.65} - 0.01\, (1+d)^{0.98}\, C^{0.46}\ \text{(m)} \tag{90}$$

$$S = 18.25\, (Q_{p50}/Q)^{-0.35} - 0.88\, (1+d)^{1.13}\, C^{0.36}\ \text{(m)} \tag{91}$$

Q_{p50} is the peak discharge for a return period of 50 years, Q is the mean annual discharge, Q_{p50} is in hundreds of m^3/s, c is the mean suspended sediment concentration (kg/m^3) and d is the sediment size.

Similarly, Clark and Davies (1988) [40] developed regime equations based on investigation in arid conditions of Yemen. They analysed the measured slopes and estimated dominant discharges by using multiple regression that gave the relationship,

$$S = 0.028\, Q_d^{-0.18}\, d_{50}^{0.09} \tag{92}$$

Where Q_d is the dominant discharge (m^3/s), and d_{50} is the particle diameter (m).

3.24 Klassen and Vermeer's Development

Klassen and Vermeer [36] investigated the braiding sand-bed Jamuna river in Bangladesh. They presented in the following regime equations.

$$D = 0.23\, Q^{0.23} \quad \text{(m)} \tag{93}$$

$$W = 161.1\, Q^{0.53} \text{ (m)} \tag{94}$$

3.25 Julien's Development

Julien[16] proposed four fundamental relationships of downstream hydraulic geometry of non-cohesive alluvial channels. He considered four fundamental concepts of hydraulics and sediments: flow continuity, flow resistance, longitudinal sediment mobility and radial sediment mobility. By combining those fundamental relationships, he derived theoretical hydraulic geometry relationships that can be written as a power function of discharge, bed sediment size and two mobility factors as follows.

$$D \propto Q^{1/(2+3a)}\, d_s^{(6a-1)/(4+6a)}\, \tau_*^{-3/(4+6a)}\, \tau_{cr}^{1/(2+3a)} \tag{95}$$

$$W \propto Q^{(1+2a)/(2+3a)} d_s^{-(1+4a)/(4+6a)}\, \tau_*^{1/(4+6a)}\, \tau_{cr}^{-(1+a)/(2+3a)} \tag{96}$$

$$S \propto Q^{-1/(2+3a)}\, d_s^{5/(4+6a)}\, \tau_*^{(7+6a)/(4+6a)}\, \tau_{cr}^{-1/(2+3a)} \tag{97}$$

Where a is the exponent of the resistance equation and τ_* is longitudinal mobility factor that equals the downstream shields parameter as follows.

$$\tau_* = \frac{\tau}{(\rho_s - \rho)\mathrm{g}d_s} = \frac{\rho \mathrm{g} D s}{(\rho_{s-\rho})\mathrm{g}d_s} = \frac{DS}{(G-1)d_s}$$

Where, τ_* is longitudinal shear stress. τ_* is radial mobility factor used in curved channels that equals to radial Shields parameter.

$$\tau_{cr} = \frac{\tau_r}{(\rho_s - \rho)\mathrm{g}d_s}$$

Where τ_r is radial shear stress.

4. Summary and Conclusions

4.1 Comments on Available Regime Theories

There seems to be no agreement in the various regime equations, on the choice of the width or depth parameter; the hydraulic radius or the mean depth is used as the depth parameter, while the wetted perimeter, water surface width, or average width is used as the characteristic width parameter. Kennedy used the depth D but Lindley used the bottom width and depth. In 1928 Lacey used hydraulic radius and wetted perimeter. Inglis-Lacey formulae are expressed in terms of width and depth. Blench recommends the use of average width and mean depth. It seems that if the roughness coefficient for the bed and the sides is the same, use of hydraulic radius is preferable to the depth, at least in the resistance equation and in the equation connecting mean velocity and hydraulic radius. However, if the roughness coefficients for the bed and sides are different, use of W and D, as advocated by Blench, seems more appropriate.

Another aspect of regime method of channel design that needs consideration is the fact that the regime equations do not include sediment load as an independent variable in stable channel design. The importance of sediment load as a variable in stable channel design has been well emphasized by Lane. At present, the exponents of the regime method as well as the tractive force method are fully convinced that the rate of sediment transport is an important variable and, as such, should be taken into consideration. This can be seen from the equations of Inglis-Lacey and Blench; and yet in most cases the design of stable channels (by regime method) is carried out without consideration to the sediment transport rate. Satisfactory performance of such a channel is, however, ensured by removing at the head works the coarse materials entering the channel and thereby maintaining the sediment concentration in the vicinity of 500 ppm or even less.

It must also be emphasized that regime equations were never intended for application to a stream where the discharge varies. In other words, the regime equations are strictly valid for only one discharge condition. The discharge to be used in regime equations is either the sustained discharge (for irrigation channels) or the dominant discharge (for alluvial streams). In this study review of the historical development of the regime geometry equations since 1895 to 1988 was presented, considering as part-I and the part-II may be later presented, consisting of the review of work done on regime equations from 1989 to 2018.

4.2 Conclusions

Three main factors primarily influence the geometry of alluvial channels: flow discharge, sediment size, and bed-load sediment in terms of shields parameter. Although discharge exerts a dominant control on channel geometry, the hydraulic geometry relationships indicate that sediment size and bed-load sediment strongly influence the channel slope, moderately influence the flow velocity, and slightly influence the channel depth and the channel width. This study shows that the exponents of discharge, sediment size, and sediment transport, in terms of Shields parameter, are dependent on the relative submergence (D/d_{50}).

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