# Subpixel Edge Localization with Statistical Error Compensation

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**ABSTRACT**

Subpixel Edge Localization (EL) techniques are often affected by an error that exhibits a systematic character. When this happens, their performance can be improved through compensation of the systematic portion of the localization error. In this paper we propose and analyze a method for estimating the EL characteristic of subpixel EL techniques through statistical analysis of appropriate test images. The impact of the compensation method on the accuracy of a camera calibration procedure has been proven to be quite significant (44%), which can be crucial especially in applications of low-cost photogrammetry and 3D reconstruction from multiple views.

## 1 Introduction

Several applications of 3D scene reconstruction from multiple views or camera calibration, are crucially sensitive to the accuracy with which certain image features are detected and localized on the image plane. The most common features that need to be precisely located on the image plane are image edges, as they usually carry significant information about the imaged scene. As very high resolution CCD cameras are currently too expensive to be used in 3D reconstruction applications, subpixel Edge Localization (EL) algorithms are becoming more and more popular as their aim is to offer super-resolution performance with low-cost CCD cameras [1, 2].

In this article we propose and evaluate a method for improving the performance of sub-pixel edge localization techniques, which is based on the correction of the EL error (ELE) associated to nearly-horizontal or nearly-vertical edges. The method is based on a statistical analysis of appropriate test images, therefore we do not need any *a-priori* information either on the camera system or on the adopted subpixel EL technique.

## 2 Model of the Acquisition System

The system we adopted for image acquisition consists of a standard TV-resolution CCD camera and a frame-grabber. The lens model [5] is obtained by combining ideal perspective projection with low-pass filtering. The filter models the limited lens bandwidth due to the finite lens aperture and the *aperture* of the photosensitive area of the CCD cells. The impulse intensity response of the lens (Modulation Transfer Function – MTF) can be obtained, through an appropriate change of variables, from the autocorrelation of the *pupil function* [6], while the impulse response of the low-pass filter that models the integration of the light over the photo-sensitive cells corresponds to the light sensitivity map of the pixel cell.

As geometric nonlinear distortions can be accurately estimated and compensated for through an appropriate camera calibration procedure, we will ignore it in what follows. As far as other types of aberration are concerned, modern good quality lenses are normally designed in such a way that blurring due to aberration is negligible with respect to that due to its limited bandwidth [7]. Finally, cameras are often equipped with a clock output for frame-grabber synchronization, in which case the acquisition system is equivalent to a digital camera.

## 3 Edge Localization Error

The 1D ELE corresponding to an abrupt luminance transition is the distance between the sharp transition that would form on the image plane when using an *ideal* (unlimited bandwidth) optical lens and the edge that has been actually detected. Besides depending on the acquisition system, the ELE critically depends on the subpixel Edge Localization (EL) technique under exam.

In Fig. 1, for example, a sharp luminance transition located in the point $p$ is being localized at subpixel precision through simple linear interpolation. Due to the limited aperture of the lens, the ideal edge profile is smoothened by the MTF, as shown in Fig. 1$b$. The luminance samples that are collected by the CCD sensor depend on the light that falls on the whole photosensitive area of the pixel, therefore they are given by the area of the shaded regions in Fig. 1$b$. A simple and frequently used way of estimating the subpixel location $p$ of the ideal edge from the samples collected from the CCD array consists in linearly interpolating (see Figure 1$c$)

the collected samples, and determining the intersection between the resulting piecewise linear profile and an appropriate threshold. The threshold is set equal to half the amplitude $W$ of the luminance discontinuity, and the resulting intersection can be taken as an estimate of the edge location.

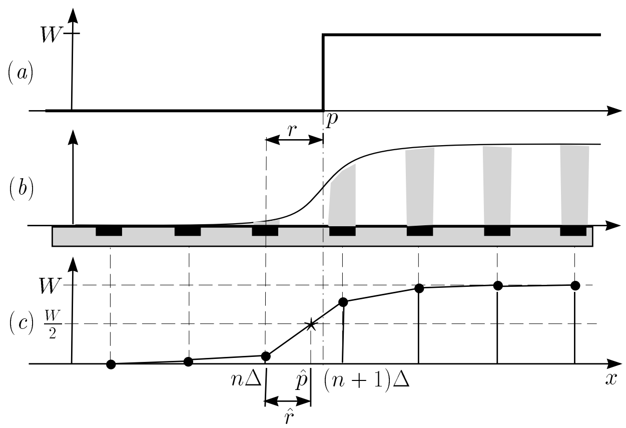



Figure 1: Subpixel edge detection based on linear interpolation. Ideal luminance profile (a), luminance profile incident on the image plane (b), linear interpolation of the image samples (c).

Such an example of subpixel EL method is simple enough to visualize the ELE associated to it, in fact the estimated edge location $\hat{p}$ differs from the ideal location $p$ of a quantity called Edge Localization Error $e$. If some conditions of regularity in the acquisition system are satisfied, then the ELE is a periodic function of the edge location.

In what follows, the function that maps the ideal relative edge location $r$ into the estimated one $\hat{r}$ is called Edge Localization Function (ELF), $\hat{r} = F_{EL}(r)$, and the ELE can be written in terms of the ELF as $e = \hat{r} - r = E_{ELE}(r) = F_{EL}(r) - r$.

## 4 Estimation of the Error Characteristic

If the Edge Localization (EL) characteristic $\hat{r} = F_{EL}(r)$ is available and invertible, then compensation of the Edge Localization Error (ELE) is possible.

As the response of the CCD camera can be considered space-invariant, the *ELE* function $e = \hat{r} - r = E_{EL}(r)$, must be periodic of period 1 pixel, therefore we can limit our analysis, for example, to any interval like $r_0 \leq r \leq 1 + r_0$. The periodicity of the ELE results in $1 + \hat{r} = F_{EL}(1 + r)$.

If $F_{EL}(r)$ is monotonic, then it is also bijective, in which case its inverse function, $r = F_{EL}^{-1}(\hat{r})$ is bijective as well and the output range corresponding to $r_0 \leq r \leq 1 + r_0$ results as $\hat{r}_0 \leq \hat{r} \leq 1 + \hat{r}_0$, where $\hat{r}_0 = F_{EL}(\hat{r}_0)$. The inverse EL function $F_{EL}^{-1}(\cdot)$ can thus be used as an *error compensation function.*

As the EL function maps ideal edge locations onto detected locations, we can derive information on this map from the joint statistics of both its input and its output. The estimation of the error compensation function, in fact, can be done through statistical analysis of an appropriate test image. The statistical distribution of the estimated edge locations can be quite easily extracted from the test image, while the statistics of the ideal edge location can be inferred from the pattern characteristics in particular cases. From a practical viewpoint it is convenient to choose test images whose ideal edge points (referred to the center of the pixel area that they fall on) are uniformly distributed over pixel areas.

If the probability density function (p.d.f.) $f_r(a)$ of the ideal edge point position $r$ is uniform in $(r_0, 1 + r_0)$, then the p.d.f. of $\hat{r} = F_{EL}(r)$ can be expressed as

$$f_{\hat{r}}(b) = \frac{f_r(a)}{F'_{EL}(a)} \ , \quad b = F_{EL}^{-1}(a) \ , \tag{1}$$

where $\hat{r}_0 \leq b < 1 + \hat{r}_0$ and $F'_{EL}(a) > 0$ is the first derivative of $F_{EL}(a)$. As a consequence, we can write

$$f_{\hat{r}}(b) = \left. \frac{1}{F'_{EL}(a)} \right|_{a=F_{EL}(b)} = \frac{d}{db} F_{EL}^{-1}(b) \ , \tag{2}$$

where $r_0 \leq a < 1 + r_0$ and $\hat{r}_0 \leq b < 1 + \hat{r}_0$.

By integrating the p.d.f. of the subpixel edge locations $\hat{r}$ detected from the image, we obtain the compensation function

$$r = C(\hat{r}) = F_{EL}^{-1}(\hat{r}) = F_{EL}^{-1}(\hat{r}_0) \ + \ \int_{\hat{r}_0}^{\hat{r}} f_{\hat{r}}(a) \ da \ . \tag{3}$$

Notice that the value of $\hat{r}_0$ is not a known parameter, therefore all that we can obtain from the analysis of the image is the statistical distribution of $\hat{r}$, computed over an arbitrary pixel-wide interval like $(A, 1 + A)$, generally not entirely contained in the interval $(\hat{r}_0, 1 + \hat{r}_0)$. Eq. (3) could thus be expressed as follows:

$$\begin{aligned} r &= \int_{\hat{r}_0}^{A} f_{\hat{r}}(b) \ db \ + \int_{A}^{\hat{r}} f_{\hat{r}}(b) \ db \ + \ r_0 \\ &= \int_{A}^{\hat{r}} f_{\hat{r}}(b) \ db \ + K_A \ . \end{aligned} \tag{4}$$

It is quite clear from eq. (4) that different choices of the interval of definition of $\hat{r}$ result in different vertical offsets $K_A$ for the compensation function.

It is worth emphasizing that the fact that the compensation function is derived from a p.d.f. through integration gives us no information on the offset $K_A$, which means that we can linearize the EL function (i.e. eliminate its ripple) but we still need to determine its offset. The extra unknown can be determined by using further *a-priori* information on the test image, or through an appropriate estimation procedure.

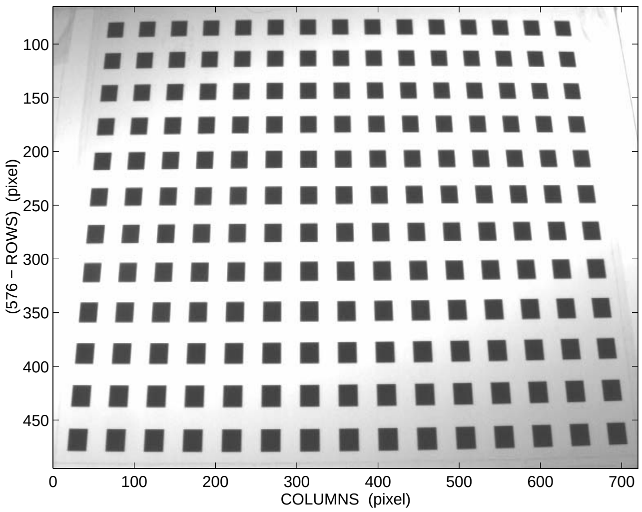



Figure 2: Calibration pattern used as a test image.

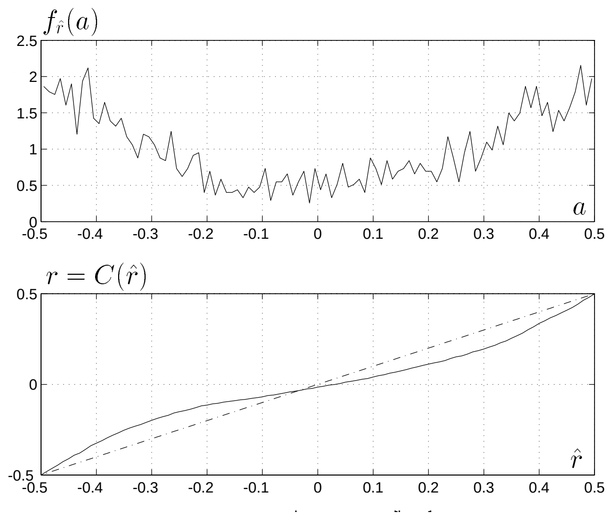



Figure 3: Estimated p.d.f. of the detected subpixel residuals *(a)* and relative compensation function *(b)*.

## 5 Error Compensation

In order to test the proposed algorithm, we have carried out some experiments of ELE correction on a test image.

All edge points of the test image are localized with sub-pixel accuracy by using an edge localization algorithm, e.g. cubic interpolation with edge location at the flex point. From each edge coordinate $x$, we compute the local edge coordinate $\hat{r} = \hat{x} - n\Delta$, where $n\Delta$ is the nearest pixel center to $\hat{x}$. Assuming that the above lengths are measured in pixels, we have $-\frac{1}{2} \leq e \leq \frac{1}{2}$ and $A = -\frac{1}{2}$.

The p.d.f. $f_{\hat{r}}(b)$ of the detected subpixel relative locations is estimated by building a histogram for $\hat{r}$. This operation corresponds to building a piecewise constant approximation of the desired p.d.f., and then normalizing its amplitude. The number of histogram intervals depends on the number of available samples of $\hat{r}$. Finally, we integrate $f_{\hat{r}}(b)$ in order to compute the first term of eq. (4). As far as the offset $K_A$ is concerned, its determination depends on the specific application.

Fig. 3*a* shows the p.d.f. $f_{\hat{r}}(b)$ estimated from the test image of Fig. 2 (vertical edges). Compensation is performed by using eqs. (4) and the resulting *compensation function* $C(\hat{r}) = F_{EL}^{-1}(\hat{r})$, is shown in Fig. 3b. The compensated edge position $r$ is then obtained by simply applying the compensation $C(\hat{r})$ to the detected position $\hat{r}$

$$r = F_{EL}^{-1}(\hat{r}) = C(\hat{r}) \qquad (5)$$

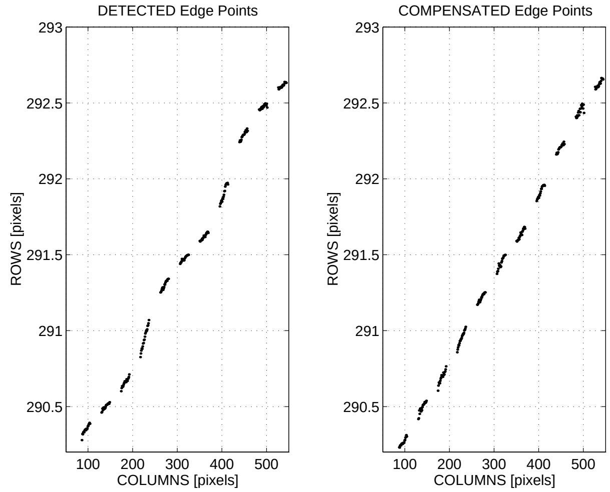
## 6 An Example of Application

The ELE compensation method of Section 4 can be organized as follows:

1. Perform subpixel EL on the *test* image
2. Estimate the compensation curve from the edge points of the test image
3. Perform subpixel EL on the *scene* image
4. Correct the edge coordinates through the estimated compensation function

Notice that, if the edge locations in the scene image are uniformly distributed, then the scene image can be used as a test image, and step 3 can be skipped.

In order to evaluate the impact of the above compensation technique on the performance of a subpixel edge localizator, we have embedded the method into a camera calibration procedure [3, 4] and compared the accuracy with and without compensation. Camera calibration consists in estimating the *intrinsic parameters* (optical center, focal length, nonlinear distortion coefficients) and the *extrinsic parameters* (relative position and orientation of the camera with respect to the target) of an image acquisition system through the analysis of the views of a *calibration pattern*. The reliability of the calibration procedure critically depends on how accurately certain *fiducial marks* of the calibration pattern are localized. The calibration pattern used in the experiment is planar and exhibits a set of regularly spaced black squares on a white background, as shown in Fig. 2. The position of the fiducial marks, i.e. the corner points of the squares, is known with a precision of $\pm 5 \mu m$. In order to perform an accurate camera calibration, it is necessary to localize the fiducial marks of the test image with the best achievable precision. Being the fiducial marks corner points of squares, they can be localized by intersecting edges detected with subpixel accuracy.




*(a)* *(b)*

Figure 4: Magnification of rows 290-293 of the horizontal edges of the calibration target. *a)* Linear interpolation and threshold crossing; *b)* Same as *(a)* with error compensation.

The adopted calibration procedure [4] estimates the camera parameters and provides us with a measure of the estimate accuracy, based on the standard deviation of the error between the detected position of fiducial marks on the image plane, and their position computed through the camera model. The accuracy measurement has been used as an evaluation of the performance of the edge localization algorithm, and a comparative evaluation of the results with and without ELE compensation has been done. As the ELE compensation requires the determination of offset parameters, the offsets have been added to the list of intrinsic parameters of the CCD camera and estimated by the calibration procedure.

The ELE statistics associated to the test image of Fig. 2 can be assumed uniform with good approximation, therefore the calibration target is suitable also for the estimation of the compensation curve.

The edge points of the test image are localized with a technique based on cubic interpolation and flex point search. From such edges it is quite straightforward to visualize the ELE associated to the adopted subpixel technique. In fact, by magnifying all horizontal edges of one row of squares, we obtain the curve of Fig. 4*a*, whose oscillations are mainly caused by the ELE. When ELE compensation is performed, we obtain the curve of Fig. 4*b*, where the ELE ripples are now quite evidently reduced.

The standard deviation of the calibration points results as being 0.045 pixel with error compensation, i.e. approximately 44% less than the accuracy we have obtained without compensation (0.082 pixel). This improvement in the performance shows that the impact of the ELE compensation technique can be significant in certain applications where precision is crucial.

## 7 Conclusions

In this paper we have proposed and analyzed a method for improving the performance of sub-pixel edge localization techniques, which is based on the compensation of their Edge Localization Error (ELE). In particular, we have shown how to estimate the EL function and how to derive the ELE compensator from it. We have also evaluated the performance of the ELE compensation method in a concrete situation, by determining its impact on the accuracy of a camera calibration procedure.

The improvement in the calibration accuracy due to ELE compensation has been shown to be quite significant (44%), which can be crucial especially in applications of low-cost photogrammetry and 3D reconstruction from multiple views, and justifies its adoption whenever it is important to maximize the precision of the edge localization without significantly affecting the total cost of the acquisition system.

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