FEQIS: A new equilibrium solver for the Fenix flight simulator

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Introduction

A new generation of modeling tools is being developed for application to magnetic confinement fusion research, e.g. for tokamak plasmas, to model a full plasma discharge, as realistically as possible, but with sufficient speed to run between discharges during operation days. These tools can be referred to as “flight simulators”. Fenix [1-3] is such a tool, developed for the ASDEX Upgrade tokamak and under development for the EU-DEMO reactor prototype. Fenix takes the Pulse Schedule as input and provides a virtual realization of the complete discharge in a few minutes. To achieve this, all models are simple but realistic. However, the equilibrium solver is still the bottleneck.

In this work, a new fast equilibrium solver FEQIS (Fast EquIlibrium Solver) was developed to be used in the Fenix flight simulator, by coupling it directly with the ASTRA transport solver [4]. The newly developed equilibrium solver has been specifically written to be both fast and flexible in the choice of the operating mode, such that transition from the pre-plasma to the plasma phase is accomplished in the smoothest and most reasonable way. In this respect, it differs from existing equilibrium solvers that typically model only the evolved plasma phase (i.e. existence of nested closed field lines is assumed). In this paper, the new code is presented.

Acceleration of the non-linear Grad-Shafranov solution

It is well known that the Grad-Shafranov equation [6] is a non-linear equation, since the solution variable ψ (the poloidal magnetic flux) also appears as “coordinate” of the r.h.s. of the equation:

$$(1) \qquad \nabla^2\psi + (additional\,geometrical\,terms) \approx c_1 dP/d\psi + c_2 dF/d\psi + (external\,currents)$$

where P and F are the profiles of plasma pressure and diamagnetic current.

Moreover, the contribution of the external currents to equation (1) depends on ψ through the mutual inductance between the plasma and the coils. The first rule to speed up equation (1) is:

- Rule FEQIS1: Equation (1) is called on a different time vector than the circuit equation, whereas at each time step, only 1 instance of either equation being called at each time step. Caution: the time step must be small enough as to not artificially influence the plasma motion. In practice this means that dt must be much smaller than the typical plasma resistive time scale, i.e. the time that it takes the plasma to move a pre-defined fraction of the plasma radius. That is, the time variable acts effectively as an "iteration" variable for the non-linear equation solution. In the very first iteration, the non-linear iterations are performed in any case to ensure an initial self-consistent solution, using the same method as described in [7].

FEQIS numerical scheme: more details

FEQIS is written in Fortran 90. In the code, equation (1) is solved on a rectangular grid (R,Z) (toroidal axis symmetry is assumed), using the well-known sine-transform method in the Z direction.

As for the plasma shape, once the flux map solution from (1) is obtained, the magnetic axis is first found by searching the O-point in the surrounding of the previous solution (an initial guess is usually given in the chamber center). In this way the value of ψ_a is set. Then, by a complete map swing, all the O and X-points (including multiple order X-points) are found. The O-points are discarded. The X-points are then tracked in time (using a 9-point bi-quadratic local X-point search) until they either merge, disappear, or new ones are born from the boundary (a boundary sweep is done at every equilibrium call). Once the boundary is found comparing X-points and the limiter position and its flux, the value of ψ_b is set. Now, the plasma current density is distributed inside the region (ψ_b – ψ_a), using a four-quadrant search method as it is also done in the SPIDER equilibrium solver [7].

Then, the boundary is passed to a fix-boundary Grad—Shafranov solver (directly included in FEQIS), that solves equation (1) on a flux grid inside the plasma boundary, but does only a few iterations to reach convergence in the flux surfaces with coarse resolution.

Structure of the circuit equations

The circuit equations are represented in matrix form as:

$$(2)\ L\frac{d\vec{I}}{dt}+R\vec{I}=\vec{V}+\frac{d\vec{\Psi}}{dt}$$

where L, R are inductance and resistance matrices, I is the vector of coil / eddy currents, V is the voltage vector (0 in the passive structures), and the last term is the variation of flux caused by the plasma onto the coils and passive elements.

The system (2) is solved implicitly in the variable I, using voltages and plasma term as known terms on the r.h.s.

The novelty in FEQIS is that the matrices L and R can change in time, in the sense that it is allowed to “re-wire” the circuits at any given time. This does not mean that L goes under time derivative, but that the present coil currents are re-calculated (using circuit equations laws) to be consistent with the new circuit schematics. Moreover, the circuit connections can be also nested, such that L and R are actually computed from the given coils geometry and connections, rather than assigned from outside.

- Rule FEQIS2: Equation (2) has differential time steps between the current term and the plasma term. Moreover, matrices L and R can change at any time (circuits re-wiring).

Applications to a DEMO equilibrium and its evolution

To display the status of FEQIS, an application is done on an equilbrium of the EU-DEMO reactor prototype [8]. The equilibrium is supposed to represent the flat-top burning phase at the nominal current and field (I_p = 17.1 MA, B_T = 4.79 T).

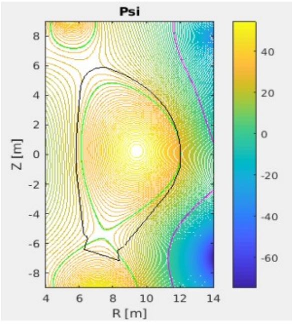

Figure 1: Initial equilibrium (green: plasma boundary)

Once the first free-boundary equilibrium is calculated, depicted in figure 1, the system is let free to evolve, setting all input voltages to the coils to 0. That is, the plasma position or shape are not actively controlled, only the passive response of the wall currents (eddy currents) is considered.

The evolution is subsequently characterised by an upward VDE. The simulated dynamic is compared between FEQIS and SPIDER in figure 2. The new code FEQIS predicts an initial equilibrium close to SPIDER, however the time evolution is hampered by numerical noise which is coming from the not yet fully optimized algorithm that searches for the boundary and assigns the new plasma current inside the boundary.

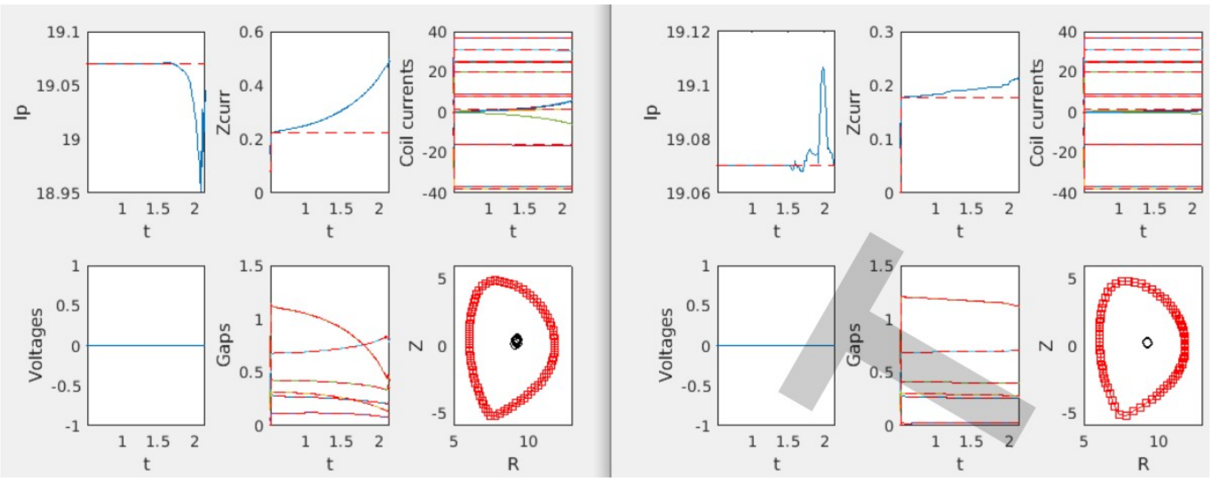

Figure 2: comparison between SPIDER (left) and FEQIS (right) for a natural VDE.

The next step is to solve these numerical issues and perform the same and more validation exercises before porting the code fully inside the Fenix flight simulator.

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