Properties, Collections, and the Successive Addition Argument: A Reply to Malpass

Abstract

The Successive Addition Argument (SAA) is one of the key arguments espoused by William Lane Craig for the thesis that the universe began to exist. Recently, Alex Malpass (2021) has developed a challenge to the SAA by way of constructing a counterexample that originates in the work of Fred Dretske. In this paper, I show that the Malpass-Dretske counterexample is in fact no counterexample to the argument. Utilizing a distinction between properties of members and properties of collections, I argue that Malpass' counterexample has no bearing on the soundness of the SAA. I also develop a novel parity argument against Malpass' argument that I demonstrate can only be resolved by way of the aforementioned analysis.

Keywords: Kalām; Successive Addition Argument; Infinity; Philosophy of Time; Cosmological Arguments for Theism

1. Introduction

The Kalām cosmological argument, as championed by William Lane Craig (1979), is an argument largely concerned with demonstrating that the temporal series of events cannot regress infinitely. One argument Craig offers in support of this thesis, as titled by Alex Malpass (2021), is the 'Successive Addition Argument', or the SAA. The SAA is stated by Craig (1979, p. 103) as follows:

(1) The temporal series of events is a collection formed by successive addition;
(2) A collection formed by successive addition cannot be an actual infinite;
(3) Therefore, the temporal series of events cannot be an actual infinite.

Malpass (2021) has recently produced a novel challenge to the SAA, based on the work of Dretske (1965). Malpass argues that there exists a clear counterexample to (2), and thus the argument is not sound. After investigating six potential objections Craig might offer, and finding them all wanting, Malpass concludes that his argument constitutes a genuine counterexample to (2), and thus that the SAA is unsound.

In this paper, I respond to Malpass on behalf of the SAA, arguing his counterexample does not render the argument unsound. My contention is that Malpass' argument is either invalid in virtue of a shift in scope, or else irrelevant to the truth of (2) in virtue of making a claim about the properties of members successively added, rather than the properties of collections formed by successive addition.

I will proceed as follows. In §2, I explicate Malpass' argument from his counterexample. In §3, I argue that Malpass' counterexample is either of an invalid form or else irrelevant to the truth of (2). In §4, I further this claim by developing a parity argument that can only be satisfactorily resolved by accepting the disjunction in §3. I conclude in §5.

2. Malpass' argument

Malpass begins building his counterexample to (2) by asking us to imagine a man named George who starts to count numbers at some time t.[1] George counts at a constant rate of one number per second, and never stops. Thus, each finite number n is such that George will count n. So, the cardinality of the numbers that George will count is just the cardinality of the natural numbers, $\aleph_0$. From this setup, Malpass (2021, p. 3) constructs the following argument:

(4) It is possible that George starts counting now and will never stop;
(5) If George starts counting now and will never stop, then for each natural number n, George will count n;
(6) If George will count each natural number, then George will count $\aleph_0$-many numbers;
(7) Therefore, it is possible that George will count $\aleph_0$-many numbers.

However, (7) appears to be a clear counterexample to (2). *Prima facie*, if it is possible that I will count x-many numbers, then it is also possible that I will form a collection by successive addition with x-many members. Thus, the fact that it is possible that George will count $\aleph_0$-many numbers means that George can form an actually infinite collection by successive addition. Yet, (2) is the claim that collections formed by successive addition cannot be actually infinite. So, Malpass' argument seems to undermine (2).

3. The scope shift fallacy

Malpass' argument has two critical premises. These are that:

(5) If George starts counting now and will never stop, then for each natural number n, George will count n;
(6) If George will count each natural number, then George will count $\aleph_0$-many numbers.

Both of these will be important in our discussion, and thus it is important to be clear on what they mean: especially the consequents. The consequent of (5) is fairly simple to formalize. Let G(n) abbreviate 'n is counted by George'. Then what Malpass has in mind in the consequent of (5) is that each n is such that there is a future time t at which n is counted by George. In terms of the tense operator 'F' for 'it will at some future time t be the case that', this can be stated as:

(8) $(\forall n)\ F(G(n))$

[1] The setup for this counterexample originates in Dretske (1965).

The antecedent of (6) is the same. But what about the consequent of (6)? Here things are not so clear. It's not immediately obvious what Malpass means by the phrase 'George will count $\aleph_0$-many numbers'. On the one hand, if we take 'it will be that ϕ' to mean that 'there is a future time t such that ϕ', the consequent of (6) can be read as the claim that 'there is a future time t at which George counts $\aleph_0$-many numbers'. More formally,

(9) $F\ (\forall n)(G(n))$

But this cannot be the correct analysis. For Malpass explicitly notes that there is *no time* at which George counts $\aleph_0$-many numbers, as his counting literally 'takes up all the time in the world' (2021, p. 15). Furthermore, moving from (8) to (9) blatantly commits a scope-shift fallacy, and Malpass notes that 'if the Dretske argument involved a shift like this it would be bad news' (Malpass 2021, p. 7). Thus, (9) cannot be what Malpass has in mind in the consequent of (6).

What, then, does (6) amount to? Another analysis of (6) shows itself if we examine how the premise is defended. Malpass notes that (6) is an application of the following broader rule of inference:

> *(R)* If each element in a set S has property P, and the cardinality of the elements of S is X, then the cardinality of the elements that are P is also [at least] X. (2021, p. 8)

(R), in the context of (6), is applied as such: if each natural number in the set of natural numbers instantiates the property 'will be counted by George' and the cardinality of the set of natural numbers is $\aleph_0$, then the cardinality of the natural numbers that instantiate the property 'will be counted by George' is also at least $\aleph_0$. As Malpass says: 'if each natural number will be counted, and there are $\aleph_0$-many natural numbers, then $\aleph_0$-many natural numbers will be counted' (2021, p. 15).

So, it appears that when Malpass affirms that the cardinality of the natural numbers that will be counted by George is also $\aleph_0$, Malpass is not falsely claiming that there is some future time where George counts $\aleph_0$-many natural numbers. Rather, what is being said is that there exist $\aleph_0$-many natural numbers each of which instantiates the property 'will be counted by George'. Otherwise, both (6) and the broader rule *(R)* are clearly faulty.

Thus, let us suppose the proper reading of (6) is just that there are $\aleph_0$-many natural numbers instantiating this property. If this is what is meant by Malpass, then his argument faces a serious problem. Namely, (6) is not telling us anything importantly *over and above* what is said by (5), the premise that each number is such that George will count it. Indeed, it seems all (6) tells us that is not already contained in (5) is that there exist $\aleph_0$-many such numbers.

But if all (6) does in the context of Malpass' argument is alter the relevant quantification—such that we are entitled to the further claim that there are $\aleph_0$-many numbers instantiating the property 'will be counted by George'—then we are left wondering how the argument can be considered in any sense a counterexample to (2). Recall that (2), the initial premise Malpass is attempting to challenge, is the claim that no collection formed by successive addition can be actually infinite. What (2) amounts to is *not* the claim that there cannot be an actually infinite number of *x*'s such that each *x* instantiates the property 'is successively added'. This claim is demonstrably false. *Any* actually infinite number of elements is such that each element instantiates the property 'will be successively added' (provided the elements occur at future times). Instead, (2) must be understood as the stronger claim that there cannot be a *collection* of an actually infinite number of *x*'s that instantiates the property 'is formed by successive addition'.

This should not come as a surprise—after all, Craig is talking about collections and what properties we may ascribe to them throughout the SAA. (1) is the claim that the temporal series of events is a *collection* that is formed by successive addition. (2) is the claim that such *collections* cannot be actually infinite. His conclusion is that the *collection* of the temporal series of events is not actually infinite.

Thus, a counterexample to (2) cannot merely be one where $\aleph_0$-many *x*'s each instantiate the property 'is successively added'—rather, it must be one where a collection of $\aleph_0$-many *x*'s is formed by successive addition. One might rightfully ask at this point what it means to say of a collection that *it* is formed by successive addition. To my mind, it is just our earlier (9): a collection of *x*'s is formed by successive addition just in case there is a future time t at which, for all *x*'s in the collection, *x* has been successively added. Craig is claiming that a certain *collection*, namely, the temporal series of events, cannot have a certain *property*, namely, being formed by successive addition, whilst having a cardinality of $\aleph_0$. What Malpass' argument shows (if it is not interpreted to be invalid) is that it is indeed possible for $\aleph_0$-many *x*'s to instantiate this property. That, though, is perfectly compatible with the SAA.

To defeat the claim that a collection that is formed by successive addition cannot be actually infinite it does not suffice to give an example of $\aleph_0$-many *x*'s that each are successively added. One must give an example of a collection of $\aleph_0$-many members which is formed by successive addition. That George will count $\aleph_0$-many numbers, in the sense that $\aleph_0$-many numbers instantiate the property 'will be counted by George', entails that it is possible that there are $\aleph_0$-many elements that instantiate the property 'will be successively added'. But that is all it entails. It certainly does not entail that a *collection* with $\aleph_0$-many members instantiates the property 'will be formed by successive addition'. Put simply, if Craig is arguing that *collections* cannot exemplify certain properties, a proper counterexample must be one where a collection exemplifies those properties. No premise in (4)–(7) entails that any collection has any such property.

Thus, the problem for Malpass' argument is disjunctive: either it is invalid in virtue of moving from a claim about the properties of members to a claim about the properties of collections, or else falls short of the mark of a proper counterexample to (2) because it is concerned *only* with a claim about the properties of members.

4. A parity

Malpass' argument relies crucially on the claim that (7) is a counterexample to the premise that a collection formed by successive addition cannot be an actual infinite. To put more forcefully the idea that Malpass' counterexample is not relevant to the truth of (2), consider how one would argue against the more evidently false claim that it is possible that the numbers George *will have* counted are actually infinite. Indeed, Malpass agrees that 'it is false that George will have counted infinitely many numbers' and makes it explicit that 'George will not *have counted* every number' (2021, pp. 12, 14). So, the following is a claim Malpass must accept:

(C) The numbers that George *will have* counted cannot be an actual infinite.

But now imagine that someone attempts to challenge (C) by way of the following familiar line of argumentation:

(4*) It is possible that George starts counting now and will never stop;
(5*) If George starts counting now and will never stop, then for each natural number n, George *will have* counted n;
(6*) If George *will have* counted each natural number, then George *will have* counted $\aleph_0$-many numbers;
(7*) Therefore, it is possible that George *will have* counted $\aleph_0$-many numbers.

(4*)–(7*), despite paralleling (4)–(7), form an unpalatable argument for Malpass, as (7*) is agreed to be blatantly false. And it also seems as though the guilty premise in this argument is (6*)—it does not follow from the mere fact that each number *will have been* counted by George that George *will have* counted $\aleph_0$-many numbers. But if this is correct, then (6*) and (6) employ the same faulty form of inference, and so Malpass' own argument is unsound.

However, like Malpass, the defender of (6*) can claim that the phrase 'George will have counted $\aleph_0$-many numbers' simply means that $\aleph_0$-many numbers instantiate the property of 'will have been counted by George'.

The obvious response here is that if (6*) is to be understood as such, (7*) will not constitute a relevant counterexample to (C): that each natural number instantiates the property 'will have been counted by George', such that there are $\aleph_0$-many numbers with this property, does not undermine the fact that the numbers that George *will have counted* cannot be actually infinite. Again, this is because what is meant by 'the numbers that George *will have* counted cannot be actually infinite' is that there is no time where the *collection* of numbers George has formed by his counting is actually infinite. Pointing out that each natural number is such that *it* will have been counted is irrelevant.

It might be objected here that I have treated the sentences 'George will count $\aleph_0$-many numbers' and 'George will have counted $\aleph_0$-many numbers' as broadly symmetrical: under either interpretation they will both be false or both be true. However, perhaps this is problematic, since Malpass understands these phrases to be asymmetric:

> The reason George *will* count every number, but George will not *have counted* every number, isn't just that there are infinitely many numbers; it is because there is no point in time which follows his counting. (Malpass 2021, pp. 14-15, emphasis in original)

Dretske concurs:

> It is true that at any stage of his task George will not yet have counted some numbers. But, clearly this fact is not relevant to whether he *will* count to infinity; it only shows that he never *will have* counted to infinity. (Dretske 1965, p. 100, emphasis in original)

But in the context of (4)–(7) and (4*)–(7*), whence the difference between the fact that George *will* count every number and the fact that George *will have* counted every number? If Malpass thinks that the latter is false because there is no time *following* George's counting, it can equally be said that the former is false because there is no time *terminating* George's counting, and so it is not the case that George will count every number. Unless, of course, what Malpass has in mind is that there need be no time *terminating* George's counting: it suffices that there are infinitely many numbers, each of which instantiates the property 'will be counted by George'. Yet, again, by the same token it can be said that there need be no time *following* George's counting: it suffices that there are infinitely many numbers, each of which instantiates the property 'will have been counted by George'. The point here is that Malpass is faced with two possible options with respect to (6) and (6*): either they make the fallaciously-motivated claim that there is a time at which George counts (or will have counted) $\aleph_0$-many numbers, or they are simply claiming that there are $\aleph_0$-many numbers that each instantiate the property 'will be counted by George' or 'will have been counted by George'.

The salient point here is that Malpass cannot consistently hold that it is not possible that George *will have* counted $\aleph_0$-many numbers and that (4)–(7) successfully defeat the SAA. I have proposed a disjunctive remedy: either both (6*) and (6) are false, or both (7*) and (7) are not relevant counterexamples.

5. Conclusion

In total, then, the SAA is not defeated by Malpass' argument. As outlined herein, the crucial third premise of the argument—that if George will count every number then he will count $\aleph_0$-many numbers—can either be read such that it is a fallacious rule of inference, or otherwise facilitates a conclusion that is no way a counterexample to (2). By my lights, this provides ample reason for defenders of the SAA to reject Malpass' counterexample to (2).

References

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