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Optimizing a Time-Sensitive Supply Chain with a Power Function Penalty Cost

Jinyu Yang[1] and Xiaoyu Xing[2]
[1]MS Candidate, School of Science, Hebei University of Technology, Beichen district, Tianjin, 300401, China, E-mail: 1781839033@qq.com
[2]Professor, School of Science, Hebei University of Technology, Beichen district, Tianjin, 300401, China, E-mail: xingxiaoyutj@163.com (corresponding author).

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Abstract: In a cost-based supply chain delivery performance model, accurately depicting the penalty cost associated with an untimely delivery is an important component of the supply chain's delivery performance. This paper introduces a two-stage time-sensitive supply chain, where the penalty cost caused by early and late delivery is treated as a power function of the default time. This default time is the deviation in time from the actual delivery time to the delivery window. Both the buyer and the supplier want to reduce the default time. After considering the product's value over time and the buyers' attitude with respect to a breach of contract, the penalty cost is constructed as the product of a nonlinear function of default time and as a linear function of the penalty factor. This yields the conditions associated with the optimal delivery window, to minimize the expected penalty with the power function. The paper shows the effect of the delivery window, penalty factor, and time sensitivity factor on supply chain performance, leading to recommendations for improved performance. Numerical results are provided to demonstrate the applicability of the proposed model. The model helps buyers more accurately determine costly losses, enabling them to reduce the impact of any default caused by suppliers. For exponentially distributed lead time, choosing the optimal delivery window position for delivery can reduce the expected penalty cost by as much as seven times. The average delivery time is shortened from 30 days to 20 days, and the expected penalty cost will be reduced by about 50%. This is a powerful tool for use in improving supply chain performance and also provides directions for improving strategies and a theoretical basis for buyers and suppliers wanting to jointly optimize supply chain performance.

Keywords: Supply chain management, power function penalty cost, delivery performance, delivery window.

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1. Introduction

Performance measurement has been recognized by many scholars for its important role in supply chain management and operations (Zakir et al., 2023a). Measuring delivery performance in a supply chain can quantify the reliability of the supply chain and provide a basis for management's decision-making. In order to improve supply chain performance and achieve common interests, some partners introduce external resources to form inter-organizational systems (IOS) (Asamoah et al., 2021). In the performance evaluation system, fuzzy logic can be used to deal with the incomplete and qualitative information contained in the supply chain system (Qorri et al., 2022).

In the overall process of the supply chain, variability exists in any link; accurately controlling variability is difficult for any senior manager (Talay and Zdemir-Akyldrm, 2018). In actual production, advanced equipment can be replaced, and advanced technology can be used to reduce the uncertainty of production time (Zhai et al., 2020). The delivery window provides a buffer period for various uncertainties and a reasonable time frame for buyers and sellers to ensure the on-time delivery of products. Abdelsadek and Kacem (2022) used an interactive information visualization decision support tool to improve delivery problems with delivery window constraints. Some algorithms, such as genetic algorithms (Ongcunaruk et al., 2021), Tabu Search (Gmira et al., 2021), Ant Colony (Neogi et al., 2018), etc., have been applied to optimize cost optimization problems involving time windows, and these algorithms have achieved good results.

The costs associated with untimely (early and late) delivery are referred to as penalty costs, which are paid by the supplier to the buyer. Some industries have very strict time requirements. For example, clothing and luxury goods industries generally charge high liquidated damages if there is a breach of contract (Roy and Sarker, 2021). Buyers have specific attitudes with respect to breaches of contract, leading to penalties as well as cost losses. This situation is not limited to the luxury goods industry; it extends across all types of buyers. The one-size-fits-all nature of the penalty function is designed to cover a wide range of products. However, suppliers and buyers generally want to combine the unique characteristics of the product itself and the buyers' attitudes towards breach of contract to customize the penalty function. This paper applies a power function delivery penalty cost model to emphasize the characteristics of product lifecycles and attitudes towards a breach of contract. A power function penalty gives the buyer flexibility, by altering the power index to reflect both their attitudes towards time and the associated losses incurred by untimely delivery. To summarize, the contributions of this paper are as follows:

- The time preference is introduced into the two-stage supply chain in the form of the power function of the penalty cost, and the exact expression of the expected penalty in the time-sensitive supply chain is given.
- Analytical expressions are gained for the optimal position of the delivery window to minimize the supply chain's expected penalty cost.
- This paper analyses the impact of the delivery window, the penalty factor for early and late delivery, the ratio of the penalty factor for early delivery and the penalty factor for late delivery, and the time preference factor on the optimal window position and the minimum expected penalty.

2. Literature Review

A supply chain performance system uses multi-company data integration and process performance metrics to achieve supply chain management (Zakir et al., 2023b). In industrial practice, circular economy (CE) indicators are used to evaluate circular supply chain performance (Calzolari et al., 2021). In food supply chains, among the 34 sustainability indicators analyzed, production accounted for the largest number (up to 17) of tools and quantitative indicators (Desiderio et al., 2021). Due to the increasing degree of globalization and the changes in customer demand, the supply chain has become quite complex. Meta-analysis can be used to explain the impact of supply chain complexity on performance (Ates et al., 2021).

Delivery performance is a key supply chain operations metric in every performance indicator hierarchy; this highlights the high priority of measuring delivery performance (Karamouz et al., 2020). Meng and Qian (2018) designed a blockchain framework called DelivChain for use as a real-time predictive delivery performance metric. Jun and Lee (2022) solved the pickup and delivery problem with time windows, using the evolutionary neural network. The goal here was to minimize total tardiness. However, Aki et al. (2018) emphasized that collaboration performance metrics (such as early or late delivery penalties) often lead to suboptimal results in the two-way communication process, due to the difficulty of meeting the technical prerequisites.

A supplier's policy for improving supply chain performance is motivated by the goal of reducing financial expenses. Shortening the lead time is an effective way of improving supply chain performance, and can increase the accuracy of demand forecasting, thus helping to reduce unnecessary inventory surplus and production capacity waste (Ed et al., 2019). Roy and Sana (2021) devised an iterative algorithm to solve the optimal solution of the revised expected total cost. The study showed that shortening the lead time can significantly reduce the total cost. Supply chain performance can also be improved if the suppliers can optimize production structures (Terzioglu et al., 2022). The widely-used Gaussian distribution has been criticized for its symmetric distribution and negative delivery times. This has led to the proposed use of approaches such as the gamma distribution, asymmetric Laplace distribution, and exponential distribution (Bushuev and Guiffrida, 2012; Roy and Sarker, 2021; Tao et al., 2021). Variance optimization performed using Gaussian distribution has also been viewed as a primary measure to increase delivery time accuracy and supply chain stability. This method can also be used to optimize the supply chain for other distribution approaches (Bushuev, 2018).

On-time delivery significantly impacts customer satisfaction and delivery reliability. The rapid development of the logistics industry has led customers in the decentralized supply chain to have high requirements for timely deliveries (Choi et al., 2018). The global COVID-19 pandemic has created severe challenges for global supply chains (GSCs) (Sarkis et al., 2020). The rapid responsiveness of GSCs enables companies to quickly recover and remain competitive in the face of sudden disruptions (Nayeri et al., 2022). Kazancoglu et al. (2022), using a partial least squares (PLS) method, analyzed 200 responses from companies, emphasizing the importance of rapid responsiveness in GSCs.

Actual delivery time is considered random. A delivery may miss the promised target time, resulting in a penalty for non-contract delivery (Chevroton et al., 2021). An on-time delivery made within an agreed-upon window does not incur loss costs. To study this cost-driving problem, several researchers have developed models with a deterministic lead time (Han et al., 2019) and controllable lead time (Dey et al., 2021). Sajadieh and Jokar (2009) relaxed control of the lead time and expanded it to be an uncertain lead time; this approach is also a key method used in this study. Majumder et al. (2018) used classical optimization techniques to minimize the joint expected cost of the supply chain system in the context of normally-distributed lead time. Li (2020) obtained the exact total cost equation, using the normally distributed form of lead time.

Untimely deliveries, including early and late deliveries, often generate losses for the supplier; these are called penalty cost. Previous studies have generally considered the penalty cost to be a linear function of time. However, this assumption of a linear penalty may underestimate the potential loss incurred due to untimely delivery. For example, late delivery is the most common type of untimely delivery. The consequence of late deliveries includes stock-out, declining sales, poor customer experience, and loss of future orders (Makinde and Munyai, 2020). Niemi et al. (2020) found that a 15-day delay led to an estimated 6% of lost sales, while a 45-day delay of 6.9% of total deliveries resulted in an estimated 30% of lost sales. In most studies, given that an early delivery increases the buyer's inventory cost, the penalty function is assumed to be linear in time and volume. In reality, costs extend far beyond this, as there may also be product damage, additional human resources required to reconcile inventories, and other costs. In the just-in-time (JIT) truck routing problem, where an excessively early or late delivery can impose an extremely high penalty, the large neighborhood search (LNS) metaheuristic method was developed to minimize the total earliness-tardiness cost (Baals et al., 2022). In the digital restaurant industry, static threshold policies are used to manage the queue problem, as both early and late fulfillment of orders will cause customer dissatisfaction (Farahani et al., 2022). Given the above, applying a power function penalty cost to reflect the real numerous costs caused by early and late deliveries is reasonable.

2. Model Settings and Problem Formulations

2.1. Model Settings and Optimum Position of the Delivery Window

This paper evaluates supply chain delivery performance using a time-sensitive penalty cost with a power function model. Before describing the model, it is important to state the following assumptions about the supply chain. There is: (1) only one supplier and one buyer; (2) a single product with a fixed delivery lot; and (3) a one-time delivery without replenishment or chargeback.

Penalties are closely related to time and penalty coefficients in the supply chain. The supplier's actual delivery time x usually follows a specific distribution, such as a uniform distribution, normal distribution, or exponential distribution. The delivery window Δc represents the acceptable interval of time for the delivery, during which the shipment does not generate a penalty. The start time of the delivery window is recorded as c_1, with deliveries before this window which a delivery generates an early delivery penalty cost. Delivery after the delivery tolerance period $c_1 + \Delta c$ generates a late delivery penalty cost.

Both penalties increase as the distance between the actual delivery time and the delivery window increases. However, different buyers have different levels of sensitivity to time, as well as to the interval of time between early delivery and late delivery. The power function is used to express the buyer's sensitivity to default. The length of time for early delivery is expressed as $(c_1 - x)^+$, so the lead time sensitive function is $[(c_1 - x)^+]^p$, where p is the lead time preference coefficient with $p \geq 1$. The delay time sensitive function is expressed as $[(x - c_1 - \Delta c)^+]^q$, where q is the delay time preference coefficient with $q \geq 1$. As such, the total penalty in the time-sensitive two-stage supply chain is:

$$Y(c_1, x) = M \cdot [(c_1 - x)^+]^p + N \cdot [(x - c_1 - \Delta c)^+]^q \quad (1)$$

In Eq. (1) M is the penalty factor for early delivery, and N is the penalty factor for late delivery. If $p = 1$, $q = 1$, and this model degenerates to a traditional two-stage serial supply model, which was studied by Bushuev and Guiffrida (2012).

Supply chain performance is measured based on the expectation of a penalty; this is also widely assumed in supply chain problems. The time-sensitive expected penalty cost in a two-stage supply chain is expressed as:

$$Y = E[Y(c_1, x)] = \int_0^{c_1} M \cdot (c_1 - x)^p f(x)dx + \int_{c_1+\Delta c}^{\infty} N \cdot (x - c_1 - \Delta c)^q f(x)dx \quad (2)$$

In Eq. (2), $E[\cdot]$ is the expectation operator, and $f(x)$ is the probability density function for the distribution of delivery time x.

In general, different buyers have different levels of sensitivity with respect to time. For many buyers, however, there is an increase in the marginal loss as the delay time in late delivery lengthens. In other words, the penalty for the last unit of delay time should be more than the previous unit of delay time. For example, if the delivery time is three days later than the delivery window deadline, then the penalty generated on the third day should be more than the penalty on the second day. This is because the loss created from a default is usually not merely linear with respect to time; rather, it sets off a chain reaction. The penalty may also include a punishment for the suppliers' breach of contract. Similarly, there may be an increase in the marginal penalty for early delivery. The choice of these two sensitive functions depends on the default's impact on the buyer, as well as the buyer's attitude toward the default.

The expected penalty function depends on M, N, c, and $f(x)$. In general, these parameters cannot be arbitrarily changed. The delivery window and penalty factor are stipulated by the contract, and the parameters related to the distribution of delivery time cannot be changed in the short term. Therefore, the only decision variable is c_1. This variable can be changed by the supplier, to decrease the total expected penalty in the supply chain. Previous approaches have set the penalty costs of early and late delivery as being linearly related to default time. This study posits that this approach is imperfect, and instead use a time-sensitive penalty cost function to incorporate both parties' preferences for time and tolerance for non-on-time delivery. The goal is to make the transaction fairer, by giving buyers and suppliers more flexibility, taking care of both parties, and providing more choices when signing the contract. The supplier's objective is to find the optimal position of delivery window c_1^* to minimize the expected penalty Y.

Theorem 2.1 The expected penalty Y is convex with respect to c_1, which in turn represents the start of the on-time delivery window. Then, the optimal position of the delivery window c_1^* minimizing Y should satisfy:

$$\frac{M}{N} = \frac{q \int_{c_1^*+\Delta c}^{\infty} (x - c_1^* - \Delta c)^{q-1} f(x)dx}{p \int_0^{c_1^*} (c_1^* - x)^{p-1} f(x)dx} \quad (3)$$

Proof For $p > 1$, $q > 1$, taking the first-order partial derivative of Y with respect to c_1 yields Eq. (4):

$$\frac{dY}{dc_1} = Mp \int_0^{c_1} (c_1 - x)^{p-1} f(x)dx - Nq \int_{c_1+\Delta c}^{\infty} (x - c_1 - \Delta c)^{q-1} f(x)dx \quad (4)$$

Taking the second-order partial derivative of Y with respect to c_1 gets Eq. (5):

$$\frac{d^2Y}{dc_1^2} = Mp(p-1) \int_0^{c_1} (c_1 - x)^{p-2} f(x)dx + Nq(q-1) \int_{c_1+\Delta c}^{\infty} (x - c_1 - \Delta c)^{q-2} f(x)dx > 0 \quad (5)$$

Here, Eq. (5) implies that Y is convex with respect to c_1. The optimal position of the delivery window c_1^* satisfies the condition that Eq. (4) equals 0. Simplifying $\frac{dY}{dc_1} = 0$ yields Eq. (3). Eq. (3) remains true even when at least one of p and q is 1.

When the delivery parameters are determined, the supplier can minimize the expected penalty by determining the optimal position of the delivery window using Theorem

2.1. This theorem can be applied across different distribution functions. At the same time, $\frac{M}{N}$ emerges as a decisive factor in determining the value of c_1^*. This is illustrated in Proposition 2.3.

2.2. Effect of Parameters

This section investigates the impact of each parameter of penalty on c_1^* and Y^*. This helps both suppliers and buyers determine the direction of negotiation and determine the best trade-off between delivery reliability and penalties. Suppliers and buyers have opposite preferences for some parameters. For example, a tight delivery window is appealing to the buyer, but is associated with a higher probability of delay and tardiness penalties. A wide delivery window gives the supplier more flexibility to arrange both production and delivery, but harms the buyer with a high demand for time and incurs excess cost.

This paper focuses on the simple case of the impact of a single parameter change on the optimal position of the delivery window and the expected penalty. This paper does not present a correlation analysis between variables, as the complexity of that approach is beyond the scope of this paper.

Proposition 2.1 Given that M, N, p, q, and $f(x)$ are fixed, increasing the width of the delivery window Δc decreases the optimal delivery window position c_1^* and the minimum total expected penalty cost Y^*.

Proof The optimal position of the delivery window c_1^* satisfies the following condition:

$$Mp\int_0^{c_1^*}(c_1^*-x)^{p-1}f(x)dx - Nq\int_{c_1^*+\Delta c}^{\infty}(x-c_1^*-\Delta c)^{q-1}f(x)dx = 0 \quad (6)$$

Calculating the derivative on both sides of Eq. (6) with respect to Δc yields the following results:

Case 1. $p = 1, q = 1,$

$$Mf(c_1^*)\cdot\frac{dc_1^*}{d\Delta c} + Nf(c_1^*+\Delta c)\left(\frac{dc_1^*}{d\Delta c}+1\right) = 0 \quad (7)$$

Eq. (7) yields Eq. (8):

$$\frac{dc_1^*}{d\Delta c} = -\frac{Nf(c_1^*+\Delta c)}{Mf(\ _1^*)+Nf(\ _1^*+\Delta c)} < 0 \quad (8)$$

Case 2. $p = 1, q > 1,$

$$Mf(c_1^*)\cdot\frac{dc_1^*}{d\Delta c} + Nq(q-1)\int_{c_1^*+\Delta c}^{\infty}(x-c_1^*-\Delta c)^{q-2}f(x)dx\cdot\left(\frac{dc_1^*}{d\Delta c}+1\right) = 0 \quad (9)$$

Eq. (9) yields Eq. (10):

$$\frac{dc_1^*}{d\Delta c} = -\frac{Nq(q-1)\int_{c_1^*+\Delta c}^{\infty}(x-c_1^*-\Delta c)^{q-2}f(x)dx}{Mf(c_1^*)+Nq(q-1)\int_{c_1^*+\Delta c}^{\infty}(x-c_1^*-\Delta c)^{q-2}f(x)dx} < 0 \quad (10)$$

Case 3. $p > 1, q = 1,$

$$Mp(p-1)\int_0^{c_1^*}(c_1^*-x)^{p-2}f(x)dx\cdot\frac{dc_1^*}{d\Delta c} + Nf(c_1^*+\Delta c)\left(\frac{dc_1^*}{d\Delta c}+1\right) = 0 \quad (11)$$

Eq. (11) yields Eq. (12):

$$\frac{dc_1^*}{d\Delta c} = -\frac{Nf(c_1^*+\Delta c)}{Mp(p-1)\int_0^{c_1^*}(c_1^*-x)^{p-2}f(x)dx\cdot\frac{dc_1^*}{d\Delta c}+Nf(c_1^*+\Delta c)} < 0 \quad (12)$$

Case 4. $p > 1, q > 1,$

$$Mp(p-1)\int_0^{c_1^*}(c_1^*-x)^{p-2}f(x)dx\cdot\frac{dc_1^*}{d\Delta c} + Nq(q-1)\int_{c_1^*+\Delta c}^{\infty}(x-c_1^*-\Delta c)^{q-2}f(x)dx\cdot\left(\frac{dc_1^*}{d\Delta c}+1\right) = 0 \quad (13)$$

Eq. (13) yields Eq. (14):

$$\frac{dc_1^*}{d\Delta c} = -\left[Nq(q-1)\int_{c_1^*+\Delta c}^{\infty}(x-c_1^*-\Delta c)^{q-2}f(x)dx\right] \Big/ \left[Mp(p-1)\int_0^{c_1^*}(c_1^*-x)^{p-1}f(x)dx + Nq\int_{c_1^*+\Delta c}^{\infty}(x-c_1^*-\Delta c)^{q-1}f(x)dx\right] < 0 \quad (14)$$

All four cases indicate that $\frac{dc_1^*}{d\Delta c} < 0$ .The optimal position of the delivery window c_1^* decreases in line with Δc. The wider the delivery window is, the more forward is the optimal position.

The next step is to assess the effect of Δc on the optimal value function Y^*. Applying the envelope theorem indicates that the partial derivative of Y with respect to Δc is:

$$\frac{\partial Y}{\partial \Delta c} = -Nq\int_{c_1+\Delta c}^{\infty}(x-c_1-\Delta c)^{q-1}f(x)dx \quad (15)$$

Eq. (15) yields:

$$\frac{dY^*}{d\Delta c} = \frac{\partial Y_1}{\partial \Delta c}\{c_1 = c_1^*(\Delta c)\} = -Nq\int_{c_1^*+\Delta c}^{\infty}(x-c_1^*-\Delta c)^{q-1}f(x)dx < 0 \quad (16)$$

Eq. (16) indicates that Y^* decreases as the width of the delivery window Δc narrows.

Proposition 2.2 (1) With N, p, q, $f(x)$, and Δc fixed, an increase in the early delivery penalty factor M increases the optimal position c_1^* and leads Y^* to increase. (2) With $M, p, q, f(x)$, and Δc fixed, an increase in the late delivery penalty factor K decreases the optimal position c_1^* and leads Y to increase.

Proof Calculating the derivation on both sides of Eq. (6) with respect to M (or N) yields the following results (Eqs. (17)-(24)):

Case 1. $p = 1, q = 1,$

$$\frac{dc_1^*}{d(M)} = -\frac{\int_0^{c_1}f(x)dx}{Mf(c_1^*)+Nf(\ _1^*+\Delta c)} < 0 \quad (17)$$

$$\frac{dc_1^*}{dN} = \frac{\int_{c_1^*+\Delta c}^{\infty}f(x)dx}{Mf(c_1^*)+Nf(c_1^*+\Delta c)} > 0 \quad (18)$$

Case 2. $p = 1, q > 1,$

$$\frac{dc_1^*}{d(M)} = -\frac{\int_{c_1^*+\Delta c}^{\infty}f(x)dx}{Mf(c_1^*)+Nq(q-1)\int_{c_1^*+\Delta c}^{\infty}(x-c_1^*-\Delta c)^{q-2}f(x)dx} < 0 \quad (19)$$

$$\frac{dc_1^*}{dN} = \frac{q\int_{c_1^*+\Delta c}^{\infty}(x-c_1^*-\Delta c)^{q-1}f(x)dx}{Mf(c_1^*)+Nq(q-1)\int_{c_1^*+\Delta c}^{\infty}(x-c_1^*-\Delta c)^{q-2}f(x)dx} > 0 \quad (20)$$

Case 3. $p > 1, q = 1,$

$$\frac{dc_1^*}{d(M)} = -\frac{p\int_0^{c_1}(c_1-x)^{p-1}f(x)dx}{Mp(p-1)\int_0^{c_1}(c_1-x)^{p-2}f(x)dx+Nf\ \ _1^*+\Delta c)} < 0 \quad (21)$$

$$\frac{dc_1^*}{dN} = \frac{\int_{c_1^*+\Delta c}^{\infty}f(x)dx}{Mp(p-1)\int_0^{c_1^*}(c_1^*-x)^{p-2}f(x)dx+Nf\ \ _1^*+\Delta c)} > 0 \quad (22)$$

Case 4. $p > 1, q > 1,$

$$\frac{dc_1^*}{d(M)} = -\left[p\int_0^{c_1}(c_1-x)^{p-1}f(x)dx\right]\Big/\left[Mp(p-1)\int_0^{c_1}(c_1-x)^{p-2}f(x)dx + Nq(q-1)\int_{c_1^*+\Delta c}^{\infty}(x-c_1^*-\Delta c)^{q-2}f(x)dx\right] < 0 \quad (23)$$

$$\frac{dc_1^*}{dN} = \left[q\int_{c_1^*+\Delta c}^{\infty}(x-c_1^*-\Delta c)^{q-1}f(x)dx\right]\Big/\left[Mp(p-1)\int_0^{c_1}(c_1-x)^{p-2}f(x)dx + Nq(q-1)\int_{c_1^*+\Delta c}^{\infty}(x-c_1^*-\Delta c)^{q-2}f(x)dx\right] > 0 \quad (24)$$

All four cases show that $\frac{dc_1^*}{d(M)} < 0$ and $\frac{dc_1^*}{dN} > 0$; that is, as the early (late) delivery penalty factor M (N) increases, the optimal delivery window position c_1^* decreases (increases).

To assess the effect of M (N) on Y^*, applying the envelope theorem yields:

$$\frac{dY^*}{d(M)} = \frac{\partial Y_1}{\partial(M)}\{c_1 = c_1^*(M)\} = \int_0^{c_1^*}(c_1^*-x)^p f(x)dx > 0 \quad (25)$$

$$\frac{dY^*}{dN} = \frac{\partial Y_1}{\partial N}\{c_1 = c_1^*(N)\} = \int_{c_1^*}^{\infty}(x-c_1^*-\Delta c)^q f(x)dx > 0 \quad (26)$$

Eq. (25)-(26) shows that the minimal expected penalty Y^* increases whenever M or N increases.

It is important to carefully observe the conditions satisfied by the optimal location of the delivery window in Theorem 2.1. When the ratio of M and N is determined, the optimal position of the delivery window is also determined. Proposition 2.2 indicates that increasing the value of M decreases c_1^*, and increasing the value of N increases c_1^*. However, when M and N are doubled at the same time, c_1^* does not change. This yields the following proposition:

Proposition 2.3 When p, q, and $f(x)$ are fixed, increasing $\frac{M}{N}$ increases the optimal position of the delivery window c_1^*.

Proof For the convenience of calculation, the conditional expression Eq. (7), satisfied by c_1^*, is transformed into Eq. (27):

$$\frac{M}{N}p\int_0^{c_1^*}(c_1^*-x)^{p-1}f(x)dx - q\int_{c_1^*+\Delta c}^{\infty}(x-c_1^*-\Delta c)^{q-1}f(x)dx = 0 \quad (27)$$

The process and calculation of this proof are similar to Proposition 2.2. As such, only the results are showed here. First, simultaneously calculate the derivation on both sides of Eq. (7) with respect to $\frac{M}{N}$. Then, continue to analyze the four cases.

When $p = 1, q = 1$, there is:

$$\frac{dc_1^*}{d\frac{M}{N}} = -\frac{\int_0^{c_1}f(x)dx}{\frac{M}{N}f(c_1^*)+f(c_1^*+\Delta c)} < 0 \quad (28)$$

When $p = 1, q > 1$, there is:

$$\frac{dc_1^*}{d\frac{M}{N}} = -\frac{\int_{c_1^*+\Delta c}^{\infty}f(x)dx}{\frac{M}{N}f(c_1^*)+q(q-1)\int_{c_1^*+\Delta c}^{\infty}(x-c_1^*-\Delta c)^{q-2}f(x)dx} < 0 \quad (29)$$

When $p > 1, q = 1$, there is:

$$\frac{dc_1^*}{d\frac{M}{N}} = -\frac{p\int_0^{c_1}(c_1-x)^{p-1}f(x)dx}{\frac{M}{N}p(p-1)\int_0^{c_1}(c_1-x)^{p-2}f(x)dx+f(\ {}_1^*+\Delta c)} < 0 \quad (30)$$

When $p > 1$, $q > 1$, there is:

$$\frac{dc_1^*}{d\frac{QH}{K}} = -\frac{p\int_0^{c_1}(c_1-x)^{p-1}f(x)dx}{\frac{M}{N}p(p-1)\int_0^{c_1}(c_1-x)^{p-2}f(x)dx+q(q-1)\int_{c_1^*+\Delta c}^{\infty}(x-c_1^*-\Delta c)^{q-2}f(x)dx} < 0 \quad (31)$$

Eqs. (28)-(31) take the same form as Eqs. (17)-(24) in Proposition 2.2, with only the coefficients changing. In these four cases, there are $\frac{dc_1^*}{d\frac{M}{N}} < 0$. This leads to the conclusion that as the ratio of M and N increases, the optimal delivery window position increases.

This proposition emphasizes that determining the ratio of M to N also determines the optimal position of the delivery window c_1^*. The buyer can indirectly adjust the delivery time by changing the ratio of the two parameters. For example, if the buyer wants the goods to arrive as early as possible, the buyer can lower the value of $\frac{M}{N}$. Based on Propositions 2.2 and 2.3, one can also decrease the value of the early delivery penalty coefficient M or increase the value of the late delivery penalty parameter N. This method can be used to decrease c_1^*. These two cases are special cases involving a decrease in the ratio of M to N. It is feasible that simultaneously decreasing the early delivery penalty parameter M and increasing the late delivery penalty parameter N will achieve the same goal.

This study has discussed discussed the effect of Δc, M, and N, but the preference parameters p and$q$ remain to be discussed. Intuitively, an increase in p should indicate that the early delivery penalty is more serious. As a result, the optimal delivery position c_1^* should be smaller. However, the reality is not this simple. To determine the effect of p on c_1^*, the derivation of Eq. (6) with respect to p is calculated to yield Eq. (32):

$$M\int_0^{c_1^*}(c_1^*-x)^{p-1}f(x)dx + Mp\int_0^{c_1^*}(c_1^*-x)^{p-1}\left[ln(c_1^*-x) + \frac{p-1}{c_1^*-x}\cdot\frac{dc_1^*}{dp}\right]f(x)dx + Np(p-1)\int_{c_1^*+\Delta c}^{\infty}(x-c_1^*-\Delta c)^{q-2}f(x)dx\cdot\frac{dc_1^*}{dp} = 0 \quad (32)$$

Here, the following derivation rule is applied:

$$((c_1^*-x)^{p-1})' = \left(e^{(p-1)ln(c_1^*-x)}\right)' = e^{(p-1)ln(c_1^*-x)}\left[ln(c_1^*-x) + \frac{p-1}{c_1^*-x}\cdot\frac{dc_1^*}{dp}\right] \quad (33)$$

Eq. (33) yields Eq. (34):

$$\frac{dc_1^*}{dp} = -\frac{M\int_0^{c_1^*}(c_1^*-x)^{p-1}f(x)dx+Mp\int_0^{c_1^*}(c_1^*-x)^{p-1}ln(c_1^*-x)f(x)dx}{Mp\int_0^{c_1^*}(c_1^*-x)^{p-1}\cdot\frac{p-1}{c_1^*-x}f(x)dx+Nq(q-1)\int_{c_1^*+\Delta c}^{\infty}(x-c_1^*-\Delta c)^{q-2}f(x)dx} = -\frac{M\int_0^{c_1^*}(c_1^*-x)^{p-1}(p\cdot ln(c_1^*-x)+1)f(x)dx}{Mp\int_0^{c_1^*}(c_1^*-x)^{p-1}\cdot\frac{p-1}{c_1^*-x}f(x)dx+N\ \ (q-1)\int_{c_1^*+\Delta c}^{\infty}(x-c_1^*-\Delta c)^{q-2}f(x)dx} \quad (34)$$

It is possible for p or q to take the value of 1, however, the value of a single point does not affect the overall monotonicity. As such, the unnecessary classified discussion is omitted. The denominator in $\frac{dc_1^*}{dp}$ is greater than 0, and the sign of $\frac{dc_1^*}{dp}$ depends on the sign of the numerator $M\int_0^{c_1^*}(c_1^*-x)^{p-1}(p\cdot ln(c_1^*-x)+1)f(x)dx$. Thus, only the positive and negative values of the numerator need to be discussed. Here, c_1^* is viewed as a variable. For clarity, this study define

$$g(y)=M\int_0^{y}(y-x)^{p-1}(p\cdot ln(y-x)+1)f(x)dx \quad (35)$$

In particular, $g(c_1^*)$ is equal to the numerator in $\frac{dc_1^*}{dp}$ for $y=c_1^*$. In Eq. (35), $(y-x)^{p-1}$ and $f(x)$ are always positive; only $p\cdot ln(y-x)+1$ may be positive or negative. The zero point of $p\cdot ln(y-x)+1$ is denoted as c_0; this yields $c_0=y-e^{-\frac{1}{p}}$.

The positive part of $g(y)$is

$$M\int_0^{c_0}(y-x)^{p-1}(p\cdot ln(y-x)+1)f(x)dx \quad (36)$$

The negative part is

$$M\int_{c_0}^{y}(y-x)^{p}-1(p\cdot ln(y-x)+1)f(x)dx \quad (37)$$

The positive or negative value of $g(y)$ depends on a comparison of the positive part, Eq. (36), and negative part, Eq. (37), which are determined by y and the probability density function $f(x)$ of the delivery time. In other words, the positive or negative value of $g(y)$ depends on the relationship between y and the 0 position of the numerator.

Turning to a discussion of $\frac{dc_1^*}{dp}$, the sign of this parameter is determined by the relationship between c_1^* and the zero position. Therefore, it is possible for $\frac{dc_1^*}{dp}$ to be positive or negative, because the determining factor c_1^* can take any value. The value of c_1^*, determined by the ratio of M and N for a fixed value of p according to Proposition 2.3, can be modified by different value of K, which is not present in the numerator of $\frac{dc_1^*}{dp}$.

Similarly, we can analyze the effect of q on c_1^* can be analyzed and yields:

$$\frac{dc_1^*}{dq}=\left[N\int_{c_1^*+\Delta c}^{\infty}(x-c_1^*-\Delta c)^{q-1}(q\cdot ln(x-c_1^*-\Delta c)+1)f(x)dx\right]/\left[Mp(p-1)\int_0^{c_1^*}(c_1^*-x)^{p-2}f(x)dx+Nq\int_{c_1^*+\Delta c}^{\infty}(x-c_1^*-\Delta c)^{q-2}\cdot\frac{q-1}{x-c_1^*-\Delta c}f(x)dx\right] \quad (38)$$

The positive or negative value of $\frac{dc_1^*}{dq}$ depends on the relationship between the positive and negative parts of the numerator, which is still determined by c_1^* and $f(x)$; c_1^* is called the threshold $A(p)$ (indicating that the threshold is related to p), when the numerator in Eq. (34) is equal to 0. The variable c_1^* is called the threshold $B(q)$ (indicating that the threshold is related to q) when the numerator in Eq. (38) is equal to zero. This yields the following assumption.

Assumption 2.1 (1) When $0<A(p)\leq c_1^*$, as p increases, the optimal position of delivery window c_1^* first increases and then decreases. When $A(p)>c_1^*$, c_1^* decreases as p increases. (2) When $B(q)>c_1^*$, with the increase of q, the optimal position of delivery window c_1^* first decreases and then increases; when $B(q)\leq c_1^*$, c_1^* decreases as q increases.

The analysis of the positive and negative values of $\frac{dc_1^*}{dp}$ and $\frac{dc_1^*}{dq}$ determines that as the time preference coefficient p and q change, the optimal position of the delivery window does not simply increase or decrease as hypothesized. Rather, the optimal position is closely related to the actual delivery parameters. The effect of p and q on the minimum expected penalty cannot be calculated by just using the envelope theorem. This is because the calculation is too complicated. Hence, a specific analysis is conducted using a numerical example.

2.3. A Special Case: Mixed Linear and Quadratic Time Dependent Penalty

This study considers the expected penalty to be linear-quadratic time dependent, where p and q take the value of either 1 or 2. In other words, there are four cases ($p=1,2;\ q=1,2$) as indicated later in this section. These reflect the combination of 1 and 2 with respect to the index of early delivery and late delivery. In this special case, the optimal delivery window position c_1^* is expressed in a simple form.

When $p=1$, $q=1$, the supply chain is called a traditional two-stage supply chain (Bushuev and Guiffrida, 2012). The expected penalty is:

$$Y_1=\int_0^{c_1}M(c_1-x)f(x)dx+\int_{c_1}^{\infty}N(x-c_1-\Delta c)f(x)dx \quad (39)$$

When $p=2, q=1$, the supply chain is called a lead time sensitive supply chain. The expected penalty is:

$$Y_2=\int_0^{c_1}M(c_1-x)^2f(x)dx+\int_{c_1}^{\infty}N(x-c_1-\Delta c)f(x)dx \quad (40)$$

When $p=1, q=2$, the supply chain is called a delay time sensitive supply chain. The expected penalty is:

$$Y_3=\int_0^{c_1}M(c_1-x)f(x)dx+\int_{c_1}^{\infty}N(x-c_1-\Delta c)^2f(x)dx \quad (41)$$

When $p=2, q=2$, the supply chain is called a two-stage time-sensitive supply chain. The expected penalty is:

$$Y_4=\int_0^{c_1}M(c_1-x)^2f(x)dx+\int_{c_1}^{\infty}N(x-c_1-\Delta c)^2f(x)dx \quad (42)$$

These four cases above cover all possible delivery preferences and cost compensations. Applying Theorem 2.1 yields the conditions satisfied by the optimal positions of the delivery window.

Theorem 2.2 Denote c_{1i}^* as the optimal positions of the delivery window, which minimize $Y_i (i=1,2,3,4)$. Then, c_{1i}^* satisfy the following conditions:

$$\frac{M}{N}F(c_{11}^*)-1+F(c_{11}^*+\Delta c)=0 \quad (43)$$

$$\frac{2M}{N}G(c_{12}^*)-1+F(c_{12}^*+\Delta c)=0 \quad (44)$$

$$\frac{M}{2N}F(c_{13}^*)-E(x)-G(c_{13}^*+\Delta c)+(c_{13}^*+\Delta c)=0 \quad (45)$$

$$\frac{M}{N}G(c_{14}^*)-E(x)-G(c_{14}^*+\Delta c)+(c_{14}^*+\Delta c)=0 \quad (46)$$

where $F(x)$ is a cumulative distribution function, and $G(x)=\int_0^{x}F(y)dy$.

Proof Let $g_i(c_1)$ be the derivatives of Y_i with respect to the delivery window position, then

$$g_1(c_1) = \frac{dY_1}{dc_1} = M\int_0^{c_1} f(x)dx - N\int_{c_1}^{\infty} f(x)dx = MF(c_1) - N\big(1 - F(c_1)\big) \quad (47)$$

$$g_2(c_1) = \frac{dY_2}{dc_1} = 2M\int_0^{c_1}(c_1 - x)f(x)dx - N\int_{c_1}^{\infty} f(x)dx = 2M\int_0^{c_1}(c_1 - x)f(x)dx - N\big(1 - F(c_1)\big) \quad (48)$$

$$g_3(c_1) = \frac{dY_3}{dc_1} = M\int_0^{c_1} f(x)dx - 2N\int_{c_1}^{\infty}(x - c_1 - \Delta c)f(x)dx = MF(c_1) - 2N\int_{c_1}^{\infty}(x - c_1 - \Delta c)f(x)dx \quad (49)$$

$$g_4(c_1) = \frac{dY_4}{dc_1} = 2M\int_0^{c_1}(c_1 - x)f(x)dx - 2N\int_{c_1}^{\infty}(x - c_1 - \Delta c)f(x)dx \quad (50)$$

This completes the processing of $g_1(c_1)$. For the integral term above with x in Eqs. (48)-(50), this study applies the cumulative integral transformation:

$$\int_0^{c_1} xf(x)dx = \int_0^{c_1}\int_0^{x} f(x)dydx = \int_0^{c_1}\int_y^{c_1} f(x)dxdy = \int_0^{c_1} F(c_1) - F(y)dy = c_1F(c_1) - G(c_1) \quad (51)$$

For a continuous random variable x with a probability density function $f(x)$, the expectation of x can be expressed as $E(x) = \int_0^{\infty} xf(x)dx$. To compute $\int_{c_1+\Delta c}^{\infty} xf(x)dx$, we use the interval additivity of the definite integral, yielding Eq. (52):

$$\int_{c_1+\Delta c}^{\infty} xf(x)dx = \int_0^{\infty} xf(x)dx - \int_0^{c_1+\Delta c} xf(x)dx = E(x) - (c_1 + \Delta c)F(c_1 + \Delta c) + G(c_1 + \Delta c) \quad (52)$$

We then rewrite $g_2(c_1), g_3(c_1), g_4(c_1)$:

$$g_2(c_1) = 2M\int_0^{c_1}(c_1 - x)f(x)dx - N\big(1 - F(c_1 + \Delta c)\big) = 2M\big[c_1F(c_1) - \big(c_1F(c_1) - G(c_1)\big)\big] - N\big(1 - F(c_1 + \Delta c)\big) = 2MG(c_1) - N\big(1 - F(c_1 + \Delta c)\big) \quad (53)$$

$$g_3(c_1) = MF(c_1) - 2N\int_{c_1}^{\infty}(x - c_1 - \Delta c)f(x)dx = MF(c_1) - 2N\big[E(x) - (c_1 + \Delta c)F(c_1 + \Delta c) + G(c_1 + \Delta c) - (c_1 + \Delta c)\big(1 - F(c_1 + \Delta c)\big)\big] = MF(c_1) - 2M[E(x) + G(c_1 + \Delta c) - (c_1 + \Delta c)] \quad (54)$$

$$g_4(c_1) = 2M\int_0^{c_1}(c_1 - x)f(x)dx - 2N\int_{c_1}^{\infty}(x - c_1 - \Delta c)f(x)dx = 2MG(c_1) - 2N[E(x) + G(c_1 + \Delta c) - (c_1 + \Delta c)] \quad (55)$$

When the derivatives Eqs. (53)-(55) equal 0, the coefficients can be simplified to obtain Eqs. (44)-(46).

Theorem 2.2 shows that, due to the choice of the quadratic form of the time-sensitive penalty cost function, the effective forms that do not include the explicit integral form are generated ultimately. This involves using the derivation of the quadratic form, the first-order of which corresponds to the expectation of the actual arrival time x and the integral form of the cumulative distribution function.

3. Results and Discussion

3.1. The Necessity of Finding the Optimal Delivery Window Position

Two numerical examples using penalty-based mixed linear and quadratic time dependence are presented to build the general model in this section. The parameter values refer to examples of Tao et al. (2021) and Roy and Sarker (2021).

Example 1. Uniformly distributed lead time

Consider a toy manufacturer that produces toys to supply to retailers. Delivery time is uniformly distributed between 0 days and 35 days, that is, $x \sim U(0,35)$. The remaining parametric values are: $M = 1507$, $N = 1320$, $\Delta c = 15$. The probability density function of delivery time x is $f(x) = \frac{1}{35}$, the cumulative distribution function of x is $\mathrm{F}(x) = \frac{x}{35}$, and the cumulative integral of the distribution function is $G(x) = \frac{x^2}{70}$. Eqs. (43)-(46) in Theorem 2.2 are applied to calculate the optimal position of the delivery window: $c_{11}^* = 9$ $(p = 2, q = 1)$, $c_{12}^* = 4$ $(p = 2, q = 1)$, $c_{13}^* = 16$ $(p = 1, q = 2)$, and $c_{14}^* = 10$ $(p = 2, q = 2)$. Based on Eqs. (39)-(42), the corresponding minimum expected penalty are: $Y_1 = 4025.53$, $Y_2 = 5745.98$, $Y_3 = 6415.36$, and $Y_4 = 26923.81$. Different expected penalty cost resulting from different delivery window positions (different deviation from the optimal delivery window) are shown in Table 1.

Table 1. Expected penalty cost for different delivery window positions (Uniformly distributed lead time)

	Y_1	Y_2	Y_3	Y_4
$c_1^* - 5$	5172	-	11769	44223
$c_1^* - 3$	4471	6821	7950	32542
$c_1^* - 1$	4093	5837	6415	27195
c_1^*	4026	5746	6315	26924
$c_1^* + 1$	4038	6036	6561	28267
$c_1^* + 3$	4306	8109	7784	35844
$c_1^* + 5$	4898	12745	9494	50011

It is observed that the delivery window position is only one day later than the optimal delivery window position. The expected penalty Y_2 will increase the most, reaching 5.05%. A delivery window position extension of up to five days will cost 1.2 times the additional penalty for Y_2. Even Y_1, with the least amount of loss, will cause an additional loss of 21.66%.

Example 2. Exponentially distributed lead time

The delivery time of a type of sofa from A-Zenith Furniture Co., Ltd. follows an exponential distribution, with mean $\beta = 30$ (an analysis of the collected data shows that the two-tailed significance of the test obtained after the one-sample Kolmogorov-Smirnov goodness-of-fit test is 0.564 (>0.05), and therefore, the delivery time follows exponential distribution). The remaining parametric values are: $M = 80$, $N = 200$, $\Delta c = 4$. The probability density function of delivery time x is $f(x) = \frac{1}{30}exp\left(-\frac{x}{30}\right)$; the cumulative distribution function of x is $F(x) = 1 - exp\left(-\frac{x}{30}\right)$; the expected x value is 30, and the cumulative integral of the distribution function is $G(x) =$

$x + 30exp\left(-\frac{x}{30}\right) - 30$. Theorem 2.2 enables the calculation of the optimal position of the delivery window in four cases as: $c_{11}^* = 35$, $c_{12}^* = 7$, $c_{13}^* = 147$, and $c_{14}^* = 40$. Table 2 shows the expected penalty cost for different deviations of the delivery window position from c_{i1}^*.

Table 2. Expected penalty cost for different delivery window positions (exponentially distributed lead time)

	Y_1	Y_2	Y_3	Y_4
$c_1^* - 5$	2817	4851	11759	127741
$c_1^* - 1$	2783	4461	11725	125174
c_1^*	2782	4442	11723	125069
$c_1^* + 1$	2784	4463	11725	125174
$c_1^* + 5$	2814	5019	11755	127672
$c_1^* + 10$	2902	7044	11843	135354
$c_1^* + 30$	3665	36624	12606	213917

It is noted from Table 2 that, unlike Example 1, a one-day delay would cause considerable additional losses, and the impact of a one-day deviation from the optimal position of the delivery window in the exponential distribution is almost negligible. The main reason for this is that the exponential distribution has a large delivery time span and a small delivery window. However, the delivery window position is 10 days later than the optimal delivery window position, and the expected penalty increases significantly. A 10-day delay will result in additional cost losses of 4.31%, 58.58%, 1.02%, and 8.22%, respectively, for the four expected penalties. In addition, a deviation of up to 30 days has the most serious impact on Y_2, which will cause a 7-times additional loss.

By completing a longitudinal comparison of the data in Table 1 and Table 2, when the values of p and q are determined, the more the position of the delivery window deviates from the target value, the higher the additional penalty cost is. By a horizontal comparison, the larger the index is, the larger is the expected penalty. Reasonable use of delivery windows by suppliers can minimize unnecessary financial expenses. This is a cost-free optimization that ensures that resources are used to the fullest extent possible.

Due to space limitations, Subsections 3.2-3.5 only show the parameter analysis of Example 2.

3.2. Effect of Delivery Window Width

Fig. 1 illustrates the effect of changing the width of the delivery window Δc from 5 to 60 on the optimal position of that delivery window. When the delivery window is widened, the optimal positions of the delivery window trend downward. The rate of that decrease is relatively slow in all four cases. When the delivery window is doubled, that is, from 4 to 8, the optimal position of the delivery window is advanced by two days, zero days, four days, and two days in the four cases, which is less than 7%. At the same Δc the optimal position of the delivery window for the delay time sensitive expected penalty is the largest. This is because when the late delivery penalty increases, the delivery window position becomes larger, and the probability increases that the delivery will not be delayed.

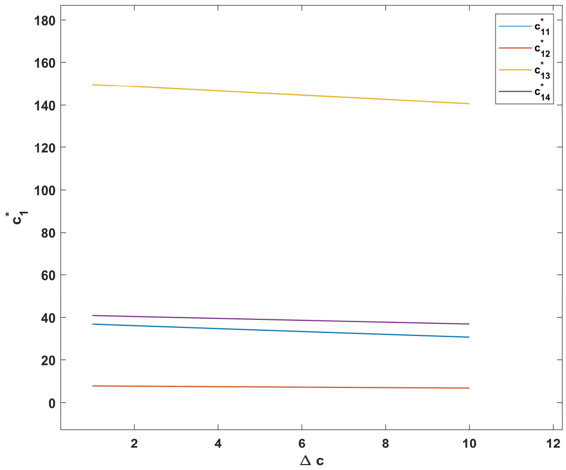

Fig. 1. The effect of Δc on c_1^*

The values and trends of c_{11}^* and c_{14}^* are similar, because both have a similar penalty policy for early and late delivery. Fig. 2 shows the effect of the width of the delivery window Δc on the minimum expected penalty Y^*. The scale of the y-axis scale is very large, so the effect of Δc on Y^* is also large, although the slope of the line in Fig. 2 does not appear to be large. Increasing the value of Δc significantly reduces Y^*. When the delivery window doubles, the expected penalty in the four cases decreases by 8.27%, 12.4%, 2.7%, and 8.64%, respectively. In particular, when $p = 2$, $q = 2$, the expected penalty is reduced from 125069 to 114268, a reduction of 10801. This is quite a substantial cost saving. The results support Proposition 2.1, namely that the wider the Δc is, the smaller the optimal position is and the less the penalty costs are.

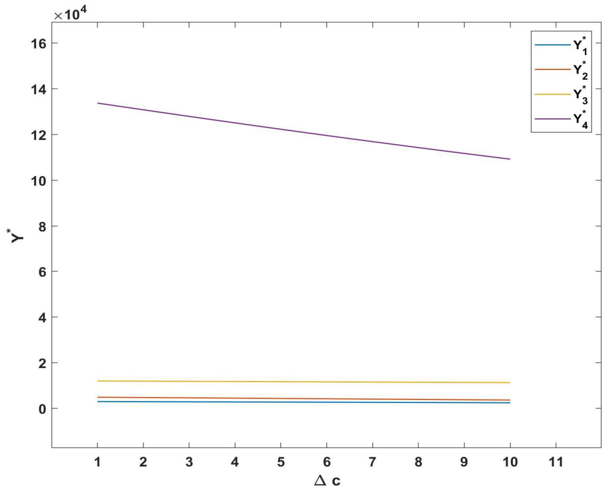

Fig. 2. The effect of Δc on Y^*

3.3. The Effect of the Ratio of M To N

Fig. 4 illustrates the effect on the optimal position of the delivery window when the ratio of M and N varies between 0.01 and 10. From Proposition 2.3, one can know that increasing the ratio $\frac{M}{N}$ will decrease the optimal position c_1^*. As the ratio of $\frac{M}{N}$ increases, with a value around 1.5 as the critical point, first, c_1^* sharply decreasesand then decreases increasingly slowly. For example, for $p = 2$ and $q = 1$, when the ratio decreases from 0.2 to 0.1, the delivery window optimal position increases by 25%, from 10 to 14; when decreasing to 0.4, c_1^* increases to 20 and doubles. The critical values for the four cases are around 1, 0.2, 2, and

0.6. The buyer assesses the ratio $\frac{M}{N}$. When its value falls between 0 and a value around 1.5, adjusting the ratio is a powerful tool that the buyer can use to adjust the delivery time. However, when the ratio exceeds a certain range, adjusting the ratio has very little impact on the delivery time.

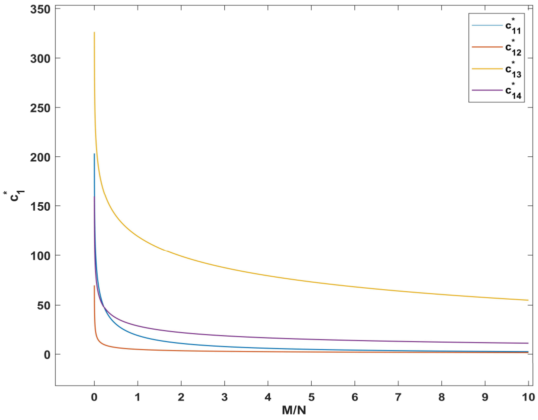

Fig. 3. The effect of $\frac{M}{N}$ on c_1^*

3.4. The Effect of p and q

Figs. 4 and 5 show the effect of the time preference coefficient on the optimal position of the delivery window. When q (p) is fixed, c_1^* decreases (increases) as p (q) increases. Small changes in p or q result in significant changes in c_1^*. For example, when $q = 2$, p increases from 2 to 3, and then c_1^* decreases from 40 to 14, a reduction of 65%. When $p = 2$, q increases from 2 to 3, and c_1^* increases from 39 to 124, an increase of more than 2-times.

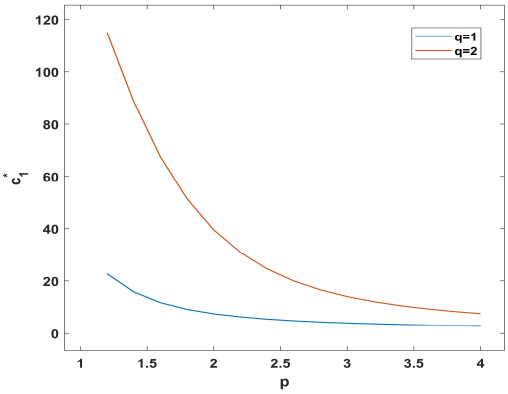

Fig. 4. The effect of p on c_1^*

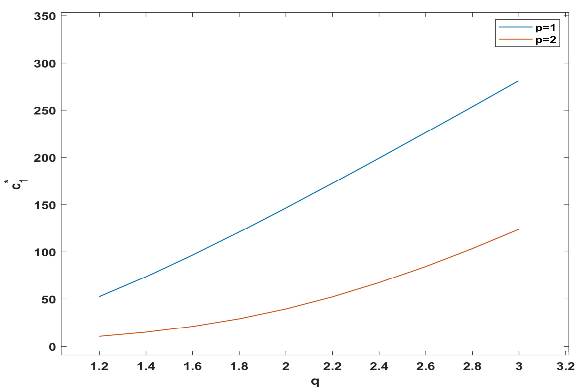

Fig. 5. The effect of q on c_1^*

As shown in Figs. 6 and 7, when combined with the effect of the time preference coefficient on Y^*, small changes in p or q can significantly impact Y^*. One might take for granted that increasing the time preference coefficient would increase the expectation penalty. However, Fig. 6 shows that when $q = 2$, Y^* first increases until reaching 2, and then decreases as p increases. This also indirectly proves the rationality of Assumption 2.1. The effects of p and q on c_1^* and Y^* are large and complex. As such, buyers should be careful and thorough in determining the values of p and q, and should not modify them without careful consideration.

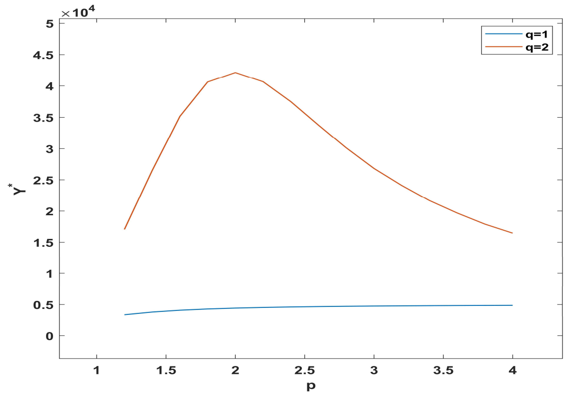

Fig. 6. The effect of p on Y^*

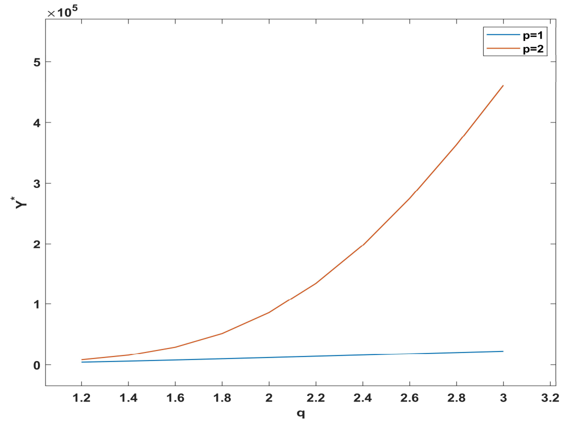

Fig. 7. The effect of q on Y^*

3.5. Sensitivity Analyses

Figs. 8 and 9 show the results of a sensitivity test, where the value of parameter β changes from 5 to 50. In the four different penalty cases, theposition of the optimal delivery window increases as β increases. When β is reduced from 30 to 20, the optimal positions of the delivery window are reduced from 35, 7, 147, and 40, to 22, 6, 88, and 26, with decreases of 37.14%, 14.29%, 40.14%, and 35%, respectively. The parameter β has little effect on c_{12}^*, has a large effect on c_{13}^*, and has similar effects on $c_{11}^*$and c_{14}^*. This outcome is reasonable, because the late delivery penalty coefficient is larger than the early delivery penalty coefficient in this example, so naturally, the delay time sensitive supply chain is more sensitive to the value of β. As such, the optimal position of the delivery window should increase as the mean delivery time (β) increases, and the supplier should avoid high compensation levels due to late delivery. Meanwhile, the minimum expected penalty cost increases as β increases. The values of Y_1^*, Y_2^*, and Y_3^* are not significantly different; Y_4^* experiences the most significant increase. When $\beta = 50$, Y_4^*=360117 is 75, 45 and 17 times of $Y_1^* = 4775$, $Y_2^* = 8040$, and $Y_3^* = 21783$, respectively. A penalty this high is also consistent with the original intention of the time-sensitive penalty: to encourage on-time delivery without default. When the average delivery time β decreases from 30 to 20, the expected penalty decreases by 36.48%, 40.12%, 39.7%, and 57.49%, respectively. The results show that reducing the average delivery time will significantly decrease the supplier's expected penalty cost.

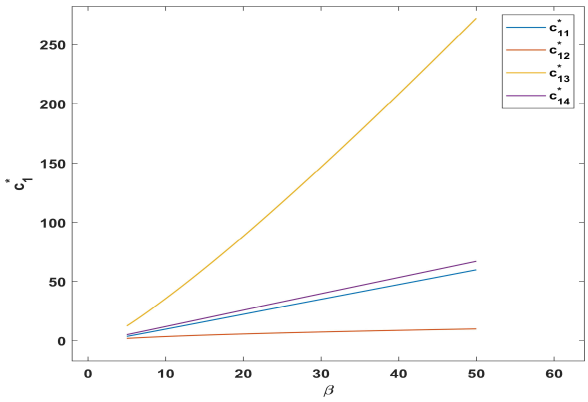

Fig. 8. The effect of β on c_1^*

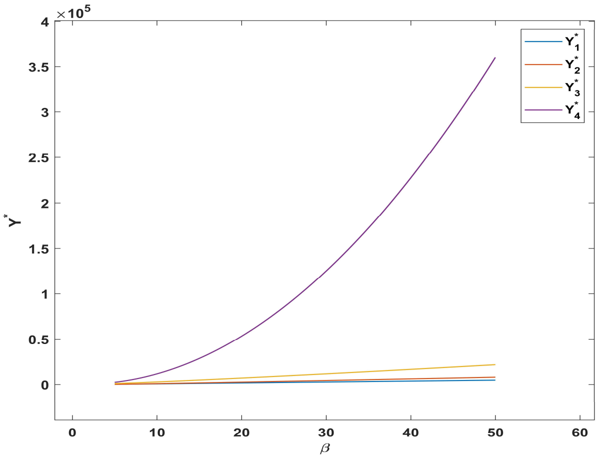

Fig. 9. The effect of β on Y^*

4. Conclusion

This paper discusses the optimal position of delivery windows in a two-stage time-sensitive supply chain, as well as the effect of different parameters on the optimal position of the delivery window and the minimum expected penalty. Penalties are generated when suppliers default. In general, the penalty is not purely linear per unit of time, as that approach only considers the cost loss. In addition, it is important to consider the penalties for breach of contract and subsequent losses. Given these facts, this study proposes a special case of expectation penalty with a power function. This involves expectation penalties that mix linear and quadratic functions over time. This special case applies to many situations in the actual supply chain. The exact type of penalty depends on the buyer's industrial structure and attitude towards a breach of contract. The supplier, however, can determine the optimal delivery start time to minimize the expected penalty.

Coordination between suppliers and buyers is strategic with respect to supply chain operations. The time-sensitive penalty proposed in this paper may satisfy the buyer's demand for punctuality and delivery preferences. The supplier can determine the optimal position of the delivery window, by minimizing the expected penalty based on available information, using Theorem 2.1. Proposition 2.1 demonstrates that increasing the delivery window Δc decreases the optimal position of delivery window c_1^* and the expected penalty cost. Suppliers and buyers can cooperate to set a mutually-agreeable delivery window width. Given an acceptable width, the buyer can make appropriate concessions, as the width has a rather limited impact on the delivery window position. In contrast, suppliers generally prefer a delivery window that is as large as possible, because of the large impact of the delivery window width on the penalty. Propositions 2.2 and 2.3, which focus on the penalty factor for early delivery factor M and late delivery factor N, recommend increasing the ratio $\frac{M}{N}$ to decrease the optimal position of delivery window c_1^*. They also show that c_1^* is fixed if $\frac{M}{N}$ is determined. The conclusion, that increasing lead and delay time preference coefficient p, q will increase the expected penalty within a certain range, and decrease outside the range, is given by Assumption 2.1. Buyers can determine their preference for default through a time preference coefficient. However, determining the value of different parameters needs to be done carefully. The sensitivity analysis shows that decreasing lead time can significantly decrease the optimal position of delivery window c_1^* and the expected penalty cost. Decision-makers could determine whether to improve supply chain performance by comparing the cost of improving the supply chain with the cost savings of optimizing the delivery window.

This study has limitations and requires future research. First, the choice of penalty with power function in this paper is an effective choice for an idealized simulation. However, the real cost-time relationship is not necessarily continuous. As such, the relationship between penalty and time could be extended to use a piecewise function, because sellers may not significantly change the punishment strategy for a small period of time. Second, when a breach of contract will generate a significant penalty, suppliers should consider ways to avoid this risk. Therefore, researchers should consider adding the risk-averse attitude of the supplier. Finally, the model could be extended to include multi-vendor and multi-seller systems.

Author Contributions

Jinyu Yang contributes to methodology, software, validation, analysis, investigation, draft preparation, manuscript editing, and visualization. Xiaoyu Xing contributes to writing - review and editing, conceptualization, supervision, and project administration.

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Jinyu Yang is a MS. Candidate at the School of Science, Hebei University of Technology, Beichen district, Tianjin, China. She was awarded scholarships for 3 years. Her research interests include Portfolio Management, Investment Strategy Analysis, and Supply Chain Management.

Xiaoyu Xing is Professor of the School of Science, Hebei University of Technology, Beichen district, Tianjin, China. Her research interests include Supply Chain Management, Stochastic Optimal Control, Risk Measurement in Insurance Actuarial and Financial Engineering, Optimal Reinsurance-Investment Strategy, Financial Asset Pricing and Hedging, Stochastic Processes and their Applications, and Reinforcement Learning.