QUARK MASS FUNCTION FROM AN OGE-TYPE INTERACTION IN MINKOWSKI SPACE*

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We present results for the quark mass function in Minkowski space calculated from an interaction kernel that consists of an effective one-gluon-exchange and a constant interaction. We analyze the gauge dependence of our results and compare them in the spacelike region to the available lattice QCD data.

1. Introduction

QCD at low energies requires an essentially non-perturbative treatment which makes the theoretical description of hadrons difficult. For the light mesons, in particular the pion, the implementation of dynamical chiral symmetry breaking (DχSB) is indispensable. In this work, we present recent results on the quark self-energy obtained in the Covariant Spectator Theory (CST) [1, 2], which has already been applied successfully to mesons previously [3–16]. The CST is a covariant approach formulated in Minkowski space and related to the Bethe–Salpeter/Dyson–Schwinger formalism (for a recent review, see [17]). It uses a quark–quark interaction kernel that includes, in addition to the one-gluon-exchange (OGE) kernel, a covariant phenomenological generalization of a linear confining potential.

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2. Quark self-energy in CST

The dynamical quark mass generation is described by the Dyson equation for the dressed quark propagator given by

$$S(p) = S_0(p) - S_0(p)Z_2\Sigma(\not{p})S(p), \tag{1}$$

where $S_0(p) = (m_0 - \not{p} - i\epsilon)^{-1}$ and $S(p) = (m_0 - \not{p} + Z_2\Sigma(\not{p}) - i\epsilon)^{-1}$ are the bare and dressed quark propagators, respectively, with m_0 the bare (current) quark mass, Z_2 a renormalization constant, and $\Sigma(\not{p})$ the quark self-energy which can be written in terms of invariant functions as

$$\Sigma(\not{p}) = A(p^2) + \not{p}\, B(p^2). \tag{2}$$

The quark mass function and wave function normalization are related to the self-energy by

$$M(p^2) = Z(p^2)[m_0 + Z_2 A(p^2)] \quad \text{and} \quad Z(p^2) = \frac{1}{1 - Z_2 B(p^2)}, \tag{3}$$

respectively. One of the central assumptions of the CST is the existence of a real mass pole of the dressed quark propagator at $p^2 = m^2$, identified as the constituent quark mass m, such that $M(p^2)$ and $Z(p^2)$ satisfy

$$M(m^2) = m \quad \text{and} \quad Z(m^2) = Z_2\left[1 - 2m\frac{dM(p^2)}{dp^2}\bigg|_{p^2=m^2}\right], \tag{4}$$

respectively. In CST, the zero-components of loop momenta are integrated by calculating only the residues of the quark propagator poles, such that the CST self-energy (times Z_2) is given by

$$Z_2\Sigma(\not{p}) = \frac{Z_2^2}{2}\sum_{\sigma=\pm}\int_{\boldsymbol{k}}\mathcal{V}\left(p, \hat{k}_\sigma\right)\left(\frac{m + \hat{\not{k}}_\sigma}{2m}\right), \tag{5}$$

where $\int_{\boldsymbol{k}} \equiv \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{m}{E_k}$, $\sigma = \pm$ labels the positive- and negative-energy on-shell momenta (corresponding to the positions of the quark propagator poles), $\hat{k}_\sigma = (\sigma E_k, \boldsymbol{k})$, with $E_k = \sqrt{m^2 + \boldsymbol{k}^2}$ and $\mathcal{V}(p, \hat{k}_\sigma)$ is the interaction kernel given by

$$\mathcal{V}\left(p, \hat{k}_\sigma\right) = \left(\frac{1}{4}\sum_a \lambda_a \otimes \lambda_a\right)\left\{\left[(\mathbf{1}\otimes\mathbf{1} + \gamma^5\otimes\gamma^5)\right]V_\ell\left(p, \hat{k}_\sigma\right)\right.$$
$$\left. - \gamma_\mu\otimes\gamma_\nu\left[\Delta_g^{\mu\nu}(q_\sigma^2)V_g\left(p, \hat{k}_\sigma\right) + t\Delta_c^{\mu\nu}(q_\sigma^2)V_c\left(p, \hat{k}_\sigma\right)\right]\right\} \tag{6}$$

Here, $q_\sigma = p - \hat{k}_\sigma$, V_ℓ is a covariant generalization of a linear confining potential,

$$V_g\left(p, \hat{k}_\sigma\right) = -4\pi\alpha_s \frac{g(y)}{\left(p - \hat{k}_\sigma\right)^2} \tag{7}$$

is the OGE interaction with α_s the *unrenormalized* strong coupling constant, $g(y)$ a regularization form factor depending on the covariant variable $y^2 = \frac{(p \cdot k)^2}{p^2 k^2}$,

$$V_c\left(p, \hat{k}_\sigma\right) = \frac{CE_k}{2m}(2\pi)^3 \delta^3\left(\boldsymbol{k} - \frac{m}{\sqrt{p^2}}\boldsymbol{p}\right) h\left(p^2\right), \tag{8}$$

is a covariant form of a constant potential with C the unrenormalized strength and h is a strong quark form factor normalized as $h(m^2) = 1$. The $\Delta^{\mu\nu}$'s in Eq. (6) are factors given in general linear covariant gauge specified by the gauge parameter ξ and chosen in this work as

$$\Delta_c^{\mu\nu}\left(q_\sigma^2\right) = g^{\mu\nu} - (1 - \xi)\frac{q_\sigma^\mu q_\sigma^\nu}{q_\sigma^2} \quad \text{and} \quad \Delta_g^{\mu\nu}\left(q_\sigma^2\right) = -\frac{q_\sigma^2}{M_g^2 + |q_\sigma^2|}\Delta_c^{\mu\nu}\left(q_\sigma^2\right). \tag{9}$$

Notice that this choice effectively gives the gluon a finite mass M_g and replaces $q^2 \to -|q^2|$, which removes the singularity in the gluon propagator. Further recall [11] that the linear confining part of $\mathcal{V}$ gives no contribution to the CST self-energy (5), which simplifies the calculation substantially.

3. Results and discussion

In the quark's rest frame, where $p = \{p_0, \mathbf{0}\}$, the OGE contributions to the invariant self-energy functions are

$$Z_2 A_g\left(p_0^2\right) = \frac{8\pi\alpha_s^r}{3} m \sum_\sigma \int_{\boldsymbol{k}} \frac{(3+\xi)g(y)}{M_g^2 + |q_\sigma^2|},$$

$$Z_2 B_g\left(p_0^2\right) = -\frac{8\pi\alpha_s^r}{3} \sum_\sigma \int_{\boldsymbol{k}} \frac{g(y)}{M_g^2 + |q_\sigma^2|}\left\{\frac{\sigma(3-\xi)E_k}{p_0} + \frac{2(1-\xi)\boldsymbol{k}^2}{q_\sigma^2}\right\}, \tag{10}$$

where $\alpha_s^r = Z_2^2 \alpha_s$ is the *renormalized* strong coupling constant. The renormalization of α_s arises from a factor of $\sqrt{Z_2}$ attached to each quark line either entering or leaving an interaction vertex. For M_g, we take the value $M_g = 0.6$ GeV and for $g(y)$, we chose the form

$$g(y) = \frac{\Lambda_g^8}{\Lambda_g^8 + m^8(y^2 - 1)^4}, \tag{11}$$

where Λ_g is an adjustable scale parameter. It can be shown analytically that the on-shell equations (4) for the OGE contributions (10) are *independent* of ξ, and so are the constituent quark mass m and Z_2. When Eq. (4) is solved in the chiral limit ($m_0 = 0$) with $m = 0.3$ GeV and $\Lambda_g \lesssim 2$ GeV for the OGE kernel alone, then the value for α_s^r turns out to be unnaturally large as compared to the approximate value known from experiment. The additional constant kernel, which can be regarded as a correction to the OGE contribution, solves this issue and leads to realistic values of our model parameters. Its contributions are

$$Z_2 A_c\left(p^2\right) \;=\; \tfrac{1}{4}(3 + \xi)C^r\, h\left(p^2\right)\,, \qquad Z_2 B_c\left(p^2\right) = 0\,, \tag{12}$$

where $C^r = Z_2^2 C$ is the *renormalized* strength. If $Z_2 A_c(p^2)$ is to satisfy (4) in the chiral limit in arbitrary gauge, then $C^r \to 4m/(3 + \xi)$, and if the constant kernel is to supplement the OGE kernel, then it is appropriate to choose $h(p^2) = A_g(p^2)/A_g(m^2)$.

Since each contribution (10) and (12) satisfies (4) separately, this suggests to write the total result as a linear combination of these contributions

$$Z_2 A\left(p^2\right) = \left[\eta + \frac{m(1 - \eta)}{Z_2 A_g(m^2)}\right] Z_2 A_g\left(p^2\right)\,, \quad Z_2 B\left(p^2\right) = \eta Z_2 B_g\left(p^2\right)\,, \tag{13}$$

where η is a mixing parameter chosen to maintain the effective OGE strength $\eta\alpha_s^r = 0.5$ regardless of the choice of Λ_g, which is roughly adjusted to agree with the lattice QCD data [18]. The results for the chiral-limit quark mass function are summarized in Fig. 1.

Notice that in Landau gauge ($\xi = 0$), there is a pole in the mass function at some small timelike p^2, where $Z_2 B(p^2) = 1$. Such poles are not present in the Feynman ($\xi = 1$) and Yennie ($\xi = 3$) gauge results. Further notice that, in order to make the dressed gluon propagator non-singular, we have introduced particular gluon dressing functions in Eq. (9). The disadvantage of this choice is that the mass function develops a discontinuity at $p^2 = 0$ (not displayed in the figure). Fortunately, the size of the discontinuity depends on the gauge and we find that the quark mass function is continuous at $p^2 = 0$ only for the Yennie gauge, which, therefore, constitutes the preferred gauge in this calculation. In the timelike region, there is a strong dependence of the quark mass function results on the gauge, however, on-mass-shell results are gauge-independent — a natural feature of the CST. In the spacelike region, the gauge dependence is very weak, allowing us to use the existing lattice QCD data for the mass function to calibrate the model. We now have a mass function that can be used in meson calculations where the quark dressing and the quark–quark interaction are completely consistent.
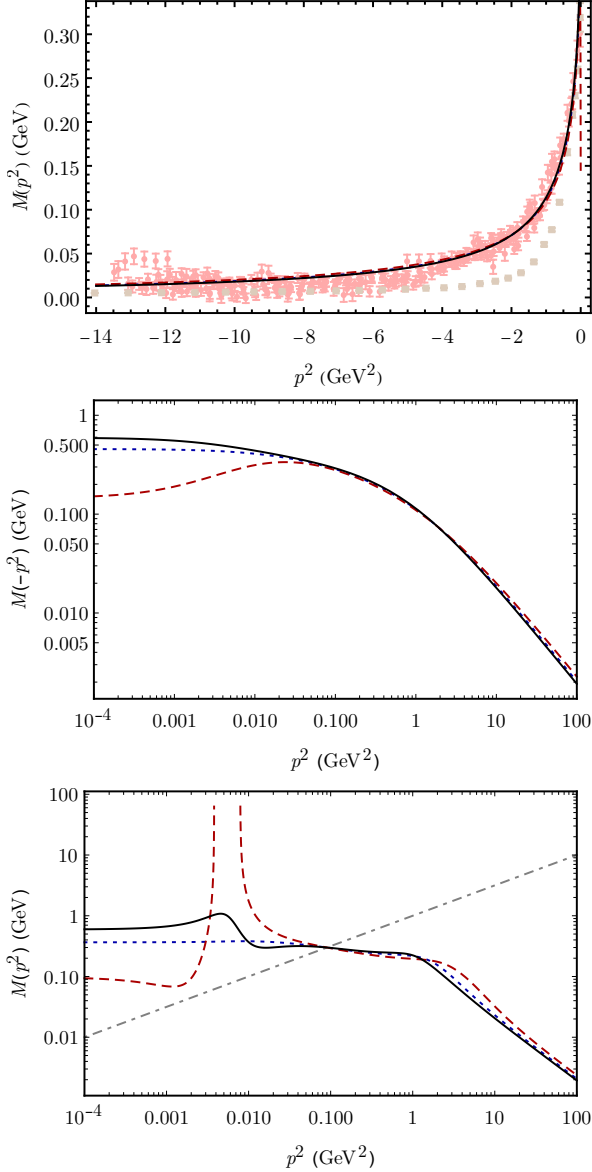
Fig. 1. (Color online) The mass function (in GeV) *vs.* p^2 (in GeV^2) for $\xi = 0$ and $\Lambda_g = 0.9$ GeV (dashed red line), $\xi = 1$ and $\Lambda_g = 0.6$ GeV (dotted blue line), and $\xi = 3$ and $\Lambda_g = 0.45$ GeV (black solid line). The top panel shows the spacelike region with lattice QCD data taken from [18] (red discs) and [19] (brown squares). The middle and bottom panels show the spacelike and timelike regions on logarithmic scales, respectively, where the intersection of the curves with $\sqrt{p^2}$ (gray dot-dashed line) marks the ξ-independent on-shell point $p^2 = m^2$.

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