ANALYSIS OF THE SAFETY EFFICIENCY OF A ROAD NETWORK: A REAL CASE STUDY

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Abstract
In this paper, recently introduced topological measures of interconnection and efficiency of network systems are applied to the safety analysis of the road transport system of the Province of Piacenza in Italy. The vulnerability of the network is evaluated with respect to the loss of a road link, e.g. due to a car accident, road work or other jamming occurrences. Eventually, the improvement in the global and local safety indicators following the implementation of a road development plan is evaluated.

1. Introduction

Complexity Science [5], [7]-[8], [10], [15] offers a promising approach to the analysis of technological network systems and infrastructures, such as computer and communication systems [1], [11], [14], power transmission and distribution systems [9], [17], rail and road transportation systems [3], oil/gas systems [3], [4]. The underlying idea is to study the robustness of network systems by analyzing the structure of interconnection of their components (hereafter also called nodes).

In particular, by defining 'reliability distances' accounting for the probabilities of failure of the links interconnecting the nodes of the network, global and local reliability efficiency indicators have been introduced for evaluating the network robustness and vulnerability to faults [18].

In this paper, these concepts are extended to the analysis of the safety of a section of the road network of Piacenza Province in Italy. The safety feature of the road sections of the network is analyzed with respect to the probability of car accidents, which depends on the traveling speed and traffic flow. The vulnerability of the network to the loss of a road link can then be evaluated, e.g. due to a blocking car accident, road work or other jamming occurrences. As a result, a ranking of the links is obtained according to their contribution to the decrease of the overall system safety. Eventually, the improvement in the safety global and local indicators following the introduction of a road development plan is evaluated.

The paper is structured as follows. Section 2 briefly introduces the topological and safety indicators used in the analysis of the network system. Section 3 contains the description of the original topology of the Piacenza's road network and its corresponding abstract graph modeling. The vulnerability analysis is reported in Section 4 together with the effects of modifications to the network topology by a road development plan. Conclusions on the outcomes of the analysis are drawn in Section 5.
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2. Topological and safety efficiency indicators for road networks

Let us consider a network of roads (hereafter also called links or edges, as in graph theory) connecting a number of locations (hereafter also called nodes or vertices, as in graph theory).

From the point of view of the topological characterization of the network, the characteristic path length L and the clustering coefficient C are typically used to measure the average distance (number of edges) between two generic vertices (a global property of the network topology) and the connectivity of the subgraph formed by each node (a local property of the network topology), respectively [16]. For studying the global properties of the network topology, the probability distribution $P\left(d_{ij}\right)$ of the shortest path lengths d_{ij} between any two nodes i and j in the network can be considered. The shortest path length distribution is synthesized by a point value, the characteristic path length, which represents the average of the shortest distances d_{ij} between all pairs of N nodes in the network

$$L = \frac{1}{N(N-1)} \sum_{i \neq j} d_{ij} \, . \tag{1}$$

Also the connectivity of the network is typically synthesized at a local level by a single point value, the average clustering coefficient, C The clustering coefficient C_i is a local property of node i defined as follows: if node i has k_i neighbors, then at most $\frac{k_i \cdot (k_i - 1)}{2}$ edges can exist between them; C_i is the fraction of these edges that actually exist, and C is the average value

$$C = \frac{1}{N} \sum_i C_i \; .$$

From the point of view of the characterization of the road safety, the probability of no car accidents p_{ij} (or its complement q_{ij}=1- p_{ij}) on the path γ_{ij} linking the pair of nodes i and j is used. Assuming independence of the accidents occurrence in the various links forming path γ_{ij}, such probability is given by the product of the probabilities of accident on the individual edges of γ_{ij}.
The *safest* path lengths $\{d_{ij}\}$ can be computed as

$$d_{ij} = \min_{\gamma_{ij}} \left(\frac{1}{\prod\limits_{mn \in \gamma_{ij}} p_{mn}} \right) = \min_{\gamma_{ij}} \left(\frac{1}{\prod\limits_{mn \in \gamma_{ij}} \left(1 - q_{mn}\right)} \right), \forall ij \tag{2}$$

where the minimization is done with respect to all paths γ_{ij} linking nodes i and j and the product extends to all the edges of each of these paths [18]. Note that $1 \leq d_{ij} \leq \infty$, the lower value corresponding to the existence of a totally safe path connecting i and j (no accident will occur in the road section from i to j, i.e., $p_{mn} = 1, q_{mn} = 0 \ \forall mn \in ij$) and the upper value corresponding to the situation of no paths connecting i and j (which is equivalent to having in all connections from i to j at least one edge where certainly an accident occurs, $p_{mn} = 0, q_{mn} = 1$).

The safety *efficiency* between nodes i and j is then defined to be inversely proportional to the length of the safest path linking them

$$s_{ij} = \frac{1}{d_{ij}} \;\; \textit{if there is at least one path connectingi i and j} \tag{3}$$

$$= 0 \; \textit{otherwise} \; (d_{ij} = \infty)$$

The average safety efficiency of the road network **G** is then

$$S_{glob}(G) = \frac{\sum\limits_{i \neq j \in G} s_{ij}}{N(N-1)} = \frac{\sum\limits_{i \neq j \in G} \frac{1}{d_{ij}}}{N(N-1)} \, . \tag{4}$$

This quantity plays a role similar to that of L in defining the network connection characteristics on a global scale, the difference being that it also accounts for the safety of the edges through which the network's nodes are connected. More precisely, whereas the characteristic path length takes into account only the steps required for getting from one node to another through a sequential path along the network, the safety efficiency measure (4) retains also the information about the safety of the path [12]. Since $s_{ij} = 1$ when there is at least one perfect path γ_{ij} in the graph which connects nodes i and j through a sequence of accident-free edges, $S_{glob}(G)$ is equal to one in case of a perfectly connected accident-free network.

As for the local properties of the graph G, these can be quantified by specializing the definition of the average safety efficiency (4) on the subgraph G_i of the k_i neighbors of each node i in the network

$$S(G_i) = \frac{\sum_{n \neq m \in G_i} s_{nm}}{k_i (k_i - 1)}. \tag{5}$$

Averaging the efficiency of the local neighborhoods of all nodes in the network a measure of the network *local safety efficiency* is defined

$$S_{loc}(G) = \frac{1}{N} \sum_{i=1 \in G}^{N} S(G_i). \tag{6}$$

Since $i \notin \mathbf{G_i}$, this parameter reveals how much the road network is fault tolerant in that it shows how safe the connection remains between the first neighbors of i when i is removed.

The quantity defined by (6) has a similar local character as the clustering coefficient C; yet, these two different indicators convey complementary information.

3. Analysis of a real case study

3.1. System description

The road transportation system of the Province of Piacenza depicted in

*Figure **10*** has been modeled as a stochastic, weighted, undirected, connected graph which takes into account its structural, functional and operative features. Each road section is characterized by several features such as typology, i.e. city road, national road or highway, speed limit, traffic capacity, average vehicular flux and average travel speed. Physical points in the network where the road features change, e.g. road junctions and points of road typology change, are defined as nodes.

The road network of N=687 nodes connected by K=789 edges can be represented by a graph G(N, K) defined by its N×N adjacency (connection) matrix $\{a_{ij}\}$ whose entries are 1 if there is an edge joining node i to node j or 0, otherwise.

Figure 10. The road network of Piacenza Province.
Highways are represented by thicker lines

It is assumed that car accidents occur along the stretch from node i to node j according to an exponential distribution with constant rate of accident per km, λ_{ij}. Hence, the probability of an accident in the link ij of length l_{ij} is

$$q_{ij} = 1 - e^{\lambda_{ij} \cdot l_{ij}} . \tag{7}$$

The accident failure rate λ_{ij} is computed as the product of two factors

$$\lambda_{ij} = u_{ij}\left(v\right) \cdot \alpha_{ij}\left(c\right). \tag{8}$$

where the first factor is the probability per unit distance that an accident occurs when traveling at an average speed v on the stretch of road from node i to node j and the second is a multiplicative factor depending on the average level of traffic congestion c.

As for the first contribution, the accident probability per km is modeled by a sigmoidal function of parameters a_{ij} and b_{ij}

$$u_{ij}\left(v\right) = \frac{1}{1 + e^{a_{ij} \cdot \left(b_{ij} - v\right)}} . \tag{9}$$

The reason behind the arbitrary choice of the accident rate (9) lies in the attempt to reproduce the fact that beyond a certain speed the accident probability increases sensibly, mainly due to the decrease in car steering and to the shortening of the reaction time available to the driver to take countermeasures avoiding crashes.

In the specific application which follows, the parameters a_{ij} and b_{ij} have been taken equal for all sections belonging to the same speed limit class; more precisely, the road sections have been divided into the three speed limit classes 50 km/h, 90 km/h and 130 km/h and for each class the parameter values have been set by imposing two constraints on the mean distance to accident $L_{ij} = \frac{1}{u_{ij}}$ in correspondence of two different speeds, as reported in *Table 9* below.

The numbers in *Table 1* have been chosen with the following general philosophy: focusing for example on the 90 km/h speed limit, it is unlikely that an accident occurs if one drives at a speed (e.g. 40 km/h) far below the speed limit but, conversely, accidents become quite likely when driving at a speed doubling the limit (180 km/h). A similar reasoning applies to the cases of 50 km/h and 130 km/h speed limits.

Table 9. Mean distance to accident for different speed limits

Speed limit [km/h]	Mean distance to accident at the speed indicated in parameters	
50	L(30km/h) ~ 70000km	L(150km/h) ~ 6km
90	L(40km/h) ~ 200000km	L(180km/h) ~ 30km
130	L(70km/h) ~ 500000km	L(250km/h) ~ 12km

As for the multiplicative factor α_{ij} accounting for the traffic flow intensity c_{ij} on link ij, it is given by

$$\alpha_{ij} = \frac{\alpha_{MAX} - \alpha_{\min}}{c_{MAX} - c_{\min}} \cdot c_{ij} + 1 , \tag{10}$$

where the traffic flow intensity $c_{ij} \in [c_{min}, c_{MAX}]$ is computed as the ratio between the average vehicular flux and the traffic capacity of the road. Note that $\alpha_{ij} \in [\alpha_{min}, \alpha_{MAX}]$, where α_{min} and α_{MAX} are the values taken by α_{ij} in correspondence of the minimum (c_{min}) and maximum (c_{MAX}) value of traffic intensity on the road links of a given speed limit class. In this work, α_{min} and α_{MAX} have been arbitrarily set equal to 1 and 10 for all links, independently of the speed limit class. *Figure 2* shows the behaviour of q_{ij} as a function of v and parameterized on α_{ij}.

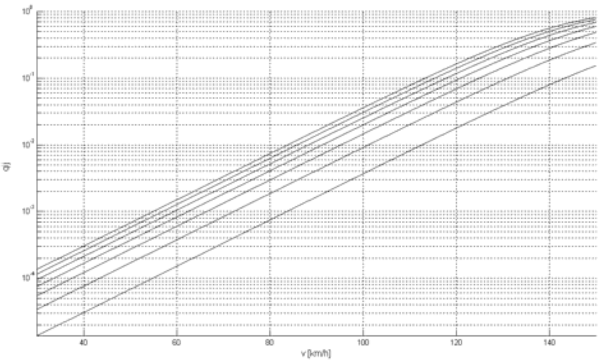

Figure 11. Accident probability q_{ij} as a function of v, for c_{ij} values in the range [0, 1.815]. The curves refer to the case of 50 km/h speed limit and a link of unitary length l_{ij}=1

A limitation of the proposed model is that no relation is accounted for between travel speed and traffic intensity; on the other hand, this is artificially reflected in the data itself which are such that roads with high traffic have a low average travel speed.

3.2. Evaluation of the safety of the road network connection

The values of the topology indicators *L* and *C* and of the safety efficiencies of the road network of the Piacenza Province in

Figure 10 are reported in column 2 of *Table 2.* The network degree distribution of *Figure 3* (the distribution *P(k)* of the number of links *k* departing from a node of the network) shows that the predominant series structure of the network connection is responsible for the large number of sparse subgraphs around the nodes, a phenomenon which leads to the small values of the average clustering coefficient and local efficiency.

As a result, if a node is disconnected many nodes become no longer connected to each other by relatively short paths. Thus, the network does not present the desirable small world characteristics of good global and local connectivity. On the other hand, such topological structure of the network reflects solely the change in road typology since, as previously explained in Section 3.1, points of change in the road features, e.g. junctions and points of typology change, are taken as nodes. Actually, only road junctions are significant for the physical connectivity of the network topology whereas typology changes are relevant with respect to the travel and, thus, the safety characteristics of the network. These are quantified by the global and local safety efficiencies which turn out to be very low, the values mainly driven by the serial topology of the network connections, as just explained.

Table 2. Topological indicators *L*, *C* and safety efficiencies $S_{glob}(G)$, $S_{loc}(G)$, for different network configurations. I: original road network; II: road network without the 10 most vulnerable links; III: the original network augmented with the whole road development plan: IV: the original network augmented with the bypass alone

	I (Figure *10*)	II (Figure *10*)	III (*Figure*)	IV (*Figure*)
L	21	∞	19	20
C	$1.06\cdot10^{-2}$	$1.12\cdot10^{-2}$	$2.20\cdot10^{-2}$	$2.08\cdot10^{-2}$
$S_{glob}(G)$	$6.61\cdot10^{-2}$	$5.68\cdot10^{-2}$	$7.24\cdot10^{-2}$	$6.89\cdot10^{-2}$
$S_{loc}(G)$	$1.15\cdot10^{-2}$	$1.22\cdot10^{-2}$	$2.31\cdot10^{-2}$	$2.19\cdot10^{-2}$

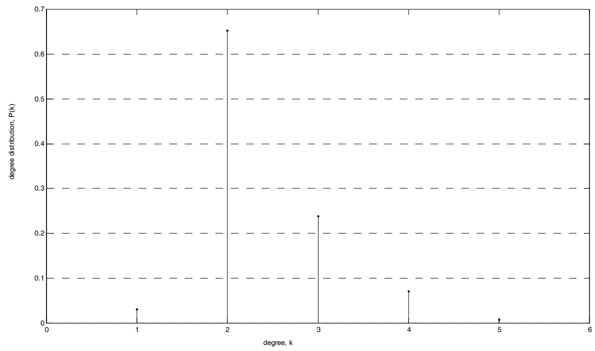

Figure 3. Degree distribution for the road network of *Figure 1*

4. Vulnerability analysis

Further insights in the properties of the road network of *Figure 1* can be inferred from an analysis of the most vulnerable road sections, i.e. those edges most crucial for the safe connectedness of the network [13]. When some edges are unavailable to travel, due to some road blockage, the safest paths between nodes change due to forced detours around the blockages. In this view, the vulnerability of the network is defined in terms of the degradation in the global safety efficiency of the network due to the disconnection, i.e. the interruption, of a set of its road links

$$V^* = \frac{S_{glob}\left(G\right) - S_{glob}\left(G^*\right)}{S_{glob}\left(G\right)}, \tag{11}$$

where G^* is the new graph resulting from G when the disconnected connections are taken out. By construction, V^* takes values in the range [0, 1].

Ranking the links in the network by decreasing vulnerability, it turns out that the 10 most critical roads form the highway path marked with circles in

*Figure **10***. The vulnerability values for these road sections are in the range $\left[2.09 \cdot 10^{-2},\ 3.20 \cdot 10^{-2}\right]$. Taking the whole path of 10 most critical links out of the network leads to the values of topological and efficiency parameters reported in the third column of *Table 2*. First of all, it is not surprising that the characteristic path length becomes infinite, since the nodes previously crossed by the now disconnected path are no longer reachable. Also, when the highway path is disconnected from the rest of the network there is the maximum decrease in global efficiency which stands for an increase in the accident probability in the whole system. This is caused by the need to substitute the missing links with paths along local and secondary streets which are on average longer and less safe. Actually, the vulnerability related to the disconnection of the highway path may even be larger in reality, due to the increase in traffic on the substitute paths. As for the increase in the network local properties (*Table 2*), this is due to the fact that the links deleted are in series in the network and thus bear null contribution to local parameters, whereas they contribute to the averaging in (4) and (6): thus, this increasing is fictitious and meaningless from a physical point of view.

4.1. Evaluation of a road development plan

Let us consider the road development plan concerning the bypass of the town of Piacenza, shown in *Figure 4.*

Figure 4. The road network with the bypass development plan: streets are drawn with dotted lines; the bypass is marked with circles.

In the fourth column of *Table 2*, the values of the global and local topological and safety indicators for the road network inclusive of the planned bypass are presented. At a global scale, an increase of 10 percent is noticed in the global safety efficiency, while an increase of 100% occurs at a local scale with respect to the original road network configuration. This was to be expected since the added roads create new alternative paths of increased safety efficiency, i.e. characterized by lower probabilities of car accident.

It is worth noticing that a relevant part of the improvement of the road development plan is due to the bypass (half of the increase in the global safety efficiency and almost the whole improvement in the local efficiency, as shown in *Table 2*, column 5). Consequently, this part should be considered of highest priority when implementing the developed plan.

Eventually, a vulnerability analysis of the augmented network shows that none of the added roads is among the 10 most vulnerable connections and that the maximum vulnerability of a link has decreased with respect to the original scenario ($V^{*} \in \left[1.83 \cdot 10^{-2}, 2.61 \cdot 10^{-2}\right]$, for the 10 most vulnerable connections). Thus, the system has become more resilient to faults in that the increase in accident probability following a road blockage is mitigated by the presence of the bypass.

5. Conclusion

In this paper, recent developments in the study of network systems from the point of view of Complexity Science have been exploited to study the global and local safety features associated to a complex road network. Newly defined, safety *efficiency* measures have been introduced considering the probabilities of car accident along the network interconnecting links. These indicators allow the analysis of the robustness and vulnerability of network systems, for optimal planning and operation.

The proposed approach has been applied to the evaluation of the road network system of the province of Piacenza, Italy. Roads are modelled as edges and points of road feature changes are marked as nodes. The predominant sequential structure of the network leads to a sparse adjacency matrix and affects the global and local connectivity properties in a negative sense. The vulnerability of the network in terms of the degradation of its safety when a road link is blocked has been used for identifying the most vulnerable paths which most contribute to the peril of car accidents.

Eventually, the method has been used to evaluate the safety improvement obtained with the introduction of a given road development plan.

Although the work is methodological in nature, it is of practical relevance that the proposed approach allows bringing the safety features into the analysis of the topology of a road network system.

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