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REGULAR PAPER

A novel hybrid of Nonlinear Sine Cosine Algorithm and Safe Experimentation Dynamics for model order reduction

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ABSTRACT

The current study introduces the hybridization of the Nonlinear Sine Cosine Algorithm (NSCA) and Safe Experimentation Dynamics (SED) as a novel optimization method for model order reduction of high-order single-input single-output (SISO) systems. Reciprocated synergism between both meta-heuristic algorithms is achieved by appropriating the nonlinear position-updated mechanism of NSCA for enhanced exploration/exploitation competencies and proficiency of SED in maximizing stagnation avoidance within the local optima. Named the NSCA-SED algorithm, the applicability of the proposed method is assessed by scholastic adoption of a sixth-order numerical transfer function towards two independent high-order systems enclosing Double-Pendulum Overhead Crane and Flexible Manipulator. Experimentation results further suggested NSCA-SED as the superior alternative in terms of execution robustness and consistency excellence against other available optimization-based methods for tackling model order reduction. Exemplified simulations sequentially demonstrated considerable improvements by the employment of NSCA-SED over conventional SCA following respective enhanced proportions of 97.17%, 13.17% and 29.03% for Example 1, Example 2 and Example 3.

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1. Introduction

Accelerated technological growth and advancement within current industries and societies have driven the development of scientific mechanisms, which streamline humanized tasks. Fundamentally depicted by the expression of complicated high-order equations, updated structures enclosing urban traffic, modern power and digital communication networks are vastly fabricated. However, the emergence of engineering and scientific challenges within system design, analysis, synthesizing and modelling facing consecutive trends of upward scaling owing to the systems' intrinsic high dimensions and reciprocated nonlinearities further stipulates the requirement for a specific algorithmic procedure like model order reduction (MOR) towards resolution.

MOR is conceptualized within the engineering sector for lowering the complexity of otherwise immensely complicated high-order dynamic structures. Being a mechanism that guides scholars and practitioners alike on apprehensions simplified systems and enables the reduction of required simulation and computation investments, the application has received widespread adaptation across numerous academic areas including engineering, renewable energy, track handling and electromagnetic systems. Revolutionary improvement has also been presented by many academicians in the previous decade for greater estimating precision of the developed reduction models, with the implementation of optimization algorithms, such as Krylov subspace [1], Hankel norm approximation method [2], Padé approximation [3] and Routh Hurwitz [4]. Because of the adequate outcomes from these approaches, prevalent setback ensues in the form of declined operational precision and consistency facing high-order dynamic models [5].

Thereafter, an exemplary solution has been pursued by academicians through the development of updated meta-heuristics algorithms, which undergo reduced-order modelling upon prior fixation of cost/objective functions. Due to their extensive competencies in maintaining systems' stability ensuing transition between high-order and reduced-order systems, employment of modernized meta-heuristics approaches towards settling encountered challenges across MOR structures has indeed received vast attention among scholastic communities in recent years. This is primarily exemplified by Mukherjee et al. [6] regarding the implementation of a Genetic Algorithm (GA) for systematic model order reduction. The Big Bang Big Crunch (BBBC) method was then recommended by [7] for a similar purpose without foregoing the existing stability of the initial system. The complexity of a MOR structure was also addressed via [8] through

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the adoption of Particle Swarm Optimization (PSO), with the employment of the Cuckoo Search Algorithm (CSA) alongside its derived algorithms being subsequently explored within the research by [9]. Following this, Bhatnagar and Gupta [10] in the context of high-order linear time-invariant systems have seen the application of Grey Wolf Optimizer (GWO) towards parametric optimization of their reduced-order models. A comparative study was then attempted by Nair et al. [11] in 2017 centering the efficacy of Ant Lion Optimizer (ALO) against alternative reduced-order approaches, including CSA and Gravitational Search Algorithm (GSA).

On another note, the GOA method was examined by Guha et al. [12] as a model reduction technique. Such endeavour has, nonetheless, been overshadowed by subsequent publication from Singh [13] in 2018, which successfully proposed the Sine Cosine Algorithm (SCA) as a superior MOR algorithm for high-order continuous structure against other efficacious order-reduction methods, including PSO, Elephant Herding Optimization (EHO) and Nelder-Mead Simplex Algorithm (NMSA), while maintaining an admirable operational simplicity. Moreover, optimization approaches, such as Harris Hawk Optimization (HHO) [14] and Salp Swarm Optimization (SSO) [15], have simultaneously emerged as exploitable algorithmic alternatives within modern academic and real-time MOR applications. With efficiency being established as a fundamental basis for the presumed effectiveness of the aforementioned MOR algorithms, estimating the precision of an order-reduced model would, therefore, be determined by the theoretical robustness of the specified optimization method. However, similar setbacks have been principally unveiled for most of the discussed techniques on extensive computation interval and premature convergence.

Uncovered shortcomings have consequentially driven scholastic examinations of better optimization alternatives through attempts of meta-heuristics hybridization, enclosing integrated algorithmic approaches, such as Particle Swarm Optimization-Differential Evolution Algorithm (PSO-DV) [16], Grey Wolf Optimizer-Chaotic Firefly Algorithm (GWO-CFA) [17], Particle Swarm Optimization-Bacterial Foraging (PSO-BF) [18], Bacterial Foraging-Modified Particle Swarm Optimization (BF-MPSO) [19], Particle Swarm Optimization-Gravitational Search Algorithm (PSO-GSA) [20], and Average Multi-Verse Optimizer and Sine Cosine Algorithm (AMVO-SCA) [21]. While such methods exploit the aggregated competencies of diverse optimization algorithms in resolving issues concerning model order reduction, their efficacious performances in the fabrication of lower-order systems are, nonetheless, offset by elevated complicatedness from existing coefficients that demand increased computational efforts and temporal investments. With this in mind, successive academic directions should acknowledge the need for hybridized optimization framework with reduced complexity and fewer foreseeable coefficients.

Ensuing previous discussion, this research is especially set to explore a contemporary hybridized algorithm known as Nonlinear SCA-Safe Experimentation Dynamics (NSCA-SED) with a definite mission of developing an efficient and uncomplicated optimization method for model order reduction. Being an essential segment to the proposed hybridization, the NSCA approach, as primarily reported by Suid et al. [22] in 2018, has been an updated variation inspirited by the conventional SCA from [23]. Upon revising execution regulation of the original algorithm through the incorporation of a nonlinear decreasing position-updated procedure, the updated algorithmic variant has succeeded in the resolution of optimization setbacks, including the identification of liquid slosh system [24] and energy generation of wind plant [25]. On the other hand, the SED method, which falls within an alternative subset of single-agent algorithms, is understood as a game-inspired optimization method that capitalizes on repetitions of multi-player stages towards strategic selections among existing solutions in conformity with its formerly specified strategic modification mechanism. Initially introduced within the founding paper by Marden et al. [26], an individual agent or player within the SED method would rely on its specific probability to maneuver in a random pattern. Supported by a stable performance record and convenient implementation among other single-agent algorithms, the applicability of said approach has since transpired across numerous engineering-related functions, including pantograph-catenary system [27], DC/DC buck–boost converter [28], and hybrid electric vehicles energy management system [29]. As such, an integration between NSCA and SED algorithms is anticipated to combine the nonlinear exploration/exploitation competency of the former and the random perturbation nature of the latter for increased outcome precision at appropriated operational simplicity. Currently proposed NSCA-SED approach has been principally preferred to tackle challenges encountered amidst the MOR process, with concurrent accounts for the problem's heightened complexity and the potential emergence of multimodal error, which further complicates the minimization of cost functions. This study would then present as founding research, regarding the implementation of multi-agent single-agent hybridization, for model order reduction.

The efficacy of the introduced NSCA-SED design as a robust model order reduction algorithm has been fundamentally validated above the structural mechanisms of one common high-order numerical system and two real-time high-order systems. A comparison has subsequently proceeded between the simulated performance of the proposed approach

Table 1. List of notations used in this paper.

$G_h(s)$	High-order system.
$G_r(s)$	Reduced-order system.
c_i and d_i	Numerator and denominator scalar constants of the high-order system.
v_i and w_i	Numerator and denominator scalar constants of the reduced-order system.
$u(t)$	Input signal.
$y_h(t)$ and $y_r(t)$	Output of the original high-order system and the reduced-order system.
f_j	Objective function of agent j.
$X_{ij}(k)$	Position of the agent j in the i-th dimension.
R^d	Set of real number vector.
P_i	Destination position in the i-th dimension.
$\boldsymbol{P}$	Current best solution obtained.
n	Number of agents.
r_1	Parameter to determine the next position's stage.
r_2	Random numbers between [0, 2π].
r_3	Random numbers between [0, 2].
r_4, r_5, r_6	Random numbers between [0, 1].
$\hat{r}_1$	Newly proposed parameter to determine the next position's stage.
a	Coefficient in r_1 and $\hat{r}_1$
k and $K_{\max}$	Current iteration and final iteration.
α and β	Coefficients to adjust the ratio of exploration and exploitation phases.
Λ	Coefficient that defines the probability of using the newly updated design variable.
t_f	Final simulation time.
ξ	Step size gain of the design variable.
z_{lo} and $z_{up}lb$ and ub	Lower and upper bound values of design variable.
z_i and $\bar{z}_i$	Current design variable and current best design variable.

and previously published optimization techniques for model reduction, including Padé approximation [3], GWO [10], ALO [11], SCA [13] GOA [12], AMVO-SCA [21], and iSCA [22] by benchmarked error indices of Integral-Square-Error (ISE), Integral-Time-Square-Error (ITSE), Integral-Absolute-Error (IAE), and Integral-Time-Absolute-Error (ITAE). Appraisals of nonparametric statistics in accordance with Wilcoxon's signed-rank analysis and block plot illustrations were further conducted for uncovering the comparable effectiveness of the proposed algorithm against its predecessor. Hereafter, notations and symbols as necessarily employed within this paper have been listed and described in Table 1.

The sectional distribution of the current paper has been systematically organized as follows: Formulated problem as encountered within the MOR process is described in Section 2. A comprehensive explanation of the NSCA-SED algorithm as contemporarily introduced for resolution of encountered setbacks within MOR is further outlined in Section 3. Following this, results acquired from the experimented simulation are discussed in Section 4, with conclusive remarks and recommended future scholastic direction of the current study being subsequently explained in Section 5.

2. Problem formulation

The MOR process is purposed for model order reduction facing an examined continuous-time SISO system for relieving the involved arithmetic simulation, while diminishing the required investment for numerical computation.

A secured high-order SISO system appropriating an accurate transfer function ($\ell \geq m$) is hereby considered:

$$G_h(s) = \frac{N_h(s)}{D_h(s)} = \frac{c_m s^m + \cdots + c_2 s^2 + c_1 s + c_0}{d_\ell s^\ell + \cdots + d_2 s^2 + d_1 s + d_0} \quad (1)$$

where scalar constants for the numerator and denominator are individually represented by respective notations of $c_i (i = 0, 1, \ldots, m)$ and $d_i (i = 0, 1, \ldots, \ell)$, respectively. As such, this study is aimed to exhaustively lower the order of transfer function $\mathrm{G}_h(\mathrm{s})$ without unnecessarily jeopardizing the precision and conceptual fundamentals of its conventional model. Specified transfer function towards the system's order reduction is, therefore, derived as follows:

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{v_p s^p + \cdots + v_2 s^2 + v_1 s + v_0}{w_q s^q + \cdots + w_2 s^2 + w_1 s + w_0} \quad (2)$$

where scalar constants for numerator and denominator of the reduced-order system (ROS) are independently represented by the respective notations of $v_i (i = 0, 1, \ldots, p)$ and $w_i (i = 0, 1, \ldots, q)$. $q < \ell$ is subsequently affiliated to remark a higher value of order $G_h(s)$ over order $G_r(s)$. Graphical representation enclosing undertaken mechanism towards model order reduction is then thoroughly outlined in Figure 1, with the input, the output as registered from the conventional system $G_h(s)$, and the output as registered from the reduced-order system $G_r(s)$ being independently represented by respective notations of $u(t)$, $y_h(t)$ and $y_r(t)$. In this case, an identical input signal has been applied towards operationalization of $G_h(t) = \mathcal{L}^{-1}\{G_h(s)\}$ and $G_r(t) = \mathcal{L}^{-1}\{G_r(s)\}$ in yielding their consequential outputs of $y_h(t) = G_h(t)u(t)$ and $y_r(t) = G_r(t)u(t)$. Inverse Laplace for the transfer function is further represented by notation $\mathcal{L}^{-1}$. Whereas the objective function would be executed upon discovering the error between $y_h(t)$ and $y_r(t)$.

The current study essentially uncovers unknown scalar constants for the respective numerator and denominator of ROS within the order $G_r(s)$ by the implementation of specified meta-heuristics approach for minimization of the following objective function:

$$\boldsymbol{F}(\theta) = \int_0^{t_f} [y_h(t) - y_r(t)]^2 dt. \quad (3)$$

for the design variable of

$$\theta = [v_0, v_1, \ldots, v_p, w_0, w_1, \ldots, w_q] \in \mathrm{R}^d \quad (4)$$

where $d = p + q + 2$, with a vector comprising real numbers within its element being represented by notation R^d. Notation $\boldsymbol{F}$ typically denotes the Integral Square Error (ISE) concerning outputs between transfer functions of examined high-order and reduced-order systems. Scholastic predisposition is deliberately recognized on the criterion of ISE minimization given

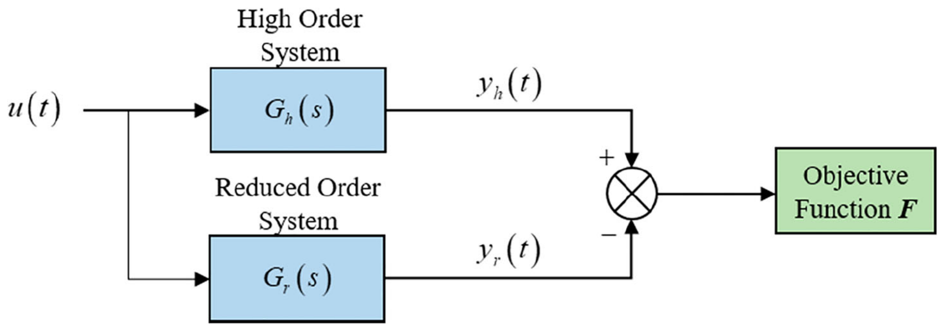

Figure 1. Illustration of the model order reduction process.

its nature for swift erroneous elimination within transient and steady-state responses. The main problem as formulated for the current paper is, therefore, described as follows:

Problem 2.1: Under the circumstance where system $G_h(s)$, input data $u(t)$ and output data $y_h(t)$ are known, design variables in (4) conforming $G_r(s)$, which minimizes the objective function $\boldsymbol{F}$ should be identified.

3. Model order reduction method

This section focuses detailed explanation of the specified technique employed for order reduction through the implementation of Nonlinear SCA-SED (NSCA-SED) as the contemporarily introduced hybridized optimization algorithm.

3.1. Overview of Sine Cosine Algorithm (SCA)

SCA is a multi-agent-based optimization approach known to operate in accordance with simple sine and cosine trigonometric functions. It directly identifies the common nature of multi-agent-based algorithms, arrays of search agents would be fabricated within the SCA, while speculatively distributed across an established search area as denoted by

$$\arg \min_{X_j(1), X_j(2), \ldots} f_j(X_j(k)) \tag{5}$$

where the objective function and position vector of j are represented by f_j and X_j, respectively. Destination position is eventually determined by algorithmic retention of superior positions throughout the updated iterations. Operationalized fine-tuning process would cease upon reaching the maximum literation, in which the position of the individual agent would be revised based on the sine and cosine functions as follows:

$$X_{ij}(k+1) = \begin{cases} X_{ij}(k) + r_1 \times \sin(r_2) \\ \quad \times |r_3 P_i - X_{ij}(k)| \text{ if } r_4 < 0.5, \\ X_{ij}(k) + r_1 \times \cos(r_2) \\ \quad \times |r_3 P_i - X_{ij}(k)| \text{ if } r_4 \geq 0.5, \end{cases} \tag{6}$$

for the iterations $k = 0, 1, 2, \ldots$, with the updated position of agent j and destination position within the i-th dimension being separately represented by respective notations of $X_{ij}(k)$ and P_i. The expression is hereby simplified by denoting the vector of the destination position by a notation $\boldsymbol{P}$ as the updated current best solution. SCA's core parameters that have been arbitrarily deposited are then represented by r_2, r_3 and r_4, with the adopted actual magnitude of operational value being subsequently represented by the symbol $||$. Notably, the registered range following positional movement as arbitrarily yielded across the designated span of $[0, 2\pi]$, which deserts or approaches, the destination position is defined by the parameter r_2. Whereas arbitrary weight of the destination position as randomized across the designated span of [0, 2], which is purposed towards stochastic expansion ($r_3 > 1$) and reduction ($r_3 < 1$) of registered range approaching the current position is defined by the parameter r_3. Interchanging sine and cosine functions as outlined within Equation (6) as arbitrarily obtained across the designated span of [0, 1] are further defined by the parameter r_4. Among others, the aforementioned explanation has, therefore, ascertained the parameter r_1 as the coefficient, which manoeuvres the algorithm's ensuing position between both independent stages of exploration and exploitation. Required equilibrium between exploration and exploitation competencies would be maintained in adherence to the equation r_1 as given by

$$r_1 = a - k\frac{a}{K_{\max}} \tag{7}$$

where the total sum of iterations and the updated iteration is independently represented by notations $K_{\max}$ and k, with $a$as a constant. Under the scenario where $r_1 > 1$, SCA would be probed to explore the globalized search region. Alternatively, the algorithm would limit its exploration within the local search region should $r_1 \leq 1$.

3.2. Overview of Safe Experimentation Dynamics (SED)

Safe Experimentation Dynamics (SED) is a game-theoretic method that characterizes individual elements within the design variable as independent players [26]. Random movements of individual players would then rely on formerly established probability by mean of loss function minimization towards succeeding specified optimized objective or design variable. On the accounts where z signifies the design variable, it shall be iteratively revised by SED in accordance with the updated criteria of

$$z_i(k+1) = \begin{cases} h(\bar{z}_i - \xi r_6) & \text{if } r_5 \leq \Lambda, \\ \bar{z}_i & \text{if } r_5 > \Lambda, \end{cases} \quad (8)$$

for $k = 0, 1, \ldots$, where independent arbitrary numbers, as consistently segregated across the span of [0, 1], are represented by symbols r_5 and r_6, coefficient, which describes the employment probability of the contemporarily revised design variable, is represented by the symbol Λ, while having its step size gain being represented by the symbol ξ. With the vector of the current best design variable as updated amidst iterative progression being denoted by notation $\bar{z}$, the i-th elements for z and $\bar{z}$ are further represented by symbols z_i and $\bar{z}_i$, respectively. Function h from Equation (8) is, therefore, given by

$$h(\cdot) = \begin{cases} z_{up} & \text{if } \bar{z}_i - \xi r_6 > z_{up}, \\ \bar{z}_i - \xi r_6 & \text{if } z_{lo} \leq \bar{z}_i - \xi r_6 \leq z_{up}, \\ z_{lo} & \text{if } \bar{z}_i - \xi r_6 < z_{lo}, \end{cases} \quad (9)$$

where formerly determined values for the design variable's lower and upper bounds are independently represented by notations z_{lo} and z_{up}, respectively.

The main aptitude of SED essentially capitalizes on its ability in retaining superior design variables throughout the tuning process towards enabling stabilized convergence. Further reinforced by the algorithm's adoption of fixed interval step size, which operatively differs from the sum of iterations, its potential practicality for the optimization purpose of order reduction is confirmed.

3.3. Proposed NSCA-SED optimization algorithm

An equal number of iterations has been especially appropriated to exploration and exploitation for the conventional SCA algorithm through linear reduction of r_1 Equation (7) from 2 to 0. Such phenomena have, nonetheless, prove detrimental to the algorithmic performance of SCA because of its overly restrained nature for operators' regulatory attempts to specify the desired exploration and exploitation proportions. An increasingly universal equation is, therefore, preferred to enclose vaster industrial applicability. Such revelation propels subsequent employment of decreasing nonlinear curve within Equation (7) which corresponds to the exponential function. The revised equation for r_1 following the undertaken adjustment is then written as

$$\hat{r}_1 = a\left(1 - \left(\frac{k}{K_{\max}}\right)^{\alpha}\right)^{\beta} \quad (10)$$

with contemporarily introduced nonlinear conversion indexes as purposed to modify exploration and exploitation proportions throughout the optimization process being denoted by respective notations of β and α. As such, notation $\hat{r}_1$ has been adopted in Equation (10) for the updated NSCA method as a direct substitution to the initial equation from Equation (7). The criteria for β and α would be determined at respective numerical arrays of, e.g. $\beta = 1.0$, $\alpha > 1.0$ or $\beta < 1.0$, $\alpha = 1.0$ to achieve greater exploration, with the contraries (e.g. $\beta = 1.0$, $\alpha < 1.0$ or $\beta > 1.0$, $\alpha = 1.0$) inversely generate greater exploitation.

Nevertheless, SCA is faced with the setback of performance deterioration due to local optima stagnation. Probable local optima entrapment has been particularly emanated from operational fundamentals of the algorithm for entirely overlooking fitness values that supersede the best global values while abandoning its potential set of solutions. Such deficiency is currently tackled by the incorporation of SED's random perturbation endeavour alongside the recommended NSCA framework. The decision is made in response to the abilities of design parameters for the current best agent and destination position through random perturbation in guiding the withdrawal of entrapped search agents from their local optima for consecutive venture of alternative search tracks. Random perturbation within SED's mechanism as outlined in Equation (8) has been graphically illustrated via a 2D representation in Figure 2 to enable extended simplicity and clarification, where separate design parameters (i.e. elements) as positioned within an established contour plot are represented by the x- and y-axes. Under the circumstance, where the agent X_j as denoted by a red-colored rectangle entered the local optima region, its withdrawal from said region is futile by the implementation of conventional SCA due to the algorithm's mechanism, which based its revised position on the agent's current position (refer to Equation (6)). With this in mind, the inclusion of random perturbation would prevail as a viable solution upon altering portions of the elements in adherence to the design parameters of destination position $\boldsymbol{P}$. Such occurrence is observable through random alteration in the second element of the agent X_j for the second design parameter of $\boldsymbol{P}$ at the x-axis in perturbing the former to a contemporary position as denoted by the green-colored rectangle, while it dismisses the challenge of local stagnation.

Moving forward, the general procedural layout of model order reduction with implementation of the introduced NSCA-SED method and its identified pseudocode have been separately illustrated in Figures 3 and

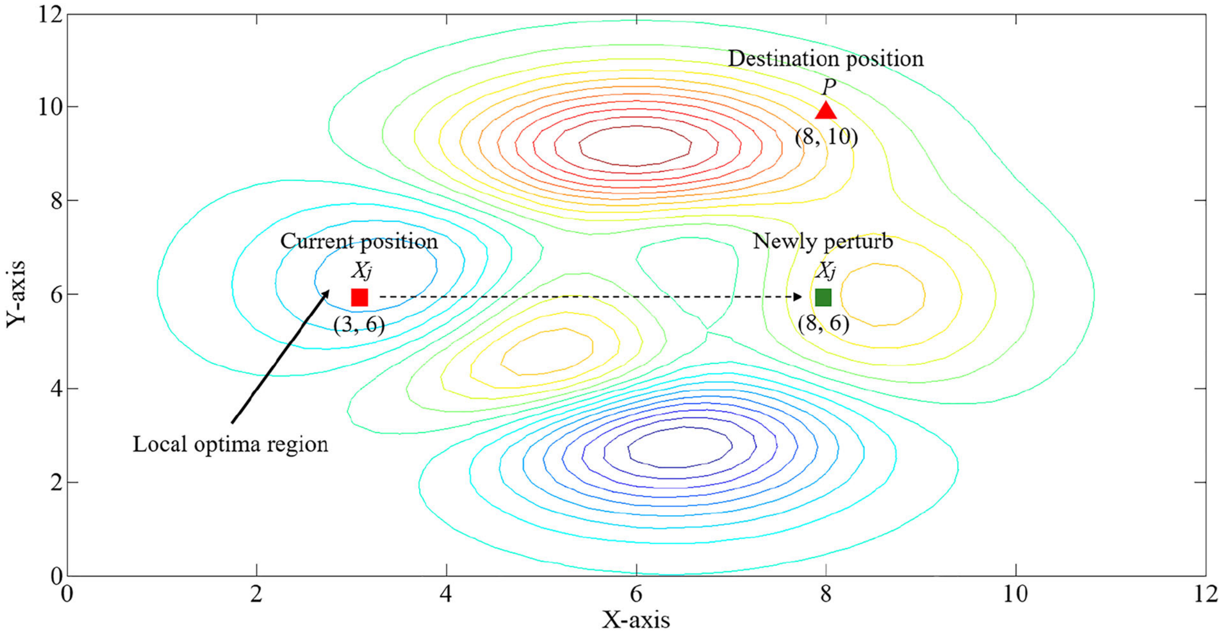

Figure 2. Illustration of SED's random perturbation mechanism.

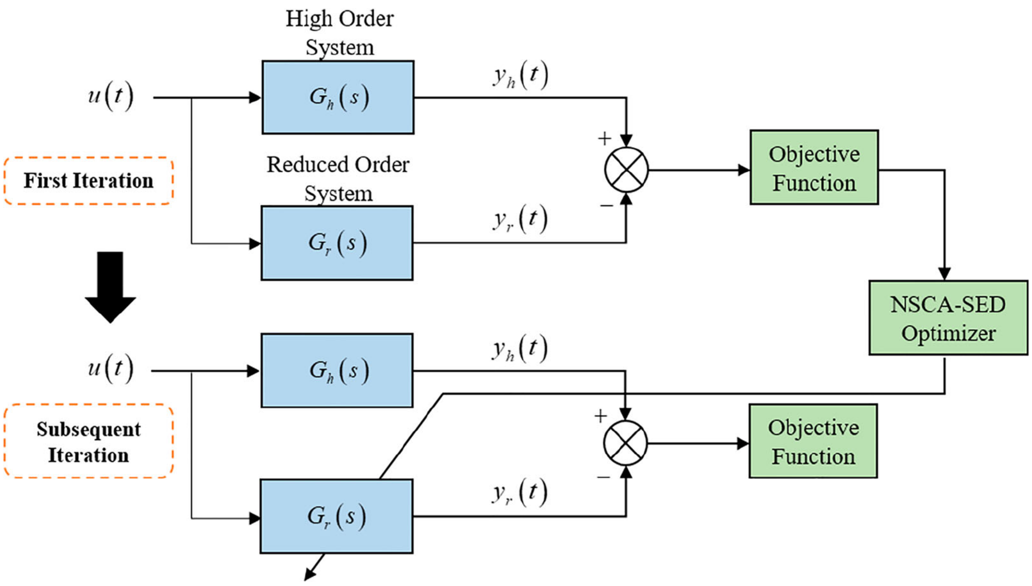

Figure 3. Diagram of NSCA-SED implementation for model order reduction problem.

4, respectively. Presumption ensuing the given pseudocode proclaims direct substitution of SCA's revised element in Equation (5) by design parameter of destination position P_i under the condition where the random number r_4 falls below probability Λ and vice versa. Substitution attempts between elements of SCA and destination position would, therefore, be executed conforming value of probability Λ. Integrated traits of NSCA-SED as proposed within the current study are further anticipated to prevent a setback of hasty convergence while ensuring improved optimization precision for the estimation of the reduced-order transfer function.

The systematic procedure for the implementation of NSCA-SED towards order-reduction of the examined system has been detailed as follows:

Step 1: Values of a, α and β from Equation (10), and Λ the SED algorithm are initially identified.

Step 2: NSCA-SED as outlined in Algorithm 1 is executed on the criteria of $\boldsymbol{F} := f_j$ and $\theta := X_j$ for individual agent j.

Step 3: Operationalization of the algorithm halts upon reaching the maximum iterations $K_{\max}$ at an acquired solution of $\theta^* := \boldsymbol{P}$, and a corresponding order-reduced system $G_r(s)$.

4. Experimental results and discussion

Effectiveness appraisal concerning order reduction of the examined system by the employment of the introduced NSCA-SED approach has been thoroughly outlined in this section. Generated performance from

Algorithm 1: Proposed NSCA-SED Algorithm

```
1. Set the initial parameters setting: upper bound (ub) and lower bound (lb), maximum
   iterations number (K_max), number of agent (n) and coeffecients a, β, α and Λ
2. Randomly initialize the position of each agents X_j (j = 1,2,3,...,...,n)
3. Evaluate the objective function of all agents f_j(X_j(0)) and determine the destination
   position P.
4. for k =1: K_max
5.     update r̂_1 using Equation (10)
6.     for i =1: d
7.         for j =1: n
8.             generate random numbers r_2, r_3, r_4 independently
9.             obtain the updated position X_ij(k+1) using Equation (6)
10.        end for
11.    end for
12.    for i =1: d
13.        for j =1: n
14.            if r_5 ≤ Λ
15.                X_ij(k+1) = P_i
16.            end if
17.        end for
18.    end for
19.    evaluate the objective function f_j(X_j(k+1))
20.    if f_j(X_j(k+1)) < f(P)
21.        P = X_j(k+1)
22.    end if
23. end for
24. Return P best solution obtained so far
```

Figure 4. Pseudocode of the proposed NSCA-SED optimization algorithm.

the proposed algorithm was fundamentally compared against other order reduction techniques as widely studied within the preceding literature, including Padé approximation [3], GWO [10], ALO [11], SCA [13], GOA [12], AMVO-SCA [21], and iSCA [22]. Circumstantial considerations were further allocated to three key scenarios, enclosing one sixth-order system numerical example, followed by the Double-Pendulum Overhead Crane and Flexible Manipulator as two alternative experimental-based systems. Except for the Padé approximation, experimented simulations were conducted across a span of 30 independent trials towards uncovering the optimal design parameters of the examined system. Whereas, the R2014a version of the Matlab software was adopted for simulated implementation of all compared methods, while adhering to the criteria as follows:

1. Precision analysis for objective function $\boldsymbol{F}$ followed the expression as given in Equation (3).
2. Arithmetic outcomes of each algorithm were assessed in accordance with obtained data for the box plot graphical and the nonparametric statistics of Wilcoxon's signed rank.
3. The robustness of each algorithm was assessed for the generated findings for ITSE, IAE, and ITAE. Herewith, performance indices for these components are arithmetically expressed by

$$\text{ITSE} = \int_0^{t_f} t([y_h(t) - y_r(t)]^2)dt, \tag{11}$$

$$\text{IAE} = \int_0^{t_f} |y_h(t) - y_r(t)|dt, \tag{12}$$

$$\text{ITAE} = \int_0^{t_f} t|y_h(t) - y_r(t)|dt. \tag{13}$$

4. Improvement ratios by percentage between the average objective function from the proposed algorithm and its compared MOR counterparts were recorded with the employment of the following computation approach:

$$\%\bar{\boldsymbol{F}} = \left(\frac{\bar{\boldsymbol{F}}_{\text{NSCA-SED}} - \bar{\boldsymbol{F}}_{\text{others}}}{\bar{\boldsymbol{F}}_{\text{others}}}\right) \times 100. \tag{14}$$

where average objective functions for NSCA-SED and its compared algorithmic approaches as recorded from 30 simulated trials are independently denoted by respective notations of $\bar{\boldsymbol{F}}_{\text{NSCA-SED}}$ and $\bar{\boldsymbol{F}}_{\text{others}}$.

4.1. Example 1

Aspired by the scholastic structure as formerly studied in Ref. [30], a sixth-order system has been accounted for as the first example of the reduced transfer function to a high-order system, with said transfer function being expressed by

$$G_h(s) = \frac{8s^5 + 3s^4 + 16s^3 + 20s^2 + 8s + 1}{\begin{array}{c}2s^6 + 33.6s^5 + 155.94s^4 + 209.46s^3 \\ +102.42s^2 + 18.3s + 1\end{array}}.$$

Under such circumstance, unit step was especially adopted as the input signal, with respective coefficients of NSCA-SED being pre-determined at $a = 2$, $\beta = 0.4$, $\alpha = 0.03$, and $\Lambda = 0.6$. Coefficients of other examined optimization approaches were simultaneously coordinated to the corresponding values from their founding publications. A standardized setting of 100 agents and 20 maximum iterations were, nonetheless, established for the simulations of all investigated algorithms to ensure unbiased comparability. With the upper and lower bounds being pre-determined at the values of $ub = 210$ and $lb = 1$, ROS for the current numerical example was then fixed as the given second- order system of

$$G_r(s) = \frac{v_1 s + v_0}{w_2 s^2 + w_1 s + w_0}.$$

On this note, the aforementioned best-fitted second-order reduced models, as yielded by the implementation of each investigated algorithm under the sampling interval of 0.001 s and simulation interval t_f of 80 s are cumulatively tabulated in Table 2. Step responses for the best design parameters across 30 independent trials as per registered from each investigated optimization approach are further demonstrated in Figure 5. Observable through the magnified section within the figure, the highest proximity between yielded responses from NSCA-SED as represented by the red-coloured line and the conventional full-order system as represented by the dashed black-coloured line has manifested considerably superior performance of the contemporarily introduced algorithm over its preceding ROS techniques. All the more so when such results are benchmarked against SCA [13] and GOA [12] where the precision of their generated responses was incomparable to the full-order response signals.

Further verification has been subsequently revealed by the execution of box plot analyses to the respective objective function $\boldsymbol{F}$ as generated by each investigated algorithm in Figure 6. Observably, unsatisfactory responses have prevailed for ALO [11], GOA [12], AMVO-SCA [21], and iSCA [22] from the aspect of operational consistency. With a value of 0.016, a higher median was, therefore, yielded by AMVO-SCA over ALO, GOA and iSCA at their respective values of 0.014, 0.01 and 0.006. An overly extensive interquartile range further acknowledged ALO [11] as the most underperformed optimization algorithm. While GWO [10] and SCA [13] have outperformed the previously mentioned algorithms, prolonged dotted lines at the upper domain of their box plots subsequently challenged the excellence of both meta-heuristics methods on a considerable number of outliers within their distributed data. Nevertheless, responses from the contemporarily introduced NSCA-SED approach have outshined the performances of its compared algorithms for having the minimal interquartile range of ≈0.000035 in the absence of apparent outliers. With possessing an exceptionally small median of ≈0.000018, the results of the proposed algorithm have demonstrated a remarkable level of consistency against other investigated methods.

Table 2. Reduced-order model obtained using different meta-heuristics optimization methods for Example 1.

Algorithms	Reduced model
Padé [3]	$\frac{31750s + 50}{327026s^2 + 32265s + 50}$
GWO [10]	$\frac{13.0407s + 18.2578}{114.3312s^2 + 202.9874s + 18.2309}$
GOA [12]	$\frac{23.1210s + 14.5917}{106.5665s^2 + 188.0288s + 14.3217}$
ALO [11]	$\frac{1.0131s + 1.0005}{9.4932s^2 + 11.3498s + 1}$
SCA [13]	$\frac{s + 20.8385}{1.3089s^2 + 210s + 20.8371}$
AMVO-SCA [21]	$\frac{24.1642s + 17.8441}{106.3492s^2 + 210s + 17.7771}$
iSCA [22]	$\frac{5.4781s + 18.1979}{81.5373s^2 + 210s + 17.9063}$
NSCA-SED	$\frac{2.7883s + 19.9813}{6.7473s^2 + 210s + 19.9433}$

Numerical representations of the acquired responses for each optimization algorithm are comprehensively outlined in Table 3. Notably, NSCA-SED has steadily outperformed its preceding counterparts for generating the smallest values of $\boldsymbol{F}$ in terms of average, maximum, and standard deviation statistics. Yielded averages of these statistical components for all investigated algorithms further revealed respective improvement ratios of 83.14%, 99.72%, 99.74%, 97.17%, 99.72% and 99.50% by NSCA-SED against the GWO [10], GOA [12], ALO [11], SCA [13], AMVO-SCA [21] and iSCA [22] methods. Wilcoxon's signed rank at an established significant threshold of $\sigma_w = 0.05$ was further executed within this study for ratifying existing differential significance between a pair of ROS methods, with the computed p-values being systematically outlined in Table 4. The sum of ranks by, which the proposed approach outperformed and underperformed against the capacities of its compared algorithms are hereby denoted by notations $\boldsymbol{S}^+$ and $\boldsymbol{S}^-$, respectively. As such, p-values, as recorded below 0.05 by the implementation of the NSCA-SED method have suggested its superior robustness among the currently examined techniques.

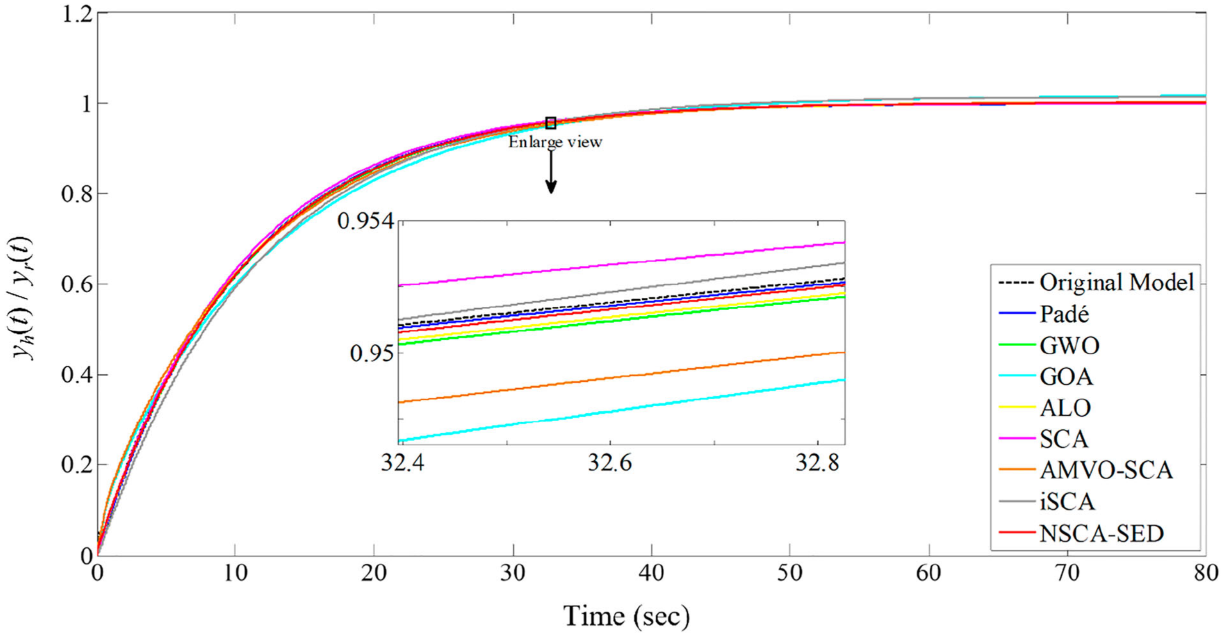

Figure 5. The response of the proposed NSCA-SED technique with other existing optimization-based model order reduction techniques for Example 1.

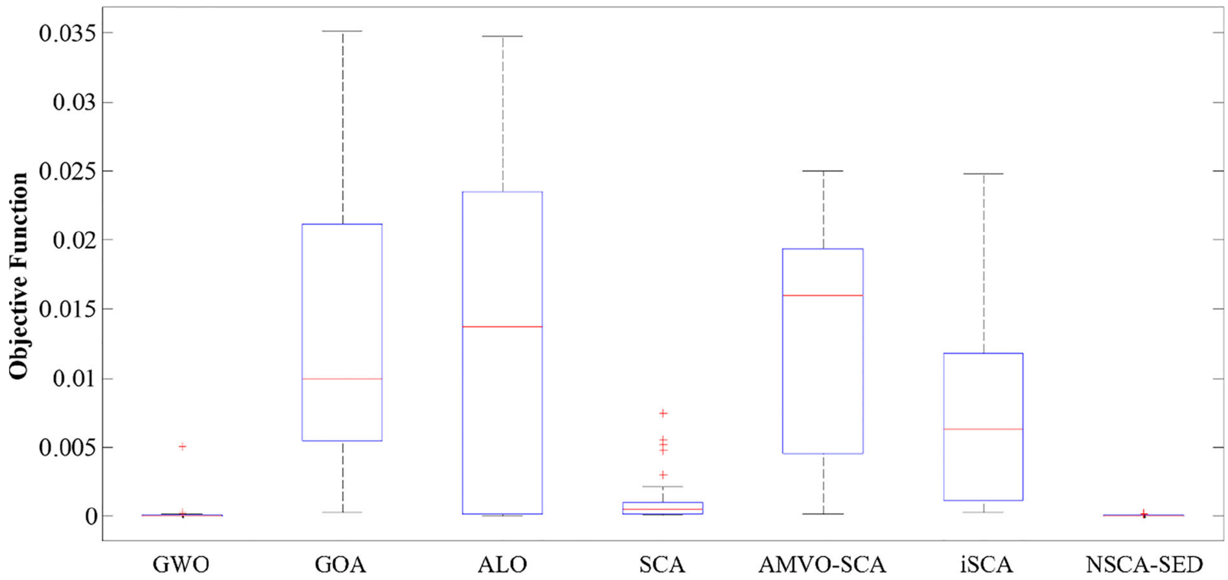

Figure 6. Box plot of the objective function *F* produced by different meta-heuristics optimization methods for Example 1.

Table 3. Statistical results of the objective function ***F*** for Example 1.

Algorithms	Average value	Min. value	Max. value	Std.
Padé [3]	–	9.26E−06	–	–
GWO [10]	2.13E−04	6.88E−06	5.05E−03	9.15E−04
GOA [12]	1.31E−02	2.86E−04	3.51E−02	1.11E−02
ALO [11]	1.41E−02	6.28E−06	3.47E−02	1.27E−02
SCA [13]	1.27E−03	3.53E−05	7.44E−03	1.93E−03
AMVO-SCA [21]	1.31E−02	1.09E−04	2.50E−02	8.20E−03
iSCA [22]	7.27E−03	2.51E−04	2.48E−02	6.67E−03
NSCA-SED	3.59E−05	6.46E−06	1.86E−04	4.17E−05

Table 4. Wilcoxon's signed rank test for the pairwise comparison between NSCA-SED with other meta-heuristics optimization methods in comparison in solving Example 1.

NSCA-SED vs	S^+	S^-	*p*-value
GWO [10]	344	121	2.93E−02
GOA [12]	465	0	1.86E−09
ALO [11]	355	100	2.05E−07
SCA [13]	406	59	9.31E−09
AMVO-SCA [21]	465	0	1.86E−09
iSCA [22]	465	0	1.86E−09

Such as on the accounts of a non-stochastic nature as exhibited by Padé approximation [19], which merely manifested minimal value in Table 3 without subsequent box-plot and Wilcoxon's signed-rank results.

Furthermore, recorded values for the common performance indices of ITSE, IAE and ITAE as generated by NSCA-SED and its meta-heuristics predecessors have been contrasted through their respective tabulations in Table 5. Such analysis has similarly considered

Table 5. Performance of ITSE, IAE and ITAE in comparisons for different meta-heuristics optimization methods for Example 1.

Algorithms	ITSE	IAE	ITAE
Padé [3]	4.55E−05	1.40E−03	2.01E−02
GWO [10]	1.63E−03	8.10E−03	2.15E−01
GOA [12]	1.58E−02	2.20E−02	5.53E−01
ALO [11]	1.66E−03	8.08E−03	2.04E−01
SCA [13]	1.87E−02	2.40E−02	6.16E−01
AMVO-SCA [21]	7.37E−02	4.89E−02	1.54E+00
iSCA [22]	1.17E−01	1.01E+00	5.98E−02
NSCA-SED	9.49E−04	5.76E−03	1.64E−01

minimal value as yielded by the conventional Padé approximation [19] on the evaluated indices. Acquired statistics have, yet again, showcased overwhelming achievements of the proposed algorithm for producing the smallest ITSE, IAE and ITAE values. Seemingly outperformed by the results from Padé approximation, sole reliance of such figures on a single indicator has, nonetheless, impaired the approach's convergence precision. Paired alongside previously discovered findings, NSCA-SED is fundamentally evidenced as the preferred optimization algorithm with admirable consistency over other investigated MOR techniques.

4.2. Example 2

Further inspired by the high-order system transfer function as studied in Ref. [31], the second scenario then considered an experimental-based sixth-order system for a Double-Pendulum Overhead Crane (DPOC). In this case, DPOC typically identifies an under-actuated nonlinear structure that comprises trolley force as a control input, and the trolley's displacement, hook and oscillated angles of the payload as three separate handling components. The transfer function of a DPOC system is, therefore, theoretically specified by

$$G_h(s) = \frac{0.1631s^4 + 9.114s^2 + 96.04}{1.06s^6 + 67.08s^4 + 874s^2}.$$

Graphically illustrated through Figure 7, a [−0.6, 0.6] amplitude bang-bang force signal has been determined as designated input to the examined full-order DPOC system by the implementation of the NSCA-SED approach at its established coefficients of $a = 2$, $\beta = 0.5, \alpha = 0.8$ and $\Lambda = 0.8$. Numerical settings of [0 0 0 0 0 10 0 0], [0.2 0.2 2 1 0.2 20 0.2 0.2], and 10 s were further established as the specifications for respective notations of ub, lb and t_f, with the sum of agents and maximum iterations being independently determined by $n = 20$ and $K_{\max} = 100$ for each investigated MOR technique. With other accounted components being essentially maintained as per the previous scenario (i.e. Example 1), undertaken preliminary simulation subsequently expressed the currently considered ROS as follows:

$$G_r(s) = \frac{v_2s^2 + v_1s + v_0}{w_4s^4 + w_3s^3 + w_2s^2 + w_1s + w_0}.$$

On a similar note, registered findings in terms of best design parameters and models of best-estimated reduced-order across 30 independent trials for individual optimization algorithms as explored within this study have been detailed in Figure 8 and Table 6. Mirroring the results of the first scenario, the estimated model as generated by NSCA-SED has shown commendable proximity to the model of a real-time full-order system, while having GOA [12] demonstrated an inadequate response outcome.

Arithmetic findings for objective function $\boldsymbol{F}$ by box plot representations and numerical descriptions

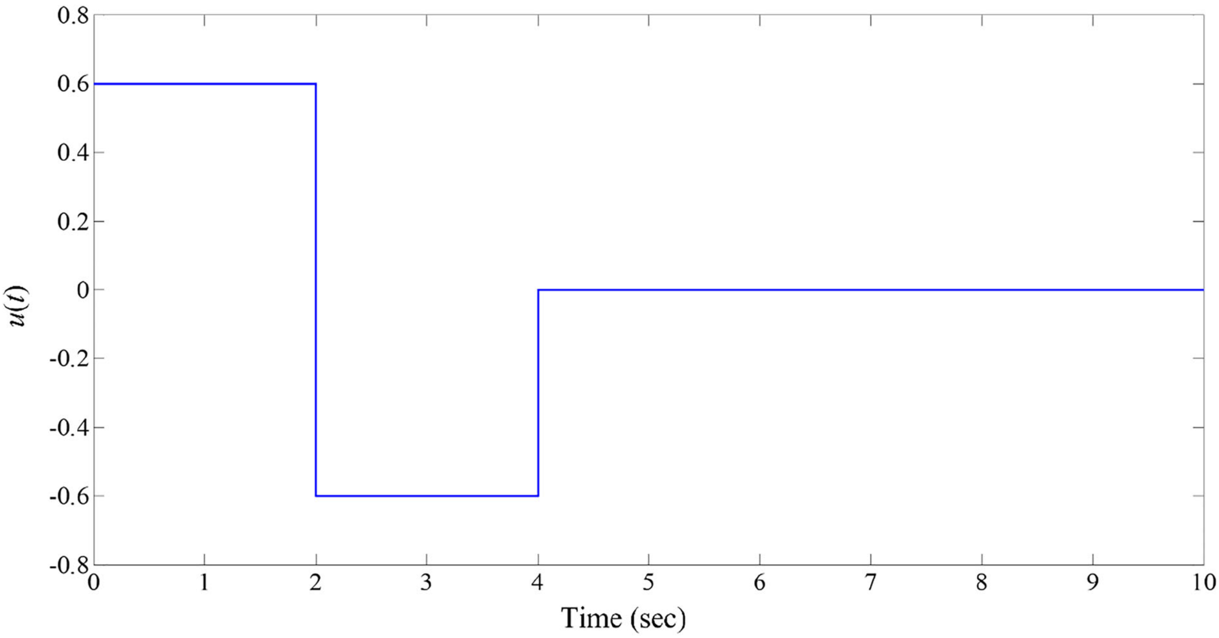

Figure 7. Bang-bang input signal $u(t)$.

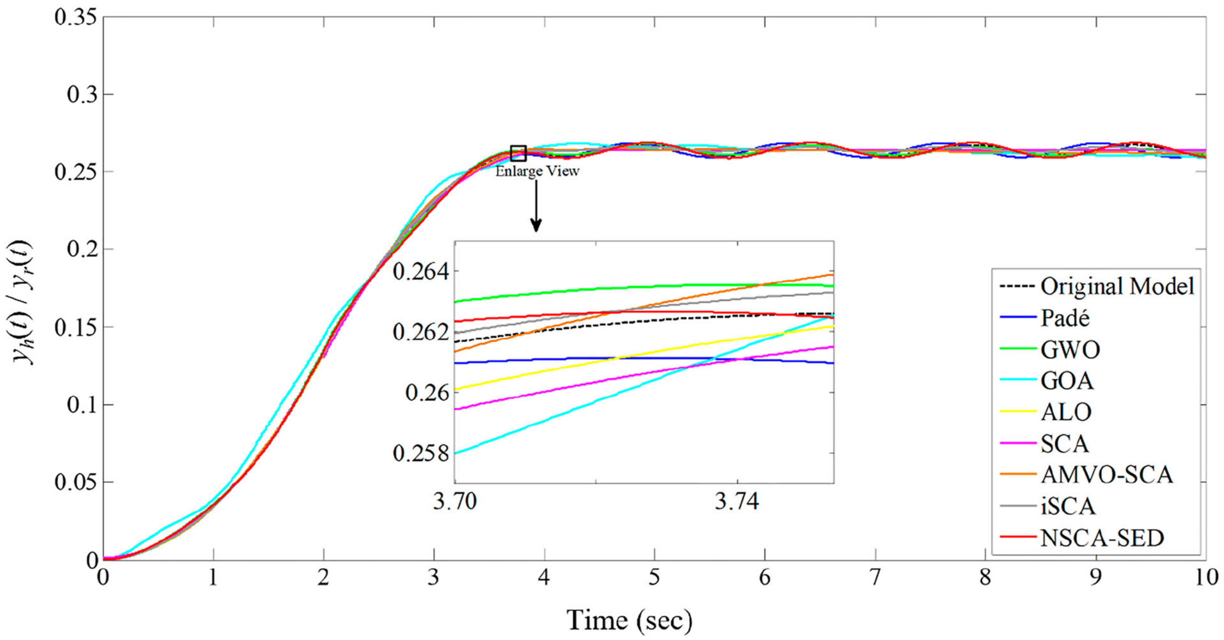

Figure 8. The response of the proposed NSCA-SED technique with other existing optimization-based model order reduction techniques for Example 2.

Table 6. Reduced-order model obtained by different meta-heuristics optimization methods for Example 2.

Algorithms	Reduced model
Padé [3]	$\frac{4.451E08s^2 + 6.531E09}{2.972E09s^4 + 5.943E10s^2}$
GWO [10]	$\frac{0.1364s^2 + 0.0221s + 1.7085}{0.8593s^4 + 0.2s^3 + 15.4371s^2 + 0.0244s + 3.8494E - 04}$
GOA [12]	$\frac{0.1545s^2 + 0.1206s + 1.7653}{0.4466s^4 + 0.1954s^3 + 15.8258s^2 + 0.0131s + 0.0121}$
ALO [11]	$\frac{0.0094s^2 + 0.0013s + 1.1537}{2.1993E - 07s^4 + 0.0017s^3 + 10.5034s^2 + 90.0820E - 09s + 1.8744E - 09}$
SCA [13]	$\frac{0.0227s^2 + 1.1121}{10.1187s^2}$
AMVO-SCA [21]	$\frac{0.1272s^2 + 0.0258s + 1.7481}{0.8530s^4 + 0.1066s^3 + 15.7453s^2 + 0.0193s + 0.0082}$
iSCA [22]	$\frac{0.0959s^2 + 1.6554}{0.7679s^4 + 14.9957s^2 + 0.0063s}$
NSCA-SED	$\frac{0.1562s^2 + 2}{s^4 + 0.0083s^3 + 18.21s^2}$

of average, minimum, maximum and standard deviation for each investigated algorithm are further outlined in Figure 9 and Table 7, respectively. Likewise, performance from Padé approximation [3] is solely indicated via minimal value as reported in Table 7 due to its non-stochastic nature. With possessing medians that approach the benchmarked figure of 0.000005, illustrated box pots have demonstrated immense competencies by most of the investigated meta-heuristics algorithms, such as GWO [10], GOA [12], SCA [13], AMVO-SCA [21], and iSCA [22] in consistency maintenance for the operationalization of model estimation. However, such proficiencies as demonstrated by GOA [12] and ALO [11] have been especially jeopardized by the existence of potential outliers on their distributed data. This phenomenon is all the more apparent for GOA [12] in exhibiting the longest box plot span as cumulatively formed by an extensive upper/lower range, interquartile range and median. Nevertheless, performances of preceding algorithms have been outshined by the contemporarily introduced NSCA-SED approach for statistically achieving the smallest average, minimum and maximum values. Table 7 consequently demonstrates improvement ratios by the implementation of the proposed algorithm against its compared MOR techniques by respective percentages of 22.04%, 98.15%, 52.19%, 13.17%, 50.22% and 9.75% on results obtained from the GWO [10], GOA [12], ALO [11], SCA [13], AMVO-SCA [21] and iSCA [22] methods. Whereas, the precision of minimal objective function

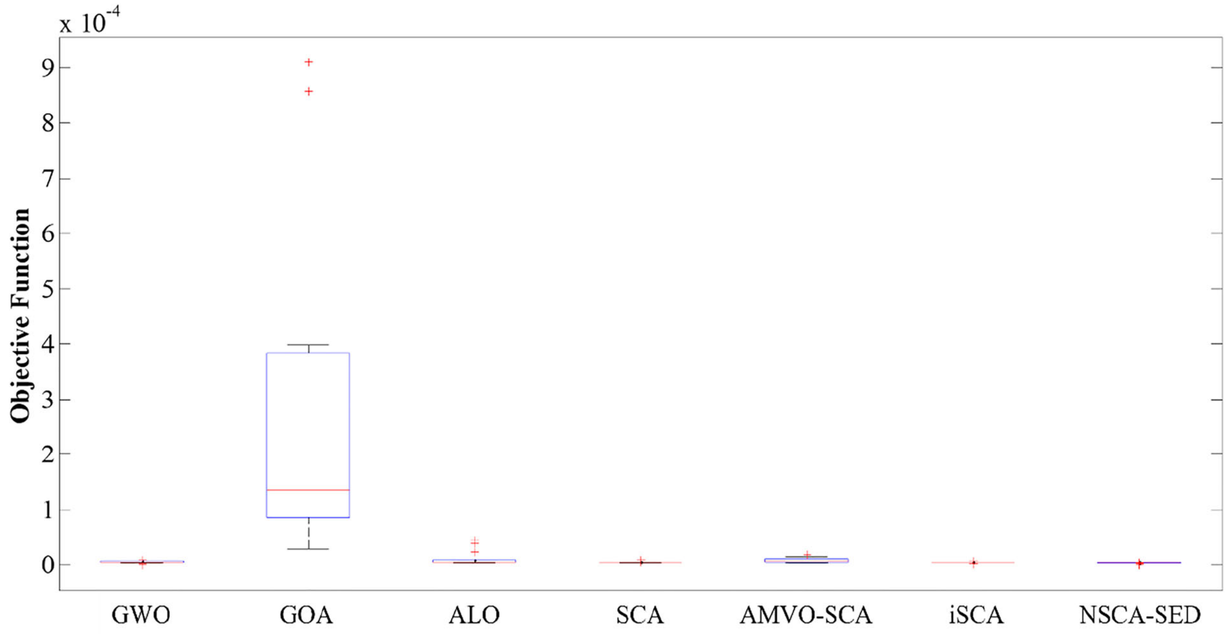

Figure 9. Box plot of the objective function F produced by different meta heuristics optimization methods for Example 2.

Table 7. Statistical results of the objective function $\boldsymbol{F}$ for Example 2.

Algorithms	Average value	Min. value	Max. value	Std.
Padé [3]	–	4.52E−06	–	–
GWO [10]	5.58E−06	9.16E−07	9.87E−06	1.65E−06
GOA [12]	2.36E−04	2.79E−05	9.10E−04	2.18E−04
ALO [11]	9.10E−06	4.45E−06	4.47E−05	1.00E−05
SCA [13]	5.01E−06	4.46E−06	8.81E−06	7.61E−07
AMVO-SCA [21]	8.74E−06	3.81E−06	1.79E−05	3.35E−06
iSCA [22]	4.82E−06	2.77E−06	6.64E−06	5.50E−07
NSCA-SED	4.35E−06	4.94E−07	4.93E−06	1.06E−06

Table 8. Wilcoxon's signed rank test for the pairwise comparison between NSCA-SED with other meta-heuristics optimization methods in solving Example 2.

NSCA-SED vs	S^{+}	S^{-}	p-value
GWO [10]	467	38	2.72E−04
GOA [12]	505	0	1.86E−09
ALO [11]	284	221	3.32E−04
SCA [13]	365	140	1.47E−04
AMVO-SCA [21]	505	0	1.82E−06
iSCA [22]	186	319	9.59E−02

as yielded by Padé approximation [3] is also overshadowed by the majority of other investigated meta-heuristics techniques because of its mere dependence on a single indicator.

Upon following up with the Wilcoxon's signed-rank test towards comparing the significance of NSCA-SED against other investigated optimization algorithms which demonstrated adequate consistency with the absence of potential outliers, obtained results as per outlined in Table 8 have supported the aforementioned discussion regarding the prevalence of the proposed algorithm forwarding compelling enhancement over respective outcomes of 2.72E−04, 1.86E−09, 3.32E−04, 1.47E−04, 1.82E−06 by GWO [10], GOA [12], ALO [11], SCA [13], and AMVO-SCA [21] at p-values below the pre-defined significance threshold of $\sigma_W = 0.05$. Such discrepancy is similarly observed for the statistical responses of ITSE, IAE and ITAE as tabulated in Table 9, with NSCA-SED being confirmed as the superior approach over its MOR predecessors. As opposed to haphazard predictions, combined findings in the second example have, therefore, validated the proposed NSCE-SED approach as a robust technique for model estimation of the currently specified DPOC system.

Table 9. Performance of ITSE, IAE and ITAE in comparisons for different meta-heuristics optimization methods for Example 2.

Algorithms	ITSE	IAE	ITAE
Padé [3]	3.49E−05	1.60E−03	1.09E−02
GWO [10]	3.16E−05	1.98E−03	1.09E−02
GOA [12]	6.95E−03	2.20E−02	1.52E−01
ALO [11]	3.74E−05	2.46E−03	1.19E−02
SCA [13]	2.92E−05	1.91E−03	1.08E−02
AMVO-SCA [21]	3.35E−05	1.90E−03	1.10E−02
iSCA[22]	2.75E−05	1.89E−03	1.04E−02
NSCA-SED	2.79E−05	1.89E−03	1.05E−02

4.3. Example 3

Aspired by the attempted endeavour in Ref. [32], the robustness of the NSCA-SED algorithm for model order reduction was ultimately assessed through the examination of an experimental-based high-order system transfer function for a Flexible Manipulator (FM) system within the current scenario. High-order system transfer function for the 10th-order FM system as published within the inspired paper is especially described by

$$G_h(s) = \frac{\begin{array}{c}3.141E-04s^8+1.287E-06s^7+5.708E-09s^6\\+1.528E-11s^5+2.873E-14s^4+4.419E-15s^3\\+3.254E-18s^2+5.364E-18s+2.915E-19\end{array}}{\begin{array}{c}s^{10}+2.37E-05s^9+2.37E-05s^8+1.21E-07s^7\\+1.727E-10s^6+5.915E-11s^5+3.861E-14s^4\\+6.764E-15s^3+2.63E-18s^2+4.33E-18s\end{array}}$$

As illustrated in Figure 10, a [−0.1, 0.1] amplitude-shaped bang-bang torque has been employed as the input signal within this example, with main coefficients of NSCA-SED being pre-established at respective values of $a = 2$, $\beta = 0.5$, $\alpha = 0.88$ and $\Lambda = 0.8$. While the upper and lower bounds of the current scenario were purposefully established as respective figures of $ub = 5$ and $lb = -4400$, the total number of agents, maximum iterations and simulation interval for all investigated MOR algorithms have been further decided at the values of $n = 40$, $K_{\max} = 1000$ and $t_f = 10$ s. The sampling interval for this scenario was then determined at 0.001 s to resemble prior settings within Example 1 and Example 2. In light of the mechanism of a real-time FM system, third order system has, therefore, been fundamentally employed as the current ROS which defines:

$$G_r(s) = \frac{v_2s^2 + v_1s + v_0}{w_3s^3 + w_2s^2 + w_1s + w_0}.$$

Herewith, estimated models for the best reduced-order as registered through the implementation of the investigated MOR techniques are thoroughly tabulated in Table 10, while having their responses centering the best parameters throughout 30 simulated trials being further illustrated in Figure 11. As observed through the magnified section of the recorded model responses, disparate proximities between the reduced-order models as predicted by each optimization method and the standard full-order model have once again elevated NSCA-SED as the superior estimation technique against its compared counterparts. Such excellence is broadened upon extensive discrepancies between the response of the proposed algorithm, and ominously unreliable outcomes as recorded from conventional Padé approximation [3], GOA [12] and ALO [11] which yielded the farthest responses to the benchmarked full-order model.

Further outlined in Figure 12, statistical attributes as exhibited from the numerical findings of the individual meta-heuristics approach are contrasted through conducted reviews on each fabricated box plot. except for AMVO-SCA [21], presented diagrams consecutively

Table 10. Reduced-order model obtained by different meta-heuristics optimization methods for Example 3.

Algorithms	Reduced model
Padé [3]	$\frac{9.382E86s^2+6.673E84s-1.024E83}{9.315E87s^3+1.262E86s^2-1.522E84s}$
GWO [10]	$\frac{-472.3083s^2-775.5699s-4384.6472}{-0.6548s^3-374.0298s^2-658.6744s+3.0091}$
GOA [12]	$\frac{-994.7226s^2-2027.4272s-4390.4791}{-1.3637s^3-833.6422s^2-1258.8069s-10.0422}$
ALO [11]	$\frac{-1909.1506s^2-2237.7168s-4395.2544}{-0.0001504s^3-1651.1546s^2-694.5873s-41.1992}$
SCA [13]	$\frac{-490.9549s^2-809.0557s-4400}{-1.0056s^3-397.8967s^2-592.3753s-1.2345}$
AMVO-SCA [21]	$\frac{-472.3083s^2-775.5699s-4384.6472}{-0.6548s^3-374.0298s^2-658.6744s+3.0091}$
iSCA [22]	$\frac{-460.2543s^2-757.3375s-4277}{-0.6548s^3-374.0298s^2-643.7179s+3.6480}$
NSCA-SED	$\frac{-467.4444s^2-753.2327s-4400}{-0.6799s^3-369.8236s^2-638.4794s-0.0044}$

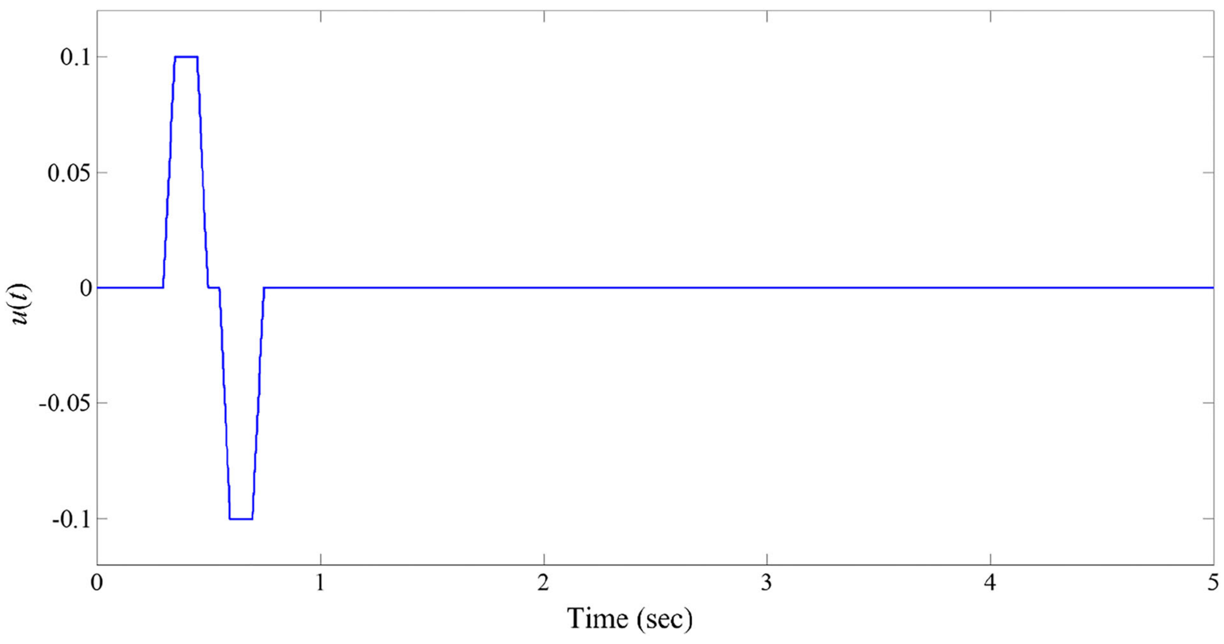

Figure 10. Shaped bang-bang input signal $u(t)$.

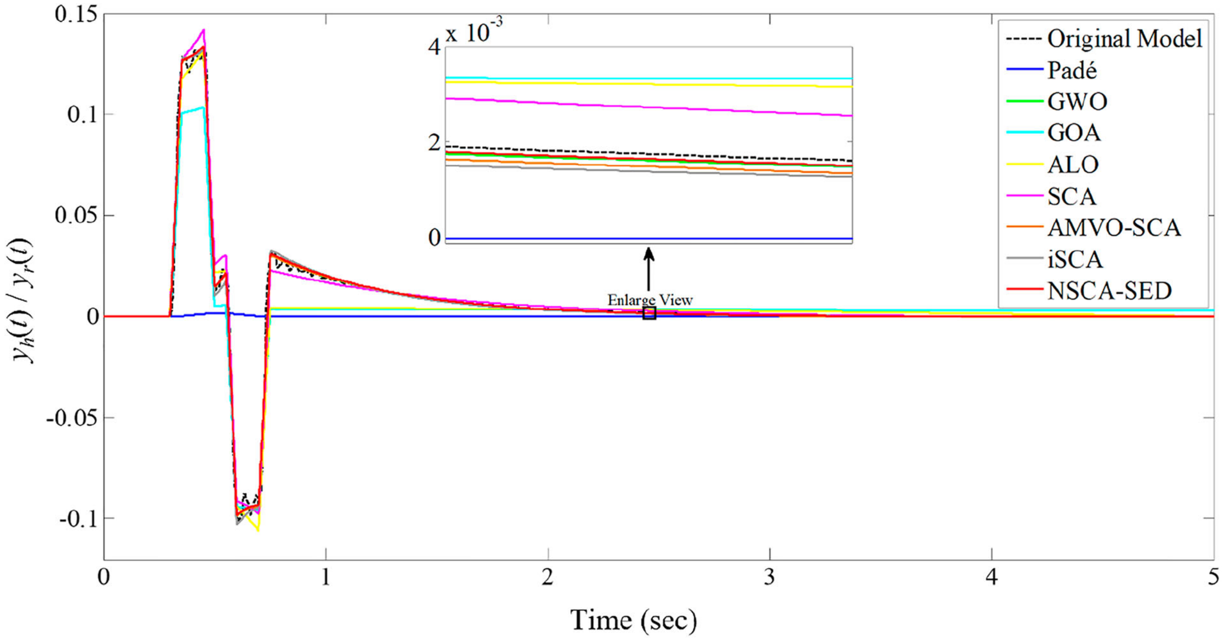

Figure 11. The response of the proposed NSCA-SED technique with other existing optimization-based model order reduction techniques for Example 3.

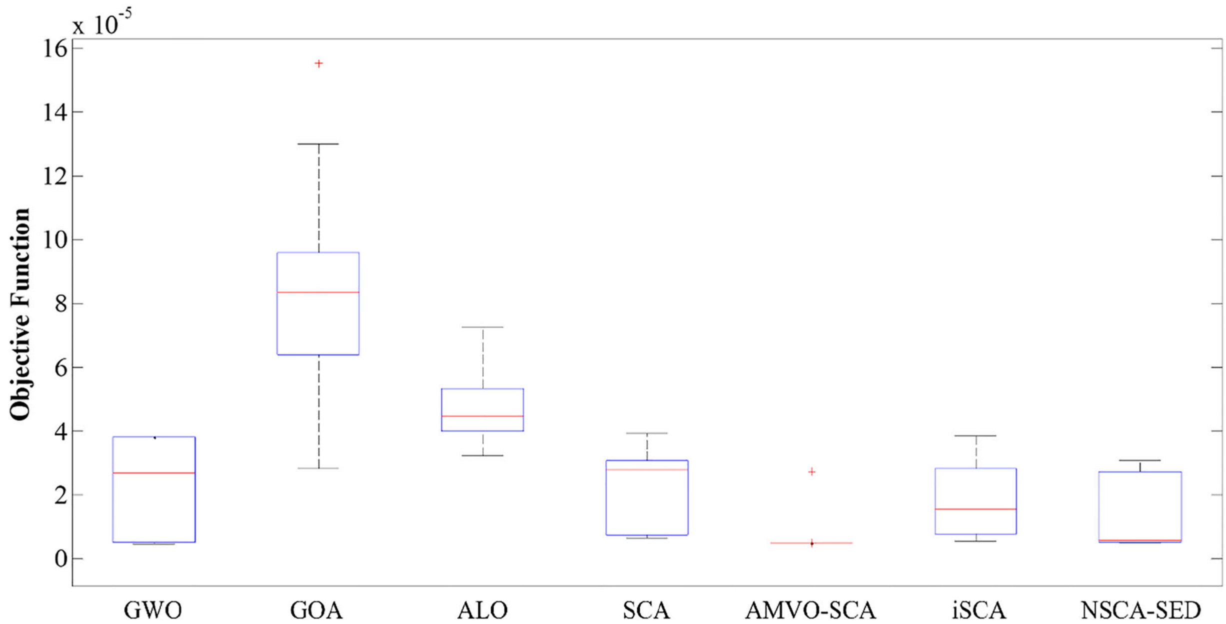

Figure 12. Box plot of the objective function *F* produced by different meta-heuristics optimization methods for Example 3.

suggested most of the investigated algorithms to possess heightened variation in terms of their estimated optimal solutions. Higher upper values, interquartile ranges and medians as recorded from GWO [10] and GOA [12] have notably positioned these algorithms as techniques with the greatest inconsistency. Consistent performance from AMVO-SCA [21] has, nonetheless, been adversely offset by the existence of potential outliers. On the contrary, a lower interquartile range of, i.e. $\approx$0.000022 as generated by the contemporarily introduced NSCA-SED approach with the absence of apparent outliers has demonstrated enhanced data distribution for its optimized resolution. Such superiority is subsequently backed by a considerable smaller median value of 0.0000051 by the proposed algorithms against ALO, GOA, SCA and iSCA at respective values of 0.000045, 0.000084, 0.000028 and 0.000015.

Undertaken comparison centering statistical findings of all investigated algorithms through components of average, minimum, maximum and standard deviation for objective function ***F*** are then systematically outlined in Table 11. With AMVO-SCA [21] achieving the smallest values for aspects of average, maximum and standard deviation, the proposed NSCA-SED approach has, thus, outperformed other MOR algorithms including GWO [10], GOA [12], ALO [11], SCA [13] and iSCA [22] to secure the runner-up position. In particular, the average value as yielded

Table 11. Statistical results of the objective function ***F*** for Example 3.

Algorithms	Average value	Min. value	Max. value	Std.
Padé [3]	–	7.44E−04	–	–
GWO [10]	2.06E−05	4.56E−06	3.82E−05	1.44E−05
GOA [12]	8.19E−05	2.82E−05	1.55E−04	2.80E−05
ALO [11]	4.81E−05	3.22E−05	7.28E−05	9.77E−06
SCA [13]	2.17E−05	6.28E−06	3.92E−05	1.19E−05
AMVO-SCA[21]	6.38E−06	4.85E−06	2.72E−05	5.65E−06
iSCA [22]	1.86E−05	5.56E−06	3.84E−05	1.11E−05
NSCA-SED	1.54E−05	4.92E−06	3.05E−05	1.12E−05

by NSCA-SED has demonstrated substantial improvements against the results of respective GWO [10], GOA [12], ALO [11], SCA [13] and iSCA [22] by the ratios of 25.24%, 81.19%, 67.98%, 29.03% and 17.20%. Following the non-stochastic nature of Padé approximation [3], which addressed the technique's statistical output by minimum value as a single indicator in Table 11, its performance is inevitably outclassed by similar reading from GWO [10].

Conducted experimentations have essentially confirmed overshadowing competencies of meta-heuristics techniques towards improved execution precision against the traditional Padé approximation [3] technique. Gathered findings within the current scenario alongside results from previous examples (i.e. Example 1 and 2), therefore, cumulatively substantiated the operational excellence of NSCA-SED against its algorithmic predecessors within the context of the model order reduction.

By the exclusion of the non-stochastic Padé approximation [3], additional pieces of evidence are acquired by paired comparisons between the introduced NSCA-SED approach and its preceding MOR algorithms above the statistical basis of Wilcoxon's signed-rank test. Registered responses in Table 12 that displayed p-values of the proposed algorithm beneath the established significance threshold of $\sigma_w = 0.05$ have, yet again, explained apparent improvements against all compared algorithmic approaches. Recorded findings for the aspects of ITSE, IAE and ITAE by the implementation of individual meta-heuristics techniques in Table 13 further demonstrated good proficiency of GWO [10] for ITSE and IAE components in the handling of an FM system. However, the robustness of the said algorithm is outperformed by NSCA-SED for ITAE at a recorded value of 5.00E−04. Except for traditional Padé approximation [3] that merely accounts for the minimal values as acquired for each evaluated component, substantial improvement at individual differences of 0.008, 0.00006, 0.00036 and 0.007 against respective MOR techniques of GOA [12], GWO [10], AMVO-SCA [21] and iSCA [22] have prevailed through the employment of the contemporarily proposed algorithm. Mirroring trends on the formerly discussed statistical figures, results as comprehensively registered from the majority of the investigated meta-heuristics approaches have outshined rendered efficacy of Padé approximation [3]. Simulated findings from Example 3 have, thus, collectively manifested NSCA-SED as the utmost competitive technique for model reduction of FM structures.

Table 12. Wilcoxon's signed rank test for the pairwise comparison between NSCA-SED with other meta-heuristics optimization methods in solving Example 3.

NSCA-SED vs	S^+	S^-	p-value
GWO [10]	400	65	2.74E−02
GOA [12]	465	0	1.86E−09
ALO [11]	465	0	1.86E−09
SCA [13]	386	79	1.01E−02
AMVO-SCA [21]	27	478	5.08E−05
iSCA [22]	383	82	4.27E−02

Table 13. Performance of ITSE, IAE and ITAE in comparison with different meta-heuristics optimization methods for Example 3.

Algorithms	ITSE	IAE	ITAE
Padé [3]	3.98E−04	1.04E−02	8.20E−03
GWO [10]	2.67E−06	6.60E−04	5.57E−04
GOA [12]	6.13E−05	5.04E−03	8.48E−03
ALO [11]	3.82E−05	3.59E−03	5.20E−03
SCA [13]	6.40E−06	1.39E−03	1.59E−03
AMVO-SCA [21]	2.77E−06	7.47E−04	8.60E−04
iSCA [22]	3.69E−05	3.66E−03	7.47E−03
NSCA-SED	2.72E−06	6.75E−04	5.00E−04

5. Conclusion

NSCA-SED algorithm has been introduced within this paper as a contemporarily hybridized optimization approach for addressing the shortcoming in model order reduction of a high-order SISO system. The system's robustness was hereby elevated using the proposed meta-heuristic technique by exploiting aggregated proficiencies of random perturbations of SED and efficacy of the NSCA algorithm. The system's performance in terms of model order reduction as generated from the implementation of the NSCA-SED approach was further measured against outcomes yielded from the Padé approximation and other prevalent optimization approaches including GWO, GOA, ALO, SCA, AMVO-SCA, and iSCA-based methods. Recorded statistics were then appraised for the common indicators of ISE, ITSE, IAE and ITAE, besides the quantified significance of each algorithm through the assessment of Wilcoxon's signed rank and box-plot analyses. Securing over 13% improvement towards the operational performance of the conventional SCA, average objective function as obtained from 30 independent simulated trials ultimately demonstrated excellent coherence and precision of the NSCA-SED approach in resolving the issue as encountered amidst model reduction against its compared algorithmic predecessors. Motivated by the technological insurgence of Industrial Revolution 4.0, the upcoming scholastic direction within this area of study should subsequently emphasize the tunable parameters of NSCA-SED to seek extensive generalizability on alternative real-time challenges, such as smart home energy dispatch and charging of electric vehicles at a larger scale.

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