The transit time of sound in a phononic crystal

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ABSTRACT

Electron material waves cannot permeate a periodic atomic lattice at each energy or frequency. There exists a forbidden gap due to the periodicity of the atoms. In analogy, acoustic waves cannot penetrate a phononic or sonic crystal at each frequency. A two-dimensional sonic crystal consists of a periodic lattice of cylinders. The periodicity is adjusted according to the wavelength of sound. Depending on the frequency, there exist "allowed" bands with a propagation of the waves as well as a "forbidden" band without propagation corresponding to the bandgap in a semiconductor. The mathematical description of the phenomena in the sonic crystal and in the atomic crystal is technically similar. Here, we investigate experimentally the velocity of sound in a sonic crystal by measurement of the wave's transit time through the crystal. The velocity in the crystal depends on the frequency and is smaller than the velocity in air.

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A phononic or sonic crystal is comprised of a periodic arrangement of cylinders in a host medium, i.e., in the present case, in air. Cylinders and air have different acoustic properties in such a way that the waves are scattered at the interface of the two materials. The sonic crystal exhibits properties for the propagation of acoustic waves similar to the propagation of material waves, i.e., electron waves, in an atomic crystal.^{1,2} The periodicity of the cylinders or the atoms causes that, depending on the energy or the frequency, the wave can permeate the material, or the wave is damped to zero, in an ideal case. This corresponds in semiconductors to the conduction and the valence band, as well as to the forbidden band, the gap. In an elementary manner, this is deduced with the Kronig–Penney model.³ In the sonic crystal, the periodicity of the cylinders causes bands of propagating sound waves and a forbidden band, where the waves are damped to zero.

The aim of this investigation is to determine the phase velocity of sound in the phononic crystal in dependence of the frequency.

Early experiments with sonic crystals have been reported in Refs. 4 and 5. Here, the sonic crystal is comprised of acrylic cylinders in a regular periodic array in air. The cylinder's diameter and their center distance are usually of the order of centimeter.⁶ These measures define the bandgap in the spectrum of the wavelengths. A mathematical derivation of the relation between geometry and propagation properties is found in Ref. 4. In this investigation, we use an array of 14×14 acrylic cylinders with a length of 37 cm,

diameter of 1 cm, and center distance of 2.4 cm. The cylinders are fixed in polyvinylchloride (PVC) plates, in direction [100].⁷ To suppress wave reflections, mineral wool is sticked to the plates.⁶

A schematic plan of the arrangement is shown in Fig. 1. The signal of a wave generator is amplified and fed into a loudspeaker placed in front of the first row of acrylic cylinders. At a distance s = 33 cm from the loudspeaker, a microphone is mounted. Between the loudspeaker and the microphone, either the phononic crystal, d = 32.2 cm, is installed or the sound crosses free air. The signal received by the microphone is evaluated with respect to the amplitude and phase. The amplitude is either measured by a voltmeter (Keithley Multimeter 2000) or the signal is recorded by a sampling oscilloscope (Rigol DS 1202Z-E) to evaluate the amplitude and phase.

The experiments are conducted as follows: For the measurement with the voltmeter, a continuous signal is generated and its amplitude at the microphone is measured. For the measurement with the sampling oscilloscope, bursts with 10 sinusoidal waves are created by the generator. The signals correspond to frequencies between 1.5 and 10 kHz. After 6 periods, i.e., 6 maxima, the signals are in a stationary state. Periods 7–10 are recorded to confirm the stationarity. To reach the stationary state, the transient behavior of the instruments, i.e., the loudspeaker, the microphone, and the microphone's amplifier, as well as the transient behavior of the sound in the sonic crystal itself play a role. Since the measurements



FIG. 1. The experimental arrangement with signal generator, loudspeaker, test track, i.e., either the sonic crystal or air, microphone, and sampling unit or voltmeter. s: distance between the loudspeaker and microphone; d: length of the phononic crystal.

are carried out either with the sonic crystal or with air between the loudspeaker and the microphone, the transient behavior of the instruments can be canceled out. The air spaces of total length (s-d)between the loudspeaker and crystal as well as between the microphone and crystal increase the transit time of sound in both cases also. Their influence can be canceled out as well. Figure 2 depicts typical signals at a nominal frequency of 5 kHz either recorded with air between the loudspeaker and microphone [Fig. 2(a)] or with the sonic crystal [Fig. 2(b)]. The input is the electrical signal at the loudspeaker, and the output is the electrical signal at the microphone's amplifier.

The height of maximum number 6, the amplitude A_6 , measured in air, A_6^{air} , and measured with the crystal, A_6^{cry} , yields the attenuation by the crystal [Eq. (1a)]. For different nominal frequencies, the ratio

$$\alpha_6 = \frac{A_6^{cry}}{A_6^{air}} \tag{1a}$$

is plotted in Fig. 3. For comparison, the ratios of the amplitudes for continuous waves

$$\alpha = \frac{A^{cry}}{A^{air}} \tag{1b}$$

recorded with the voltmeter are also shown. They confirm the measurement using the burst. The transmission gap, also reported by several other authors, is visible.^{5–9}

To determine the velocity of sound in the sonic crystal, we measure the time t between the beginning of the burst at the loudspeaker's input and the zero crossing of the microphone's signal after the maximum number 6, see Fig. 2. This is carried out at different nominal frequencies, i.e., each 500 Hz, for air, yielding t_1 , and for the sonic crystal, yielding t_2 . t_1 comprises the transit time of the sound along the distance d, see Fig. 2(a), in air, the time to cross (s–d) as well as the settling time of the instruments, i.e., the loudspeaker and the microphone. Due to the instruments settling time t_{instr} and with c as the velocity of sound in air, c = 340 m/s, we have



FIG. 2. The transmission of a burst with a nominal frequency of 5 kHz for air (a) and for the sonic crystal (b).



FIG. 3. The ratio of the amplitudes in air and with the crystal measured by the sampling oscilloscope and by the voltmeter for comparison. The arrow indicates the sonic gap.



FIG. 4. The phase velocity of sound in the sonic crystal c_{cry} and the velocity of sound in air c. The arrow indicates the sonic bandgap.

$$t_1 = \frac{d}{c} + \frac{s-d}{c} + t_{instr.}$$
(2)

When the sound propagates through the crystal, we measure t₂ [Fig. 2(b)] as follows:

$$t_2 = \frac{d}{c_{cry}} + \frac{s - d}{c} + t_{instr},\tag{3}$$

where c_{crv} is the velocity of sound in the crystal to be evaluated,

$$c_{cry} = \frac{d}{t_2 - t_1 + \frac{d}{c}}.$$
(4)

In this way, the influence of the instruments and the air space on the measurements cancel out. The result is plotted in Fig. 4.

It was already experimentally found and theoretically predicted¹⁰ that the phase velocity of sound in sonic crystals is generally lower than the velocity in air. This is confirmed, but here the velocity is resolved with respect to frequency. Both, for frequencies below the bandgap and above the bandgap, the speed of sound slightly decreases.

After the end of the transient effects, the sound wave is monochromatic. Within this monochromatic wave, the zero crossing following the sixth maximum is looked at and the time to this crossing is measured. Since, for the measurement, the wave is in a







FIG. 6. The signal, 7 kHz, within the bandgap of the crystal at the microphone.

monochromatic state, the result obtained with Eq. (4) is independent of the number of passed periods or maxima. Therefore, the time to a constant phase is determined. Thus, Eq. (4) yields the phase velocity of the signal in the crystal. The phase velocity is directly related to the radian frequency $2\pi f$ using the wavevector k as follows:

$$2\pi f = c_{cry} \cdot k. \tag{5}$$

With the experimental result (Fig. 4), the relation between f and k is plotted in Fig. 5,

$$k(2\pi f) = \frac{2\pi f}{c_{cry}}.$$
(6)

Within the gap, the measurement of t_2 is not possible. This becomes evident with Fig. 6. The figure depicts the signals for restricted propagation in the crystal at a frequency of 7 kHz, i.e., in the gap. The zero crossings recorded for the crystal, necessary to determine t2, are irregular. A well-defined velocity of sound so far

52 µs microphone voltage outputs 2 air crystal n 1050 1150 900 950 1000 1100 1200 time [µs]

FIG. 7. An expansion of the time axis of Fig. 2 around 1000 μ s. The signal of the crystal is delayed by 52 μ s against the signal through the air.

cannot be obtained in the sonic gap. A nonlinear distortion of the waves is observed.

Qualitatively, the slowing down of the sound's velocity can be seen also with an expansion of the time axis of Fig. 2 around the time the microphone signals start, i.e., close to $1000 \ \mu s$ after the beginning of the burst, see Fig. 7. The signal, which crossed the air, is compared to the signal through the crystal. With the crystal, the signal is delayed by about 52 μs . However, this is only a qualitative statement since at the beginning the signal is not in a stationary state. The tendency to delay the signal is confirmed.

It is evident from Fig. 2(b) that the microphone signal received after crossing the crystal has an echo. This experimental finding can only be obtained employing this direct measurement of the received wave. It cannot result from measurements where only the phase shift between input (loudspeaker signal) and output (microphone signal) is determined.

The velocity of sound waves in a sonic crystal is smaller than the velocity of sound in air, and it depends on the frequency, i.e., it has a dispersion. A bandgap for the propagation of sound is measured. Within the gap, the amplitude is damped, and we find nonlinear distortions of the wave. These results are a consequence of the periodicity of the sonic crystal's elements. In electrical conductors with their electron material waves, the periodicity of the atoms yields allowed and forbidden bands for the movement of electrons similar to the gap in the sonic crystal. In addition, in electronic conductors, the relation between energy and wavevector, the band structure, which is determined by the lattice structure, yields a dispersive velocity of the electron waves.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflict of interest to disclose.

Author Contributions

H.K. and R.B. contributed equally to this work.

Herbert Kliem: Conceptualization (equal); Investigation (equal); Methodology (equal). **Roxanne Bohdjalian**: Conceptualization (equal); Investigation (equal); Methodology (equal).

DATA AVAILABILITY

The data that support the findings of this study are available within the article.

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